



Chapter 6

Priority Queues

6.1 Introduction

- A priority queue is a collection of zero or more elements. Each element has a priority or value.
- Operations:
 - 1)find an element
 - 2)insert a new element
 - 3)delete an element

6.1 Introduction

- In a min priority queue the find operation finds the element with minimum priority, while the delete operation delete this element.
- In a max priority queue, the find operation finds the element with maximum priority, while the delete operation delete this element.

6.1 Introduction

ADT of a max priority queue

AbstractDataType MaxPriorityQueue

{

instances

finite collection of elements, each has a priority

operations

Create(): create an empty priority queue

Size(): return number of element in the queue

Max(): return element with maximum priority

Insert(x): insert x into queue

DeleteMax(x): delete the element with largest priority

from the queue; return it in x;

}

6.2 Linear List Representation



Use an unordered linear list

Insertions are performed at the right end of the list, $\theta(1)$

A deletion requires a search for the element with largest priority, $\theta(n)$

6.3 Heaps

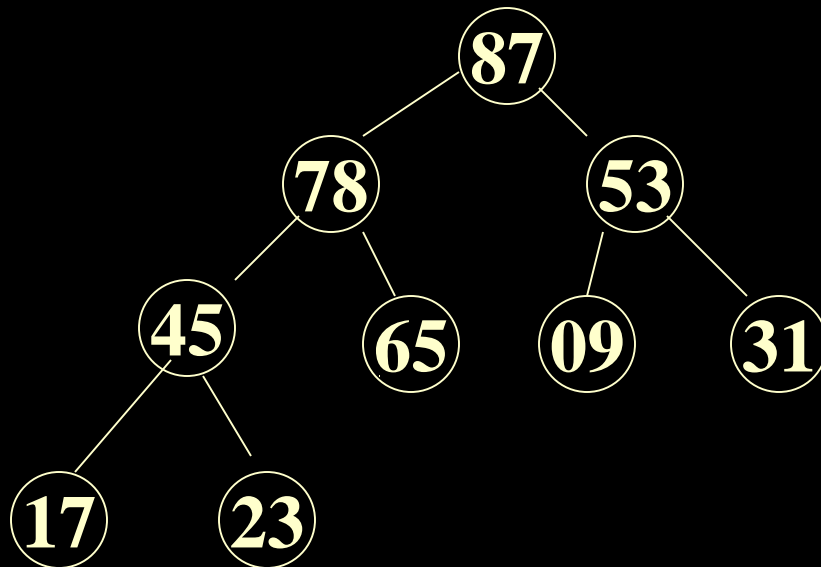
1.definition: A max heap(min Heap)

- **is A complete binary tree**
- **The value in each node is greater(less) than or equal to those in its children(if any).**

6.3 Heaps

Example of a max heap

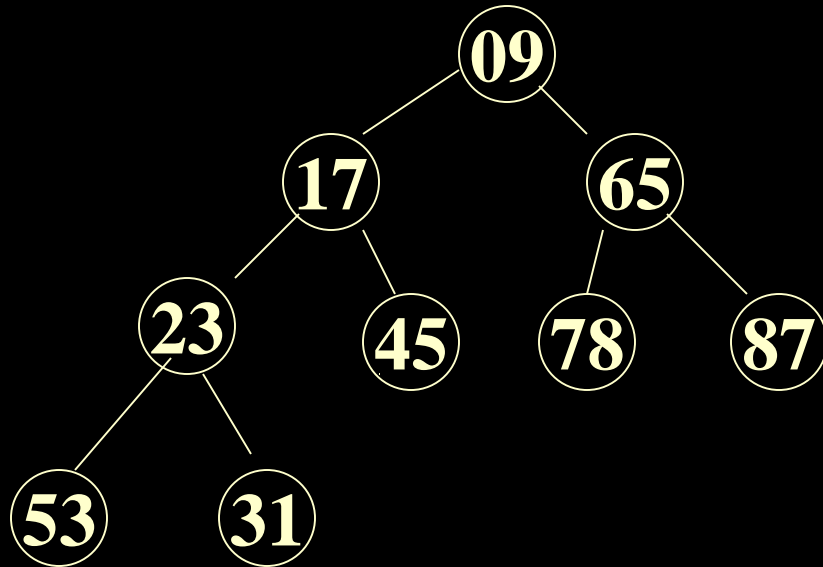
$k = \{87, 78, 53, 45, 65, 09, 31, 17, 23\}$



6.3 Heaps

Example of a min heap

$k = \{09, 17, 65, 23, 45, 78, 87, 53, 31\}$




6.3 Heaps

2. class MaxHeap

Data member of heap: T * heap, int MaxSize, CurrentSize

```
template<class T>class MaxHeap
{ public:
    MaxHeap(int MaxHeapSize=10);
    ~MaxHeap(){delete[]heap;}
    int size()const{return CurrentSize;}
    T Max(){ if (CurrentSize==0)throw OutOfBounds();
             return heap[1];}
    MaxHeap<T>&insert(const T&x);
    MaxHeap<T>& DeleteMax(T& x);
    void initialize(T a[], int size, int ArraySize);
private:
    int CurrentSize, MaxSize;
    T * heap;
}
```

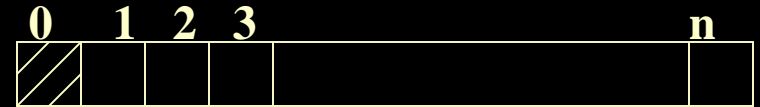


6.3 Heaps

3.member function of MaxHeap

1)Constructor for MaxHeap

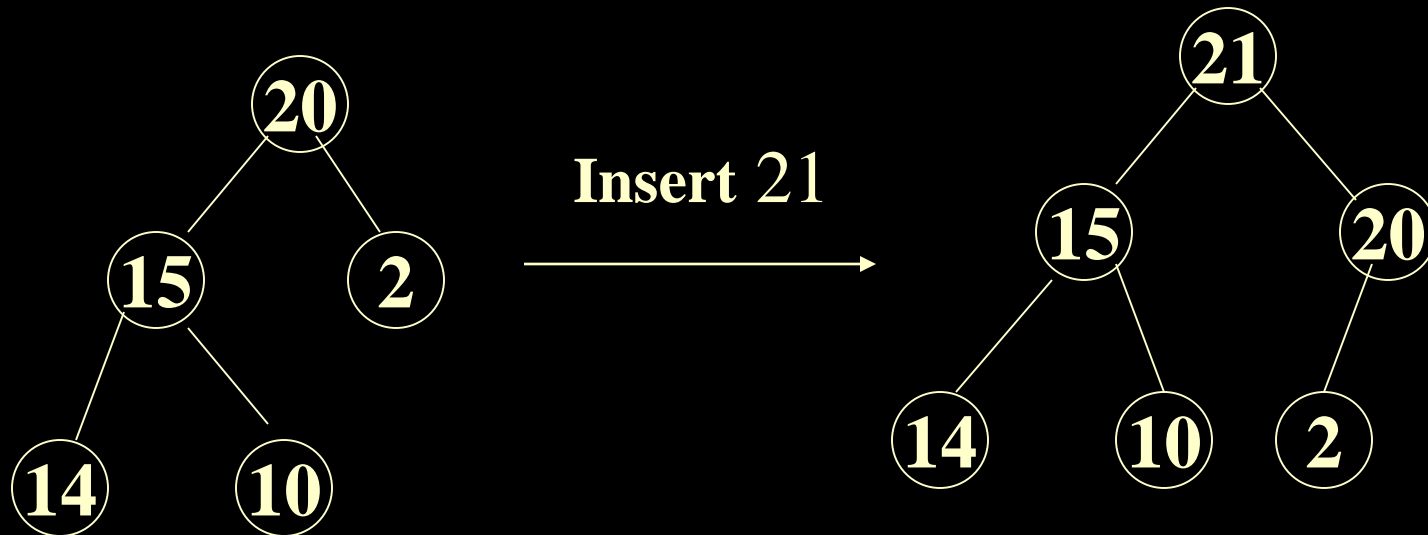
```
template<class T>
MaxHeap<T>::MaxHeap(int MaxHeapSize)
{
    MaxSize=MaxHeapSize;
    Heap=new T[MaxSize+1];
    CurrentSize=0;
}
```



6.3 Heaps

2) Insertion

Example:



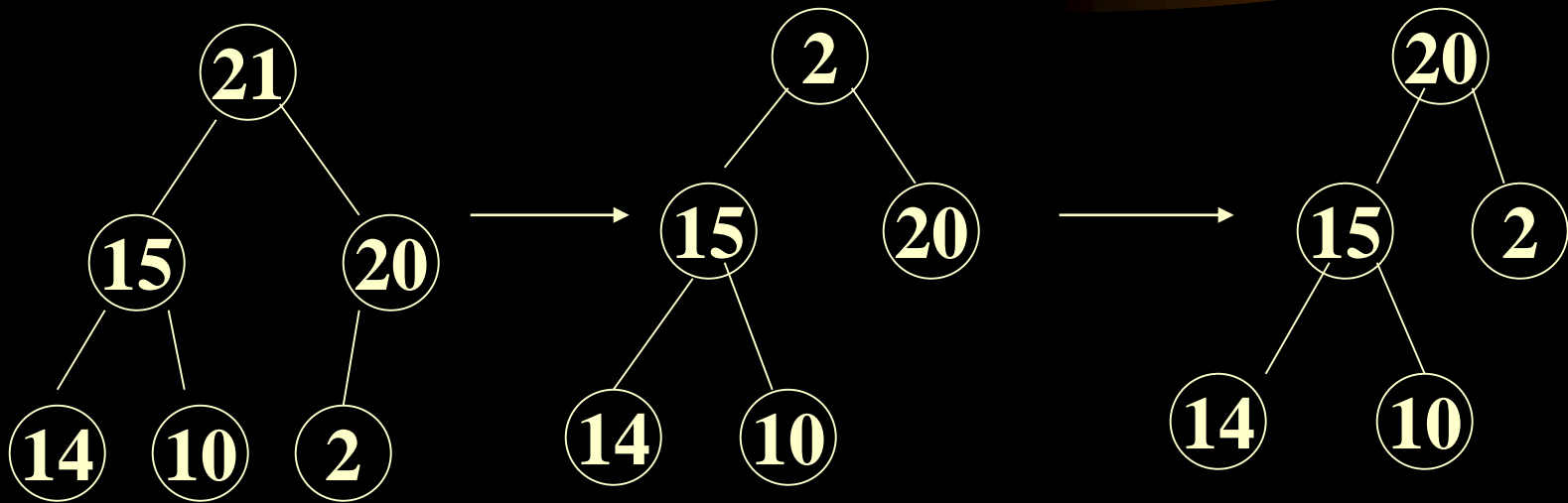
6.3 Heaps

Insertion

```
template<class T>MaxHeap<T>& MaxHeap<T>::  
    Insert(const T& x)  
{ if(CurrentSize==MaxSize)throw NoMem();  
  int i=++CurrentSize;  
  while(i!=1 && x>heap[i/2])  
    { heap[i]=heap[i/2]; i/=2; }  
  heap[i]=x;  
  return *this;  
}  
time complexity is  $O(\log_2 n)$ 
```

6.3 Heaps

3) deletion



6.3 Heaps

deletion from a max heap

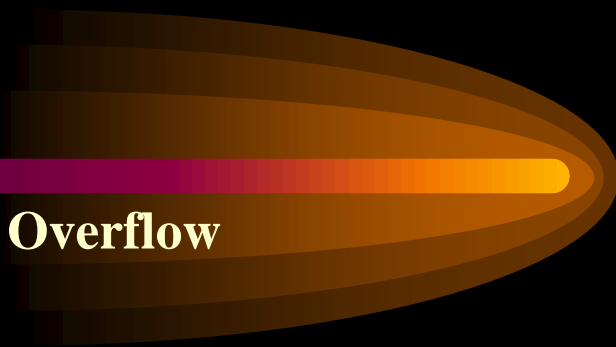
```
template<class T>MaxHeap<T>& MaxHeap<T>:: DeleteMax(T& x)
{ if (CurrentSize==0)throw OutOfBounds();
  x=heap[1];
  T y=heap[CurrentSize--];
  int i=1; ci=2;
  while(ci<=CurrentSize)
  { if(ci<CurrentSize&&heap[ci]<heap[ci+1]) ci++;
    if(y>=heap[ci]) break;
    heap[i]=heap[ci];
    i=ci; ci*=2;
  }
  heap[i]=y;  return *this;
}    Time complexity is  $O(\log_2 n)$ 
```

6.3 Heaps

```
java program(MinHeap)
public class BinaryHeap
{
    public BinaryHeap( )
    public BinaryHeap( int capacity )
    public void insert( Comparable x ) throws Overflow
    public Comparable findMin( )
    public Comparable deleteMin( )

    public boolean isEmpty( )
    public boolean isFull( )
    public void makeEmpty( )

    private static final int DEFAULT_CAPACITY = 100;
    private int currentSize;
    private Comparable [ ] array;
    private void percolateDown( int hole )
    private void buildHeap( )
}
```



6.3 Heaps

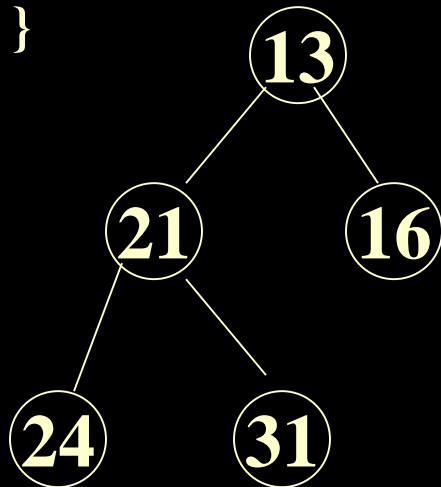
```
public BinaryHeap( )  
{  this( DEFAULT_CAPACITY );  
}
```

```
public BinaryHeap( int capacity )  
{  currentSize = 0;  
    array = new Comparable[ capacity + 1 ];  
}
```

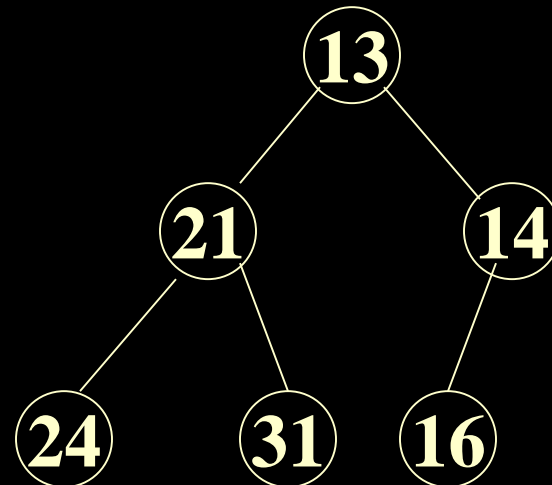
```
public void makeEmpty( )  
{  currentSize = 0;  
}
```


6.3 Heaps

```
public void insert( Comparable x ) throws Overflow
{ if( isFull( ) )
    throw new Overflow( );
  int hole = ++currentSize;
  for( ; hole > 1 && x.comparebleTo( array[ hole / 2 ] ) < 0;
      hole /= 2 )
    array[ hole ] = array[ hole / 2 ];
  array[ hole ] = x;
}
```



Insert 14



6.3 Heaps

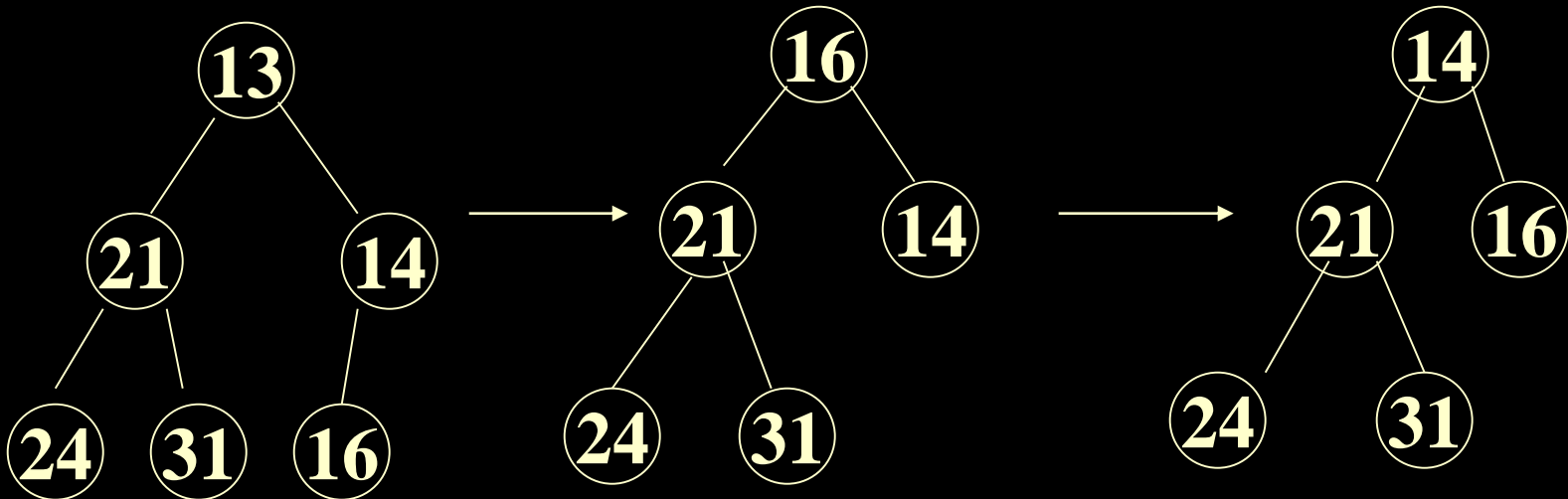
```
public Comparable deleteMin()  
{ if( isEmpty( ) )  
    return null;
```

```
    Comparable minItem = findMin( );  
    array[ 1 ] = array[ currentSize-- ];  
    percolateDown( 1 );
```

```
    return minItem;  
}
```

6.3 Heaps

```
private void percolateDown( int hole )
{
    int child;
    Comparable tmp = array[ hole ];
    for( ; hole * 2 <= currentSize; hole = child )
    {
        child = hole * 2;
        if ( child != currentSize && array[ child + 1 ].compareTo( array[ child ] ) < 0 )
            child++;
        if( array[child].compareTo( tmp ) < 0 )
            array[ hole ] = array[ child ];
        else
            break;
    }
    array[ hole ] = tmp;
}
```

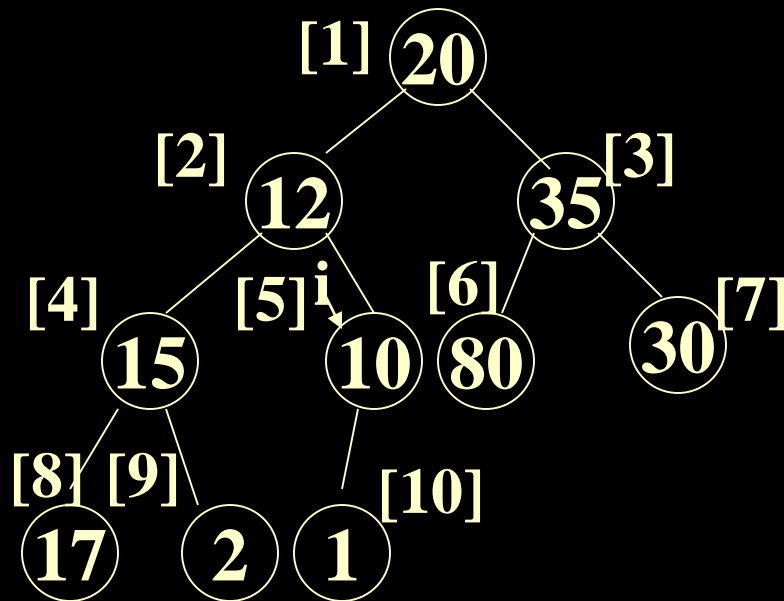


6.3 Heaps

4) Initialize a nonempty max heap

Example: {20,12,35,15,10,80,30,17,2,1}

书中称为由底向上:



$i = [n/2], [n/2]-1, \dots, 1$

Turn into max heap from
these subtree roots

6.3 Heaps

initialize (C++ program)

Template<class T> void MaxHeap<T>::

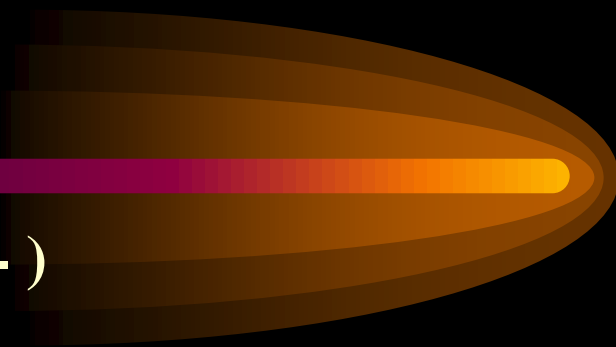
Initialize (T a[],int size,int ArraySize)

```
{ delete[] heap;
  heap=a; CurrentSize=Size;  MaxSize=ArraySize;
  for( int i=CurrentSize/2; i>=1; i--)
    { T y=heap[i];  int c=2*i;
      while(c<=CurrentSize)
        { if(c<CurrentSize && heap[c]<heap[c+1]) c++;
          if(y>=heap[c]) break;
          heap[c/2] = heap[c];
          c*=2;
        }
      heap[c/2]=y;
    }
}
```

6.3 Heaps

java initialize

```
private void buildHeap()  
{ for( int i = currentSize / 2; i > 0 ; i-- )  
    percolateDown( i );  
}
```



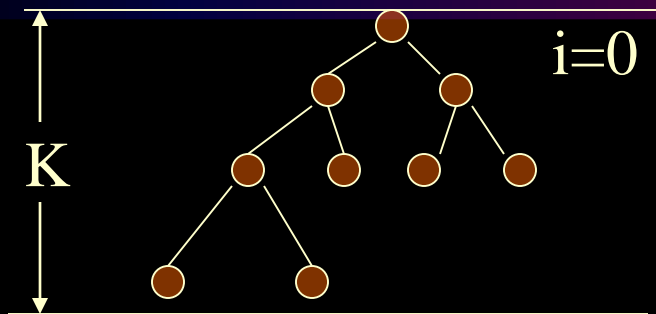
6.3 Heaps

Complexity of Initialize:

Create Heap time complexity:

算法分析

初始建堆：n个结点， $K=\lfloor \log_2 n \rfloor$ ，从0层开始



第*i*层交换的最大次数为*k-i*
第*i*层有 2^i 个结点

$$\text{总交换次数: } \sum_{i=0}^{k-1} 2^i \cdot (k-i) = \sum_{j=1}^k j \cdot 2^{k-j} = \sum_{j=1}^k j(2^k \cdot 2^{-j})$$

\uparrow
令 $k-i=j$

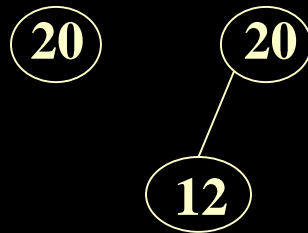
$$= 2^k \cdot \sum_{j=1}^k j \cdot 2^{-j} \leq 2^k \cdot 2 \leq 2^{\log n} \cdot 2 = 2n = O(n)$$

6.3 Heaps

4) Initialize a nonempty max heap

Example: {20,12,35,15,10,80,30,17,2,1}

还可以这样做: 依次插入一个元素到堆中. 书中称为由顶向下(也可见本书中例子).



Complexity of Initialize:

$$\sum_{K=1}^n \lfloor \log_2 k \rfloor \leq \sum_{K=1}^n \log_2 k = \log_2 1 + \dots + \log_2 n = \log_2 (1 \cdot 2 \dots n) =$$

$$\log_2 (n!) \leq O(n \log_2 n)$$

(提示: 因 $n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$

显然 该n 项都小于n, 所以 $\log_2 (n!) \leq \log_2 (n \cdot n \cdot n \cdot \dots \cdot n)$)

6.4.Applications of Priority Queues

1.heap sort

Method:

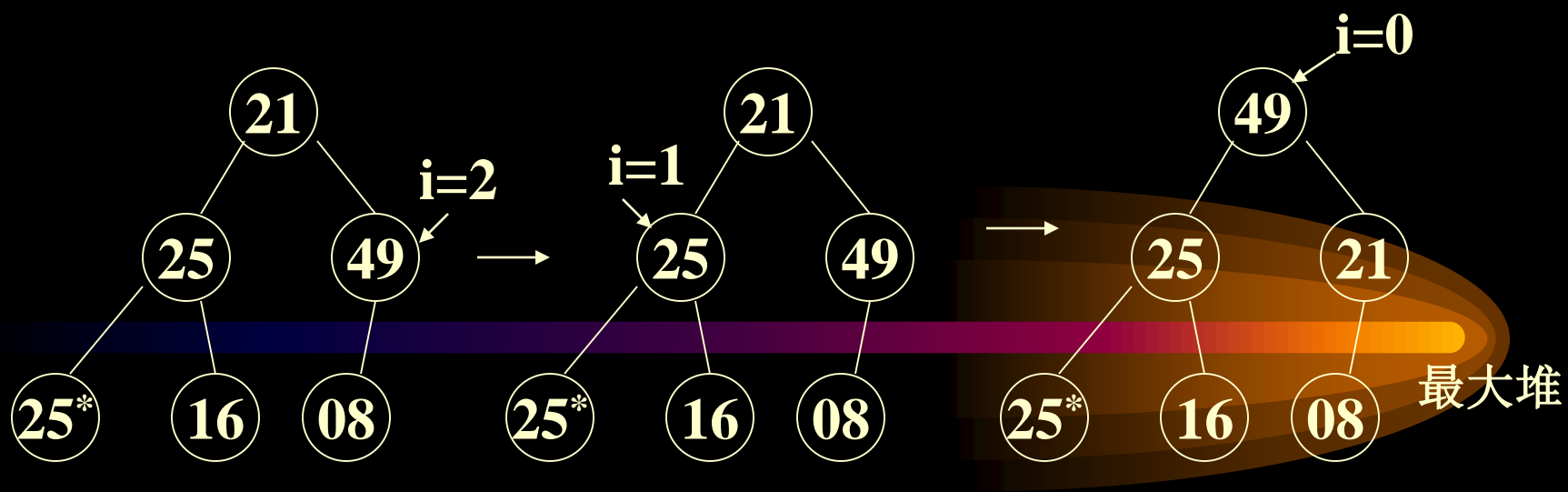
- 1)initialize a max heap with the n elements to be sorted $O(n)$
- 2)each time we delete one element, then adjust the heap $O(\log_2 n)$

Time complexity is $O(n)+O(n*\log_2 n)=O(n*\log_2 n)$

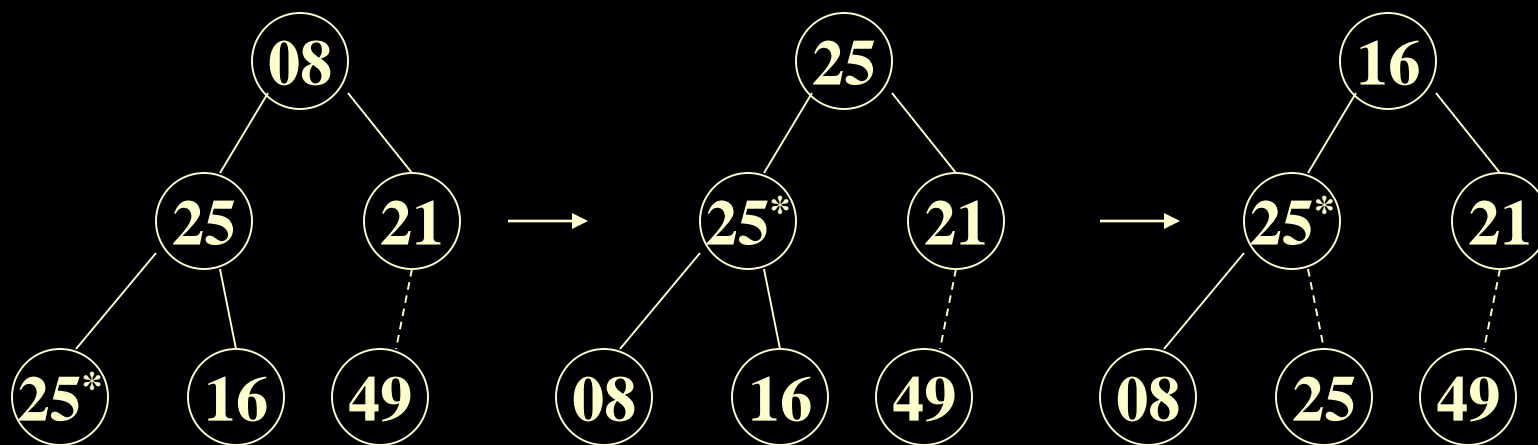
6.4.Applications of Priority Queues

heap sort

Example :{21,25,49,25*,16,08}



调整





从以上例子可以看出堆排序是不稳定的

heap sort

算法: c++

```
Template<class Type>void HeapSort(datalist<Type>&list)
{for(int i=(list.currentsize)/2;i>=1;i--)
    FilterDown(i,list.currentsize);
  for(i=list.currentsize;i>1;i--)
    {Swap(list.Vector[1],list.vector[i]);
      FilterDown(1,i-1);
    }
}
```

heap sort

java program

```
public static void heapsort( Comparable [ ] a )  
{ for( int i = a.length / 2; i >= 1; i-- )  
    percDown( a, i, a.length );  
  for( int i = a.length ; i > 1; i-- )  
  { swapReferences( a, 1, i );  
    percDown( a, 1, i-1);  
  }  
}
```

6.4.Applications of Priority Queues

2. The Selection Problem

A decorative graphic consisting of several overlapping, horizontal, teardrop-shaped ellipses. The ellipses are colored in a gradient from dark brown to bright orange, with the brightest orange at the rightmost tip. They are arranged in a way that they appear to be moving or expanding from left to right.

6.4.Applications of Priority Queues

2. The Selection Problem

在N个元素中找出第K个最大元素。

1A算法：读入N个元素放入数组，并将其选择排序，返回适当的元素。
运行时间： $O(N^2)$

1B算法：

1) 将K个元素读入数组，并对其排序（按递减次序）。
最小者在第K个位置上。

2) 一个一个地处理其余元素：

每读入一个元素与数组中第K个元素(在K个元素中为最小)比较，
如果 $>$ ，则删除第K个元素，再将该元素放在合适的位置上。
如果 $<$ ，则舍弃。
最后在数组K位置上的就是第K个最大元素。

例如：3, 5, 8, 9, 1, 10 找第3个最大元素。

6.4.Applications of Priority Queues

运行时间(1B 算法):

$$O(K^2 + (N - K) * K)$$

$$= O(N * K)$$

$$\text{当 } K = \lceil N / 2 \rceil, O(N^2)$$

试验：在 $N = 100$ 万个元素中，找第 500,000 个最大元素。

以上两个算法在合理时间内均不能结束，都要处理若干天才算完。

用堆来实现：

6A算法：假设求第K个最小元素

1) 将N个元素建堆（最小）

$$O(N)$$

2) 执行K次delete

$$O(K * \log N)$$

$$\left. \begin{array}{l} O(N) \\ O(K * \log N) \end{array} \right\} O(N + K * \log N)$$

如果 $K = \lceil N / 2 \rceil$,

$$\theta(N * \log N)$$

如果 $K = N$,

$$O(N * \log N) \text{ —— 堆排序}$$

6.4.Applications of Priority Queues

6B算法：假设求第K个最大元素

1) 读入前K个元素，建立最小堆 $O(K)$

2) 其余元素一一读入：

每读入一个元素与堆中第K个最大元素比（实际上是堆中最小元素）

$O(1)$

大于，则将小元素去掉（堆顶），该元素进入，进行一次调整。

$O(\log K)$

小于，则舍弃。

$$O(K + (N-K) * \log K) = O(N * \log K)$$

$$\text{当 } K = \lceil N / 2 \rceil, \theta(N * \log N)$$

对6A, 6B,用同样的数据进行测试，只需几秒钟左右给出问题解。

Chapter 6

2009年统考题:

8. 已知关键字序列 5,8,12,19,28,20,15,22 是最小根堆(最小堆), 插入关键字3,调整后得到的小根堆是

- A. 3, 5, 12, 8, 28, 20, 15, 22, 19 B. 3, 5, 12, 19, 20, 15, 22, 8, 28**
C. 3, 8, 12, 5, 20, 15, 22, 28, 19 D. 3, 12, 5, 8, 28, 20, 15, 22, 19

exercises:

- 1. a. Show the result of inserting 10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13, and 2, one at a time, into an initially empty binary heap.**
b. Show the result of using the linear-time algorithm to build a binary heap using the same input.
- 2. Show the result of performing three deleteMin operations in the heap of the previous exercise.**

Chapter 6

3.判别以下序列是否是堆？如果不是，将它调整为堆。

1) { 100, 86, 48, 73, 35, 39, 42, 57, 66, 21 }

2) { 12, 70, 33, 65, 24, 56, 48, 92, 86, 33 }

3) { 103, 97, 56, 38, 66, 23, 42, 12, 30, 52, 06, 20 }

4) { 05, 56, 20, 23, 40, 38, 29, 61, 35, 76, 28, 100 }

4.设待排序的关键码序列为{ 12, 2, 16, 30, 28, 10, 16*, 20, 6, 18 }, 使用堆排序方法进行排序。写出建立的初始堆，以及调整的每一步。