Electric Charge and electric current

i = current

q = Charge

A = Ampere

$$i(t) = \frac{dq(t)}{dt} = \frac{\text{Charged Particles}}{\text{unit of time}}$$

$$q(t) = \int_{-\infty}^{t} i(x)\,dx$$

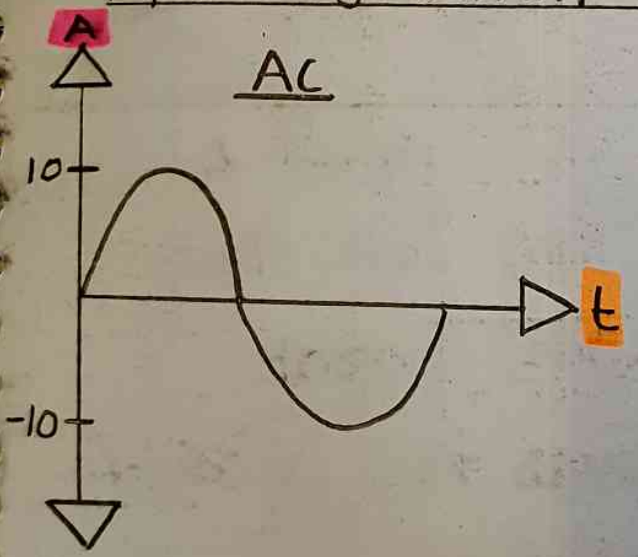
$i = 1A$

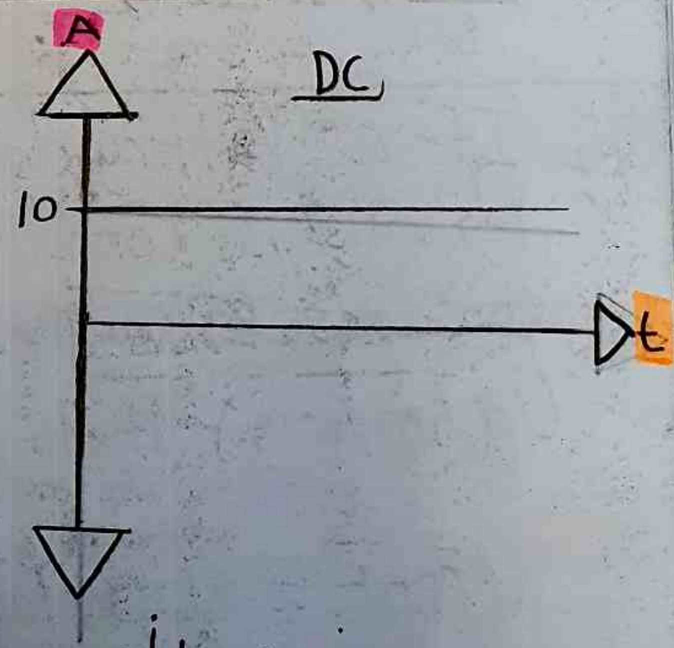
$$1A = \frac{1C}{S}$$

Coulomb

second

Alternating Current (AC) vs. Direct Current (DC)





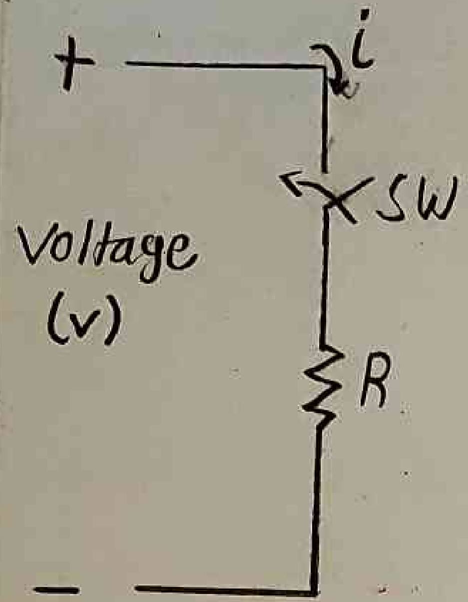
$i_{ac} = 10\sin(t)A$

$i_{dc} = 10A$

A = Amps

t = time

Definition of Voltage



Coulomb = Unit of charge

J = Joule

W = Watt

Current = watts / vots

$1v = 1j/c$

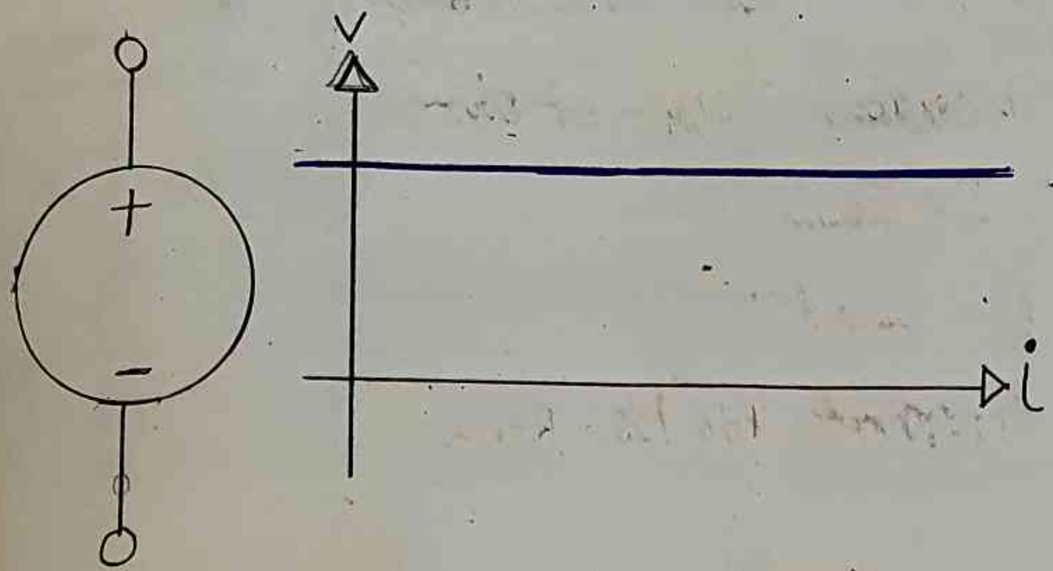
$1w = \frac{1J}{s} = 1v(1A)$

Q1. If a battery with a voltage equal to 100 volts is connected to a light bulb that consumes 60 watts, what is the current (in Amperes) that flow through the circuit??

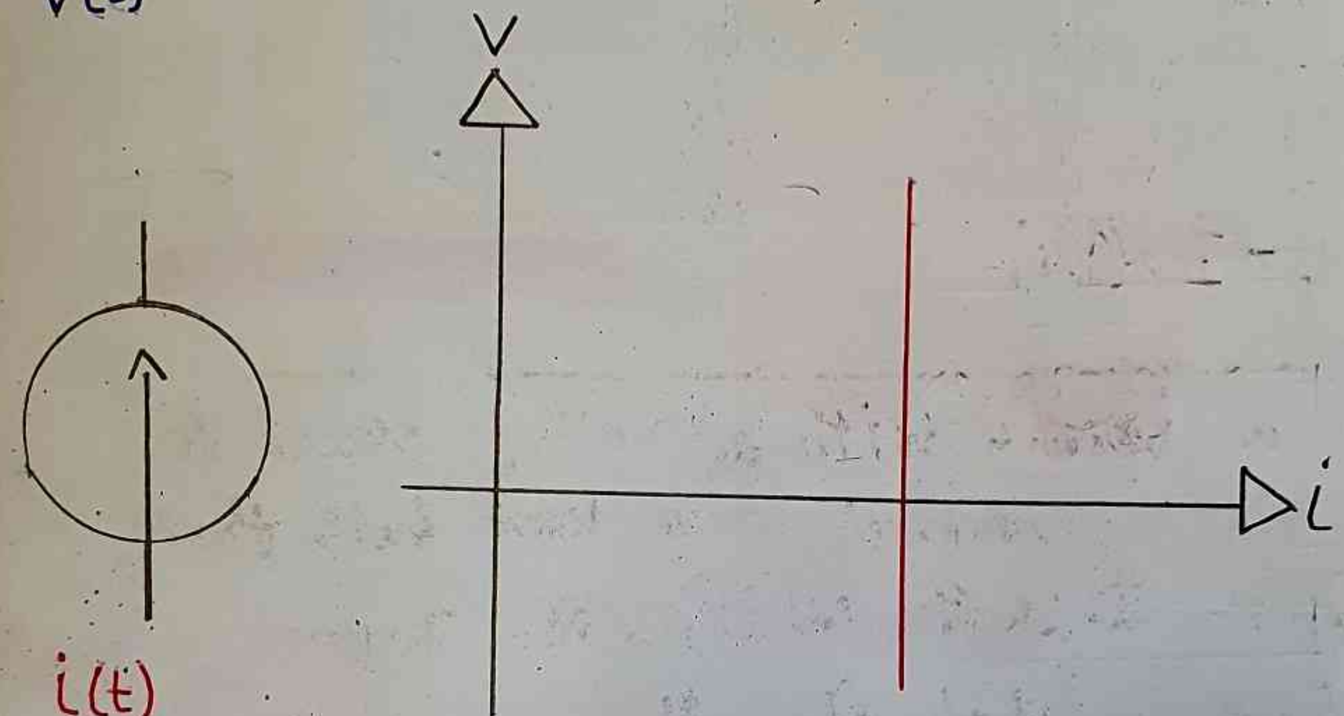
Current = 60w / 100v

= 0.6 Amps

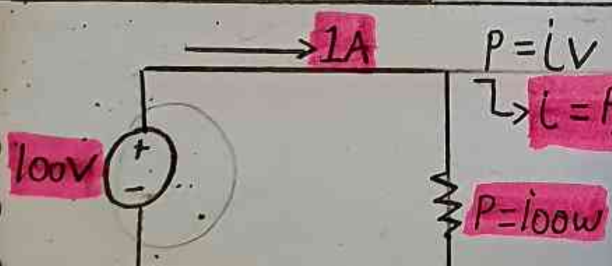
Independant Sources



V(t)



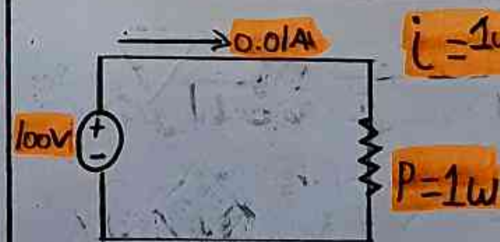
i(t)



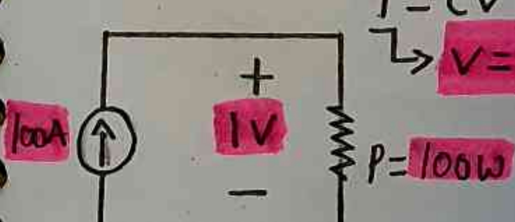
$P = iV$

$\hookrightarrow i = P/V = \frac{100W}{100V}$

$= 1A$



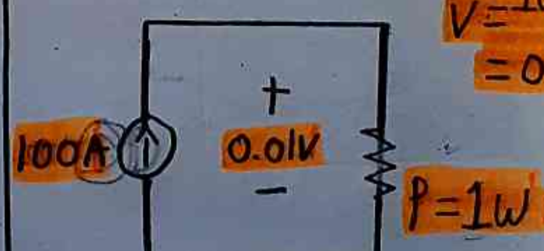
$i = 1W/100V = 0.01A$



$P = iV$

$\hookrightarrow V = P/i = \frac{100W}{100A}$

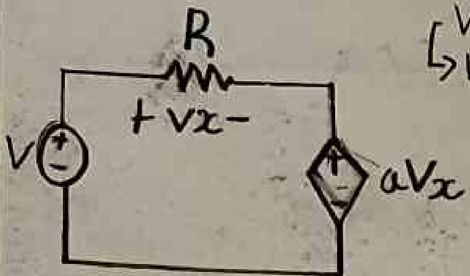
$= 1V$



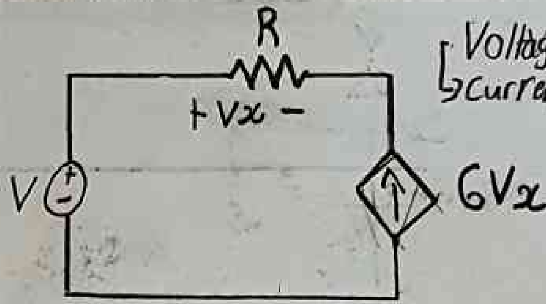
$V = 1W/100A$

$= 0.01V$

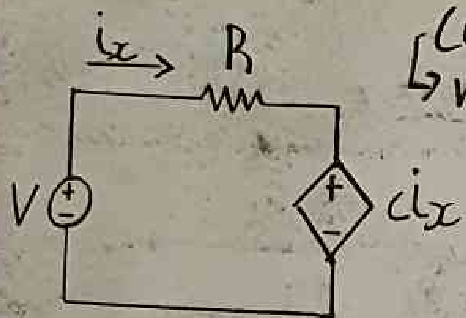
Dependant Sources



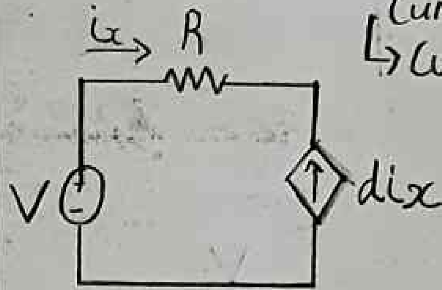
Voltage-dependant
↳ Voltage Source



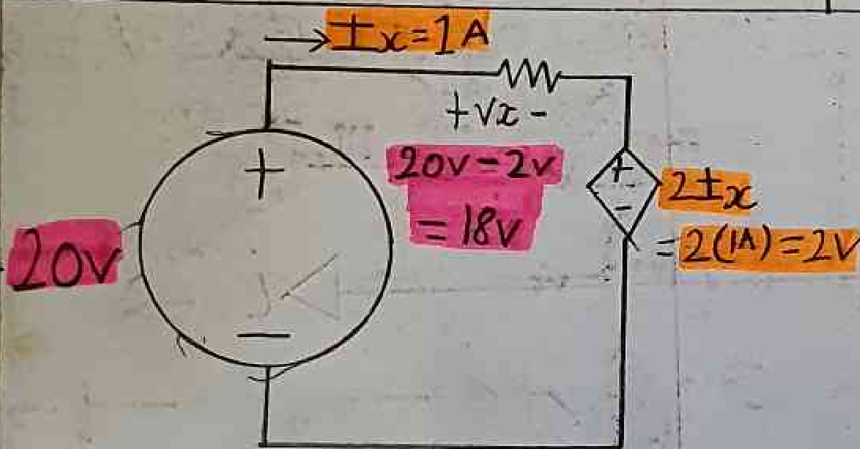
Voltage-dependant
↳ Current source



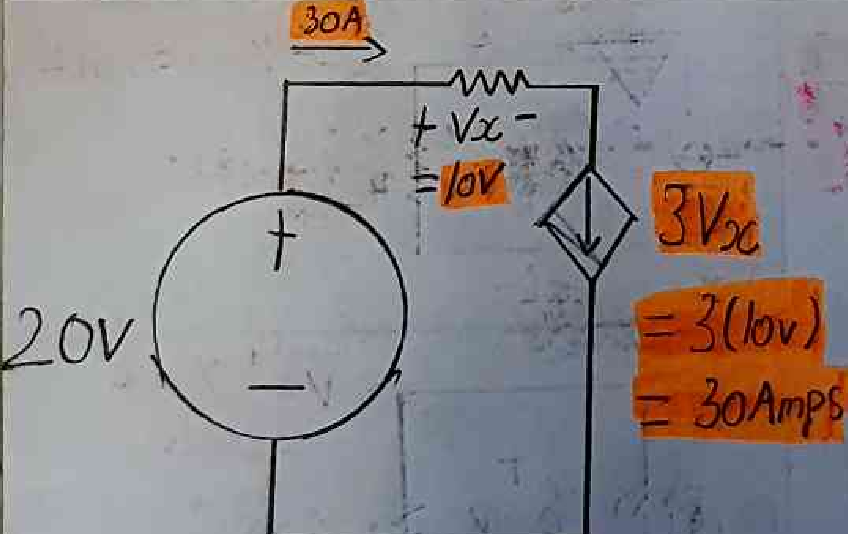
Current-dependant
↳ voltage Source



Current-dependant
↳ Current Source

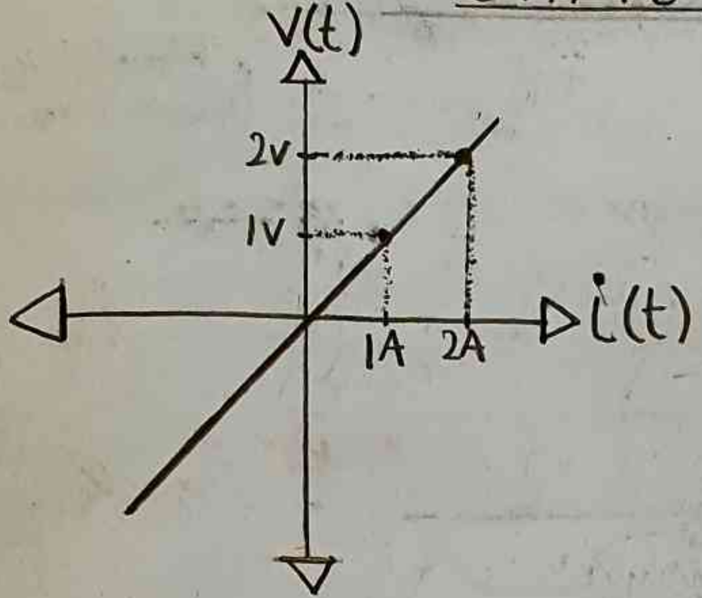


$P = iV = 1A(18v) = 18w$



$P = 30A(10v) = 300w$

Ohm's Law



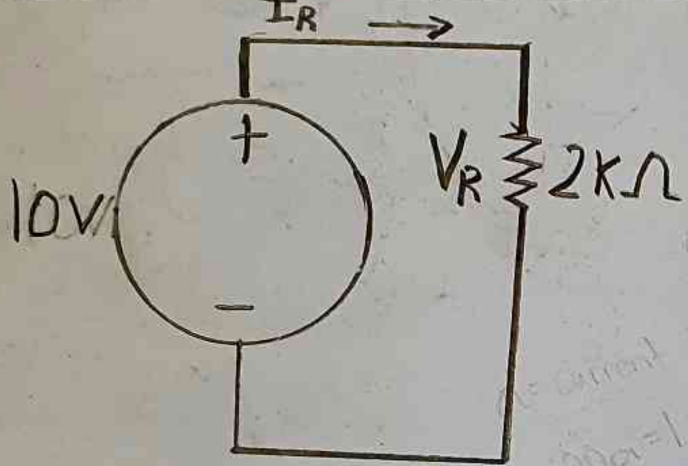
$P = iv \rightarrow P = i(Ri) = i^2R$

$P = \frac{V}{R}(v) \rightarrow P = \frac{V^2}{R}$

$R \equiv \frac{V}{i} \rightarrow 1\Omega = \frac{1V}{1A}$

$G = 1/R \rightarrow 1/\Omega = 1S$

$v(t) = Ri(t)$



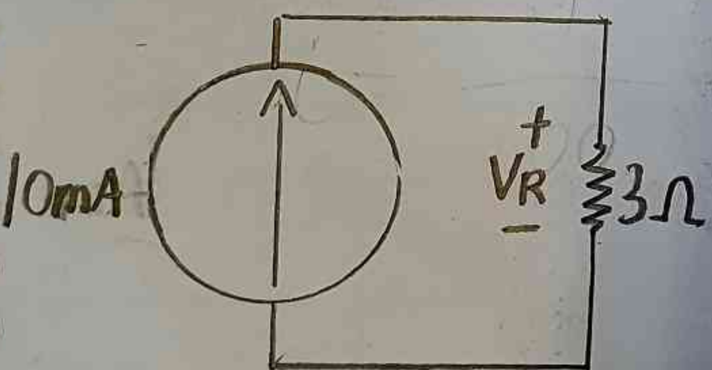
$V = iR \rightarrow i = V/R = 10V/2k\Omega = 5mA$

$P = \frac{V^2}{R} = \frac{10V^2}{2k\Omega} = 0.05W$

$P = i^2(R) = (5 \times 10^{-3})^2(2k\Omega)$
$= 0.05W$

$5mA = 5 \times 10^{-3}$
$= 0.005A$

$P = iv = (5 \times 10^{-3})(10v) = 0.05W$



$V = iR = (10 \times 10^{-3}) \times 3\Omega = 0.03V$

$P = \frac{V^2}{R} = \frac{0.03}{3} = 0.3mW$

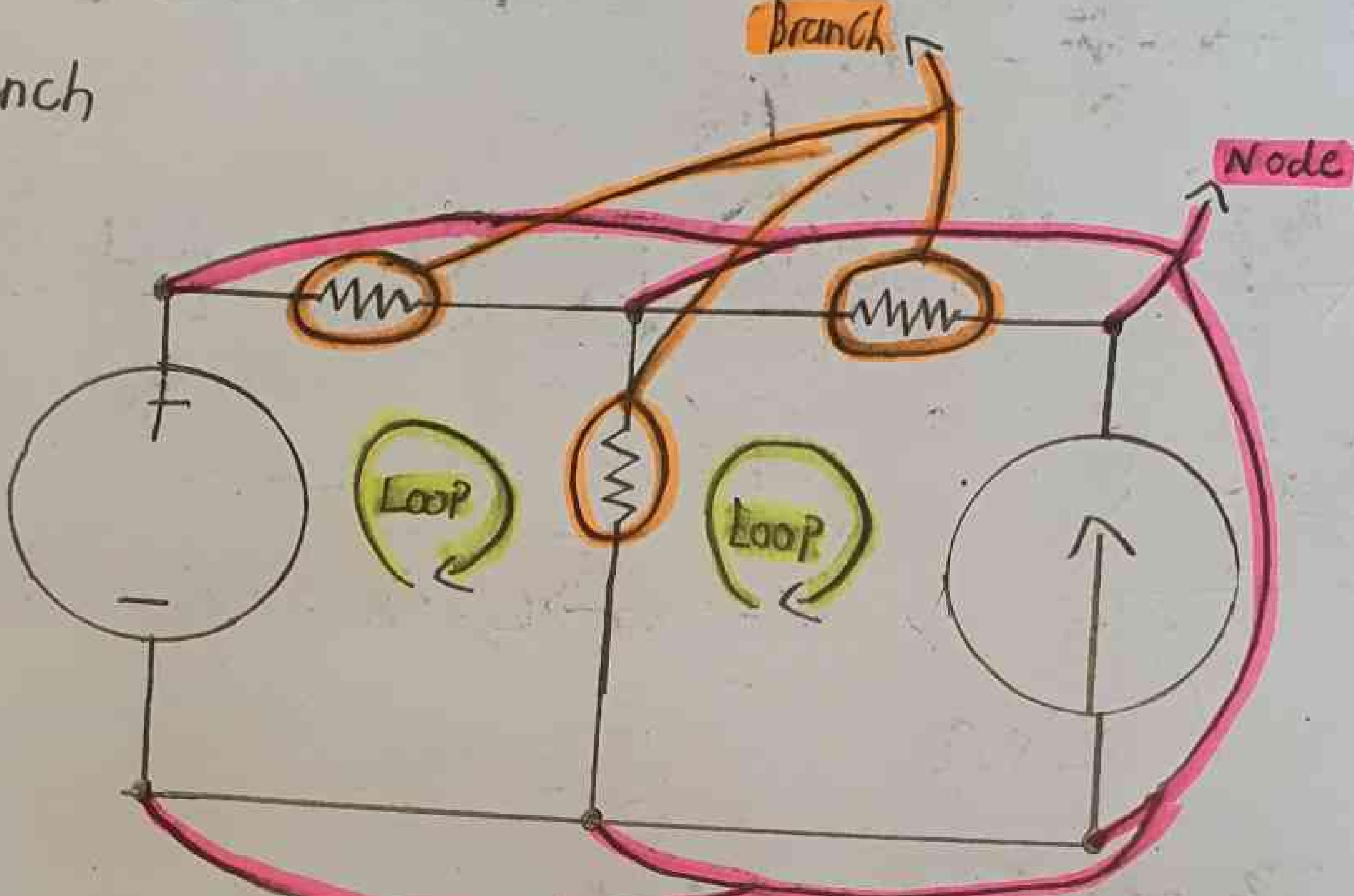
$P = i^2R = (10 \times 10^{-3})^2 \times (3\Omega)$
$= 0.3mW$

$10mA = 10 \times 10^{-3}$
$= 0.01A$

$P = iv = (10 \times 10^{-3}) \times 0.03V = 0.3mW$

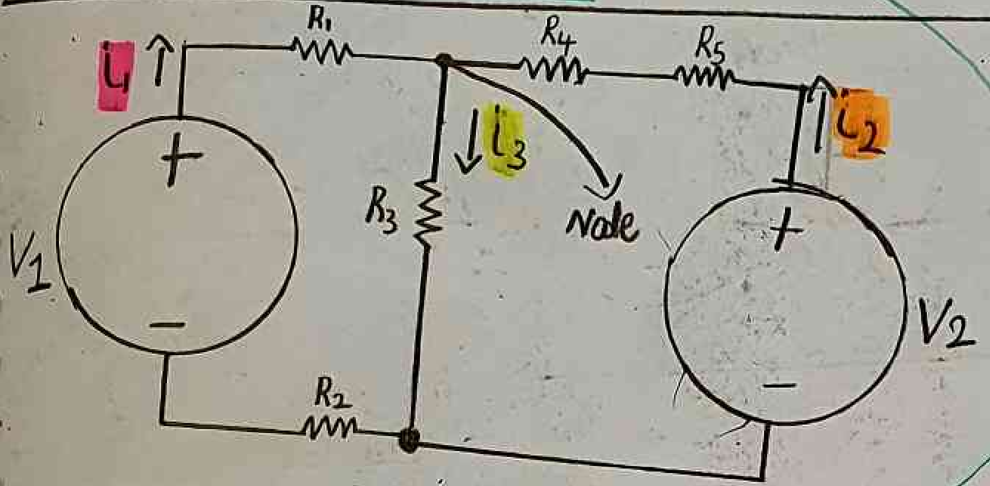
Circuit terminology

- Nodes
- Loops
- Branch



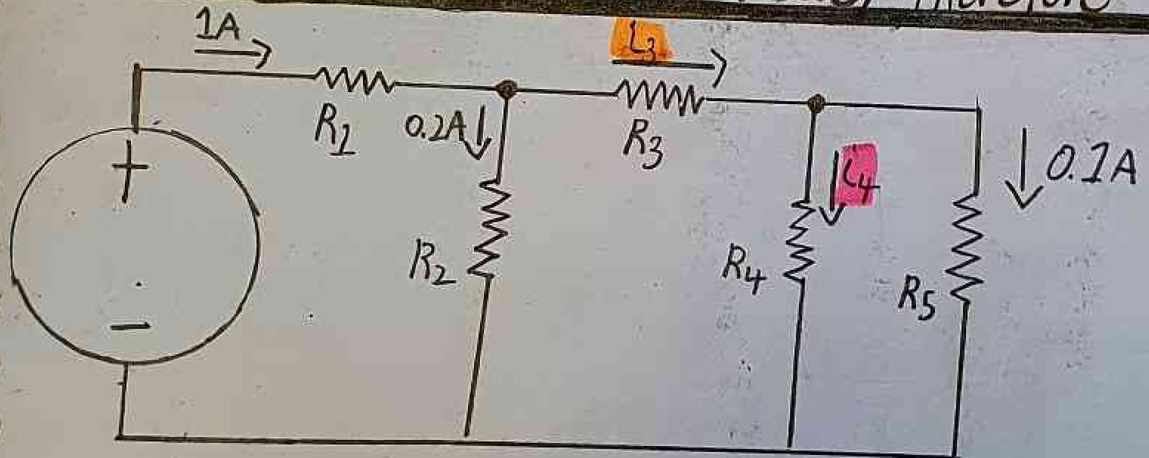
Kirchhoff's Current Law

Equation: $\boxed{\sum_{j=1}^{n} i_j(t) = 0}$



$\boxed{i_1 + i_2 - i_3 = 0}$

i_1 and i_2 are entering the node so it's Positive
While i_3 is exiting the node, therefore it's negative.

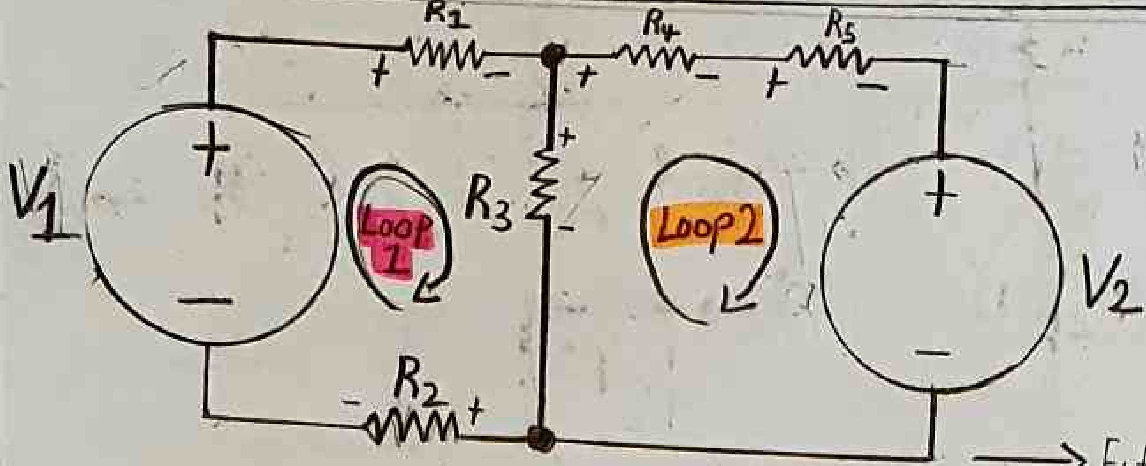


$1A - 0.2A - i_3 = 0 \rightarrow i_3 = 1A - 0.2A = 0.8A$

$0.8A - i_4 - 0.1A = 0 \rightarrow i_4 = 0.8A - 0.1A = 0.7A$

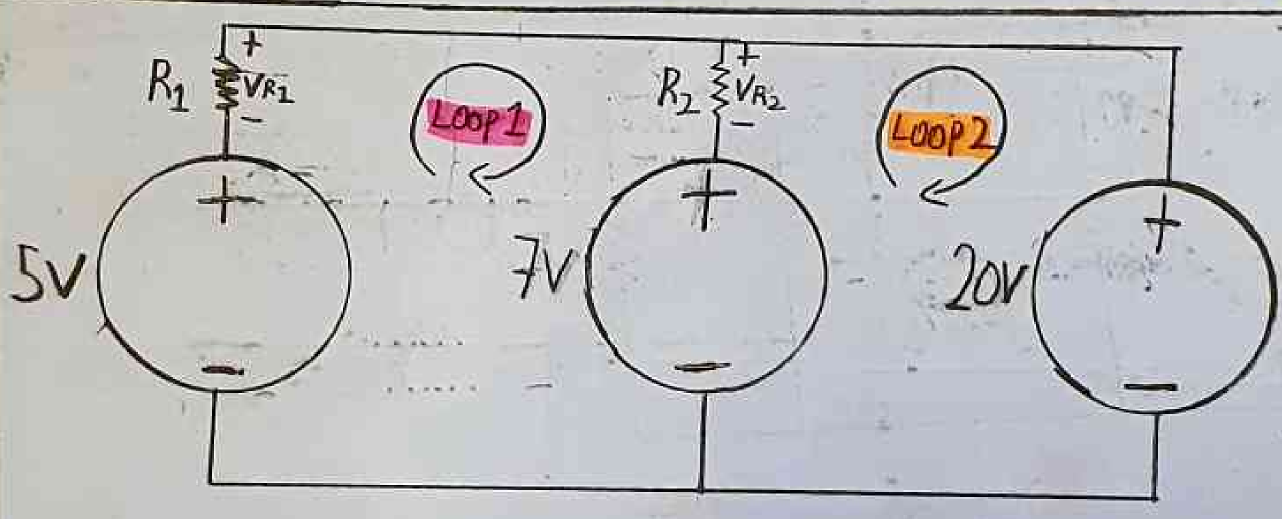
Kirchhoff's Voltage Law

Equation: $\sum_{j=1}^{n} vj(t) = 0$



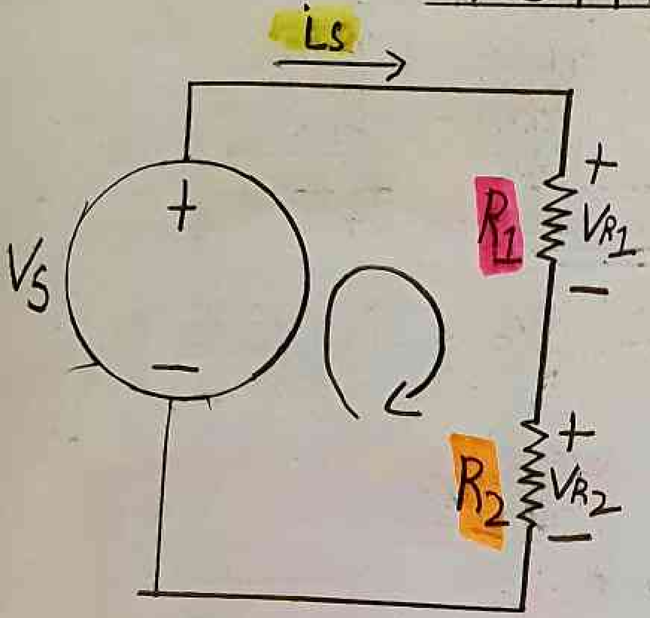
→ Entire circuit is also a loop
↳ Loop3

- Loop 1 $= -V_1 + V_{R_1} + V_{R_3} + V_{R_2} = 0$
- Loop 2 $= V_2 - V_{R_3} + V_{R_4} + V_{R_5} = 0$
- Loop3 $= -V_1 + V_{R_1} + V_{R_4} + V_{R_5} + V + V_{R_2} = 0$



- Loop 1 $= -5V - V_{R_1} + V_{R_2} + 7V = 0 \longrightarrow V_{R_1} = 15V$
- Loop 2 $= -7V - V_{R_2} + 20V = 0 \longrightarrow V_{R_2} = 13V$

Voltage Division



$$-V_s + V_{R1} + V_{R2} = 0 \rightarrow V_s = V_{R1} + V_{R2}$$

$$V = iR$$

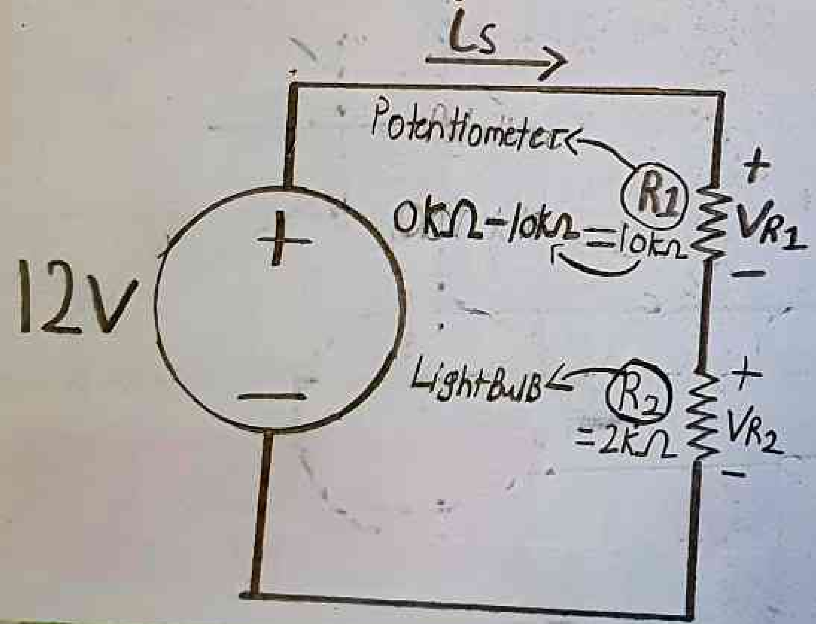
$$V_s = i_s R_1 + i_s R_2 = i_s(R_1 + R_2)$$

$$\rightarrow i_s = \frac{V_s}{R_1 + R_2}$$

$$V_R = \frac{V_s R}{R(\text{total})}$$

$$V_{R1} = \frac{V_s}{R_1 + R_2}(R_1)$$

$$V_{R2} = \frac{V_s}{R_2 + R_2}(R_2)$$



$R1 = 1k\Omega$

$$V_{R_2} = \frac{12V(2k\Omega)}{1k\Omega + 2k\Omega} = 8V$$

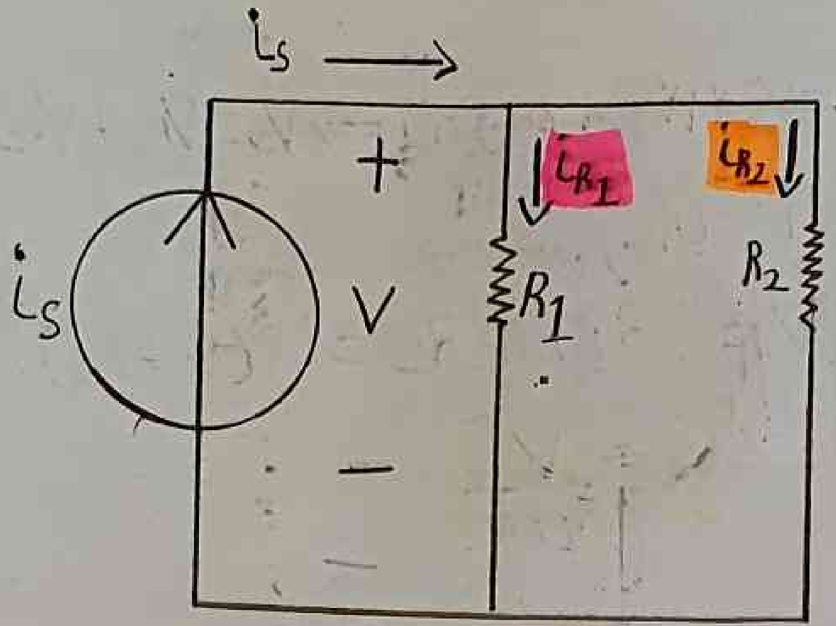
$$P = \frac{V^2}{R} = \frac{(8V)^2}{2k\Omega} = 32mW$$

$R1 = 10k\Omega$

$$V_{R_2} = \frac{12V(2k\Omega)}{10k\Omega + 2k\Omega} = 2V$$

$$P = \frac{V^2}{R} = \frac{(2V)^2}{2k\Omega} = 2mW$$

Current Division



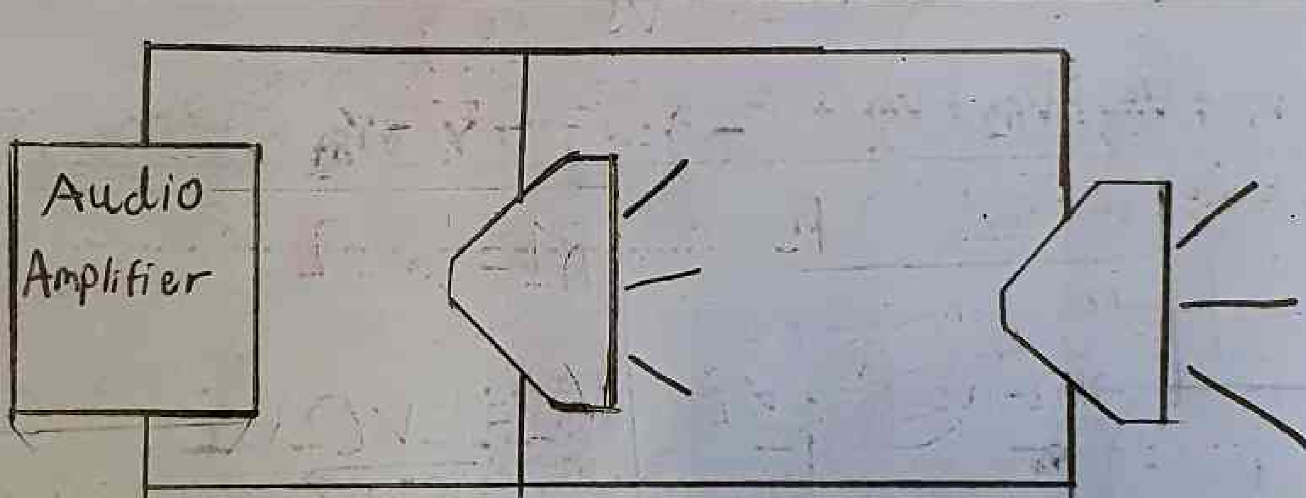
$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

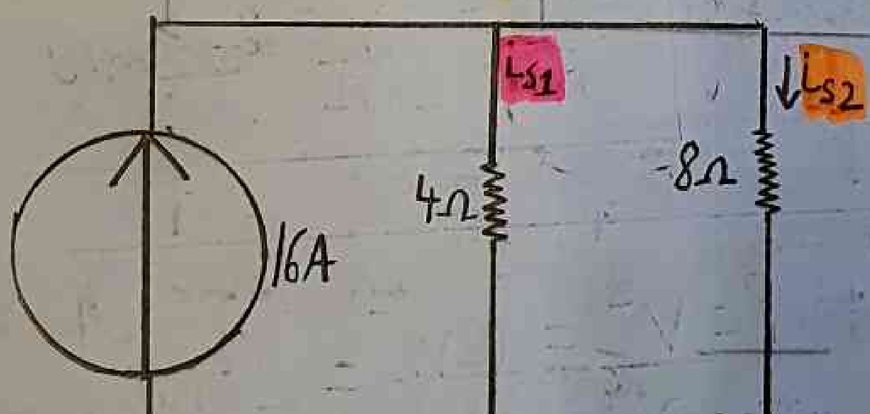
Resistor Parallel

$$V = iR = i_s\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$i_{R_1} = V/R_1 = \frac{i_s\left(\frac{\cancel{R_1} R_2}{R_1 + R_2}\right)}{\cancel{R_1}} = \frac{i_s R_2}{R_1 + R_2}$$

$$i_{R_2} = \frac{i_s\left(\frac{R_1 \cancel{R_2}}{R_1 + R_2}\right)}{\cancel{R_2}} = \frac{i_s R_1}{R_1 + R_2}$$





$$i_{S_2} = \frac{16A(8\Omega)}{4\Omega + 8\Omega} = 10.67A$$

$$i_{S_2} = \frac{16A(4\Omega)}{4\Omega + 8\Omega} = 5.33A$$

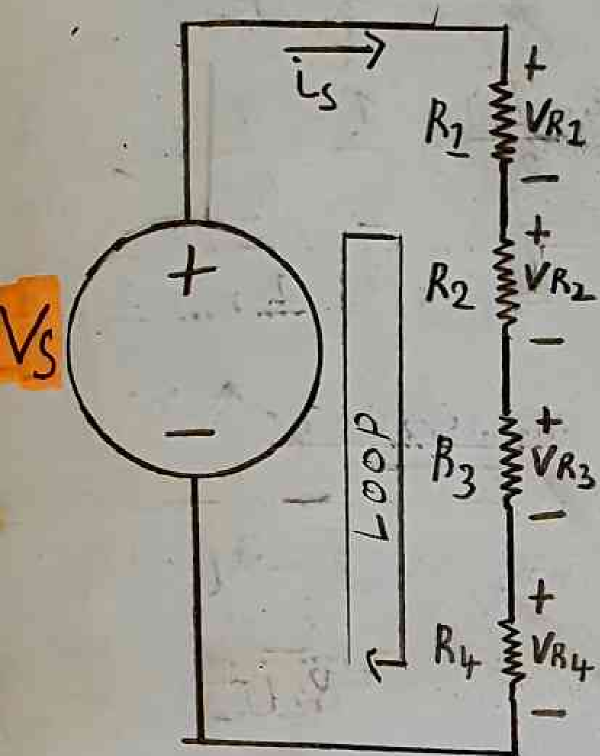
$$P = i^2 R = (10.67A)^2(4\Omega) = 455.40W$$

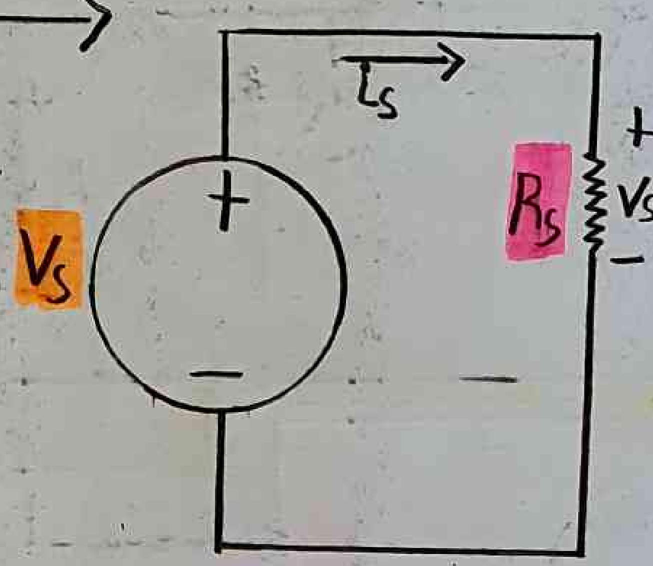
$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{4\Omega(8\Omega)}{4\Omega + 8\Omega} = 2.67\Omega$$

$$P_{total} = 16^2(2.67) = 683.52W$$

$$P = i^2 R = (5.33A)^2(8\Omega) = 227.27W$$

Series Resistors





$R_s = R_1 + R_2 + \dots R_n$

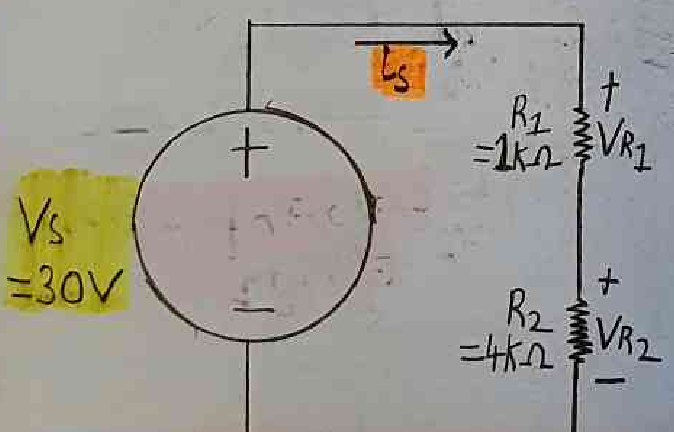
$\hookrightarrow R_s = R_1 + R_2 + R_3 + R_4$

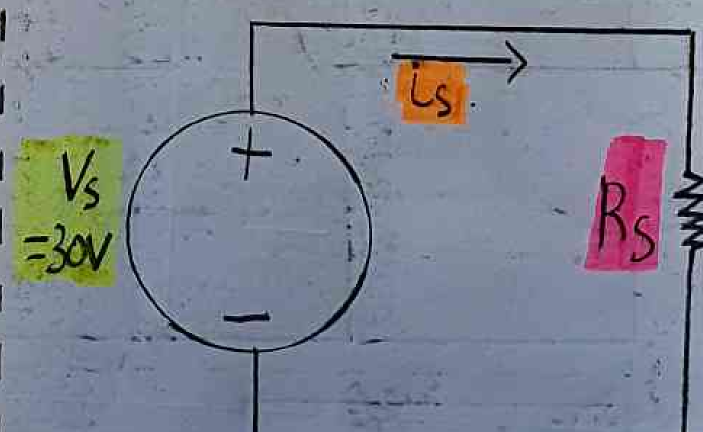
$V = iR$

$\hookrightarrow -V_s + V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4} = 0 \rightarrow V_s = V_{R_1} + V_{R_2} + V_{R_3} + V_{R_4}$

$\hookrightarrow V_s = i_s R_1 + i_s R_2 + i_s R_3 + i_s R_4 = i_s (R_1 + R_2 + R_3 + R_4)$

$\hookrightarrow \dfrac{V_s}{i_s} = R_s$





Case 1:

$-30V + V_{R_1} + V_{R_2} = 0$

$\hookrightarrow 30V = V_{R_1} + V_{R_2}$

$\hookrightarrow 30V = i_s (R_1 + R_2)$

$\rightarrow i_s = \dfrac{30V}{R_1 + R_2}$

$\rightarrow = \dfrac{30V}{1k\Omega + 4k\Omega}$

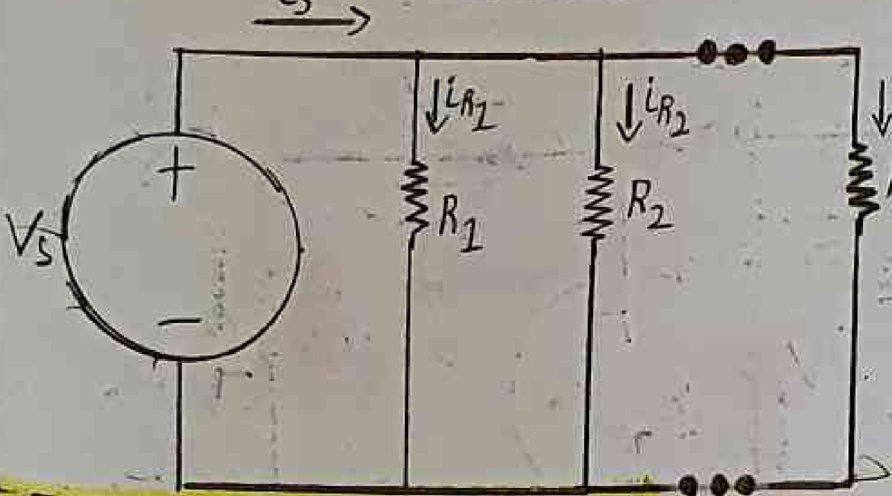
$\rightarrow = 6mA$

Case 2:

$R_s = 1k\Omega + 4k\Omega = 5k\Omega$

$V = iR \rightarrow i = V/R \Rightarrow i_s = 30V/5k\Omega = 6mA$

Parallel Resistors

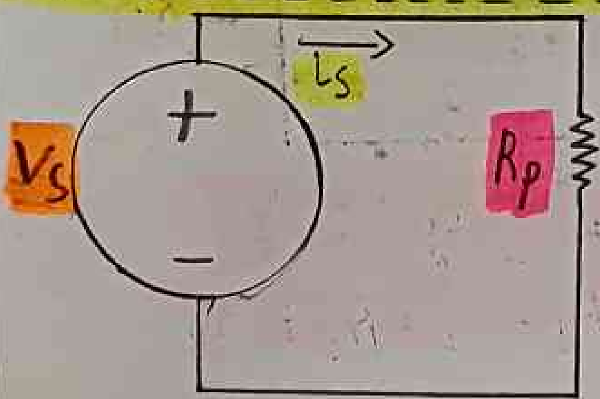


Equation: $\frac{1}{R_p} = \sum_{j=1}^{n} \frac{1}{R_j}$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\hookrightarrow R_p = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1}$$

means infinite

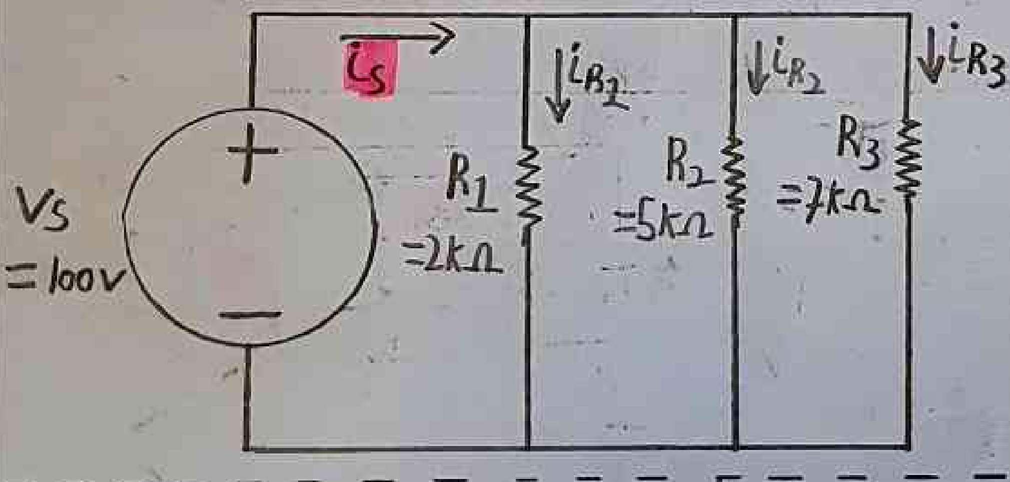


$i_s = i_{R_1} + i_{R_2} + \cdots + i_{R_n}$, $V = iR \rightarrow i = V/R$

$$\hookrightarrow i_s = V_s/R_1 + V_s/R_2 + \cdots + V_s/R_n = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)$$

$$\hookrightarrow \frac{i_s}{V_s} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$V_s = i_s R_p \rightarrow \frac{i_s}{V_s} = \frac{1}{R_p}$, $R_p = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{R_2 + R_1}{R_1 R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$



Case 1:

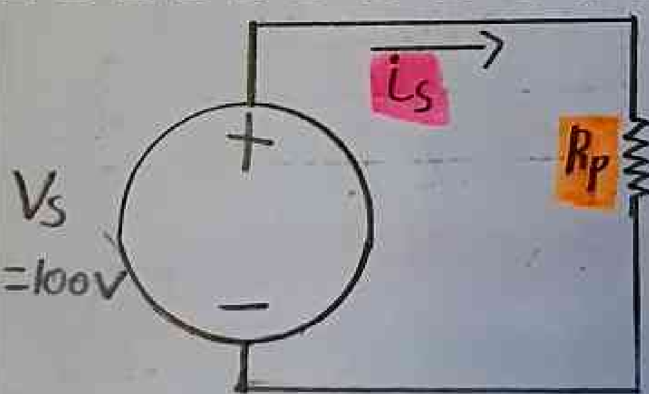
$$i_s - i_{R_1} - i_{R_2} - i_{R_3} = 0$$

$$\hookrightarrow i_s = i_{R_1} + i_{R_2} + i_{R_3}$$

$$\hookrightarrow i_s = \frac{100V}{2k\Omega} + \frac{100V}{5k\Omega} + \frac{100V}{7k\Omega}$$

$$\hookrightarrow i_s = 50mA + 20mA + 14.29mA = 84.29mA$$

$V = iR \rightarrow i = V/R$

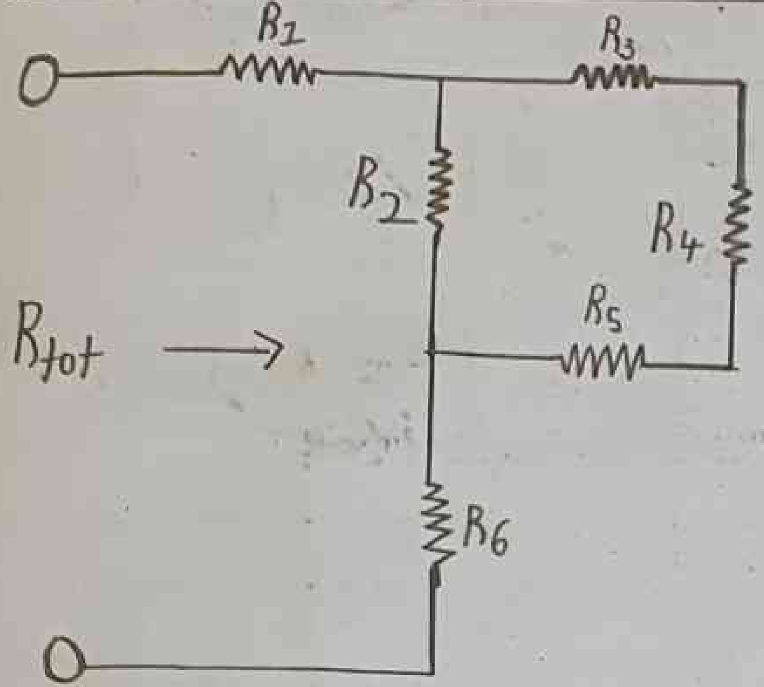


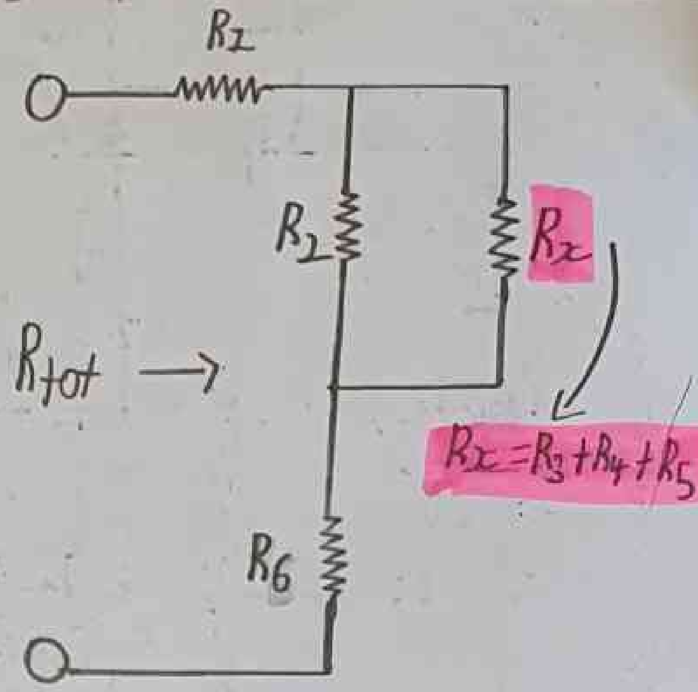
Case 2:

$$R_p = \left(\frac{1}{2k\Omega} + \frac{1}{5k\Omega} + \frac{1}{7k\Omega}\right)^{-1} = 1.186k\Omega$$

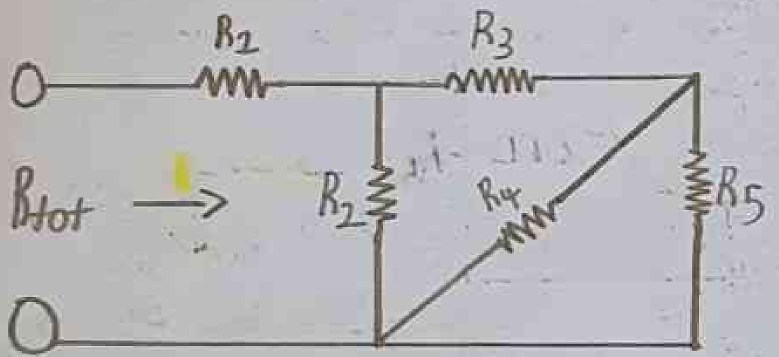
$$i_s = 100V/1.186k\Omega = 84.32mA$$

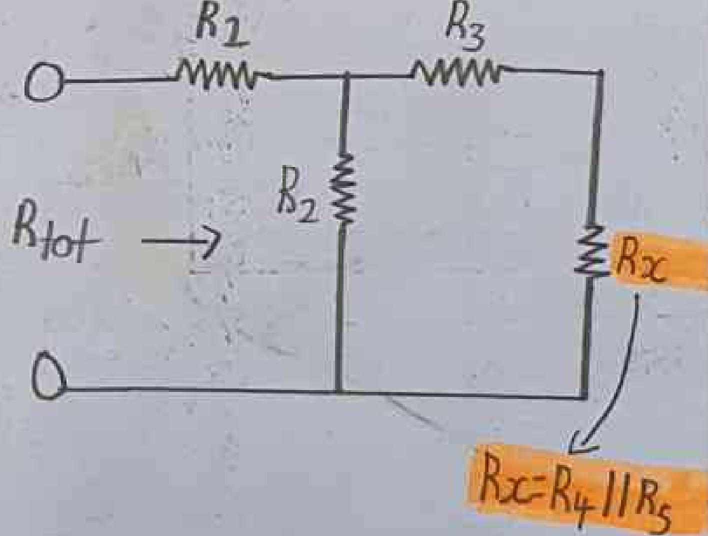
Series/Parallel Combination of Resistors





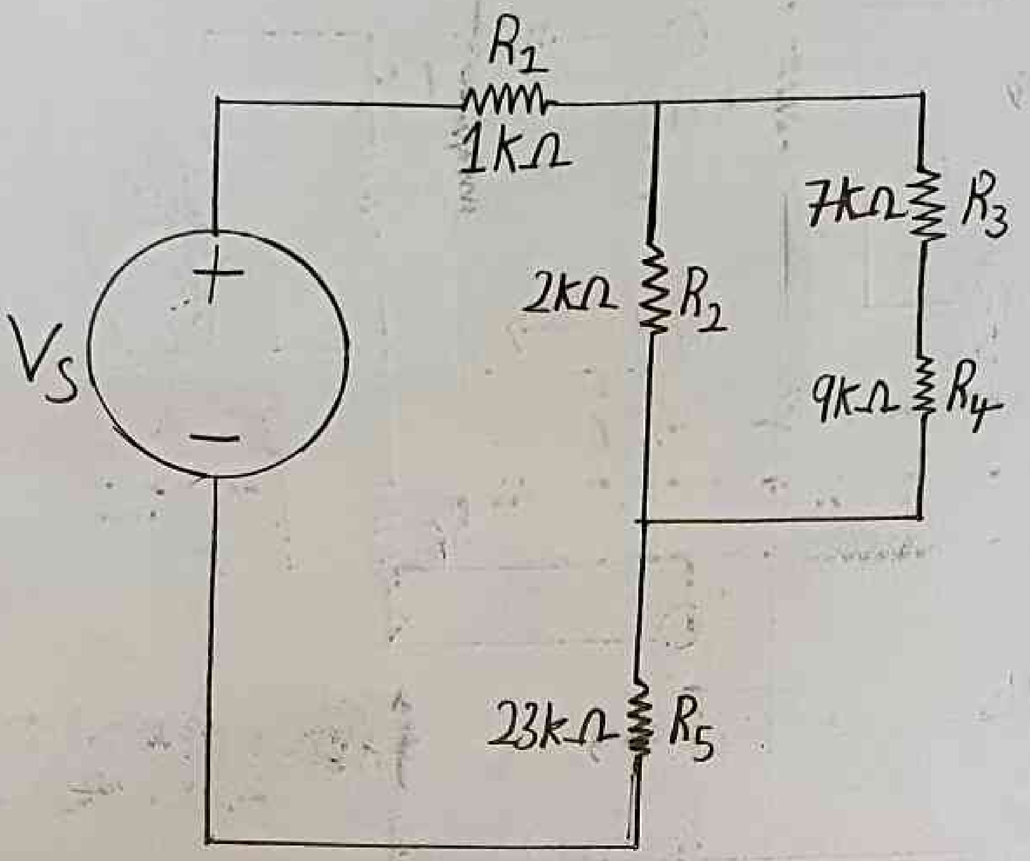
1 $\Longrightarrow$ 2

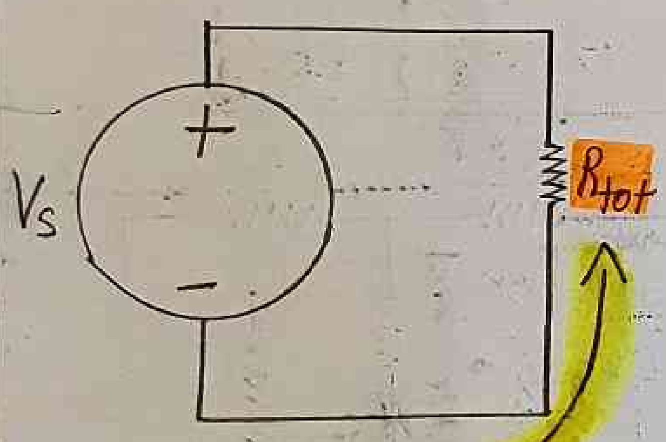


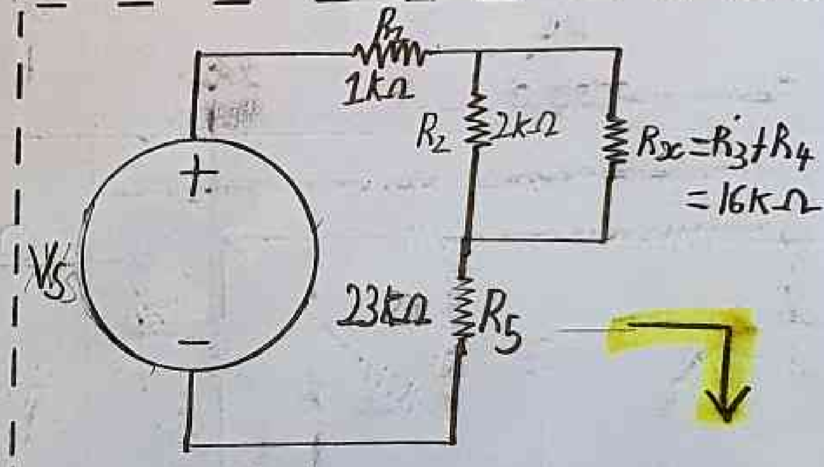


1 $\Longrightarrow$ 2

Example – Series / Parallel Combinations of Resistors

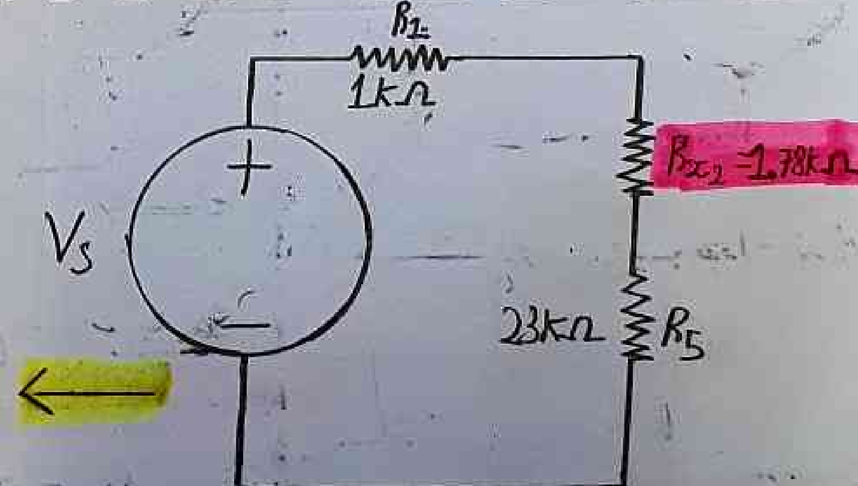






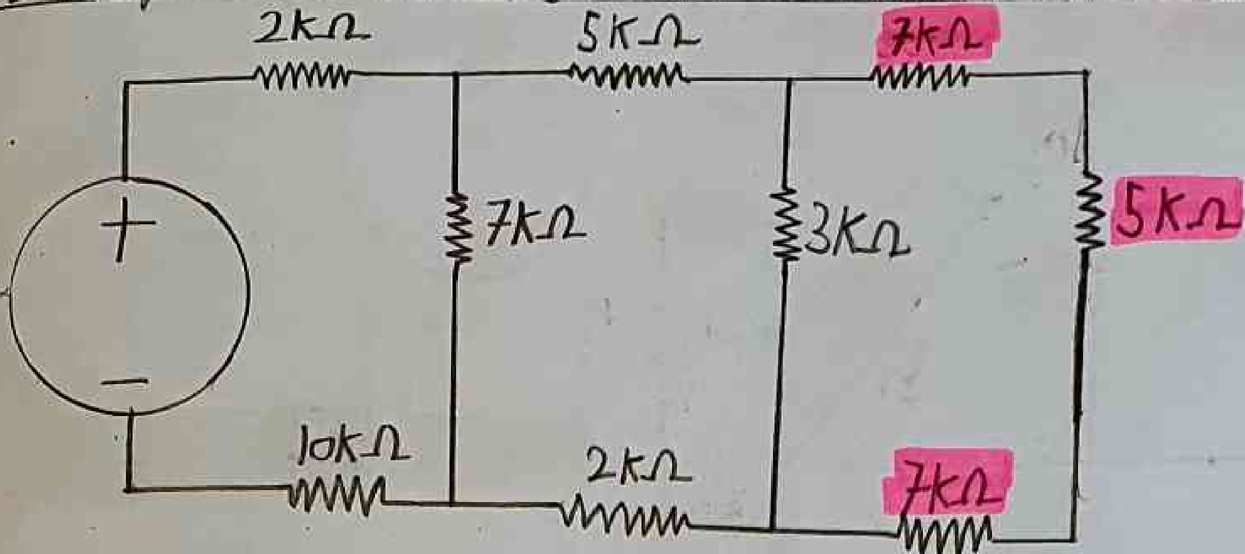
$R_{x_3} = 1k\Omega + 1.78k\Omega + 23k\Omega$

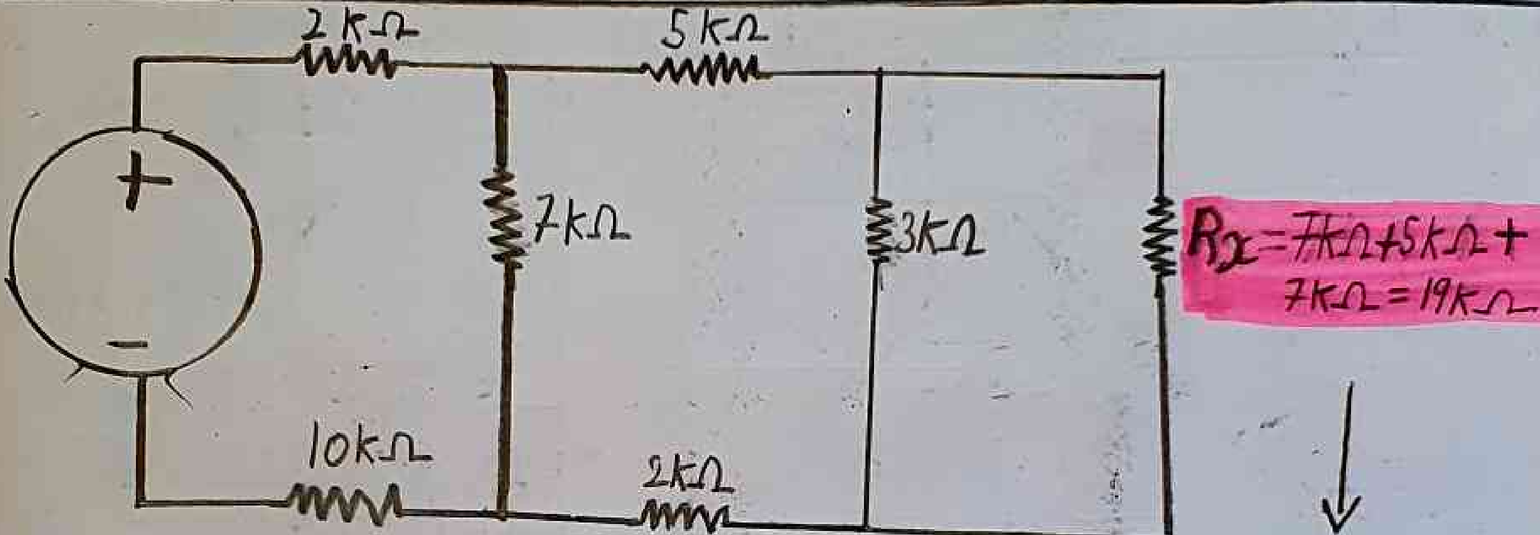
$= 25.78k\Omega$

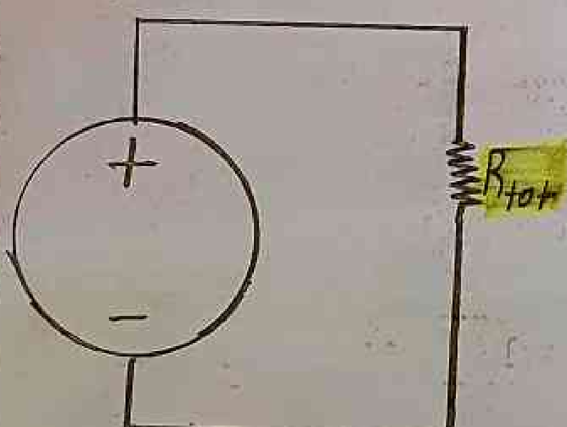


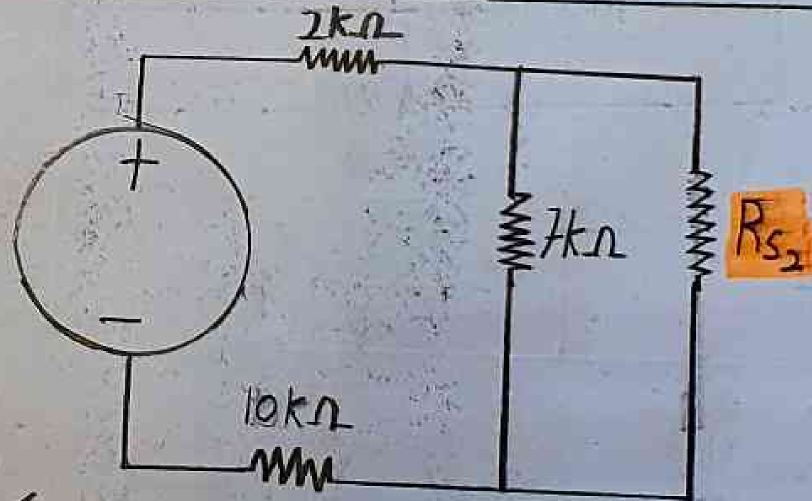
$$R_p = \left(\frac{1}{2k\Omega} + \frac{1}{16k\Omega} \right)^{-1} = 1.78k\Omega$$

Example - Series/Parrallel Combinations of Resistors









$$R_{tot} = R_p = \left(\frac{1}{7k\Omega} + \frac{1}{9.59k\Omega}\right)^{-1}$$

$$= 2k\Omega + 4.05k\Omega + 10k\Omega$$

$$= 16.05k\Omega$$

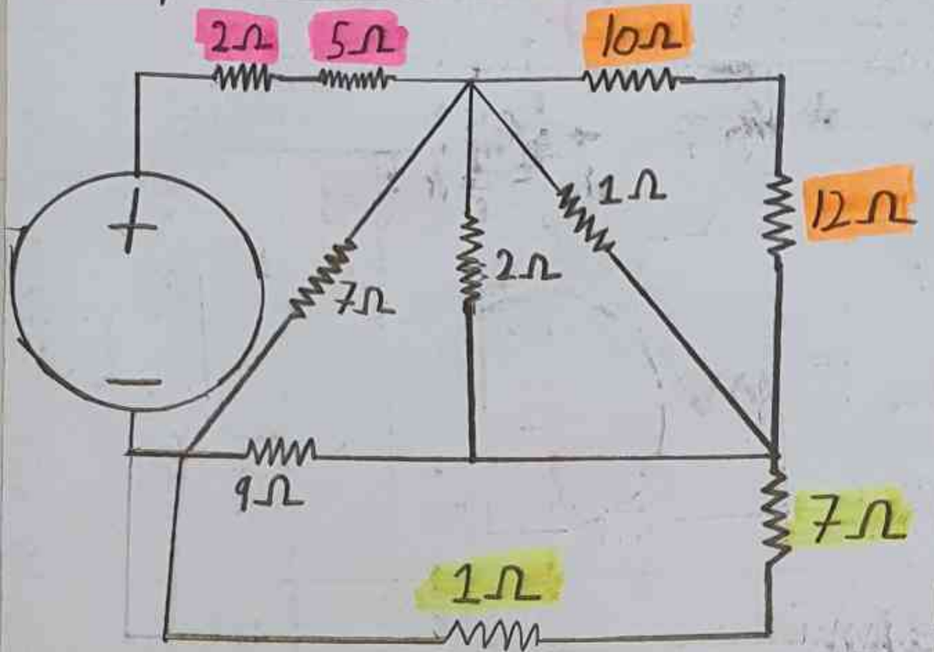
$$R_{tot} = 16.05k\Omega$$

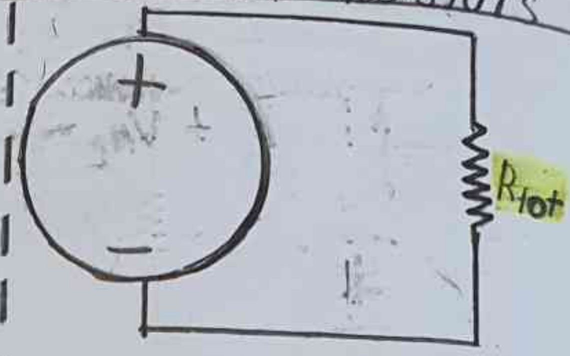
$$R_{x_2} = R_p = \left(\frac{1}{3k\Omega} + \frac{1}{\underbrace{19k\Omega}_{R_x}}\right)^{-1} = 2.59k\Omega$$

$$R_{x_2} = 2.59k\Omega + 5k\Omega + 2k\Omega$$

$$= 9.59k\Omega$$

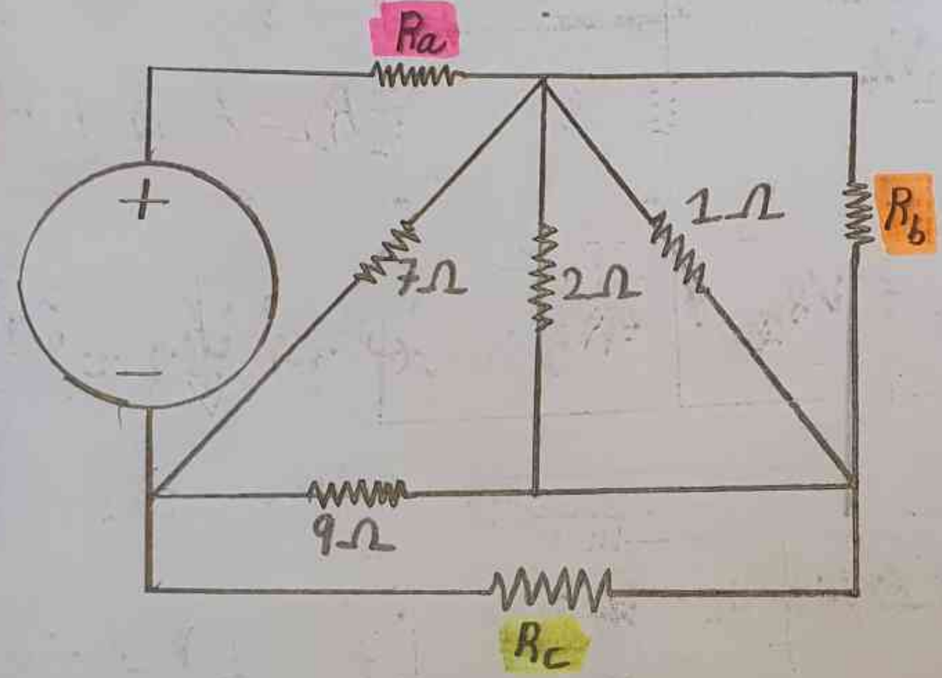
Example - Series/Parallel Combinations of Resistors

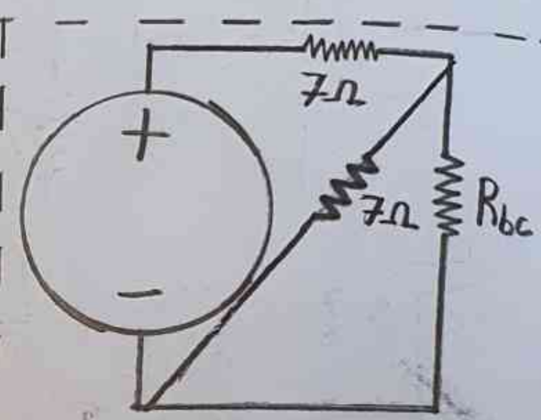




$$R_{tot} = 7\Omega + \left(\frac{1}{7\Omega} + \frac{1}{4.882\Omega}\right)^{-1}$$

$$= 9.876\Omega$$





$$R_{bc} = R_{c_2} + R_{b_2}$$

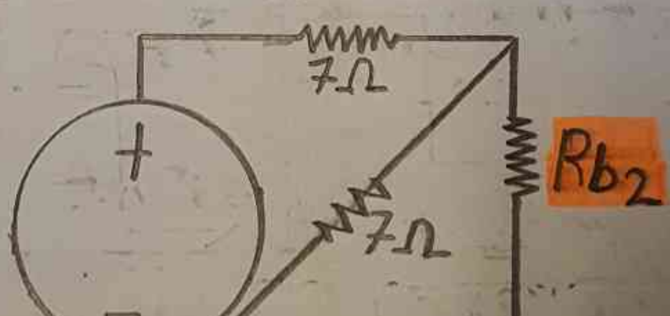
$$= 0.647\Omega + 4.235\Omega$$

$$= 4.882\Omega$$

$R_a = 5\Omega + 2\Omega = 7\Omega$

$R_b = 10\Omega + 12\Omega = 22\Omega$

$R_c = 7\Omega + 1\Omega = 8\Omega$



R_{c_2} → R_P

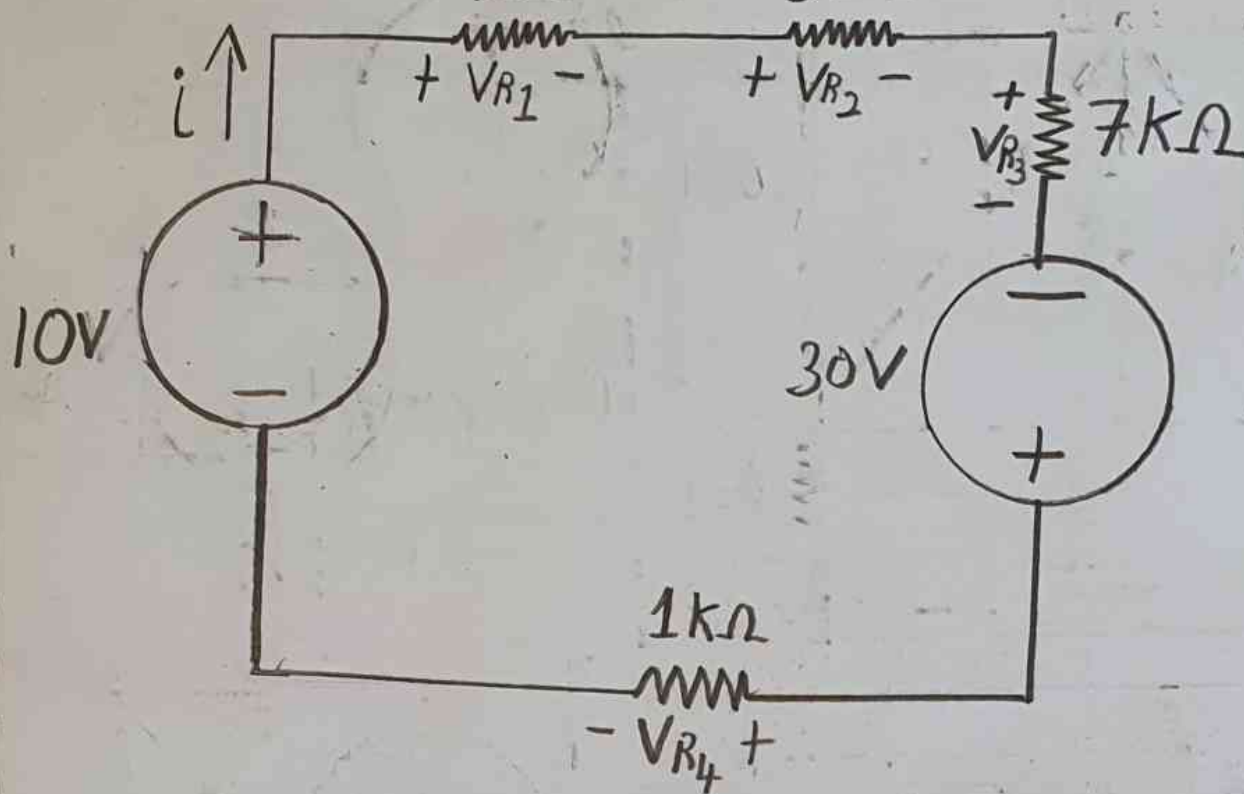
$\left(\frac{1}{9\Omega} + \frac{1}{8\Omega}\right)$

$= 4.235\Omega$

$$R_{b_2} = R_p = \left(\frac{1}{2\Omega} + \frac{1}{1\Omega} + \frac{1}{22\Omega}\right)^{-1}$$

$$= 0.647\Omega$$

Example - Resistive Circuit Analysis



- Loop 1: $-10V + V_{R_1} + V_{R_2} + V_{R_3} - 30V + V_{R_4} = 0$

$\underline{V = iR}$

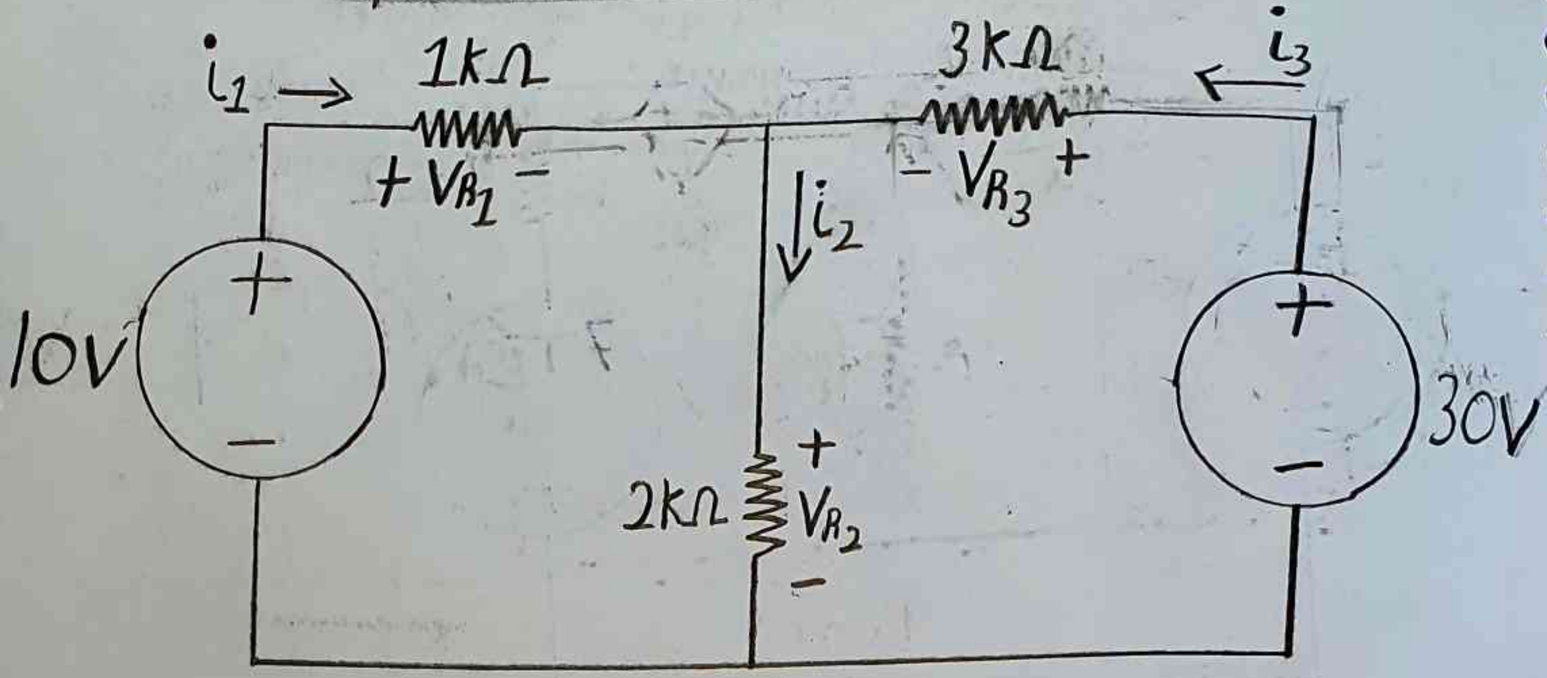
$\rightarrow -10V + i(2k\Omega) + i(5k\Omega) + i(7k\Omega) - 30V + i(1k\Omega) = 0$

$\rightarrow i(2k\Omega + 5k\Omega + 7k\Omega + 1k\Omega) = 40V$

$\rightarrow i = \dfrac{40V}{15k\Omega} = 2.667mA$

$V_{R_1} = (2.667mA)(2k\Omega) = 5.334V$	$-10V + 5.334V + 13.335V +$
$V_{R_2} = (2.667mA)(5k\Omega) = 13.335V$	$18.669V - 30V + 2.667V$
$V_{R_3} = (2.667mA)(7k\Omega) = 18.669V$	$= 0$
$V_{R_4} = (2.667mA)(1k\Omega) = 2.667V$	

Example - Resistive Circuit Analysis



$$\boxed{i_1 + i_3 - i_2 = 0}$$

- Loop 1: $-10V + V_{R_1} + V_{R_2} \rightarrow -10 + 1k\Omega\, i_1 + 2k\Omega\, i_2 = 0$

- Loop 2: $-V_{R_2} - V_{R_3} + 30V \rightarrow -2k\Omega\, i_2 - 3k\Omega\, i_3 + 30V = 0$

$V = iR$

$i_1 = \frac{10V - 2k\Omega\, i_2}{1k\Omega}$, $i_3 = \frac{30V - 2k\Omega\, i_2}{3k\Omega}$

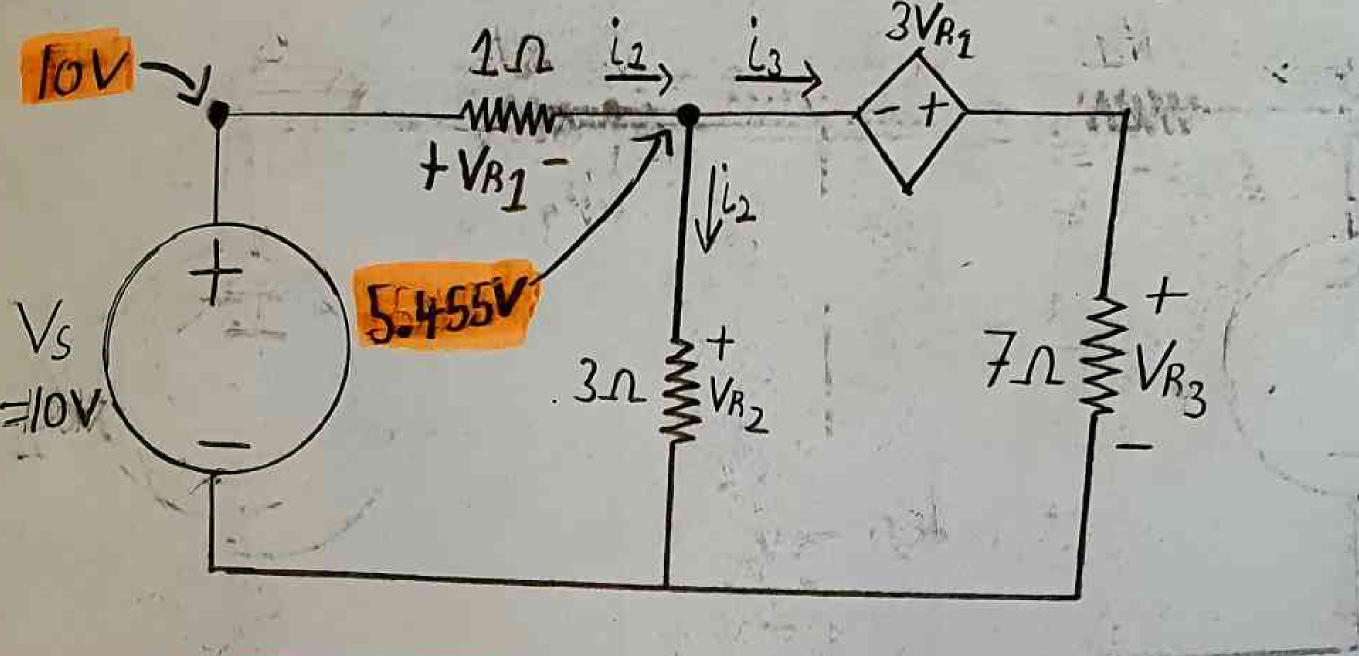
$$\boxed{\frac{10V - 2k\Omega\, i_2}{1k\Omega} + \frac{30V - 2k\Omega\, i_2}{3k\Omega} - i_2 = 0}$$

$\hookrightarrow 10mA - 2i_2 + 10mA - \frac{2}{3} i_2 - i_2 = 0$

$\hookrightarrow 20mA = 3.667 i_2 \rightarrow i_2 = 5.454mA$

$i_1 = 10mA - 2(5.454mA) = \boxed{-0.908mA}$, $i_3 = 10mA - \frac{2}{3}(5.454mA) = \boxed{6.364\,mA}$

Example - Resistive Circuit Analysis



$$i_1 - i_2 - i_3 = 0$$

Loop 1: $-10V + V_{R_1} + V_{R_2} = 0 \longrightarrow V_{R_1} + V_{R_2} = 10V$

Loop 2: $-V_{R_2} - 3V_{R_1} + V_{R_3} = 0 \longrightarrow -3V_{R_1} - V_{R_2} + V_{R_3} = 0V$

$V = iR \longrightarrow i = V/R$

① $\dfrac{V_{R_1}}{1\Omega} - \dfrac{V_{R_2}}{3\Omega} - \dfrac{V_{R_3}}{7\Omega} = 0V \longrightarrow$

$V_{R_1} = 4.545V$

$V_{R_2} = 5.455V$

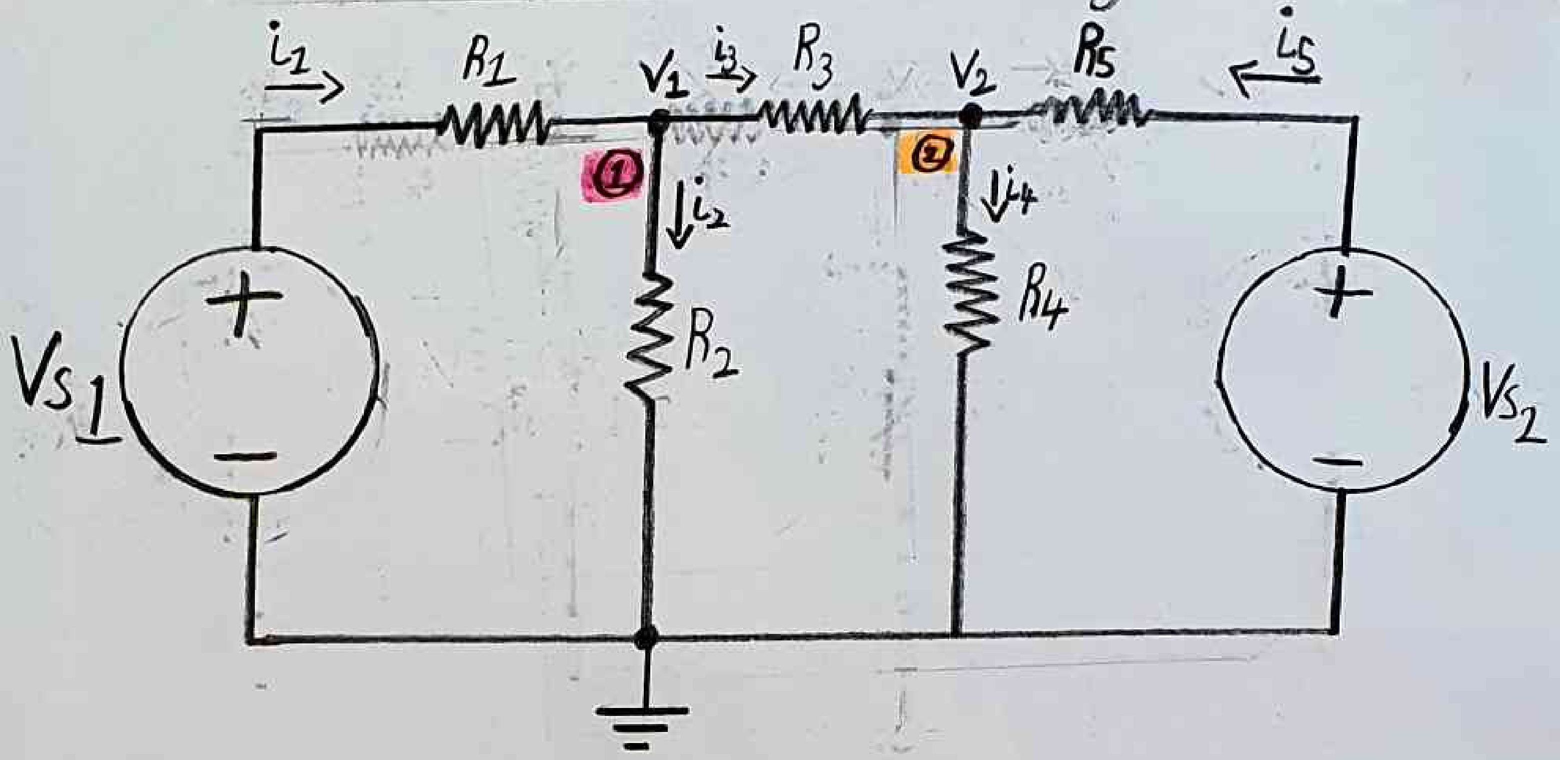
$V_{R_3} = 19.091V$

$i_1 = 4.545A$

$i_2 = 1.818A$

$i_3 = 2.727A$

Definition of Nodal Analysis



At node 1: $i_1 - i_2 - i_3 = 0$

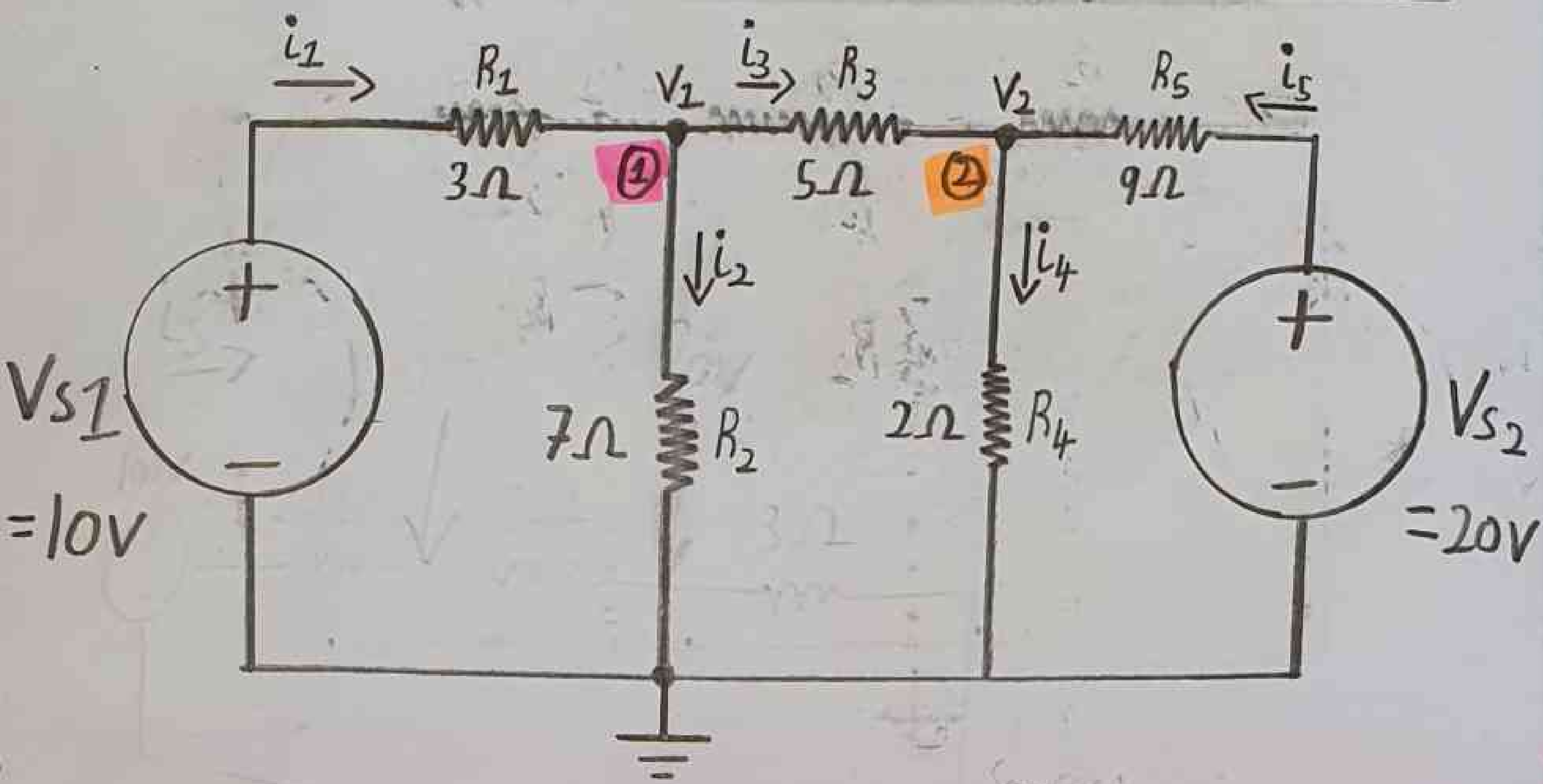
$i = V/R$, $\dfrac{V_{s1} - V_1}{R_1} - \dfrac{V_1}{R_2} - \dfrac{V_1 - V_2}{R_3} = 0$

At node 2: $i_3 - i_4 + i_5 = 0$

$i = V/R$, $\dfrac{V_1 - V_2}{R_3} - \dfrac{V_2}{R_4} + \dfrac{V_{s2} - V_2}{R_5} = 0$

$i_1 = \dfrac{V_{s1} - V_1}{R_1}$, $i_2 = \dfrac{V_1}{R_2}$, $i_3 = \dfrac{V_1 - V_2}{R_3}$, $i_4 = \dfrac{V_2}{R_4}$, $i_5 = \dfrac{V_{s2} - V_2}{R_5}$

Example - Nodal Analysis with Independant sources



At node 1: $i_1 - i_2 - i_3 = 0 \rightarrow \frac{V_{s1}-V_1}{R_1} - \frac{V_1}{R_2} - \frac{V_1 - V_2}{R_3} = 0$

↳ $\frac{V_{s1}}{R_1} - \frac{V_1}{R_1} - \frac{V_1}{R_2} - \frac{V_1}{R_3} + \frac{V_2}{R_3} = 0$

↳ $\frac{V_{s1}}{R_1} + V_1\left(-\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}\right) + V_2\left(\frac{1}{R_3}\right) = 0$

↳ $\frac{10V}{3\Omega} + V_1\left(-\frac{1}{3\Omega} - \frac{1}{7\Omega} - \frac{1}{5\Omega}\right) + V_2\left(\frac{1}{5\Omega}\right) = 0$

↳ $V_1(-0.676) + V_2(0.200) = -3.333$ ①

Full Breakdown
↳ Node Voltage Method Circuit analysis with Current Sources

Youtube

At node 2: $i_3 - i_4 + i_5 = 0 \rightarrow \frac{V_1 - V_2}{R_3} - \frac{V_2}{R_4} + \frac{V_{s2} - V_2}{R_5}$

↳ $\frac{V_1}{R_3} - \frac{V_2}{R_3} - \frac{V_2}{R_4} + \frac{V_{s2}}{R_5} - \frac{V_2}{R_5} = 0$

↳ $V_1\left(\frac{1}{R_3}\right) + V_2\left(-\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5}\right) + \frac{V_{s2}}{R_5} = 0$

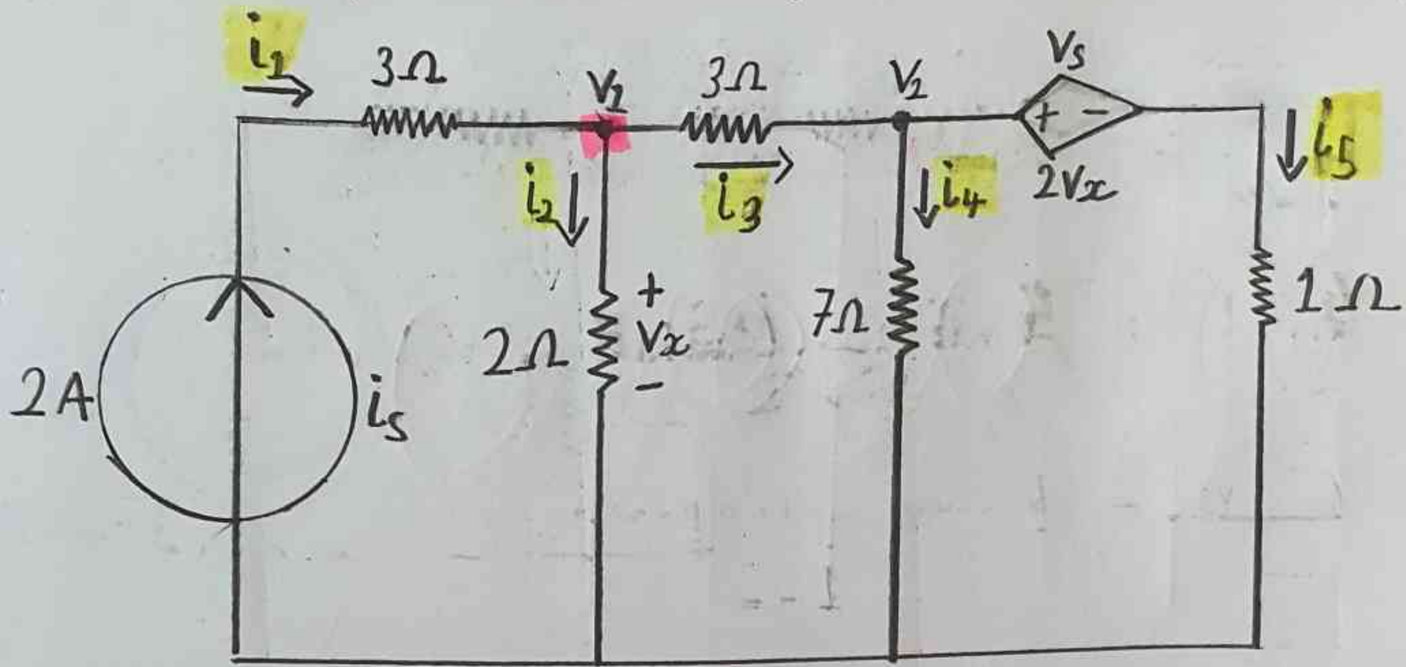
↳ $V_1\left(\frac{1}{5\Omega}\right) + V_2\left(-\frac{1}{5\Omega} - \frac{1}{2\Omega} - \frac{1}{9\Omega}\right) + \frac{20V}{9\Omega} = 0$

↳ $V_1(0.200) + V_2(-0.811) = -2.222$ ②

$V_1 = 6.193V$

$V_2 = 4.267V$

Example - Nodal Analysis With Dependant Sources



At node 1: $i_1 - i_2 - i_3 = 0 \longrightarrow 2A - \frac{V_1}{2\Omega} - \frac{V_1 - V_2}{3\Omega} = 0$

$\hookrightarrow 2A - \frac{V_1}{2\Omega} - \frac{V_1}{3\Omega} + \frac{V_2}{3\Omega} = 0$

$\hookrightarrow V_1\left(-\frac{1}{2\Omega} - \frac{1}{3\Omega}\right) + V_2\left(\frac{1}{3}\right) = -2A$

$\hookrightarrow V_1(-0.833) + V_2(0.333) = -2A$ ①

At node 2: $i_3 - i_4 - i_5 = 0 \longrightarrow \frac{V_1 - V_2}{3\Omega} - \frac{V_2}{7\Omega} - \frac{V_2 - V_s}{1\Omega} = 0$

but $V_s = 2V_x = 2V_1$

$\hookrightarrow \frac{V_1 - V_2}{3\Omega} - \frac{V_2}{7\Omega} - \frac{V_2 - (2V_1)}{1\Omega} = 0$ $\longrightarrow$

$\hookrightarrow \frac{V_1}{3\Omega} - \frac{V_2}{3\Omega} - \frac{V_2}{7\Omega} - \frac{V_2}{1\Omega} + \frac{2V_1}{1\Omega} = 0$

$\hookrightarrow V_1\left(\frac{1}{3\Omega} - \frac{2}{1\Omega}\right) + V_2\left(-\frac{1}{3\Omega} - \frac{1}{7\Omega} - 1\right) = 0$

$\hookrightarrow V_1(2.333) + V_2(-1.476) = 0$ ②

$V_1 = 6.522V$

$V_2 = 10.309V$

$$i_1 = 2A$$

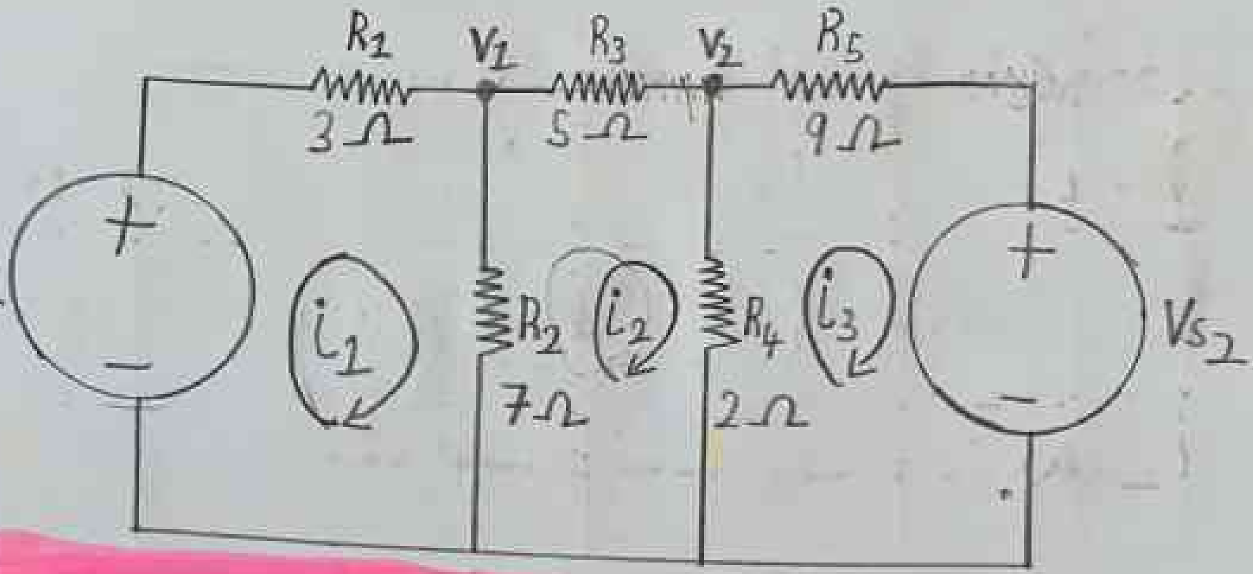
$$i_2 = \frac{V_1}{2\Omega} = \frac{6.522V}{2\Omega} = 3.261A$$

$$i_3 = \frac{V_1 - V_2}{3\Omega} = \frac{6.522V - 10.309V}{3\Omega} = -1.262A$$

$$i_5 = \frac{V_2 - V_s}{1\Omega} = \frac{10.309V - 2(6.522V)}{1\Omega} = -2.735A$$

$$i_4 = \frac{V_2}{7\Omega} = \frac{10.309V}{7\Omega} = 1.473A$$

Definition of Loop Analysis



Loop1: $-V_{s1} + i_1 R_1 + (i_1 - i_2) R_2 = 0$

$\hookrightarrow -V_{s1} + i_1 R_1 + i_1 R_2 - i_2 R_2 = 0$

$\hookrightarrow i_1 (R_1 + R_2) + i_2(-R_2) = V_{s1}$ ①

Loop2: $(i_2 - i_1) R_2 + i_2 R_3 + (i_2 - i_3) R_4 = 0$

$\hookrightarrow i_2 R_2 - i_1 R_2 + i_2 R_3 + i_2 R_4 - i_3 R_4 = 0$

$\hookrightarrow i_1(-R_2) + i_2(R_2 + R_3 + R_4) + i_3(-R_4) = 0$ ②

Loop3: $(i_3 - i_2) R_4 + i_3 R_5 + V_{s2} = 0$

$\hookrightarrow i_3 R_4 - i_2 R_4 + i_3 R_5 + V_{s2} = 0$

$\hookrightarrow i_2(-R_4) + i_3(R_4 + R_5) = -V_{s2}$ ③

Next Page

Example - Loop Analysis with Independent Sources

$$i_1(R_1+R_2)+i_2(-R_2)=V_{S1} \quad ①$$

$$i_1(-R_2)+i_2(R_2+R_3+R_4)+i_3(-R_4)=0 \quad ②$$

$$i_2(-R_4)+i_3(R_4+R_5)=-V_{S_2} \quad ③$$

Loop 1: $i_1(10\Omega)+i_2(-7\Omega)=10V$ ①

Loop 2: $i_1(-7\Omega)+i_2(14\Omega)+i_3(-2\Omega)=0$ ②

Loop 3: $i_2(-2\Omega)+i_3(11\Omega)=-20V$ ③

① $i_1=\dfrac{10V+7i_2}{10\Omega}$, ③ $i_3=\dfrac{-20V+2i_2}{11\Omega}$

↳ Plug in equation ②:

$$-7\Omega\left(\frac{10V+7i_2}{10\Omega}\right)+14i_2+-2\left(\frac{-20V+2i_2}{11\Omega}\right)=0$$

↳ $-7A-\frac{49}{10}i_2+14i_2+\frac{40}{11}A-\frac{4}{11}i_2=0$

↳ $-3.364A=-8.736i_2 \rightarrow i_2=0.385A$

$(i_1-i_2)R_2 \rightarrow (1.269-0.385)7\Omega$

↳ $V_1=6.188V$

$i_1=1.269A$

$i_3=-1.748A$

$(i_2-i_3)R_4 \rightarrow (0.385+1.748)2\Omega$

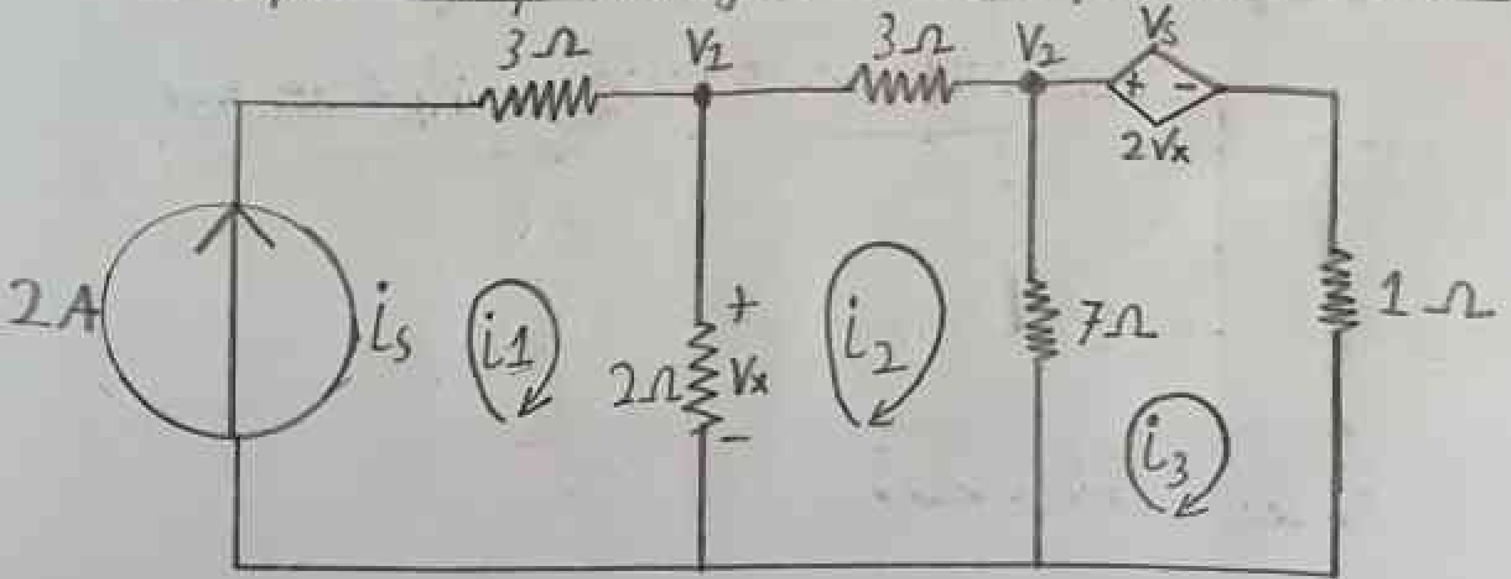
↳ $V_2=4.266V$

From Nodal Analysis:

$V_1=6.193V$

$V_2=4.267V$

Example - Loop Analysis With Dependant Sources



Loop 1: $i_1 = 2A$ ①

Loop 2: $(i_2 - i_1)2\Omega + i_2(3\Omega) + (i_2 - i_3)7\Omega = 0$

$\Rightarrow 2i_2 - 2i_1 + 3i_2 + 7i_2 - 7i_3 = 0$

$\Rightarrow i_1(-2\Omega) + i_2(2\Omega + 3\Omega + 7\Omega) + i_3(-7\Omega) = 0$

$\Rightarrow i_1(-2\Omega) + i_2(12\Omega) + i_3(-7\Omega) = 0$ ②

Loop 3: $(i_3 - i_2)7\Omega + 2V_x + i_3(1\Omega) = 0$

but $V_x = (i_1 - i_2)2\Omega$

$\Rightarrow (i_3 - i_2)7\Omega + 2(i_1 - i_2)2\Omega + i_3(1\Omega) = 0$

$\Rightarrow 7i_3 - 7i_2 + 4i_1 - 4i_2 + i_3 = 0$

$\Rightarrow i_1(4\Omega) + i_2(-7\Omega - 4\Omega) + i_3(7\Omega + 1\Omega) = 0$

$\Rightarrow i_1(4\Omega) + i_2(-11\Omega) + i_3(8\Omega) = 0$ ③

$i_1 = 2A$

$i_2 = -1.263A$

$i_3 = -2.737A$

From Nodal Analysis:

$V_1 = 6.522V$, $V_2 = 10.309V$

$V_1 = (i_1 - i_2)2\Omega$

$\Rightarrow (2A + 1.263A)2\Omega$

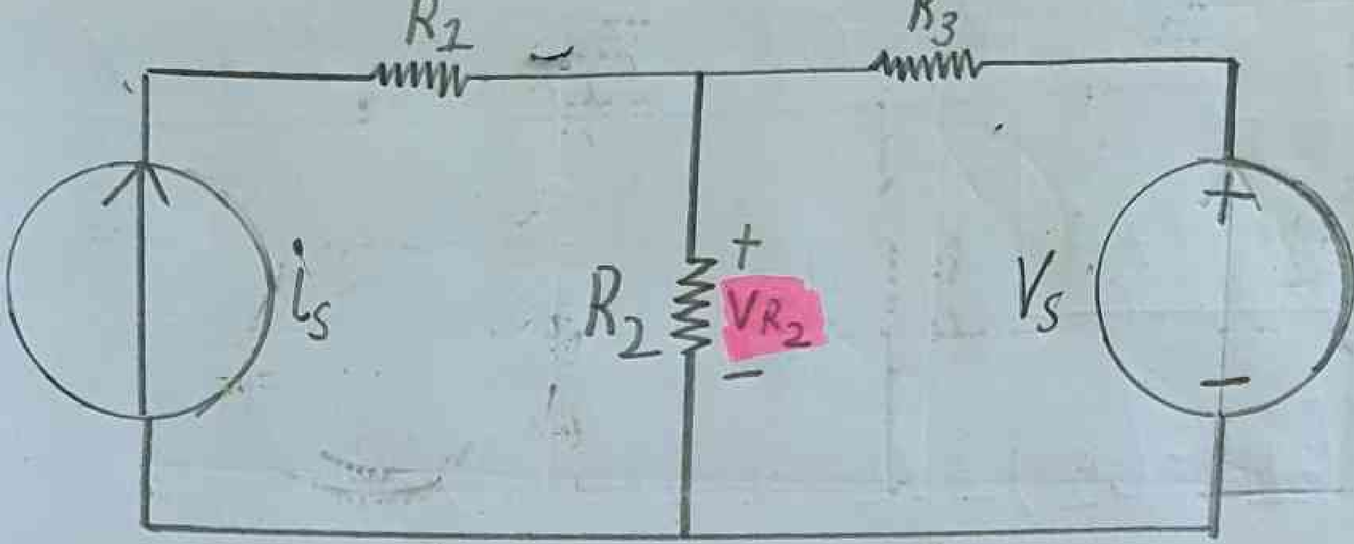
$\Rightarrow V_1 = 6.526V$

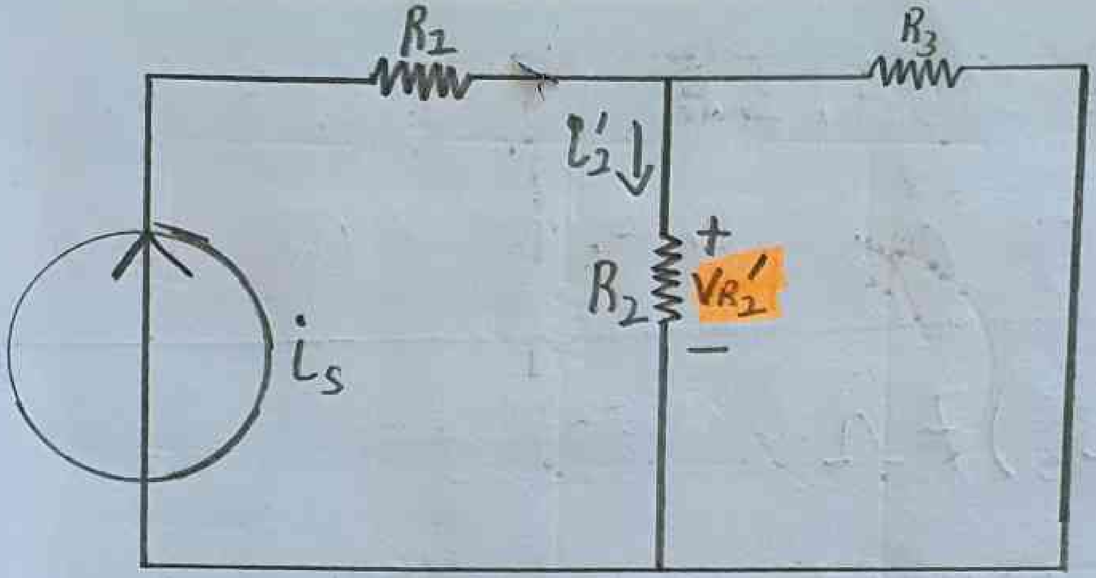
$V_2 = (i_2 - i_3)7\Omega$

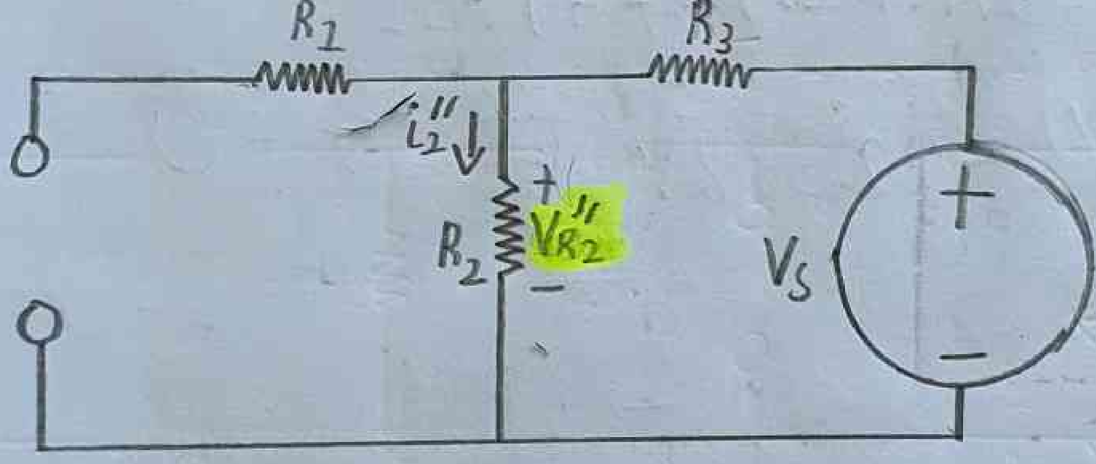
$\Rightarrow (-1.263A + 2.737A)7\Omega$

$\Rightarrow V_2 = 10.318V$

Superposition Theorem







$$\boxed{V_{R_2} = V_{R_2}' + V_{R_2}''}$$

Solving for V_{R_2}':

$$i_2' = \frac{i_s(R_3)}{R_2 + R_3} \longrightarrow V_{R_2}' = i_2'(R_2)$$

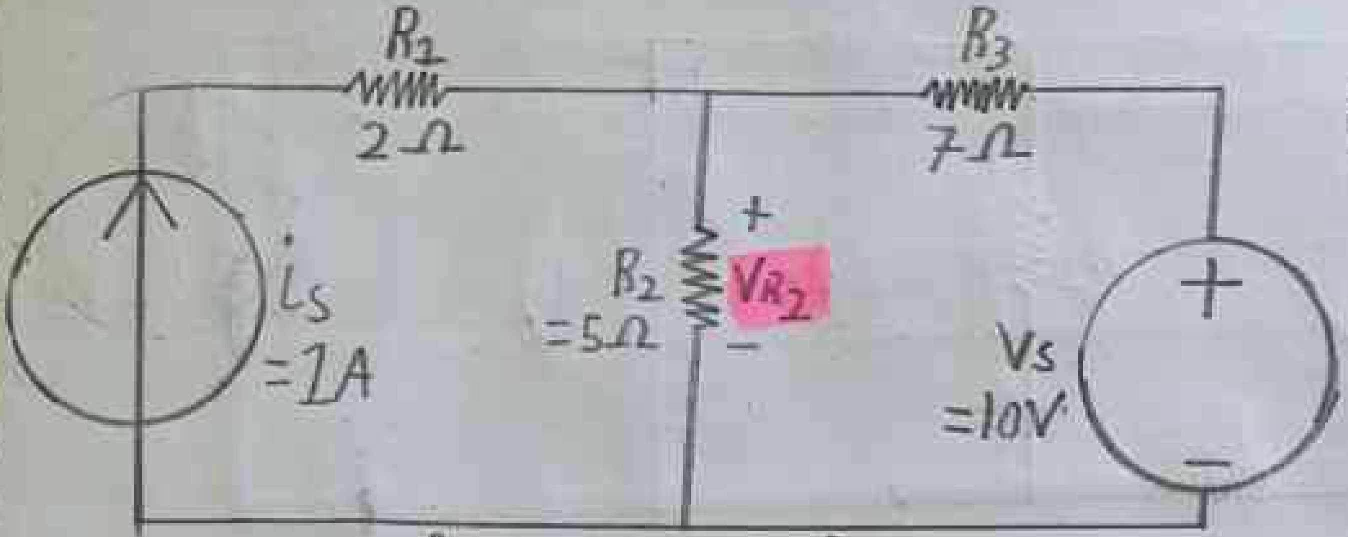
Current Division

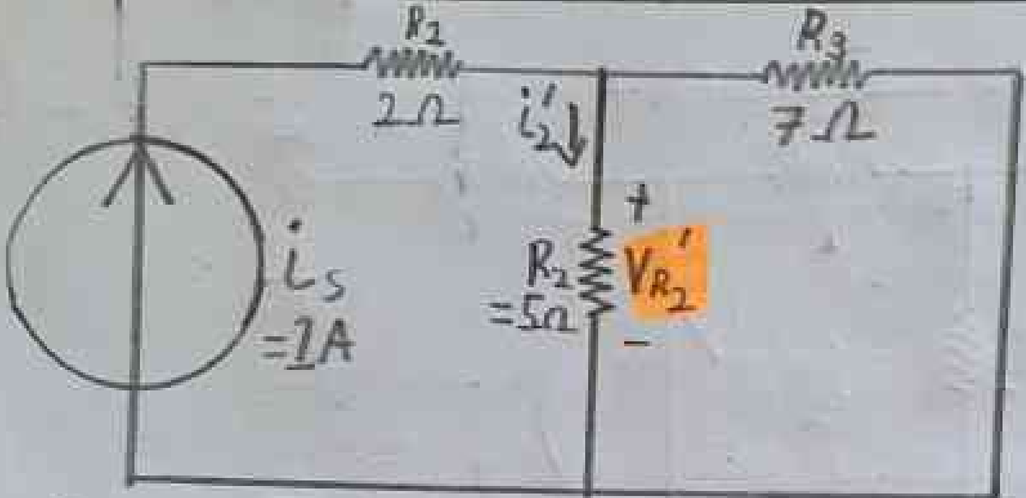
Solving for V_{R_2}'':

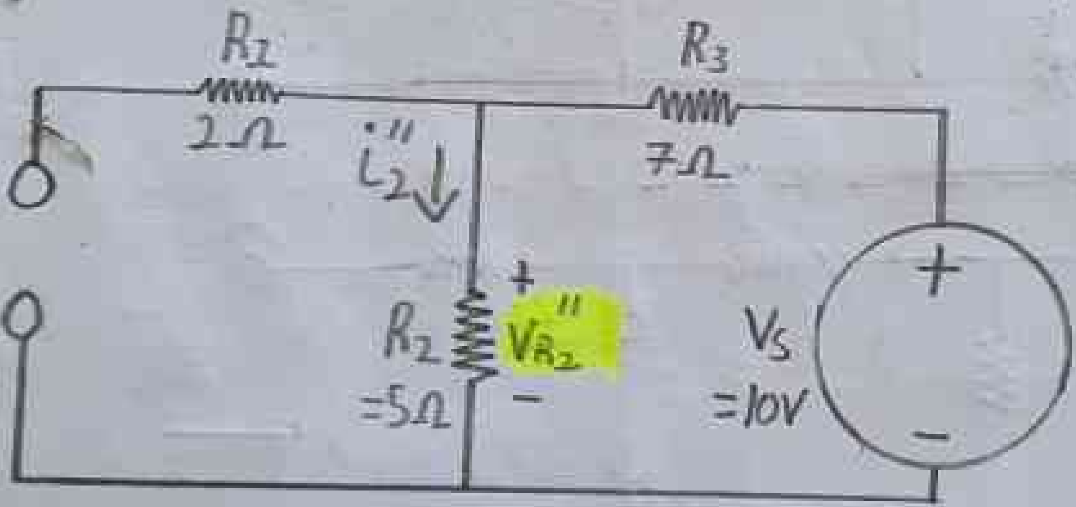
$$V_{R_2}'' = \frac{V_s(R_2)}{R_2 + R_3}$$

Voltage Division

Example – Superposition Theorem







$V_{R_2} = V_{R_2}' + V_{R_2}''$

Solving for V_{R_2}':

$$i_2' = \frac{1A\,(7\Omega)}{5\Omega + 7\Omega} = 0.583A$$

$V_{R_2}' = 0.583A \times 5\Omega$

$V_{R_2}' = 2.915V$

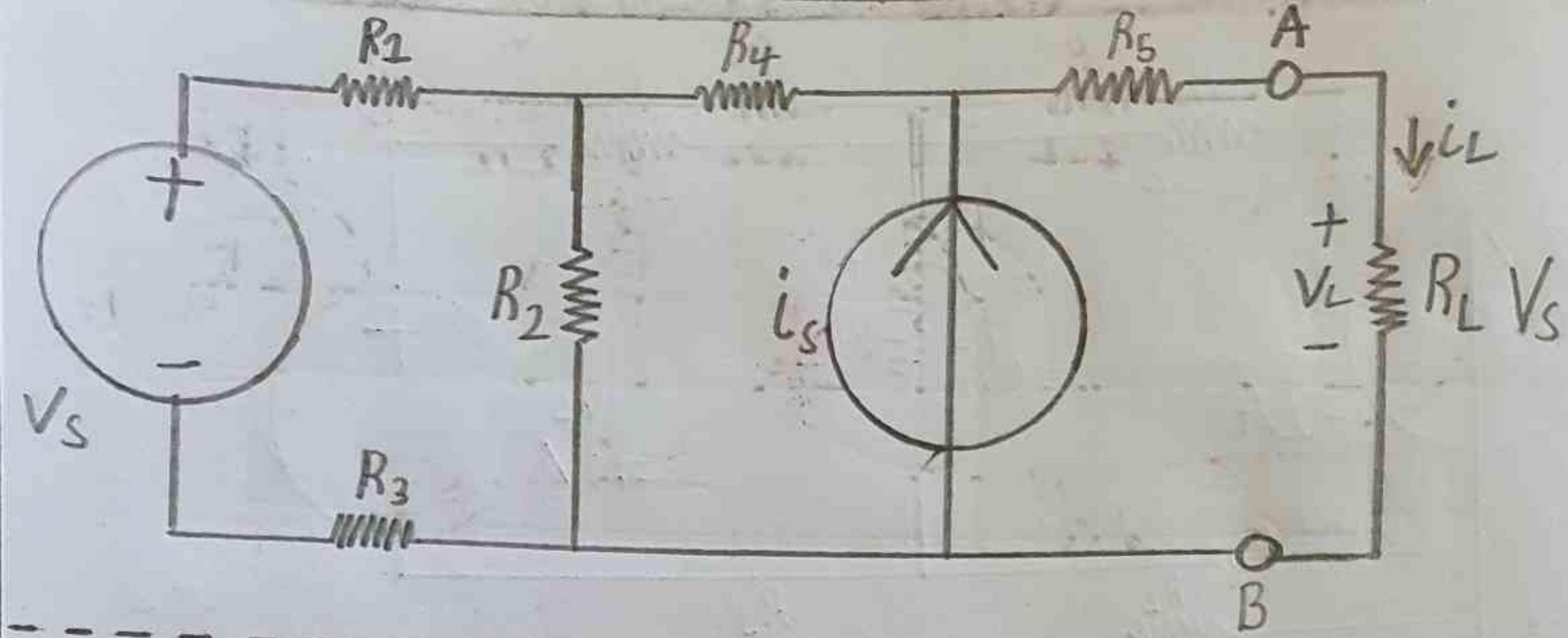
Solving for V_{R_2}'':

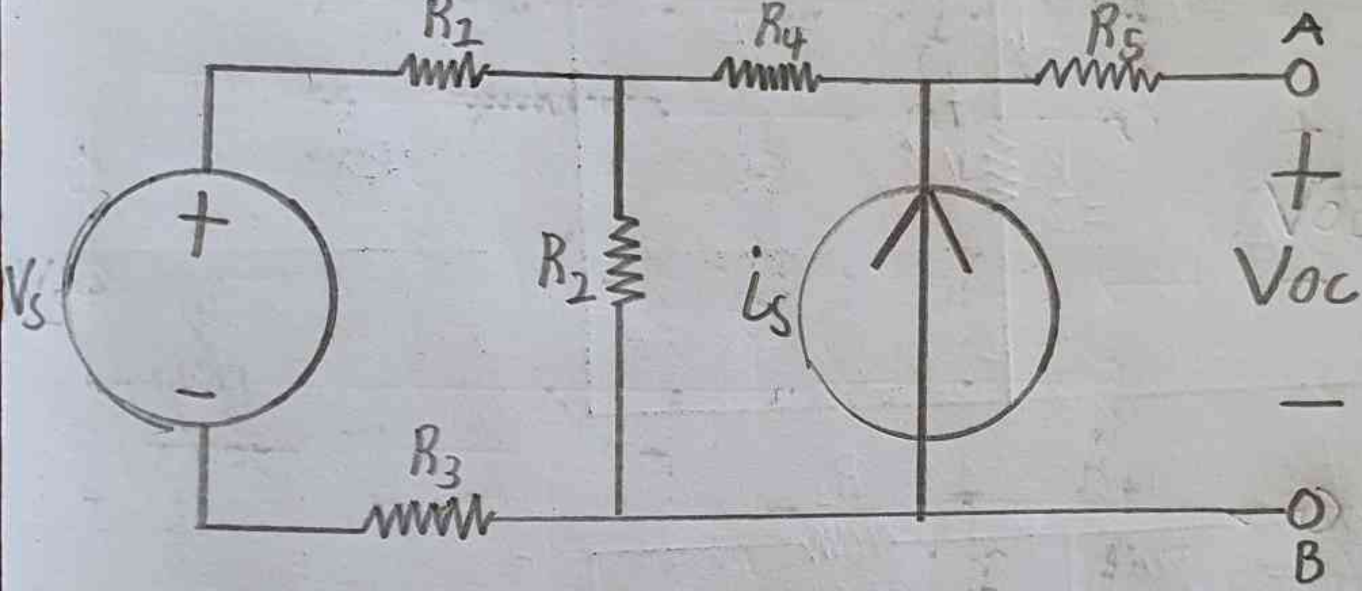
$$V_{R_2}'' = \frac{10V\,(5\Omega)}{5\Omega + 7\Omega}$$

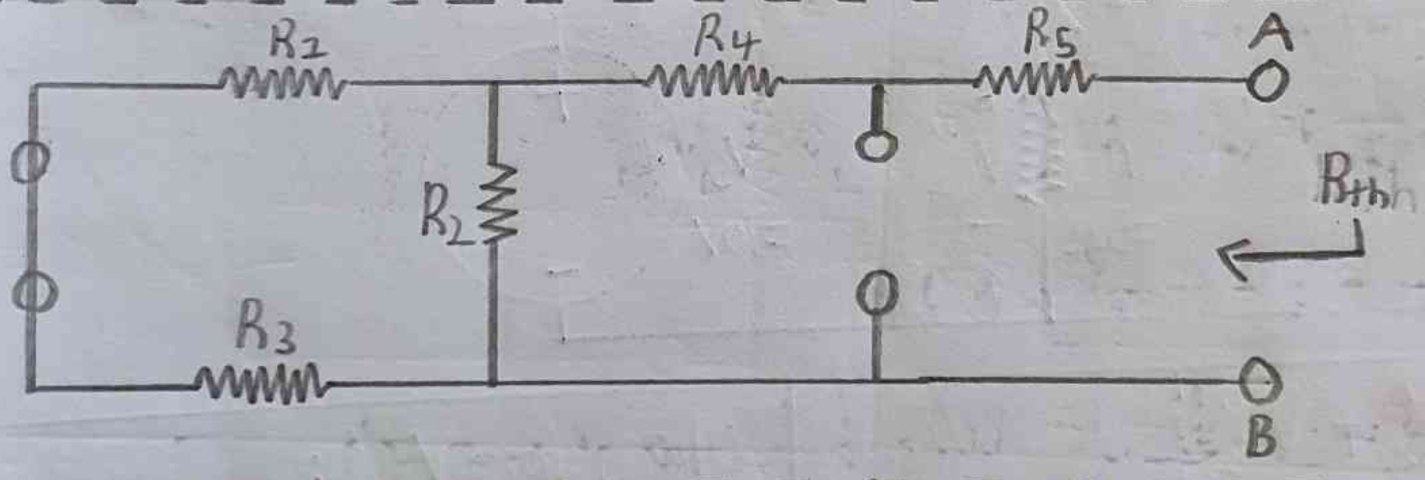
$V_{R_2}'' = 4.167V$

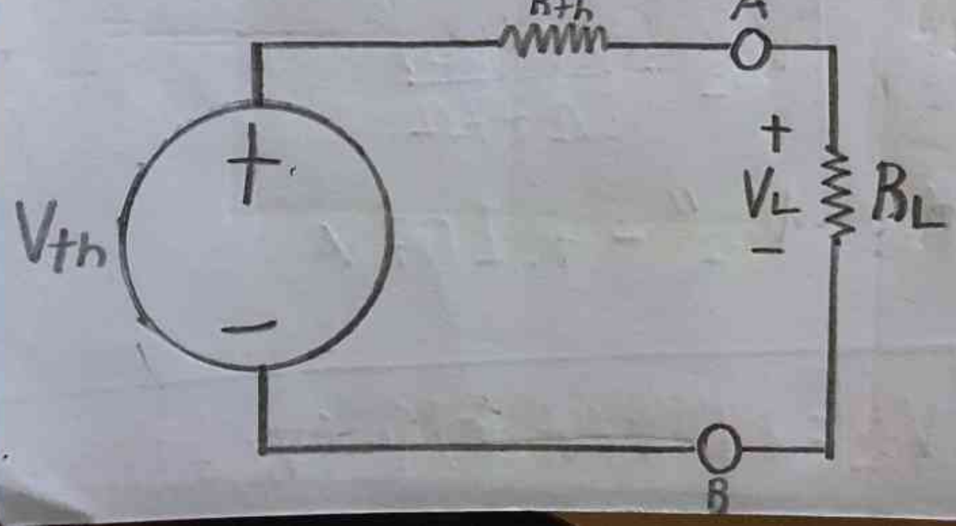
$V_{R_2} = 2.915V + 4.167V \rightarrow V_{R_2} = 7.082V$

Thévenin's Theorem

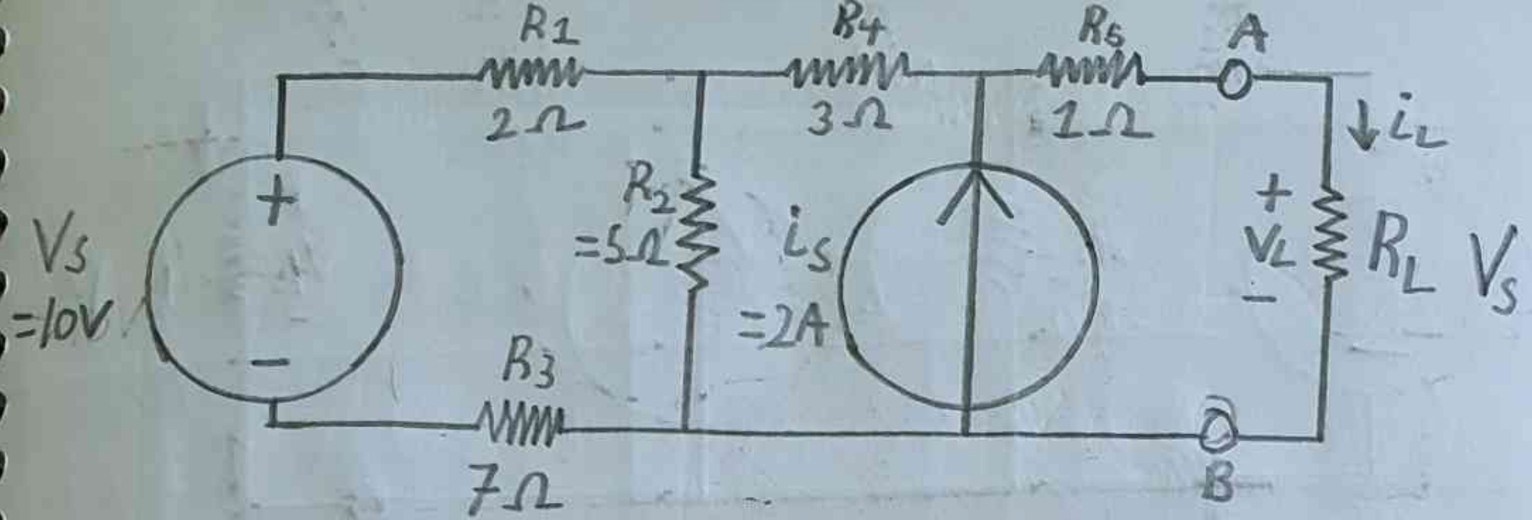


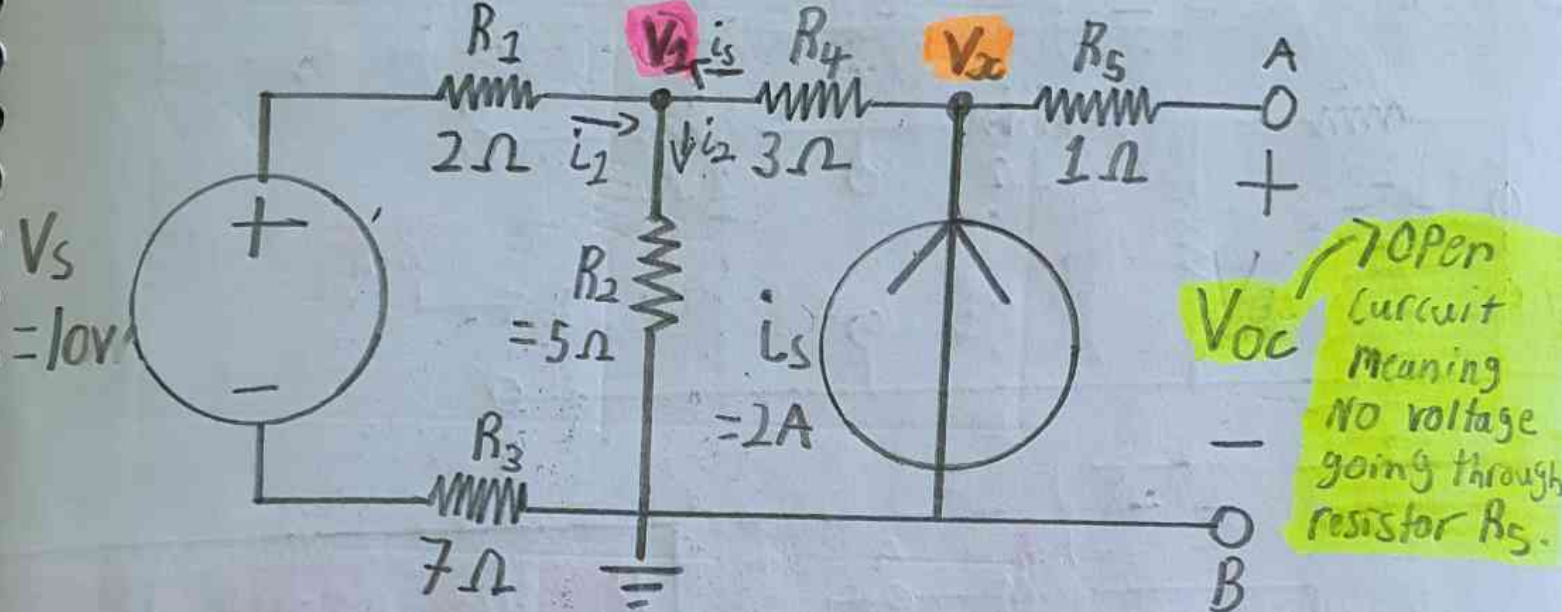






Example - Thévenin's Theorem Part 1





$i_1 - i_2 + i_s = 0 \longrightarrow \frac{V_s - V_1}{2\Omega} - \frac{V_1}{5\Omega} + 2A = 0$

$\frac{V_s}{2\Omega} - \frac{V_1}{2\Omega} - \frac{V_1}{5\Omega} + 2A = 0$

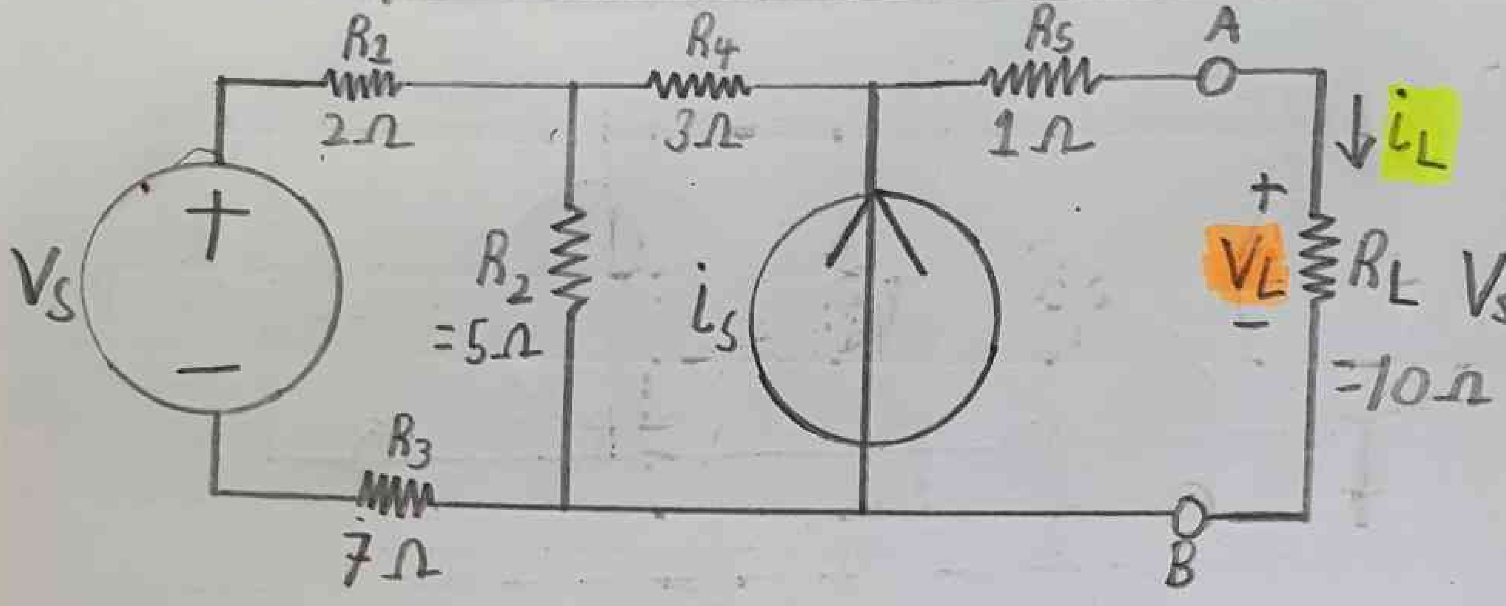
$V_1\left(-\frac{1}{2\Omega} - \frac{1}{5\Omega}\right) = -2A - \frac{10V}{2\Omega} \longrightarrow V_1(-0.7) = -7A$

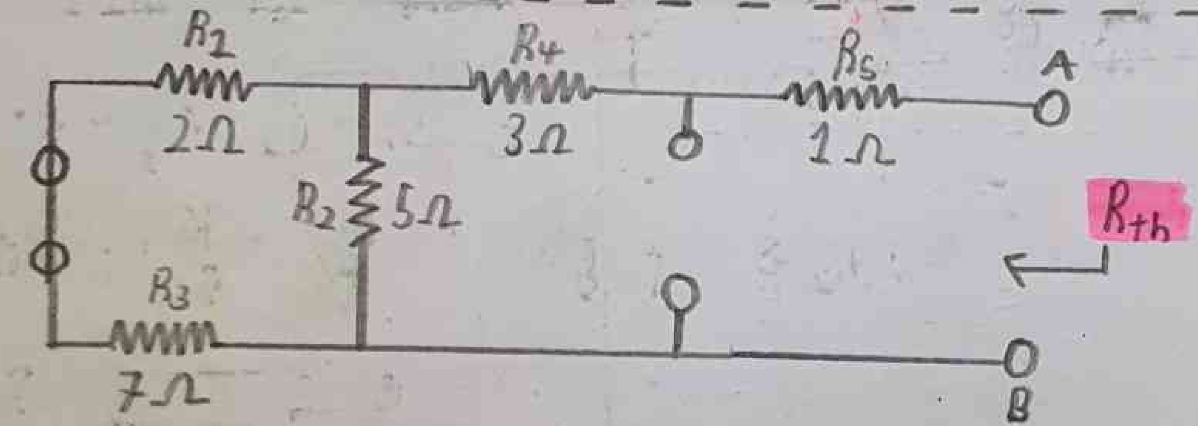
$V_1 = \frac{-7}{-0.7}$, $V_1 = 10V$

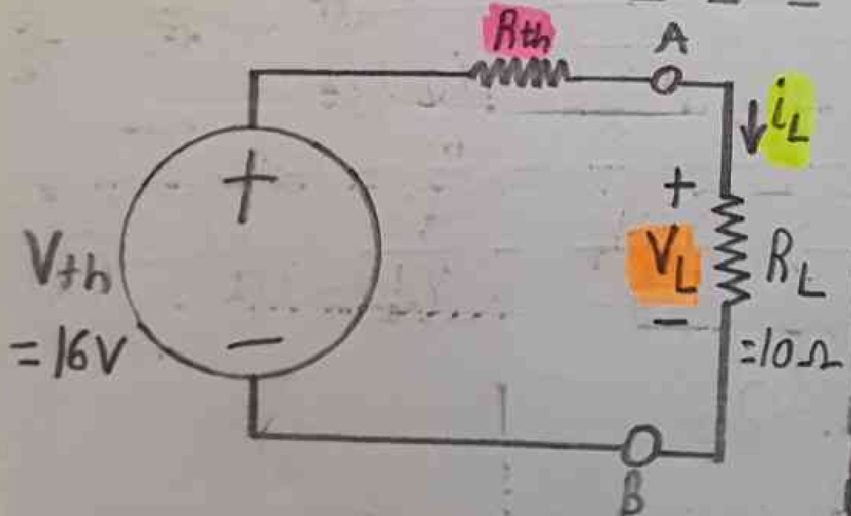
$V_x = 10V + 2A(3\Omega) = 16V$

$V_x = V_{oc} = 16V = V_{th}$

Example - Thévenin's Theorem Part 2







$R_{th} = ((2\Omega + 7\Omega) \parallel 5\Omega) + 3\Omega + 1\Omega$

$\rightarrow 9\Omega \parallel 5\Omega = \left(\frac{1}{9\Omega} + \frac{1}{5\Omega}\right)$

$R_{th} = 3.214\Omega + 3\Omega + 1\Omega$

$R_{th} = 7.214\Omega$

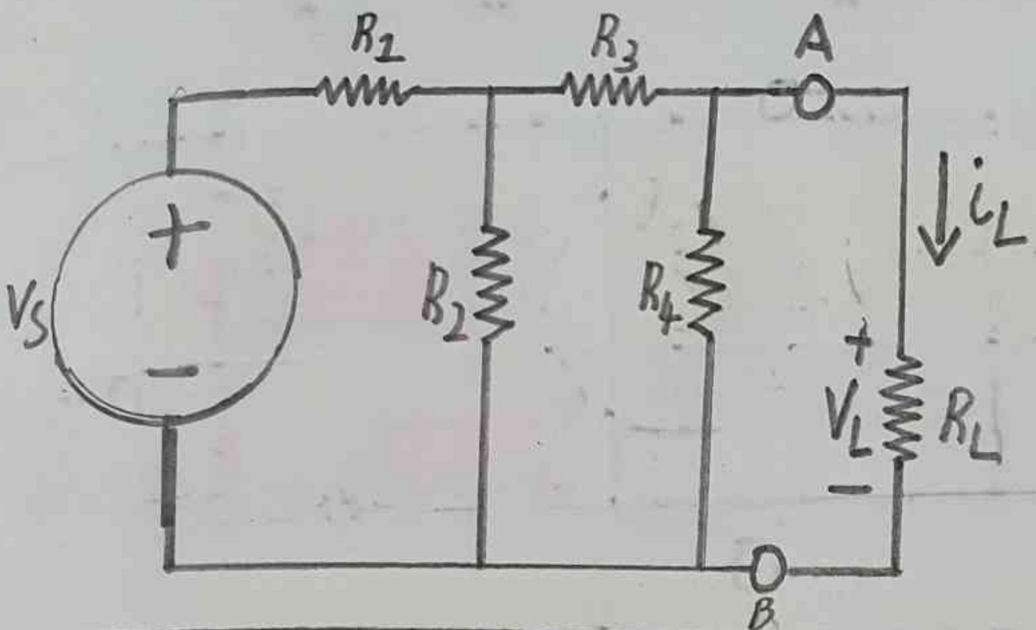
$V_L = \frac{16V(10\Omega)}{R_{th} + 10\Omega} = 9.295V$

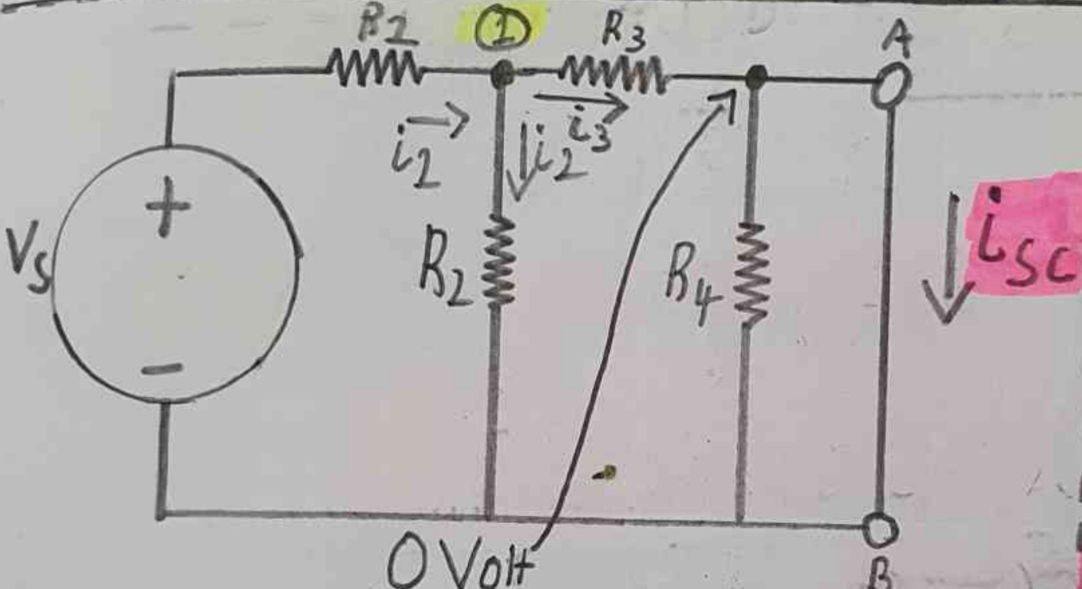
$i_L = V_L / R_L$

$\rightarrow i_L = 9.295V / 10\Omega$

$\rightarrow i_L = 0.9295 A$

Norton's Theorem





At node 1:

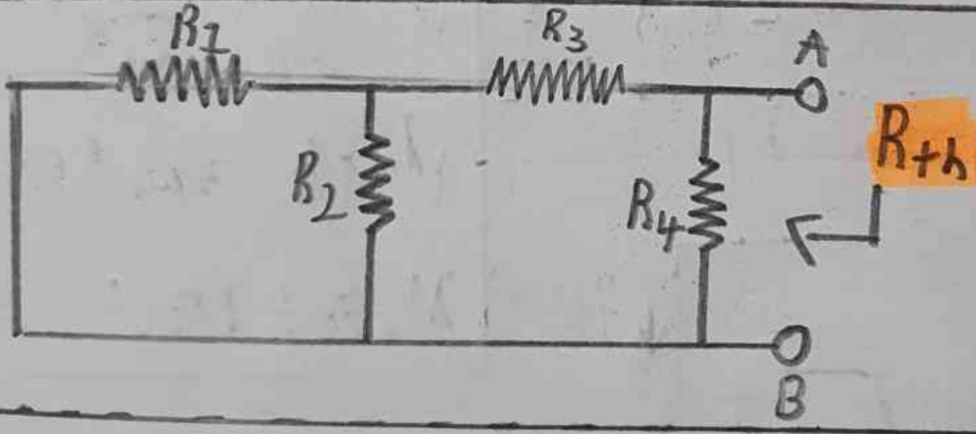
$$i_1 - i_2 - i_3 = 0$$

$$\hookrightarrow \frac{V_s - V_1}{R_1} - \frac{V_1}{R_2} - \frac{V_1}{R_3} = 0$$

$$\hookrightarrow \frac{V_s}{R_1} - \frac{V_1}{R_1} - \frac{V_1}{R_2} - \frac{V_1}{R_3} = 0$$

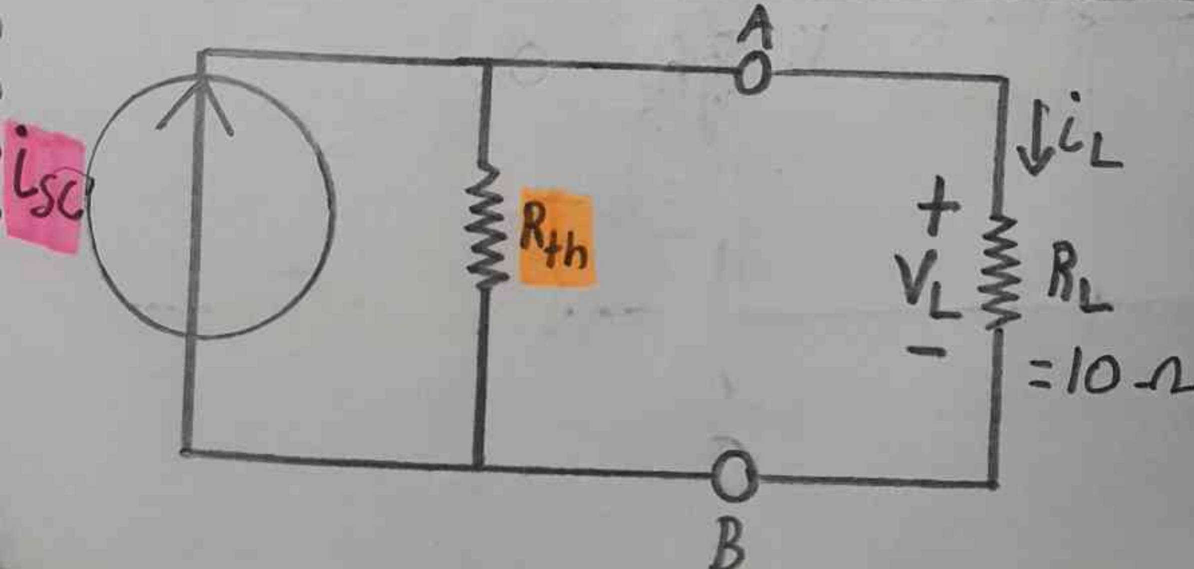
$$\hookrightarrow V_1\left(\frac{-1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}\right) = \frac{-V_s}{R_1}$$

$$i_{sc} = \frac{V_1}{R_3}$$



$$R_{th} = (R_1 \| R_2 + R_3) \| R_4$$

$$\left(\left(\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + R_3\right) + \frac{1}{R_4}\right)^{-1}$$



Example - Norton's Theorem

Refer to last Page:

$V_1\left(-\frac{1}{R_1}-\frac{1}{R_2}-\frac{1}{R_3}\right)=\frac{-V_s}{R_1}$	$i_{sc}=\frac{V_1}{R_3}$
$R_{th}=(R_1 \parallel R_2+R_3)\parallel R_4$	$i_{sc}=\frac{5.931V}{5\Omega}=1.186A$

$V_1\left(\frac{-1}{2\Omega}-\frac{1}{7\Omega}-\frac{1}{5\Omega}\right)=\frac{-10V}{2\Omega} \rightarrow V_1(-0.843)=-5$

$\Rightarrow V_1=5.931V$

$R_{th}=\left(\frac{1}{2\Omega}+\frac{1}{7\Omega}\right)^{-1}+5\Omega=6.556\Omega$

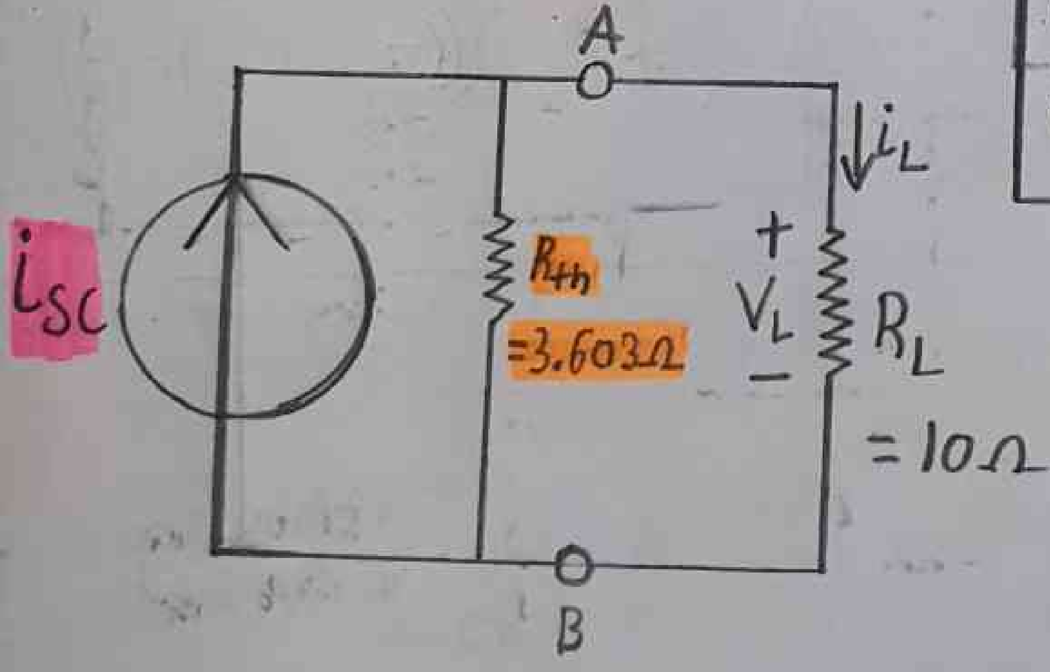
$\Rightarrow \left(\frac{1}{6.556\Omega}+\frac{1}{8\Omega}\right)^{-1}=3.603\Omega=R_{th}$

$i_L=\frac{R_{th}\, i_{sc}}{R_{th}+R_L}=\frac{3.603\Omega(1.186A)}{3.603\Omega+10\Omega}=0.314A$

$V_L=(i_L)(R_L)$

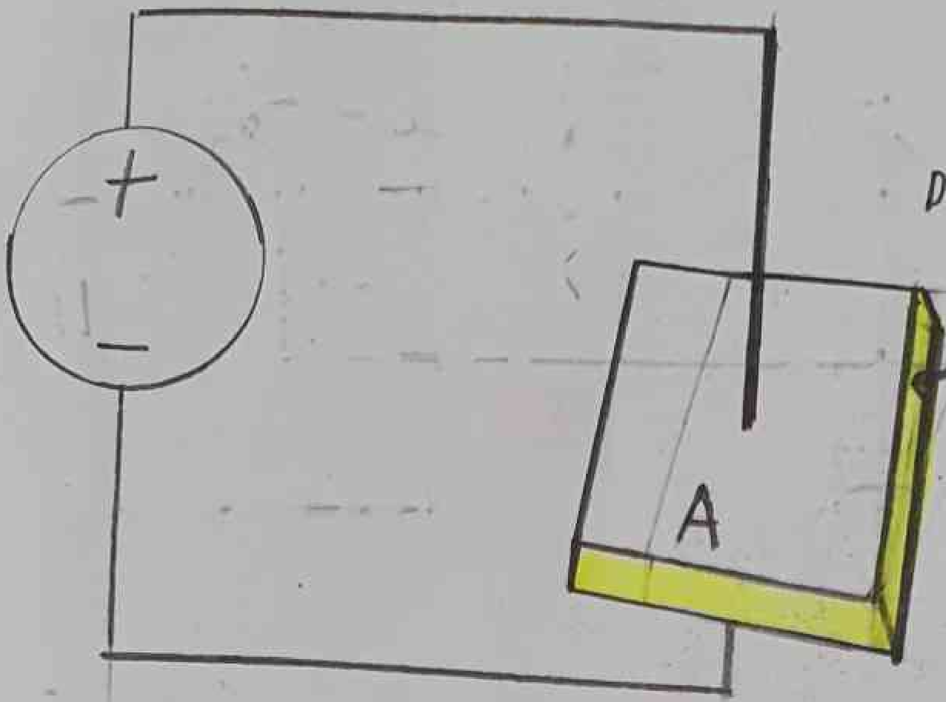
$V_L=(0.314A)(10\Omega)$

$V_L=3.14V$



Definition of capacitor

$$\boxed{q = Cv}$$

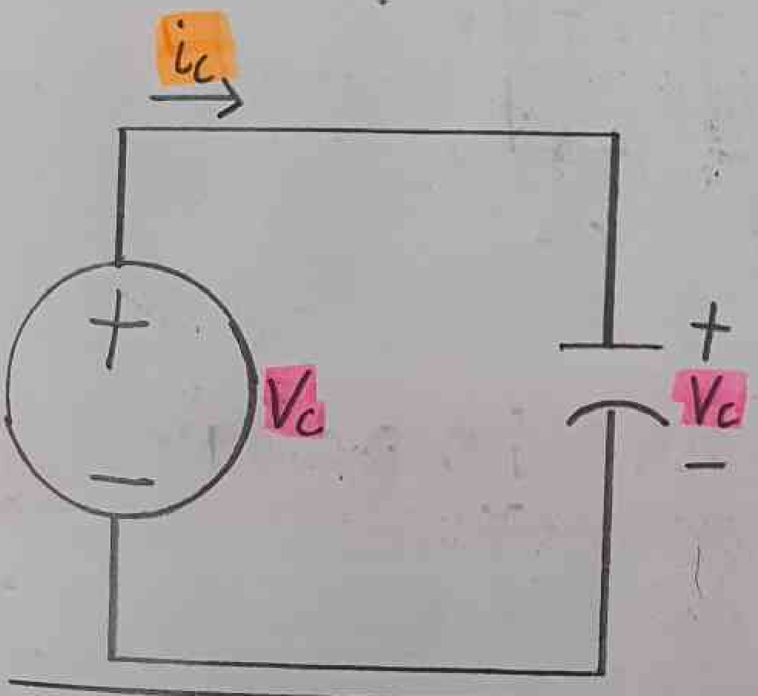


$$\boxed{W_C(t) = \frac{1}{2} C v^2(t) \, J}$$

Energy stored in a capacitor

Charge in a capacitor

$$\boxed{C = \frac{\varepsilon A}{d}}$$



$$i = \frac{dq}{dt} = \frac{d(Cv)}{dt}$$

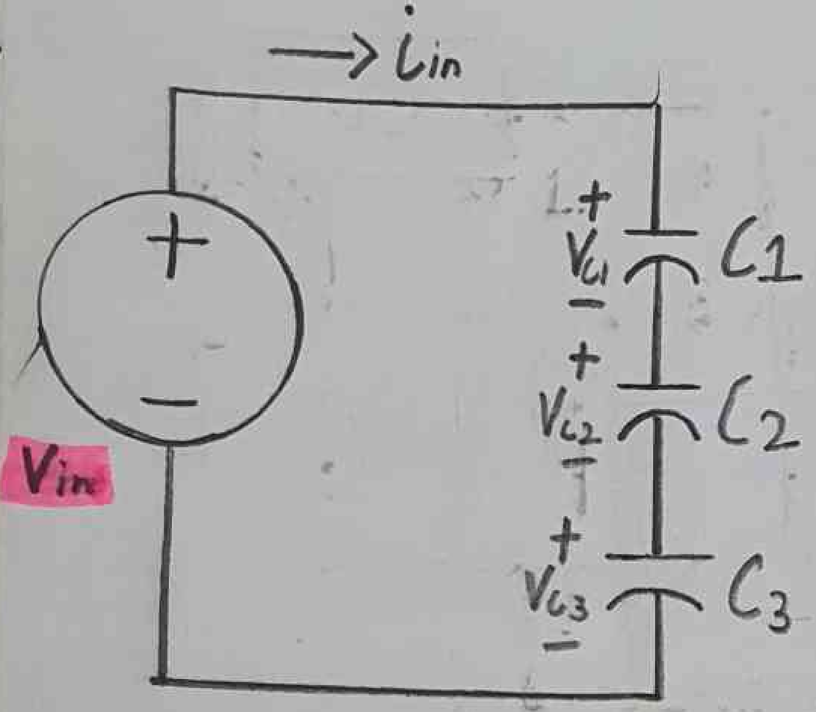
$$\hookrightarrow i_C = C \frac{dv}{dt}$$

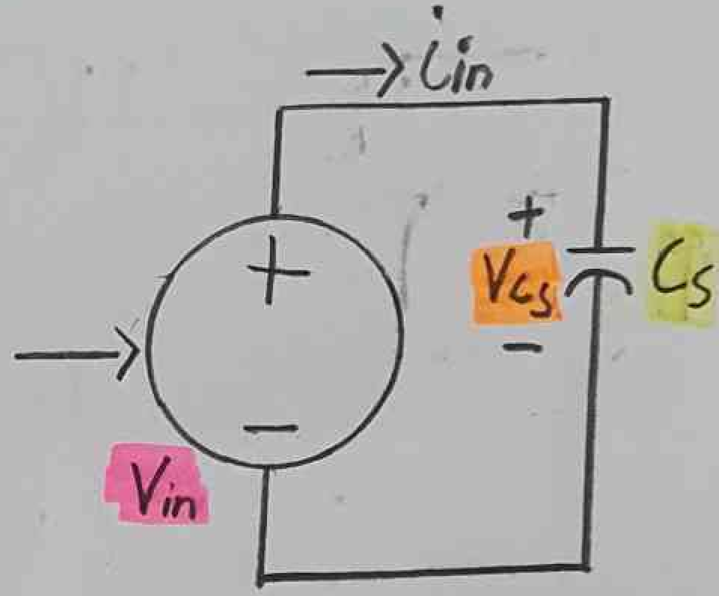
Current through a capacitor

$$\boxed{V_C(t) = V_C(t_0) + \frac{1}{C} \int_{t_0}^{t} i(x) \, dx}$$

Voltage Across a capacitor

Series Capacitors





$$V_C = \frac{1}{C}\int_{t_0}^{t} i_c(t)\,dt + V_c(t_0)$$

$$V_{in} = V_{c_1} + V_{c_2} + V_{c_3}$$

$$V_{in} = \left[\frac{1}{C_1}\int_{t_0}^{t} i_{in}\,dt + V_{c_1}(t_0)\right] + \left[\frac{1}{C_2}\int_{t_0}^{t} i_{in}\,dt + V_{c_2}(t_0)\right]$$

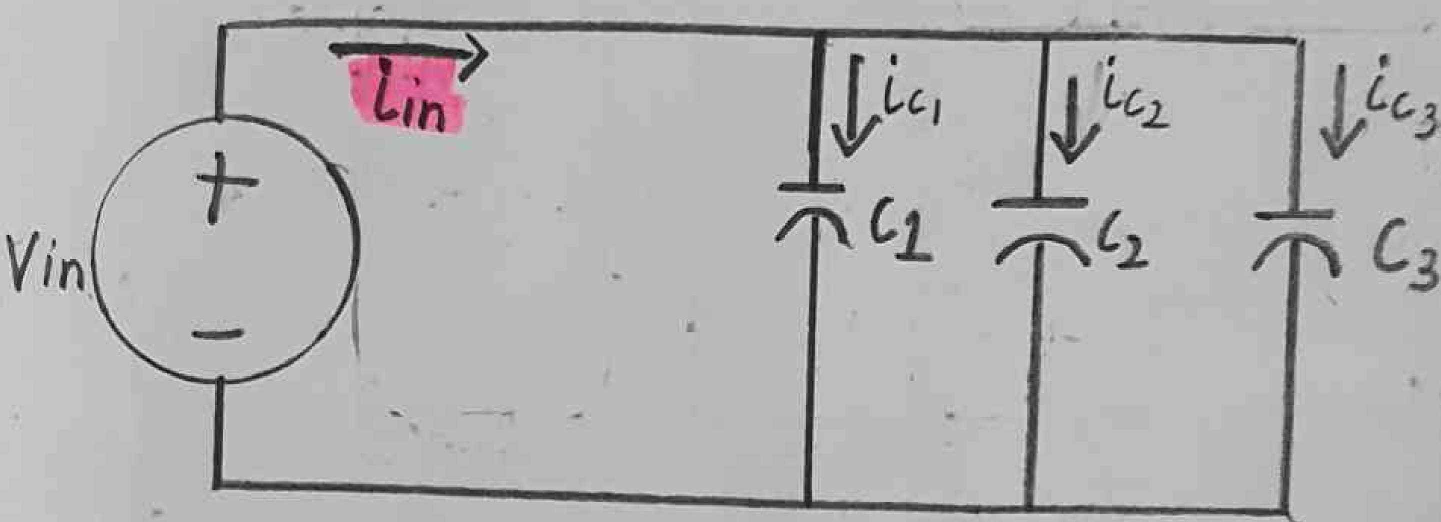
$$+ \left[\frac{1}{C_3}\int_{t_0}^{t} i_{in}\,dt + V_{c_3}(t_0)\right]$$

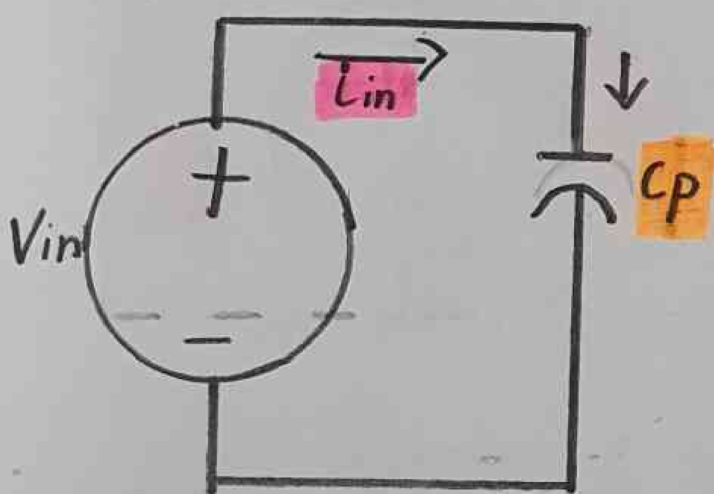
$$= \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]\int_{t_0}^{t} i_{in}\,dt + V_{c_1}(t_0) + V_{c_2}(t_0) + V_{c_3}(t_0)$$

$$V_{cs} = \frac{1}{C_s}\int_{t_0}^{t} i_{in}\,dt + V_{cs}(t_0)$$

$$\hookrightarrow C_s = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} \longrightarrow \frac{1}{C_s} = \sum_{i=1}^{\wedge} \frac{1}{C_i}$$

Parallel Capacitors





$$i_c = C\frac{dV_c}{dt}$$

$$i_{in} = i_{c_1} + i_{c_2} + i_{c_3}$$

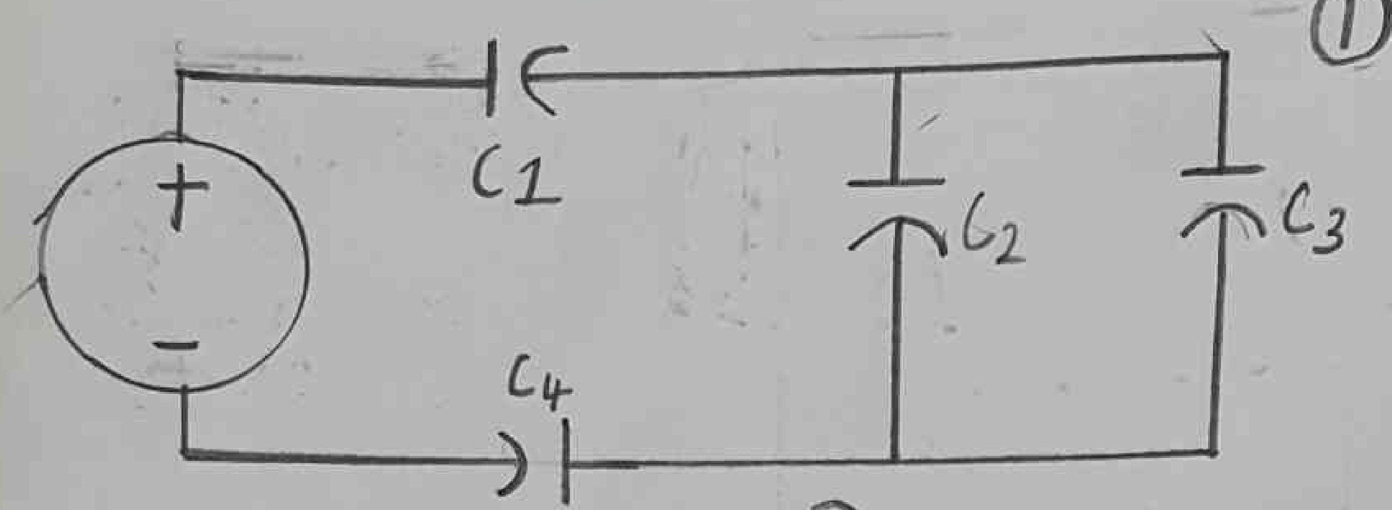
$$\rightarrow C_1\frac{dV_{in}}{dt} + C_2\frac{dV_{in}}{dt} + C_3\frac{dV_{in}}{dt}$$

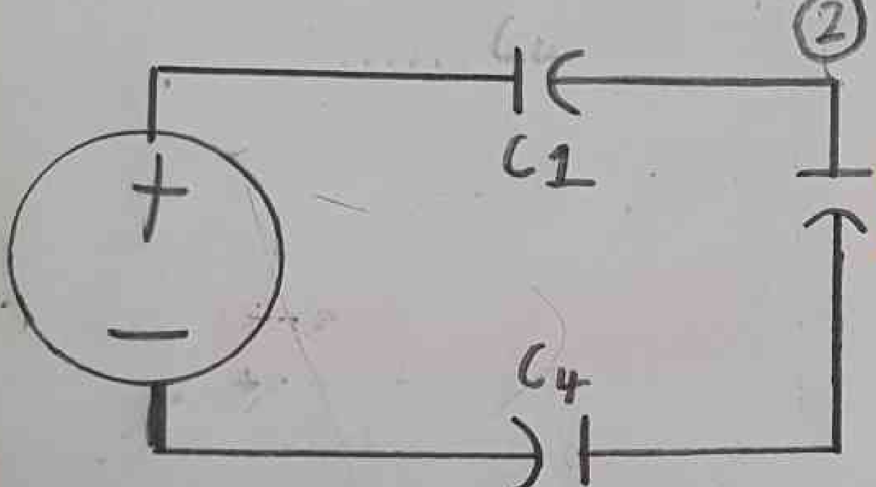
$$i_{in} = (C_1 + C_2 + C_3)\frac{dV_{in}}{dt}$$

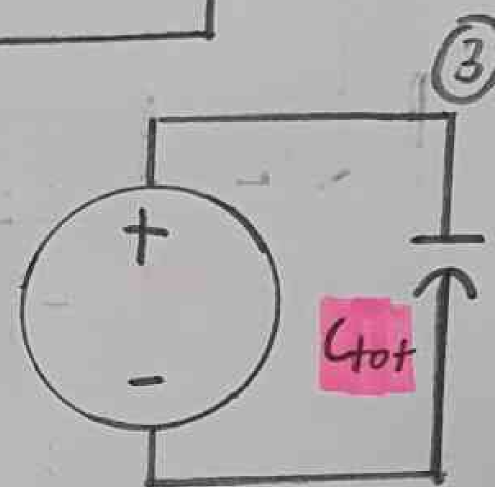
$$i_{in} = C_p\frac{dV_{in}}{dt}$$

$$\rightarrow C_p = C_1 + C_2 + C_3 \longrightarrow C_p = \sum_{i=1}^{n} C_n$$

Series/Parallel combination of capacitors





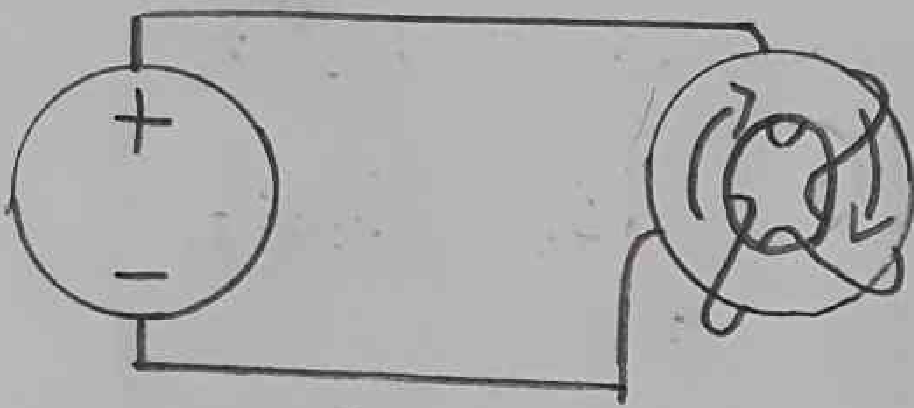


$$C_p=\sum_{i=1}^{n} C_i, \quad \frac{1}{C_s}=\sum_{i=1}^{n}\frac{1}{C_i}$$

$$C_p = C_2 + C_3$$

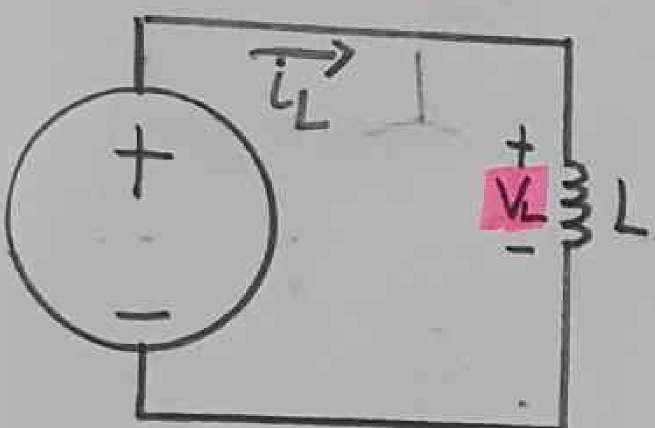
$$\hookrightarrow C_{tot} = \left[\frac{1}{C_1} + \frac{1}{(C_2+C_3)} + \frac{1}{C_4}\right]^{-1}$$

Definition of Inductors



$$\boxed{V_L = L\frac{di}{dt}}$$

Voltage across an Inductor



$$V_L = L\frac{di}{dt} \longrightarrow di = \frac{V_L}{L}dt \longrightarrow i(t) = \int_{-\infty}^{t}\frac{V_L}{L}dt$$

$$\hookrightarrow i(t) = \frac{1}{L}\int_{-\infty}^{t} V_L\,dt \longrightarrow \boxed{i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} V_L(x)\,dx}$$

Current through an Inductor

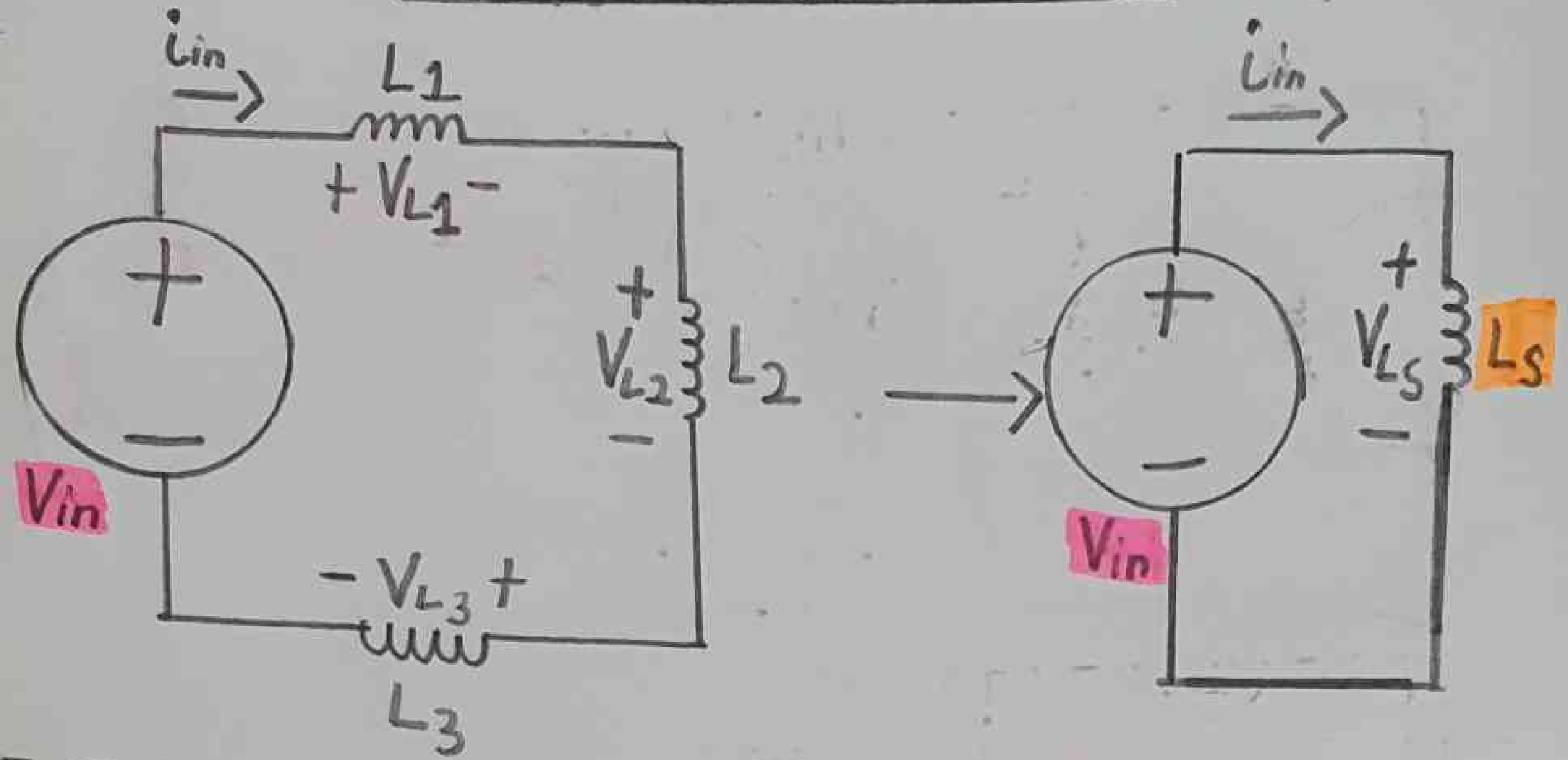
$$V_L = L\frac{di}{dt}, \quad P(t) = V(t)\,i(t)$$

$$P_L(t) = L\frac{di}{dt}i, \quad W_L(t) = \int_{-\infty}^{t} L\frac{di(x)}{dx}i(x)\,dx$$

$$\hookrightarrow W_L(t) = L\int_{-\infty}^{t} i(x)\,di(x)$$

$$\boxed{= \frac{1}{2}L\,i^2(t)\ J} \longrightarrow \text{Energy Stored in a Inductor}$$

Series Inductors



$$V_L = L \frac{di}{dt}$$

$$V_{in} = V_{L_1} + V_{L_2} + V_{L_3}$$

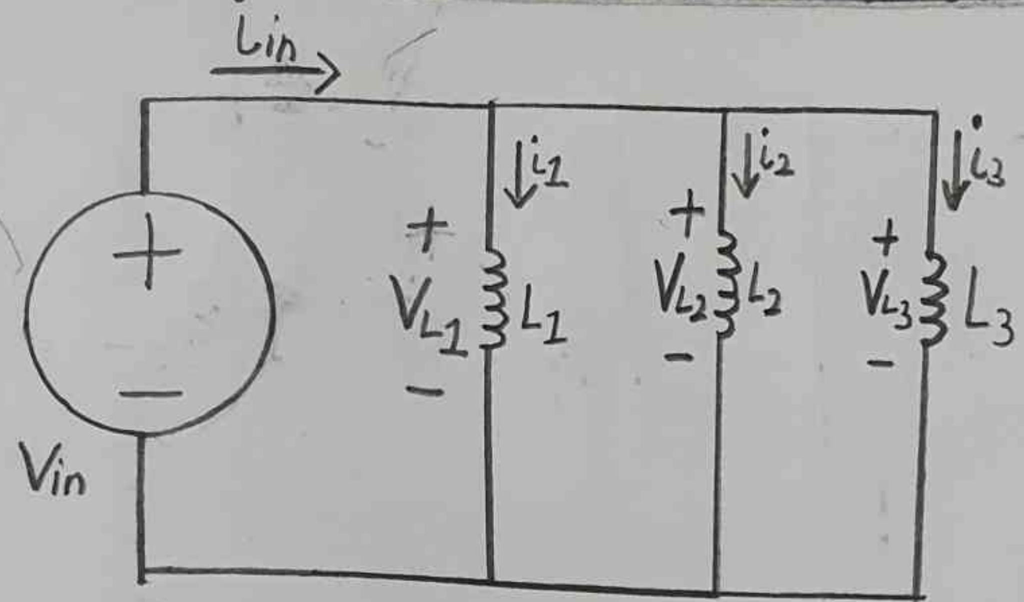
$$= L_1 \frac{di_{in}}{dt} + L_2 \frac{di_{in}}{dt} + L_3 \frac{di_{in}}{dt}$$

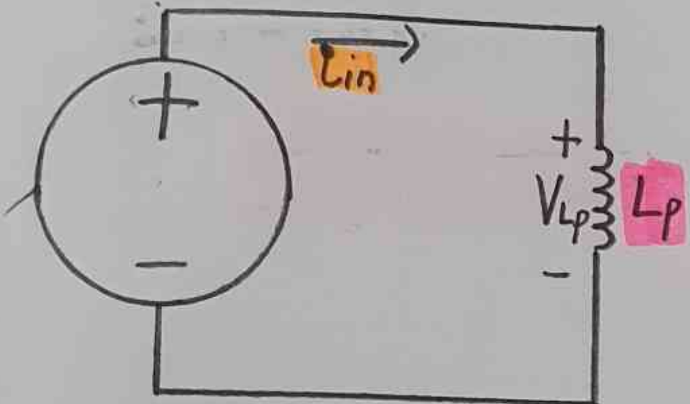
$$= (L_1 + L_2 + L_3) \frac{di_{in}}{dt}$$

$$V_{in} = L_S \frac{di_{in}}{dt}$$

$$L_S = L_1 + L_2 + L_3 \longrightarrow L_S = \sum_{i=1}^{n} L_i$$

Parallel Inductors





$$i_L = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} V_L(x)\,dx$$

$$i_{in} = i_1 + i_2 + i_3$$

$$\hookrightarrow \left[i_1(t_0) + \frac{1}{L_1}\int_{t_0}^{t} V_{L_1}(x)\,dx\right] + \left[i_2(t_0) + \frac{1}{L_2}\int_{t_0}^{t} V_{L_2}(x)\,dx\right]$$

$$+ \left[i_3(t_0) + \frac{1}{L_3}\int_{t_0}^{t} V_{L_3}(x)\,dx\right], \quad i_{in}(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

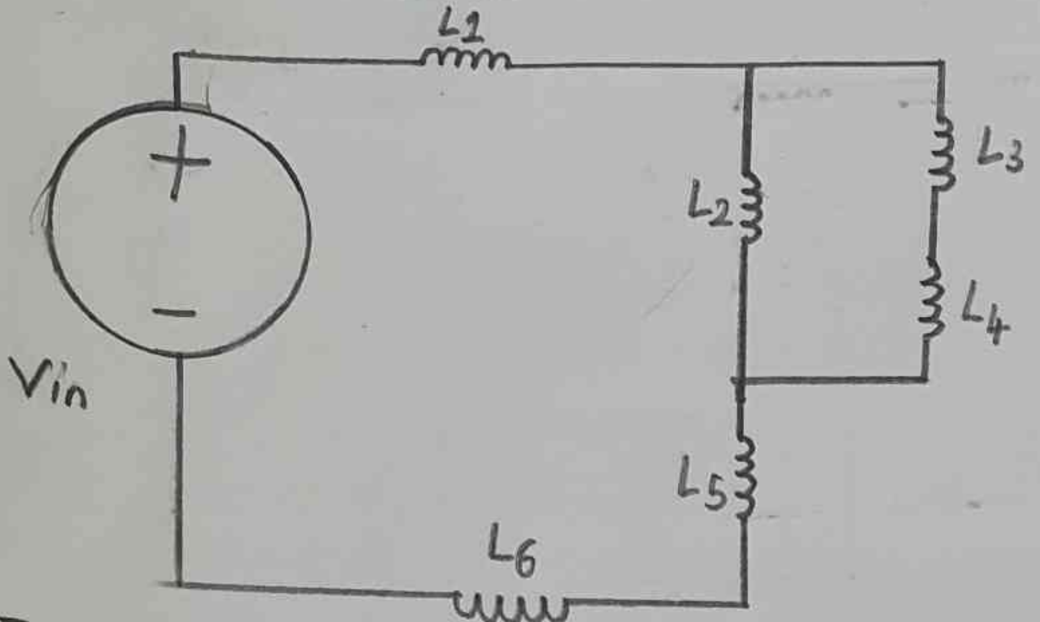
$$= i_{in}(t_0) + \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} V_{in}(x)\,dx$$

$$i_{in}(t) = i_{in}(t_0) + \frac{1}{L_P}\int_{t_0}^{t} V_{in}(x)\,dx$$

$$L_P = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

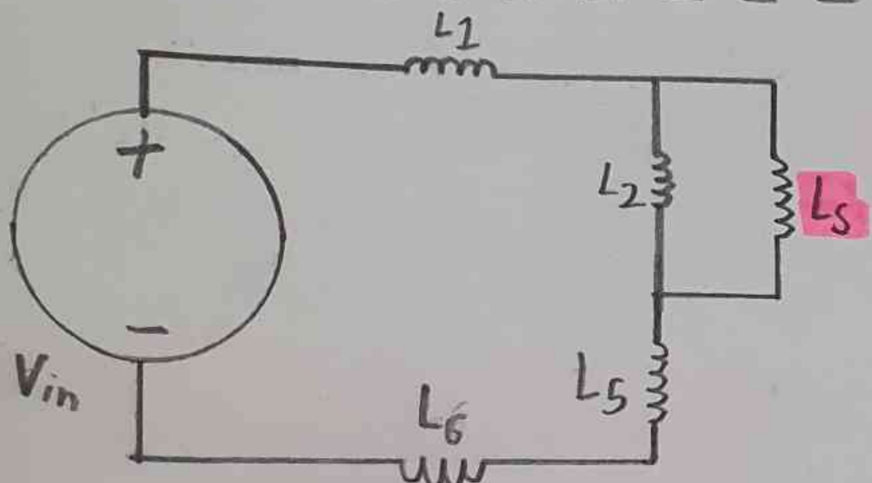
$$\frac{1}{L_P} = \sum_{i=1}^{n} \frac{1}{L_i}$$

Series / Parallel combination of Inductors

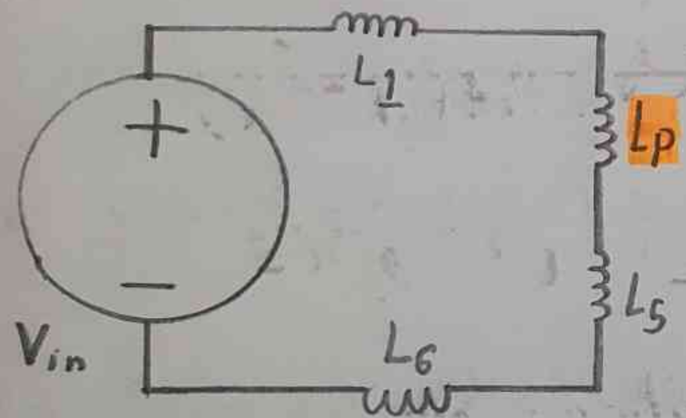


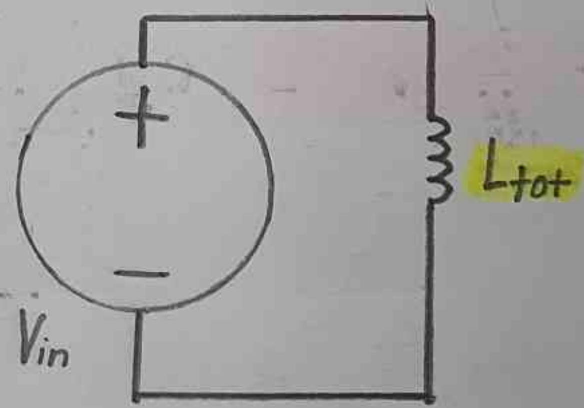
$$L_s = \sum_{i=1}^{n} L_i$$

$$\frac{1}{L_p} = \sum_{i=1}^{n} \frac{1}{L_i}$$



$$L_s = L_3 + L_4$$

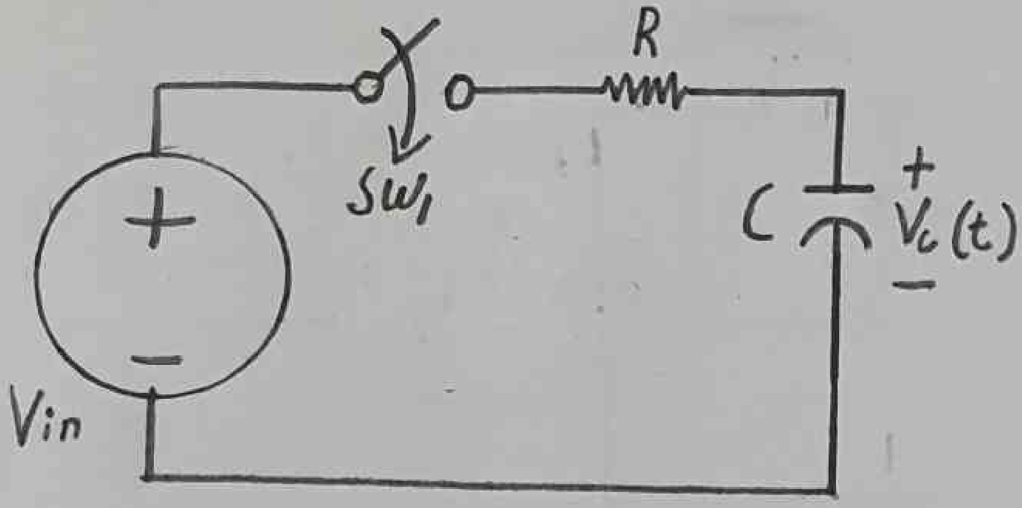


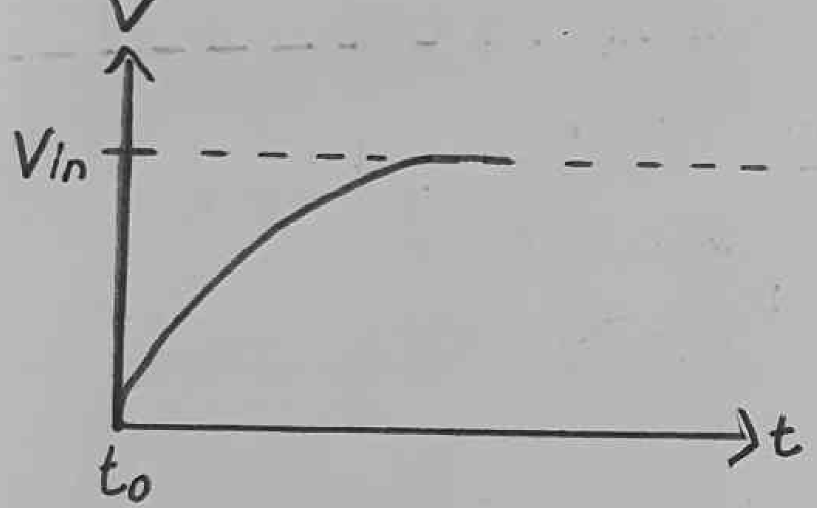


$$L_p = \left(\frac{1}{L_2} + \frac{1}{L_s}\right)^{-1}$$

$$L_{tot} = L_1 + L_p + L_5 + L_6$$

First-Order Transient Circuits - Introduction





$$i_c = C\frac{dv}{dt}, \quad V_L = L\frac{di}{dt}$$

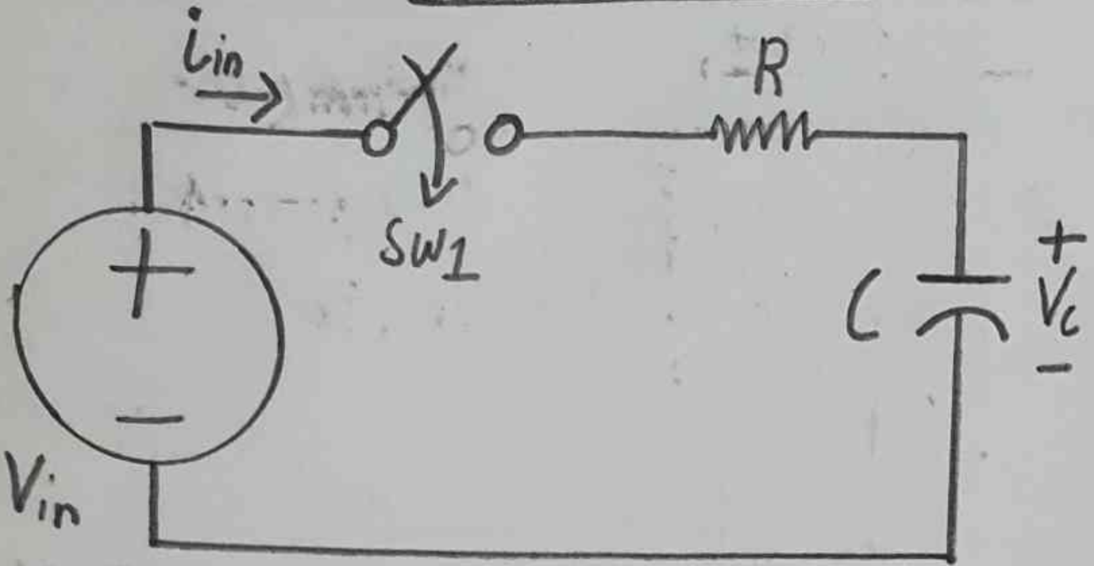
$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

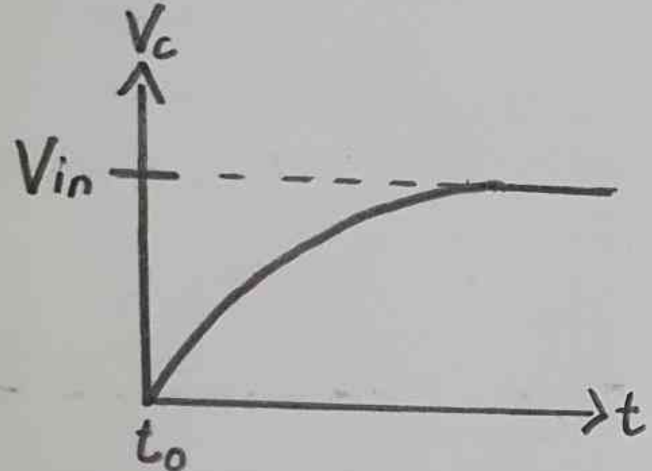
$$x(t) = k_1 + k_2 e^{-t/\tau}$$

k_1 = Steady State

τ = time Constant

Series RC Circuits





$$\frac{dx(t)}{dt} + a\,x(t) = f(t)$$

$$x(t) = k_1 + k_2 e^{-t/\tau}$$

$$V_c = k_1 + k_2 e^{-t/\tau}$$

$$i_c = C\frac{dv}{dt}$$

$$\hookrightarrow \frac{V_{in}(t) - V_c(t)}{R} = C\frac{dV_c(t)}{dt} \longrightarrow \frac{dV_c(t)}{dt} + \frac{V_c(t)}{RC} = \frac{V_{in}(t)}{RC}$$

$$\hookrightarrow \frac{d}{dt}\left(k_1 + k_2 e^{-t/\tau}\right) + \frac{k_1 + k_2 e^{-t/\tau}}{RC} = \frac{V_{in}(t)}{RC}$$

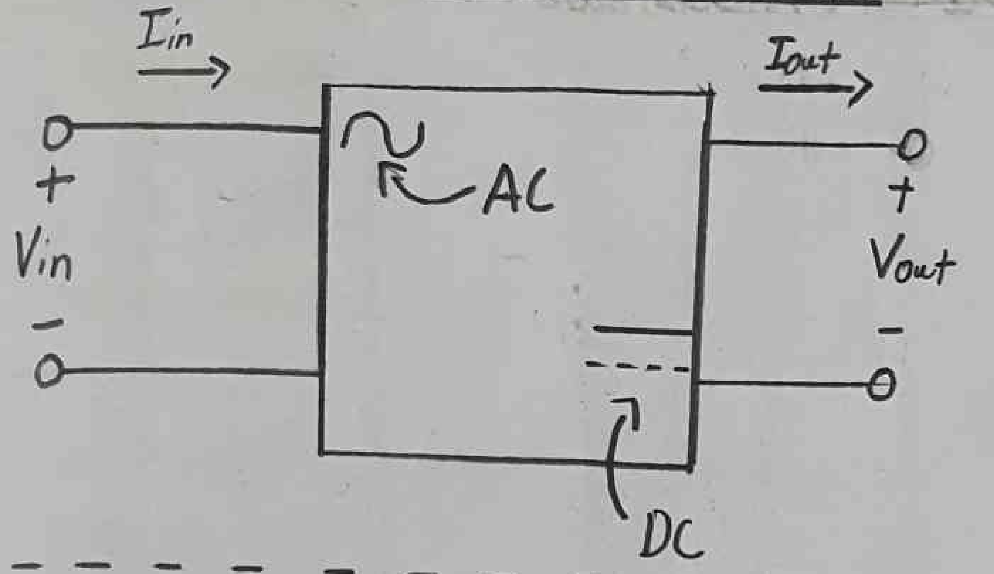
$$\hookrightarrow -\frac{k_2 e^{-t/\tau}}{\tau} + \frac{k_1}{RC} = \frac{V_{in}(t)}{RC} \longrightarrow \frac{k_1}{\cancel{RC}} = \frac{V_{in}}{\cancel{RC}} \longrightarrow \boxed{k_1 = V_{in}}$$

$$\frac{k_2 e^{-t/\tau}}{\tau} = \frac{k_2 e^{-t/\tau}}{RC} \longrightarrow \boxed{\tau = RC}$$

$$V_c(t_0) = 0 = k_1 + k_2 \underbrace{e^{-t_0/\tau}}_{=1} \longrightarrow 0 = V_{in} + k_2 \longrightarrow \boxed{k_2 = -V_{in}}$$

$$\boxed{\hookrightarrow V_c(t) = V_{in} - V_{in} e^{-t/RC}}$$

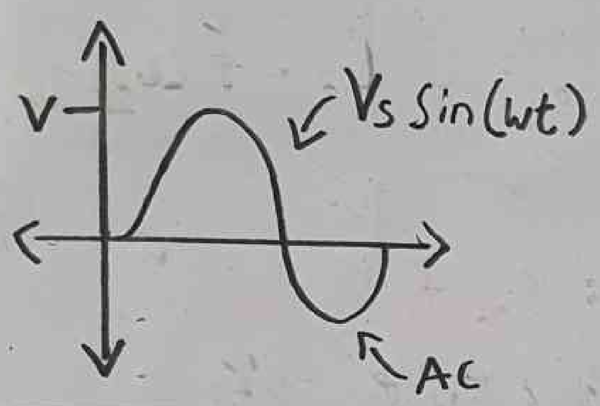
Ideal Rectifiers

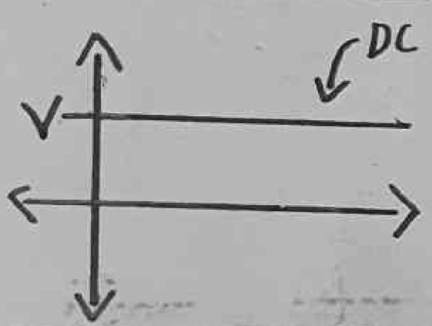


$V_{out\,pkpk} = 0V$

$I_{out\,pkpk} = 0A$

$P_{in} = P_{out}$





Q1. What is the Voltage ripple at the out Put of an ideal rectifier?

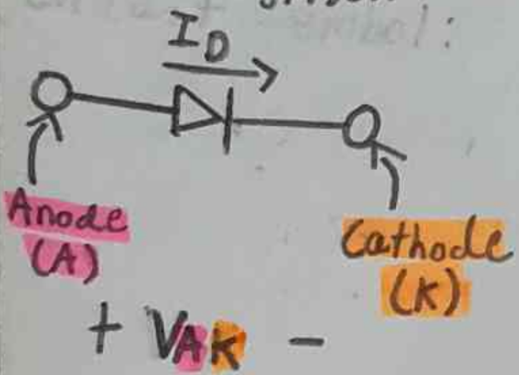
Answer = Zero because the output voltage of an ideal rectifier is a Perfect DC voltage. In techical terms, the voltage only has a 0Hz component.

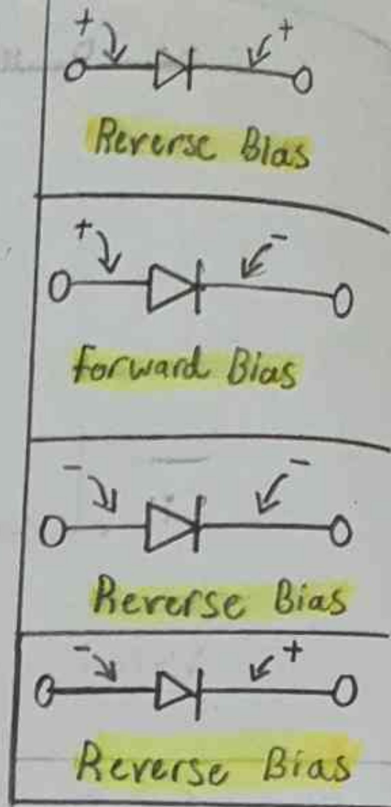
Q2. What is the efficency of and ideal rectifier?

Answer = 100% because ideally, there a no losses in an ideal rectifier, which means that the input Power and output Power are equal, hence the 100% efficency.

Review of the Diode

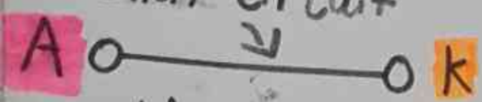
Circuit Symbol:





If Voltage from A to K is positive:

Short circuit



$V_{AK} > 0$ Forward Bias

If voltage from A to K is negative:

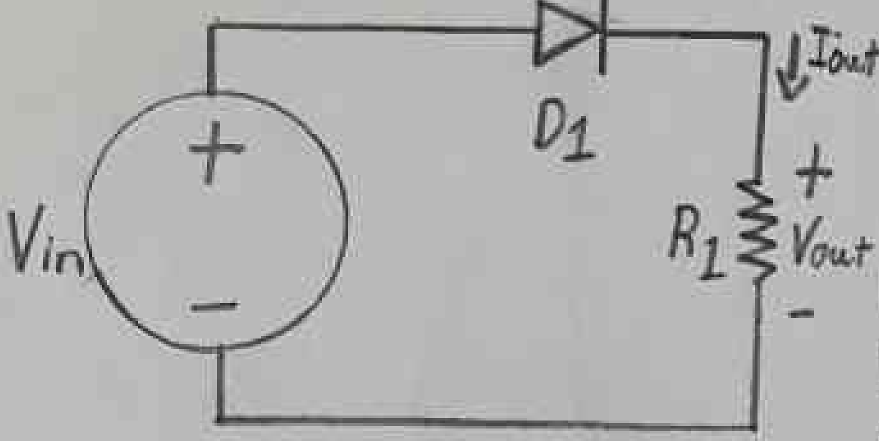
open circuit

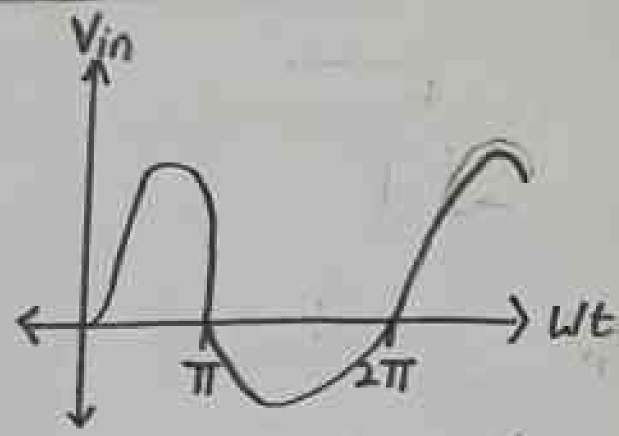
$V_{AK} < 0$ Reverse Bias

Q1. When a Diode is OFF, the anode - cathode voltage (V_{AK}) is:

Answer: Either zero or negative because for a diode to be on, it needs to be forward biased, (V_{AK} needs to be positive)

Half-Wave Rectifiers with Resistive Load

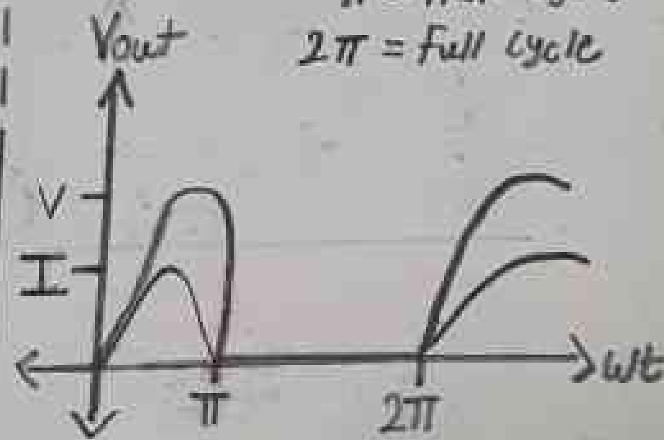




$V = IR$

↳ $I = V/R$

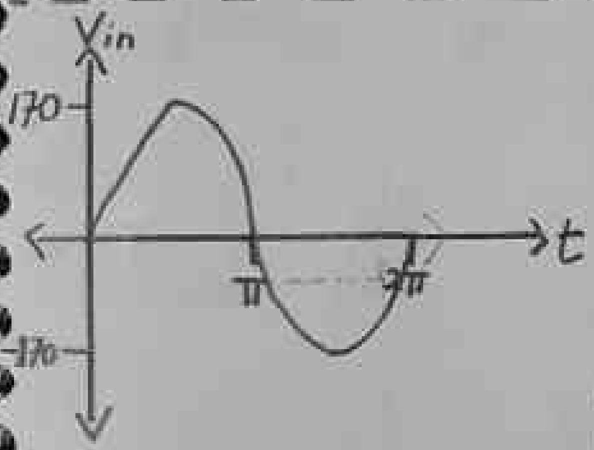
π = Half cycle
2π = Full cycle

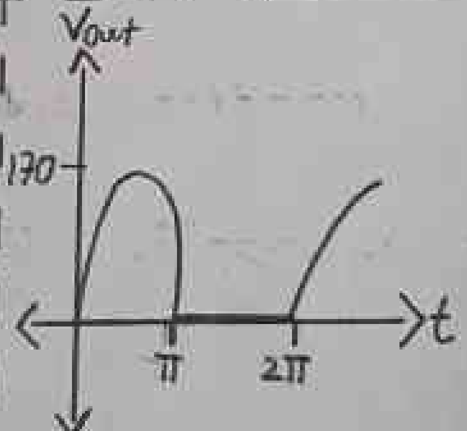


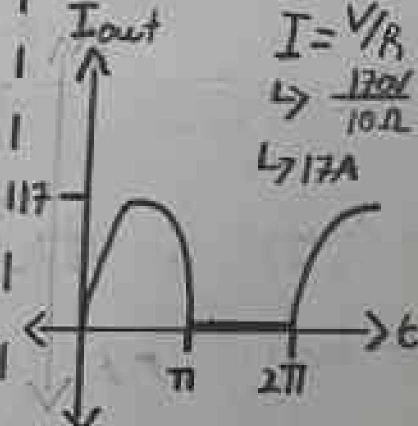
$V_{in} = 120 V_{rms}$ at 60 Hz

$V_{in} = 170 \sin(377t)\, V$

$120 V_{rms} \sqrt{2} = 170V$; $2\pi f = 2\pi(60) = 377$







$I = V/R$
↳ $\frac{170V}{10\Omega}$
↳ 17A

Average of output voltage

$$\langle V_{out} \rangle = \frac{1}{T}\int_0^T V_{in}\,dt \rightarrow \frac{1}{16.6}\int_0^{8.3} 170 \sin(377t)\,dt$$

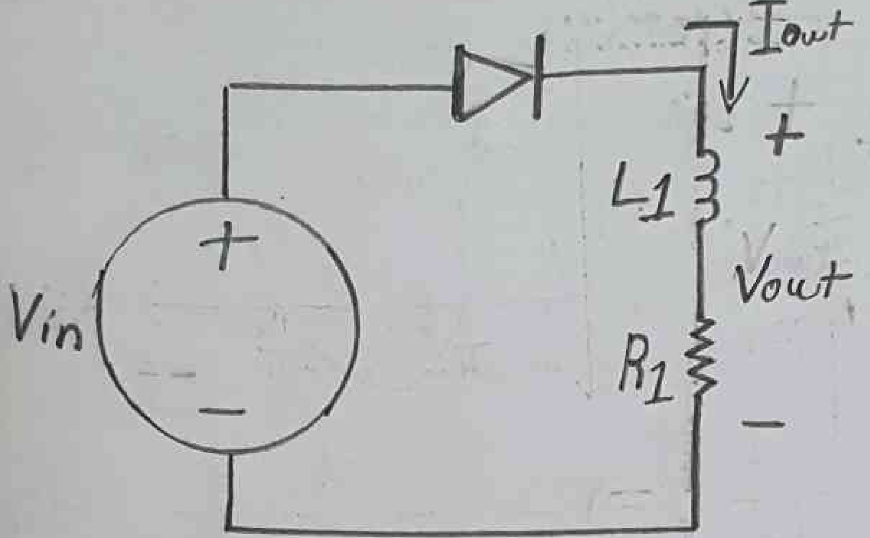
$T = \frac{1}{60} = 16.6ms$

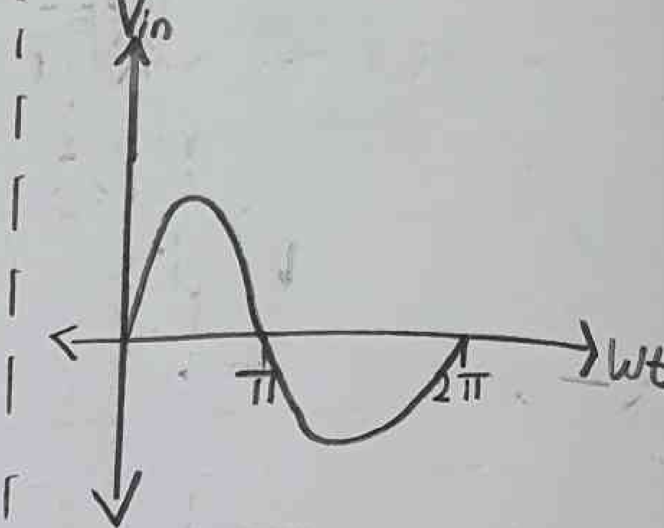
$= 0.319(170) = 54.23V$

$$\langle V_{out} \rangle = \frac{\text{Voltage Peak}}{\pi}$$

↳ $\frac{170V}{\pi} = 54.23V$

Half - Wave Rectifiers With Inductive Load



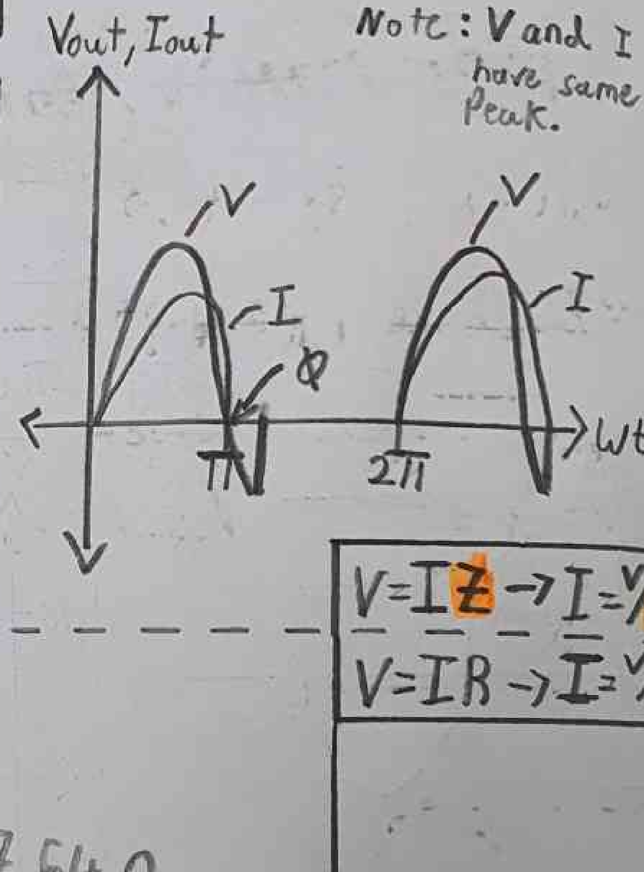


$$V_L = L\frac{di}{dt} \rightarrow \frac{di}{dt} = \frac{V_L}{L}$$

$$Z = \sqrt{R^2 + (\omega t)^2}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Note: V and I have same Peak.



$V_{in} = 170\sin(377t)V$, $60H_z$

$L_1 = 20mH$, $R_1 = 10\Omega$

Millahenerics

$$V = IZ \rightarrow I = V/Z$$

$$V = IR \rightarrow I = V/R$$

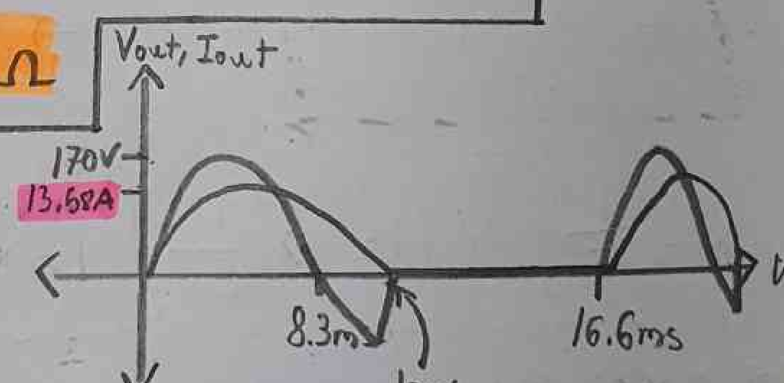
$$X = \omega L = 377(20mH \times 10^{-3}\Omega) = 7.54\Omega$$

$$Z = \sqrt{10^2 + 7.54\Omega^2} = 12.52\Omega$$

$$I_{out_{PK}} = 170/12.52 = 13.58A$$

$$\phi = \tan^{-1}\left(\frac{X}{10\Omega}\right) = 37.02^\circ$$

$$\hookrightarrow 16.6ms\left(\frac{37.02^\circ}{360^\circ}\right) = 1.7ms$$

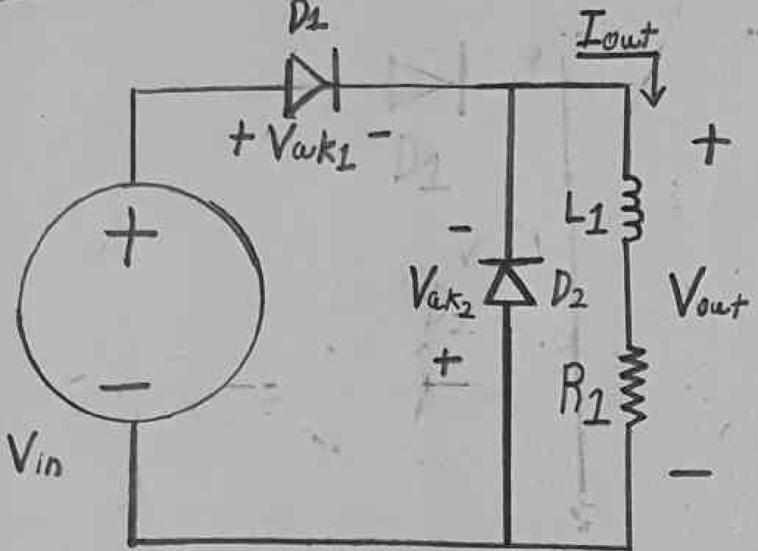


$$\langle V_{out}\rangle = \frac{1}{16.6 \times 10^{-3}}\int_0^{10\times10^{-3}} 170\sin(377t)\,dt = 0.289(170)$$

$= 49.13V$

$\hookrightarrow 54.23V$

The freewheeling Diode

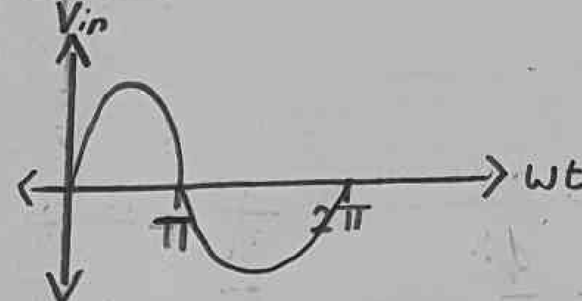


State 1:

D_1 is ON, D_2 is OFF

State 2:

D_1 is OFF, D_2 is ON



$V_{in} = 170 \sin(377t)\,V$, 60Hz

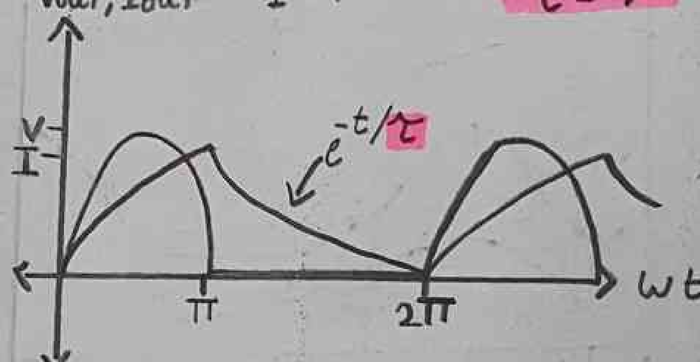
$L_1 = 20mH$, $R_1 = 10\Omega$

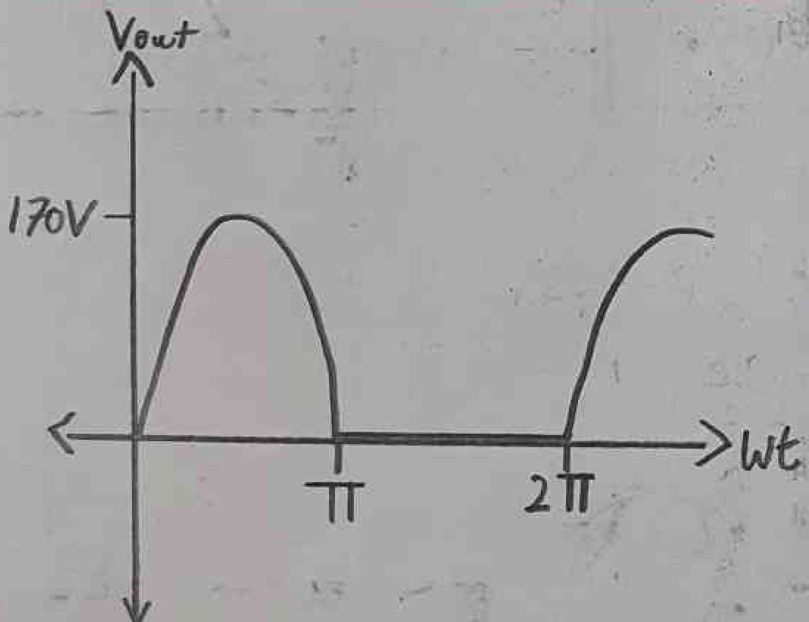
$Z = 12.52\,\Omega$, $Z = \sqrt{R^2 + (\omega t)^2}$

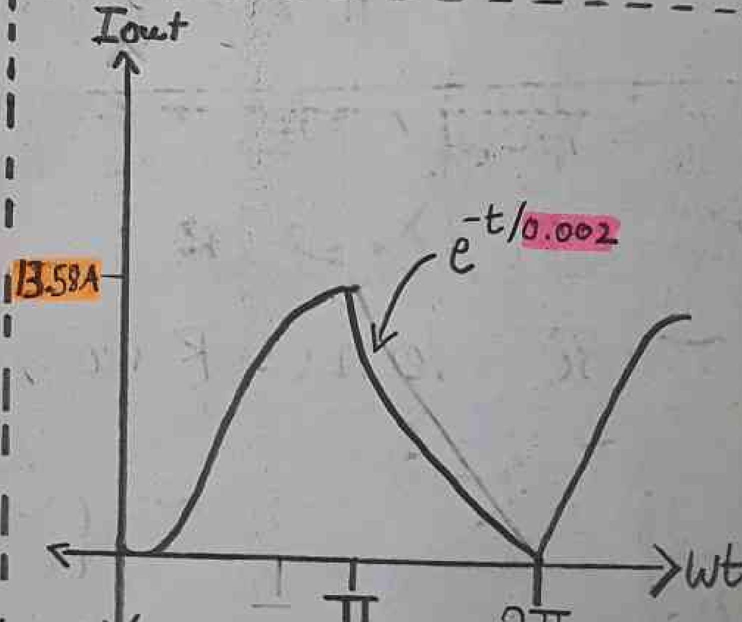
$I_{out_{pk}} = 170V / 12.52\Omega = 13.58A$

$\tau = L/R = 20mH \times 10^{-3} / 10\Omega = 0.002$

$I = V/Z$ $\tau = L/R$



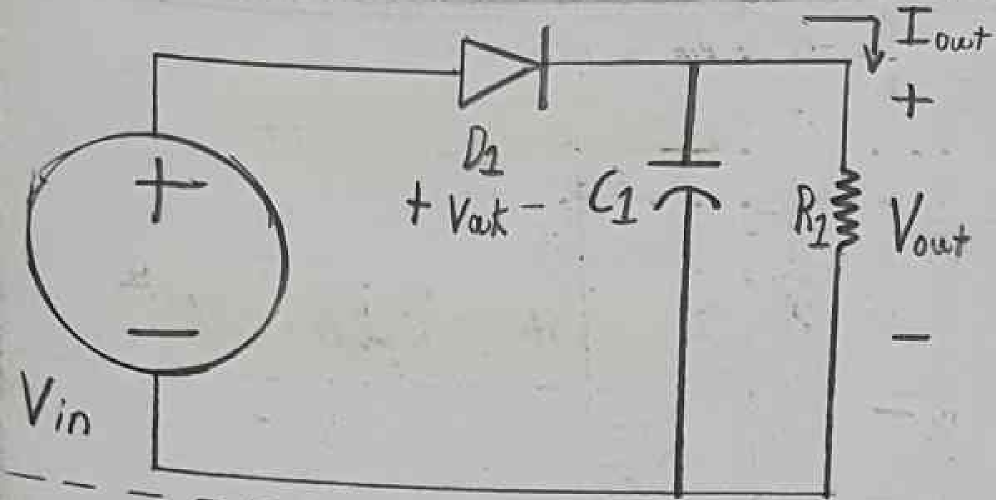


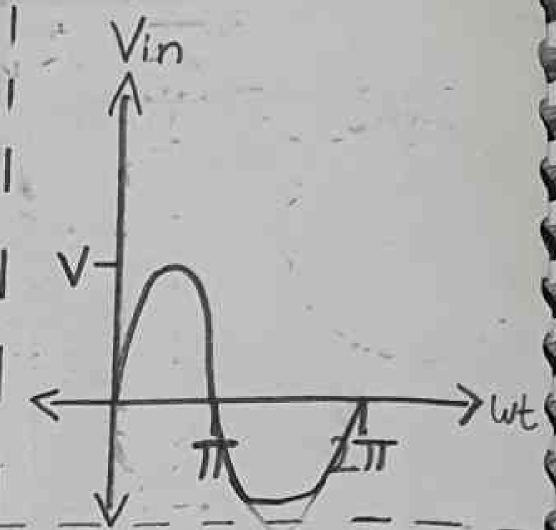


$\langle V_{out} \rangle = 49.13V$

$\hookrightarrow 54.23V$

Half-Wave Rectifiers with capacitive filters

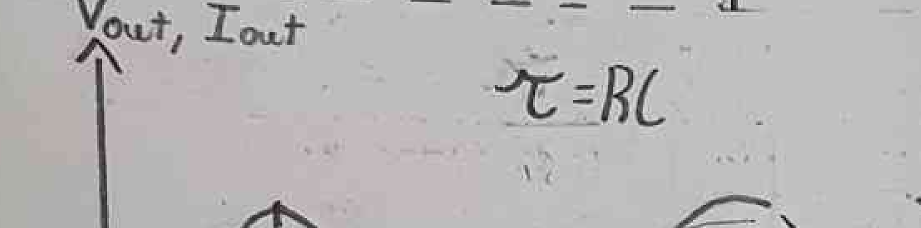




$i_c = C\dfrac{dv}{dt} \longrightarrow \dfrac{dv}{dt} = \dfrac{i_c}{C}$

wt_1 to $\pi/2$, D_1 is ON

$\pi/2$ to $2\pi + wt_1$, D_1 is off



$\tau = RC$

wt_1 to $\pi/2 = V_s \sin(wt)$

$\pi/2$ to $2\pi + wt_1 = e^{-t/\tau}$

$t = 16.6ms \times 10^{-3} + t_1$

$170\sin(337(16.6 \times 10^{-3} + t_1))$

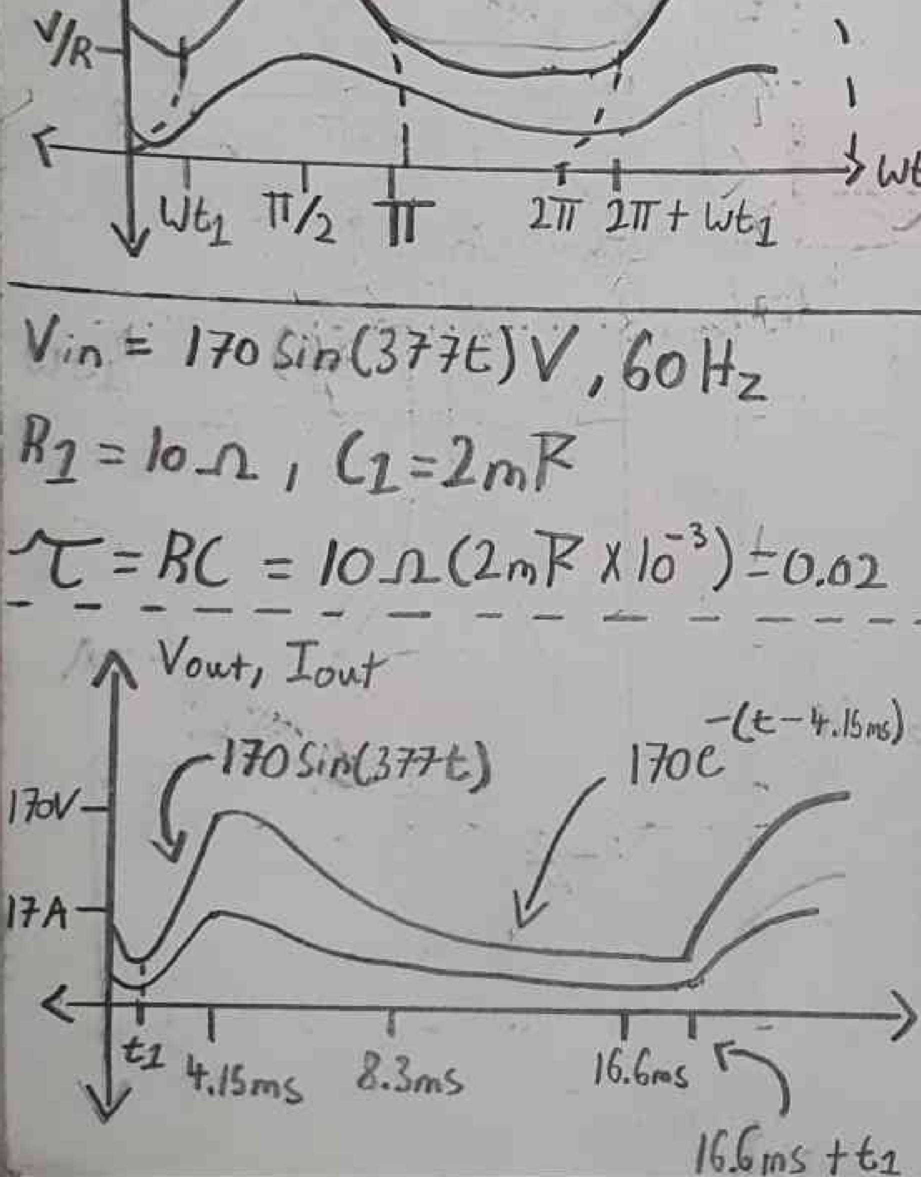
$\hookrightarrow 170e^{-(t-4.15ms)/0.02}$

$\hookrightarrow t_1 = 1.45\ ms$

$V_{in} = 170\sin(377t)V$, 60 Hz

$R_1 = 10\Omega$, $C_1 = 2mF$

$\tau = RC = 10\Omega(2mF \times 10^{-3}) \doteq 0.02$



$\langle V_{out} \rangle = \dfrac{1}{16.6 \times 10^{-3}} \left[\int_{1.45 \times 10^{-3}}^{4.15 \times 10^{-3}} 170\sin(377)dt + \int_{4.15 \times 10^{-3}}^{(16.6+1.45) \times 10^{-3}} 170e^{-(t-4.15 \times 10^{-3})/0.02}\, dt \right]$

Sin Wave Integral

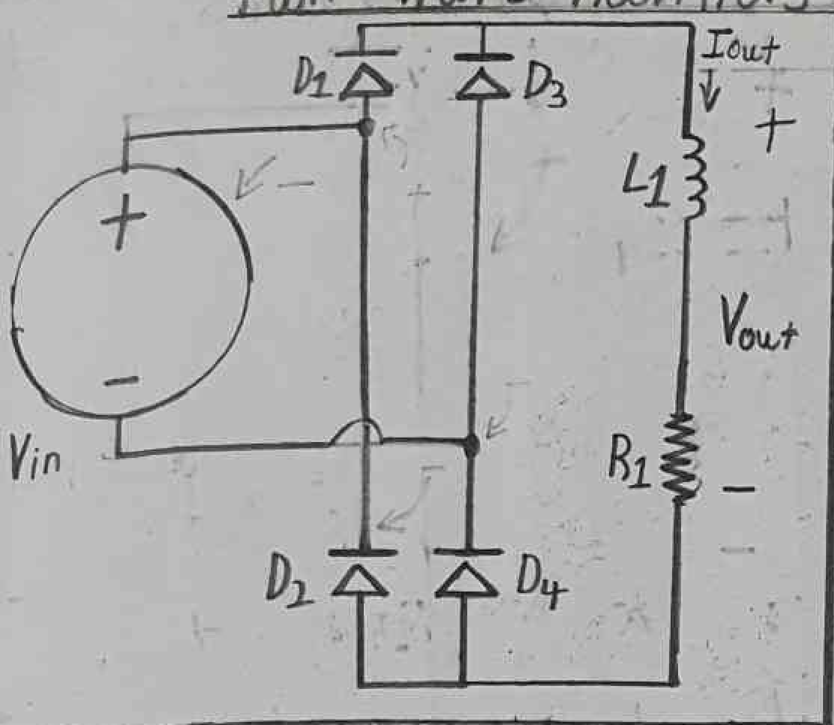
Decaying exponential function Integral

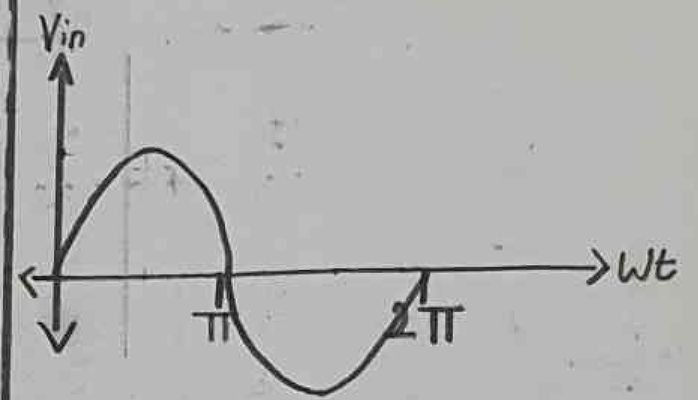
$= 0.74\ Vin$

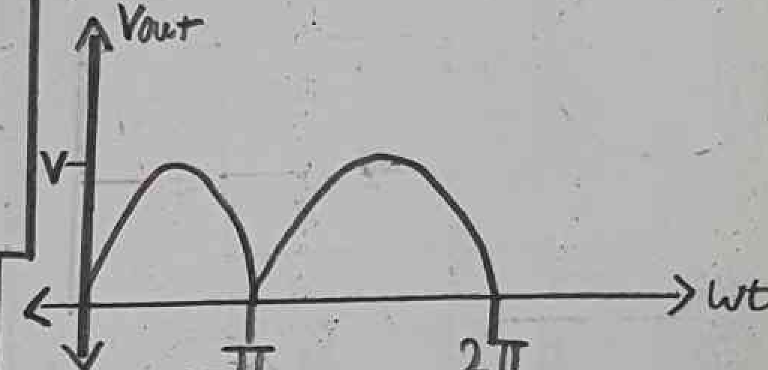
$= 0.74(170)$

$= 125.8V$

Full-Wave Rectifiers with Inductive Load







State 1, $0 \rightarrow \pi$

D_1/D_4 ON, D_2/D_3 Off

State 2, $\pi \longrightarrow 2\pi$

D_1/D_4 OFF, D_2/D_3 ON

$V_{in} = 170 \sin(377t)\,V$

$L_1 = 20mh$, $R_1 = 10\Omega$

$V_L = L\frac{di}{dt}$, $V_{out} = V_{L_1} + I_{out} \times R$

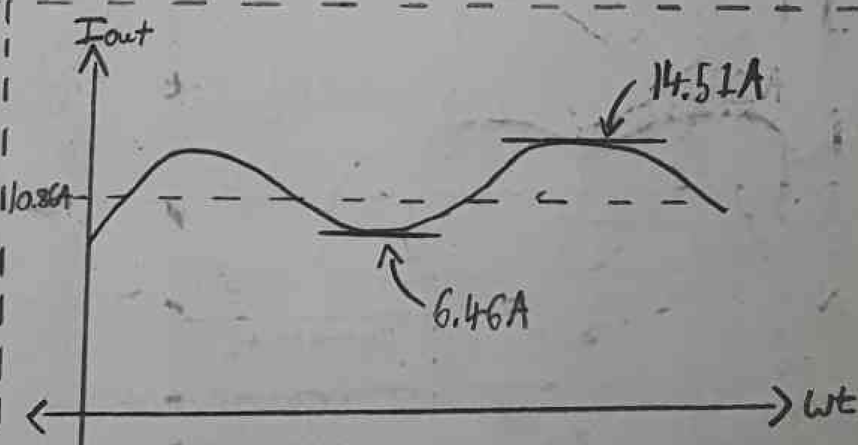
$V_{out} = |170 \sin(377t)|$

$|170 \sin(377t)| = 0.02\frac{di}{dt} + i\,(10)$

$$\langle V_{out}\rangle = \frac{1}{16.6\times10^{-3}}\left[\int_0^{8.3\times10^{-3}} 170\sin(377t)\,dt + \int_{8.3\times10^{-3}}^{16.6\times10^{-3}} -170\sin(377t)\,dt\right] = 0.639\,(170V) = 108.64V$$

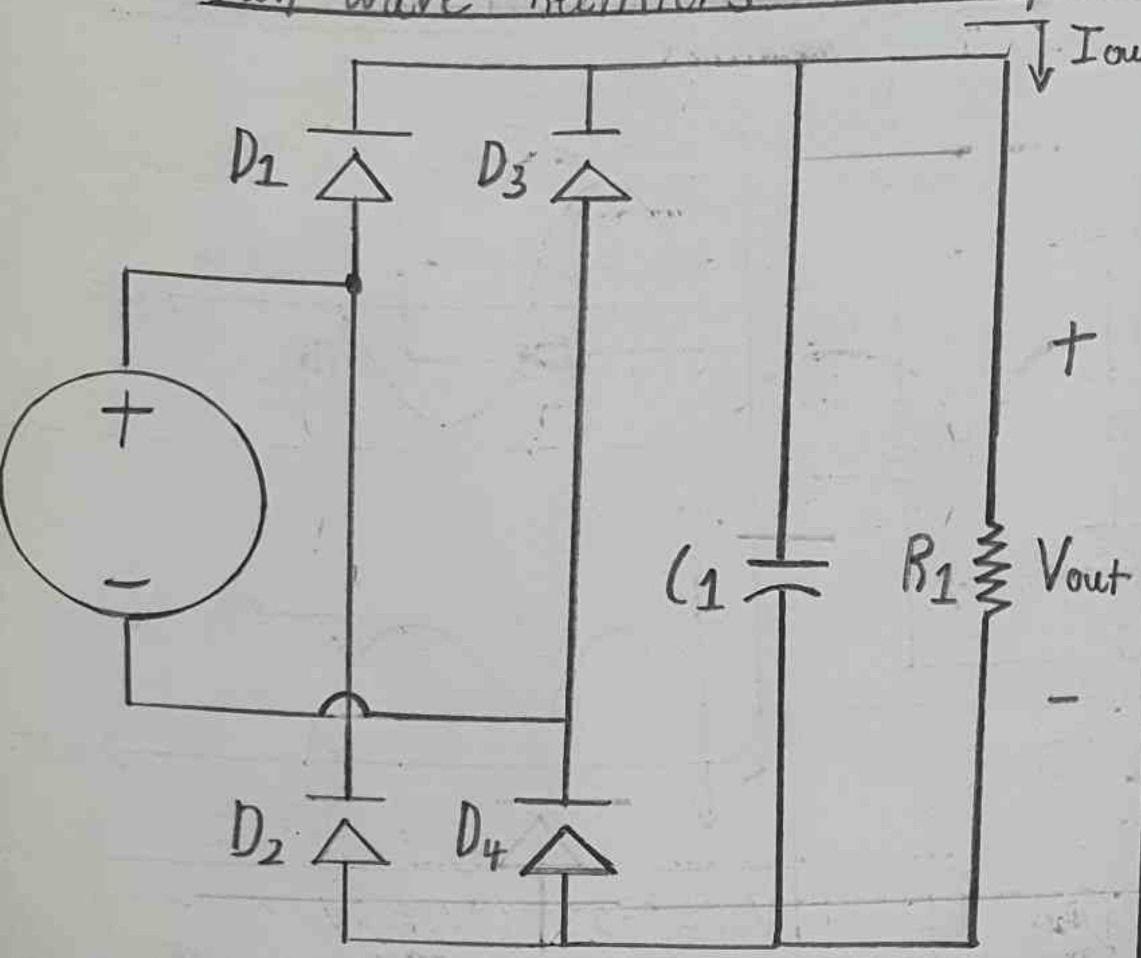
$\langle I_{out}\rangle = V_{out}/R = \frac{108.64V}{10\Omega}$

$\Rightarrow I_{out} = 10.86A$



$I_{out_{pkpk}} \cong 8A$

Full wave Rectifiers with capacitive filters



$R_1 = 10\Omega$, $C_1 = 2mF$

$V_{in} = 170\sin(377t)\,V$

t_1 to $\pi/2$:

$V_{out} = |170\sin(377t)|$

$\pi/2$ to $\pi + t_1$:

$V_{out} = 170e^{-(t-4.5ms)/0.02}$

$\tau = RC = 10(2\times10^{-3}) = 0.02$

$2\pi = 16.6ms$, $\pi = 8.3ms$,

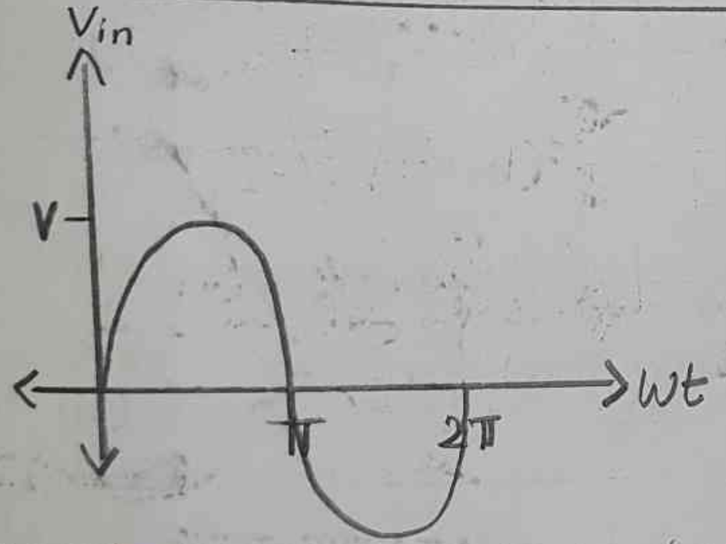
$\pi/2 = 4.15ms$

$|170\sin(377t)| = 170e$

$\hookrightarrow 170e^{-(t-4.15ms)/0.02}$,

Let $t = 8.3ms + t_1$

$\hookrightarrow t_1 \stackrel{\sim}{=} 2.2ms$

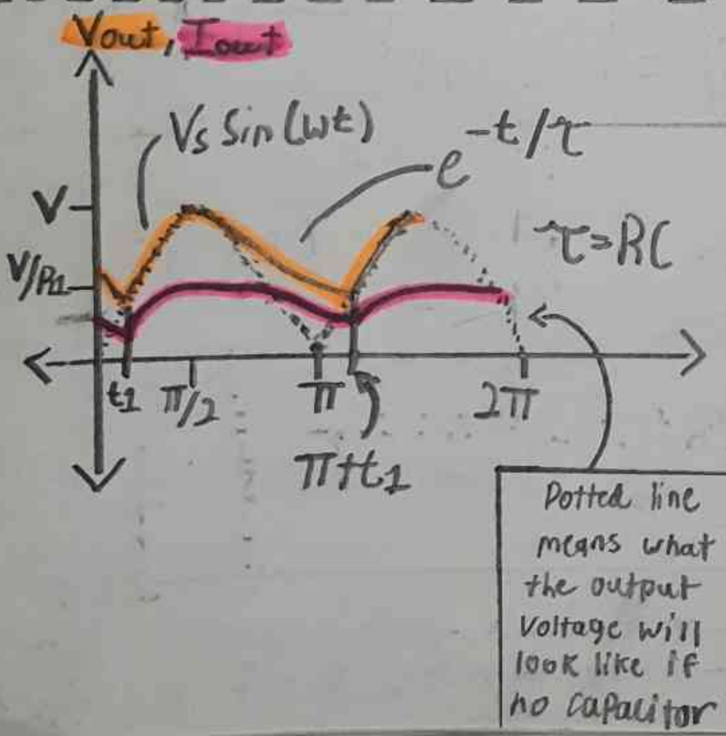


$\langle V_{out}\rangle = \frac{1}{16.6\times10^{-3}}\left[\int_{2.2ms}^{4.15ms} 170\sin(377t)\,dt + \int_{4.15ms}^{10.5ms} 170e^{-(t-4.15ms)/0.02}\right]$

Sine wave Integral

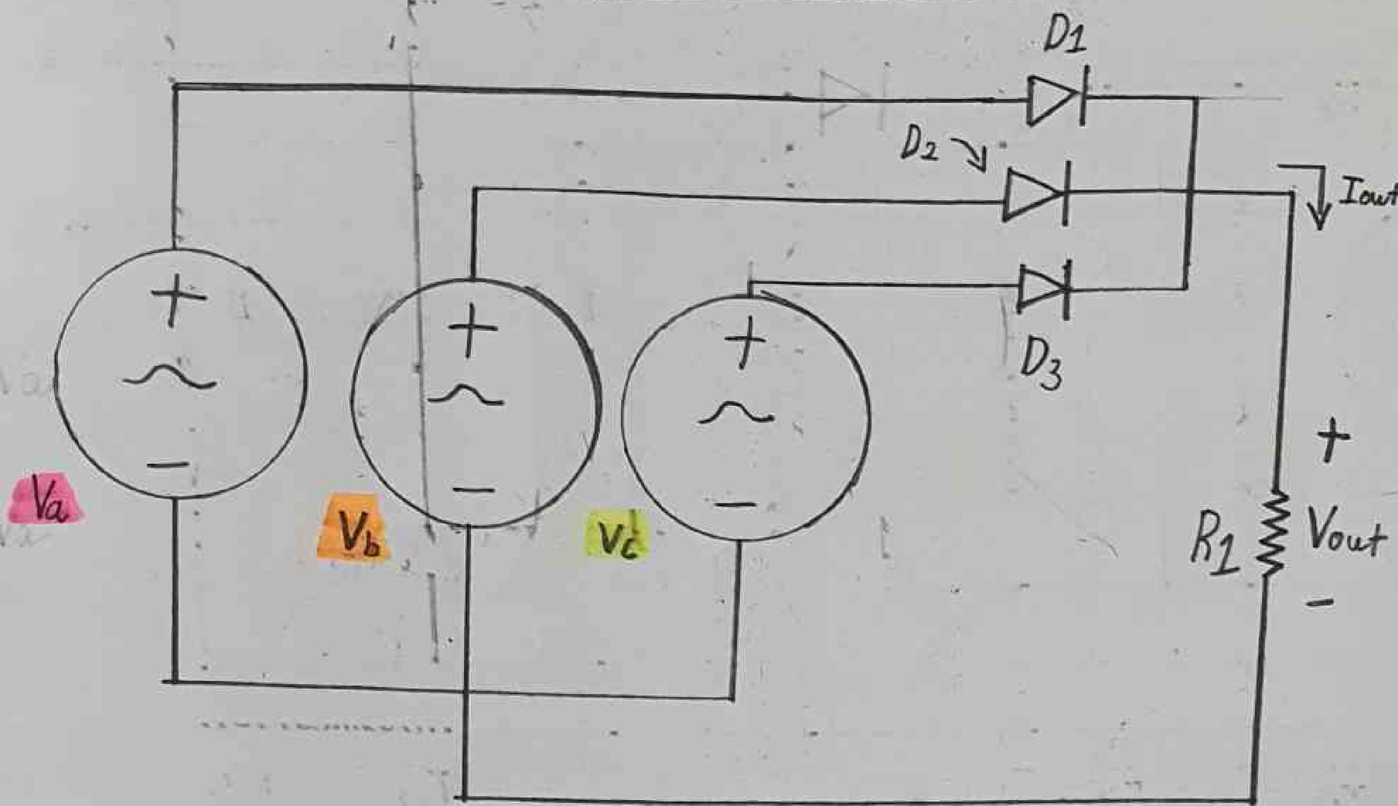
Decaying exponential function integral

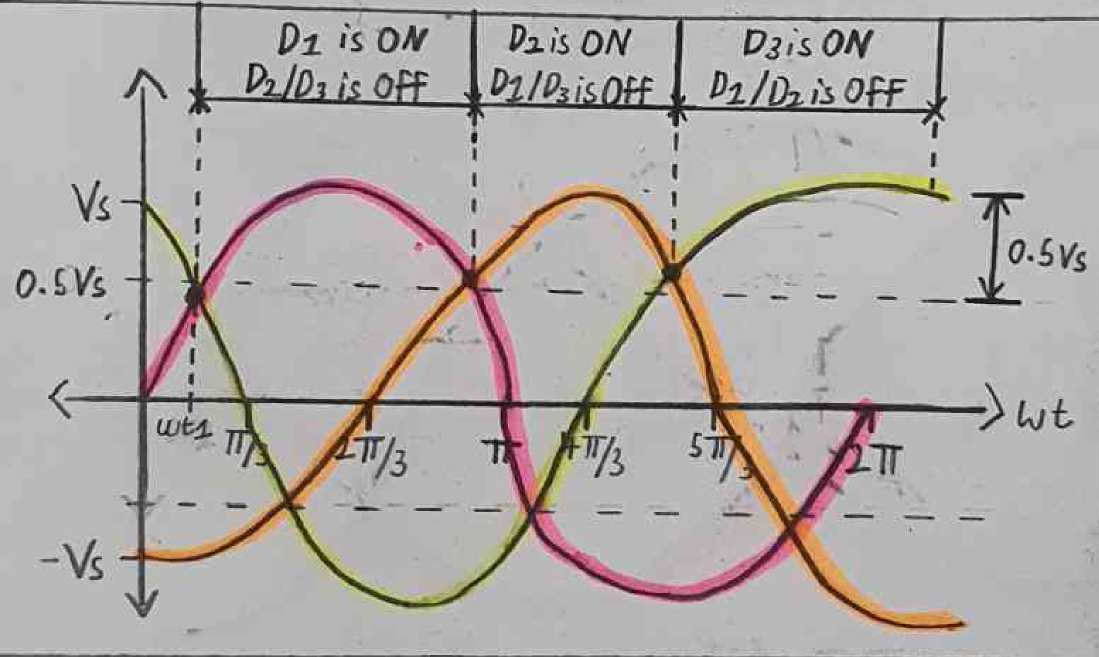
$\langle V_{out}\rangle = 0.869(170)V = 147.35V$



Half wave Rectifiers with Resistive Load :

- 3 Phase Rectifiers





$V_a = V_s \sin(\omega t)$

$V_b = V_s \sin(\omega t - 2\pi/3)$

$V_c = V_s \sin(\omega t + 2\pi/3)$

or

$V_c = V_s \sin(\omega t - 4\pi/3)$

$V_a = V_c$

$\hookrightarrow V_s \sin(\omega t) = V_s \sin(\omega t + 2\pi/3)$

$\hookrightarrow \omega t_1 = \pi/6$

$V_a = V_s \sin(\omega t_1) = 0.5 V_s$

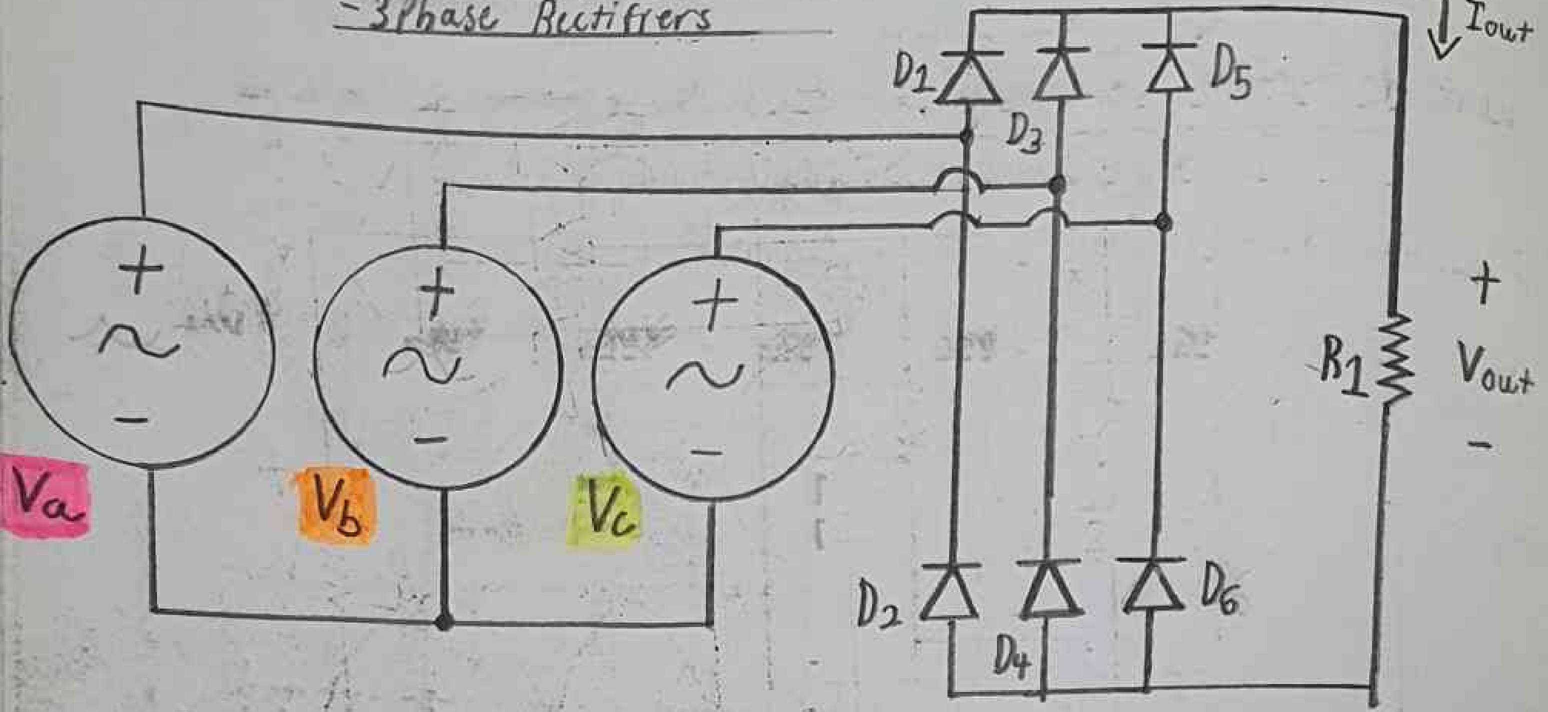
$\langle V_{out} \rangle = \frac{1}{\omega T} \int_0^T V_{out} \, d\omega t$

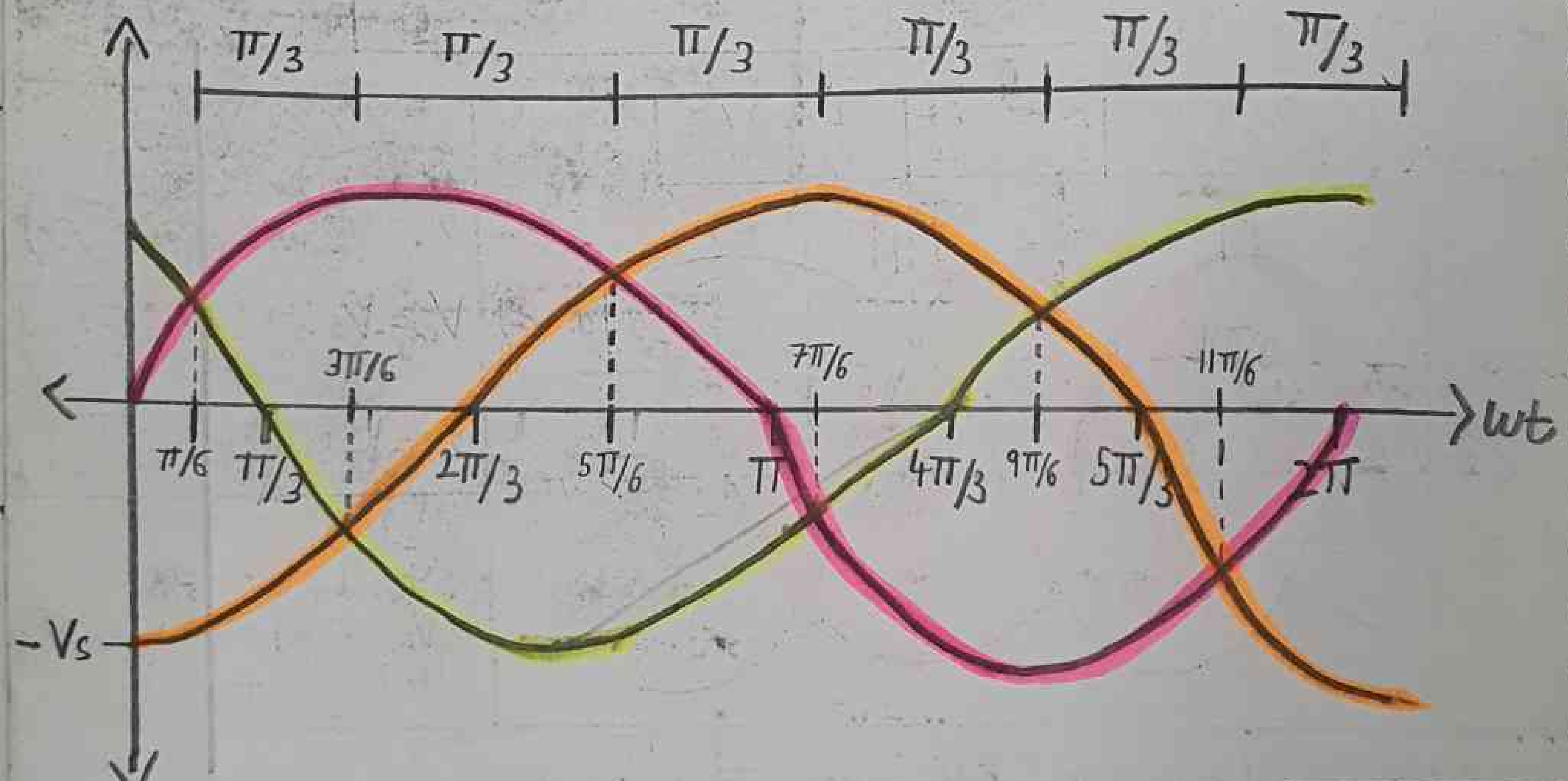
$$\langle V_{out} \rangle = \frac{1}{2\pi} \left[\int_0^{\omega t_1} V_s \sin(\omega t + 2\pi/3) \, d\omega t + \int_{\pi/6}^{2\pi/3 + \pi/6} V_s \sin(\omega t) \, d\omega t + \int_{2\pi/3 + \pi/6}^{4\pi/3 + \pi/6} V_s \sin(\omega t - 2\pi/3) \, d\omega t + \int_{4\pi/3 + \pi/6}^{2\pi} V_s \sin(\omega t + 2\pi/3) \, d\omega t \right]$$

$$\frac{V_s}{2\pi} (0.36602 + 1.73205 + 1.73205 + 1.36603) \cong 0.82699 V_s = \frac{3\sqrt{3}}{2\pi} V_s$$

Full-Wave Rectifiers with Inductive Load

-3Phase Rectifiers





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$V_a = V_s \sin(\omega t)$

$V_b = V_s \sin(\omega t - 2\pi/3)$

$V_c = V_s \sin(\omega t + 2\pi/3)$

0 to $\pi/6$, $V_{out} = V_c - V_b$, D_4/D_5 is ON

$\pi/6$ to $3\pi/6$, $V_{out} = V_a - V_b$, D_1/D_4 is ON

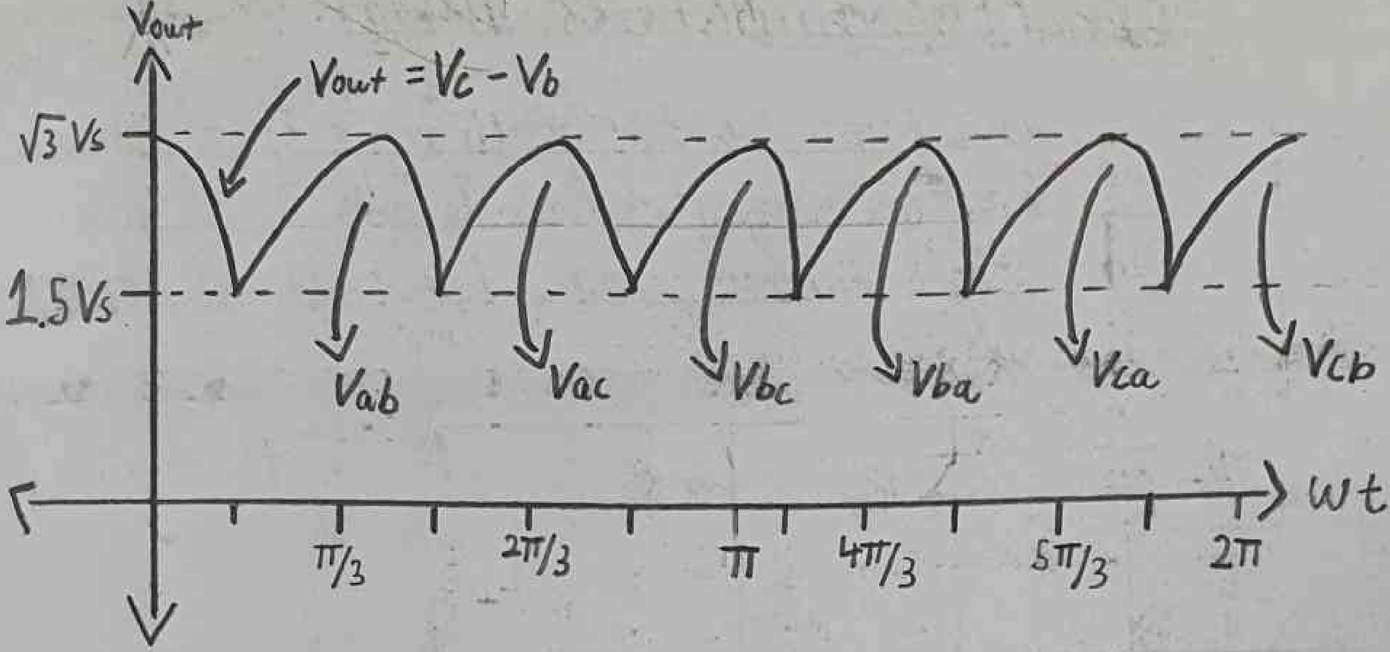
$3\pi/6$ to $5\pi/6$, $V_{out} = V_a - V_c$, D_1/D_6 is ON

$5\pi/6$ to $7\pi/6$, $V_{out} = V_b - V_c$, D_3/D_6 is ON

$7\pi/6$ to $9\pi/6$, $V_{out} = V_b - V_a$, D_3/D_2 is ON

$9\pi/6$ to $11\pi/6$, $V_{out} = V_c - V_a$, D_5/D_2 is ON

$11\pi/6$ to 2π, $V_{out} = V_c - V_b$, D_5/D_4 is ON



$$V_b = V_s \sin(wt - 2\pi/3)$$
$$V_c = V_s \sin(wt + 2\pi/3)$$

$$V_{ab} = \sqrt{3}\, V_s \sin(wt + \pi/6)$$
$$V_{ac} = \sqrt{3}\, V_s \sin(wt - \pi/6)$$
$$V_{bc} = \sqrt{3}\, V_s \sin(wt - 3\pi/6)$$

$$V_{ca} = \sqrt{3}\, V_s \sin(wt\; 5\pi/6)$$
$$V_{cb} = \sqrt{3}\, V_s \sin(wt + 3\pi/6)$$

$$V_{ba} = \sqrt{3}\, V_s \sin(wt - 5\pi/6)$$

$$V_{out} = V_c - V_b = \sqrt{3}\, V_s \sin(wt + 3\pi/6)$$

Peak Voltage

$$wt = \pi/6$$

minnimun Voltage

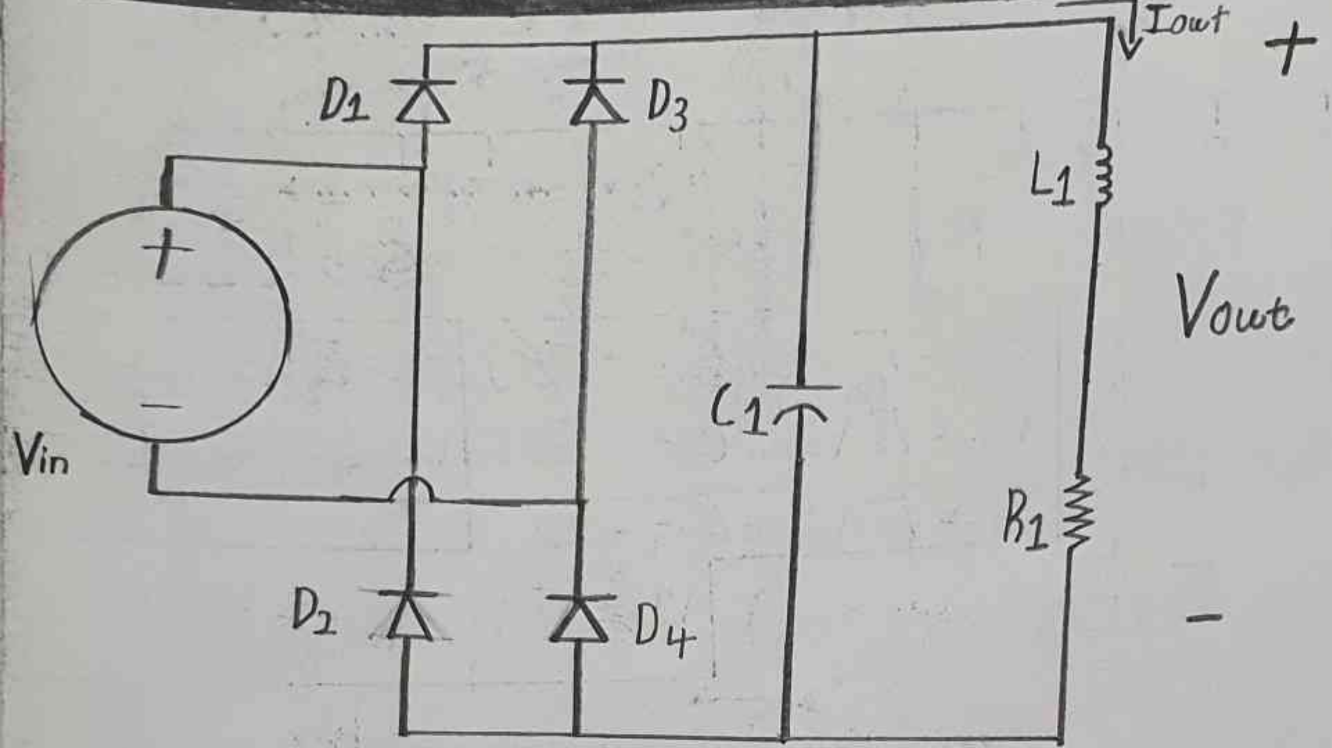
$$V_{out} = \sqrt{3}\, V_s \sin(\pi/6 + 3\pi/6) = 1.5 V_s$$

$$\langle V_{out} \rangle = \frac{1}{2\pi}\left[\int_0^{\pi/6} V_{cb}\, dwt + \int_{\pi/6}^{3\pi/6} V_{ab}\, dwt + \int_{3\pi/6}^{5\pi/6} V_{ac}\, dwt + \int_{5\pi/6}^{7\pi/6} V_{bc}\, dwt \right.$$

$$\left. + \int_{7\pi/6}^{9\pi/6} V_{ba}\, dwt + \int_{9\pi/6}^{11\pi/6} V_{ca}\, dwt + \int_{11\pi/6}^{2\pi} V_{cb}\, dwt \right] = \frac{3\sqrt{3}}{\pi} V_s \cong 1.65399 V_s$$

A Single-Phase Rectifier Design

Challenge - Design a single phase rectifier with an input voltage of 230Vrms at 50Hz an output voltage ripple lower than 20VpkPk, and an output current ripple lower than 2ApkPk the output load is 5 ohms.

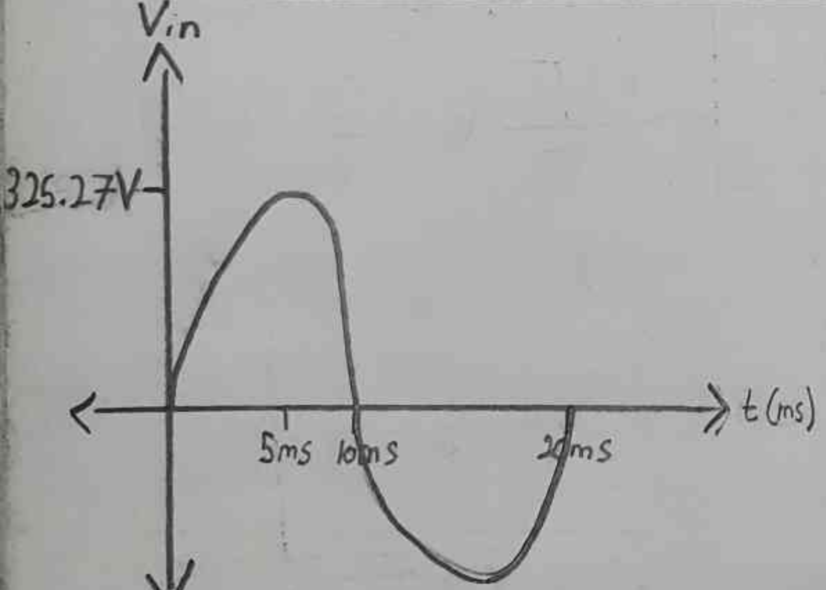


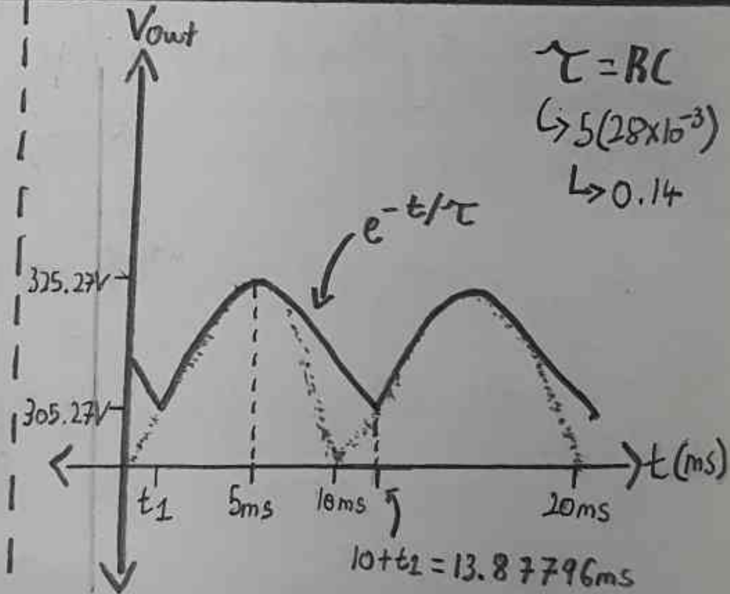
$V_{in} = 230\,V_{rms}$ at $50\,Hz$

Peak voltage $= 230\sqrt{2} = 325.269\,V$

Radiance $= 2\pi(50Hz) = 314.159$ radians/second

$T = \frac{1}{50Hz} = 20\,ms$





$\tau = RC$
$\hookrightarrow 5(28\times10^{-3})$
$\hookrightarrow 0.14$

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$V_{out} = 325.269 \sin(314.159t)$, $V = 305.269V$

$\hookrightarrow t_1 = 3.87796ms$

$325.269e^{-(t-5\times10^{-3})/5C} = 305.269V$, $t = 13.8779ms$ $\longrightarrow C = 27.9801mF$

$\hookrightarrow 28mF$

$$V_{out}(t) = \begin{cases} 325.269 \sin(314.159t)V, & 3.87796ms \text{ to } 5.0ms \\ 325.269e^{-(t-5\times10^{-3})/0.14}, & 5.0ms \text{ to } 13.87796ms \end{cases}$$

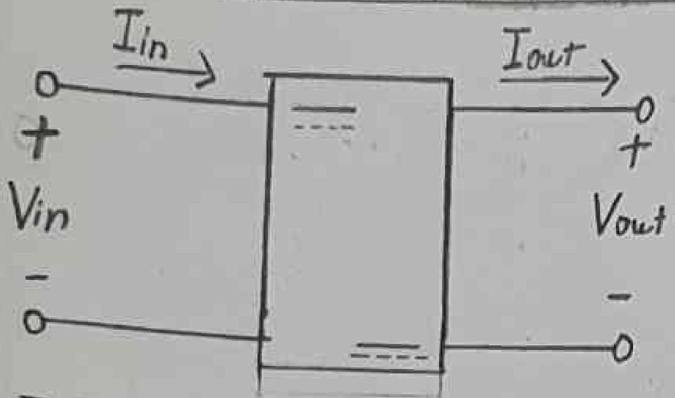
$$V_L = L\frac{di}{dt}$$

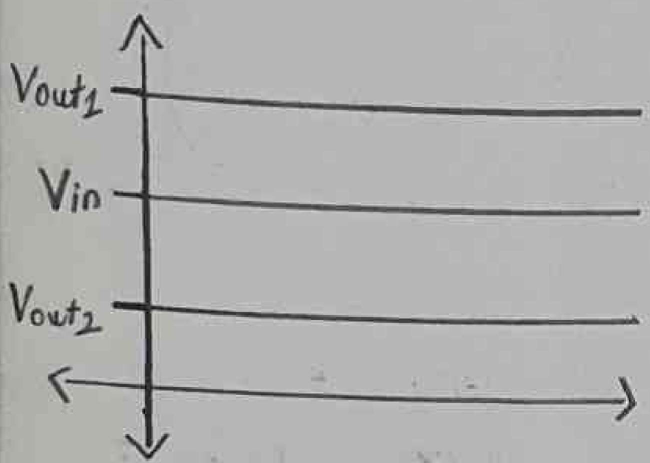
$$V_{out} = V_L + I_{out}R = L\frac{dI_{out}}{dt} + I_{out}R$$

Fourier Series, find Coeffient

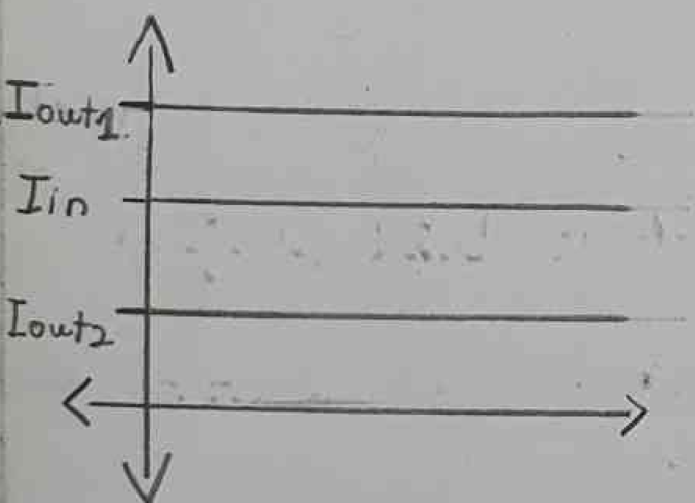
$$\frac{dI_{out}}{dt} = \frac{V_{out} - 5I_{out}}{L}, \quad L_1 = 10mH$$

The Ideal DC to DC Converter





$V_{out\,pkpk} = 0$



$I_{out\,pkpk} = 0A$

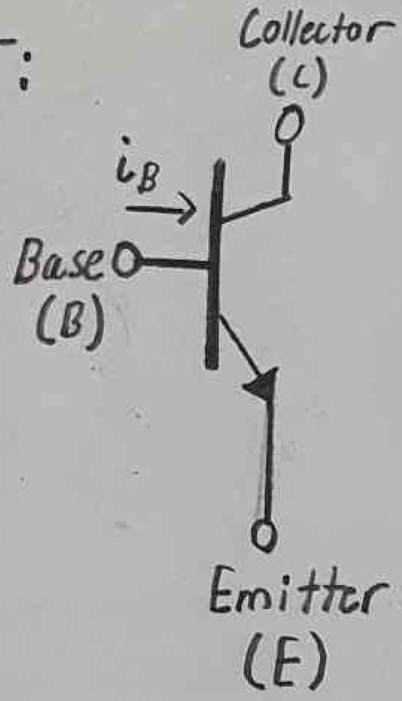
$P_{in} = P_{out}$

$P = IV$

$\hookrightarrow I = \frac{P}{V}$

Review of the Power BJT and the Power MOSFET

BJT:

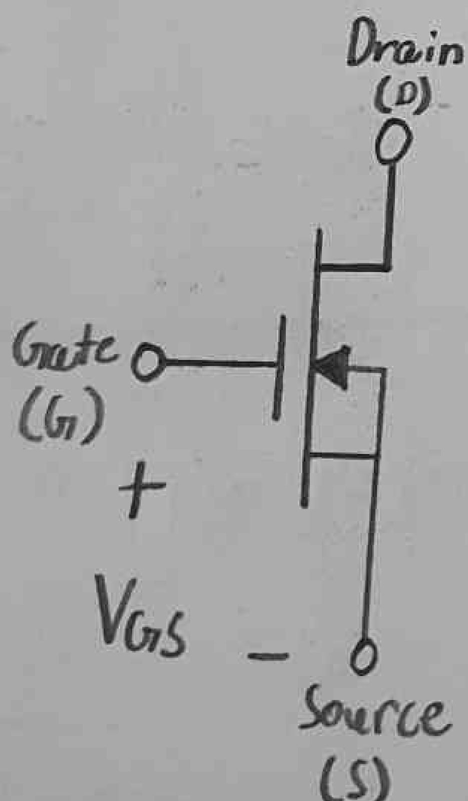


When $i_B > 0A$

When $i_B = 0$

Note: BJT is a current controlled switch.

MOSFET:

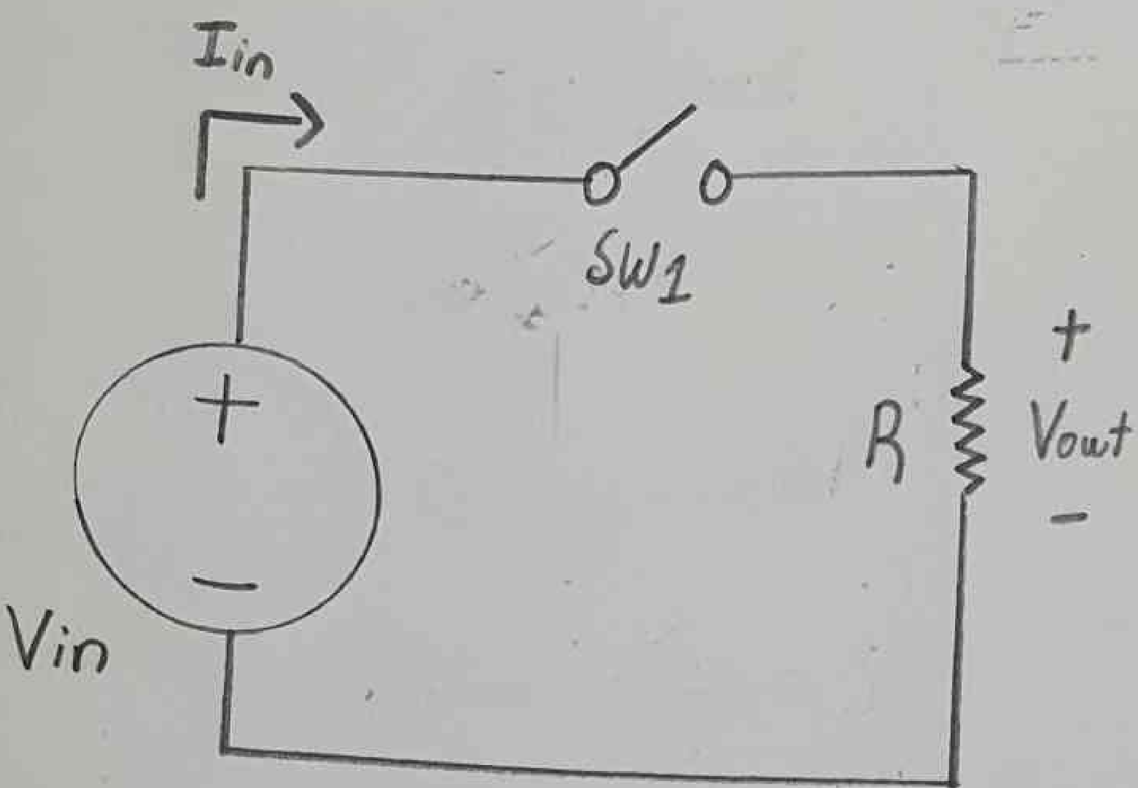


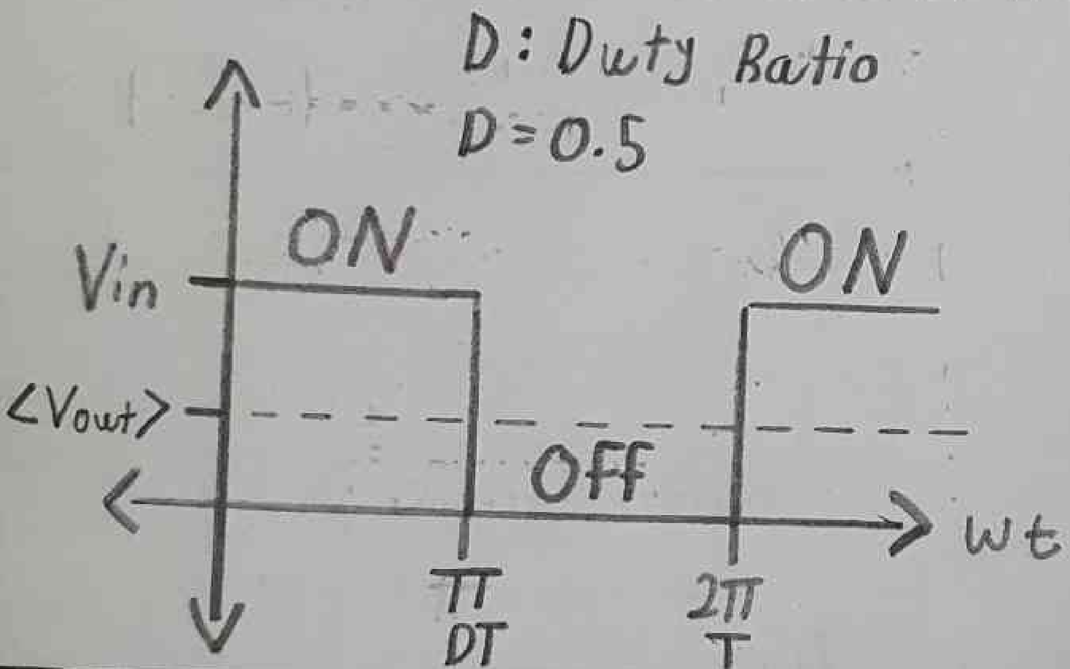
When $V_{GS} > 0V$

When $V_{GS} = 0V$

Note: Mosfet is a voltage controlled switch.

The Basic DC Chopper



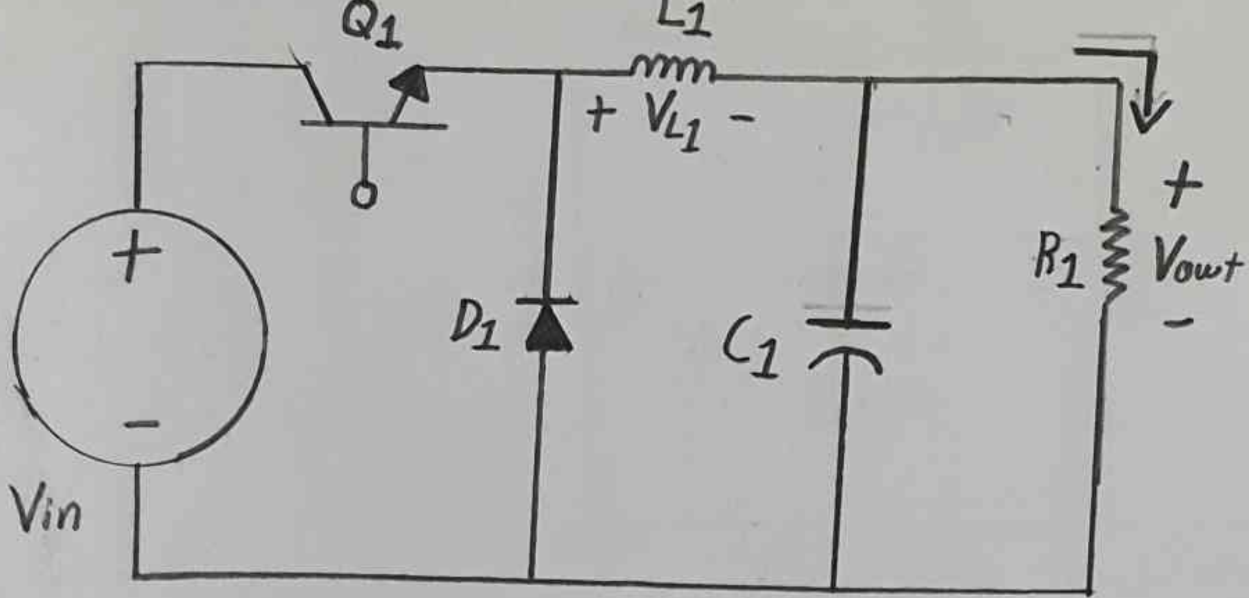


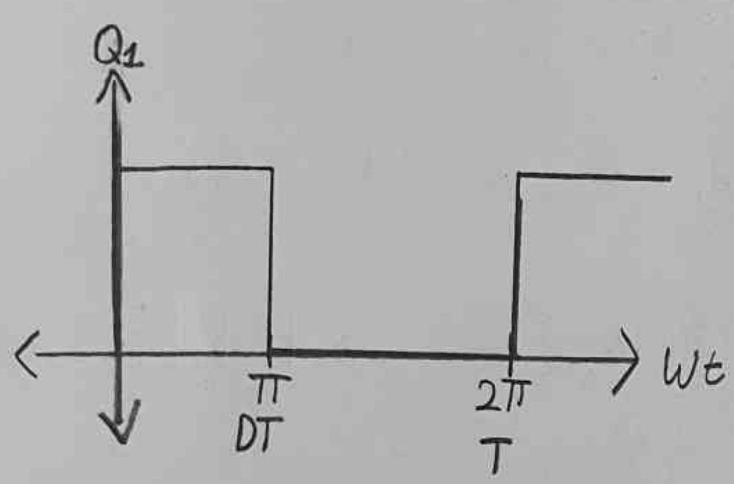
$$\langle V_{out} \rangle = \frac{1}{2\pi}\int_0^{\pi} V_{in}\, d\omega t = \frac{1}{2\pi}\left[V_{in}(\pi) - V_{in}(0)\right]$$

$$\hookrightarrow \frac{V_{in}(\cancel{\pi})}{2\cancel{\pi}} = \frac{V_{in}}{2}$$

$$\hookrightarrow \langle V_{out} \rangle = DV_{in}$$

Buck (Step-Down) Converters





State 1 0 to π:

Q_1 is ON, D_1 is OFF

$$-V_{in} + V_{L1} + V_{out} = 0$$

$$\hookrightarrow V_{L1} = V_{in} - V_{out}$$

State 2 π to 2π:

Q_1 is OFF, D_1 is ON

$$V_{L1} + V_{out} = 0 \rightarrow V_{L1} = -V_{out}$$

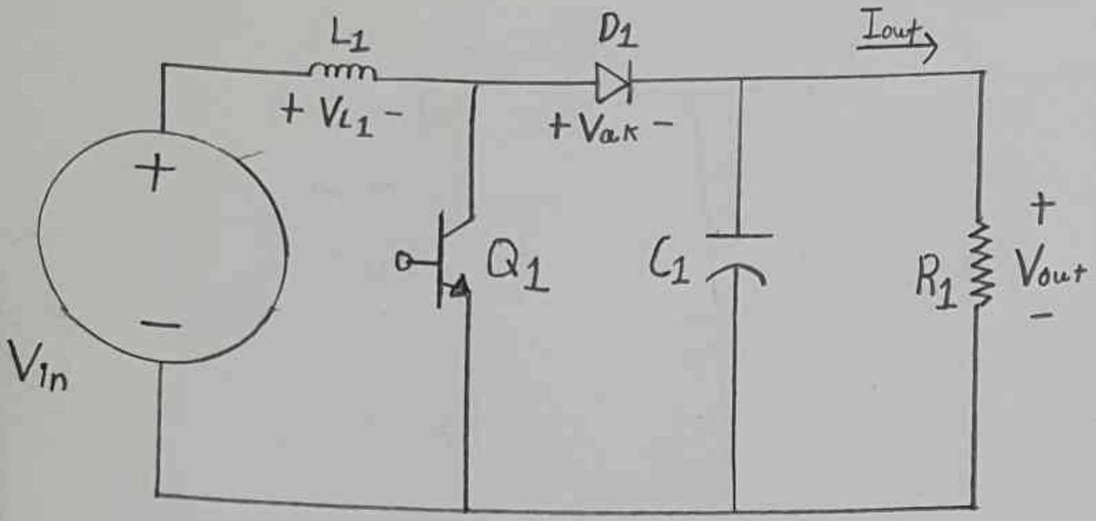
$$DT(V_{in} - V_{out}) + (T - DT)(-V_{out}) = 0$$

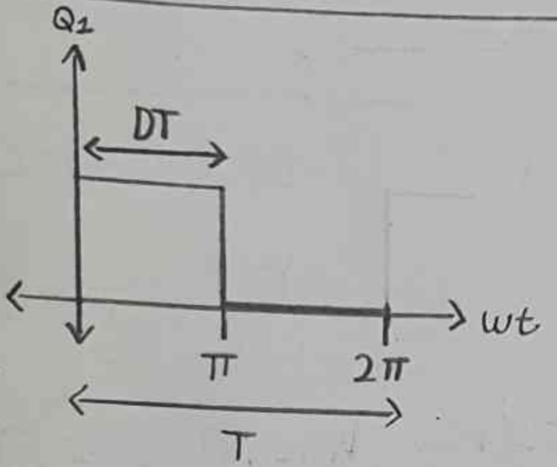
$$\hookrightarrow D\cancel{T}(V_{in} - V_{out}) + (1-D)\cancel{T}(-V_{out}) = 0$$

$$\hookrightarrow DV_{in} - DV_{out} - V_{out} + DV_{out} = 0$$

$$\hookrightarrow \boxed{V_{out} = DV_{in}}$$

Boost (Step-Up) Converters





State 1:

Q_1 is ON and D_1 is OFF

$-V_{in} + V_{L_1} = 0 \longrightarrow V_{L_1} = V_{in}$

State 2:

Q_1 is OFF and D_1 is ON

$-V_{in} + V_{L_1} + V_{out} \longrightarrow V_{L_1} = V_{in} - V_{out}$

Energy discharged from Inductor L_1 during State 2.

Energy Stored in inductor L_1 during State 1

$$\boxed{\frac{D\cancel{T}(V_{in})}{\cancel{L}}} + \boxed{\frac{(1-D)\cancel{T}(V_{in}-V_{out})}{\cancel{L}}} = 0$$

↳ $\cancel{DV_{in}} + V_{in} - V_{out} - \cancel{DV_{in}} + DV_{out} = 0$

↳ $V_{in} + V_{out}(D-1) = 0$

↳ $V_{in} = V_{out}(1-D) \longrightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1-D} \longrightarrow V_{out} = \frac{V_{in}}{1-D}$

Examples: $V_{out} = V_{in}/0.9 = 1.11V_{in}$, $V_{out} = V_{in}/0.1 = 10V_{in}$