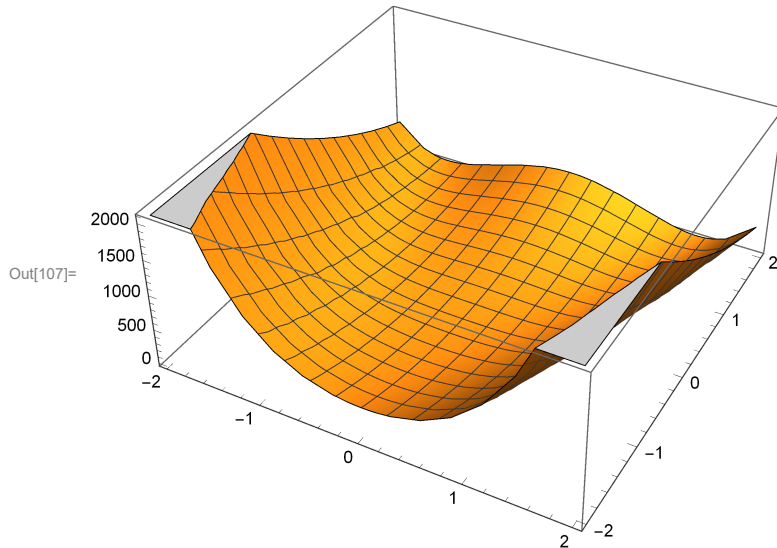```
In[105]:= x0 = {-1, 1};
         F[x1_, x2_] = 100 (x2 - x1^2)^2 + (1 - x1)^2;
```

```
In[107]:= Plot3D[F[x1, x2], {x1, -2, 2}, {x2, -2, 2}]
```

Out[107]=


```
In[187]:= G1 = ContourPlot[F[x1, x2], {x1, -3, 3}, {x2, -3, 3}];
```

```
In[110]:= x00 = {x0, {-1.0005, 0.9995}};
```

```
In[111]:= MatrixForm[x00]
```

Out[111]//MatrixForm=

$$\begin{pmatrix} -1 & 1 \\ -1.0005 & 0.9995 \end{pmatrix}$$

```
In[112]:=
         GrVToch[x1_, x2_] = Grad[F[x1, x2], {x1, x2}];
```

```
In[113]:= tabl =
         {{"Итерация", "u₁", "u₂", "x₁", "x₂", "F[x̄]", "||∇F(x̄)||"}, {"Начало", "-", "-",
           x0[[1]], x0[[2]], F[x0[[1]], x0[[2]]], Norm[GrVToch[x0[[1]], x0[[2]]], 2]}};
         Srav = {{"Метод", "Время выполнения метода", "Количество итераций"}};
```

```
In[115]:= Φ1[a_] = F[x00[[1, 1]] - a * GrVToch[x00[[1, 1]], x00[[1, 2]]][[1]],
           x00[[1, 2]] - a * GrVToch[x00[[1, 1]], x00[[1, 2]]][[2]]];
```

```
In[116]:= Φ2[a_] = F[x00[[2, 1]] - a * GrVToch[x00[[2, 1]], x00[[2, 2]]][[1]],
           x00[[2, 2]] - a * GrVToch[x00[[2, 1]], x00[[2, 2]]][[2]]];
```

```
In[117]:= Trigg1 = {};
         toch1 = {x00[[1]]};
         toch2 = {x00[[2]]};
```

```
In[120]:= Do[AppendTo[Trigg1, N[Table[Solve[D[Φ1[a], a] == 0][[i, 1, 2]],
                {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}]][[i]]],
            {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}];
```

```
In[121]:= Trigg1

Out[121]= {0.5, 0.00125631, 0.248744}

In[122]:= α1 = Trigg1[[3]];

In[123]:= Trigg2 = {};

In[124]:= Do[AppendTo[Trigg2, N[Table[Solve[D[Φ2[a], a] == 0][[i, 1, 2]],
            {i, 1, Length[Solve[D[Φ2[a], a] == 0]]}]][[i]]],
          {i, 1, Length[Solve[D[Φ2[a], a] == 0]]}];

In[125]:= Trigg2

Out[125]= {0.00118187, 0.223529, 0.448848}

In[126]:= α2 = Trigg2[[2]];

In[127]:= xk = {{}, {}};

In[128]:= xk[[1]] = x00[[1]] - α1 * GrVToch[x00[[1, 1]], x00[[1, 2]]];

In[129]:= xk[[2]] = x00[[2]] - α2 * GrVToch[x00[[2, 1]], x00[[2, 2]]];

In[130]:= AppendTo[toch1, xk[[1]]];
          AppendTo[toch2, xk[[2]]];

In[132]:= Clear[xmensh, xbolsh];

In[133]:= Which[F[xk[[1, 1]], xk[[1, 2]]] < F[xk[[2, 1]], xk[[2, 2]]], {xmensh = xk[[1]];
            xbolsh = xk[[2]];},
          F[xk[[1, 1]], xk[[1, 2]]] > F[xk[[2, 1]], xk[[2, 2]]], {xmensh = xk[[2]];
            xbolsh = xk[[1]];}];

In[134]:= H[a_] = F[xk[[2, 1]] + a (xmensh[[1]] - xbolsh[[1]]),
            xk[[2, 2]] + a (xmensh[[2]] - xbolsh[[2]])];

In[135]:= Tricks = {};

In[136]:= Do[AppendTo[Tricks, N[Table[Solve[D[H[a], a] == 0][[i, 1, 2]],
            {i, 1, Length[Solve[D[H[a], a] == 0]]}]][[i]]],
          {i, 1, Length[Solve[D[H[a], a] == 0]]}];

In[137]:= αh = Tricks[[1]];

In[138]:= For[i = 2, i ≤ Length[Solve[D[H[a], a] == 0]],
          i++, If[H[Tricks[[i]]] < H[αh], αh = Tricks[[i]]]]

In[139]:= Tricks

Out[139]= {-72.5463, -29.5885, 13.3708}

In[140]:= xtch = xk[[2]] + αh * (xmensh - xbolsh);

In[141]:= AppendTo[toch1, xtch];

In[142]:= ϵ = 0.001;
```

```mathematica
In[143]:= k = 1;

     AppendTo[tabl, {k, xk[[1]], xk[[2]], xtch[[1]], xtch[[2]],
        F[xtch[[1]], xtch[[2]]], Norm[GrVToch[xtch[[1]], xtch[[2]]], 2]}];

In[145]:= xtch = {xtch, xtch + ϵ / 2};


In[146]:= AppendTo[toch2, xtch[[2]]];

In[147]:=

In[148]:= t1 = SessionTime[];

In[149]:= While[Norm[GrVToch[xtch[[1, 1]], xtch[[1, 2]]], 2] > ϵ,
      {Φ1[a_] = F[xtch[[1, 1]] - a * GrVToch[xtch[[1, 1]], xtch[[1, 2]]][[1]],
          xtch[[1, 2]] - a * GrVToch[xtch[[1, 1]], xtch[[1, 2]]][[2]]];
       Φ2[a_] = F[xtch[[2, 1]] - a * GrVToch[xtch[[2, 1]], xtch[[2, 2]]][[1]],
          xtch[[2, 2]] - a * GrVToch[xtch[[2, 1]], xtch[[2, 2]]][[2]]];
       Trigg1 = {};
       Do[If[Element[N[Table[Solve[D[Φ1[a], a] == 0][[i, 1, 2]],
              {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}]][[i]], Reals],
         AppendTo[Trigg1, N[Table[Solve[D[Φ1[a], a] == 0][[i, 1, 2]],
              {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}]][[i]]]],
        {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}];
       α1 = Trigg1[[1]];
       For[i = 1, i ≤ Length[Trigg1],
        i++, If[Φ1[Trigg1[[i]]] < Φ1[α1], α1 = Trigg1[[i]]]];
       Trigg2 = {};
       Do[If[Element[N[Table[Solve[D[Φ2[a], a] == 0][[i, 1, 2]],
              {i, 1, Length[Solve[D[Φ2[a], a] == 0]]}]][[i]], Reals],
         AppendTo[Trigg2, N[Table[Solve[D[Φ2[a], a] == 0][[i, 1, 2]],
              {i, 1, Length[Solve[D[Φ2[a], a] == 0]]}]][[i]]]],
        {i, 1, Length[Solve[D[Φ2[a], a] == 0]]}];
       α2 = Trigg2[[1]];
       For[i = 1, i ≤ Length[Trigg2],
        i++, If[Φ2[Trigg2[[i]]] < Φ2[α2], α2 = Trigg2[[i]]]];
       xk[[1]] = xtch[[1]] - α1 * GrVToch[xtch[[1, 1]], xtch[[1, 2]]];
     xk[[2]] = xtch[[2]] - α2 * GrVToch[xtch[[2, 1]], xtch[[2, 2]]];
       AppendTo[toch1, xk[[1]]];
       AppendTo[toch2, xk[[2]]]; Clear[xmensh, xbolsh]
     Which[F[xk[[1, 1]], xk[[1, 2]]] < F[xk[[2, 1]], xk[[2, 2]]], {xmensh = xk[[1]];
          xbolsh = xk[[2]];},
        F[xk[[1, 1]], xk[[1, 2]]] > F[xk[[2, 1]], xk[[2, 2]]], {xmensh = xk[[2]];
          xbolsh = xk[[1]];}];
       H[a_] = F[xk[[2, 1]] + a (xmensh[[1]] - xbolsh[[1]]),
          xk[[2, 2]] + a (xmensh[[2]] - xbolsh[[2]])];
       Tricks = {};
       Do[If[Element[N[Table[Solve[D[H[a], a] == 0][[i, 1, 2]],
```

```
            {i, 1, Length[Solve[D[H[a], a] == 0]]}]][[i]], Reals],
        AppendTo[Tricks, N[Table[Solve[D[H[a], a] == 0][[i, 1, 2]],
            {i, 1, Length[Solve[D[H[a], a] == 0]]}]][[i]]]],
      {i, 1, Length[Solve[D[H[a], a] == 0]]}];
    αh = Tricks[[1]];
    For[i = 1, i ≤ Length[Tricks],
     i++, If[H[Tricks[[i]]] < H[αh], αh = Tricks[[i]]]];
    Clear[xtch];
    xtch = xk[[2]] + αh * (xmensh - xbolsh);
    AppendTo[toch1, xtch];
    k++;
    AppendTo[tabl, {k, xk[[1]], xk[[2]] xtch[[1]], xtch[[2]],
      F[xtch[[1]], xtch[[2]]], Norm[GrVToch[xtch[[1]], xtch[[2]]], 2]}];
        xtch = {xtch, xtch + ϵ / 2};
    AppendTo[toch2, xtch[[2]]];
   }]
  t2 = SessionTime[];
```

In[151]:= `AppendTo[Srav, {"Овражный градиентный метод", t2 - t1, k}];`

```
In[152]:= Insert[Grid[tabl], {Dividers → All, Spacings → 1.5 {0.4, 0.4}}, 2];
      AppendTo[tabl, {"Точное решение: F[x̄]=",
        xtch[[1, 1]], xtch[[1, 2]], F[xtch[[1, 1]], xtch[[1, 2]]]}];
      AppendTo[tabl, {"Time:", t2 - t1, "-", "-", "-"}];
      Grid[tabl, Dividers → All]
```

Out[155]=

Итерация	u_1	u_2	x_1	x_2	$F[\bar{x}]$	$\|\|\nabla F(\bar{x})\|\|$
Начало	−	−	−1	1	4	4
1	{−0.0050: 2525, 1.}	{0.0280444, 1.06657}	−0.414125	0.176476	2.00223	2.23742
2	{−0.0154: 737, −0.0215: 603}	{−0.0098: 422, −0.0012: 5747}	− 0.0291311	0.968081	7.37132	
3	{0.0901202, 0.008800: 75}	{0.0236841, 0.002328: 51}	0.0446538	0.601443	4.82269	
4	{0.257062, 0.0646756}	{0.105491, 0.0266088}	0.146636	0.392902	4.7717	
5	{0.402422, 0.1597}	{0.214553, 0.085263}	0.265108	0.252431	4.73518	
6	{0.525327, 0.273891}	{0.337526, 0.17614}	0.39659	0.152156	4.49181	
7	{0.635711, 0.402489}	{0.470968, 0.298389}	0.535648	0.0829747	4.03735	
8	{0.735355, 0.539574}	{0.608627, 0.446823}	0.674827	0.0388345	3.37377	
9	{0.823485, 0.677369}	{0.741925, 0.610545}	0.804486	0.0141619	2.52208	
10	{0.898027, 0.806028}	{0.859715, 0.771925}	0.912087	0.00324602	1.53788	
11	{0.955546, 0.912888}	{0.947329, 0.905335}	0.980915	0.0002538: 01	0.570151	
12	{0.990565, 0.981182}	{0.990742, 0.981657}	0.99966	7.68523×10^{-7}	0.0385651	
13	{0.999839, 0.999678}	{1.00014, 1.00028}	1.	5.21492×10^{-14}	0.0000102: 208	
Точное решение : F[x̄]=	1.	1.	5.21492×10^{-14}			
Time:	1.4870065	−	−	−		

```
In[156]:= G2 = ListPlot[toch1, PlotStyle → Black, Joined → True];
         G3 = ListPlot[toch2, PlotStyle → Red, Joined → True];
         Show[G1, G3, G2]
```

Out[158]=


```
In[159]:=
```

```
In[160]:= (*МНС*)
```
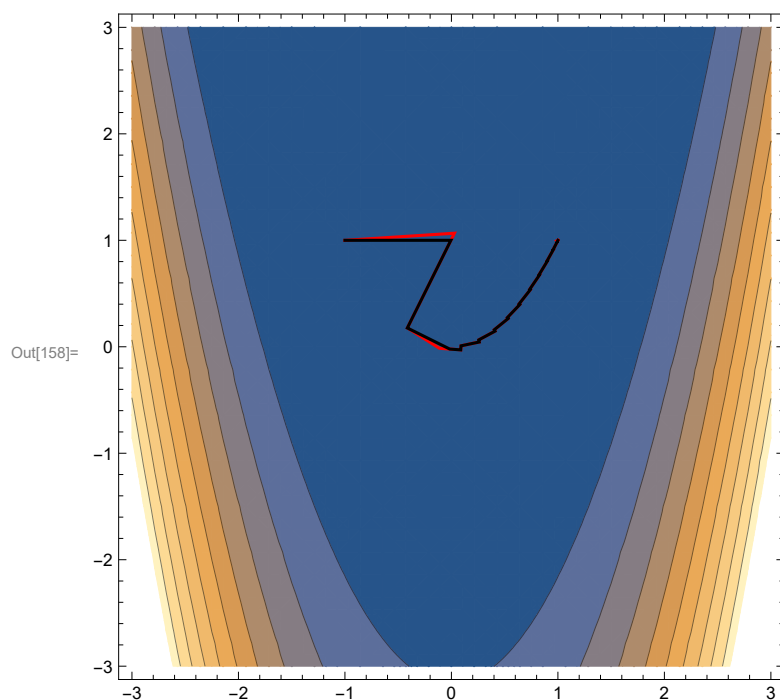
```
In[161]:= Φ1[a_] = F[x00[[1, 1]] - a * GrVToch[x00[[1, 1]], x00[[1, 2]]][[1]],
            x00[[1, 2]] - a * GrVToch[x00[[1, 1]], x00[[1, 2]]][[2]]];
```

```
In[162]:=
```

```
In[163]:= Trigg1 = {};
         toch1 = {x00[[1]]};
```

```
In[165]:= Do[AppendTo[Trigg1, N[Table[Solve[D[Φ1[a], a] == 0][[i, 1, 2]],
             {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}]][[i]]],
           {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}];
```

```
In[166]:= Trigg1
```

```
Out[166]= {0.5, 0.00125631, 0.248744}
```

```
In[167]:= α1 = Trigg1[[3]];
```

```
In[168]:= xk = x00[[1]] - α1 * GrVToch[x00[[1, 1]], x00[[1, 2]]];
```

```
In[169]:= AppendTo[toch1, xk];
```

```
In[170]:= k = 1;
```

```
In[171]:= t1 = SessionTime[];
```

```
In[172]:= While[Norm[GrVToch[xk[[1]], xk[[2]]], 2] > ϵ, {
          Φ1[a_] = F[xk[[1]] - a * GrVToch[xk[[1]], xk[[2]]][[1]],
            xk[[2]] - a * GrVToch[xk[[1]], xk[[2]]][[2]]];
          Trigg1 = {};
          Do[If[Element[N[Table[Solve[D[Φ1[a], a] == 0][[i, 1, 2]],
                  {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}]][[i]], Reals],
            AppendTo[Trigg1, N[Table[Solve[D[Φ1[a], a] == 0][[i, 1, 2]],
                  {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}]][[i]]]],
           {i, 1, Length[Solve[D[Φ1[a], a] == 0]]}];
          α1 = Trigg1[[1]];
          For[i = 1, i ≤ Length[Trigg1],
           i++, If[Φ1[Trigg1[[i]]] < Φ1[α1], α1 = Trigg1[[i]]]];
          xk = xk - α1 * GrVToch[xk[[1]], xk[[2]]];
          AppendTo[toch1, xk];
          k++;

         }]

In[186]:= Print["Точка xk = ", xk, " F(x) = ", F[xk[[1]], xk[[2]]],
          " шаг α = ", α1, " ||∇F(x̄)|| = ", Norm[GrVToch[xk[[1]], xk[[2]]], 2]]

         Точка xk = {0.999001, 0.997999} F(x) =
          9.99916×10⁻⁷ шаг α = 0.00124937 ||∇F(x̄)|| = 0.000999706
```

```
In[183]:= k

Out[183]= 4339
```

```
In[173]:= t2 = SessionTime[];
```

```
In[174]:= AppendTo[Srav, {"Метод наискорейшего спуска", t2 - t1, k}];
```

```
In[175]:= Insert[Grid[Srav], {Dividers → All, Spacings → 1.5 {0.4, 0.4}}, 2];
         Grid[Srav, Dividers → All]
```

Метод	Время выполнения метода	Количество итераций
Овражный градиентный метод	1.4870065	13
Метод наискорейшего спуска	205.2893605	4339

In[177]:= **G4 = ListPlot[toch1, PlotStyle → Black, Joined → True];**

Show[G1, G4]

Out[178]=