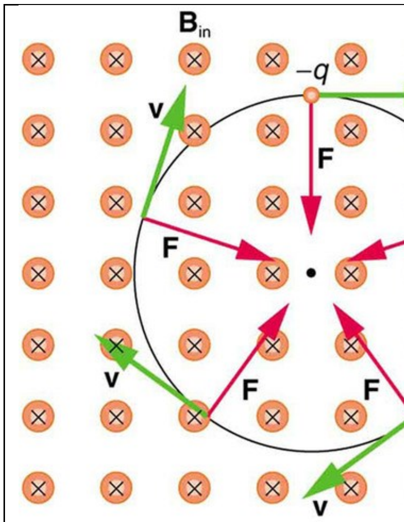
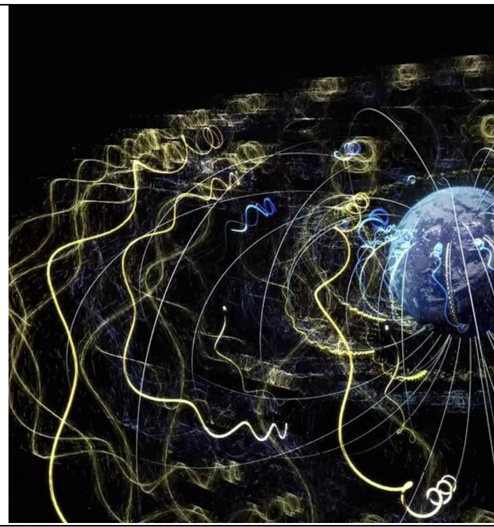
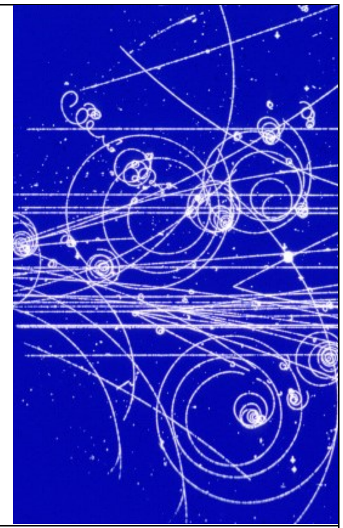
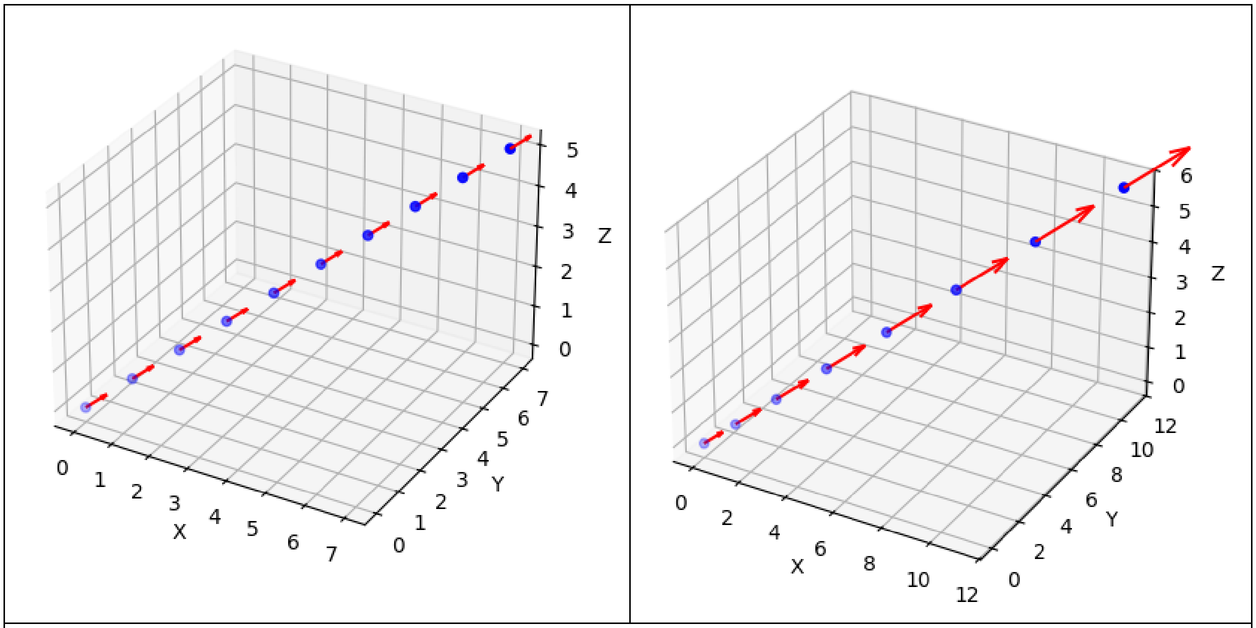
Exploring particle motion in a magnetic field

		
From 21.4: Motion of a Charged Particle in a Magnetic Field - Physics LibreTexts	From NASA SVS | Beyond Earth - Earth's Geomagnetic Activity	From The decay of a lambda particle in the 32 cm hydrogen bubble chamber - CERN Document Server
In a vacuum (where a particle won't lose energy to other material), a charged particle in a uniform magnetic field travels in a circle.	Particles traveling in earth's magnetosphere don't lose much energy (it's mostly vacuum out there). But the Earth's magnetic field gets stronger near the poles. See how the path of the particle gets "tighter" as it nears the pole?	Particles in a bubble chamber are traveling in a material and the particle will lose energy to that material.

Introductory physics textbooks will tell you that a particle in a uniform magnetic field travels in a circle. This is true, and the goal of this exploration is to become convinced that this must be the case because of the form of the magnetic force, $\vec{F} = q\vec{v} \times \vec{B}$.

Trajectory

A particle's "trajectory" is its location as a function of time. Mathematicians often write this $\vec{s}(t)$ and physicists sometimes write this $\vec{x}(t)$. Note that however you write it, this is a vector-valued function – the location of a particle has three components.



Two particle trajectories with different values of acceleration. Particle locations at times 1 s, 2 s, 3 s, etc. are shown in blue and velocity vectors are shown in red. Code at https://colab.research.google.com/drive/1hlyGDshQNkNpTYZWTGvujjdIB6BMlERx?usp=sharing

Readers may be familiar with the relationships between position, velocity, and acceleration – velocity is the derivative of position, and acceleration is the derivative of velocity. One way to see how a particle behaves in a magnetic field is to write down these differential equations and guess at solutions. Please give this a try. In this worksheet, however, we are going to focus on how to find the trajectory computationally.

EVEN IF YOU BELIEVE THE STATEMENT ABOVE, CHECK THE UNITS. DO THEY WORK?

However, in the case of a particle in a magnetic field we know the acceleration ($m\vec{a} = q\vec{v} \times \vec{B}$) and want to get the position from it. To go from the acceleration to the position, we integrate:

$$\vec{v}(t) = \int_0^t \vec{a}\,(\tau) d\tau$$

$$\vec{x}(t) = \int_0^t \vec{v}(\tau) d\tau$$

In the case of constant acceleration, this gives us the equations we use frequently in introductory physics,

$$\vec{v}(t) = \vec{\mathrm{v}}_0 + \vec{a}t$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

In the case of a particle moving in a magnetic field, the acceleration is not constant. However, if we make the time interval small then we can approximate the acceleration as constant. So

$$\vec{v}(t+\Delta t) = \vec{v}_0 + \vec{a}\Delta t$$

$$\vec{x}(t+\Delta t) = \vec{x}_0 + \vec{v}_0\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

Provided that Δt is very small. I've written the velocity at time t as v0 and the position at time t as x0. Also note that if Δt is very small, Δt^2 is even smaller, so we can drop that term. And that brings us to our classic "update equations" for calculating a trajectory:

$$\vec{v}(t+\Delta t) = \vec{v}_0 + \vec{a}\Delta t$$

$$\vec{x}(t+\Delta t) = \vec{x}_0 + \vec{v}_0\Delta t$$

Calculation 1

Consider a proton moving in a magnetic field and calculate its position and velocity after 1 ms. The magnetic field has a strength of 1 T and is pointing into the page. Assume that the proton starts at position (0, 0, 0) m and has a velocity of (1, 0, 0) m/s.

The answer should be $\vec{x}_1 = \vec{x}(0.001\ s) = (0.001,\ 0, 0)m$ and $\vec{v}_1 = \vec{v}(0.001\ s) = (1, 0.096, 0)m/s$, assuming you use the initial velocity in your position calculation.

Calculation 2

What is the position and location of the proton after another 1 ms?

The answer should be $\vec{x}_2 = \vec{x}(0.002\ s) = (0.002,\ 9.6\mathrm{E}-5, 0)m$ and $\vec{v}_2 = \vec{v}(0.002\ s) = (0.99, 0.142, 0)m/s$, assuming you use $\vec{v}_1$ in your position calculation.

Calculation 3

What is the position and location of the proton after another 1 ms?

The answer should be $\vec{x}_3 = \vec{x}(0.003\ s) = (0.00299,\ 0.000287, 0)m$ and $\vec{v}_3 = \vec{v}(0.003\ s) = (0.972, 0.286, 0)m/s$, assuming you use $\vec{v}_2$ in your position calculation.

Graphing

Graph the locations and velocity vectors of the proton at each step you've calculated so far. Do you think this will move in a circle? Why or why not?

Coding

You are welcome to continue this calculation by hand, but it's tedious! Instead, write some code that does this calculation for you. Keep working on the code until it agrees with the calculations you did by hand, and then see if you can get the code to plot out the entire circle for you. Is the particle moving in a circle?

Thinking about the code

When you choose a timestep of 1 ms, you should see the proton moving in a spiral, not a circle. Here is where we need to ask the question: is this physically accurate? That will be the topic of the next exploration! For now, look at what happens if you decrease the time step. Note that if you decrease the time step, you'll need to increase the number of steps to move the proton for the same amount of time.