1 First Level (Abridged)

In dielectric materials, electrons are bound to positively charge core atoms. When an electric field is applied to the material, the charges will polarize. This effect can be observed when placing a dielectric material within the uniform electric field of a capacitor with infinite plate size. To begin, the polarization of the charges can be expressed as a distance vector considering the sign of the charge. Across the volume of the material, this polarization ($\vec{P}$) can be treated as the polarization along the volume by considering the number (n) of charges (q) per unit volume ($\frac{1}{V}$).

$$\vec{P} = \frac{nq}{V}\vec{d} \tag{1}$$

$$\sigma_B = P \cdot \vec{n} \tag{2}$$

As a result, σ_B is used to express the polarization that occurs across the volume of the material. σ_B is most noteable at the boundaries of the material that align perpendicularly with the capacitor's electric field. Here, σ_B will appear as surface charges, while the internal of the material will result in a net neutral region. Consequently, the polarization will give rise to an induced electric field that exists within the material as if another capacitor was working in reverse to the original.

$$\vec{E}_{ind} = \frac{\sigma_B}{\varepsilon_0} \tag{3}$$

Here it is important to note that the direction of $\vec{P}$ is opposite to the direction of the field, due to the distance vector pointing from the negative to positive charge. Consequently, $\vec{E}_{ind}$ will point opposite to the capacitor's electric field. The electric field relationship between the capacitor's field (E_{ext}) and the dielectric material's field (E_{ind}) can be expressed and derived as:

$$\vec{E}_{tot} = \vec{E}_{ind} + \vec{E}_{ext} \tag{4}$$

$$\varepsilon_0\vec{E}_{tot} + \vec{P} = \varepsilon_0\vec{E}_{ext} \tag{5}$$

From these equations, a discrepency in the total electric field is caused by a sort of reduction due to the induced electric field. When modeled as a cross section of the material, this can be represented as a factor of $E_{ind} = \frac{\sigma_B}{\varepsilon_0}$

The boundary condition can be expressed similarly for a surface boundary perpendicular to the direction of E_{ext}, which happens to be the factor of difference of E_{tot}. In the case for parallel surfaces, the difference in electric field is not influenced by the dielectrics internal field. Then there is no noticeable quantity to observe for boundaries parallel to the direction of the capacitor's electric field.

$$\vec{E}^{\perp}_{above} - \vec{E}^{\perp}_{below} = \frac{\sigma_B}{\varepsilon_0} \tag{6}$$

or

$$(\vec{E}_B - \vec{E}_A) \cdot \hat{n} = \frac{\sigma_B}{\varepsilon}, \text{ and } (\vec{E}_B - \vec{E}_A) \times \hat{n} = 0 \tag{7}$$

2 Python Generated Models

All of the python scripts used to generate any plots and models are located in this github repository. All scripts were generated using Claude AI. `https://github.com/Derkula/EM_Standards_Pradhan/tree/main/Standards/EM_3`

2.1 Dielectric Model Within a Capacitor

Using python, a model of this was generated. In this scenario, a capacitor exists with the upper plate being positively charged. The plate length is modeled to be 3cm long, and 1cm apart. This was done solely for visual clarity. In essence, the script is treating the capacitor's electric field as uniform. For the sake of visual clarity, the vector representation is not accurate to the actual electric field. The polarization is modeled by utilizing equation (1) and the corresponding induced electric field using equation (3). For added complexity, the script also considers the polarization constant of the material. It is shown that the induced field within the dielectric is in the opposite direction of the external field. Consequently, the net electric field within the material is reduced. The net field is calculated using equation (4).

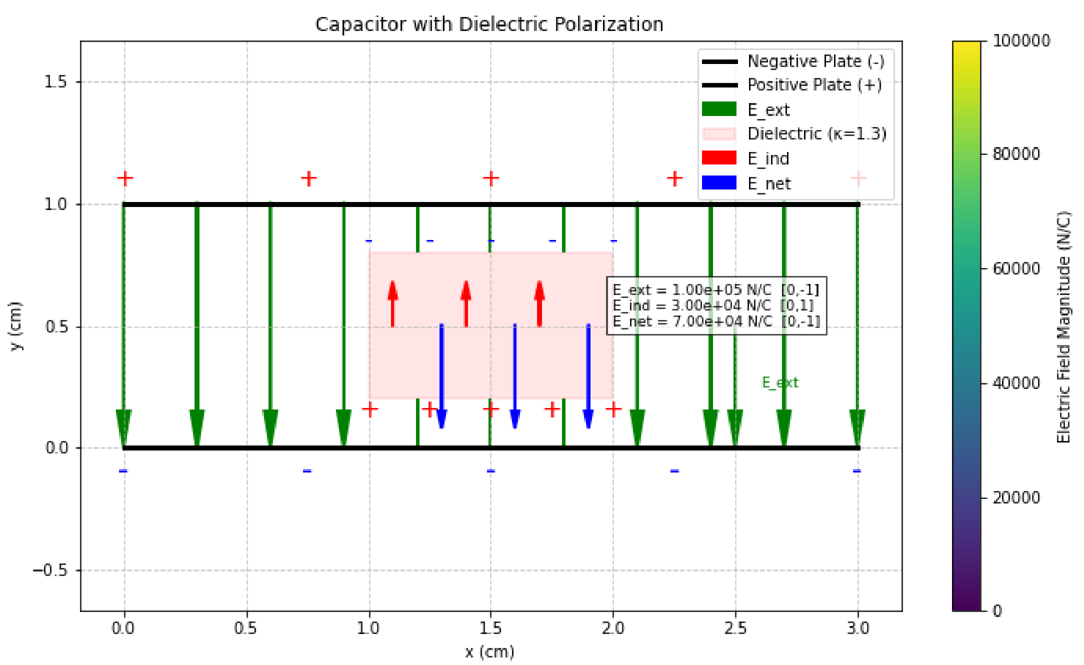


Figure 1: Model of Dielectric.

2.2 Cross Section Representation of Net Electric Field

An additional plot was created to model the cross sections of the model along the x and y axis. It is shown that while the external electric field remains constant, the total electric field is reduced while within the material. It is also shown that the induced electric field increased while within the material, as this is the region where it exists. These descriptions of magnitude align with predictions about the model. Recalling equations (4) and (7), the changes in the net electric field are not continuous, as the induced field exists only within the boundaries of the material. The reduction to the net electric field happens to be by a factor of E_{ind}. and aligns with the perpendicular expression of the material's boundary.

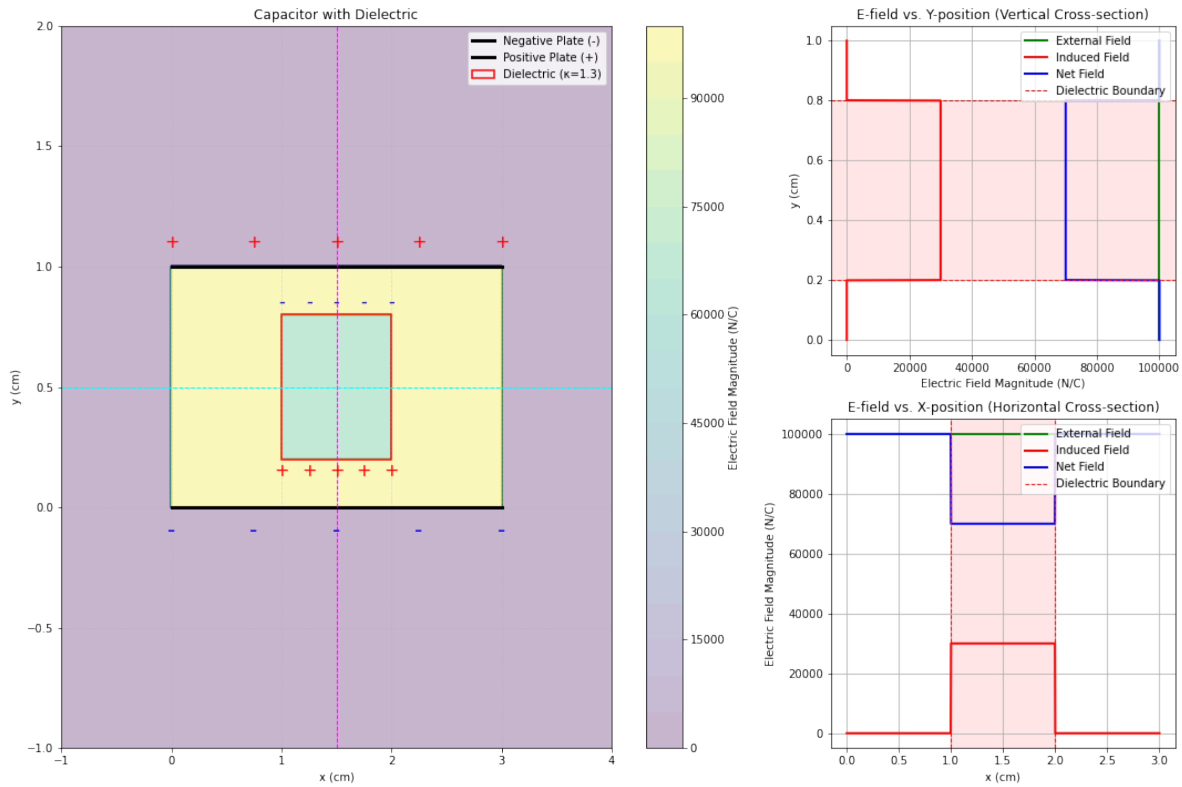


Figure 2: Cross Section of Electric Field Magnitudes.

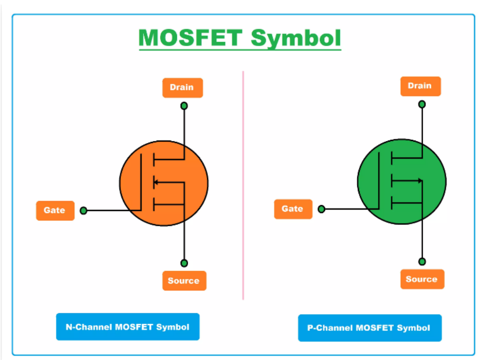
3 MOSFET's and Dielectric Materials as a Current Bridge

3.1 General Overview

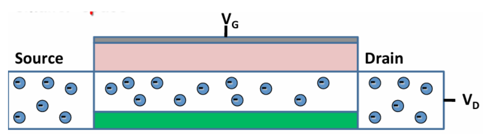
A MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor) is a semiconductor-based electronic component designed to control current flow by utilizing these concepts and effects of an applied electric field. As a voltage-driven device, MOSFETs can be used as switch or amplifiers. Further integration of MOSFETs in CMOS circuitry is an excellent form of digital logic gates because of their performance and low power consumption.

The topic of a MOSFET is an interesting use case for the polarization experienced by a dielectric material when met with an external electric field. The circuit components of a mosfet include three terminals, the gate, source, and drain (Fig 3).

For the purpose of this standard, the gate can be thought of as a capacitor with a dielectric material filling the space between plates, usually a Silicon Oxide glass dielectric, shaded pink in Fig 4 ElectroBOOM [1]. When a voltage is applied to the gate, an electric field between the plates will be generated. The thin oxide layer will experience a charge polarization, and an induced electric field will be produced. This induced field can be used to attract electrons to the gate, and create a current bridge linking the source with the drain.



(a) MOSFET in Diagrams Watson [4]



(b) Simple diagram of a MOSFET's construct Gilbert [3]

3.2 I-V Characterization

Modeling the current output of a MOSFET can be done so by considering the properties of the gate, along with the input and output voltage measured at the terminals (V_G and V_D respectively). An I-V plot can be generated and describe a MOSFET's linear regime (eqn. 8), or a saturated regime Gilbert [3].

$$I_D = \frac{\bar{\mu}_n Z C_i}{L}[(V_G - V_T)V_D - \frac{V_D^2}{2}] \tag{8}$$

$$\text{(Saturated): } I_D = \frac{Z}{L}\bar{\mu}_n C_i[(V_G - V_T)V_D^{sat.} - \frac{(V_D^{sat.})^2}{2}], \text{ with } V_D^{(sat.)} \approx V_G - V_T \tag{9}$$

The analysis of the I-V characterization isn't necessary here, but certain quantities involved are relevant for studying the electric field effects related to the gate voltage.

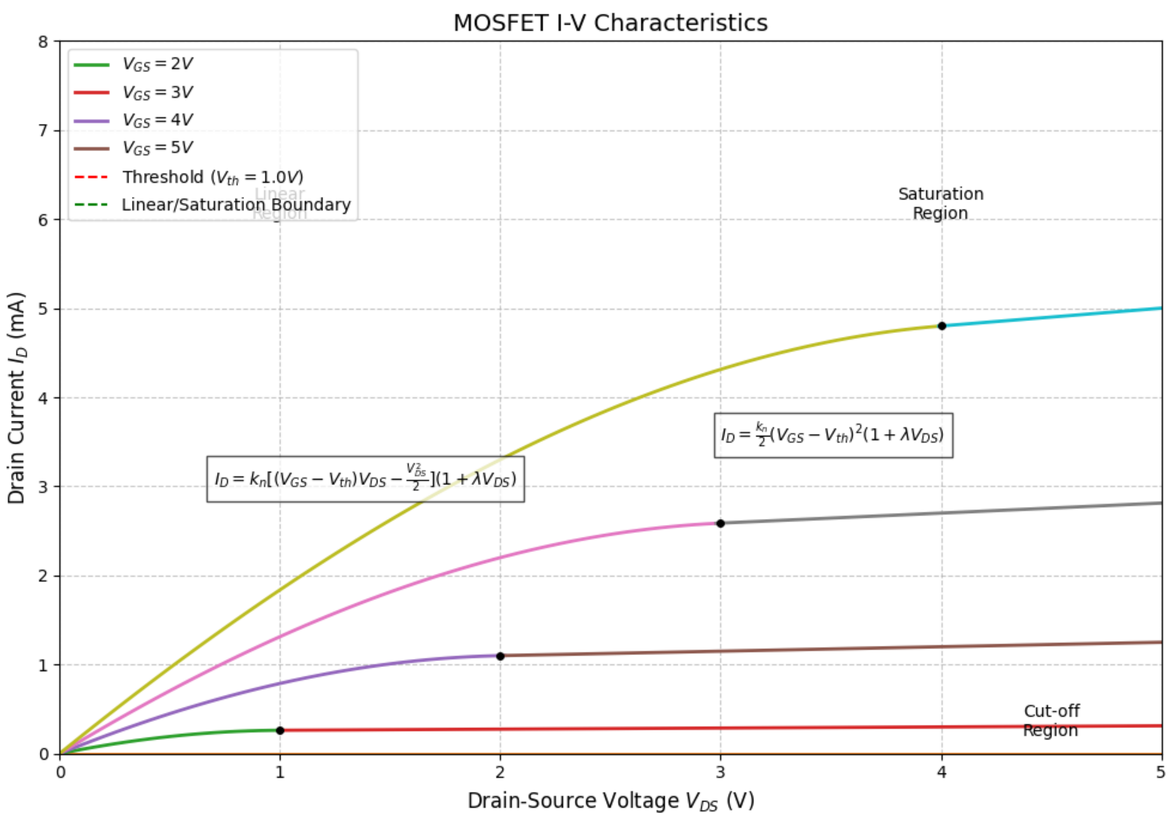


Figure 4: I-V Characteristic modeled with Python using equations (8) and (9)

3.3 Charge Polarization

Following a similar approach for the polarization of the dielectric, Its useful to define the electric field within the gate. Because a votlage is being applied, the electric field can be expressed as the potential voltage. Using the expression $E = -\nabla V$ in a uniform field we derive the following equation, where t_{ox} represents the thickness of the dielectric oxide material which is equal to the space between plates.

$$E_{net} = -\frac{V_G}{t_{ox}} \tag{10}$$

It's then observed that a steady increase in voltage a the gate results in a linearly increasing electric field. The electric field then polarizes the oxide. When a positive voltage is applied, an electric field will appear in the direction opposite to the gradient. In this case, assume that $V_G = 3V$, then the resulting electric field would point upwards as we consider the thickness to extend below the gate in a negative y direction. From equations (1) and (3), it can then be determined that the oxide will polarize, and an induced electric field will arise in the opposite direction, pointing downwards. The figure below depicts this occurrence.

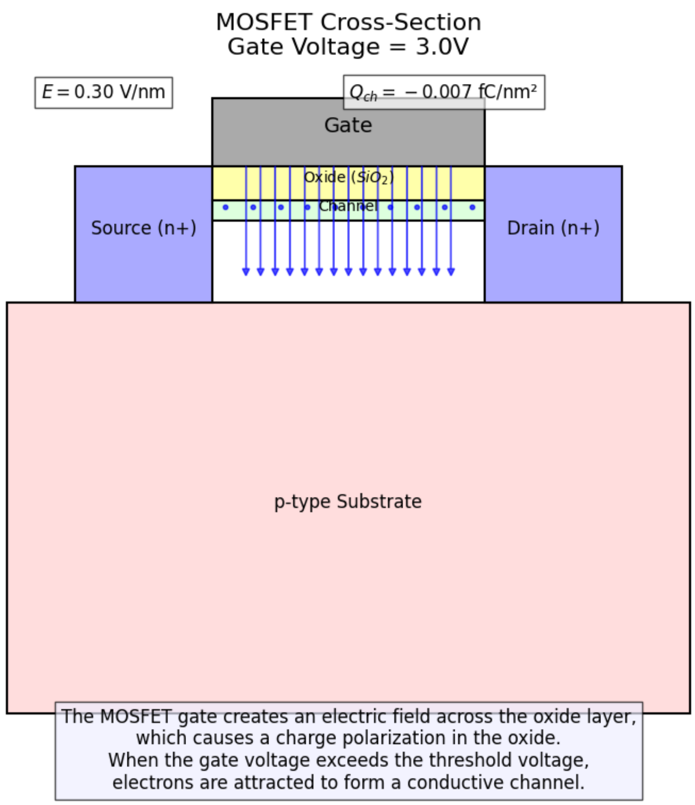


Figure 5: Cross Sectional Diagram Showing the Channel forming from the Oxide's electric Field

Consequently, electrons will be attracted by this electric field following the force $F = -qE$. Previously mentioned was a threshold for saturation to occure. When this threshold is met, a channel of charge begins to occur. The channel charge density begins to decrese in charge, as the electrons begin accumulating at the bottom surface of the gate. The formation of this channel is what allows current to pass between the Source and the Drain.

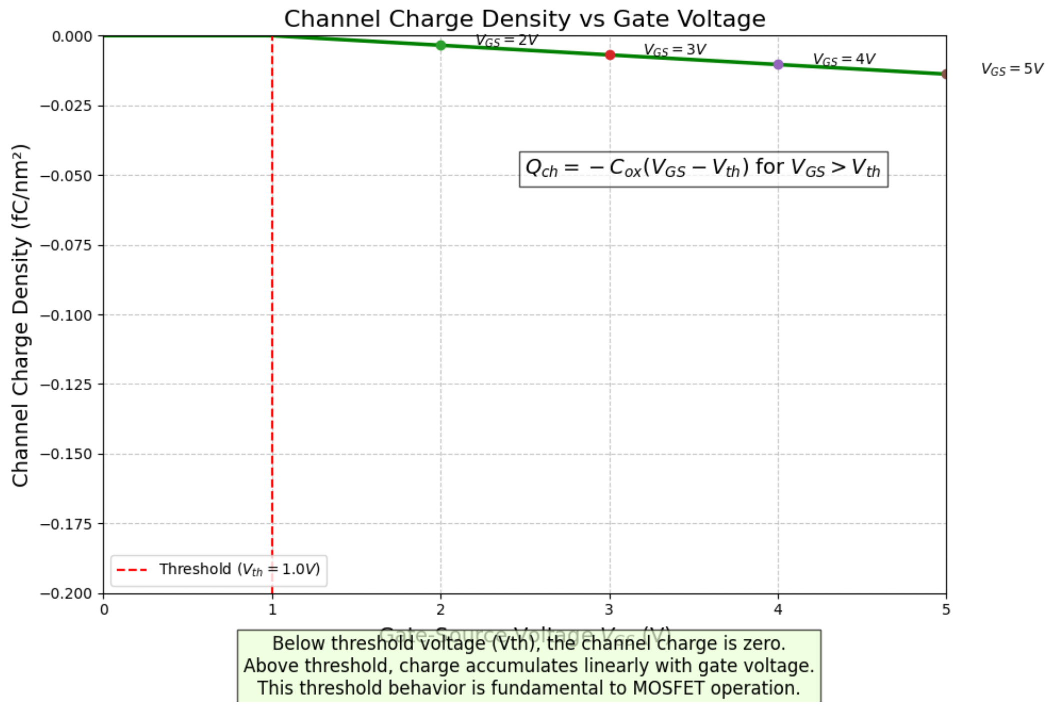


Figure 6: Channel charge density as a function of applied gate voltages. A similar trend is experienced with increasing magnitudes of E.

With the channel now formed, its useful to consider the charge mobilities of charge carriers moving along the channel. The drift velocity of charge for electrons can be expressed Gilbert [3]:

$$J_n = qn\mu_n E \tag{11}$$

The trend shows an increase of drift velocity as the Electric field increases. However, there is a limit due to an increased density in the channel. Consequently the charge mobility may decrease. Furthermore, scattering of charges may be experienced by impurities in the oxide, as well as collisions with the crystal structure. I did not delve any further than this point, but just wanted to mention it.

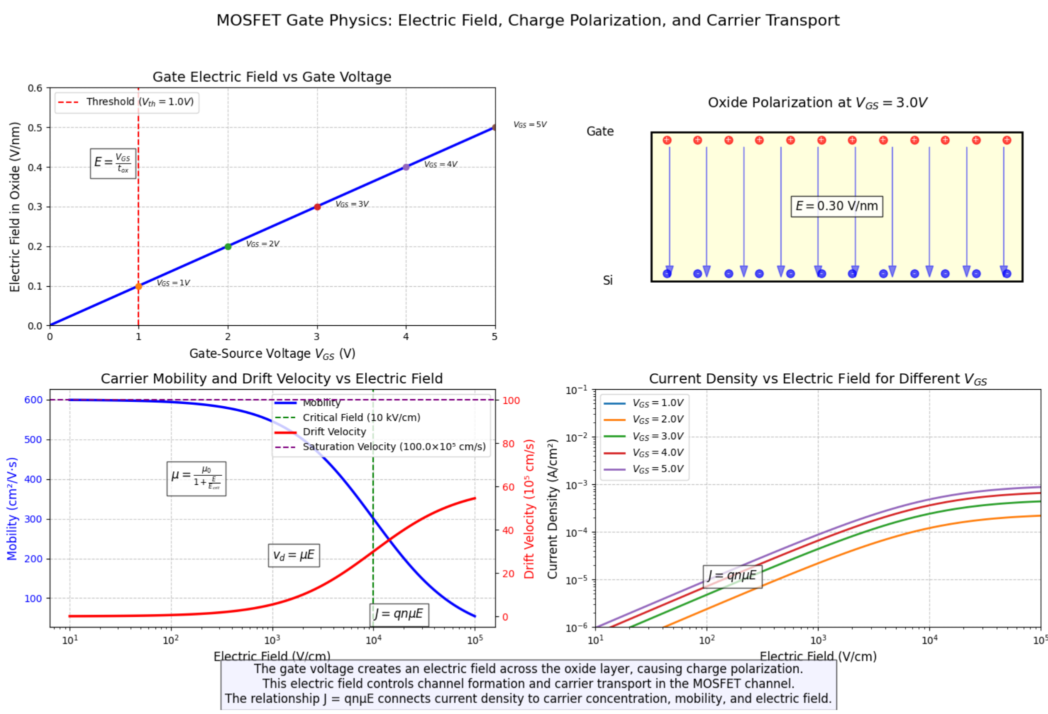


Figure 7: Plots related to the Electric field relationships with Current

References

[1] ElectroBOOM. This chip changed the world! (electroboom101 - 012), 2024. URL `https://www.youtube.com/watch?v=lyfx8CL7AkI&t=377s`.

[2] ElectronicsTutorials. the mosfet, 2021. URL `https://www.electronics-tutorials.ws/transistor/tran_6.html`.

[3] M. J. Gilbert. Ece340 lecture 41: Mosfets 2, 2012. URL `https://transport.ece.illinois.edu/ECE340F12-Lectures/ECE340Lecture41-MOSFET-II.pdf`.

[4] David Watson. What is mosfet? definition, full form, symbol working, 2018. URL `https://www.theengineeringprojects.com/2018/02/introduction-to-mosfet.html`.