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Math 172 Spring 2024

Written Assignment 4

Be sure to show all of your work. You may discuss these questions with other students or your TA, but electronic help is not permitted. This assignment will be submitted via Crowdmark.

1. Evaluate the definite integral $\int_1^{e^2} x^2 \ln(x)dx$.

Let $u = \ln(x)$ $dv = x^2 dx$

$du = \frac{1}{x} dx$ $v = \frac{1}{3}x^3$

$\int u\,dv = uv - \int v\,du$

$= \left[\ln(x) \cdot \frac{1}{3}x^3\right]_1^{e^2} - \int_1^{e^2} \frac{1}{3}x^3 \cdot \frac{1}{x} dx$

$= \left[\ln(e^2) \cdot \frac{1}{3}(e^2)^3\right] - \left[\ln(1) \cdot \frac{1}{3}(1)^3\right] - \int_1^{e^2} \frac{1}{3}x^2 dx$

$= \left[2 \cdot \frac{1}{3}e^6\right] - \left[0 \cdot \frac{1}{3}\right] - \left[\frac{1}{9}x^3\right]_1^{e^2}$

$= \frac{2}{3}e^6 - \left[\left(\frac{1}{9}(e^2)^3\right) - \left(\frac{1}{9}(1)^3\right)\right] = \frac{2}{3}e^6 - \frac{1}{9}e^6 + \frac{1}{9} = \boxed{\frac{5}{9}e^6 + \frac{1}{9}}$

2. Evaluate the indefinite integral $\int x^2 e^{3x} dx$.

Let $u_1 = x^2$ $dv_1 = e^{3x}dx$

$du_1 = 2x\,dx$ $v_1 = \frac{1}{3}e^{3x}$

$= x^2 \cdot \frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x} \cdot 2x\,dx$

Let $u_2 = 2x$ $dv_2 = \frac{1}{3}e^{3x}$

$du_2 = 2\,dx$ $v_2 = \frac{1}{9}e^{3x}$

$= \frac{1}{3}x^2e^{3x} - \left[2x \cdot \frac{1}{9}e^{3x} - \int \frac{1}{9}e^{3x} \cdot 2\,dx\right] + C$

$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

$= \boxed{e^{3x}\left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right) + C}$

3. Evaluate the indefinite integral $\int e^x \cos(4x)\, dx$.

Let $u_1 = \cos(4x)$ $dv_1 = e^x\, dx$
$du_1 = -4\sin(4x)\, dx$ $v_1 = e^x$

$$I = \cos(4x) \cdot e^x - \int e^x \cdot -4\sin(4x)\, dx$$

Let $u_2 = -4\sin(4x)$ $dv_2 = e^x\, dx$
$du_2 = -16\cos(4x)$ $v_2 = e^x$

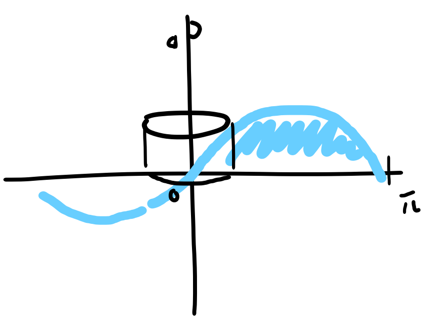
$$I = e^x\cos(4x) - \left[-4\sin(4x)\cdot e^x - \int e^x \cdot -16\cos(4x)\, dx\right] + C$$

$$I = e^x\cos(4x) + 4e^x\sin(4x) - 16\int e^x\cos(4x)\, dx + C$$

$$I = e^x\cos(4x) + 4e^x\sin(4x) - 16I + C$$

$$17I = e^x(4\sin(4x) + \cos(4x)) + C \longrightarrow I = \boxed{\frac{e^x(4\sin(4x) + \cos(4x))}{17}}$$

4. Find the volume of the solid generated when the region bounded by $f(x) = \sin(x)$ and the x-axis on $[0, \pi]$ is rotated about the y-axis.



Use shell method

Let $u = x$ $dv = \sin(x)\, dx$
$du = dx$ $v = -\cos(x)$

$$V = 2\pi\int_0^{\pi} x(\sin(x))\, dx$$

$$= 2\pi\left[\left[x \cdot -\cos(x)\right]_0^{\pi} - \int_0^{\pi} -\cos(x)\, dx\right]$$

$$= 2\pi\left\{\left[-x\cos(x)\right]_0^{\pi} + \sin(x)\Big|_0^{\pi}\right]$$

$$= 2\pi\left[\left((-\pi\overset{-1}{\cos(\pi)}) - (0)\right) + \left(\overset{0}{\sin(\pi)} - \overset{0}{\cancel{\sin(0)}}\right)\right]$$

$$= 2\pi(-\pi \cdot -1) = \boxed{2\pi^2 \text{ units}^3}$$