

Math 5125

HW 1

Due: Wednesday, September 1

1. Suppose $A \xrightarrow{f} B$ and $C \subseteq A$.
 - (a) Prove that $C \subseteq f^{-1}(f(C))$.
 - (b) Prove that $C = f^{-1}(f(C))$ if f is injective.
2. Let $a = 12$ and $b = 17$. Use the Euclidean algorithm to find $d = \gcd(a, b)$ and express d as a linear combination of a and b . Does a have a multiplicative inverse modulo b ? If so, what is $\bar{a}^{-1} \in \mathbb{Z}/b\mathbb{Z}$?
3. Let n be a positive integer. For any $\bar{a} \in \mathbb{Z}/n\mathbb{Z}$ (where $a \in \mathbb{Z}$), define the set

$$(\bar{a}) = \{\overline{ka} \mid k \in \mathbb{Z}\} \subseteq \mathbb{Z}/n\mathbb{Z}.$$

Prove that $(\bar{a}) = (\bar{d})$ where $d = \gcd(a, n)$.

4. Show that the set

$$S = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \mid a, b \in \mathbb{R} \right\}.$$

is *closed* under matrix addition and multiplication, i.e., $A, B \in S$ implies $A + B \in S$ and $AB \in S$. (Do the formulas for these operations look familiar?)