Phys 5405

HW 9

4.2 4.7(a,b) 4.8(a)

4.2 A point dipole with dipole moment $\mathbf{p}$ is located at the point $\mathbf{x}_0$. From the properties of the derivative of a Dirac delta function, show that for calculation of the potential Φ or the energy of a dipole in an external field, the dipole can be described by an effective charge density

$$\rho_{\text{eff}}(\mathbf{x}) = -\mathbf{p}\cdot\nabla\delta(\mathbf{x}-\mathbf{x}_0)$$

4.2 The potential from the dipole is given by

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int d^3x'\vec{p}(\vec{x}')\cdot\nabla'\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) \ . \tag{1}$$

The distribution of the dipole is a delta function,

$$\vec{p}(\vec{x}') = \vec{p}\,\delta(\vec{x}'-\vec{x}_0) \ . \tag{2}$$

Plug it into the formula of potential and integrate by parts,

$$\Phi(\vec{x}) = -\frac{1}{4\pi\epsilon_0}\int d^3x'\vec{p}\cdot\nabla'\delta(\vec{x}'-\vec{x}_0)\frac{1}{|\vec{x}-\vec{x}'|} \equiv \frac{1}{4\pi\epsilon_0}\int d^3x'\frac{\rho_{\text{eff}}(\vec{x}')}{|\vec{x}-\vec{x}'|} \ . \tag{3}$$

Therefore, the dipole can be described by an effective charge density,

$$\rho_{\text{eff}}(\vec{x}) = -\vec{p}\cdot\nabla\delta(\vec{x}-\vec{x}_0) \ . \tag{4}$$

4.7 A localized distribution of charge has a charge density

$$\rho(\mathbf{r}) = \frac{1}{64\pi} r^2 e^{-r} \sin^2\theta$$

(a) Make a multipole expansion of the potential due to this charge density and determine all the nonvanishing multipole moments. Write down the potential at large distances as a finite expansion in Legendre polynomials.
(b) Determine the potential explicitly at any point in space, and show that near the origin, correct to r^2 inclusive,

$$\Phi(\mathbf{r}) \simeq \frac{1}{4\pi\epsilon_0}\left[\frac{1}{4} - \frac{r^2}{120} P_2(\cos\theta)\right]$$

4.8 A very long, right circular, cylindrical shell of dielectric constant ϵ/ϵ_0 and inner and outer radii a and b, respectively, is placed in a previously uniform electric field E_0 with its axis perpendicular to the field. The medium inside and outside the cylinder has a dielectric constant of unity.
(a) Determine the potential and electric field in the three regions, neglecting end effects.