5.1 We have the differential expression

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \, d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \; . \tag{1}$$

Then

$$\mathbf{B} = \int d\mathbf{B} = \frac{\mu_0 I}{4\pi} \oint d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \; . \tag{2}$$

It suffices to show that

$$\oint d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = \nabla\Omega \; . \tag{3}$$

First, we can make use of the identity,

$$\boxed{\frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = \nabla'\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|}\right) \equiv \mathbf{V}} \; . \tag{4}$$

Then, consider each components

$$\boxed{\hat{\mathbf{x}}_i \cdot \oint d\mathbf{l}' \times \mathbf{V} = \oint d\mathbf{l}' \cdot (\mathbf{V} \times \hat{\mathbf{x}}_i) = \int_S d\mathbf{a}' \cdot (\nabla' \times (\mathbf{V} \times \hat{\mathbf{x}}_i))} \; . \tag{5}$$

Now we evaluate $\nabla' \times (\mathbf{V} \times \hat{\mathbf{x}}_i)$, using the identity

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} \; , \tag{6}$$

so we have

$$\boxed{\nabla' \times (\mathbf{V} \times \hat{\mathbf{x}}_i) = -\hat{\mathbf{x}}_i(\nabla' \cdot \mathbf{V}) + (\hat{\mathbf{x}}_i \cdot \nabla')\mathbf{V} = (\hat{\mathbf{x}}_i \cdot \nabla')\mathbf{V} = \frac{\partial}{\partial x_i'}\mathbf{V}} \; , \tag{7}$$

where we have used the fact that for $\mathbf{x} \neq \mathbf{x}'$,

$$\nabla' \cdot \mathbf{V} = \nabla'^2\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|}\right) = 0 \; . \tag{8}$$

Finally, we have achieved that

$$\hat{\mathbf{x}}_i \cdot \oint d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = \int_S d\mathbf{a}' \cdot \frac{\partial}{\partial x_i'}\left(\nabla'\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|}\right)\right) = -\frac{\partial}{\partial x_i}\int_S d\mathbf{a}' \cdot \nabla'\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|}\right) \tag{9}$$

From Jackson, the equation below (1.25), we have

$$d\mathbf{a}' \cdot \nabla'\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|}\right) = -d\Omega \; . \tag{10}$$

Therefore,

$$\hat{\mathbf{x}}_i \cdot \oint d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = \frac{\partial}{\partial x_i} \int_S d\Omega \ , \tag{11}$$

which implies that

$$\oint d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = \nabla\Omega \ . \tag{12}$$

5.3 In Jackson section 5.5, it already gives the magnetic induction for a circular current loop, For the position located on the symmetric axis of the loop, the magnetic induction is given by

$$B_z = \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}} \ , \tag{13}$$

where a is the radius of the loop and z is the distance between the center of the loop and the position where we measure the magnetic induction.

In this case, we have

$$B_z = \frac{\mu_0 NI}{2} a^2 \int dz \frac{1}{(a^2 + z^2)^{3/2}} \ . \tag{14}$$

Instead of using variable z, we use the variable θ such that

$$\tan \theta = a/z \ . \tag{15}$$

Then

$$\boxed{B_z = \frac{\mu_0 NI}{2} \int_{\pi-\theta_1}^{\theta_2} d(\cos \theta) = \frac{\mu_0 NI}{2} (\cos \theta_2 + \cos \theta_1) \ .} \tag{16}$$

3