Problem 1

The metric is given by

$$ds^2 = (1 + 2V)dt^2 - dx^2 - dy^2 - dz^2. \tag{1}$$

From the definition of the Christoffel symbols,

$$\Gamma^{\mu}_{\ \rho\sigma} = \frac{1}{2}g^{\mu\nu}\left(\frac{\partial g_{\nu\sigma}}{\partial x^{\rho}} + \frac{\partial g_{\rho\nu}}{\partial x^{\sigma}} - \frac{\partial g_{\rho\sigma}}{\partial x^{\nu}}\right), \tag{2}$$

we can easily calculate that, for $i = 1, 2, 3$,

$$\Gamma^{i}_{\ 00} = -\frac{1}{2}g^{ii}\frac{\partial g_{00}}{\partial x^{i}} = -\frac{1}{2}(-1)\partial_i(1 + 2V) = \partial_i V, \tag{3}$$

$$\Gamma^{0}_{\ i0} = \frac{1}{2}g^{00}\frac{\partial g_{00}}{\partial x^{i}} = \frac{\partial_i V}{1 + 2V}. \tag{4}$$

In the weak field limit, V is very small and we have

$$\Gamma^{i}_{\ 00} = \Gamma^{0}_{\ i0} = \partial_i V. \tag{5}$$