Matrix multiplication

- Matrix multiplication

Please take your time and learn about **Matrices** and **Matrix Multiplication**!

We will use these mathematics heavily in the next chapters

QUANTGATES
Renaissance Computation At Work

Matrix multiplication

Learn how to multiply matrices with **vectors** and other **matrices**.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ae+bf \\ ce+df \end{bmatrix}$$

Matrix vector multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Matrix multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \otimes \begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} = \begin{bmatrix} a\begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} & b\begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} \\ c\begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} & d\begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} ae_1 & ae_2 & be_1 & be_2 \\ ae_3 & ae_4 & be_3 & be_4 \\ ce_1 & ce_2 & de_1 & de_2 \\ ce_3 & ce_4 & de_3 & de_4 \end{bmatrix}$$

Tensor product of two matrices

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Now Try these;

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

This matrix is called **Not Gate** or **Pauli Matrix x**

Can you see what happened here? You just applied the Not Gate, and you **Transformed** your Qubit from Zero State to One State!

$|0> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, Zero Base of the Qubit

$|1> = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, Base One of your Qubit

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

This Matrix is called Hadamard and the vector is just base Zero

$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, Hadamard Gate

Exercise

Compute below tensor products.

$$H \otimes H$$

$$H \otimes H \otimes H$$

H X H = (1/sqrt(2))[1 1 ;1 -1] X (1/sqrt(2))[1 1; 1 -1]
H X H = (1/sqrt(2))[(1/sqrt(2))[1 1; 1 -1] (1/sqrt(2))[1 1; 1 -1]; 1 -1];
(1/sqrt(2))[1 1; 1 -1] -(1/sqrt(2))[1 1; 1 -1]; 1 -1]]
H X H = [(1/2)[1 1; 1 -1] (1/2)[1 1; 1 -1];
(1/2)[1 1; 1 -1] -(1/2)[1 1; 1 -1]]

H X H X H = (H X H) X H = [(1/2)[1 1; 1 -1] (1/2)[1 1; 1 -1];
(1/2)[1 1; 1 -1] -(1/2)[1 1; 1 -1]]
X (1/sqrt(2))[1 1; 1 -1]
since H = (1/sqrt(2))[1 1; 1 -1]
H X H X H = [(1/2)[H H; H -H] (1/2)[H H; H -H];
(1/2)[H H; H -H] -(1/2)[H H; H -H]]