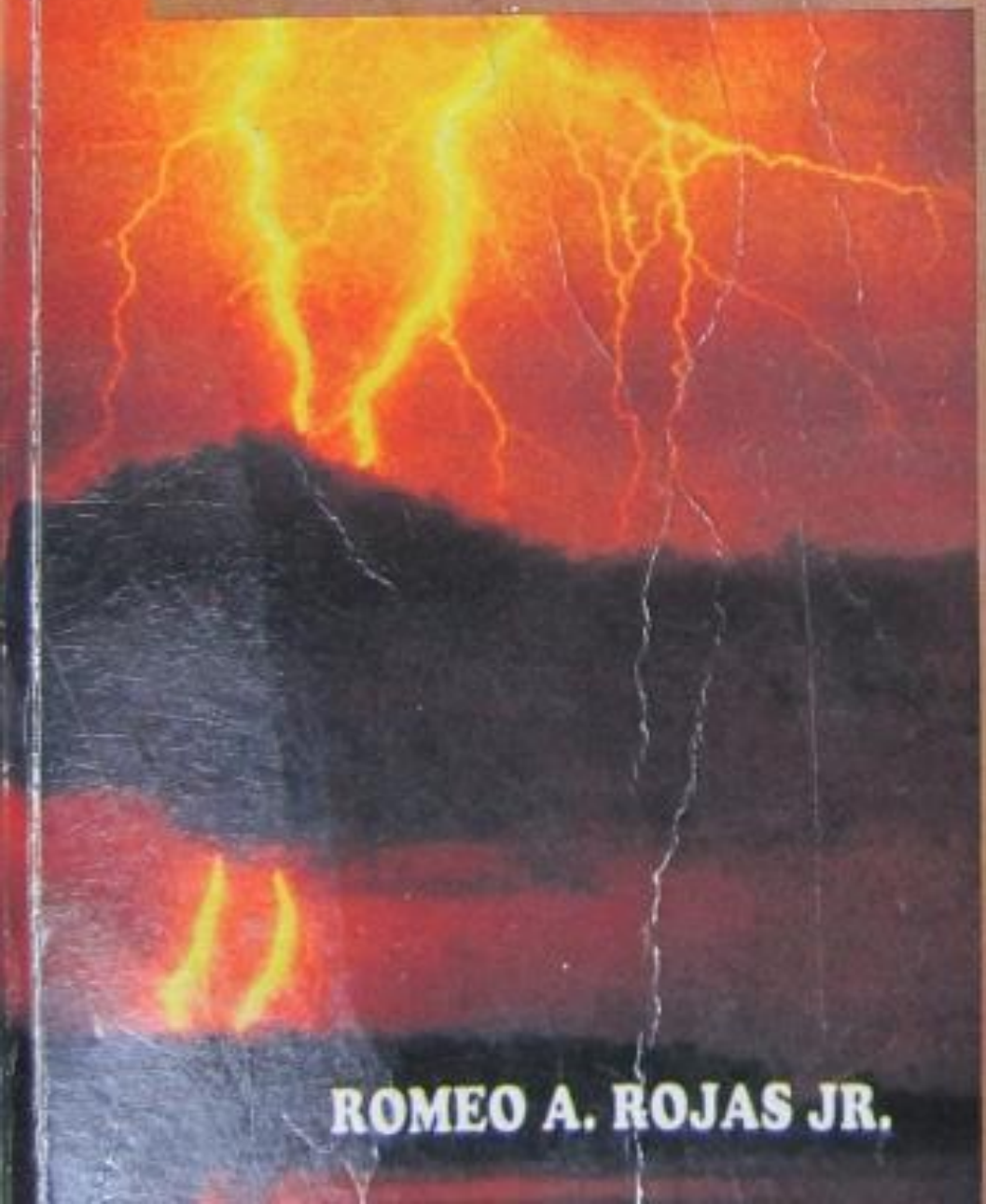
1001 Solved Problems in

ELECTRICAL ENGINEERING



Contains 1001 Solved Problems in

- DC/AC Circuits
- Electrostatics
- Electromagnetic Induction
- Magnetic Circuits
- Instrumentation
- DC Generators
- DC Motors
- Alternators
- Transformers
- Induction Motors
- Synchronous Motors
- Transmission Lines
- Faults
- Illumination
- Rectifiers & Converters
- Power Plants
- Misc. Problems

And Past Registered

ROMEO A. ROJAS JR.

IMPORTANT! PLEASE READ

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Thank You and God Bless!

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Electricity: Basic Principles

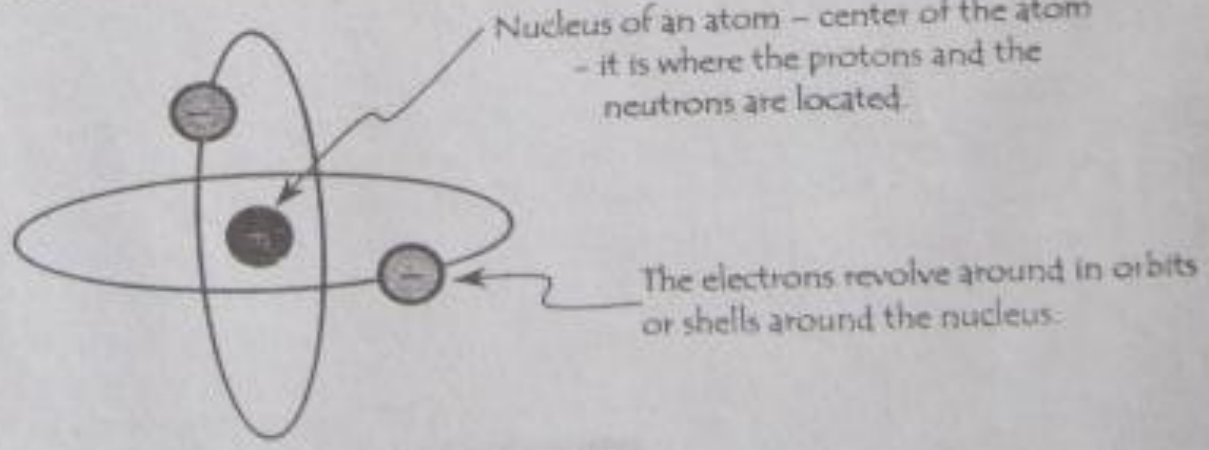
STRUCTURE OF MATTER

- **Matter** – anything that occupies space and has weight
- **Element** – a substance that cannot be decomposed any further by chemical action
- **Compound** – a combination of two or more elements
- **Molecule** – smallest particle that a compound can be reduced to before it breaks down into its elements.
- **Atom** - smallest part that an element can be reduced to and still keeping the properties of the element.

Parts of an atom:

Name	Charge	Mass (kg.)
Proton	Positive charge	1.672×10^{-27}
Electron	Negative charge	9.107×10^{-31}
Neutron	No charge	1.672×10^{-27}

The atomic structure of an atom:



$$N = 2n^2$$

where: N = total number of electrons on a given shell
n = n^{th} shell of the atom

- **Valence electrons** – electrons found in the outermost shell or orbit of an atom.
- **Atomic Number** – represents the number of electrons or protons of an atom.
- **Atomic Mass** – represents the sum of protons and neutrons of an atom.

Element	Number of electrons	Number of protons	Number of neutrons	Valence electron(s)
Copper	29	29	34	1
Aluminum	13	13	14	3
Germanium	32	32	41	4

ELECTRIC CHARGE

A body is said to be charge, if it has either an **excess or deficit of electrons** from its normal values due to sharing.

- **Coulomb** (C) – unit of electric charge, which is equivalent to **6.25×10^{18}** electrons or protons. Named after the French physicist, **Charles A. Coulomb** (1736 – 1806).

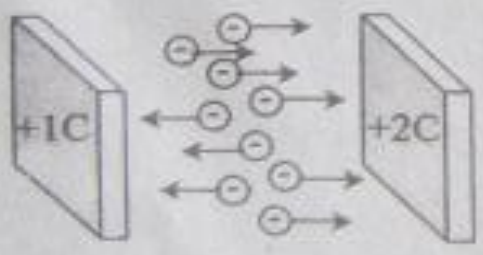
POTENTIAL DIFFERENCE

Potential – the capability of doing work

Any charge has the capability of **doing work of moving another charge** either by attraction or repulsion.

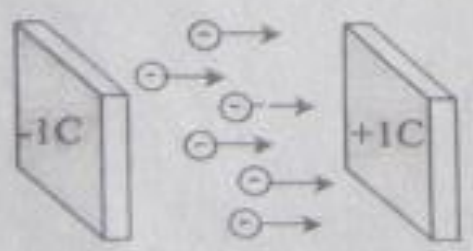
Example: Assume 1 C of charge can moved 3 electrons.

a.



6 electrons will be attracted by the +2 C plate and 3 electrons will be attracted by the +1 C plate, making a resultant motion of 3 electrons going towards the +2 C plate

b.



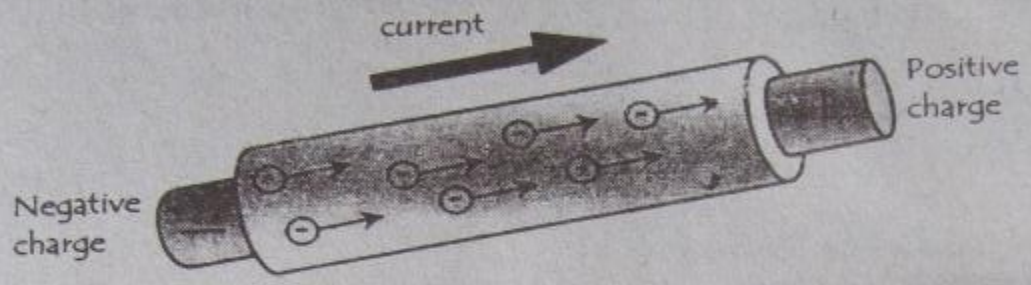
3 electrons will be attracted by the +1 C plate and 3 electrons will be repelled by the –1 C plate, making a resultant motion of 6 electrons going towards the +1 C plate

The **net number of electrons** moved in the direction of the positive charge plate depends upon the **potential difference** between the two charges.

- **Volt** (V) – unit of potential difference, which is equal to **one joule of work done per one coulomb of charge.** Named after the Italian physicist, **Alessandro C. Volta** (1754 – 1827) who invented the first electric battery.

ELECTRIC CURRENT

When a potential difference between two charges forces a third charge to move, the charge in motion is called an **electric current**.



- **Ampere** (A) – unit of charge flow equal to **one coulomb of charge** past a given point in **one second**. Named after the French physicist and mathematician, **Andre M. Ampere** (1775-1836).

RESISTANCE

The fact that a wire carrying a current can become hot, it is evident that the work done by the applied force in producing the current must be accomplished against some **opposition or resistance.**

- **Ohm** (Ω) – practical unit of resistance. Named after the German physicist, **Georg S. Ohm** (1787 – 1854).

$$R = \frac{\rho L}{A} \qquad R = \frac{\rho V}{A^2} \qquad R = \frac{\rho L^2}{V}$$

where: R = resistance (ohm)
A = cross-sectional area (square meter)
ρ = resistivity (ohm-meter)
L = length (meter)
V = volume cubic meter)

- **Specific resistance (resistivity)** – resistance offered by a unit cube of the material

- **Circular mil** (CM) – area of a circle having a diameter (d) of **one mil**.

$CM = d^2$

1,000 mil = 1 inch
1 MCM = 1,000 CM

EFFECT OF TEMPERATURE IN RESISTANCE

Experiments have shown that the resistance of all wires generally used in practice in electrical systems, increases as the temperature increases.

$$\frac{R_1}{R_2} = \frac{T + t_1}{T + t_2} \qquad \frac{R_2}{R_1} = 1 + \alpha_{t1}\Delta_t \qquad \alpha_{t1} = \frac{1}{T + t_1} \qquad \Delta_t = t_2 - t_1$$

where: R_1 = initial resistance (ohm)
R_2 = final resistance (ohm)
T = inferred absolute temperature
= temperature when resistance of a given material is zero.
t_1 = initial temperature
t_2 = final temperature
Δ_t = change in temperature
α = temperature coefficient of resistance

Material	ρ (Ω-CM / ft)	T (°C)	α at 20 °C
Silver	9.9	243	0.0038
Copper	10.37	234.5	0.00393
Aluminum	17	236	0.0039
Tungsten	33	202	0.0045
Zinc	36	250	0.0037

- **Temperature coefficient of resistance (α)** - ohmic change per degree per ohm at some specified temperature.

INSULATION RESISTANCE OF CABLES

$$R = \frac{\rho}{2\pi\ell}\ln\frac{r_2}{r_1}$$

where: ρ = resistivity of the insulating material (ohm-meter)
ℓ = length of cable (meter)

r_1 = radius of the cable
r_2 = radius to the outer surface of the insulation

CONDUCTANCE

- **Conductance** is a measure of the material's ability to conduct electric current. It is equal to the **reciprocal of resistance**.
- **Siemens** (formerly **mho**) – unit of conductance. Named after the German engineer, **Ernst Werner von Siemens** (1816-1892)

$$G = \frac{1}{R} \qquad G = \frac{\delta A}{L} \qquad \delta = \frac{1}{\rho}$$

where: δ = conductivity (siemens per meter)
L = length (meter)
A = cross-sectional area (square meter)
ρ = specific resistance (ohm-meter)
G = conductance (siemens)
R = resistance (ohm)

Test 1

Problem 1:

A positively charged dielectric has a charge of 2 coulombs. If 12.5×10^{18} free electrons are added to it, what will be the net charge on the said dielectric?

A. 4 C
B. -2 C
C. 8 C
D. Zero

Problem 2:

A battery can deliver 10 joules of energy to move 5 coulombs of charge. What is the potential difference between the terminals of the battery?

A. 2 V
B. 50 V
C. 0.5 V
D. 5 V

Problem 3:

A cloud of 2.5×10^{19} electrons move past a given point every 2 seconds. How much is the intensity of the electron flow?

A. 1 A
B. 2 A
C. 2.5 A
D. 1.5 A

Problem 4:

The current in an electric lamp is 5 amperes. What quantity of electricity flows towards the filament in 6 minutes?

A. 30 C
B. 3600 C
C. 72 C
D. 1800 C

Problem 5:

A constant current of 4 A charges a capacitor. How long will it take to accumulate a total charge of 8 coulombs on the plates?

A. 2 s
B. 32 s
C. 1/2 s
D. 12 s

Problem 6: EE Board March 1998

The substation bus bar is made up of 2-iches round copper bars 20 ft. long. What is the resistance of each bar if resistivity is 1.724×10^{-6} ohm-cm.

A. 7.21×10^{-5} ohm
B. 13.8×10^{-6} ohm
C. 5.185×10^{-5} ohm
D. 2.96×10^{-5} ohm

Problem 7: EE Board October 1997

Determine the resistance of a bus bar made of copper if the length is 10 meters long and the cross-section is a 4 x 4 cm^2. Use 1.7241 micro ohm-cm as the resistivity.

A. 2.121×10^{-4} ohm
B. 4.312×10^{-4} ohm
C. 3.431×10^{-5} ohm
D. 1.078×10^{-4} ohm

Problem 8:

Nichrome ribbon resistor elements each has a resistance of 1 ohm. The element is made from sheet of nichrome alloy, 0.025 cm thick. If the width of the ribbon is 0.3 cm, what length is required per element? Assume specific resistance of nichrome alloy to be 109 $\mu\Omega$-cm.

A. 68.8 cm
B. 62.1 cm
C. 70.7 cm
D. 67.4 cm

Problem 9: EE Board October 1991

One turn of a copper bar is produced by cutting a copper washer along a radius and spreading the ends. The washer is cut from soft-drawn copper having a resistivity at 20°C of 1.732×10^{-6} ohm-cm. The washer is 0.125 inch thick and has inside diameter and outside diameter of 1 inch and 9 inches respectively. Calculate the exact resistance between the two ends of the turn to direct current, taking into account the non-uniform current distribution. Assume the contact along the ends of the turn to be perfect over the entire cross-section.

A. 12.74×10^{-6} ohm
B. 15.53×10^{-6} ohm
C. 17.22×10^{-6} ohm

D. 14.83×10^{-6} ohm

Problem 10: EE Board October 1990

Determine the resistance of a conductor 0.10 m long, with a uniform diameter of 1.0 cm and having a resistivity which varies as a function of length L measured from one end of the conductor according to the formula: $\rho = 0.003 + 10^{-4}L^2$ ohm-cm.

A. 0.0852 ohm
B. 0.0915 ohm
C. 0.0806 ohm
D. 0.0902 ohm

Problem 11: EE Board April 1992

A coil has 6,000 turns of wire and a resistance of 380 ohms. The coil is rewound with the same quantity (weight) of wire, but has 13,400 turns. How many ohms will the new coil have?

A. 1895 ohms
B. 1825 ohms
C. 1792 ohms
D. 1905 ohms

Problem 12: EE Board April 1992

A copper wire of unknown length has a resistance of 0.80 ohm. By successive passes through drawing dies, the length of the wire is increased to 2 ½ times its original value. Assuming that resistivity remains unchanged during the drawing process, determine the new value of its resistance.

A. 4 Ω
B. 3 Ω
C. 5 Ω
D. 6 Ω

Problem 13: EE Board October 1998

A one-meter rod of 2-cm diameter is drawn until its resistance is 100 times the initial resistance. Its length afterward is?

A. 10 m
B. 100 m
C. 12.5 m
D. 5 m

Problem 14: EE Board April 1993

A kilometer of wire having a diameter of 11.7 mm and a resistance of 0.031 ohm is drawn down so that its diameter is 5.0 mm. What does its resistance become?

A. 0.85 ohm

B. 0.78 ohm
C. 0.93 ohm
D. 0.81 ohm

Problem 15:

A conductor whose diameter is 0.175 inch has a resistance of 0.5 ohm. The wire is drawn through a series of dies until its diameter is reduced to 0.08 inch. Assuming the specific resistance of the material remains constant, what is the resistance of lengthened conductor?

A. 11.45 Ω
B. 10.22 Ω
C. 12.75 Ω
D. 10.82 Ω

Problem 16: EE Board April 1995

A certain wire has a resistance R. The resistance of another wire identical with the first except for having twice its diameter is

A. 4R
B. $\frac{1}{2}R$
C. 2R
D. $\frac{1}{4}R$

Problem 17:

A given wire has a resistance of 17.5 ohms. If its length is 560 m, how much length must be cut-off from the wire in order to reduce its resistance to 12.5 ohms.

A. 160 m
B. 170 m
C. 145 m
D. 155 m

Problem 18: EE Board October 1996

What is the size in square millimeter (mm^2) is the cable of 250 MCM size?

A. 118.656 mm^2
B. 126.675 mm^2
C. 112.565 mm^2
D. 132.348 mm^2

Problem 19:

A 500 MCM ACSR cable has 37 strands. Determine the diameter in mils of each strand.

A. 116.25
B. 120.24
C. 118.34
D. 110.35

Problem 20:

A copper transmission line is to be replaced by one of aluminum having the same total resistance. If the cross sectional area of the copper wire is 500 MCM, what would be the cross sectional area of the new aluminum wire?

A. 800 MCM
B. 820 MCM
C. 850 MCM
D. 900 MCM

Problem 21: EE Board October 1998

The resistance of a copper wire at 30 °C is 50 ohms. If the temperature coefficient of copper at 0 °C is 0.00427, what is the resistance at 100 °C?

A. 72.26 ohms
B. 54.25 ohms
C. 63.24 ohms
D. 58.15 ohms

Problem 22:

The shunt field winding of a shunt generator has a resistance of 80 ohms at 20°C. After several hours of continuous operation, the winding temperature rises to 50°C. How much is the winding resistance under this condition. Assume the resistance temperature coefficient of copper to be 0.004 ohm per degree at 0°C.

A. 88.89 Ω
B. 90.12 Ω
C. 85.22 Ω
D. 92.81 Ω

Problem 23:

The field windings of a series motor has a resistance of 120 ohms at 15°C. After running for two hours, the resistance increases to 140 ohms. If the temperature coefficient of the windings at 0°C is 0.004, find the new temperature of the windings.

A. 60.4 °C
B. 59.2 °C
C. 58.4 °C
D. 53.7 °C

Problem 24: EE Board March 1998

The resistance of a wire is 126.48 ohms at 100 °C and 100 ohms at 30 °C. Determine the temperature coefficient of copper at 0 °C.

A. 0.00427
B. 0.00615
C. 0.0256
D. 0.0356

Problem 25:

Two wires A & B made from two different materials have temperature coefficient of resistance equal to 0.0025 and 0.0005 ohm per °C, respectively. It is desired to make a coil of wire having a resistance of 1200 ohms with a temperature coefficient of 0.001, using suitable lengths of the two given wires connected in series. Determine the required length of wire A?

A. 5.5 m
B. 6.2 m
C. 6.0 m
D. 5.0 m

Problem 26: EE Board October 1991

Two heating elements which is 500 ohms and 250 ohms are connected in series with temperature coefficients of 0.001 and 0.003 ohms per °C, respectively at 20 °C Calculate the effective temperature coefficient of the combination.

A. 0.00215
B. 0.00626
C. 0.00712
D. 0.00167

Problem 27:

A cylindrical rubber insulated cable has a diameter of 0.18 inch and an insulation thickness of 0.25 inch. If the specific resistance of rubber is 10^{14} ohm-cm, determine the insulation resistance per 1000-ft length of the cable.

A. 624 MΩ
B. 682 MΩ
C. 678 MΩ
D. 694 MΩ

Problem 28:

The diameter of a given bare conductor is 0.50 inch. A thermoplastic insulation with thickness of 0.1 inch is wrapped around to insulate the conductor. Determine the insulation resistance of this conductor per meter. Assume specific resistance of thermoplastic insulation to be 2×10^{14} ohm - cm.

A. $107 \times 10^9 \Omega$

B. $105 \times 10^9 \Omega$

C. $110 \times 10^9 \Omega$

D. $109 \times 10^9 \Omega$

Problem 29: EE Board October 1992

The insulation resistance of a kilometer of the cable having a diameter of 2 cm and an insulation thickness of 2 cm is 600 ohms. If the thickness of the insulation is increased to 3 cm, find the insulation resistance of the cable.

A. 725 ohms
B. 850 ohms
C. 757 ohms
D. 828 ohms

Problem 30: EE Board October 1998

The resistance of 120 meters of wire is 12 ohms. What is its conductance?

A. 0.0521 mho
B. 0.083 mho
C. 6 mhos
D. 12 mhos

ANSWER KEY

1. D	7. D	13. A	19. A	25. C
2. A	8. A	14. C	20. B	26. D
3. B	9. B	15. A	21. C	27. D
4. D	10. C	16. D	22. A	28. A
5. A	11. A	17. A	23. B	29. C
6. C	12. C	18. B	24. A	30. B

RATING

27-30 - Topnotcher

21-26 - Passer

15-20 - Conditional

0-14 - Failed

Solutions to Test 1

1

$Q_1 = +2\,C$

$Q_2 = -12.5 \times 10^{18} e \times \dfrac{1C}{6.25 \times 10^{18} e} = -2\,C$

$Q_{net} = Q_1 + Q_2 = 2 + (-2)$

$Q_{net} = 0$

2

Note: volt = joules per coulomb

$E = \dfrac{W}{Q} = \dfrac{10}{5}$

$E = 2$ volts

3

Note: ampere = coulombs per second

$Q = 2.5 \times 10^{19} \times \dfrac{1C}{6.25 \times 10^{18}} = 4\,C$

$I = \dfrac{Q}{t} = \dfrac{4}{2}$

$I = 2$ amperes

4

$Q = It = 5\left(6\,\text{min} \times \dfrac{60\,\text{sec}}{1\,\text{min}}\right)$

$Q = 1800\,C$

5

$t = \dfrac{Q}{I} = \dfrac{8}{4}$

$t = 2$ sec

6

$A = \dfrac{\pi d^2}{4} = \dfrac{\pi\left(2\,\text{inch} \times \dfrac{2.54\,\text{cm}}{1\,\text{inch}}\right)^2}{4} = 20.268\,\text{cm}^2$

$L = 20\,\text{ft} \times \dfrac{12\,\text{inch}}{1\,\text{ft}} \times \dfrac{2.54\,\text{cm}}{1\,\text{inch}} = 609.6\,\text{cm}$

$R = \rho\dfrac{L}{A} = \dfrac{(1.724 \times 10^{-6})(609.6)}{20.268}$

$R = 5.185 \times 10^{-5}$ ohm

7

$$L = 10\,m \times \frac{100\,cm}{1\,m} = 1000\,cm$$

$$R = \rho\frac{L}{A} = \frac{(1.7241 \times 10^{-6})(1000)}{(4 \times 4)}$$

$$R = 107.75 \times 10^{-6}\ ohm$$

8

$$L = \frac{RA}{\rho} = \frac{(1)(0.025 \times 0.3)}{109 \times 10^{-6}}$$

$$L = 68.8\,cm$$

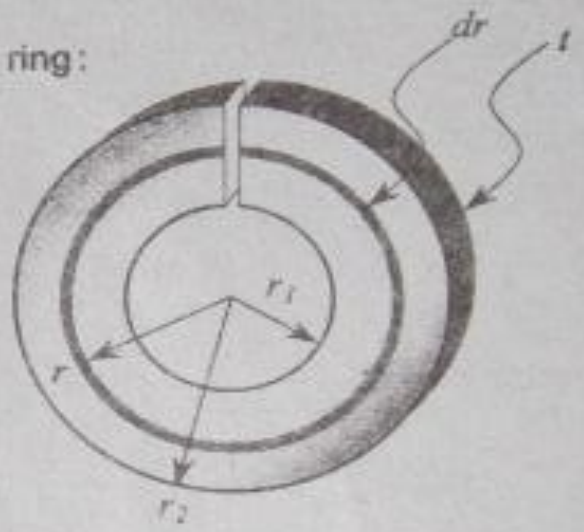
9

$$R = \rho\frac{L}{A}$$

Consider a differential element ring:

$$A = t\,dr$$

$$L = 2\pi r$$



$$dR = \frac{\rho(2\pi r)}{t\,dr}$$

$$G = \frac{1}{R}$$

$$dG = \frac{t\,dr}{\rho(2\pi r)}$$

$$G = \frac{t}{2\pi\rho}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{t}{2\pi\rho}\ln r\Big|_{r_1}^{r_2} = \frac{t}{2\pi\rho}(\ln r_2 - \ln r_1)$$

$$G = \frac{0.125\,inch \times \left(\frac{2.54\,cm}{1\,inch}\right)}{2\pi(1.724 \times 10^{-6})}(\ln 4.5 - \ln 0.5)$$

$$G = 64402.25\,mho$$

$$R = \frac{1}{G} = \frac{1}{64402.25}$$

$$R = 15.527 \times 10^{-6}\ \Omega$$

10

$$A = \frac{\pi d^2}{4} = \frac{\pi(1)^2}{4} = \frac{\pi}{4}\,cm^2$$

$$L = 0.1\,m = 10\,cm$$

$$R = \rho\frac{L}{A}$$

$$R = \frac{1}{A}\int_0^{10} \rho\, dL = \frac{1}{A}\int_0^{10} (0.003 + 10^{-4}L^2)\, dL$$

$$R = \frac{4}{\pi}\left[0.003L + 10^{-4}\left(\frac{L^3}{3}\right)\right]_0^{10} = \frac{4}{\pi}\left[0.003(10) + 10^{-4}\left(\frac{10^3}{3}\right)\right]$$

$$R = 0.0806\,\Omega$$

11 $R = \rho\dfrac{L^2}{V}$

Note: Since the coil is rewound with the same weight of wire, thus the volumes of the wire used in both conditions are constant. And with the volume constant, resistance varies directly as the square of the length.

$R = kL^2$; k = proportionality constant

$$\frac{R_2}{R_1} = \left(\frac{L_2}{L_1}\right)^2$$

$$\frac{R_2}{380} = \left(\frac{\text{length per turn} \times N_2}{\text{length per turn} \times N_1}\right)^2 = \left(\frac{13400}{6000}\right)^2$$

$$R_2 = 1895.35\,\Omega$$

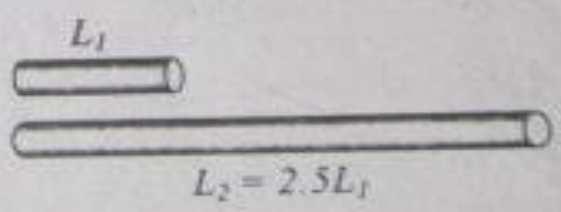
12 Note: Assuming there are no losses in the process, the volume of the material remains constant. And with the volume constant, the resistance of the material varies directly as the square of the length.

$R = kL^2$; k = proportionality constant

$$\frac{R_2}{R_1} = \left(\frac{L_2}{L_1}\right)^2$$

$$R_2 = R_1\left(\frac{L_2}{L_1}\right)^2 = 0.80\left(\frac{2.5L_1}{L_1}\right)^2$$

$$R_2 = 5\,\Omega$$

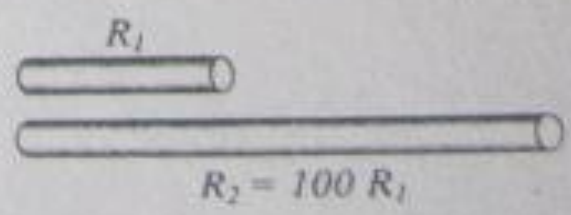


13 $R = kL^2$; k = proportionality constant

$$\frac{R_2}{R_1} = \left(\frac{L_2}{L_1}\right)^2$$

$$L_2 = L_1\sqrt{\frac{R_2}{R_1}} = 1\sqrt{\frac{100R_1}{R_1}}$$

$$L_2 = 10\,\text{m}$$



14 $R = \rho \frac{V}{A^2} = \frac{\rho V}{\left(\frac{\pi}{4} d^2\right)^2} = k' \frac{V}{d^4}$

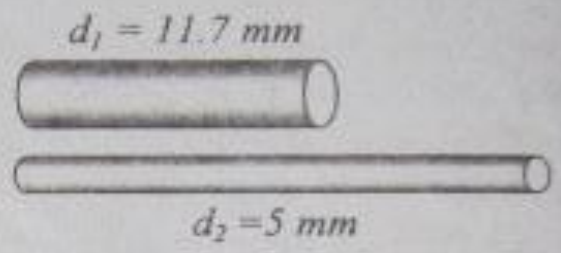
Note: Assuming there are no losses in the process, the volume of the material remains constant. And with the volume constant, the resistance of the material varies inversely as the fourth power of the diameter.

$R = \frac{k}{d^4}$; k = proportionality constant

$\frac{R_2}{R_1} = \left(\frac{d_1}{d_2}\right)^4$

$R_2 = R_1 \left(\frac{d_1}{d_2}\right)^4 = 0.031 \left(\frac{11.7}{5}\right)^4$

$R_2 = 0.93\,\Omega$



15 $R = \frac{k}{d^4}$; k = proportionality constant

$\frac{R_2}{R_1} = \left(\frac{d_1}{d_2}\right)^4$

$R_2 = R_1 \left(\frac{d_1}{d_2}\right)^4 = 0.5 \left(\frac{0.175}{0.08}\right)^4$

$R_2 = 11.45\,\Omega$

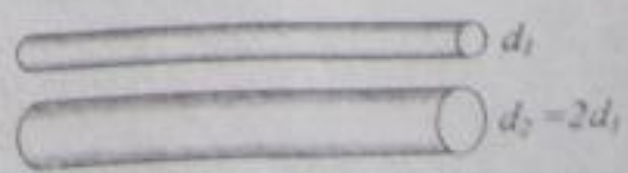
16 $R = \rho \frac{L}{A} = \frac{\rho L}{\frac{\pi}{4} d^2} = k' \frac{L}{d^2}$

Note: Assume the length of the two wires in both conditions to be constant. And with the length constant, the resistance of the wire varies inversely as the square of the diameter.

$R = \frac{k}{d^2}$; k = proportionality constant

$\frac{R_2}{R_1} = \left(\frac{d_1}{d_2}\right)^2$

$R_2 = R_1 \left(\frac{d_1}{d_2}\right)^2 = R \left(\frac{d_1}{2 d_1}\right)^2$



17 $R = \rho \frac{L}{A}$

Note: Assume the cross sectional area of the wire to be constant. And with the cross sectional area constant, resistance of the wire varies directly as the length.

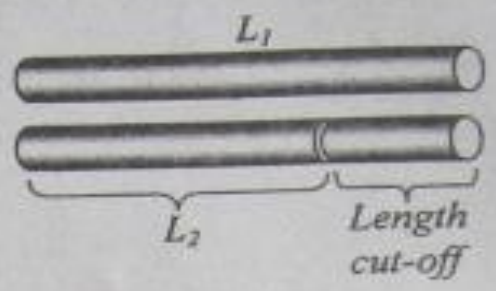
$R = kL$; k = proportionality constant

$\frac{R_2}{R_1} = \frac{L_2}{L_1}$

$L_2 = \frac{L_1 R_2}{R_1} = \frac{560(12.5)}{17.5} = 400\text{ m}$

Length cut - off $= L_1 - L_2 = 560 - 400$

Length cut - off $= 160\text{ m}$



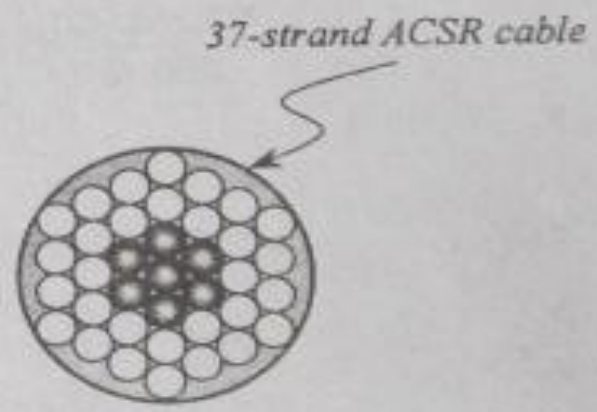
18 250 MCM = 250,000 CM

$CM = d^2$

$d = \sqrt{CM} = \sqrt{250{,}000} = 500\text{ mils} \times \frac{1\text{ inch}}{1000\text{ mils}} \times \frac{25.4\text{ mm}}{1\text{ inch}} = 12.7\text{ mm}$

$A = \frac{\pi d^2}{4} = \frac{\pi(12.7)^2}{4}$

$A = 126.67\text{ mm}^2$



19 500 MCM = 500,000 CM

$A_{strand} = \frac{500{,}000}{37} = 13513.51\text{ CM}$

$CM = d^2$

$d = \sqrt{CM} = \sqrt{13513.51}$

$d = 116.25\text{ mils}$

20 Note: Lengths in both conditions are the same.

$R_a = \frac{\rho_a L_a}{A_a}$; $\rho_a = 17\ \Omega\text{-CM per ft}$

$R_c = \frac{\rho_c L_c}{A_c}$; $\rho_c = 10.37\ \Omega\text{-CM per ft}$

$R_a = R_C$

$\frac{\rho_a L_a}{A_a} = \frac{\rho_c L_c}{A_c}$

$$A_a = \frac{A_c \rho_a}{\rho_c} = \frac{500\,(17)}{10.37} = 819.672$$

$$A_a \cong 820\ \text{CM}$$

21

$$\alpha_0 = \frac{1}{T}$$

$$T = \frac{1}{0.00427} = 234.192\ °\text{C}$$

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

$$R_2 = R_1\left(\frac{T + t_2}{T + t_1}\right) = \frac{50(234.192 + 100)}{234.192 + 30}$$

$$R_2 = 63.24\ \Omega$$

22

$$T = \frac{1}{\alpha_0} = \frac{1}{0.004} = 250\ °\text{C}$$

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

$$R_2 = R_1\left(\frac{T + t_2}{T + t_1}\right) = \frac{80(250 + 50)}{250 + 20}$$

$$R_2 = 88.89\ \Omega$$

23

$$T = \frac{1}{\alpha_0} = \frac{1}{0.004} = 250\ °\text{C}$$

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

$$t_2 = (T + t_1)\left(\frac{R_2}{R_1}\right) - T$$

$$= (250 + 15)\left(\frac{140}{120}\right) - 250$$

$$t_2 = 59.2\ °\text{C}$$

24

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

$$\frac{100}{126.48} = \frac{T + 30}{T + 100}$$

$$T + 30 = 0.7906\,T + 79.06$$

$$T = 234.288\ °\text{C}$$

$$\alpha_0 = \frac{1}{T} = \frac{1}{234.288}$$
$$\alpha_0 = 0.00427$$

25

$$R_t = R_a(1+0.0025\Delta_t) + R_b(1+0.0005\Delta_t) \Rightarrow ❶$$
$$R_t = (R_a + R_b)(1+0.001\Delta_t) \rightarrow ❷$$

Equate Eq.1 to Eq.2 :

$$R_a(1+0.0025\Delta_t) + R_b(1+0.0005\Delta_t) = (R_a + R_b)(1+0.001\Delta_t)$$
$$0.0015\,R_a\Delta_t = 0.0005\,R_b\Delta_t$$
$$R_b = 3\,R_b$$

$$R_a + R_b = 1200$$
$$R_a + 3\,R_a = 1200$$
$$R_a = 300\,\Omega$$
$$L_a = \frac{300}{50}$$
$$L_a = 6\,m$$

26

$$R_t = 500\,(1+0.001\Delta_t) + 250(1+0.003\Delta_t) \Rightarrow ❶$$
$$R_t = (500+250)(1+\alpha_{eff}\Delta_t) \Rightarrow ❷$$

Equate Eq.1 to Eq.2 :

$$500\,(1+0.001\Delta_t) + 250(1+0.003\Delta_t) = (750)(1+\alpha_{eff}\Delta_t)$$
$$500 + 0.5\Delta_t + 250 + 0.75\Delta_t = 750 + 750\,\alpha_{eff}\Delta_t$$
$$\alpha_{eff} = 0.00167$$

27

$$R = \frac{\rho}{2\pi\ell}\ln\frac{r_2}{r_1}$$
$$R = \frac{1\times10^{14}}{2\pi\left(1000\text{ ft}\times\frac{100\text{ cm}}{3.281\text{ ft}}\right)}\ln\left(\frac{0.34}{0.09}\right)$$
$$R = 694.05\,M\Omega$$

28

$$R = \frac{\rho}{2\pi\ell}\ln\frac{r_2}{r_1}$$
$$R = \frac{2\times10^{14}}{2\pi(100\text{ cm})}\ln\left(\frac{0.35}{0.25}\right)$$
$$R = 107.102\times10^{9}\,\Omega$$

29 Note: Specific resistance of the cable and length are constant in both conditions

$$R = \frac{\rho}{2\pi\ell} \ln \frac{r_2}{r_1}$$

$$\frac{R_1}{R_2} = \frac{\ln \frac{r_2}{r_1}}{\ln \frac{r_2'}{r_1'}}$$

$$R_2 = R_1 \left(\frac{\ln \frac{r_2'}{r_1'}}{\ln \frac{r_2}{r_1}} \right) = 600 \left(\frac{\ln \frac{4}{1}}{\ln \frac{3}{1}} \right)$$

$$R_2 = 757\ \Omega$$

30 $$G = \frac{1}{R} = \frac{1}{12}$$

$$G = 0.0833 \text{ mho}$$

DC Electric Circuits

OHM'S LAW

Ohm's Law states that the current flowing in an electric circuit is **directly proportional** to the **impressed emf** applied to the circuit and **inversely** to the **equivalent resistance** of the said circuit. Named after the German physicist, **Georg S. Ohm** (1787 – 1854).

$$I = \frac{E}{R} \qquad R = \frac{E}{I} \qquad E = IR$$

where: E = impressed voltage (volt)
I = current drawn (ampere)
R = resistance (ohm)

ELECTRICAL POWER

- **Electrical power** – rate of using or consuming the electrical energy
- **Watt** – unit of electrical energy equal to **one joule of energy** consumed in **one second.** Named after the British engineer and inventor **James Watt** (1736 – 1819).

$$P = EI \qquad P = I^2R \qquad P = \frac{E^2}{R}$$

where: P = electrical power (watt)
E = voltage (volt)
I = current (ampere)
R = resistance (ohm)

ELECTRICAL & HEAT ENERGY EQUIVALENTS

❖ **Energy** – the capacity to do work

$$W = Pt \qquad Q = 0.24Pt \qquad Q = mc\Delta_T$$

where: W = electrical energy (joule)
Q = heat energy (calorie)
P = electrical power (watt)
t = time (second)
m = mass of the material (gram)
c = specific heat of the material (calorie per gram -°C)
Δ_T = change in temperature of the material (°C)

Note: 1 calorie ≈ 4.186 joules
1 BTU ≈ 252 calories

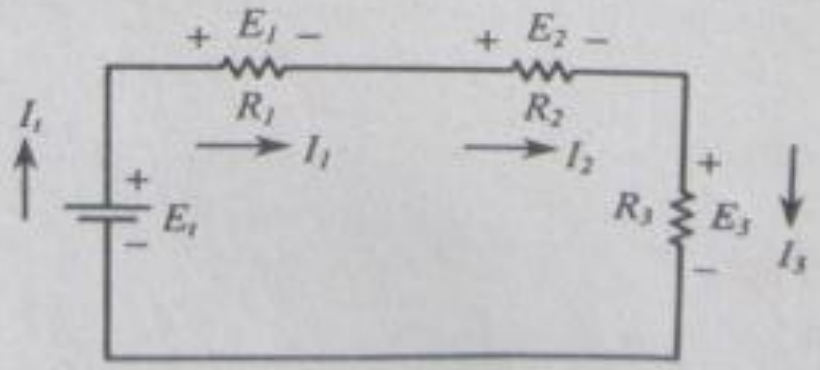
❖ **Kilowatt-hour** (kW-hr) – unit in which electrical energy is sold to a customer.

$$kW - hr = kilowatts \times hours$$

Note: 1 day = 24 hours
1 month = 30 days
1 year = 365 days = 8760 hours

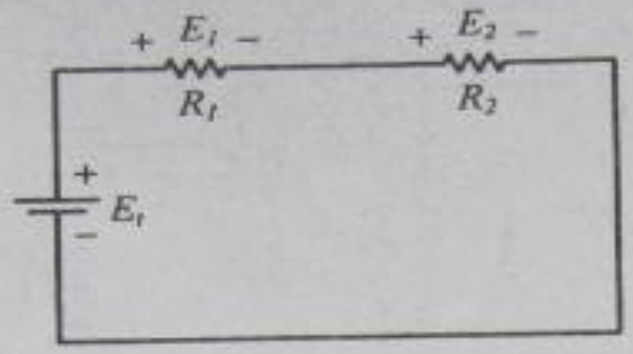
SERIES-CONNECTED RESISTORS

❖ **Series circuit** - the resistances are connected end to end.



$$R_t = R_1 + R_2 + R_3 \qquad E_t = E_1 + E_2 + E_3 \qquad I_t = I_1 = I_2 = I_3$$

VOLTAGE DIVISION THEOREM (VDT) FOR RESISTORS CONNECTED IN SERIES



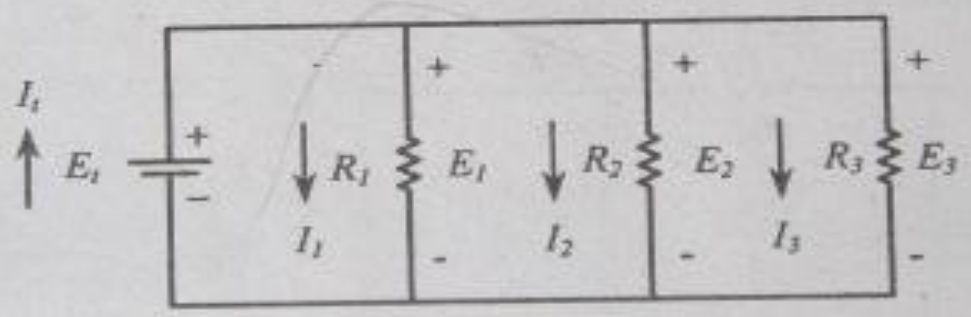
$$E_1 = \frac{E_t R_1}{R_1 + R_2}$$

$$E_2 = \frac{E_t R_2}{R_1 + R_2}$$

Note: If there are three or more resistances in series, reduce first the circuit into two resistors in series before applying the VDT.

PARALLEL-CONNECTED RESISTORS

- **Parallel circuit** – the resistances are connected across each other.

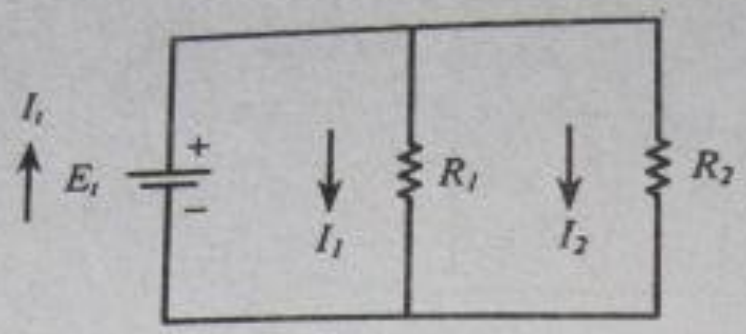


$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad E_t = E_1 = E_2 = E_3 \qquad I_t = I_1 + I_2 + I_3$$

Note: Independent of the circuit connection either series, parallel or combination of both, the total power drawn by the circuit is equivalent to the powers drawn by each load resistor.

$$P_t = P_1 + P_2 + P_3 + \ldots\ldots + P_n$$

CURRENT DIVISION THEOREM (CDT) FOR RESISTORS CONNECTED IN PARALLEL



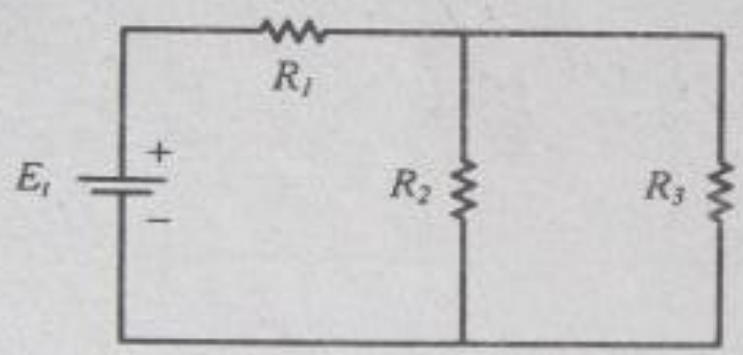
$$I_1 = \frac{I_t R_2}{R_1 + R_2}$$

$$I_2 = \frac{I_t R_1}{R_1 + R_2}$$

Note: If there are three or more resistances in parallel, reduce first the circuit into two resistors in parallel before applying the CDT.

SERIES-PARALLEL CONNECTED RESISTORS

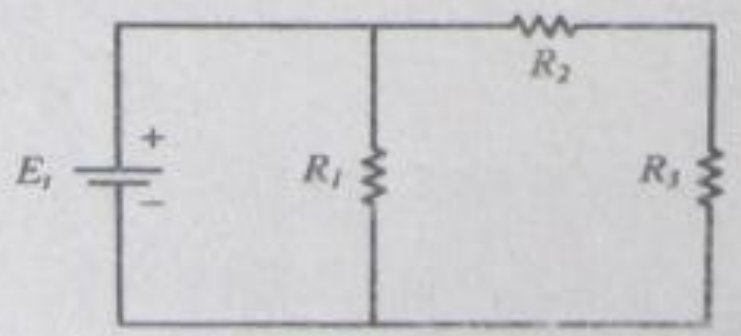
- **Series-parallel circuit** – a combinational circuit which when simplified will result into a series circuit.



$$R_t = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

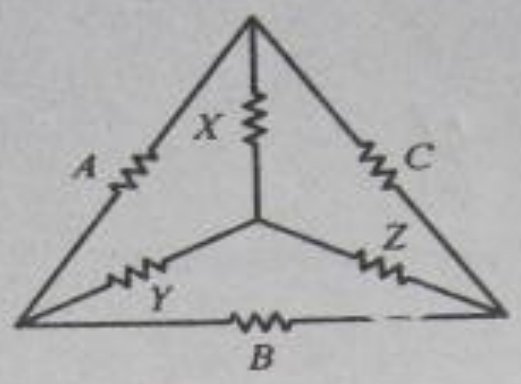
PARALLEL-SERIES CONNECTED RESISTORS

- **Parallel-series circuit** – a combinational circuit which when simplified will result into a parallel circuit.



$$R_t = \frac{R_1(R_2 + R_3)}{R_1 + (R_2 + R_3)}$$

DELTA & WYE-CONNECTED RESISTORS



- **Wye to delta transformations**

$$A = \frac{XY + YZ + ZX}{Z} \qquad B = \frac{XY + YZ + ZX}{X} \qquad C = \frac{XY + YZ + ZX}{Y}$$

- **Delta to wye transformations**

$$X = \frac{AC}{A + B + C} \qquad Y = \frac{AB}{A + B + C} \qquad Z = \frac{BC}{A + B + C}$$

- **If $A = B = C = R_\Delta$ and $X = Y = Z = R_y$**

$$R_\Delta = 3R_Y \qquad R_y = \frac{R_\Delta}{3}$$

Test 2

Problem 31: EE Board October 1997

A load of 10 ohms was connected to a 12-volt battery. The current drawn was 1.18 amperes. What is the internal resistance of the battery?

A. 0.35 ohm
B. 0.20 ohm
C. 0.25 ohm
D. 0.30 ohm

Problem 32:

The potential at the terminals of the battery falls from 9 V on open circuit to 6 volts when a resistor of 10 ohms is connected across its terminals. What is the internal resistance of the battery?

A. 5 Ω
B. 4 Ω
C. 3 Ω
D. 2 Ω

Problem 33: EE Board April 1997

The electromotive force of a standard cell is measured with a potentiometer that gives a reading of 1.3562 V. When a 1.0 megaohm resistor is connected across the standard cell terminals, the potentiometer reading drops to 1.3560 V. What is the internal resistance of the standard cell?

A. 174.5 ohms
B. 145.7 ohms
C. 147.5 ohms
D. 157.4 ohms

Problem 34:

A battery is formed of five cells joined in series. When the external resistance is 4 ohms, the current is 1.5 A and when the external resistance is 9 ohms, the current falls to 0.75 A. Find the internal resistance of each cell.

A. 0.5 ohm
B. 1.0 ohm
C. 0.2 ohm
D. 0.3 ohm

Problem 35: EE Board April 1988

A Barangay power station supplies 60 kW to a load over 2,500 ft, 100 mm^2, two-conductors copper feeder, the resistance of which is 0.078 ohm per 1000 ft. The bus bar voltage is maintained constant at 600 V. Determine the load current.

A. 105 A
B. 108 A
C. 110 A
D. 102 A

Problem 36:

A 120 V DC motor draws current of 100 A and is located 1000 ft from the supply source. If the diameter of the copper transmission line is 0.45 inch, what must be the voltage of the supply?

A. 125.32 V
B. 130.24 V
C. 129.32 V
D. 127.05 V

Problem 37: EE Board October 1986

A LRT car, 5 km distance from the Tayuman, takes 100 A over a 100 mm hard drawn copper trolley wire having a resistance of 0.270 ohm per km. The rail and ground return has a resistance of 0.06 ohm per km. If the station voltage is 750 V, what is the voltage of the car?

A. 585 V
B. 590 V
C. 585 V
D. 590 V

Problem 38:

The hot resistance of an incandescent lamp is 10 ohms and the rated voltage is 50 V. Find the series resistance required to operate the lamp from an 80 V supply.

A. 8 Ω
B. 4 Ω
C. 6 Ω
D. 10 Ω

Problem 39:

A resistive coil draws 2 A at 110 V after operating for a long time. If the temperature rise is 55 °C above the ambient temperature of 20 °C, calculate the external resistance which must be initially connected in series with the coil to limit the current to 2 A. The temperature coefficient of the material of the coil is 0.0043 per °C at 20 °C.

A. 10.12 ohms
B. 10.52 ohms

C. 11.45 ohms
D. 12.05 ohms

Problem 40:

A carbon resistor dissipates 60 W of power from a 120 V source at 20 °C. How much power will be dissipated in the resistor at 120 °C if connected across the same source? Assume the temperature coefficient of carbon at 20 °C is -0.0005 per °C.

A. 61.50 W
B. 65.21 W
C. 62.34 W
D. 63.16 W

Problem 41: EE Board April 1981, EE Board October 1984

Two (2) 115-V incandescent lamps A and B are connected in series across a 230-V source. If lamp A is rated 75 watts and lamp B is rated 50 watts, determine the current drawn by the series connection.

A. 0.52 A
B. 0.64 A
C. 0.48 A
D. 0.57 A

Problem 42:

An arc lamp takes 10 A at 50 volts. A resistance R is to be place in series so that the lamp may burn correctly from a 110 V supply. Find the power wasted in this resistor.

A. 800 watts
B. 600 watts
C. 700 watts
D. 900 watts

Problem 43:

Conductor "x" of a certain material and a given cross section has a resistance of 0.1 ohm per meter and a temperature coefficient of 0.005 per °C. Conductor "y" of another material and a given cross section has a resistance of 0.5 ohms per meter and a temperature coefficient of 0.001 per °C. Is desired to make a coil having a resistance of 500 ohms and a temperature coefficient of 0.002 by using suitable lengths of the two wires connected in series. Calculate the required length of wire "x"

A. 1225 m
B. 1250 m
C. 1240 m
D. 1210 m

Problem 44: EE Board April 1997

If a resistor rated at 5 watts and 6 volts are connected across a battery with an open circuit voltage of 6 volts. What is the internal resistance of the battery if the resulting current is 0.8 A?

A. 0.30 ohm
B. 0.26 ohm
C. 0.23 ohm
D. 0.03 ohm

Problem 45:

A high voltage DC transmission line delivers 1000 MW at 500 kV to an aggregate load over a distance of 900 km. Determine the voltage at the sending end. Assume the loop resistance of the line to be 1 mΩ per km.

A. 510.5 kV
B. 505.2 kV
C. 507.7 kV
D. 503.6 kV

Problem 46: EE Board April 1989

It is required that a loading of 3 kW be maintained in a heating element at an initial temperature of 20°C, a voltage of 220 V is necessary for the purpose. After the element has settled down to steady state, it is found that a voltage of 240 volts is necessary to maintain the 3 kW loading. The element resistance temperature coefficient is 0.0006 per degree centigrade at 20°C. Calculate the final temperature of the heating element.

A. 345.43 °C
B. 326.42 °C
C. 336.84 °C
D. 318.48 °C

Problem 47:

A 200-W, 110-V incandescent lamp has a filament having a temperature coefficient of resistance equal to 0.005 at 0 °C. If the normal operating temperature of the bulb is 2500 °C, how much current will the bulb draw at the instant it is turned on. Assume a room temperature of 20 °C.

A. 29.42 A
B. 18.37 A
C. 22.31 A
D. 28.21 A

Problem 48:

The power drawn by a resistive coil made of copper wire is 220 W at 100 V and 20 °C. Calculate the power consume by the coil at 115 V and 100 °C. The temperature coefficient of resistance at 20 °C is 0.00393.

A. 175 W
B. 211 W
C. 183 W
D. 225 W

Problem 49:

An electric water heater has a rating of 1 kW, 230 V. The coil used as the heating element is 10 m long and has a resistivity of 1.724×10^{-6} ohm-cm. Determine the required diameter of the wire in mils?

A. 2.43 mils
B. 2.52 mils
C. 3.21 mils
D. 1.35 mils

Problem 50:

When two resistors A and B are connected in series, the total resistance is 36 ohms. When connected in parallel, the total resistance is 8 ohms. What is the ratio of the resistance R_a to resistance R_b. Assume $R_a < R_b$.

A. 0.5
B. 0.4
C. 0.8
D. 0.6

Problem 51: EE Board March 1998

Three resistors of 10, 15 and 20 ohms each are connected in parallel. What is the equivalent resistance?

A. 45 ohms
B. 17.2 ohms
C. 0.22 ohm
D. 4.62 ohms

Problem: 52

The equivalent resistance of three resistors A, B and C connected in parallel is 1.714 ohms. If A is twice of B and C is half as much as B, find the equivalent resistance when the three of them are connected in series.

A. 17.5 ohms
B. 21.0 ohms
C. 24.5 ohms
D. 28.0 ohms

Problem 53:

Three resistors of 10, 12 and 15 ohms are connected in parallel. Evaluate the value of current to the parallel system that will make the current in the 10 ohm resistor equal to 2 A.

A. 5 A
B. 6 A
C. 4 A
D. None of these

Problem 54:

Three resistors of 10, 12 and "x" ohms, respectively are connected in parallel across a constant current source of 8 A. Determine "x" if this resistor draws 2.5 A.

A. 10 Ω
B. 12 Ω
C. 13 Ω
D. 11 Ω

Problem 55:

Two resistors A and B made of different materials have temperature coefficients of resistance at 20 °C of 0.004 and 0.006 respectively. When connected across a voltage source at 20 °C, they draw current equally. What percentage of the total current at 100 °C does resistor A carry?

A. 47.14%
B. 52.86%
C. 61.34%
D. 38.66%

Problem 56:

Two resistors A and B made of different materials have temperature coefficients of resistance of 0.003 and 0.005, respectively. When connected in parallel across a voltage source at 15 °C, it consumes equal power. What is the ratio of the power drawn by resistor B to that in resistor A when temperature rises to 60 °C? Assume supply voltage is constant.

A. 0.829
B. 0.926
C. 0.964
D. 0.882

Problem 57: EE Board March 1998

Three resistors of 10, 15 and 20 ohms each are connected in parallel. What is the total conductance?

A. 0.217 mho
B. 3.41 mhos
C. 4.62 mhos
D. 0.52 mho

Problem 58:

Two heating units x and y are connected across a 100 V supply. Unit y takes twice as much power as unit x. Total power supplied by the source is 2280 watts. Determine the total power drawn by the heating units if they are connected in series across the same source.

A. 615.32 W
B. 506.58 W
C. 582.45 W
D. 604.38 W

Problem 59:

Three resistors R_1, R_2 and R_3 are connected in series-parallel with R_1 in series with the parallel combination of R_2 and R_3. The whole combination is connected across a 120 V DC source. Resistors R_1, R_2 and R_3 take 750 W, 250 W and 200 W, respectively Calculate the resistance R_2.

A. 8.10 Ω
B. 8.52 Ω
C. 7.84 Ω
D. 9.22 Ω

Problem 60:

Two lamps X and Y are connected in series. Lamp X is rated 50 W at 100 V while lamp Y is rated at 100 W at 100 V also. The two lamps are to be connected in series across a 200 V source. What resistance must be connected across lamp A so that each lamp will operate properly?

A. 150 ohms
B. 100 ohms
C. 200 ohms
D. 180 ohms

Problem 61: EE Board October 1997

A 5-ohm resistance is connected in parallel with a 10-ohm resistance. Another set, a 6-ohm and an 8-ohm resistances are also connected in parallel. The two sets are connected in series. What is the equivalent resistance?

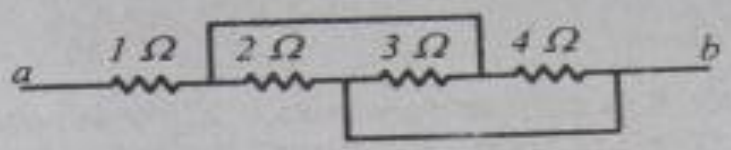
A 6.76 ohms
B. 9.25 ohms
C. 14.4 ohms
D. 21.2 ohms

Problem 62: EE Board March 1998

Two resistances of 10 and 15 ohms each respectively are connected in parallel. The two are then connected in series with a 5-ohm resistance. What is the equivalent resistance?

A. 11 ohms
B. 12 ohms
C. 10 ohms
D. 9 ohms

Problem 63:
From the circuit as shown, determine the resistance between terminals a & b



A. 1.000 Ω
B. 1.923 Ω
C. 2.083 Ω
D. 2.352 Ω

Problem 64: EE Board October 1997
A 10-ohm and a 20-ohm resistance are connected in parallel. Another resistance of 5-ohm is connected in series with the two. If the supply voltage is 48 volts, what is the current through the 10-ohm resistance?

A. 3.21 A
B. 2.74 A
C. 4.02 A
D. 5.72 A

Problem 65:
A 30-ohm resistor is connected in parallel with a variable resistance R. The parallel combination is then connected in series with a 6-ohm resistor and connected across a 120 V source. Find the minimum value of R if the power taken by R is equal to the power taken by the 6-ohm resistor.

A. 10.35 Ω
B. 12.24 Ω
C. 10.24 Ω
D. 11.46 Ω

Problem 66: EE Board March 1998
Two resistances of 10 and 15 ohms, each respectively are connected in parallel. The two are then connected in series with a 5-ohm resistance. It is then connected across a 12-V battery, what are the current and power?

A. 1.2 A, 17.28 W
B. 0.96 A, 11.52 W
C. 1.09 A, 13.1 W
D. 1.5 A, 20.25 W

Problem 67:

An 8-ohm resistor is connected in series with a parallel combination of two resistors, R and 24 ohms. Determine R if the power consumed by the parallel-connected resistors is equal to the power consumed by the 8-ohm resistor.

A. 10 ohms
B. 16 ohms
C. 12 ohms
D. 20 ohms

Problem 68:

A multi-tap resistor R is connected across a 220-V supply. A voltmeter whose internal resistance is 15-kΩ is connected across the center tap and one end of the supply terminals. If the voltmeter registers 100 V, what is the value of resistor R.

A. 5.5 kΩ
B. 5.0 kΩ
C. 6.0 kΩ
D. 6.5 kΩ

Problem 69:

A potential divider of resistance of 50 ohms is connected across a 100 V DC source. A load resistance of 10 ohms is connected across a tap in the potential divider and the negative terminal of the source. If a current of 4 A flows towards the load, what is the current supplied by the source?

A. 5.32 A
B. 5.05 A
C. 5.21 A
D. 5.48 A

Problem 70:

Two resistors A and B are connected in series across a 220 V DC source. When a voltmeter with an internal resistance of 10 kΩ ohms, is connected across resistor A, the instrument reads 100 V and when connected across resistor B, it reads 80 volts. Find the resistance of resistor A.

A. 4 kΩ
B. 3 kΩ
C. 5 kΩ
D. 6 kΩ

Problem 71: EE Board April 1992

An electric kettle was marked 500 W, 230 V found to take 15 minutes to bring 1 kilogram of water at 15 °C to boiling point. Determine the heat efficiency of the kettle.

A. 79.1%
B. 75.3%

C. 72.4%
D. 74.8%

Problem 72: EE Board October 1997

A process equipment contains 100 gallons of water at 25 °C. It is required to bring it to boiling in 10 minutes. The heat loss is estimated to be 5%. What is the kW rating of the heater?

A. 125 kW
B. 252 kW
C. 50.5 kW
D. 207 kW

Problem 73: EE Board October 1989

A total of 0.8 kg of water at 20 °C is placed in a 1-kW electric kettle. How long a time in minute is needed to raise the temperature of the water to 100°C?

A. 4.46 min.
B. 5.32 min
C. 6.34 min
D. 4.56 min

Problem 74: EE Board October 1998

How many calories does an electric heater of 100 watts generate per second?

A. 10
B. 23.88
C. 1000
D. 42.25

Problem 75: EE Board October 1997

The electric energy required to raise the temperature of water in a pool is 1000 kWh. If the heat losses are 25%, the heating energy required will be _____.

A. 1111 kWh
B. 1266 kWh
C. 1750 kWh
D. 1333 kWh

Problem 76: EE Board April 1992

An electric heater carries 12 A at 110 V, is submerged in 22.5 lbs of water for 30 minutes. What will be the final temperature of the water if its initial temperature is 35°F?

A. 135.43 °F
B. 125.42 °F
C. 133.56 °F
D. 128.33 °F

Problem 77: EE Board October 1990

In an electric heater the inlet temperature is 15 °C. Water is flowing at the rate of 300 grams per minute. The voltmeter measuring voltage across the heating element reads 120 volts and an ammeter measuring current taken reads 10 amperes. When steady state is finally reached, what is the final reading of the outlet thermometer?

A. 57.6 °C
B. 68.4° C
C. 72.6° C
D. 42.6 °C

Problem 78: EE Board October 1991

Four cubic meters of water is to be heated by means of four 1.5 kW, 230-V immersion heating elements. Assuming the efficiency of the heater as 90%, determine the time required boiling the water if the initial temperature is 20°C and if all four elements are connected in parallel.

A. 71 hrs
B. 63 hrs
C. 69 hrs
D. 66 hrs

Problem 79: EE Board October 1991

Four cubic meters of water is to be heated by means of four 1.5 kW, 230-V immersion heating elements. Assuming the efficiency of the heater as 90%, determine the time required boiling the water if the initial temperature is 20°C and if the elements are connected two in series in parallel with two in series.

A. 275.6 hrs
B. 295.3 hrs
C. 252.2 hrs
D. 264.4 hrs

Problem 80: EE Board April 1997

A circuit consisting of three resistors rated: 10 ohms, 15 ohms and 20 ohms are connected in delta. What would be the resistances of the equivalent wye connected load?

A. 0.30, 0.23 and 0.15 ohm
B. 3.0, 4.0 and 5.0 ohms
C. 3.33, 4.44 and 6.66 ohms
D. 5.77, 8.66 and 11.55 ohms

Problem 81: EE Board October 1994

The equivalent wye element of 3 equal resistors each equal to R and connected in delta is

A. R

B. $\frac{3}{2}R$

C. $\frac{R}{3}$

D. 3R

Problem 82:

Three resistors of 6-ohm resistance are connected in delta. Inside the delta another three 6-ohm resistors are connected in wye. Find its resistance between any two corners.

A. 2 ohms
B. 3 ohms
C 4 ohms
D. 1 ohm

ANSWER KEY

31.	B	42.	B	53.	A	64.	B	75.	D
32.	A	43.	B	54.	B	65.	D	76.	A
33.	C	44.	A	55.	B	66.	C	77.	C
34.	C	45.	D	56.	B	67.	C	78.	C
35.	B	46.	C	57.	A	68.	C	79.	A
36.	B	47.	C	58.	B	69.	B	80.	C
37.	A	48.	C	59.	A	70.	C	81.	C
38.	C	49.	B	60.	C	71.	A	82.	B
39.	B	50.	A	61.	A	72.	D		
40.	D	51.	D	62.	A	73.	A		
41.	A	52.	B	63.	B	74.	B		

☑ RATING

46-52 - Topnotcher

36-45 - Passer

26-35 - Conditional

0-25 - Failed

Solutions to Test 2

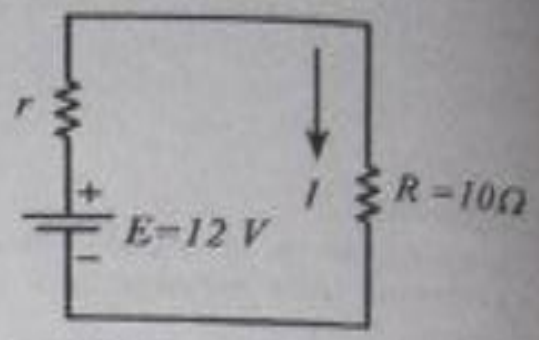
31

$$I = \frac{E}{r + R}$$

$$r = \frac{E}{I} - R = \frac{12}{1.18} - 10$$

$$= 0.169\,\Omega$$

$$r \cong 0.2\,\Omega$$



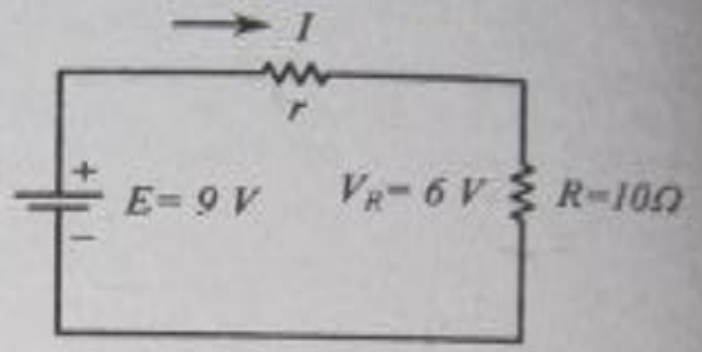
32

$$I = \frac{V_R}{R} = \frac{6}{10} = 0.6\text{ A}$$

$$V_r = E - V_R = 9 - 6 = 3\text{ V}$$

$$r = \frac{V_r}{I} = \frac{3}{0.6}$$

$$r = 5\,\Omega$$



33

$$I = \frac{V_R}{R} = \frac{1.3560}{1 \times 10^6}$$

$$I = 1.356 \times 10^{-6}\text{ A}$$

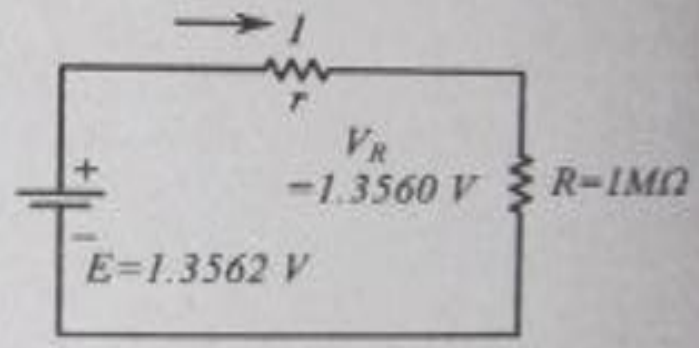
$$V_r = E - V_R$$

$$= 1.3562 - 1.3560$$

$$V_r = 0.0002\text{ V}$$

$$r = \frac{V_r}{I} = \frac{0.0002}{1.356 \times 10^{-6}}$$

$$r = 147.5\,\Omega$$



34 Let: r = internal resistance per cell.

$$I = \frac{E}{5r + R_L}$$

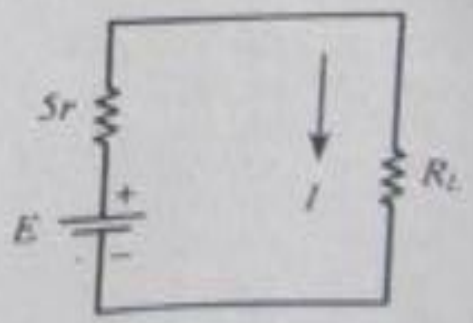
Substitute conditions :

$$1.5 = \frac{E}{5r + 4}$$

$$E = 7.5r + 6 \Rightarrow ❶$$

$$0.75 = \frac{E}{5r + 9}$$

$$E = 3.75r + 6.75 \Rightarrow ❷$$



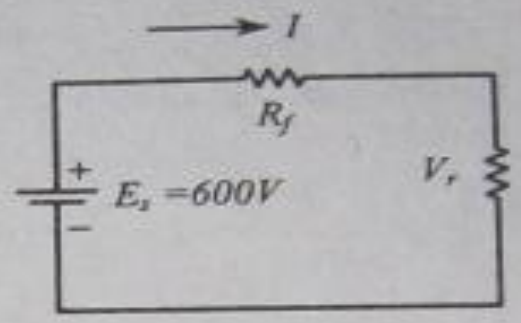
Equate Eq. 1 to Eq. 2 :

$$7.5r + 6 = 3.75r + 6.75$$

$$= 0.2\,\Omega$$

35 Let : R_f = total resistance of the feeder conductors

$$R_f = 2\left[\frac{0.078\ \Omega}{1000\ \text{ft}} \times 2500\ \text{ft}\right] = 0.39\ \Omega$$



$$E_s = V_r + IR_f$$

$$E_s = \frac{P_r}{I} + IR_f$$

$$600 = \frac{60{,}000}{I} + I(0.39)$$

$$600I = 60{,}000 + 0.39I^2$$

$$I^2 - 1538.46 + 153846 = 0$$

Using the quadratic formula :

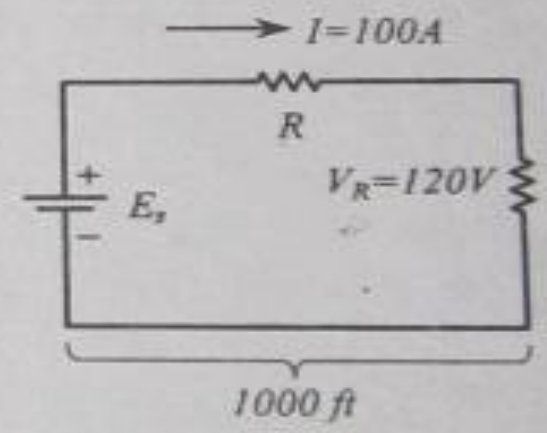
$$I = \frac{1538.46 \pm \sqrt{(1538.46)^2 - 4(1)(153846)}}{2(1)} = \frac{1538.46 \pm 1323.433}{2}$$

Note: Use negative sign for minimum current

$$I = \frac{1538.46 - 1323.433}{2} = 107.513\ \text{A}$$

$$I \cong 108\ \text{A}$$

36 Let : R = total resistance of the line



$$A = d^2 = (0.45 \times 1000)^2 = 202{,}500\ \text{CM}$$

$$R = \rho\frac{L}{A}$$

$$R = \frac{10.37(2000)}{202{,}500} = 0.1024\ \Omega$$

$$E_s = V_R + IR = 120 + 100(0.1024)$$

$$E_s = 130.24\ \text{volts}$$

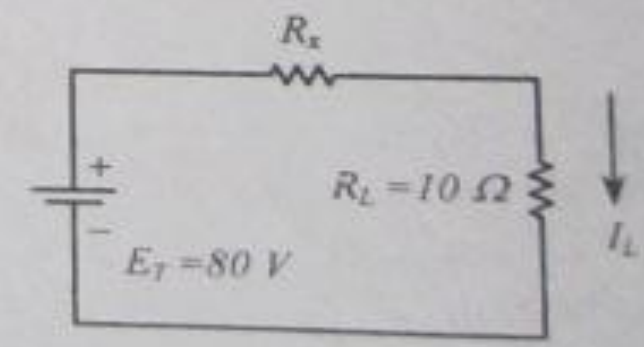
37 Assume : R = equivalent resistance of the circuit

$$V_R = E_s - IR$$

$$V_R = 750 - 100\left[(0.27 + 0.06)(5)\right]$$

$$V_R = 585\ \text{V}$$

38



$$I_L = \frac{V_L}{R_L} = \frac{50}{10} = 5\ \text{A}$$

$$R_s = \frac{E_T - V_L}{I_L} = \frac{80 - 50}{5} = 6\ \Omega$$

39

$$R_2 = \frac{E}{I} = \frac{110}{2} = 55\,\Omega$$

$$R_2 = R_1(1+\alpha\,\Delta_t)$$

$$R_1 = \frac{R_2}{1+\alpha\Delta_t} = \frac{55}{1+0.0043(55)} = 44.48\,\Omega$$

$$R = R_2 - R_1 = 55 - 44.48$$

$$R = 10.52\,\Omega$$

40

At 20 °C:

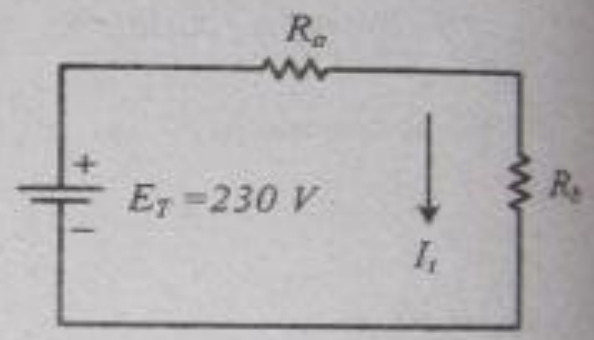
$$R_1 = \frac{V^2}{P} = \frac{120^2}{60} = 240\,\Omega$$

At 120°C:

$$R_2 = R_1(1+\alpha\Delta_t) = 240\left[1-0.0005(120^\circ - 20^\circ)\right] = 228\,\Omega$$

$$P = \frac{E^2}{R_2} = \frac{120^2}{228}$$

$$P = 63.16\text{ W}$$



41

$$R_a = \frac{E_a^{\,2}}{P_a} = \frac{(115)^2}{75} = 176.333\,\Omega$$

$$R_b = \frac{E_b^{\,2}}{P_b} = \frac{(115)^2}{50} = 264.5\,\Omega$$

$$R_t = R_a + R_b = 176.333 + 264.5 = 440.833\,\Omega$$

$$I_t = \frac{E_t}{R_t} = \frac{230}{440.833}$$

$$I_t = 0.52\text{ A}$$

42

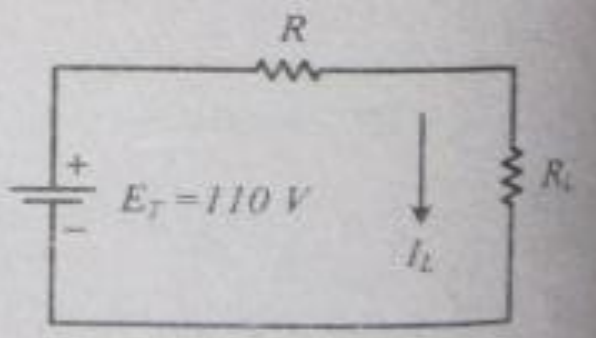
$$R_L = \frac{V_L}{I_L} = \frac{50}{10} = 5\,\Omega$$

$$R_t = \frac{E_t}{I_L} = \frac{110}{10} = 11\,\Omega$$

$$R = R_t - R_L = 11 - 5 = 6\,\Omega$$

$$P_{loss} = I_L^{\,2}R = (10)^2(6)$$

$$P_{loss} = 600\text{ watts}$$



43

$$R_t = R_x(1+\alpha_x\Delta_t) + R_y(1+\alpha_y\Delta_t)$$

$$R_t = R_x(1+0.005\Delta_t) + R_y(1+0.001\Delta_t) \Rightarrow ❶$$

$$R_t = (R_x + R_y)(1+\alpha_t\Delta_t)$$

$$R_t = (R_x + R_y)(1+0.002\Delta_t) \Rightarrow ❷$$

$R_x + R_y = 500$

$R_y = 500 - R_x \Rightarrow$ ❸

Equate Eq.1 to Eq.2:

$R_x(1+0.005\Delta_t) + R_y(1+0.001\Delta_t) = (R_x + R_y)(1+0.002\Delta_t)$

$R_x + 0.005\Delta_t R_x + R_y + 0.001\Delta_t R_y = R_x + 0.002\Delta_t R_x + R_y + 0.002\Delta_t R_y$

$0.003R_x - 0.001R_y = 0 \Rightarrow$ ❹

Substitute Eq.3 in Eq.4:

$0.003R_x - 0.001(500 - R_x) = 0$

$0.004R_x = 0.5$

$R_x = 125$ ohms

$L_x = \dfrac{125\,\Omega}{0.1\Omega/m}$

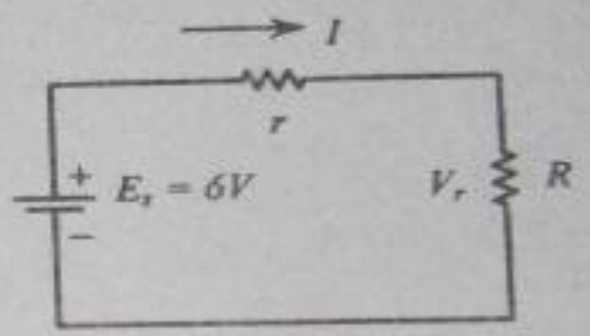
$L_x = 1250$ m

44. $R = \dfrac{V_r^2}{P} = \dfrac{6^2}{5} = 7.2\,\Omega$

$I = \dfrac{E}{r+R}$

$r = \dfrac{E}{I} - R = \dfrac{6}{0.8} - 7.2$

$r = 0.3\,\Omega$

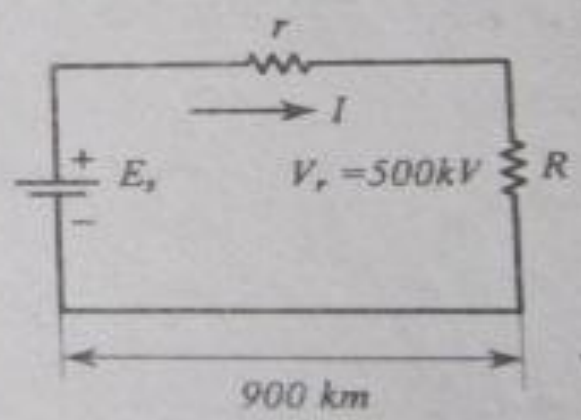


45. $r = (0.001)(2 \times 900) = 1.8\,\Omega$

$I = \dfrac{P}{V_r} = \dfrac{1000 \times 10^6}{500 \times 10^3} = 2000$ A

$E_s = V_r + Ir = 500{,}000 + 2000(0.18)$

$E_s = 503.6$ kV



46. $R_1 = \dfrac{E_1^2}{P_2} = \dfrac{220^2}{3000} = 16.133\,\Omega$

$R_2 = \dfrac{E_2^2}{P_2} = \dfrac{240^2}{3000} = 19.2\,\Omega$

$R_2 = R_1[1 + \alpha(t_2 - t_1)]$

$19.2 = 16.133\,[1 + 0.0006\,(t_2 - 20°)]$

$t_2 = 336.84\,°C$

47

$$R_1 = \frac{E^2}{P} = \frac{110^2}{200} = 60.5\ \Omega$$

$$\alpha_0 = \frac{1}{T}$$

$$T = \frac{1}{0.005} = 200\ ^\circ C$$

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

$$R_2 = R_1\left(\frac{T + t_2}{T + t_1}\right) = 60.5\left(\frac{200 + 20}{200 + 2500}\right) = 4.93\ \Omega$$

$$I = \frac{E}{R_2} = \frac{110}{4.93}$$

$$I = 22.31\ A$$

48

$$R_{20} = \frac{(E)^2}{P_{20}} = \frac{110^2}{220} = 55\ \Omega$$

$$R_{100} = R_{20}[1 + \alpha\Delta_t] = 55[1 + 0.00393(100 - 20] = 72.29\ \Omega$$

$$P_{100} = \frac{E^2}{R_{100}} = \frac{115^2}{72.29}$$

$$P_{100} = 183\ W$$

49

$$R = \frac{E^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega$$

$$R = \rho\frac{L}{A}$$

$$A = \frac{\rho L}{R} = \frac{1.724 \times 10^{-6}(10 \times 100)}{52.9} = 32.589 \times 10^{-6}\ cm^2$$

$$A = \frac{\pi}{4}d^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(32.589 \times 10^{-6})}{\pi}} = 0.00788\ cm \times \frac{1000\ mils}{2.54\ cm}$$

$$d = 2.52\ mils$$

50

When series connected :

$R_a + R_b = 36 \Rightarrow$ ❶

When parallel connected :

$$\frac{R_a R_b}{R_a + R_b} = 8 \Rightarrow ❷$$

Substitute Eq.1 to Eq.2 :

$R_a(36 - R_a) = 8(36)$

$R_a^2 - 36R_a + 288 = 0$

Using the quadratic formula :

$$R_a = \frac{36 \pm \sqrt{36^2 - 4(1)(288)}}{2(1)} = \frac{36 \pm 12}{2} = \frac{36 - 12}{2} = 12\,\Omega$$

$R_b = 36 - 12 = 24\,\Omega$

$\text{Ratio} = \frac{12}{24}$

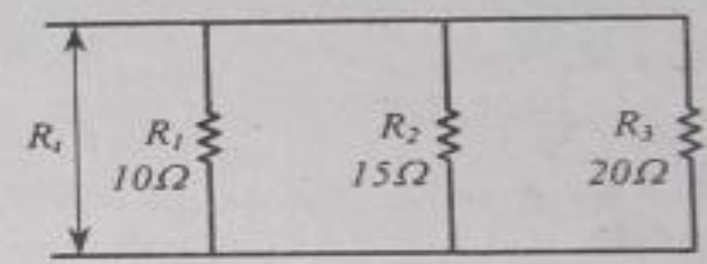
Ratio = 0.5

51

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_t} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20}$$

$R_t = 4.615\,\Omega$



52

$$\frac{1}{R_t} = \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}$$

$$\frac{1}{1.714} = \frac{1}{2R_b} + \frac{1}{R_b} + \frac{1}{0.5R_b}$$

$R_b = 6\,\Omega$

$R_a = 12\,\Omega$

$R_c = 3\,\Omega$

When connected in series :

$R_t = R_a + R_b + R_c = 12 + 6 + 3$

$R_t = 21\,\Omega$

53

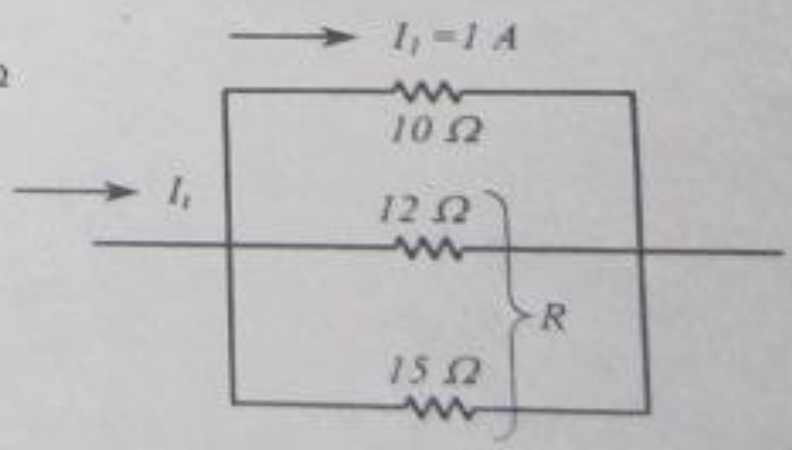
$$R = \frac{R_2R_3}{R_2 + R_3} = \frac{12(15)}{12 + 15} = 6.67\,\Omega$$

By CDT :

$$I_1 = \frac{I_t R}{R_1 + R}$$

$$i_1 = \frac{I_t(R_1 + R)}{R} = \frac{2(10 + 6.67)}{6.67}$$

$I_t = 5\,A$



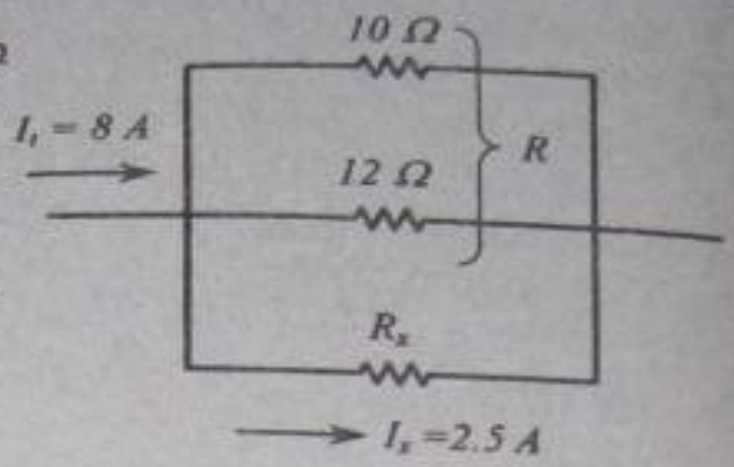
54 $R = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{10(12)}{10+12} = 5.45\,\Omega$

By CDT:

$I_x = \dfrac{I_t R}{R_x + R}$

$R_x = \dfrac{I_t R}{I_x} - R = \dfrac{8(5.45)}{2.5} - 5.45$

$R_x = 12\,\Omega$



55 At 100°C:

$R_{2a} = R_{1a}(1 + \alpha_a \Delta_t)$

$= R_{1a}[1 + 0.004(100 - 20)] = 1.32\,R_{1a}$

$R_{2b} = R_{1b}(1 + \alpha_b \Delta_t)$

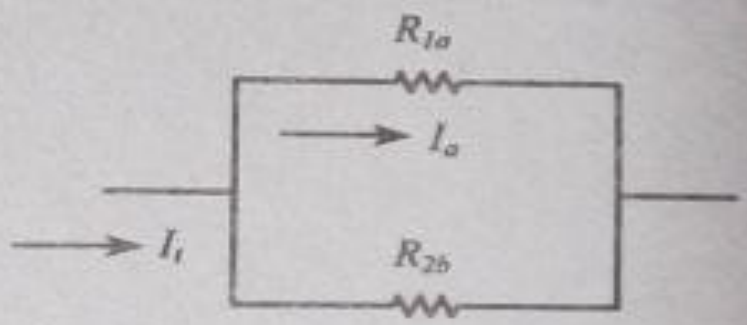
$= R_{1b}[1 + 0.006(100 - 20)] = 1.48\,R_{1b}$

Note: Since they draw equal currents at 20 °C, then $R_{1a} = R_{1b} = R$

By CDT:

$I_a = \dfrac{I_t\,(1.48R)}{1.32R + 1.48R} = 0.5286\,I_t$

$I_a = 52.86\%$ of I_t



56 At 100°C:

$R_{2a} = R_{1a}(1 + \alpha_a \Delta_t) = R_{1a}[1 + 0.003(60 - 15)] = 1.135\,R_{1a}$

$R_{2b} = R_{1b}(1 + \alpha_b \Delta_t) = R_{1b}[1 + 0.005(60 - 15)] = 1.225\,R_{1b}$

Note: Since they draw equal powers at 15 °C, then $R_{1a} = R_{1b} = R$

$P_a = \dfrac{E^2}{R_{2a}}; \quad P_b = \dfrac{E^2}{R_{2b}}$

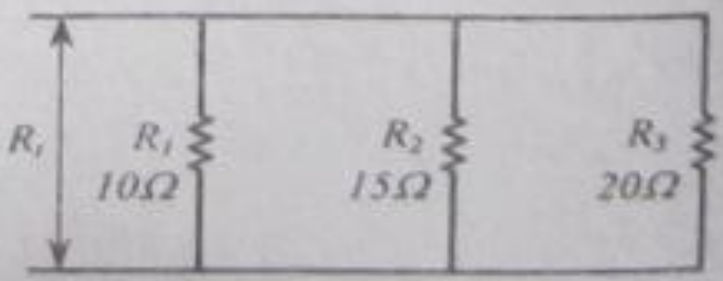
$\dfrac{P_b}{P_a} = \dfrac{R_{2a}}{R_{2b}} = \dfrac{1.135R}{1.225R} = 0.926$

57 $G_t = \dfrac{1}{R_t}$

$\dfrac{1}{R_t} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

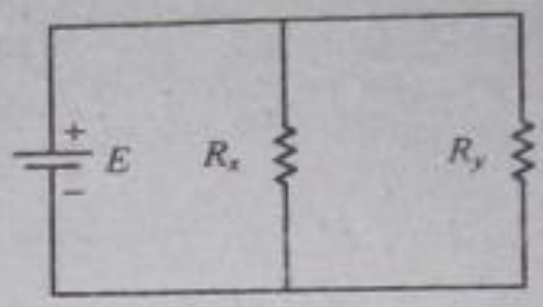
$G_t = \dfrac{1}{10} + \dfrac{1}{15} + \dfrac{1}{20}$

$G_t = 0.2167$ mho



58 $P_y = 2P_x \Rightarrow ❶$

$P_x + P_y = 2280 \Rightarrow ❷$

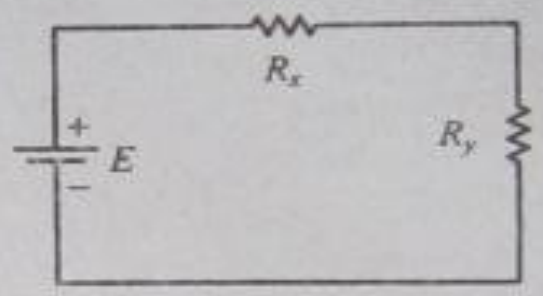


Substitute Eq.1 in Eq.2 :

$P_x + 2P_x = 2280$

$P_x = 760\ W$

$P_y = 2(760) = 1520\ W$



$$R_x = \frac{E^2}{P_x} = \frac{100^2}{760} = 13.16\ \Omega$$

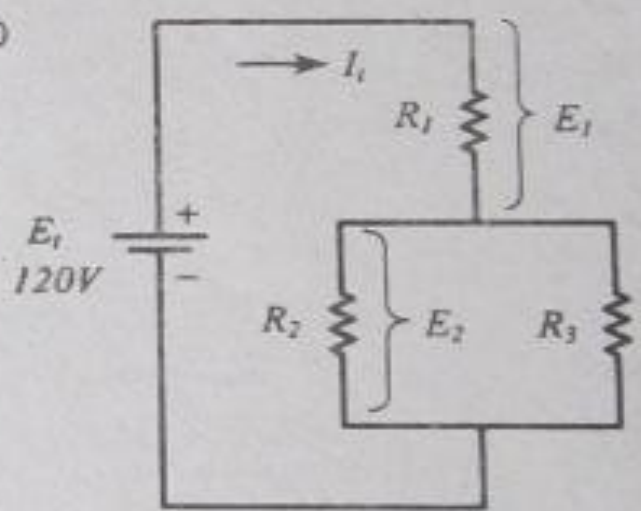
$$R_y = \frac{E^2}{P_y} = \frac{100^2}{1520} = 6.58\ \Omega$$

$$P_{series} = \frac{E^2}{R_{series}} = \frac{100^2}{13.16 + 6.58}$$

$P_{series} = 506.58\ W$

59 $P_t = P_1 + P_2 + P_3 = 750 + 250 + 200$

$P_t = 1200\ W$



$$I_t = \frac{P_t}{E_t} = \frac{1200}{120} = 10\ A$$

$$E_1 = \frac{P_1}{I_t} = \frac{750}{10} = 75\ V$$

$E_2 = 120 - 75 = 45\ V$

$$R_2 = \frac{E_2^{\ 2}}{P_2} = \frac{45^2}{250}$$

$R_2 = 8.1\Omega$

60 $$R_x = \frac{E^2}{P_x} = \frac{100^2}{50} = 200\ \Omega$$

$$R_y = \frac{E^2}{P_y} = \frac{100^2}{100} = 100\ \Omega$$

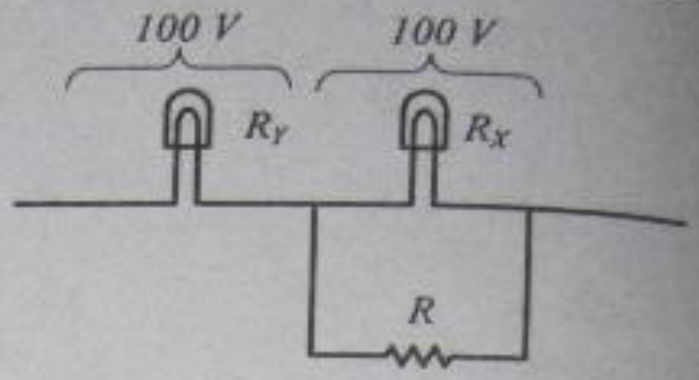
Note: So that the lamps may operate properly when connected in series across a 200 V source, the voltage across each lamp must be equal to 100 V. Thus, the resistance across the parallel branch must be equal to the resistance of lamp Y.

$$R_{equiv} = \frac{RR_x}{R + R_x}$$

$$100 = \frac{R(200)}{R + 200}$$

$$R + 200 = 2R$$

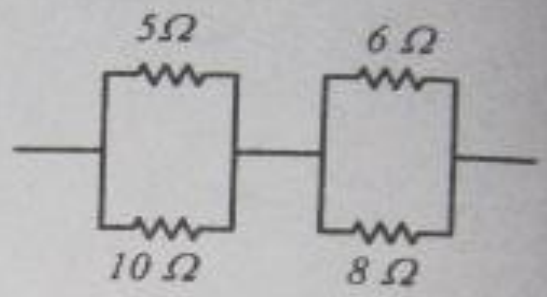
$$R = 200\,\Omega$$



61

$$R_t = \frac{5(10)}{5+10} + \frac{6(8)}{6+8}$$

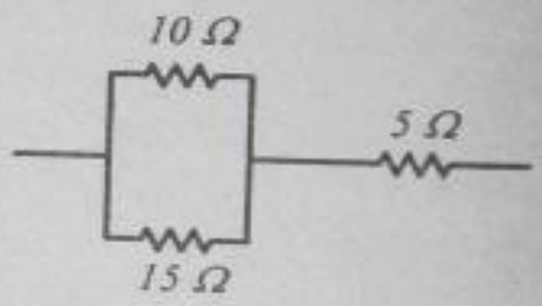
$$R_t = 6.76\,\Omega$$



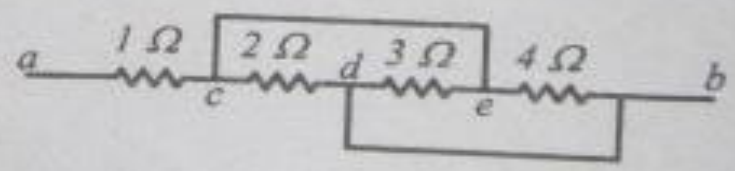
62

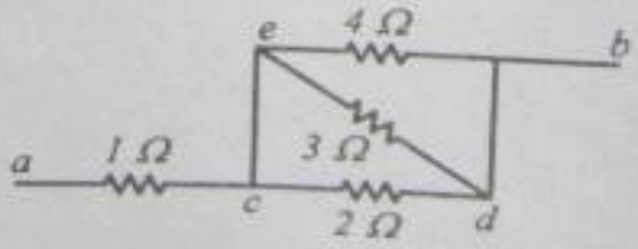
$$R_t = \frac{10(15)}{10+15} + 5$$

$$R_t = 11\,\Omega$$



63



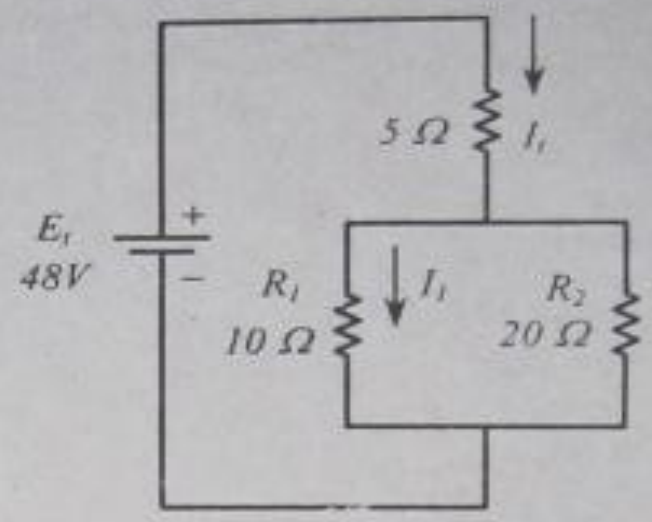


$$R_{AB} = 1 + \frac{2(3)(4)}{2(3) + 3(4) + 4(2)}$$

$$R_{AB} = 1.923\,\Omega$$

64 $R_t = \dfrac{10(20)}{10+20} + 5 = 11.67\,\Omega$

$I_t = \dfrac{E_t}{R_t} = \dfrac{48}{11.67} = 4.113\text{ A}$



By CDT :

$I_1 = \dfrac{I_t R_2}{R_1 + R_2} = \dfrac{4.113(20)}{10+20}$

$I_1 = 2.74\text{ A}$

65 By CDT, $I_R = \dfrac{I_T(30)}{30+R}$

$P_6 = P_R$

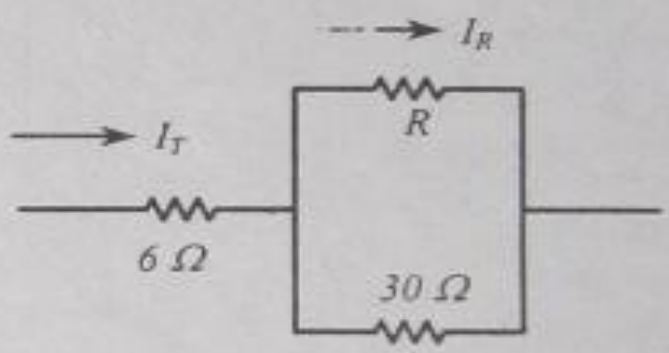
$I_T^2(6) = I_R^2(R)$

$6I_T^2 = R\left(\dfrac{I_T^2(900)}{(30+R)^2}\right)$

$(30+R)^2 = \dfrac{900R}{6}$

$900 + 60R + R^2 = 150R$

$R^2 - 90R + 900 = 0$



By quadratic formula :

$R = \dfrac{90 \pm \sqrt{(-90)^2 - 4(1)(900)}}{2(1)} = \dfrac{90 \pm 67.08}{2}$

Take (-) for minimum value of R :

$R = \dfrac{90 - 67.08}{2}$

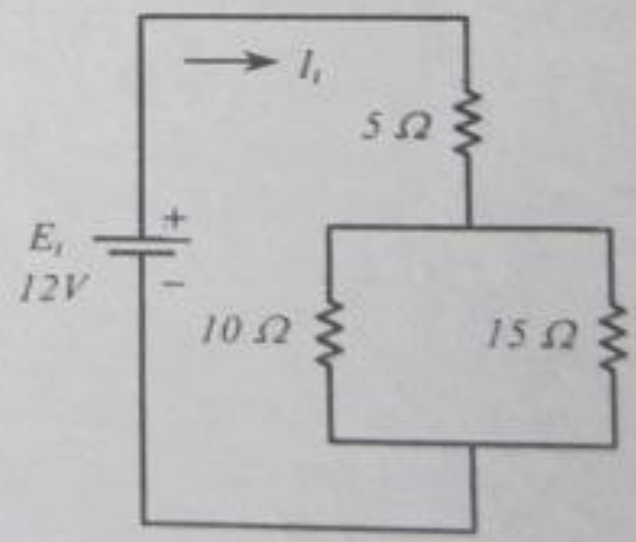
$R = 11.46\,\Omega$

66 $R_t = \dfrac{10(15)}{10+15} + 5 = 11\Omega$

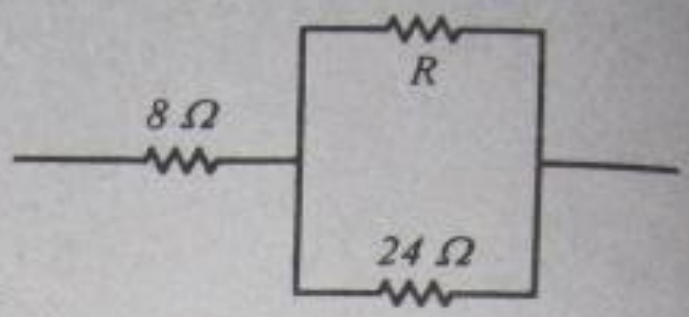
$I_t = \dfrac{E_t}{R_t} = \dfrac{12}{11} = 1.09\text{ A}$

$P_t = E_t I_t = 12(1.09)$

$P_t = 13.08\text{ W}$



67 Note: Since powers drawn are equal, the equivalent resistance of the parallel combination must be equal to 8 ohms also.



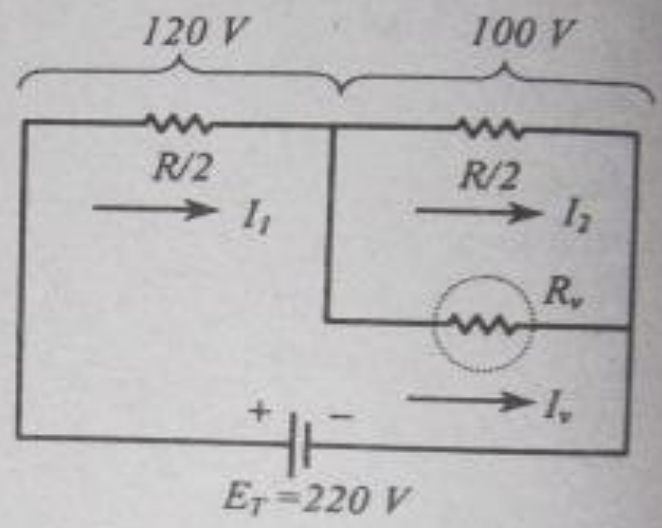
$R_{eq} = 8$

$8 = \dfrac{24R}{24+R}$

$24R = 192 + 8R$

$R = 12\,\Omega$

68 $I_1 = I_2 + I_v$



$\dfrac{120}{R/2} = \dfrac{100}{R/2} + \dfrac{100}{15{,}000}$

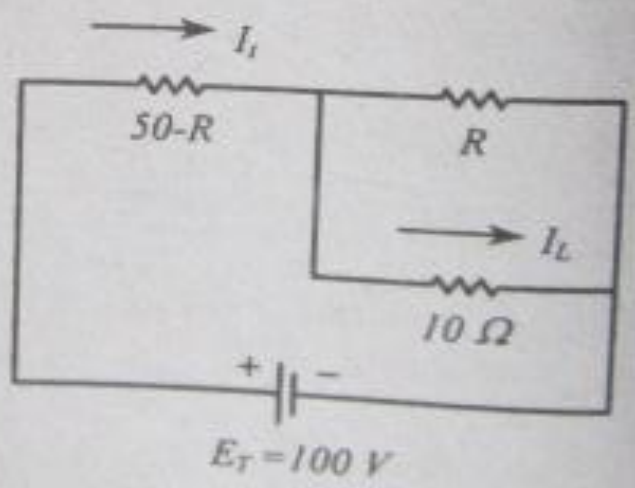
$\dfrac{240}{R} = \dfrac{200}{R} + \dfrac{100}{15{,}000}$

$\dfrac{40}{R} = \dfrac{100}{15{,}000}$

$R = \dfrac{40(15{,}000)}{100} = 6000$

$R = 6\,k\Omega$

69 by CDT:



$I_L = \dfrac{I_t R}{R + R_L}$

$I_t = \dfrac{I_L(R + R_L)}{R} = \dfrac{4(R+10)}{R} \Rightarrow$ ❶

$V_1 = E_T - V_L = V_T - I_L R_L$

$V_1 = 100 - 4(10) = 60$ V

$I_t = \dfrac{V_1}{50 - R} = \dfrac{60}{50 - R} \Rightarrow$ ❷

Equate Eq.1 and Eq.2:

$\dfrac{4(R+10)}{R} = \dfrac{60}{50 - R}$

$(R + 10)(50 - R) = 15R$

$50R - R^2 + 500 - 10R = 15R$

$R^2 - 25R - 500 = 0$

Using the quadratic formula:

$$R = \frac{25 \pm \sqrt{25^2 - 4(1)(-500)}}{2(1)} = \frac{25 \pm 51.23}{2}$$

$$R = \frac{25 + 51.23}{2}$$

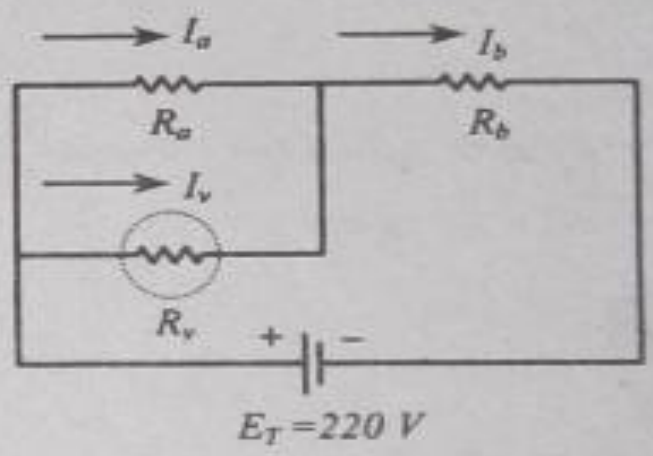
$$R = 38.12\,\Omega$$

Substitute R in Eq.2:

$$I_t = \frac{60}{50 - R} = \frac{60}{50 - 38.12}$$

$$I_t = 5.05\text{ A}$$

70 Using the first condition:



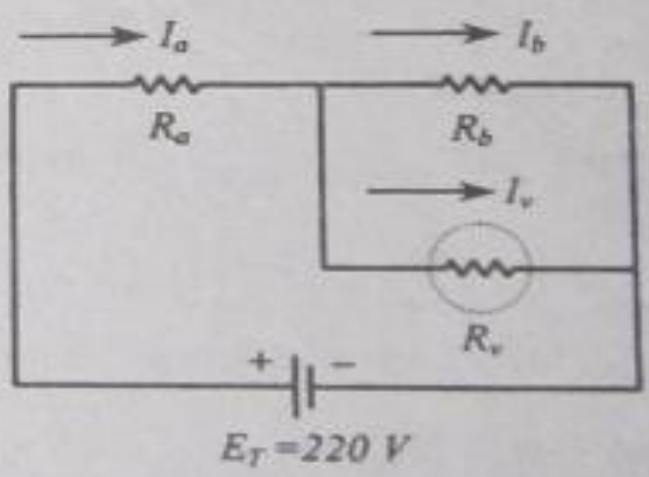
$$I_a + I_v = I_b$$

$$\frac{100}{R_a} + \frac{100}{10{,}000} = \frac{120}{R_b}$$

$$\frac{1}{R_b} = \frac{100}{10{,}000(120)} + \frac{100}{120R_a}$$

$$\frac{1}{R_b} = \frac{1}{12{,}000} + \frac{5}{6R_a} \Rightarrow ❶$$

Using the second condition:



$$I_a = I_b + I_v$$

$$\frac{140}{R_a} = \frac{80}{10{,}000} + \frac{80}{R_b} \Rightarrow ❷$$

Substitute Eq.1 in Eq.2:

$$\frac{140}{R_a} = \frac{80}{10{,}000} + 80\left(\frac{1}{12{,}000} + \frac{5}{6R_a}\right)$$

$$\frac{140}{R_a} = \frac{11}{750} + \frac{400}{6R_a}$$

$$\left(\frac{140}{R_a} = \frac{11}{750} + \frac{400}{6R_a}\right)R_a$$

$$140 = 0.0146677\,R_a + 66.6667$$

$$R_a = 4999.88\,\Omega$$

$$R_a \cong 5\text{ k}\Omega$$

71

$Q = mc\Delta_t$

$Q = (1000\ g)(4.186\ J/g\,°C)(100 - 15) = 355{,}810\ J$

$$\eta = \frac{W_{output}}{W_{input}} = \frac{355810}{500\left(15\ min \times \frac{60\ sec}{1\ min}\right)} = 0.7906$$

$\eta \cong 79.1\%$

72

$$m = 100\ gal \times \frac{3.78\ li}{1\ gal} \times \frac{1\ m^3}{1000\ li} \times \frac{1000\ kg}{1\ m^3} = 378\ kg$$

$Q_{out} = mc\Delta_t = 378\ kg\,(1\ kcal/kg\,°C)(100 - 25) = 28350\ kcal$

$Q_{in} = Q_{out} + 0.05Q_{in} = 28350 + 0.05Q_{in}$

$Q_{in} = 29842.105\ kcal$

$Q = 0.24\,Pt$

$$29842.105 = 0.24P\left(10\ min \times \frac{60\ sec}{1\ min}\right)$$

$P = 207.24\ kW$

73

$Q = mc\Delta_t$

$Q = (800\ g)(4.186\ J/g\,°C)(100 - 20) = 267904\ J$

$W = Pt$

$$t = \frac{W}{P} = \frac{267904}{1000} = 267.904\ sec \times \frac{1\ min}{60\ sec}$$

$t = 4.46\ min.$

74

Note: 1 calorie = 4.186 J

$$Q = \frac{Pt}{4.186} = \frac{100(1)}{4.186}$$

$Q = 23.88$ calories

75

$W_{in} = W_{out} + W_{loss}$

$W_{in} = 1000 + 0.25W_{in}$

$W_{in} = 1333.333\ kW\text{-}hr$

76

$Q = 0.24Pt = 0.24EIt$

$$Q = 0.24(110)(12)(30\ min) \times \frac{60\ sec}{1\ min} = 570.24\ kcal$$

$$m = 22.5\ \text{lb} \times \frac{1\ \text{kg}}{2.202\ \text{lb}} = 10.22\ \text{kg}$$

$$t_1 = \frac{5}{9}(°F - 32) = \frac{5}{9}(35 - 32) = 1.667\ °C$$

$$Q = mc\Delta_t = mc(t_2 - t_1)$$

$$t_2 = \frac{Q}{mc} + t_1 = \frac{570.24}{10.22(1)} + 1.667 = 57.46\ °C$$

$$t_2 = \frac{9}{5}°C + 32 = \frac{9(57.46)}{5} + 32$$

$$t_2 = 135.43\ °F$$

77

$$Q = mc\Delta_t \Rightarrow ❶$$

$$Q = 0.24Pt = 0.24Elt \Rightarrow ❷$$

Equate Eq.1 to Eq.2 :

$$0.24Elt = mc\Delta_t$$

$$0.24El = \frac{m}{t}(c\Delta_t)$$

$$0.24(120)(10) = \left(\frac{300\ \text{g}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{1\ \text{cal}}{\text{g}\ °C}\right)(\Delta_t)$$

$$\Delta_t = 57.6\ °C$$

$$\Delta_t = t_2 - t_1$$

$$t_2 = \Delta_t - t_1 = 57.6 - 15$$

$$t_2 = 72.6\ °C$$

78

Note: The rating of the heater is always the power drawn from the supply.

$$P_{drawn} = 4(1.5) = 6\ \text{kW}$$

$$P_{effective} = P_{drawn}\,\eta = 6\,(0.9) = 5.4\ \text{kW}$$

$$Q = mc\Delta t \Rightarrow ❶$$

$$Q = Pt \Rightarrow ❷$$

$$m = 4\ \text{m}^3\left(\frac{1000\ \text{kg}}{1\ \text{m}^3}\right) = 4000\ \text{kg}$$

Equate Eq.1 to Eq.2 :

$$mc\Delta t = Pt$$

$$t = \frac{mc\Delta_t}{P} = \frac{4000\ \text{kg}\,(4.186\ \text{kJ/kg}\ °C)(100 - 20)\ °C}{5.4\ \text{kW}}$$

$$t = 248059.259\ \text{sec}$$

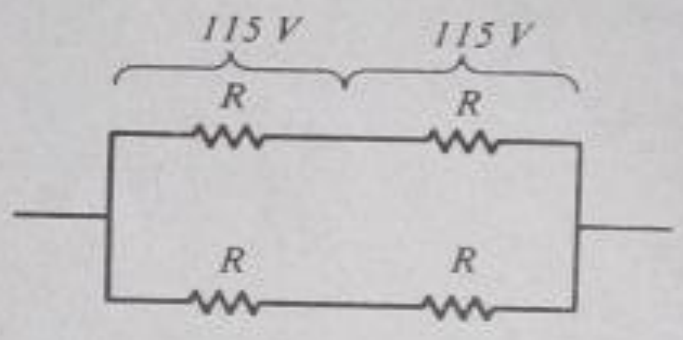
$$t = 248059.259 \times \frac{1\,hr}{3600\,sec} = 68.9\,hrs$$

$$t \cong 69\,hrs$$

79 $$R = \frac{E^2}{P} = \frac{230^2}{1500} = 35.266\,\Omega$$

Notes:

1. The rating of the heater is always the power drawn from the supply.
2. When two heater elements are connected in series, the voltage across each element will be equal to 1/2 of the total voltage.
2. Since not specified, assume the circuit connection does not affect the efficiency of the heater.



$$P_{drawn} = 4\left(\frac{E_{drop}^{\ 2}}{R}\right) = 4\left(\frac{(115)^2}{35.266}\right) = 1500\,W = 1.5\,kW$$

$$P_{effective} = P_{drawn}\,\eta = 1.5(0.9) = 1.35\,kW$$

$$Q = mc\Delta t \Rightarrow ❶$$

$$Q = Pt \Rightarrow ❷$$

$$m = 4\,m^3\left(\frac{1000\,kg}{1\,m^3}\right) = 4000\,kg$$

Equate Eq.1 to eq.2 :

$$mc\Delta t = Pt$$

$$t = \frac{mc\Delta_t}{P} = \frac{4000\,kg\,(4.186\,kJ/kg\,°C)(100-20)\,°C}{1.35\,kW}$$

$$t = 992237.037\,sec \times \frac{1\,hr}{3600\,sec}$$

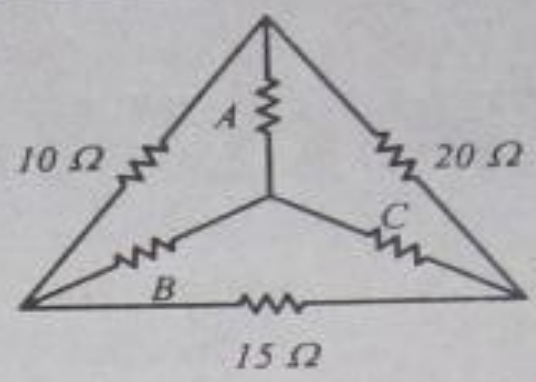
$$t = 275.6\,hrs.$$

80

$$A = \frac{10(20)}{10+15+20} = 4.44\,\Omega$$

$$B = \frac{10(15)}{10+15+20} = 3.33\,\Omega$$

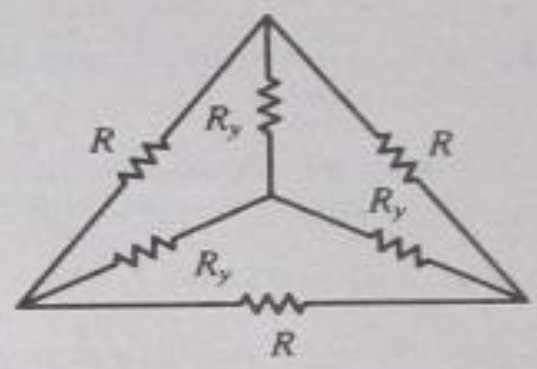
$$C = \frac{15(20)}{10+15+20} = 6.66\,\Omega$$



81

$$R_y = \frac{R(R)}{R+R+R} = \frac{R^2}{3R}$$

$$R_y = \frac{R}{3}$$



82 Convert the Y resistors to Δ:

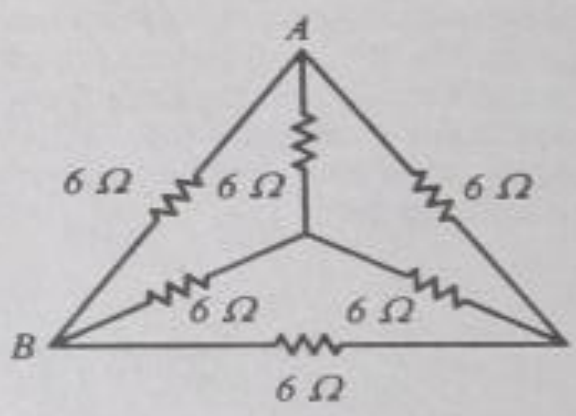
$$R_\Delta = 3R_y = 3(6) = 18\,\Omega$$

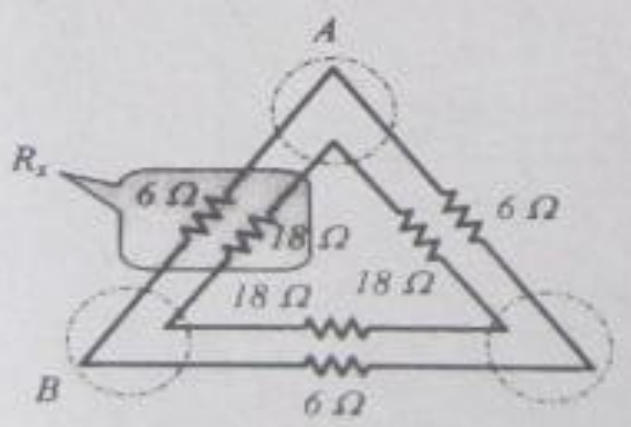
$$R_x = \frac{18(6)}{18+6} = 4.5\,\Omega$$

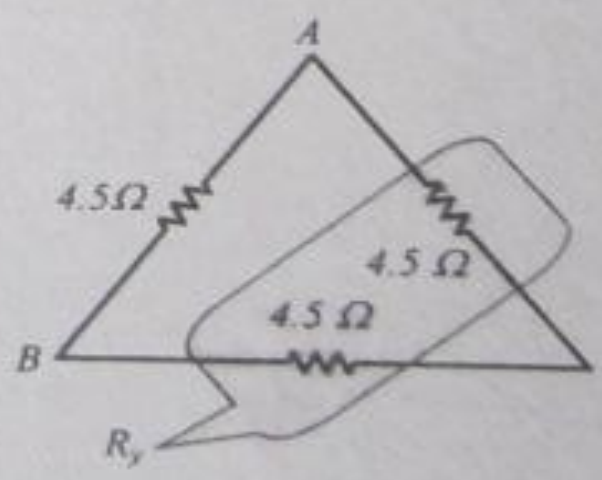
$$R_y = 4.5 + 4.5 = 9\,\Omega$$

$$R_{AB} = \frac{R_x R_y}{R_x + R_y} = \frac{4.5(9)}{4.5+9}$$

$$R_{AB} = 3\,\Omega$$







Network Laws & Theorems

KIRCHHOFF'S LAW

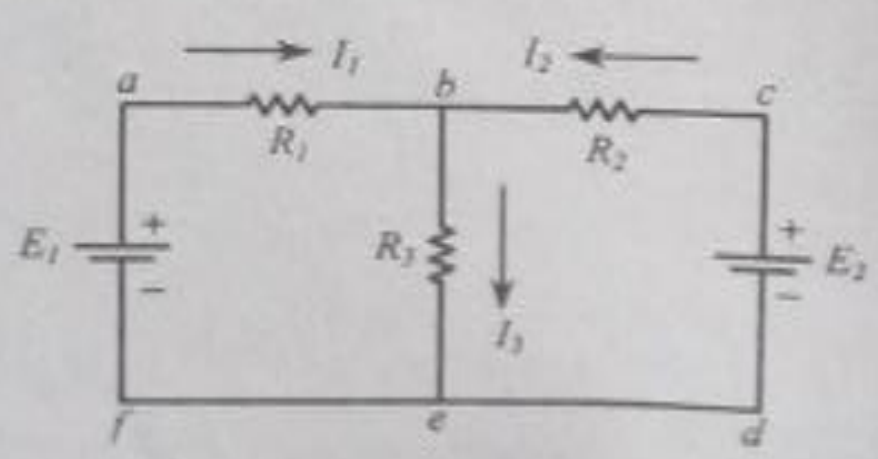
Named after the German physicist, **Gustav Robert Kirchhoff** (1824 – 1887)

- **Current Law (KCL)** - the algebraic sum of the currents at any junction or node of an electric circuit is zero.
- **Voltage Law (KVL)** - the algebraic sum of the emf's and the resistance voltage drops in any closed loop of an electric circuit is zero.

Sign Conventions for Kirchhoff's Law:

- Current towards the node, **positive current**.
- Current away from the node, **negative current**.
- In a voltage source, if loop enters on minus and goes out on plus, **positive emf**.
- In a voltage source, if loop enters on plus and goes out on minus, **negative emf**.
- In a resistance, if the loop direction is the same as the current direction, **negative resistance voltage drop**
- In a resistance, if the loop direction is opposite to the current direction, **positive resistance voltage drop**.

Example :



By KCL :

at junction b : $I_1 + I_2 - I_3 = 0$

By KVL :

for loop fabef : $E_1 - I_1R_1 - I_3R_3 = 0$

for loop dcbed : $E_2 - I_2R_2 - I_3R_3 = 0$

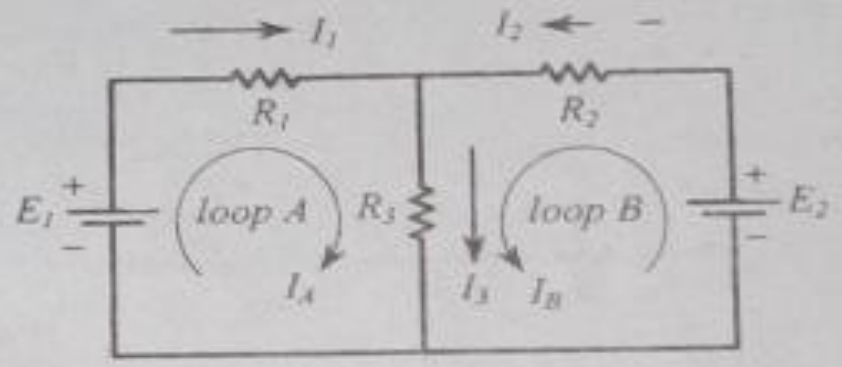
for loop fabcdef : $E_1 - I_1R_1 + I_2R_2 - E_2 = 0$

Note: Currents I_1, I_2 and I_3 are evaluated by simultaneous substitutions of the equations formulated using KCL and KVL.

MAXWELL'S MESH METHOD

This method involves a set of independent loop currents assigned to as many meshes as it exist in the circuit and these currents are employed in connection with appropriate resistances when the KVL equations are written.

Example :



By KVL :

for loop A: $E_1 - I_A(R_1 + R_3) - I_B R_3 = 0$

for loop B: $E_2 - I_B(R_2 + R_3) - I_A R_3 = 0$

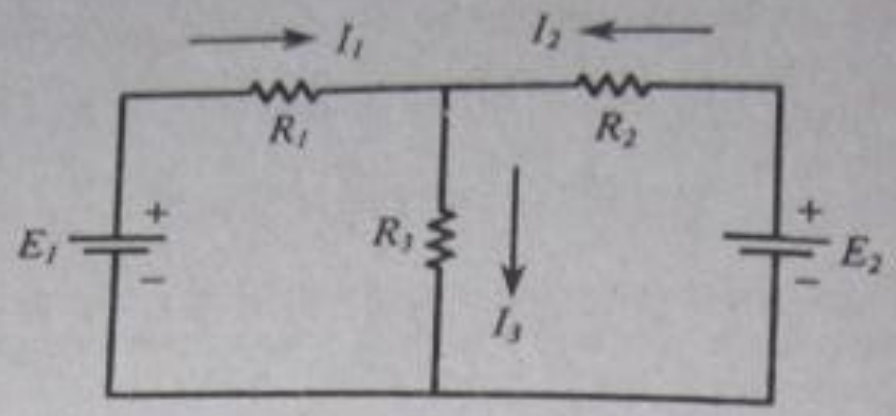
Note : Mesh currents I_A and I_B are evaluated by simultaneous substitutions of the equations formulated from each loop or mesh using KVL.

$$I_1 = |I_A| \qquad I_2 = |I_B| \qquad I_3 = |I_A + I_B|$$

SUPERPOSITION THEOREM

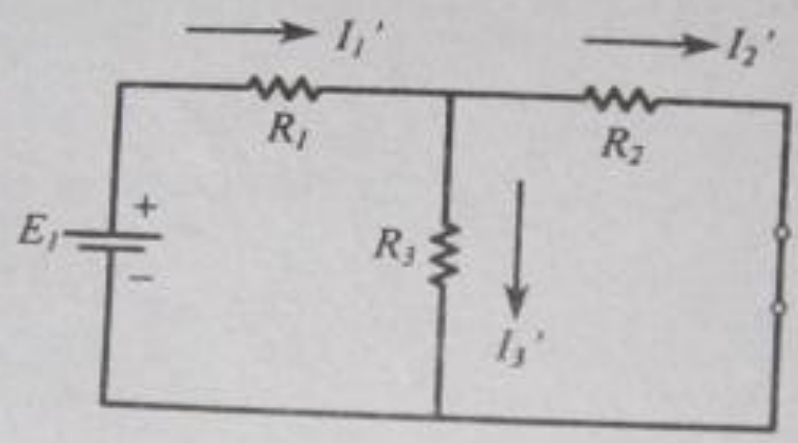
In a network of resistors, the current in any resistor is equal to the **algebraic sum of the currents** delivered by each independent sources assuming that each source is **acting alone or independently** with respect to the others.

Example :

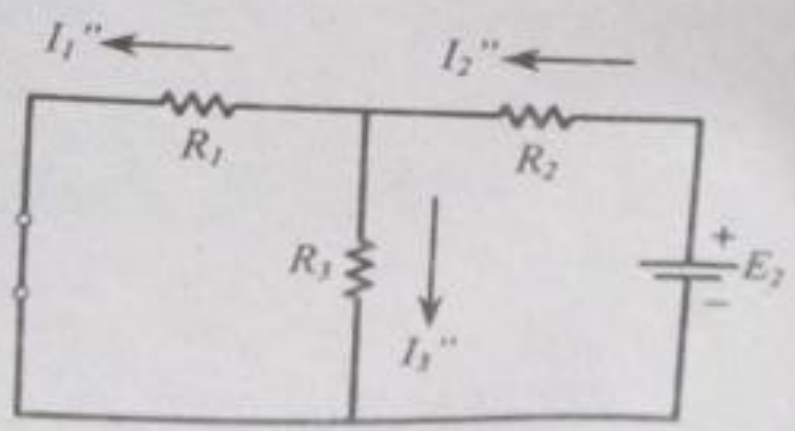


Note : If a source (either a current or a voltage source) is acting alone, the other current sources are open circuited while the other voltage sources are short circuited.

STEP 1 : If E_1 is acting alone.



STEP 2 : If E_2 is acting alone,



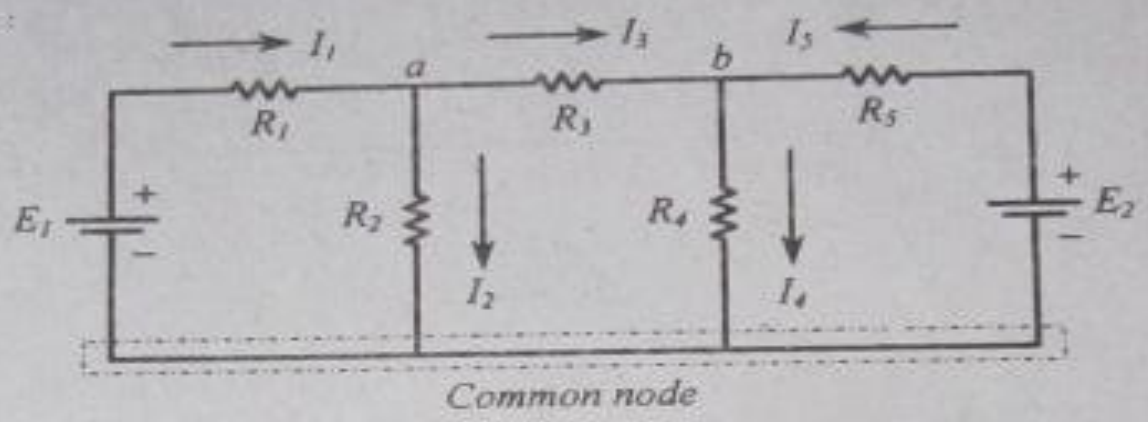
Note : The currents I_1', I_2', I_3', I_1'', I_2'' & I_3'' are evaluated using basic electric circuit principles.

$$I_1 = \left[I_1' - I_1''\right] \qquad I_2 = \left[I_2' - I_2''\right] \qquad I_3 = \left[I_3' + I_3''\right]$$

NODAL NODE METHOD

Using this method, a circuit with "n" nodes, has a solution with only "n – 1" number of equations needed.

Example :



Note : Since there are three (n = 3) nodes in this circuit, therefore only two equations are needed to solve this problem.

By KCL at node a: $I_1 = I_2 + I_3$

$$\frac{E_1 - V_a}{R_1} = \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3}$$

By KCL at node b: $I_4 = I_3 + I_5$

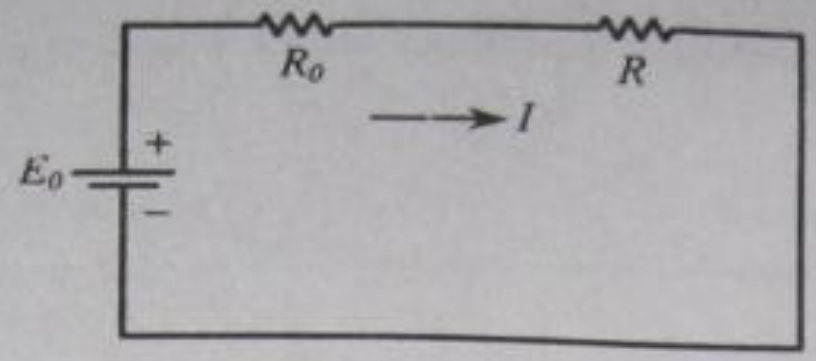
$$\frac{V_b}{R_4} = \frac{V_a - V_b}{R_3} + \frac{E_2 - V_b}{R_5}$$

Note : Node voltages V_a and V_b are evaluated by simultaneous substitutions of the equations formualted using KCL. and correspondingly, currents flowing through each resistances can be solved.

THEVENIN'S THEOREM

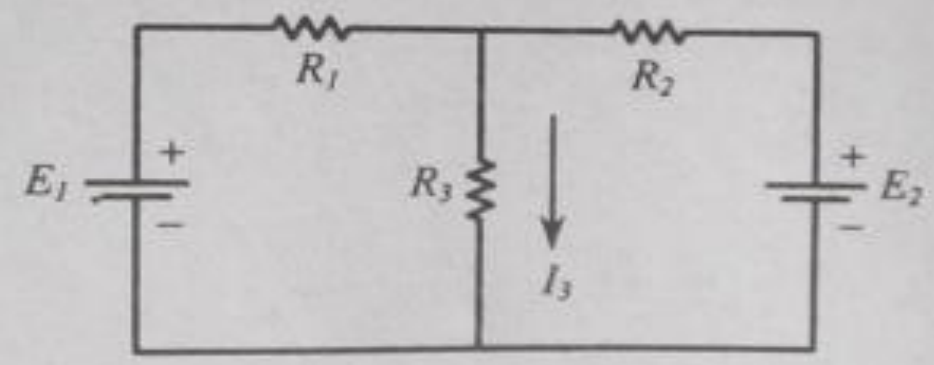
If a resistor of R ohms be connected between any two terminals of a linear network, the resulting steady state current through the resistor is the ratio of the **potential difference E_o** (between the two points prior to the connection) and the sum of the values of the **resistance R_o** (resistance between the two points) and the **connected resistance R**. Named after the French telegraph engineer, **Charles Leon Thevenin** (1857 – 1926)

Thevenin's equivalent circuit:



$$I = \frac{E_o}{R_o + R}$$

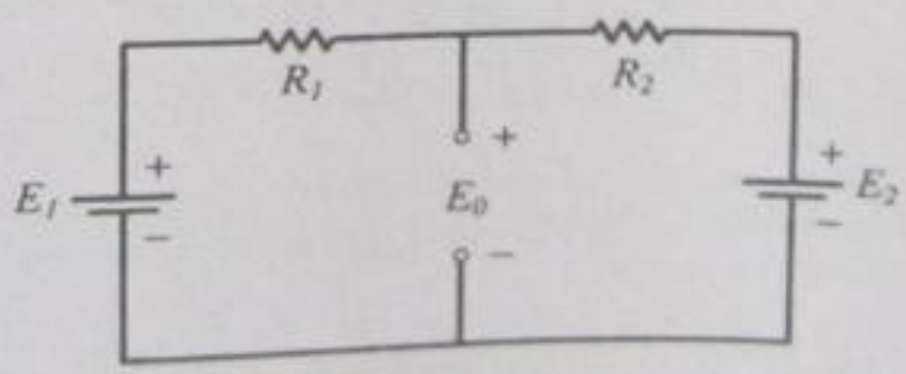
Example : Solve for I_3.



0

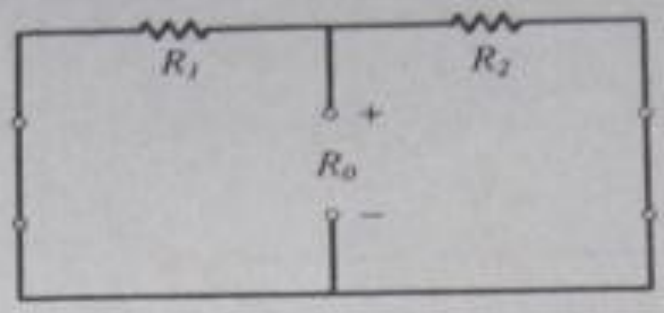
Solution :

STEP 1 : Open circuit R_3, and solve for the voltage across the open circuited terminals.



Note : E_o is computed using any methods (Kirchhoff's, Maxwell, Nodal, etc.) of analyzing network problems.

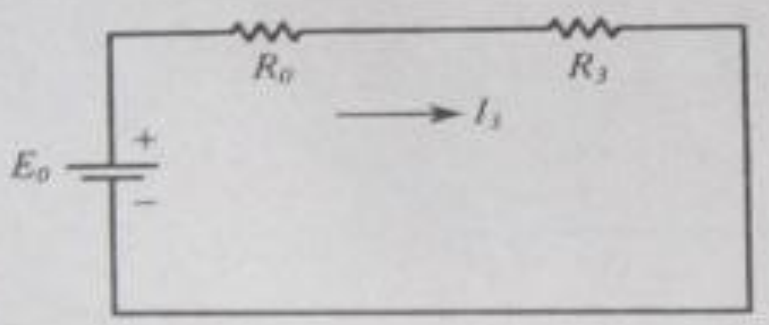
STEP 2 : Compute R_o (short circuit all independent voltage sources and open circuit all independent current sources).



$$R_o = \frac{R_1 R_2}{R_1 + R_2}$$

Note : R_o is computed using the basic principles of finding the total resistance of a given circuit.

STEP 3 : Construct the Thevenin's equivalent circuit and solve the resulting current.

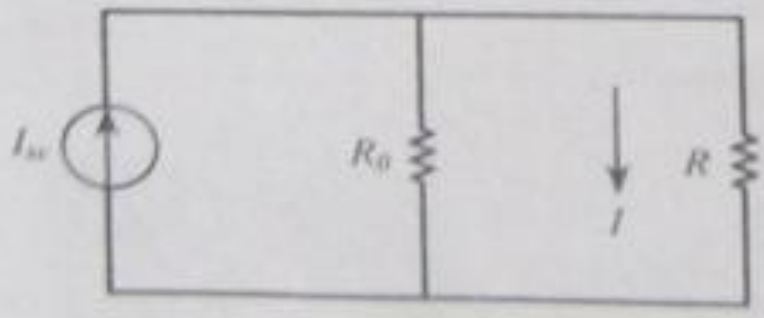


$$I_3 = \frac{E_o}{R_o + R_3}$$

NORTON'S THEOREM

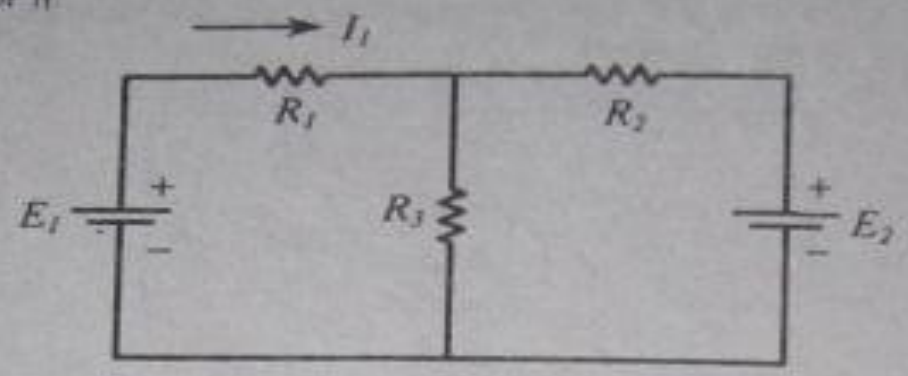
This theorem is analogous to Thevenin's theorem except that instead of the open circuit test, this theorem uses the short circuit test and the equivalent circuit is a parallel circuit. Named after the American engineer, **E. L. Norton** (1898 -)

Norton's equivalent circuit:

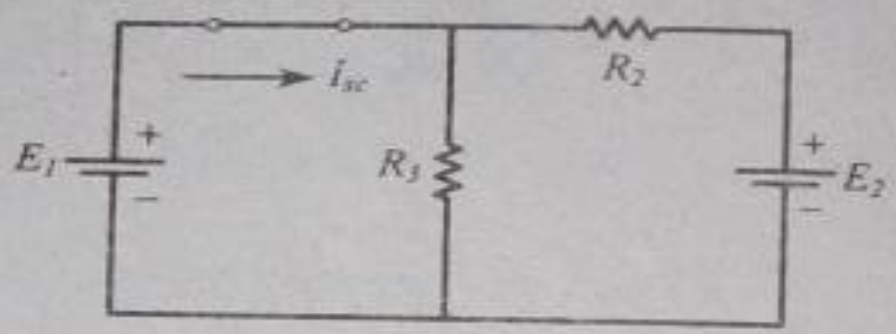


$$I = \frac{I_{sc}(R_o)}{R_o + R}$$

Example : Solve for I_1

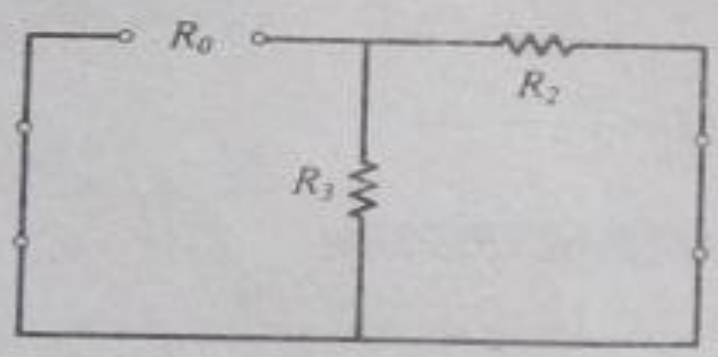


STEP 1 : Short circuit R_1 and solve the short circuit current that flows towards the branch



Note : I_{sc} is computed using any methods (Kirchhoff's, Maxwell, Nodal, etc.) of analyzing network problems.

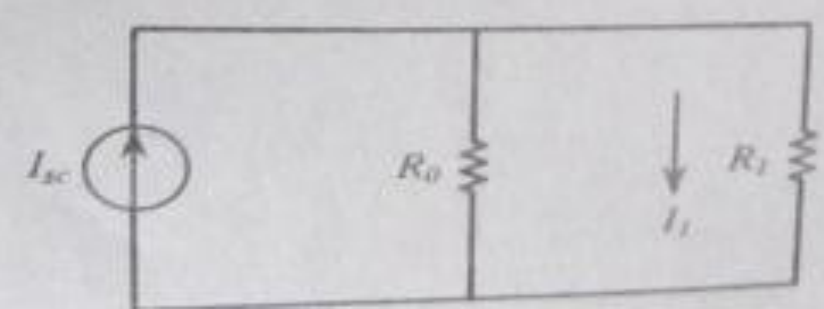
STEP 2 : Compute R_o



$$R_o = \frac{R_2 R_3}{R_2 + R_3}$$

Note : R_o is computed using the basic principles of finding the total resistance of a given circuit.

STEP 3 : Construct the Norton's equivalent circuit and solve for I_1

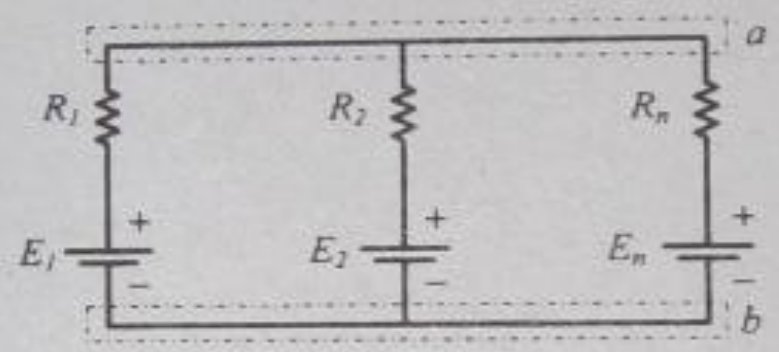


$$I_1 = \frac{I_{sc}(R_o)}{R_o + R_1}$$

MILLMAN'S THEOREM

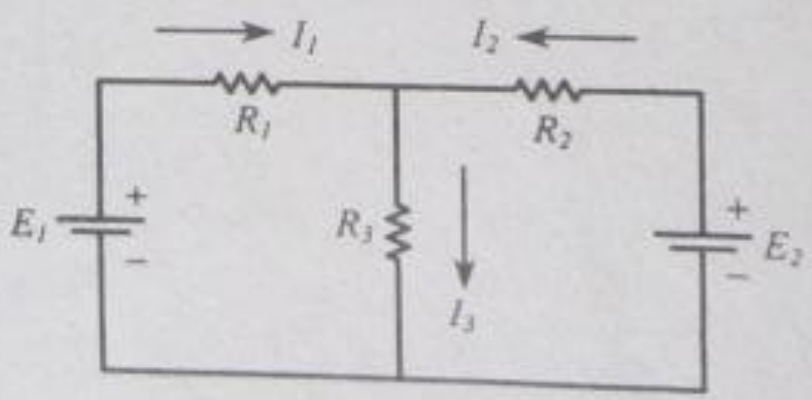
When any number of voltage sources of arbitrary generated voltage and finite internal resistance different from zero are connected in parallel, the resulting voltage across the parallel combination is the ratio of the algebraic sum of the currents that each source individually delivers when short circuited to the algebraic sum of the internal conductance.

Millman's equivalent circuit:

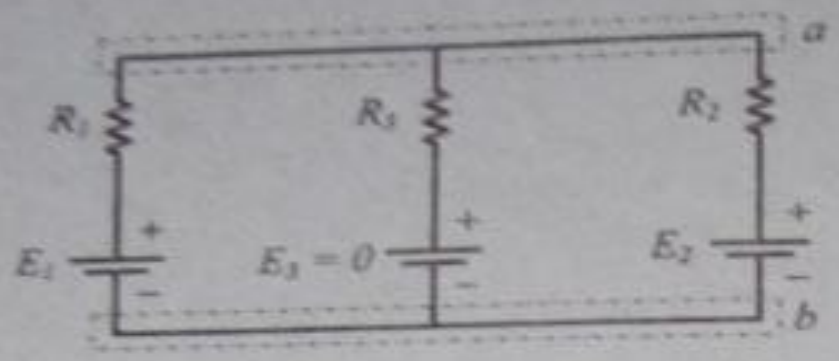


$$V_{ab} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \ldots\ldots + \frac{E_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}}$$

Example : Solve I_1, I_2 and I_3



STEP 1 : Draw the Millman's equivalent circuit and solve for V_{ab}



$$V_{ab} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

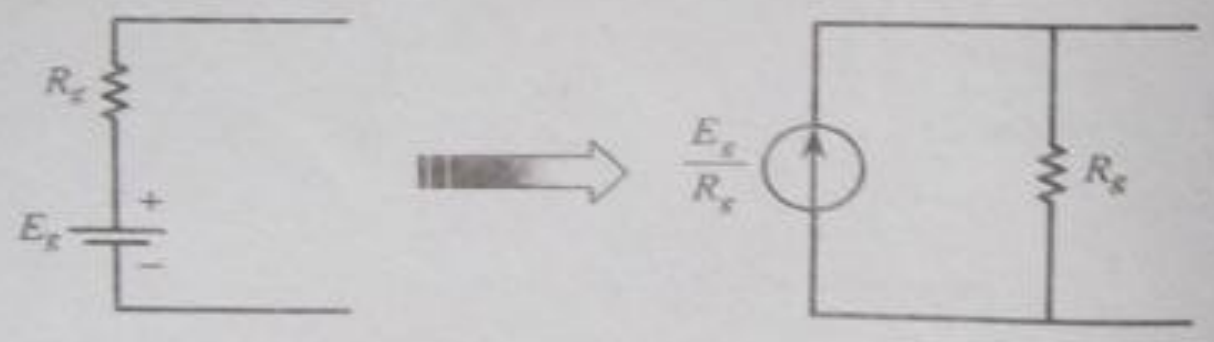
STEP 2: Solve for I_1, I_2 and I_3

$$I_1 = \frac{E_1 - V_{ab}}{R_1} \qquad I_2 = \frac{E_2 - V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

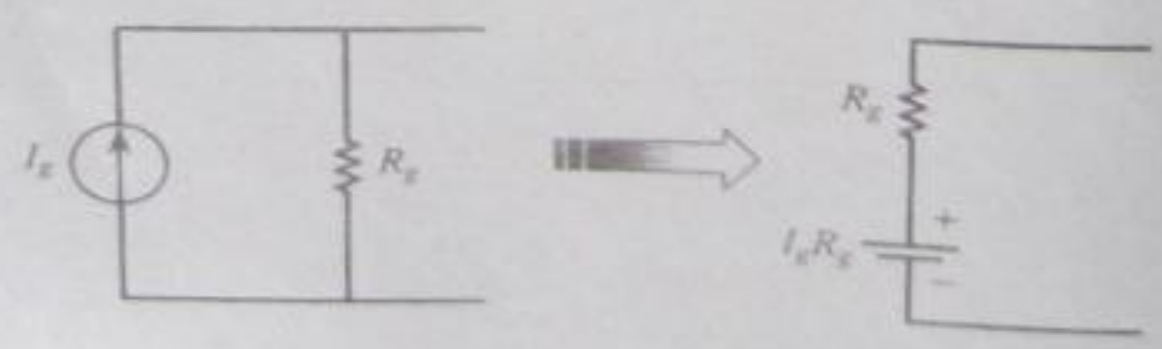
SOURCE TRANSFORMATION METHOD

This method simplifies the number of meshes in the network and thus simplifies the number of equations needed.

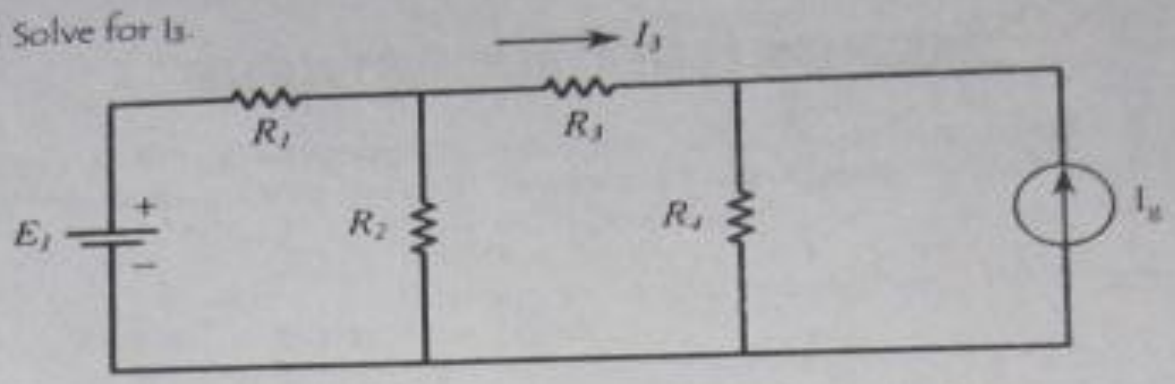
- **Voltage source to current source**



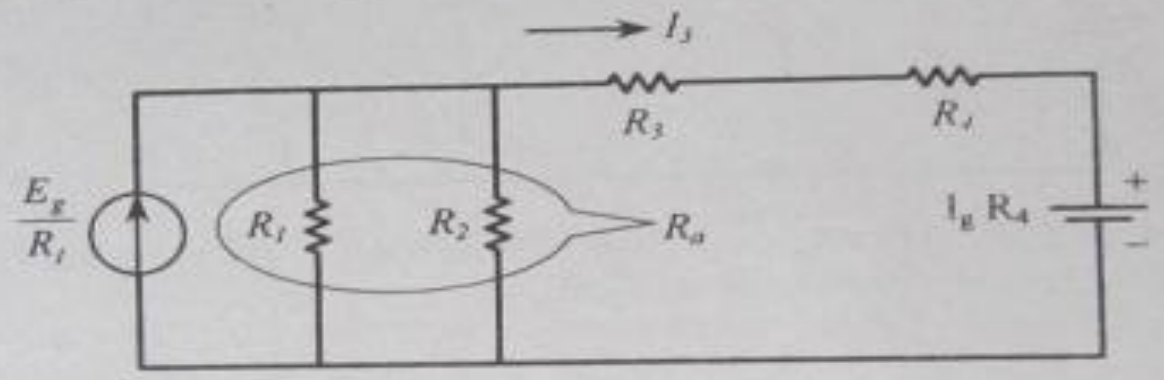
- **Current source to voltage source**



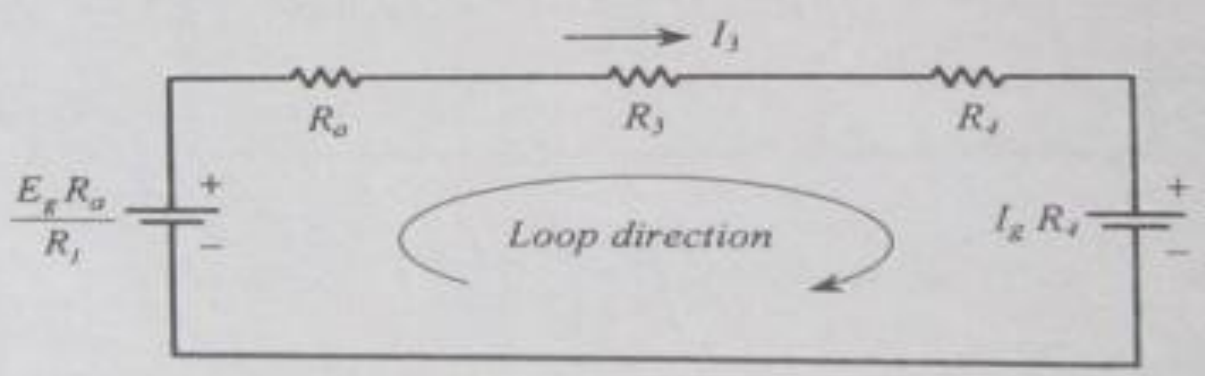
Example : Solve for I_3.



STEP 1 : Transform the voltage and current sources to their equivalents



STEP 2 : Simplify the parallel branch and transform back the current source to an equivalent voltage source.



STEP 3: By KVL (using the given loop direction), I_3 can then be solved.

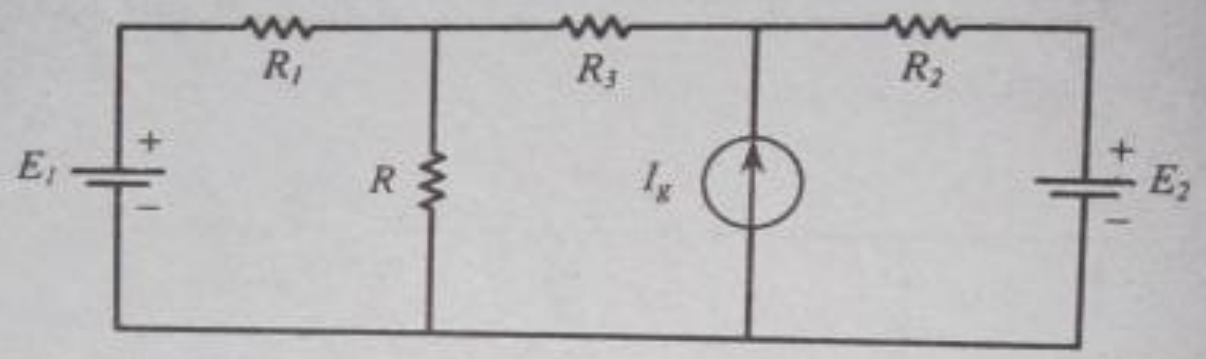
$$\frac{E_g R_a}{R_1} - I_3(R_a + R_3 + R_4) - I_g R_4 = 0$$

$$I_3 = \frac{\frac{E_g R_a}{R_1} - I_g R_4}{R_a + R_3 + R_4}$$

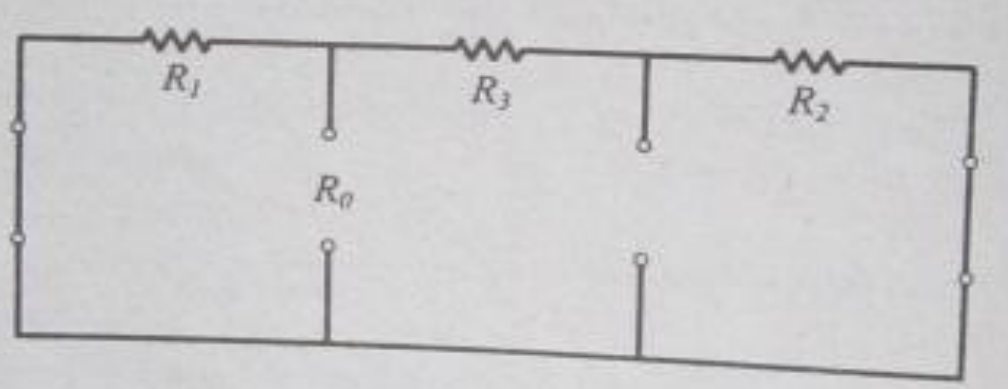
MAXIMUM POWER TRANSFER THEOREM

Maximum power transferred to a load resistor occurs **only** when the said resistor has a value **equal to the resistance (R_o)** of the network looking back from the load terminals.

Example : Find R if it is to absorbed maximum power.



Solution : Open circuit resistor R, short circuit all independent voltages sources and open all independent current sources.



$$R = R_0 = \frac{R_1(R_2 + R_3)}{R_1 + (R_2 + R_3)}$$

Note : If the corresponding current or power drawn by resistor R is to be solved, use either Thevenin's or Norton's theorem to solve the problem.

Test 3

Problem 83: EE Board October 1998

A 12 V battery of 0.05-ohm resistance and another battery of 12 V and 0.075 ohm resistance supply power to a 2-ohm resistor. What is the current through the load?

A. 5.85 A
B. 5.63 A
C. 5.72 A
D. 5.91 A

Problem 84:

Two equal resistances are connected in parallel across a 150 V DC source. What potential difference exists between the midpoint of one resistance and a point that is one-third from either end of the other resistance.

A. 24 V
B. 28 V
C. 25 V
D. 30 V

Problem 85: EE Board April 1989

The LRT trolley system 10 miles long is fed by two substations that generate 600 volts and 560 volts, respectively. The resistance of the trolley wire and rail return is 0.3 ohm per mile. If the car is located 4 miles from the 600-volt station draws 200 A from the line. How much is the current supplied by each station?

A. 133.33 A, 66.67 A
B. 123.67 A, 76.33 A
C. 117.44 A, 82.56 A
D. 125.54 A, 63.05 A

Problem 86:

Ten cells each of emf 1.5 volts and internal resistance of 0.2 ohm are joined in parallel and connected to an external circuit resistance of 3 ohms. What current the load draws?

A. 0.45 A
B. 0.65 A
C. 0.50 A
D. 0.48 A

Problem 87:

Two batteries A and B are connected across each other with terminals of the same polarity together. The open circuit emf and internal resistance of each battery is respectively 24 V and 1.5 Ω. Determine the resistance of a heating load connected across the parallel combination of batteries so that the power consumed in the load is 100 watts.

A. 4.16 Ω
B. 4.22 Ω
C. 4.02 Ω
D. 4.12 Ω

Problem 88: EE Board October 1996

The lead storage batteries "A" and "B" are connected in parallel. "A" has an open circuit voltage of 12 V and an internal resistance of 0.2 ohm. Battery "B" has an open circuit voltage of 12.2 V and an internal resistance of 0.3 ohm. If the two batteries together delivers power to a 0.5 ohm power resistor. Neglecting effects of temperature, how much current is contributed by battery "A"?

A. 29.62 A
B. 16.00 A
C. 12.85 A
D. 25.24 A

Problem 89:

Three resistors of 2-ohm resistance are connected in delta. Inside the delta another three 2-ohm resistors are connected in wye. Six batteries of negligible internal resistance and of different emf are inserted into each branch. Using an ammeter, the current in one of the delta branch was found out to be 3 A. If a 4-ohm resistance is inserted into that branch, what will be the new current?

A. 2.00 A
B. 1.75 A
C. 1.50 A
D. 1.25 A

Problem 90: EE Board April 1992

In Manila, the LRT runs between Gil Puyat Station and Tayuman Station, which is 4 km apart and maintains voltages of 420 volts and 410 volts respectively. The resistance of go and return is 0.05 ohm per km. The train draws a constant current of 300 A while in motion. What are the currents supplied by the two stations if the train is at the distance of minimum potential?

A. 175 A, 125 A
B. 183 A, 117 A
C. 164 A, 136 A
D. 172 A, 128 A

Problem 91: EE Board October 1986

A LRT car, 5 km distance from the Tayuman, takes 100 A over a 100 mm hard drawn copper trolley wire having a resistance of 0.270 ohm per km. The rail and ground return has a resistance of 0.06 ohm per km. If the station voltage is 750 V, what is the efficiency of transmission?

A. 78%
B. 81%
C. 74%
D. 79%

Problem 92:

A resistance R is connected across two batteries A and B connected in parallel. The open circuit emfs and internal resistances of the batteries are 12 V, 2 ohms and 8 V, 1 ohm, respectively. Determine the ohmic value of R if the power absorbed by R is 7.656 watts.

A. 10 Ω
B. 12 Ω
C. 9 Ω
D. 8 Ω

Problem 93: EE Board October 1981

A charger, a battery and a load are connected in parallel. The voltage across the charger is 12.5 volts and the battery has an emf of 12 volts and internal resistance of 0.1 ohm. The load consists of a 2 ohms resistor. Find the current through the charger.

A. 6.61 A
B. 6.25 A
C. 6.42 A
D. 6.50 A

Problem 94: EE Board October 1996

A lead storage battery is rated at 12 volts. If the internal resistance is 0.01 ohm, what is the maximum power that can be delivered to the load?

A. 1,200 W
B. 3,600 W
C. 7,200 W
D. 1,800 W

Problem 95: EE Board April 1995

A 120-V battery having an internal resistance of 0.5 ohm, is connected through a line resistance of 9.5 ohms to a variable load resistor. What maximum power will the battery deliver to the load resistor?

A. 36 watts

B. 63 watts
C. 630 watts
D. 360 watts

Problem 96:

Two resistors x and y are connected in parallel. Resistor x having a value of 6 ohms while resistor y being adjustable over a wide range. The parallel combination is then connected through a 2-ohm line resistor to a 120 V DC source. For what resistance y will its power be the greatest?

A. 8.0 Ω
B. 1.5 Ω
C. 2.0 Ω
D. 1.8 Ω

Problem 97: EE Board April 1988

A barangay power station supplies 60 kW to a load over 2,500 ft, 100 mm^2, two-conductor copper feeder, the resistance of which is 0.078 ohm per 1000 ft. The bus bar voltage is maintained constant at 600 V. Determine the maximum power which can be transmitted.

A. 230.77 kW
B. 220.35 kW
C. 242.73 kW
D. 223.94 kW

Problem 98:

A variable resistor R is connected in parallel with a fixed resistor of 1.25 ohms. The combination is then connected across a 12 V battery with internal resistance 0.25 ohm. Solve for the maximum power that can be delivered to resistor R.

A. 130.20 W
B. 115.52 W
C. 120.21 W
D. 142.42 W

Problem 99: EE Board April 1991

Twelve similar wires each of resistance 2 ohms are connected so as to form a cube. Find the resistance between the two diagonally opposite corners.

A. 1.45 ohms
B. 1.66 ohms
C. 2.01 ohms
D. 1.28 ohms

Problem 100: EE Board April 1991

Twelve similar wires each of resistance 2 ohms are connected so as to form a cube. Find the resistance between the two corners of the same edge.

A. 1.133 Ω
B. 1.102 Ω
C. 1.125 Ω
D. 1.167 Ω

Problem 101: EE Board October 1991

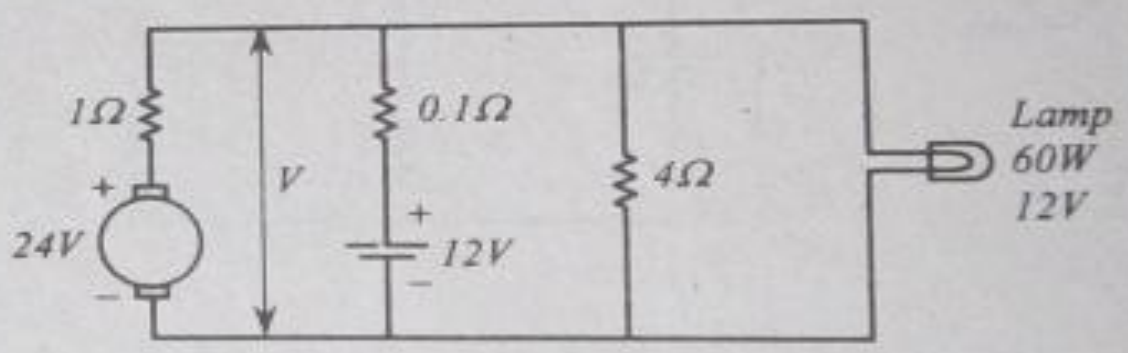
Twelve identical wires of resistance 6 ohms each are arranged to form the edge of a cube. A current of 40 mA is led into the cube at one corner and out at the other diagonally opposite corner. Calculate the potential difference developed between these corners.

A. 0.20 V
B. 0.28 V
C. 0.22 V
D. 0.24 V

Problem 102: EE Board August 1976

Find the value of the voltage V.

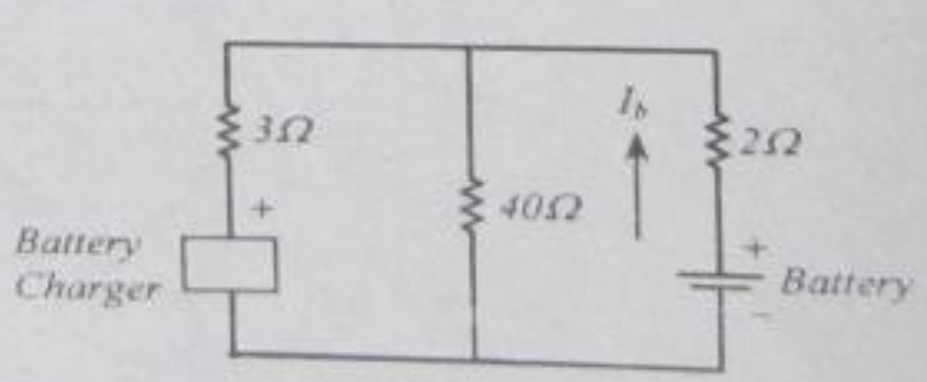
A. 12.34 V
B. 11.24 V
C. 12.19 V
D. 11.66 V



Problem 103: EE Board April 1982

Referring to the circuit diagram below, if the charger voltage is 130 volts and the battery voltage is 120 volts, solve for the current I_b.

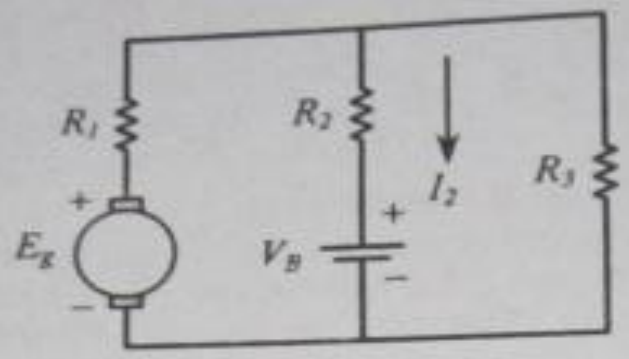
A. –0.215 A
B. 0.215 A
C. –0.306 A
D. 0.306 A



Problem 104: EE Board August 1977

In the figure below R_1 = 1 ohm, R_2 = 1 ohm, R_3 = 3 ohms, I_2 = 2 A and V_B = 120 V. Find E_g.

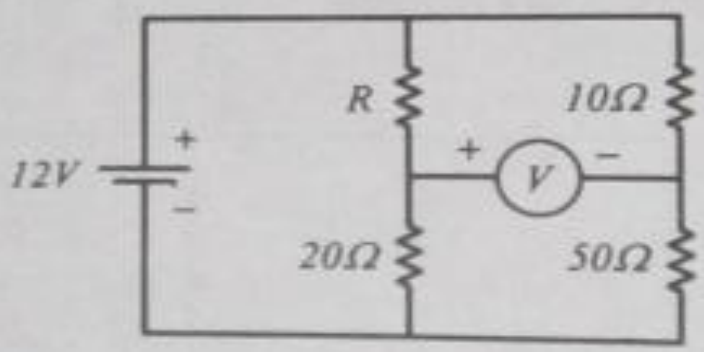
A. 182.41 V
B. 153.32 V
C. 164.67 V
D. 157.22 V



Problem 105: EE Board October 1980, EE Board April 1984

In the dc circuit as shown, the high resistance voltmeter gives a reading of +0.435 volt. What is the value of the resistance R?

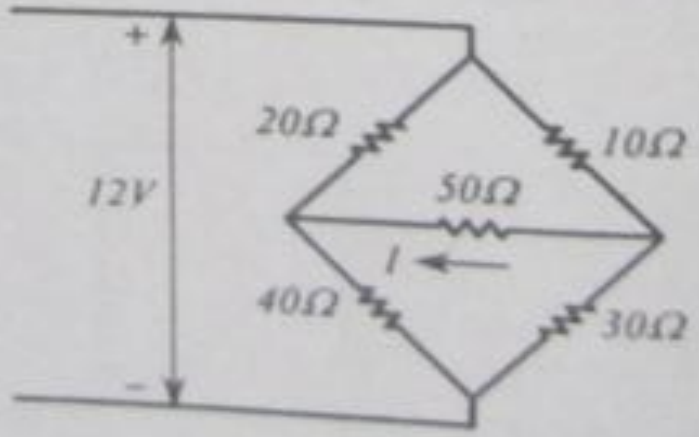
A. 4 ohms
B. 5 ohms
C. 3 ohms
D. 2 ohms



Problem 106: EE Board April 1980

Determine I in the figure.

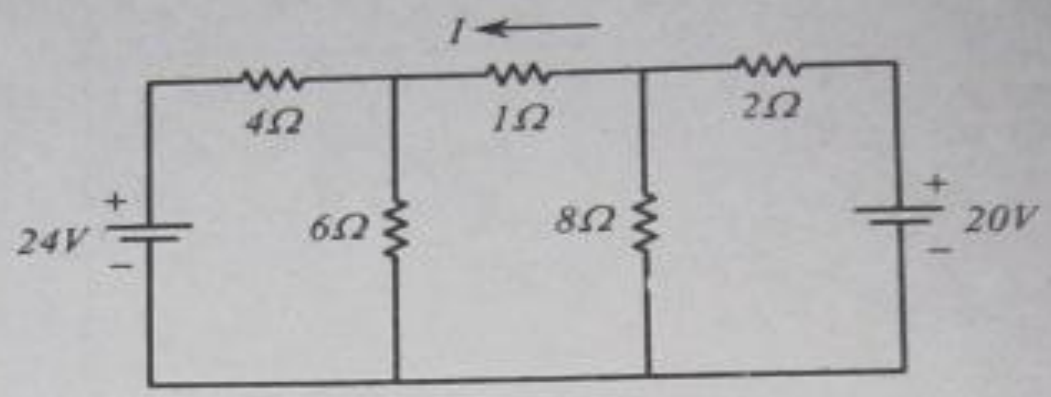
A. 0.028 A
B. 0.010 A
C. 0.025 A
D. 0.014 A



Problem 107:

Solve for I using Source transformation method.

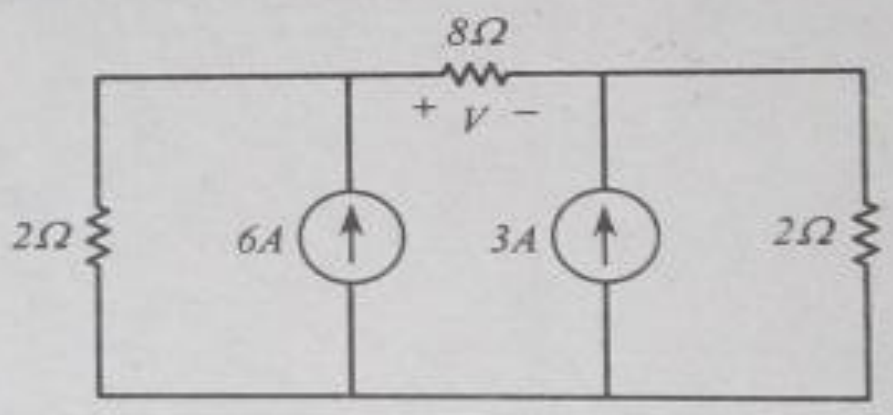
A. 0.32 A
B. 0.25 A
C. 0.38 A
D. 0.27 A



Problem 108:

Solve for V using Maxwell's mesh method.

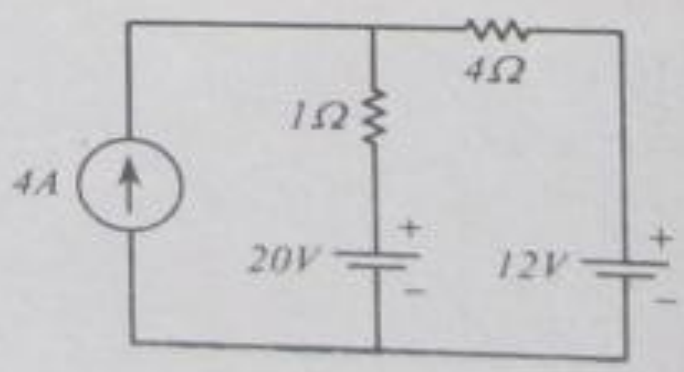
A. 4.0 V
B. 5.2 V
C. 2.6 V
D. 3.4 V



Problem 109:

Determine the current in the 1-ohm resistor using Norton's theorem.

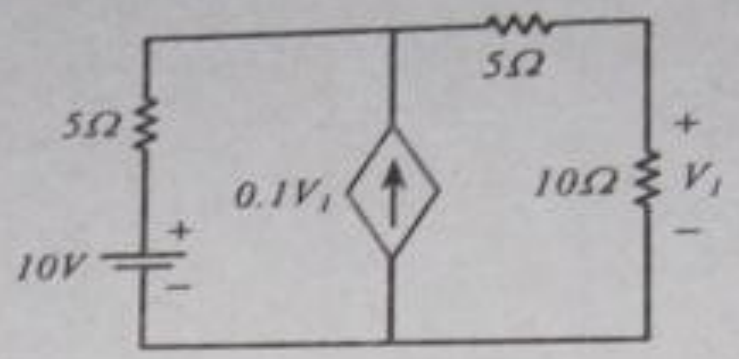
A. 1.54 A
B. 1.25 A
C. 1.60 A
D. 1.33 A



Problem 110:

Determine the current in the 10-ohm resistor using Thevenin's theorem.

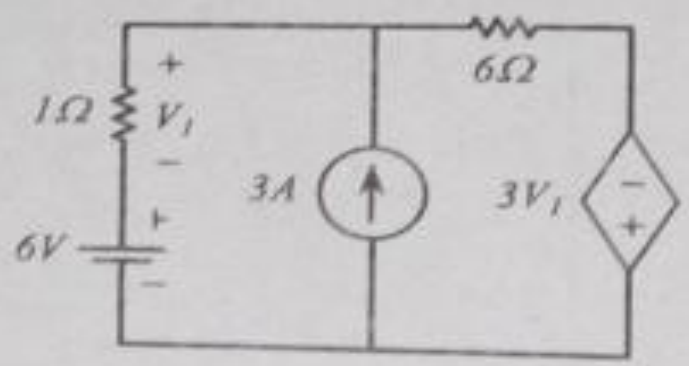
A. 0.833 A
B. 0.667 A
C. 0.707 A
D. 0.508 A



Problem 111:

Determine the current in the 6-ohm resistor by Superposition theorem.

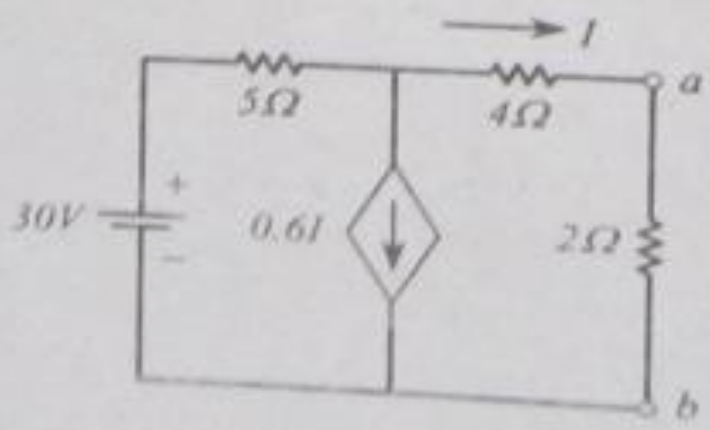
A. 3.4 A
B. 4.2 A
C. 4.0 A
D. 3.8 A



Problem 112:

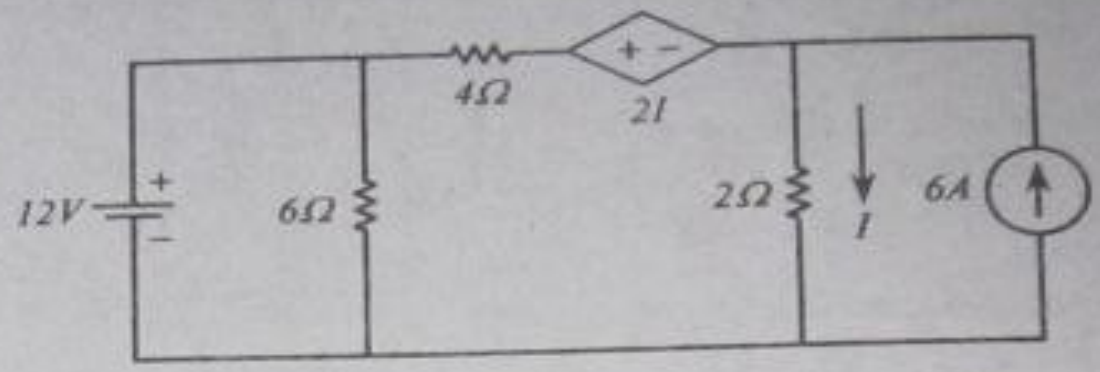
Determine the looking back resistance between terminals a & b.

A. 11 Ω
B. 9 Ω
C. 10 Ω
D. 12 Ω



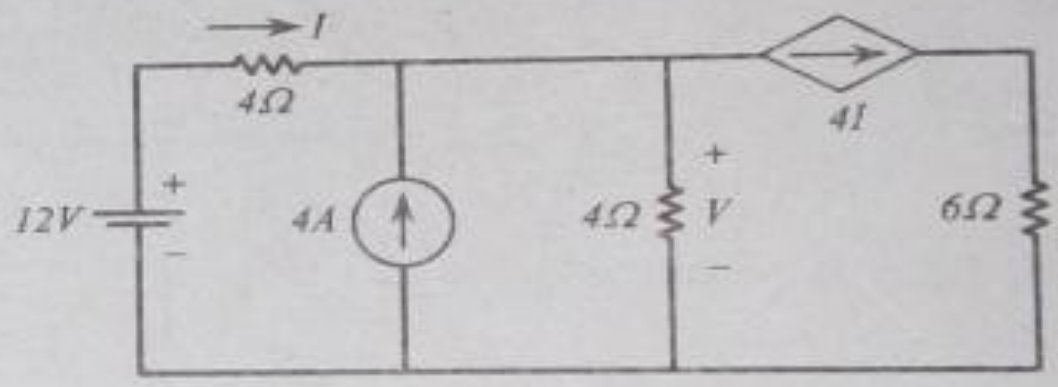
Problem 113:
Determine I.

A. 4.50 A
B. 4.28 A
C. 5.34 A
D. 3.75 A



Problem 114:
Determine the voltage V.

A. 14 V
B. 12 V
C. 10 V
D. 16 V



ANSWER KEY

83.	D	90.	A	97.	A	104.	C	111.	B
84.	C	91.	A	98.	C	105.	C	112.	D
85.	A	92.	A	99.	B	106.	D	113.	A
86.	C	93.	D	100.	D	107.	A	114.	C
87.	D	94.	B	101.	A	108.	A		
88.	C	95.	D	102.	A	109.	C		
89.	C	96.	B	103.	A	110.	B		

☑ RATING

28-32 - Topnotcher
22-27 - Passer
16-21 - Conditional
0-15 - Failed

Solutions to Test 3

83 Apply KVL on loop 'dabed':

$12 - 0.05I_1 - 2I_L = 0$

$I_1 = 240 - 40I_L \Rightarrow$ ❶

Apply KVL on loop 'fcbef':

$12 - 0.075I_2 - 2I_L = 0$

$I_2 = 160 - 26.67I_L \Rightarrow$ ❷

Apply KCL at node 'b':

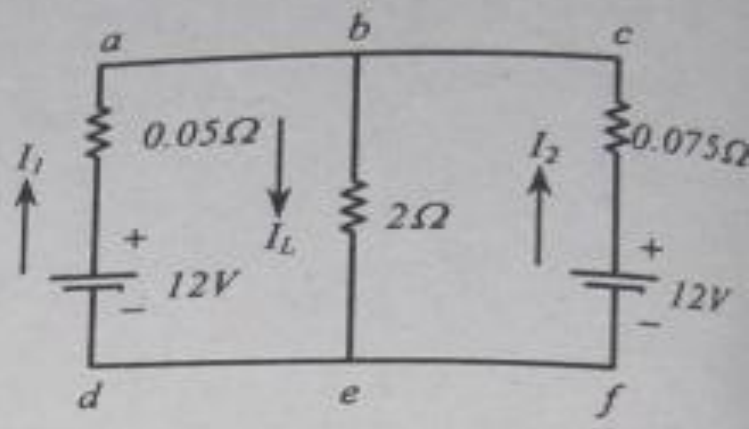
$I_1 + I_2 = I_L \Rightarrow$ ❸

Substitute Eq.1 and Eq.2 in Eq.3:

$240 - 40I_L + 160 - 26.67I_L = I_L$

$67.67I_L = 400$

$I_L = 5.91\text{A}$



84 $I = \dfrac{150}{R} \Rightarrow$ ❶

Apply KVL on loop 'cdbac':

$V + I\left(\dfrac{R}{3}\right) - I\left(\dfrac{R}{2}\right) = 0$

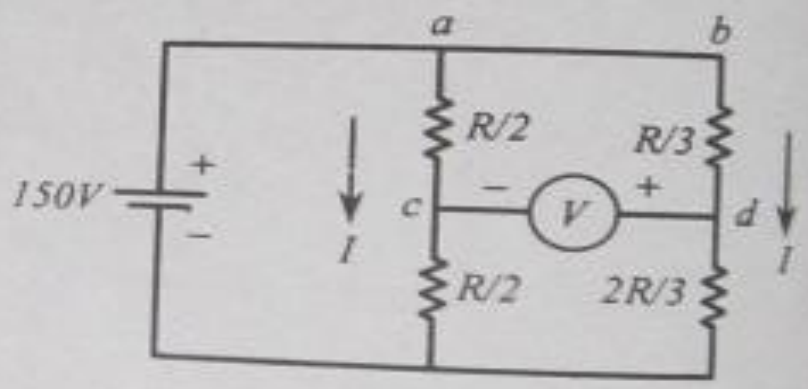
$V - 0.167\,IR = 0 \Rightarrow$ ❷

Substitute Eq.1 To Eq.2:

$V - 0.167\left(\dfrac{150}{R}\right)(R) = 0$

$V - 25.05 = 0$

$V \cong 25$ volts



85 $r_1 = 0.3(4) = 1.2\,\Omega$

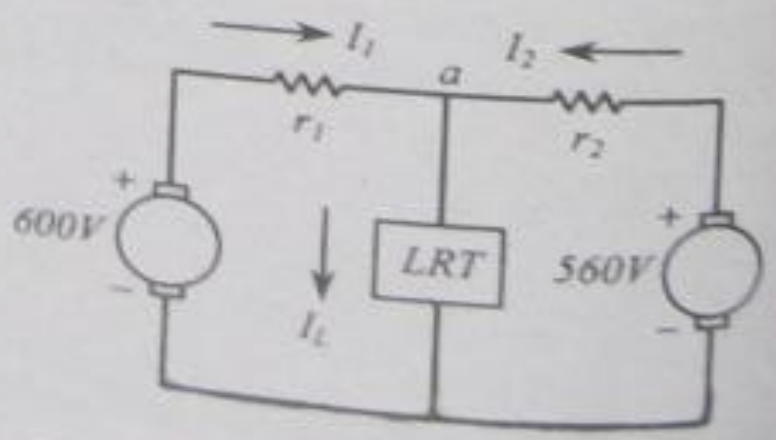
$r_2 = 0.3(6) = 1.8\,\Omega$

Use Nodal node method :

Apply KCL at node 'a':

$I_1 + I_2 = I_L$

$\dfrac{600 - V_L}{1.2} + \dfrac{560 - V_L}{1.8} = 200$



$V_L = 440$ volts

$$I_1 = \frac{600 - V_L}{1.2} = \frac{600 - 440}{1.2}$$

$I_1 = 133.333$ A

$$I_2 = \frac{560 - V_L}{1.8} = \frac{560 - 440}{1.8}$$

$I_2 = 66.667$ A

86 Use Thevenin's theorem:
Solve for the looking back resistance across the load terminals.

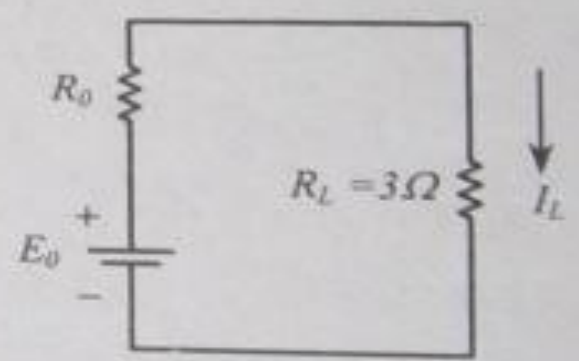
Note: Open circuit R_L. Short-circuit all the 10 cells. By inspection the looking back resistance is equal to the equivalent resistance of 10 identical resistors connected in parallel.

$$R_0 = \frac{r}{10} = \frac{0.2}{10} = 0.02\,\Omega$$

Solve the open circuit voltage across the load terminals:

Note: Since the cells are identical, there is no circulating current through the parallel combination, thus E_0 is equal to the open circuit emf of one cell.

$E_0 = 1.5$ volts



Using the Thevenin's equivalent circuit:

$$I_L = \frac{E_0}{R_0 + R_L} = \frac{1.5}{0.02 + 3} = 0.4966$$

$I_L \cong 0.5$ A

Another solution:

Note: Since the cells are identical, the load current (I_L) is divided equally ($I_L/10$) among the ten cells. Refer to the circuit diagram, consider a loop with one cell only and the load resistance.

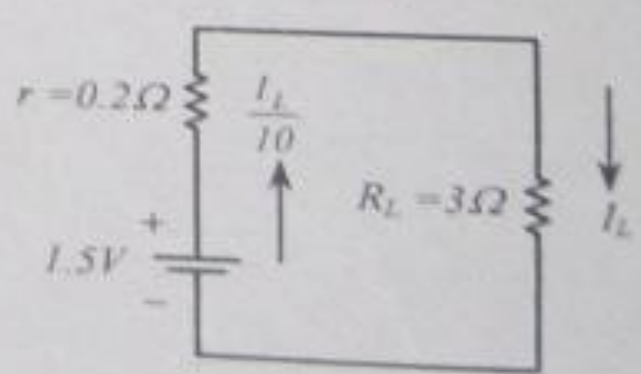
Apply KVL:

$$1.5 - \frac{I_L}{10}(0.2) - I_L\,(3) = 0$$

$1.5 - 3.02\,I_L = 0$

$I_L = 0.4966$

$I_L \cong 0.5$ A



67 **Use Thevenin's theorem**
Solve for looking back resistance across the load terminals:

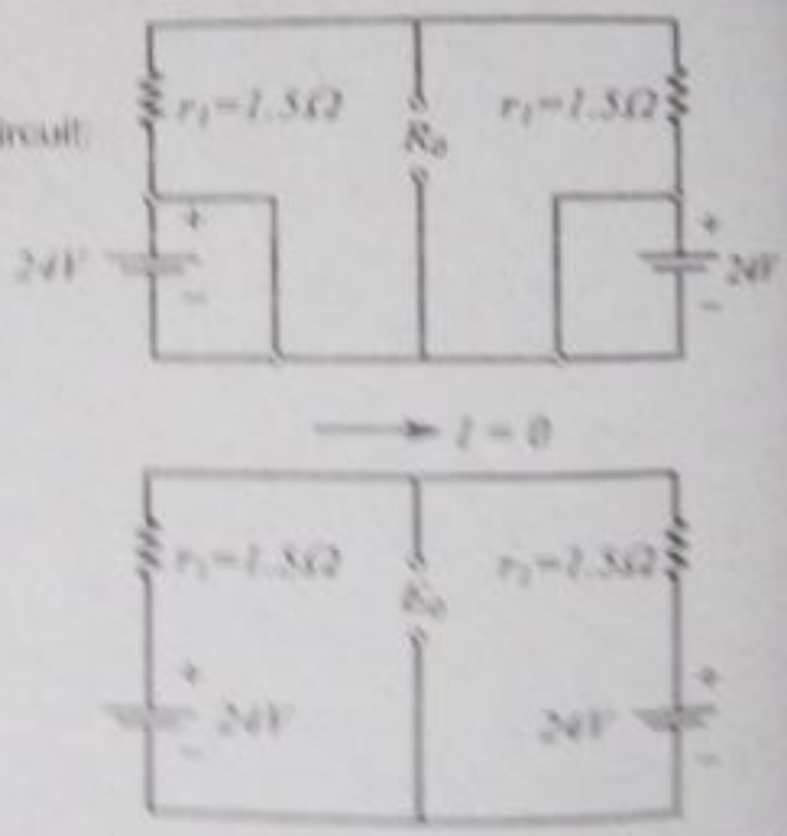
Note: Open circuit R_L. Short-circuit the two batteries. By inspection, the looking back resistance is equal to the equivalent resistance of two identical resistors connected in parallel.

$$R_o = \frac{r}{2} = \frac{1.5}{2} = 0.75\,\Omega$$

Solve the open circuit voltage across the output.

Note: Since the batteries are identical, there is no circulating current in the loop for the two batteries, thus the open circuit voltage is equal to the open circuit voltage of each battery.

$$E_o = 24\text{ V}$$



Using the Thevenin's equivalent circuit:

$$I_L = \frac{E_o}{R_o + R_L} = \frac{24}{0.75 + R_L} \rightarrow (1)$$

$$P_L = I_L^2 R_L \rightarrow (2)$$

Substitute Eq 1 to Eq 2:

$$100 = \left(\frac{24}{0.75 + R_L}\right)^2 R_L$$

$$(0.75 + R_L)^2 = \frac{576R_L}{100}$$

$$0.5625 + 1.5R_L + R_L^2 = 5.76R_L$$

$$R_L^2 - 4.26R_L + 0.5625 = 0$$

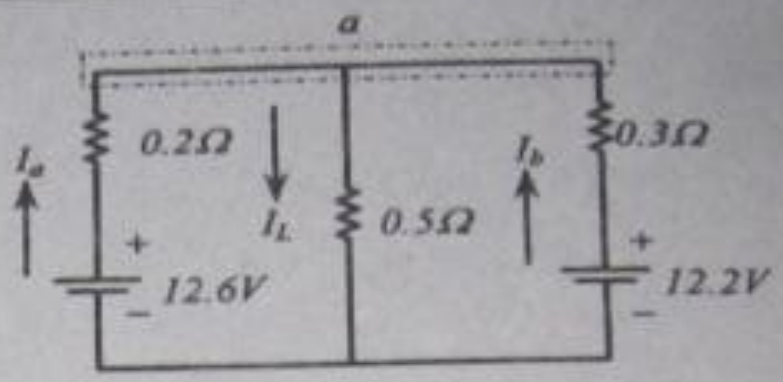
Using the quadratic formula:

$$R_L = \frac{4.26 \pm \sqrt{(4.26)^2 - 4(1)(0.5625)}}{2(1)} = \frac{4.26 \pm 3.867}{2}$$

$$R_L^{(+)} = 4.12\,\Omega$$

$$R_L^{(-)} = 0.138\,\Omega$$

88. Use Nodal node method:
Apply KCL at node 'a'.



$I_a + I_b = I_L$

$$\frac{12.6 - V_1}{0.2} + \frac{12.2 - V_1}{0.3} = \frac{V_1}{0.5}$$

$63 - 5V_1 + 40.66 - 3.33V_1 = 2V_1$

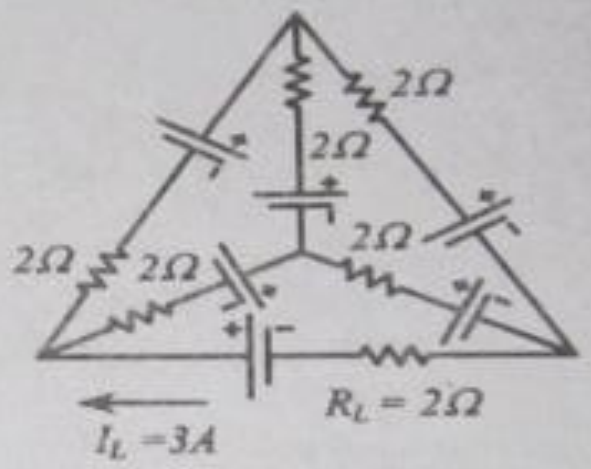
$10.33V_1 = 103.66$

$V_1 = 10.03$ volts

$$I_a = \frac{12.6 - V_1}{0.2} = \frac{12.6 - 10.03}{0.2}$$

$I_a = 12.85$ A

89. Use Thevenin's theorem.
Open circuit R_L and solve for R_0.

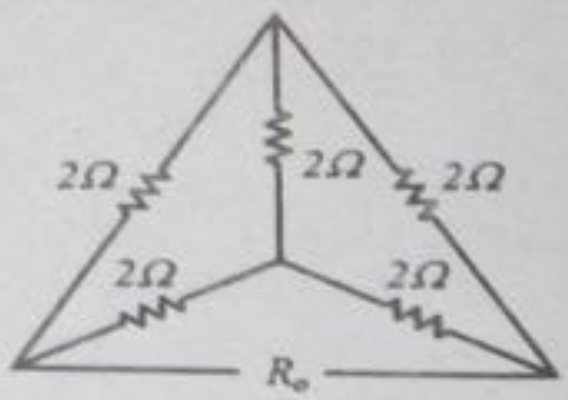


$$R_0 = R_y + \frac{2 + R_y}{2} \Rightarrow (1)$$

$$R_y = \frac{R_\Delta}{3} = \frac{2}{3}\Omega$$

Substitute R_y in Eq.1:

$$R_0 = \frac{2 + 2/3}{2} + 2/3 = 2\Omega$$



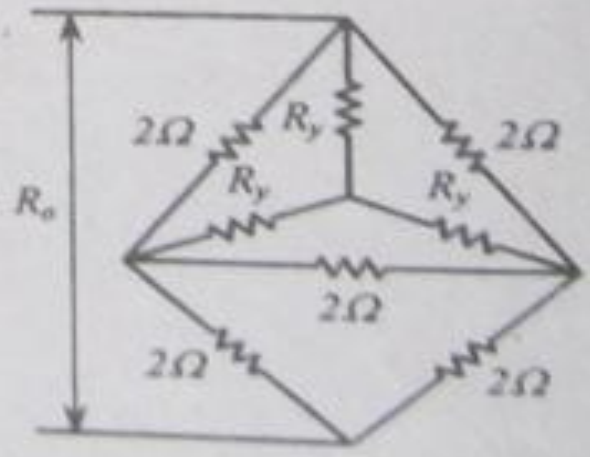
Using Thevenin's equation:

$$I_L = \frac{E_0}{R_0 + R_L}$$

If $R_L = 2\Omega$

$E_0 = I_L (R_0 + R_L) = 3(2 + 2)$

$E_0 = 12$ volts



If a 4Ω resistor is inserted:

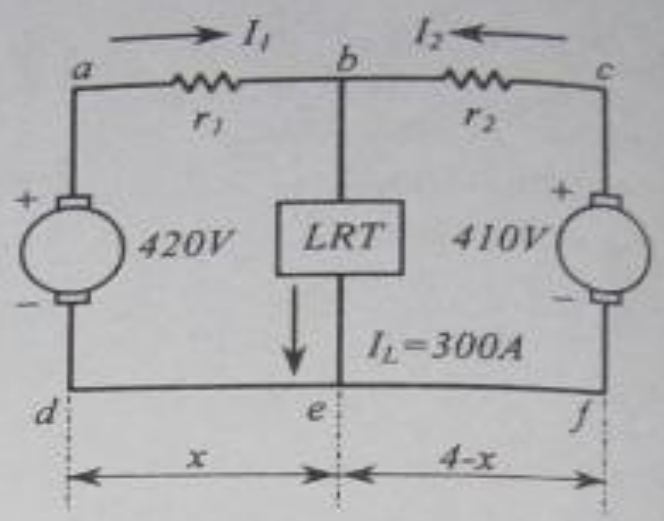
$R_L = 2 + 4 = 6\Omega$

$$I_L = \frac{E_0}{R_0 + R_L} = \frac{12}{2 + 6}$$

$I_L = 1.5$ A

90

$r_1 = 0.05x$
$r_2 = 0.05(4 - x)$



Apply KCL on node 'b':
$I_1 + I_2 = 300$
$I_1 = 300 - I_2 \Rightarrow$ ❶

Let: V_L = voltage across the train

Apply KVL on loop 'dabed':
$420 - I_1(0.05x) - V_L = 0$
$V_L = 420 - I_1(0.05x) \Rightarrow$ ❷

Apply KVL on loop 'fcbef':
$410 - I_2(0.05)(4 - x) - V_L = 0$
$V_L = 410 - I_2(0.2 - 0.05x) \Rightarrow$ ❸

Substitute Eq.1 in Eq.2:
$V_L = 420 - (300 - I_2)(0.05x)$
$V_L = 420 - 15x + 0.05xI_2 \Rightarrow$ ❹

Equate Eq.3 to Eq.4:
$410 - 0.2I_2 + 0.05xI_2 = 420 - 15x + 0.05xI_2$

$$I_2 = \frac{410 - 420 + 15x}{0.2}$$

$I_2 = 75x - 50$

Substitute I_2 in Eq.4:
$V_L = 420 - 15x + 0.05x(75x - 50)$
$V_L = 420 - 15x + 3.75x^2 - 2.5x$
$V_L = 420 - 17.5x + 3.75x^2$

$$\frac{dV_L}{dx} = 0 - 17.5 + 7.5x$$

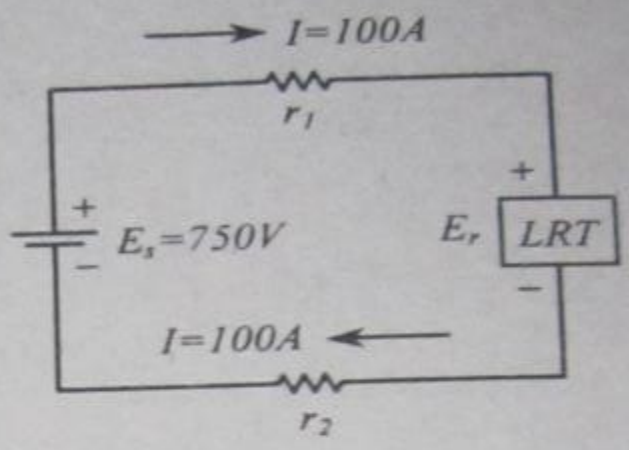
$0 = 7.5x - 17.5$
$x = 2.333$

$I_2 = 75x - 50 = 75(2.333) - 50$
$I_2 = 125$ A
$I_1 = 300 - I_2 = 300 - 125$
$I_1 = 175$ A

91

$r_1 = 0.27(5) = 1.35\ \Omega$

$r_2 = 0.0.06(5) = 0.3\ \Omega$



Apply KVL :

$750 - 100r_1 - E_r - 100r_2 = 0$

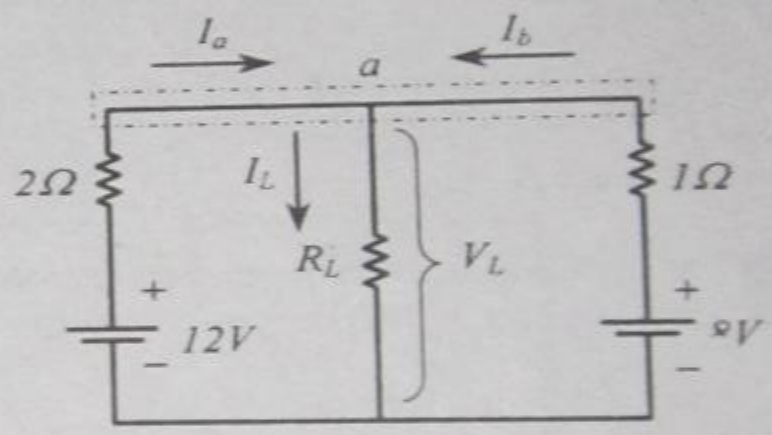
$750 - 100(1.35) - E_r - 100(0.3) = 0$

$E_r = 585\ V$

$$\eta = \frac{P_r}{P_s} = \frac{E_r I}{E_s I} = \frac{E_r}{E_s} = \frac{585}{750} \times 100\%$$

$\eta = 78\%$

92



Use Nodal node method:
Apply KCL at node 'a'.

$I_a + I_b = I_L$

$$\frac{12 - V_L}{2} + \frac{8 - V_L}{1} = I_L$$

$6 - 0.5V_L + 8 - V_L = I_L$

$I_L = 14 - 1.5V_L \Rightarrow$ ❶

$P_L = V_L I_L$

$$I_L = \frac{P_L}{V_L} = \frac{7.656}{V_L} \Rightarrow ❷$$

Equate Eq.1 to Eq.2:

$$14 - 1.5V_L = \frac{7.656}{V_L}$$

$14V_L - 1.5V_L^2 = 7.656$

$V_L^2 - 9.33V_L + 5.104 = 0$

Using the quadratic formula:

$$V_L = \frac{9.33 \pm \sqrt{(9.33)^2 - 4(1)(5.104)}}{2(1)} = \frac{9.33 \pm 8.162}{2}$$

$V_L^{(+)} = 8.75$ volts

$V_L^{(-)} = 0.584$ volts

Substitute $V_L = 8.75$ in Eq.2:

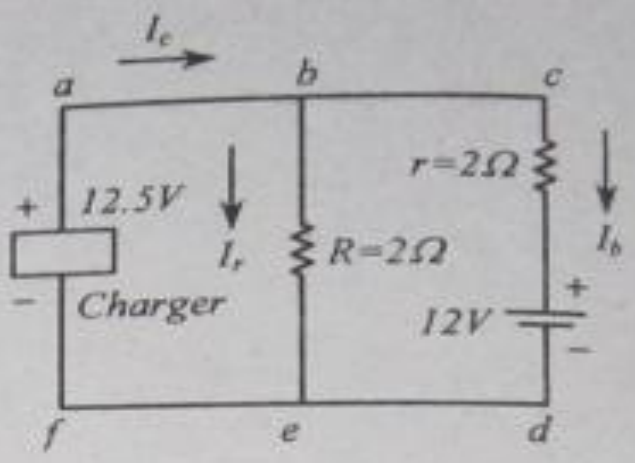
$$I_L = \frac{7.656}{8.75} = 0.875\text{ A}$$

$$R = \frac{V_L}{I_L} = \frac{8.75}{0.875}$$

$R = 10$ ohms

93

$$I_r = \frac{12.5}{2} = 6.25\text{ A}$$



Apply KVL on loop 'abcdefa':

$12.5 - I_b(2) - 12 = 0$

$I_b = 0.25\text{ A}$

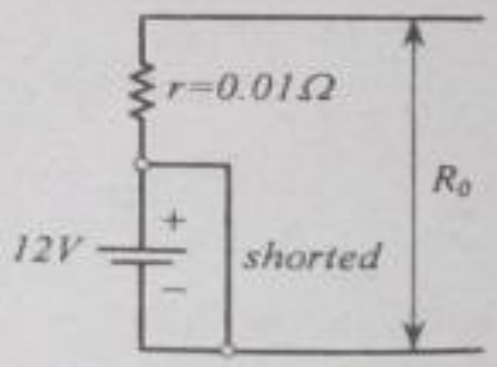
Apply KCL at node 'b':

$I_c = I_r + I_b = 6.25 + 0.25$

$I_c = 6.5\text{ A}$

94 Note: For maximum power transfer, load resistance must be equivalent to the looking back resistance measured at the load terminals. In this problem, $R_0 = r$.

$R_L = R_0 = r = 0.01\Omega$



Using Thevenin's equation:

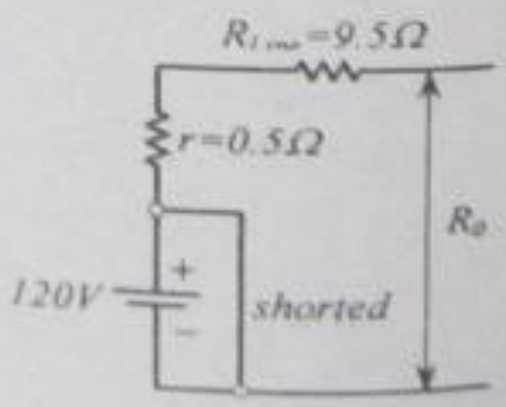
$$I_L = \frac{E_0}{R_0 + R_L} = \frac{12}{0.01 + 0.01} = 600\text{ A}$$

$P_L = I_L^2 R_L = (600)^2(0.01)$

$P_L = 3600$ watts

95 Note: For maximum power transfer, load resistance must be equivalent to the looking back resistance measured at the load terminals. In this problem, $R_0 = r + R_{Line}$

$R_L = R_0 = r + R_{Line} = 0.5 + 9.5 = 10\,\Omega$



Using Thevenin's equation:

$$I_L = \frac{E_0}{R_0 + R_L} = \frac{120}{10 + 10} = 6\text{ A}$$

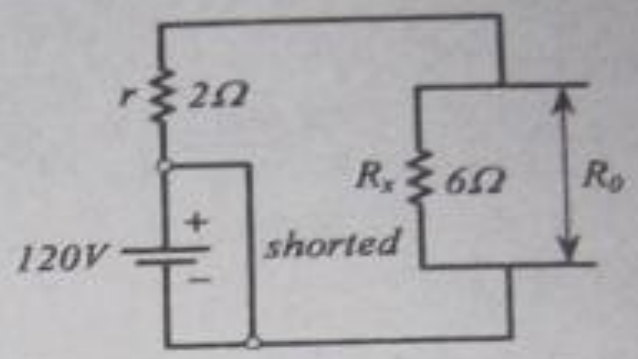
$P_L = I_L^2 R_L = (6)^2(10)$

$P_L = 360$ watts

96 Note: For maximum power transfer, load resistance must be equivalent to the looking back resistance measured at the load terminals.

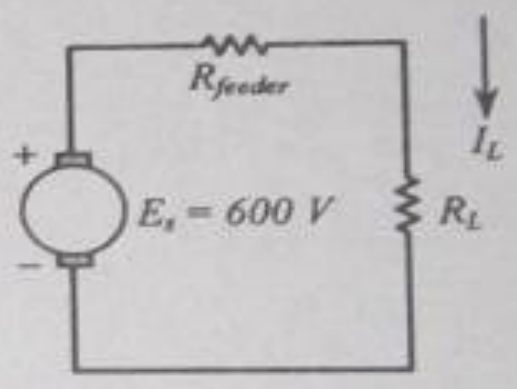
Open circuit R_y and solve the looking back resistance across its terminals.

$$R_0 = \frac{2(6)}{2+6} = 1.5\,\Omega$$
$$R_y = R_0 = 1.5\,\Omega$$



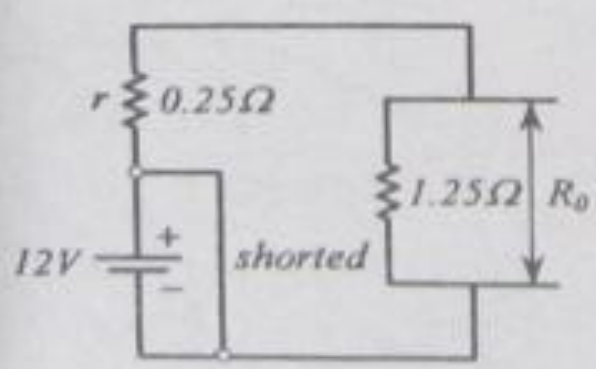
97 Note: For maximum power transfer, load resistance must be equivalent to the looking back resistance measured at the load terminals.

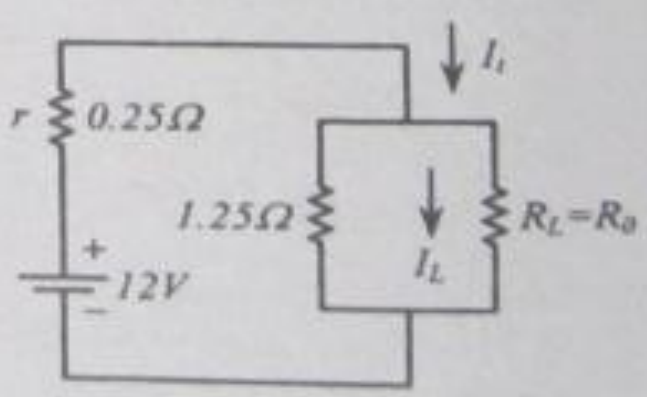
$$R_{feeder} = 2\left(\frac{0.078\,\Omega}{1000\text{ ft}} \times 2500\text{ ft}\right)$$
$$= 0.39\,\Omega$$
$$R_L = R_{feeder}$$
$$I_L = \frac{E_s}{R_{feeder} + R_L} = \frac{600}{0.39 + 0.39}$$
$$I_L = 769.23\text{ A}$$
$$P_L = I_L^2 R_L = (769.23)^2(0.39)$$
$$P_L = 230.77\text{ kW}$$



98 Note: For maximum power transfer, load resistance must be equivalent to the looking back resistance measured at the load terminals.

Open circuit R_y and solve the looking back resistance across its terminals.





$$R_L = R_0 = \frac{0.25(1.25)}{0.25 + 1.25} = 0.208\,\Omega$$

$$R_1 = 0.25 + \frac{1.25(0.208)}{1.25 + 0.208} = 0.428\,\Omega$$

$$I_1 = \frac{E_1}{R_1} = \frac{12}{0.428} = 28.04\text{ A}$$

By CDT:

$$I_L = \frac{I_1(1.25)}{1.25 + R_L} = \frac{28.04(1.25)}{1.25 + 0.208} = 24.04\text{ A}$$

$$P_L = I_L^2 R_L = (24.04)^2(0.208)$$

$$P_L = 120.21\text{ W}$$

99 Note: Independent of the circuit connection, the total power drawn by a circuit is equal to the sum of the powers taken by each resistance.

Let: R = resistance of each resistor

$$P_t = P_1 + P_2 + P_3 + \ldots\ldots + P_{12}$$

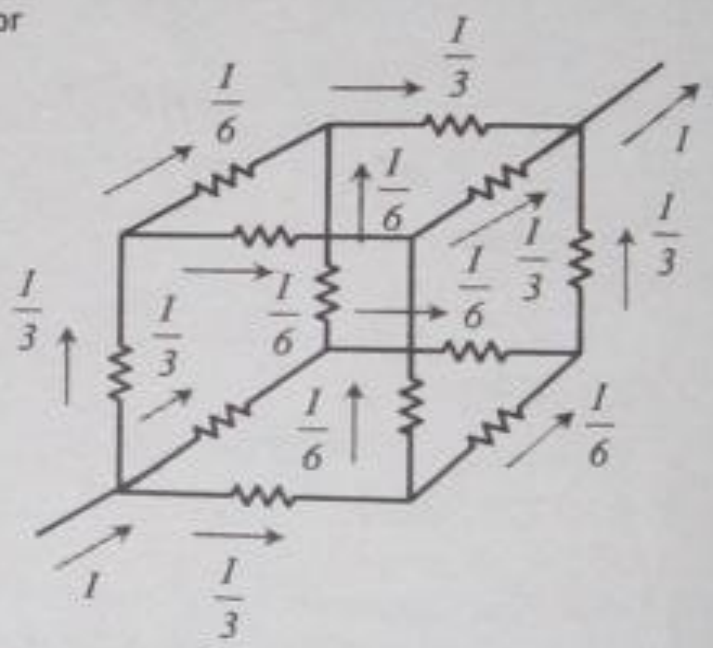
$$I^2R_t = 6\left(\frac{I}{3}\right)^2 R + 6\left(\frac{I}{6}\right)^2 R$$

$$I^2R_t = \frac{6}{9}\left(I^2R\right) + \frac{6}{36}\left(I^2R\right)$$

$$R_t = \frac{6}{9}R + \frac{6}{36}R$$

$$R_t = \frac{5}{6}R = \frac{5(2)}{6}$$

$$R_t = 1.66\,\Omega$$



100 Let: R = resistance of each resistor

Apply KVL on loop 'abcda':

$$(I - 2I_1)R - I_1R - (I_1 - I_2)R - I_1R = 0$$

$$IR - 2I_1R - 2I_1R - I_1R + I_2R = 0$$

$$I - 5I_1 + I_2 = 0$$

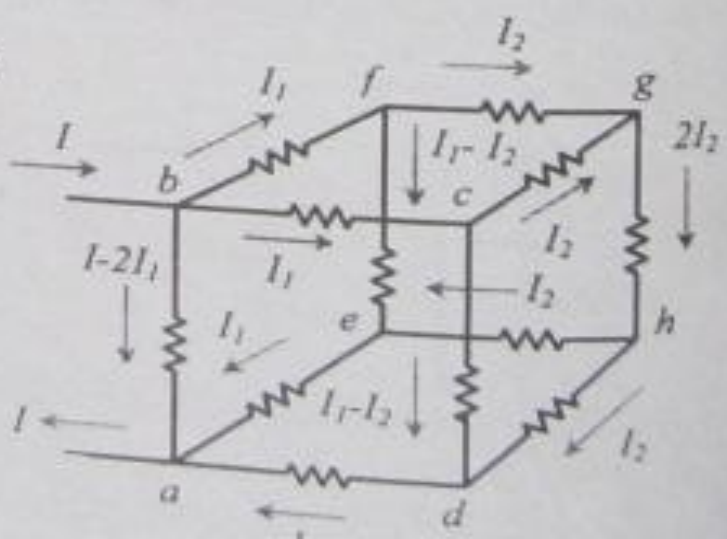
$$I_2 = 5I_1 + I \Rightarrow ❶$$

Apply KVL on loop 'efghe':

$$(I_1 - I_2)R - I_2R - 2I_2R - I_2R = 0$$

$$I_1R - I_2R - I_2R - 2I_2R - I_2R = 0$$

$$5I_2 - I_1 = 0 \Rightarrow ❷$$



Substitute Eq.1 in Eq.2:

$5(5I_1 + I) - I_1 = 0$

$25I_1 + 5I - I_1 = 0$

$I_1 = \frac{5I}{24}$

Solve for the voltage across branch 'ab':

$V_{ab} = (I - 2I_1)R = \left[I - 2\left(\frac{5I}{24}\right)\right]R = \frac{7}{12}IR \Rightarrow ❸$

Solving for the total current:

$I = \frac{V_{ab}}{R_t}$

$V_{ab} = IR_t \Rightarrow ❹$

Equate Eq.4 to Eq.3:

$IR_t = \frac{7}{12}IR$

$R_t = \frac{7}{12}R$

Substitute R = 2:

$R_t = \frac{7}{12}(2)$

$R_t = 1.167\,\Omega$

101 Note: Refer to the problem #99 for the formula to solve for R_t.

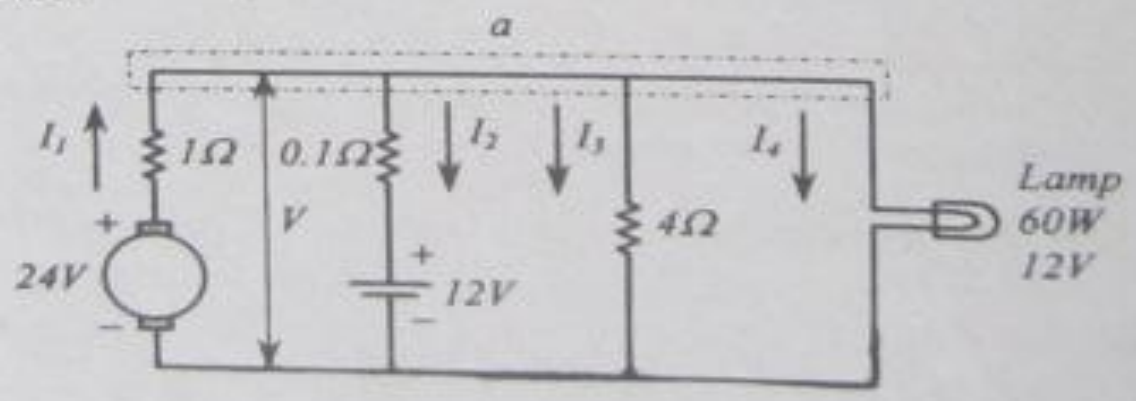
Let: R = resistance of each resistor

$R_t = \frac{5}{6}R = \frac{5}{6}(6) = 5\,\Omega$

$E_t = I_tR_t = (40 \times 10^{-3})(5)$

$E_t = 0.20\,V$

102



Solving for the resistance of the lamp:

$$R_L = \frac{V_L^2}{P_L} = \frac{(12)^2}{60} = 2.4\,\Omega$$

Use Nodal node method:
Apply KCL on node 'a':

$I_1 = I_2 + I_3 + I_4$

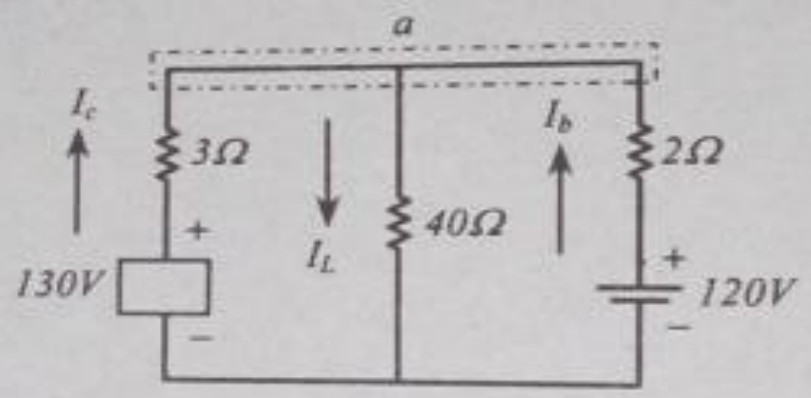
$$\frac{24 - V}{1} = \frac{V - 12}{0.1} + \frac{V}{4} + \frac{V}{2.4}$$

$24 - V = 10V - 120 + 0.25V + 0.416V$

$11.666V = 144$

$V = 12.34$ volts

103



Use Nodal node method:
Apply KCL at node 'a':

$I_c + I_b = I_L$

$$\frac{130 - V_L}{3} + \frac{120 - V_L}{2} = \frac{V_L}{40}$$

$43.333 - 0.333V_L + 60 - 0.5V_L = 0.025V_L$

$0.858V_L = 103.333$

$V_L = 120.43$ volts

$$I_b = \frac{120 - 120.43}{2}$$

$I_b = -0.215$ A

Note: The negative sign denotes only that the actual current is opposite in the given direction (charging the battery).

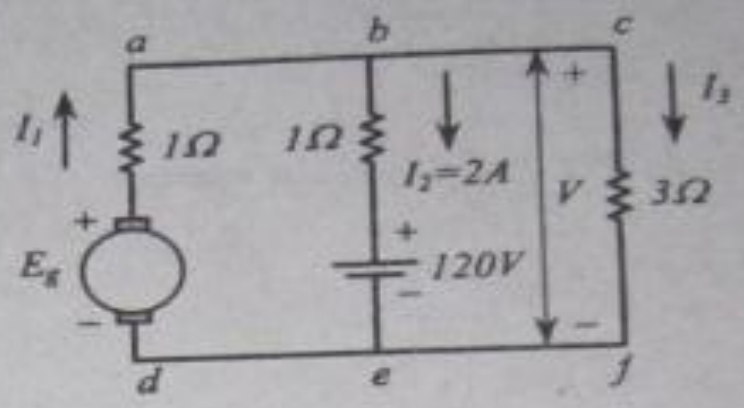
104 Apply KVL on loop 'ebcfe':

$120 + I_2R_2 - V = 0$

$120 + 2(1) - V = 0$

$V = 122$ volts

$$I_3 = \frac{V}{R_3} = \frac{122}{3} = 40.67\text{ A}$$



Apply KCL at node 'b':

$I_1 = I_2 + I_3 = 2 + 40.67$

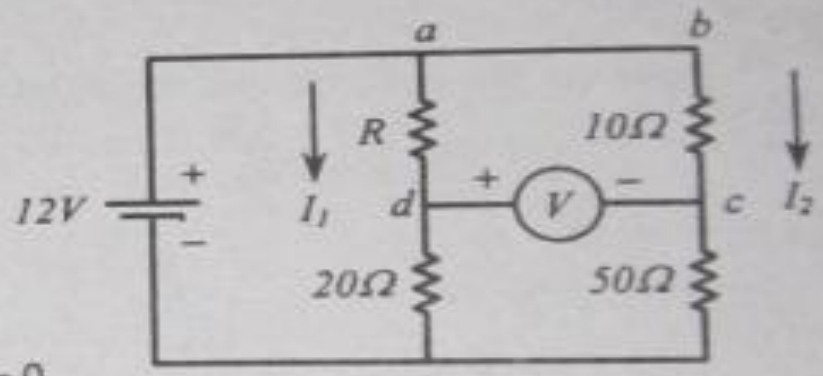
$I_1 = 42.67\text{ A}$

Apply KVL on loop 'dabed':

$E_g - 42.67(1) - 2(1) - 120 = 0$

$E_g = 164.67$ volts

105 Note: Since the voltmeter has a very high resistance compared to the resistances in the circuit, the current flow towards the voltmeter is negligible.



$$I_1 = \frac{12}{20 + R}$$

$$I_2 = \frac{12}{10 + 50} = 0.2\text{ A}$$

Apply KVL on loop 'cdabc':

$0.435 + I_1R - I_2(10) = 0$

$$0.435 + \left(\frac{12}{20 + R}\right)R - (0.2)(10) = 0$$

$$\frac{12R}{20 + R} = 1.565$$

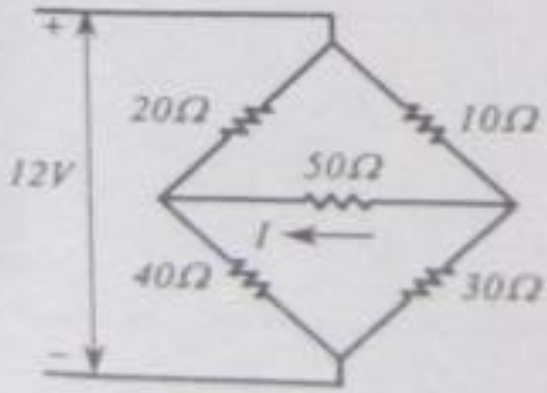
$12R = 31.3 + 1.565R$

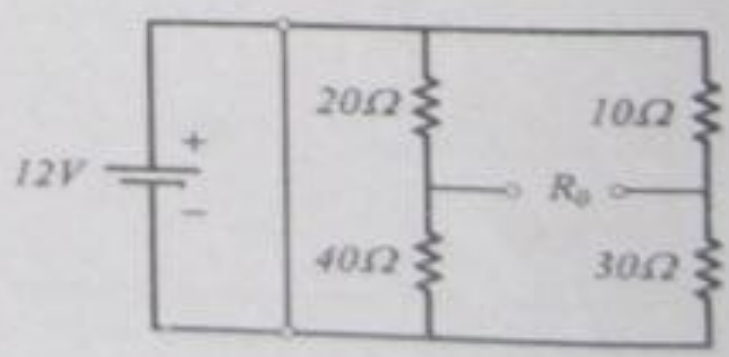
$10.435R = 31.3$

$R = 3\Omega$

106 Use Thevenin's theorem.

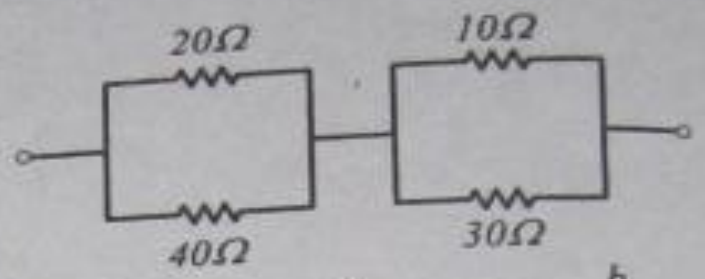
Note: Short circuit the 12-V supply, open circuit the 50-Ω resistor and solve the looking back resistance of the circuit at these terminals.





$$R_0 = \frac{20(40)}{20+40} + \frac{10(30)}{10+30}$$

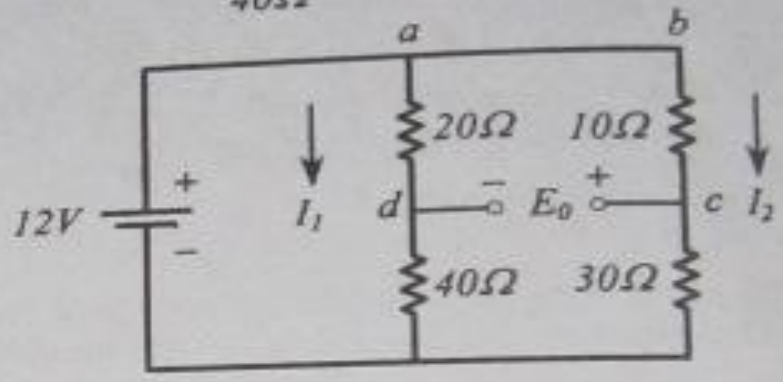
$$R_0 = 20.833\,\Omega$$



Solve for the open circuit voltage:

$$I_1 = \frac{12}{20+40} = 0.2\text{ A}$$

$$I_2 = \frac{12}{10+30} = 0.3\text{ A}$$



Apply KVL on loop 'dcbad':

$$E_0 + I_2(10) - I_1(20) = 0$$

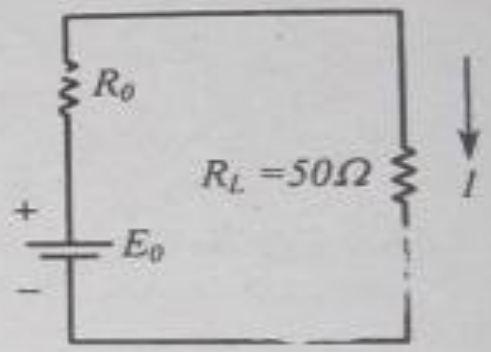
$$E_0 = 0.2(20) - 0.3(10)$$

$$E_0 = 1\text{ volt}$$

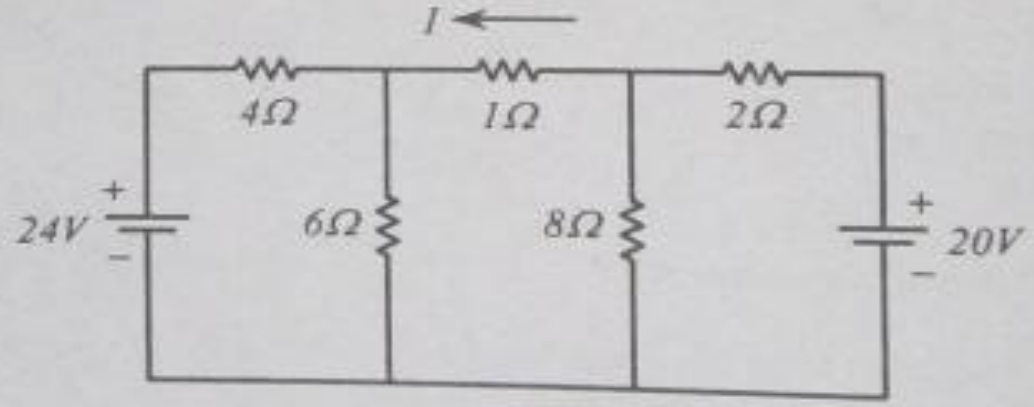
Using the Thevenin's equivalent circuit:

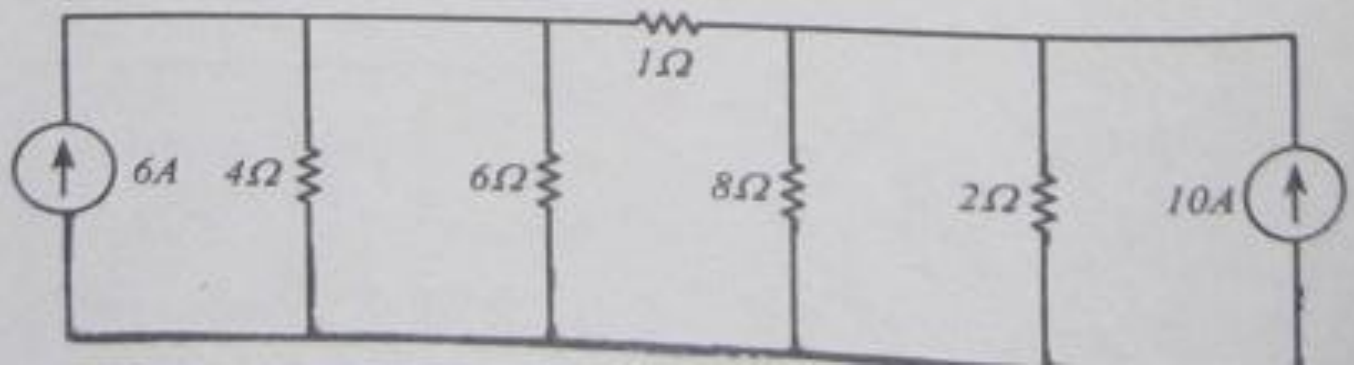
$$I = \frac{E_0}{R_0 + R_L} = \frac{1}{20.833 + 50}$$

$$I = 0.014\text{ A}$$



107 Transform the voltage sources to their equivalent current sources.

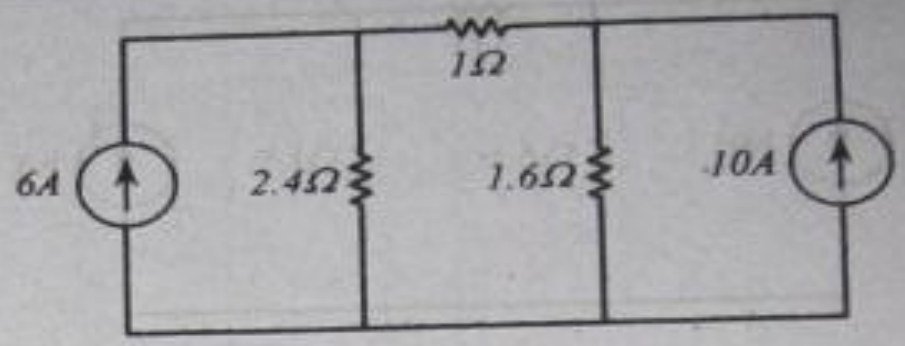




Simplify the parallel connected resistors:

$$r_1 = \frac{4(6)}{4+6} = 2.4\,\Omega$$

$$r_2 = \frac{8(2)}{8+2} = 1.6\,\Omega$$



Transform the current sources to their equivalent voltage sources:

$E_1 = 6(2.4) = 14.4V$

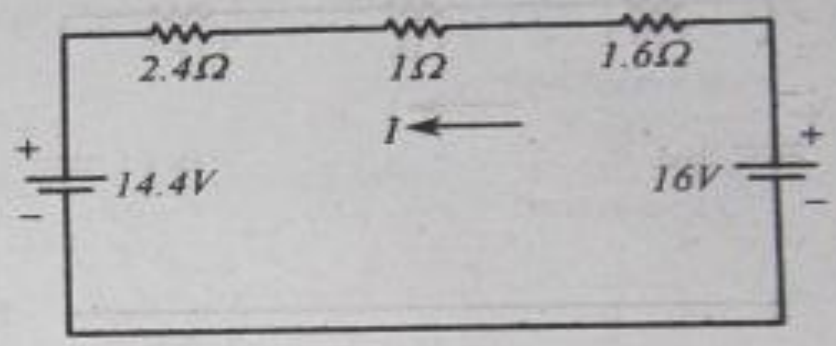
$E_2 = 10(1.6) = 16V$

Apply KVL:

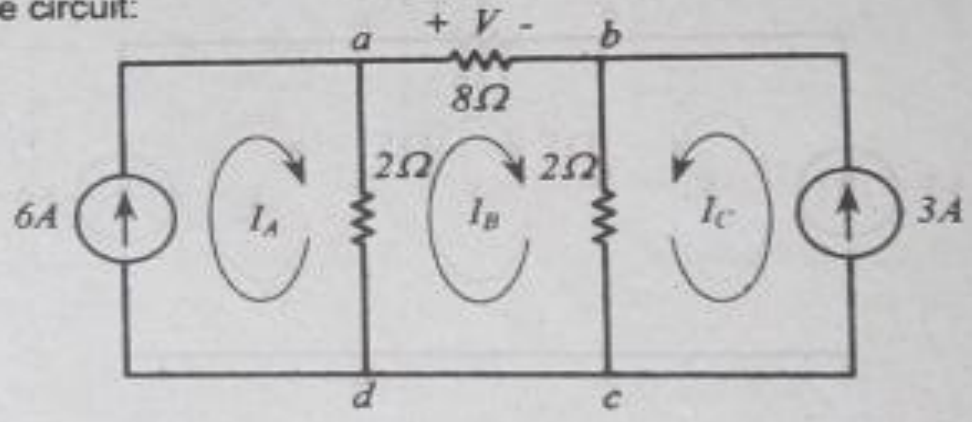
$16 - I(1.6 + 1 + 2.4) - 14.4 = 0$

$1.6 - 5I = 0$

$I = 0.32A$



108 Re-draw the circuit:



Apply KVL on loop 'dabcd':

$(I_A - I_B)2 - I_B(8) - (I_B + I_C)2 = 0$

Note: $I_A = 6A$; $I_C = 3A$

Substitute:

$(6 - I_B)2 - 8I_B - (I_B + 3)2 = 0$

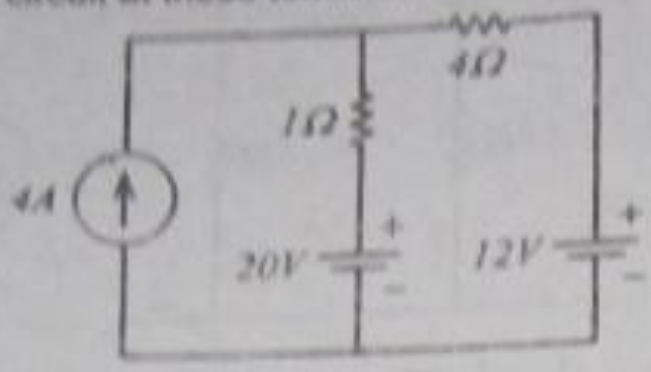
$12 - 2I_B - 8I_B - 2I_B - 6 = 0$

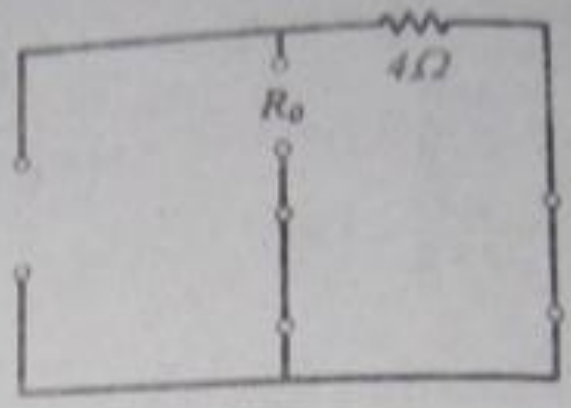
$I_B = 0.5\,A$

$V = I_B(8) = 0.5(8)$

$V = 4$ volts

109 Open-circuit the 1-ohm resistor and solve the looking back resistance of the circuit at these terminals.

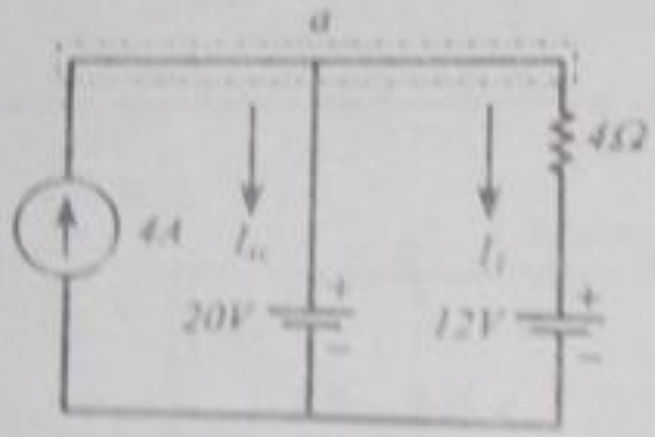


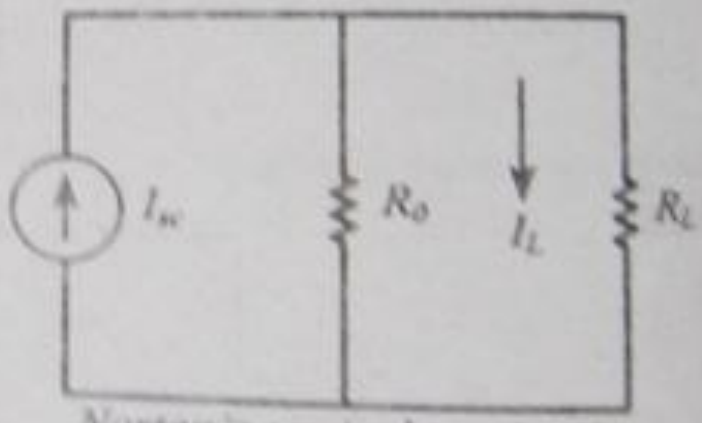


Note: The current source is opened while the voltage sources are short-circuited.

By inspection, $R_o = 4\ \Omega$

Short-circuit the 1-ohm resistor and solve the short circuit current that will flow to this branch.





Norton's equivalent circuit

Apply KCL at node 'a':

$$4 = I_{sc} + I_1$$

$$4 = I_{sc} + \frac{20 - 12}{4}$$

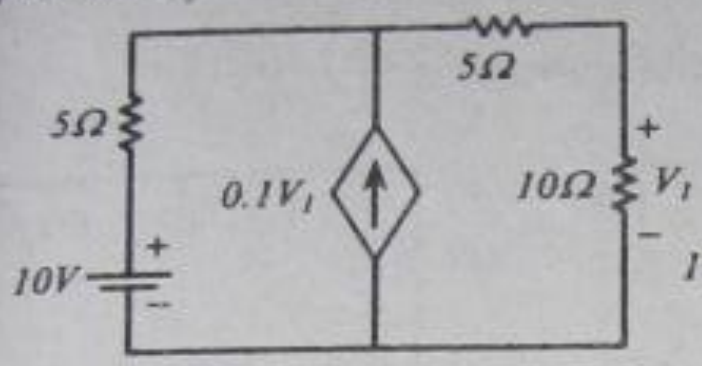
$$I_{sc} = 2\ A$$

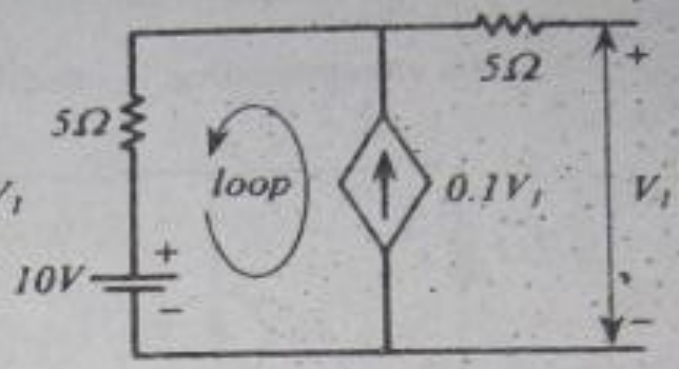
Use the Norton's equivalent circuit:

$$I_L = \frac{I_{sc}R_o}{R_o + R_L} = \frac{(2)(4)}{4 + 1}$$

$$I_L = 1.6\ A$$

110 Open-circuit the 10-ohm resistance and solve the open circuit voltage at these terminals.





Apply KVL using the loop as shown :

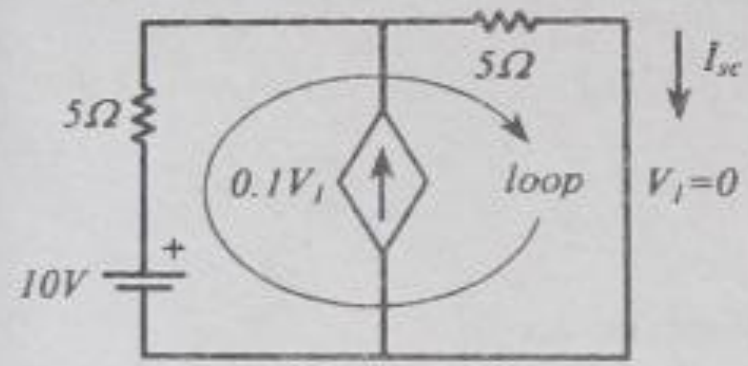
$V_1 - (0.1V_1)(5) - 10 = 0$

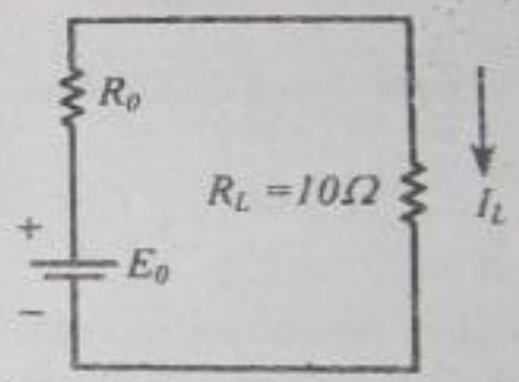
$V_1 - 0.5V_1 - 10 = 0$

$V_1 = 20$ volts

$E_0 = V_1 = 20$ volts

Short-circuit the 10-ohm resistor and solve the short circuit current.





Thevenin's equivalent circuit

Note: The voltage-controlled current source as shown will deliver zero current since V_1 is zero.

Apply KVL using the loop as shown:

$10 - I_{sc}(5 + 5) = 0$

$I_{sc} = 1\text{A}$

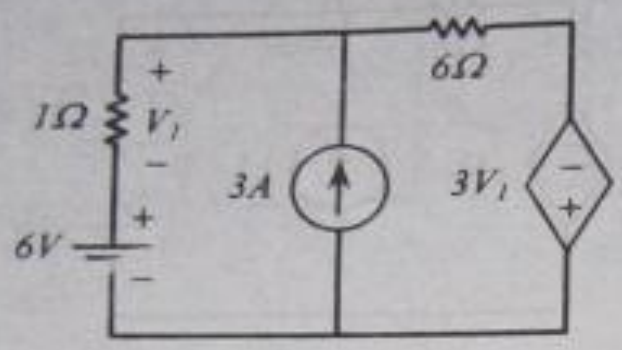
$$R_0 = \frac{E_0}{I_{sc}} = \frac{20}{1} = 20\,\Omega$$

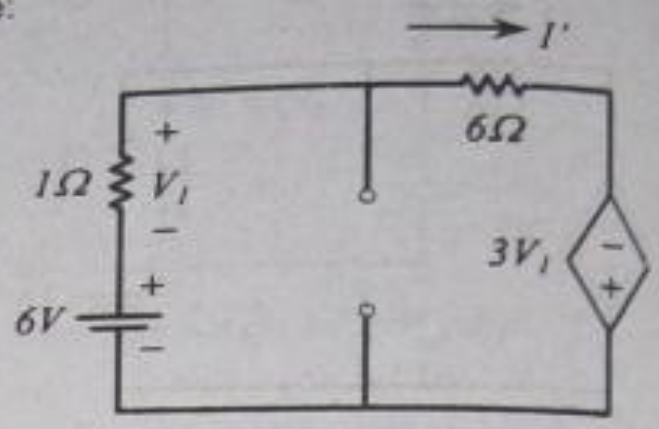
Use the Thevenin's equivalent circuit:

$$I_L = \frac{E_0}{R_0 + R_L} = \frac{20}{20 + 10} = 0.667\text{ A}$$

111 Note: When a source is operating alone, the other current sources must be opened while the other voltage sources must be short-circuited.

Let, the voltage source to operate alone:





Apply KVL:

$6 + V_1 - I'(6) + 3V_1 = 0$

$6 + 4V_1 = 6I'_1 \Rightarrow$ ❶

$V_1 = I'(1) = I' \Rightarrow$ ❷

Substitute Eq.2 in Eq.1:

$6 + 4(I') = 6I'$

$I' = \frac{6}{2} = 3\text{ A}$

Let, the current source to operate alone:

Apply KCL at node 'a':

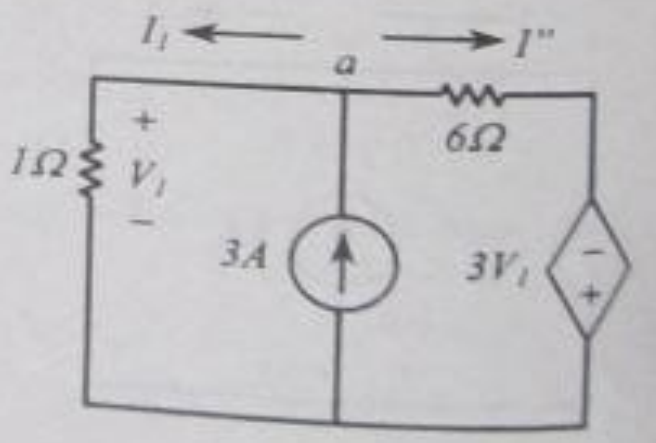
$3 = I_1 + I''$

$3 = \frac{V_1}{1} + \frac{V_1 + 3V_1}{6}$

$3 = 1.667V_1$

$V_1 = 1.8V$

$I'' = \frac{4V_1}{6} = \frac{4(1.8)}{6} = 1.2A$

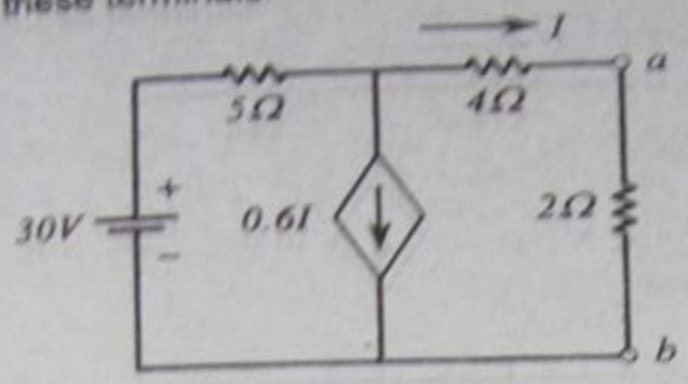


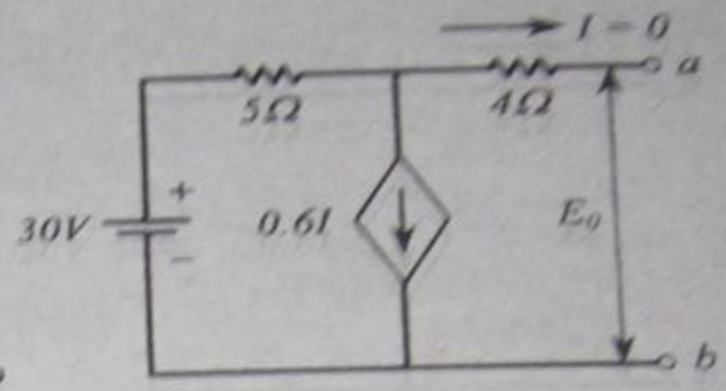
$I = I' + I'' = 3 + 1.2$

$I = 4.2\text{ A}$

Note: Since the direction of currents for both I' and I'' are the same, then the resultant current that will flow in the 6-ohm resistance, is equal to the arithmetic sum of the two currents respectively.

112 Open circuit the 2-ohm resistance and solve the open circuit voltage across these terminals.

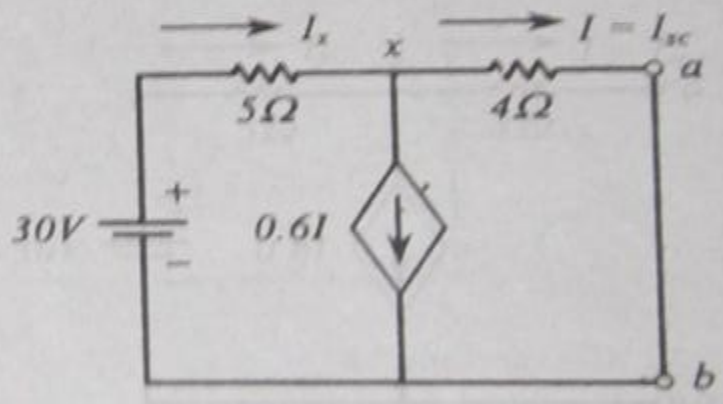




Note: Since terminals 'ab' is open-circuited (I = 0), the current controlled current source will deliver zero current (equivalent to an open circuit).

By inspection, $E_0 = 30$ V

Short circuit the 2-ohm resistance and solve the short circuit current.



Apply KCL at node 'x':

$I_x = I + 0.6I$

$I_x = 1.6I$

$$\frac{30 - V_x}{5} = 1.6\left(\frac{V_x}{4}\right)$$

$6 - 0.2V_x = 0.4V_x$

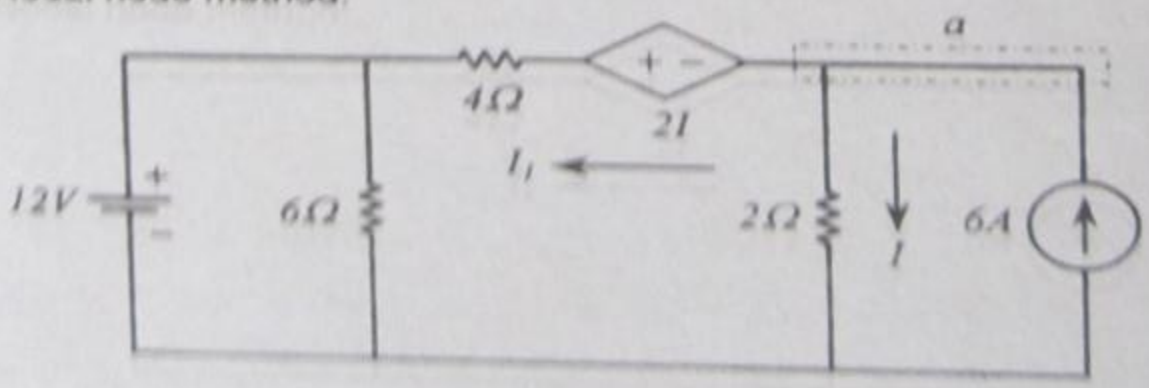
$V_x = 10$ volts

$$I_{sc} = I = \frac{V_x}{4} = \frac{10}{4} = 2.5 \text{ A}$$

$$R_0 = \frac{E_0}{I_{sc}} = \frac{30}{2.5}$$

$R_0 = 12\,\Omega$

113 Use Nodal node method:



Apply KCL at node 'a':

$6 = I + I_1$

$6 = \frac{V_a}{2} + \frac{V_a + 2I - 12}{4}$

$6 = 0.5V_a + 0.25V_a + 0.5I - 3$

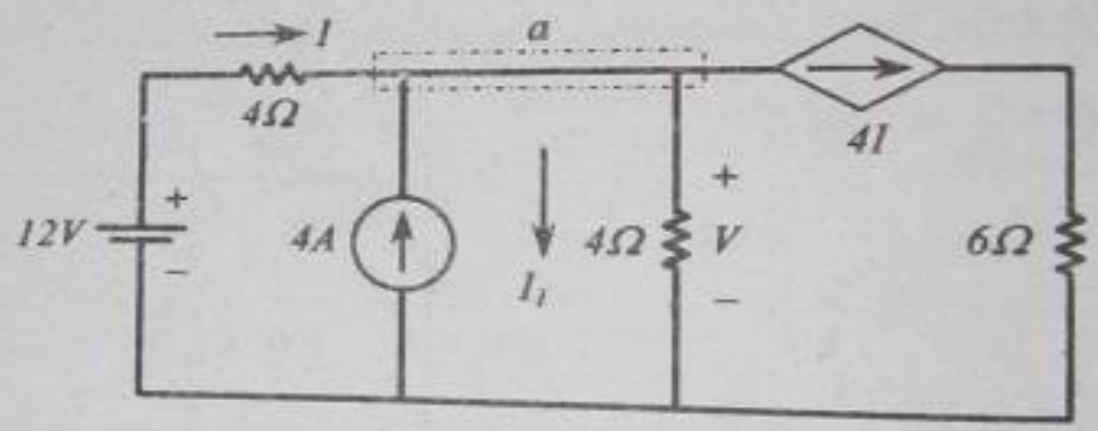
$9 = 0.75V_a + 0.5\left(\frac{V_a}{2}\right)$

$9 = 0.75V_a + 0.25V_a$

$V_a = 9V$

$I = \frac{V_a}{2} = \frac{9}{2} = 4.5\ A$

114 Use Nodal node method:



Apply KCL at node 'a':

$I + 4 = I_1 + 4I$

$4 = I_1 + 3I$

$4 = \frac{V}{4} + 3\left(\frac{12 - V}{4}\right)$

$4 = 0.25V + 9 - 0.75V$

$0.5V = 5$

$V = 10$ volts

Electrostatics

COULOMB'S LAW

- **First law of electrostatics:** Like charges repel each other and unlike charges attract each other.
- **Second law of electrostatics:** The force of attraction or repulsion between charges is directly proportional to the product of two charges and inversely proportional to the square of the distance between them.

$$F = \frac{Q_1 Q_2}{4\pi \Sigma_o \Sigma_r d^2} \qquad F = \frac{kQ_1Q_2}{d^2}$$

where: F = force (newton)
Q_1, Q_2 = charge in each body (coulomb)
Σ_o = absolute permittivity
= **8.854×10^{-12} farad per meter**
Σ_r = relative permittivity or dielectric constant
= **1(one)**, for free space
d = distance between the two bodies (meter)
k = constant in SI units equal to **9×10^9**

Note: In cgs units: F = dyne; Q = stat coulomb; d = centimeter and k = 1. Also there are 3×10^9 stat coulomb in a 1 coulomb of charge.

ELECTROSTATIC POTENTIAL

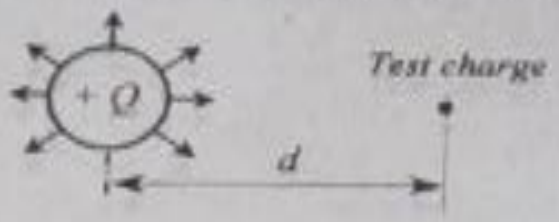
Electrostatic potential – the electric potential resulting from the location of charged bodies in the vicinity

$$V = \frac{Q}{4\pi \Sigma_o \Sigma_r d}$$

where: V = electrostatic potential (volt) at a distance of d (meter) from a charge body of charge Q (coulomb)

ELECTRIC FIELD INTENSITY

Electric field intesity - the force per unit charge that will act at a point in the field on a very small test charge placed at that location.



$$E = \frac{Q}{4\pi \Sigma_o \Sigma_r d^2}$$

where : E = electric intensity (newton per coulomb)
d = distance in meters of the test charge (1 C) to the charge (+Q) body

- **Electric field outside an isolated sphere in free space**

$$E = \frac{Q}{4\pi \Sigma_o r^2}$$

where: E = electric field intensity (volt per meter) at a distance r (meter) from the center of an isolated charged sphere located in free space.
Q = total charge (coulomb) which is distributed uniformly on the sphere

- **Electric field intensity created by an isolated, charged long cylindrical wire in free space**

$$E = \frac{\Delta}{2\pi \Sigma_o r}$$

where: E = electric field (volt per meter)
Δ = charge per unit of length (coulomb per meter) distributed uniformly over the surface of the isolated cylinder
r = distance in meters from the center of the cylinder to the point at which the electric field intensity is evaluated

ELECTRIC FLUX DENSITY

$$D = \Sigma_o \Sigma_r E \qquad D = \frac{\Psi}{A}$$

where D = electric flux density (coulomb per square meter)
Ψ = electric flux (coulomb)
A = area perpendicular to electric flux (square meter)

CAPACITANCE

Capacitance is a measure of how well a capacitor can store electrical charges.

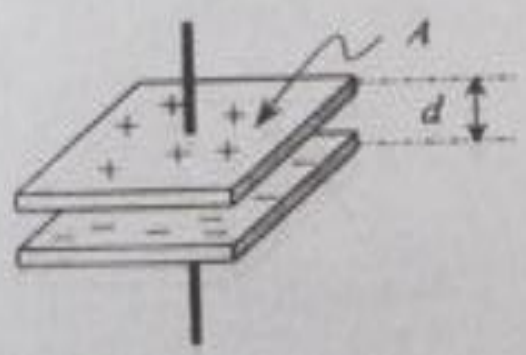
- **Capacitor** (old name is **Condenser**) - any device on which electric charges can be stored so as to posses electrical potential. It consists of **two conducting plates** separated by a layer of an insulating medium called **dielectric**.

- **Farad** (F) - unit of capacitance when one coulomb of charge given to its plates raises its potential difference by one volt. Named after the British physicist and chemist who discovered electromagnetic induction, **Michael Faraday** (1791 – 1867).

$$C = \frac{Q}{E}$$

where: C = capacitance of a charge capacitor (farad)
Q = charge stored (coulomb)
E = voltage across the capacitor (volt)

- **Capacitance of a two parallel-plate capacitor with a uniform dielectric medium**

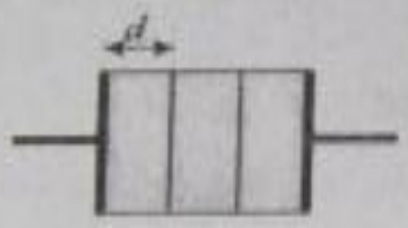


$$C = \frac{\Sigma_0 \Sigma_r A}{d}$$

$$C = \frac{kA}{d}$$

where: C = capacitance (farad)
A = area of each plate (square meter)
d = thickness of dielectric material (meter)
$k = \Sigma_0 \Sigma_r$ = dielectric constant
Σ_0 = absolute permittivity
Σ_r = relative permittivity of the dielectric

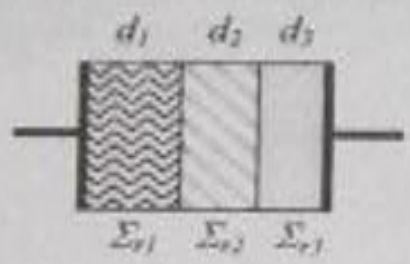
- **Capacitance of an "n" parallel-plate capacitor having the same type of dielectric material and thickness of dielectric**



$$C = \frac{\Sigma_o \Sigma_r A}{d}(n-1)$$

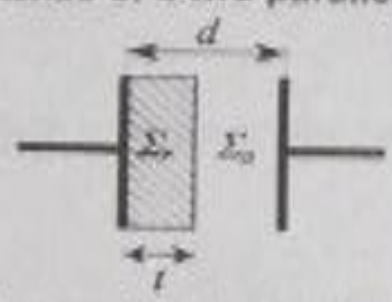
where: n = number of plates

- **Capacitance of an "n" parallel-plate capacitor with different dielectric material and thickness of insulation**



$$C = \frac{\Sigma_o A}{\frac{d_1}{\Sigma_{r1}} + \frac{d_2}{\Sigma_{r2}} + \frac{d_3}{\Sigma_{r3}}}$$

- **Capacitance of a two parallel-plate capacitor with a medium partly air.**



$$C = \frac{\Sigma_o A}{d - \left(t - \frac{t}{\Sigma_r}\right)}$$

- **Capacitance of an isolated spnere**

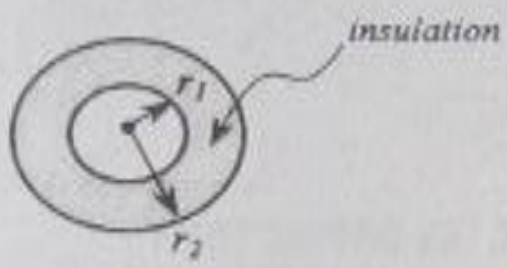
$$C = 4\pi\Sigma_o\Sigma_r r$$

- **Capacitance of a concentric spherical capacitor**

$$C = 4\pi\Sigma_o\Sigma_r \frac{r_1 r_2}{r_2 - r_1}$$

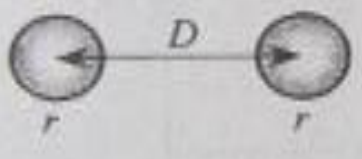
$$C = 4\pi\Sigma_0\Sigma_r \frac{r_2^2}{r_2 - r_1}$$

- **Capacitance of a coaxial cable (farad per meter)**



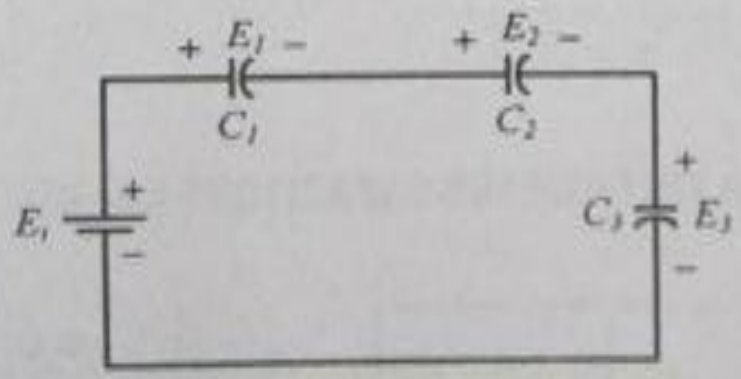
$$C = \frac{2\pi\Sigma_0\Sigma_r}{\ln\frac{r_2}{r_1}}$$

- **Capacitance of a two wire line (parallel-cylinder capacitor) with opposite charges (farad per meter)**



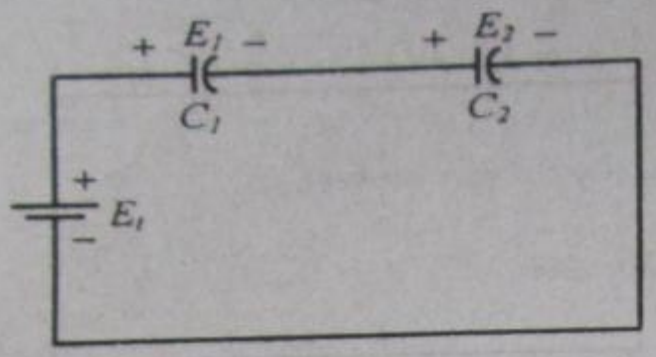
$$C = \frac{\pi\Sigma_0\Sigma_r}{\ln\left[\frac{D}{2r} + \sqrt{\left[\frac{D}{2r}\right]^2 - 1}\right]}$$

SERIES-CONNECTED CAPACITORS



$$C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \qquad Q_t = Q_1 = Q_2 = Q_3 \qquad E_t = E_1 + E_2 + E_3$$

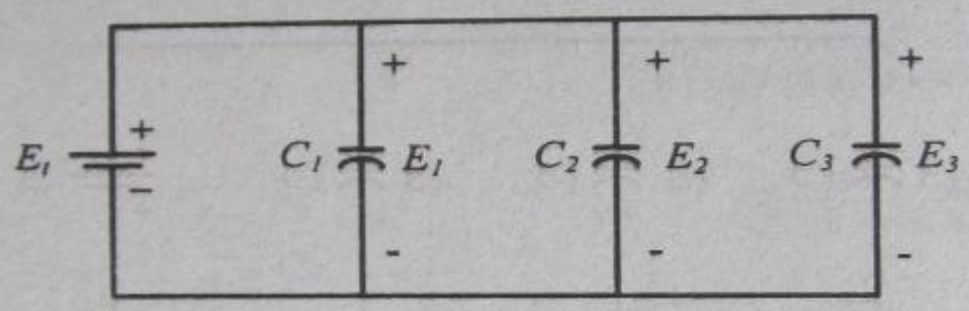
VOLTAGE DIVISION THEOREM FOR CAPACITORS CONNECTED IN SERIES



$$E_1 = \frac{E_t C_2}{C_1 + C_2}$$

$$E_2 = \frac{E_t C_1}{C_1 + C_2}$$

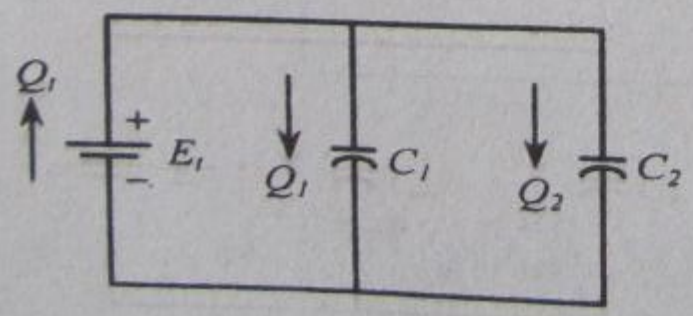
PARALLEL-CONNECTED CAPACITORS



$$C_t = C_1 + C_2 + C_3 \qquad Q_t = Q_1 + Q_2 + Q_3 \qquad E_t = E_1 = E_2 = E_3$$

where: Q_t = total charge accumulated (coulomb)
C_t = total capacitance (farad)
E_t = supply voltage (volt)

CHARGE DIVISION THEOREM FOR CAPACITORS CONNECTED IN PARALLEL



$$Q_1 = \frac{Q_t C_1}{C_1 + C_2}$$

$$Q_2 = \frac{Q_t C_2}{C_1 + C_2}$$

Note: For series-parallel and parallel-series connections, use the basic principles in series and parallel connections to simplify the circuit.

ENERGY STORED IN A CHARGE CAPACITOR

$$W = \frac{1}{2}CE^2 \qquad W = \frac{1}{2}QE \qquad W = \frac{1}{2}\left(\frac{Q^2}{C}\right)$$

where: W = energy stored (joule)
C = capacitance (farad)
E = voltage across (volt)
Q = charge accumulated (coulomb)

ELASTANCE

Elastance is the reciprocal of capacitance.

$$S = \frac{1}{C}$$

where: S = elastance (reciprocal farad or daraf)

Test 4

Problem 115: EE Board March 1998

Three equal positive charges of 10 statcoulomb each are located at the vertices of an equilateral triangle of 2 cm leg. What is the magnitude of the force acting on each charge?

A. 12.5 dynes
B. 25 dynes
C. 43.3 dynes
D. 50 dynes

Problem 116: EE Board October 1997

Two point charges 10 cm apart exert a force of 1 x 10 raised to the (-3) grams on each other. If the charges are the same value, what is the value of each charge in stat coulomb?

A. 8.90 stat coulomb
B. 9.90 stat coulomb
C. 6.90 stat coulomb
D. 7.90 stat coulomb

Problem 117: EE Board April 1997

Two electrons in a vacuum experience a force of 2×10^{-15} N. How far apart are the electrons?

A. 0.48×10^{-7} m
B. 2.84×10^{-12} m
C. 3.39×10^{-7} m
D. 5.05×10^{-12} m

Problem 118:

Two point charges of 2 C and a 5 C are 10 meters apart. A third point charge of 6 C is placed between them. At what distance from the 5 C charge must the 6 C charge is placed, in such a way that it will stay in equilibrium?

A. 6.125 m
B. 5.365 m
C. 6.025 m
D. 5.256 m

Problem 119: EE Board October 1996

Two spheres separated from each other by 10 m have charges of 0.001 coulomb and 0.003 coulomb respectively. In between the two spheres is a point of zero electric field. What is the distance from the 0.001 coulomb sphere?

A. 6.34 m
B. 7.78 m
C. 2.24 m
D. 3.66 m

Problem 120:

Calculate the magnitude of the electric field at a point that is 30 cm from a point charge $Q = -3.2 \times 10^{-6}$ C.

A. 3.2×10^{6} N/C
B. 3.0×10^{5} N/C
C. 2.8×10^{6} N/C
D. 2.4×10^{5} N/C

Problem 121: EE Board October 1998

Two metallic plates separated by 1 cm are connected across a 12 V battery. A certain plastic material is inserted completely filling the space between the plates and the charge on each plate observed to double. What is the dielectric constant of the plastic material?

A. 0.5
B. 2
C. 4
D. 6

Problem 122: EE Board October 1991

Calculate the capacitance between two parallel plates each of which is 100 cm^2 and 2 mm apart in air.

A. 0.443 μμF
B. 0.515 μμF
C. 0.452 μμF
D. 0.502 μμF

Problem 123:

A capacitor has two parallel metal plates measuring 0.45 m square each. The distance between the plates is 1 cm in free air. If a dielectric material whose permittivity is 4 and 5 mm thick is placed in the upper plate, leaving an air gap between the bottom plate and the dielectric, what is the capacitance of the capacitor?

A. 207 μμF
B. 254 μμF
C. 193 μμF
D. 287 μμF

Problem 124:

A capacitor is composed of two plates separated by a 3-mm insulation whose dielectric constant is 4. An additional piece of 6-mm insulation is inserted between the plates. What is the dielectric constant of the additional insulation if the capacitance of the capacitor decreases to half as much of its original value?

A. 5
B. 8
C. 6
D. 7

Problem 125:

A parallel-plate capacitor has an equivalent capacitance of 850 pF. The area of each plate is 6 cm^2 and the thickness of the dielectric is 0.02 cm. If the material used as the dielectric has a constant of 8, how many sections are there in parallel?

A. 4
B. 5
C. 7
D. 6

Problem 126: EE Board October 1994

A parallel plate capacitor is made of 350 plates, separated by paraffined paper 0.0010 cm thick (k = 2.5). The effective size of the plate is 15 by 30 cm. What is the capacitance of this capacitor?

A. 35 μF
B. 15 μF
C. 140 μF
D. 70 μF

Problem 127: EE Board October 1997

A capacitor is charged with 0.23 watt-second of energy at a voltage of 48 volts. What is its capacitance?

A. 180 μF
B. 240 μF
C. 200 μF
D. 220 μF

Problem 128: EE Board October 1997

A 20 μF capacitor is charged by a 12-V battery. What is the stored energy in the capacitor?

A. 1.07×10^{-3} J
B. 2.88×10^{-2} J
C. 2.88×10^{-3} J
D. 1.44×10^{-3} J

Problem 129: EE Board October 1994

A parallel plate capacitor with air between its plate is charged until the potential difference V appears across it. Another capacitor having hard rubber (dielectric constant = 3) between its plates but otherwise identical, is also charged to the same potential difference. If the energy of the first capacitor is W, that of the second is

A. 9W
B. $\frac{W}{3}$
C. W
D. 3W

Problem 130:

A capacitor whose plates is 20 cm x 3.0 cm and is separated by a 1.0-mm air gap is connected across a 12-V battery. Determine the charge accumulated on each plate after a long time.

A. 5.3×10^{-12} C
B. 7.2×10^{-9} C
C. 8.1×10^{-11} C
D. 6.4×10^{-10} C

Problem 131: EE Board October 1990

The result of capacitance C_1 = 6 microfarads and C_2 connected in series is 3 microfarads. Capacitor C_2, in microfarads is

A. 3
B. 4
C. 8
D. 6

Problem 132:

The equivalent capacitance of two capacitors in series is 0.03 μF and when connected in parallel, 0.16 μF. If the capacitor with the smaller capacitance is replaced with a capacitor whose capacitance is double as much what will be the new equivalent series capacitance of the combination.

A. 0.048 μF
B. 0.055 μF
C. 0.042 μF
D. 0.050 μF

Problem 133:

Three capacitors of 16, 15 and 12 μF, respectively are connected in series. What is the maximum voltage that can be supplied to the combination if the voltage drop across any of the three capacitors must not to exceed 100 V.

A. 260 V
B. 245 V
C. 255 V
D. 250 V

Problem 134:

Three capacitors of 20, 10 and 15 μF respectively are connected in series across a 100-V supply. Find the voltage across the 10 μF capacitor.

A. 54.52 V
B. 46.15 V
C. 38.25 V
D. 42.38 V

Problem 135:

Three condensers of 6, 10 and 15 μF are connected in series across a 200 V supply. If the condensers are disconnected, how much energy can the charged condensers provide if connected across each other with like polarities together?

A. 0.052 J
B. 0.060 J
C. 0.076 J
D. 0.068 J

Problem 136:

Two capacitors with capacitance of 6 μF and 4 μF are connected in series across a 100 V dc source. If the supply voltage is cut-off and the two capacitors are connected in parallel, what will be the final charge accumulated in the 4 μF capacitor?

A. 96 μC
B. 192 μC
C. 126 μC
D. 88 μC

Problem 137:

A 40 μF capacitor is charged to store 0.2 J of energy. An uncharged 60 μF capacitor is connected in parallel with the first one through a perfectly conducting leads. What is the energy lost due to the work done by the charges in spreading out over the two capacitors?

A. 0.08 J
B. 0.12 J
C. 0.80 J
D. 0.012 J

Problem 138:

Four capacitors A, B, C and D are connected across a 150-V DC source. The charges accumulated in A, B and C are equal to 2000, 750 and 1500 μC, respectively. If the total capacitance of the combination is known to be 35 μF, how much charge is the accumulated in capacitor D?

A. 1200 μC
B. 1000 μC
C. 1150 μC
D. 1400 μC

Problem 139: EE Board October 1993

A condenser of 1 microfarad capacitance is charged to 100 volts and then disconnected from the power supply. A second but uncharged capacitor of 3 microfarad is then connected across the first capacitor. The voltage across the parallel combination is

A. 66 2/3 volts
B. 45 volts
C. 25 volts
D. 110 volts

Problem 140: EE Board October 1994

Three capacitors A, B and C are charged as follows: A = 10 μF, 100 V; B = 15 μF, 150 V and C = 25 μF, 200 V. They are connected in parallel with terminals of like polarity together. What is the voltage across the combination?

A. 150 V
B. 120 V
C. 185 V
D. 165 V

Problem 141:

A 100 μF capacitor is charged from a 200-V supply. The charged capacitor is then connected across three uncharged capacitors connected in parallel. The charges on these capacitors after a long time become 4000, 5000 and 6000 μC. Solve the voltage across the combination.

A. 64 V
B. 50 V
C. 76 V
D. 80 V

Problem 142:

Two capacitors of 200 and 800 nF, respectively are connected in parallel. The combination is then connected in series with another two capacitors of C and 600 nF connected in parallel. If the total capacitance of the set-up is 500 nF, determine the capacitance C.

A. 650 nF
B. 550 nF
C. 400 nF
D. 500 nF

Problem 143: EE Board April 1997

A coaxial cable has a 10-mm diameter inside conductor and a metallic sheath with an inside diameter of 20 mm. If the insulating medium has a dielectric constant, k = 2, and a permittivity of 1.113×10^{-10}, what is the capacitance between the conductor and the shield per km?

A. 0.1605 μF/km
B. 0.8036 μF/km
C. 0.0803 μF/km
D. 0.0403 μF/km

Problem 144: EE Board October 1994

A lead-sheath cable for underground service has a copper conductor (diameter =0.350 inch) surrounded by 0.20-inch wall of rubber insulation. Assuming the dielectric constant of 4.3 for rubber, calculate the capacitance of the cable per mile length.

A. 1.01 μF/mile
B. 0.504 μF/mile
C. 0.76 μF/mile
D. 0.252 μF/mile

ANSWER KEY

115. C	122. A	129. D	136. B	143. A
116. B	123. D	130. D	137. B	144. B
117. C	124. B	131. D	138. B	
118. A	125. B	132. A	139. C	
119. D	126. A	133. C	140. D	
120. B	127. C	134. B	141. B	
121. B	128. D	135. A	142. C	

☑ RATING

27-30 - Topnotcher

21-26 - Passer

15-20 - Conditional

0-14 - Failed

Did you know that... the first man who demonstrated that lightning is electricity is a public official who had helped draft one of history's major documents, the Constitution of the United States of America?

Benjamin Franklin, an American public official, a writer and a scientist was the first to use the terms "positive" and "negative" charges. Franklin was one of the seventeen children. He quit school at the age of ten to become a printer. His life is the classic story of a self-made man achieving wealth and fame through handwork, determination and intelligence.

He was the first to demonstrate that lightning is electricity and prove it when he flew a kite in 1752. He picked a day when a storm is about to break. At the top of the kite, he fastened a stiff wire pointing up. At the other end of the string, he tied a metallic key so it hung close to a leyden jar. It started to rain. The moistened string began to conduct electricity. It was fortunate for Franklin that there was no lightning. The storm ahead of the storm gave him enough electricity to prove, without a doubt that electricity and lightning were the same. Sparks jumped from the key to the jar until the jar was completely filled with "electric fluid".

As a public official, he played a major rule in the American Revolution. He had negotiated French support for the colonists, signed the Treaty of Paris and had helped draft one of history's major documents, the "Constitution of the United States of America".

His numerous scientific and practical innovations include the lightning rod, bifocal spectacles, and a stove.

Solutions to Test 4

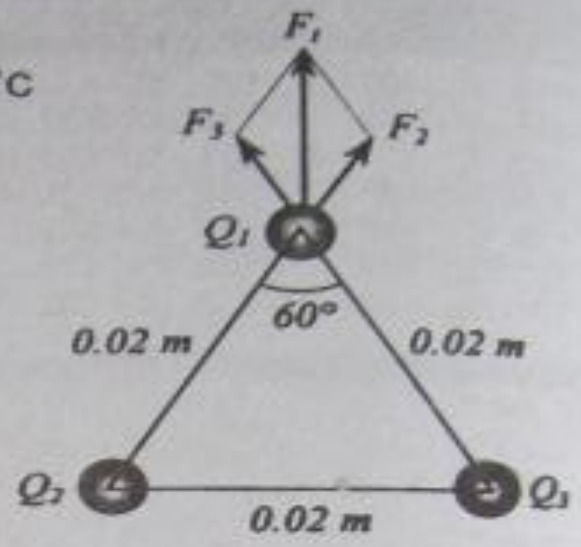
115 $Q = 10\ \text{statC} \times \dfrac{1\text{C}}{3 \times 10^9\ \text{statC}} = 3.33 \times 10^{-9}\text{C}$

$F_2 = \dfrac{kQ^2}{d^2} = \dfrac{(9 \times 10^9)(3.33 \times 10^{-9})^2}{(0.02)^2}$

$F_2 = 2.5 \times 10^{-4}\text{N}$

$= 2.5 \times 10^{-4}\ \dfrac{\text{kg}-\text{m}}{\text{sec}^2} \times \dfrac{1000\ \text{g}}{1\text{kg}} \times \dfrac{100\ \text{cm}}{1\text{m}}$

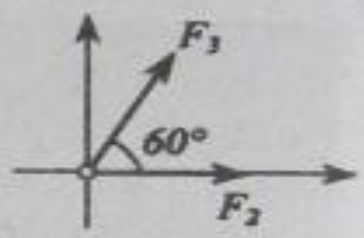
$F_2 = F_3 = 25\ \dfrac{\text{g-cm}}{\text{sec}^2} = 25\ \text{dynes}$



Ω Using cgs units to solve for F_2 or F_3

$F_2 = F_3 = \dfrac{kQ^2}{d^2} = \dfrac{1(10)^2}{(2)^2}$

$F_2 = F_3 = 25\ \text{dynes}$



Refer to the vector diagram

$F_t = F\angle 0° + F\angle 60° = 25\angle 0° + 25\angle 60°$

$= 25 + 12.5 + j21.65 = 37.5 + j21.65$

$= 43.3\angle 30°$

$|F_t| = 43.3\ \text{dynes}$

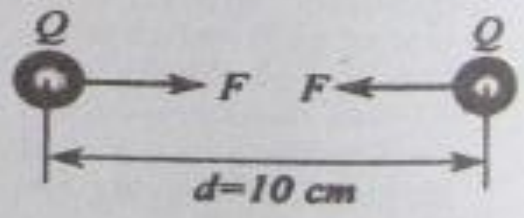
116 $F = 1 \times 10^{-3}\text{g} \times 981\ \text{cm/s}^2$

$F = 0.981\ \text{dyne}$

$F = \dfrac{kQ^2}{d^2}$

$Q = \sqrt{\dfrac{Fd^2}{k}} = \sqrt{\dfrac{(0.981)(10)^2}{1}}$

$Q = 9.9\ \text{stat C}$



117 Note: Since it requires 6.25×10^{18} electrons or protons to have a charge of one coulomb, thus the charge of one electron or proton must be 1.6×10^{-19} coulomb.

$F = \dfrac{kQ^2}{d^2}$

$$d = \sqrt{\frac{kQ^2}{F}} = \sqrt{\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{2 \times 10^{-15}}}$$

$d = 3.39 \times 10^{-7}$ m

118 $F_1 = F_2$

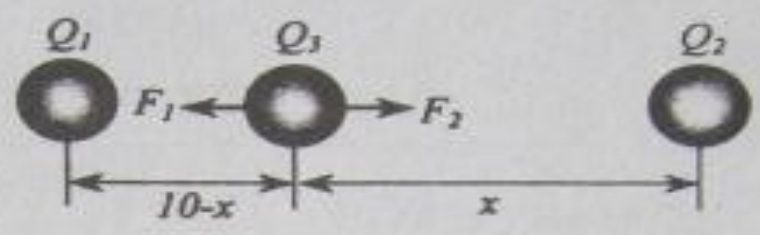
$$\frac{kQ_1Q_3}{(10-x)^2} = \frac{kQ_2Q_3}{x^2}$$

$$\frac{(2)(6)}{(10-x)^2} = \frac{(5)(6)}{x^2}$$

$$\left(\frac{x}{10-x}\right)^2 = \frac{30}{12}$$

$x = 1.581(10 - x)$

$x = 6.125$ m



119 $E_1 = E_2$

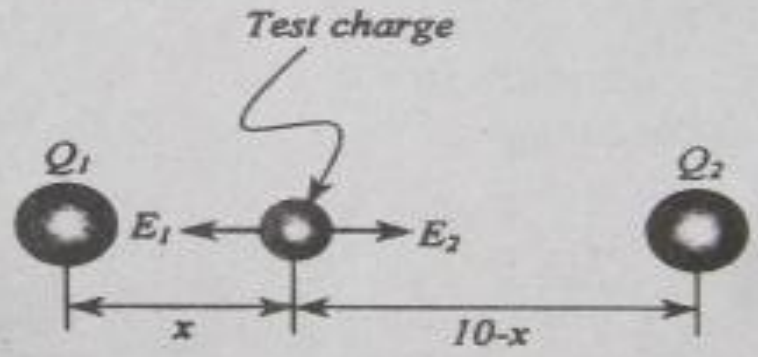
$$\frac{kQ_1}{x^2} = \frac{kQ_2}{(10-x)^2}$$

$$\frac{0.001}{x^2} = \frac{0.003}{(10-x)^2}$$

$$\left(\frac{x}{10-x}\right)^2 = 0.333$$

$x = 0.577\,(10 - x)$

$x = 3.658$ m



120 $$E = \frac{kQ}{d^2} = \frac{(9 \times 10^9)(3 \times 10^{-6})}{(0.3)^2}$$

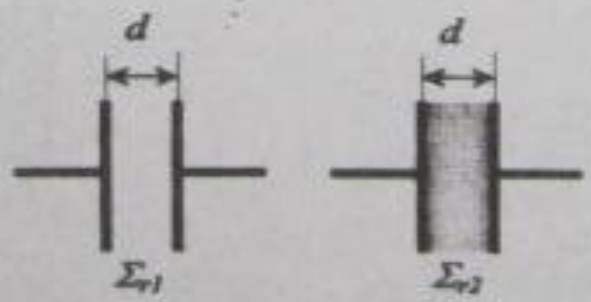
$E = 3 \times 10^5$ N/C

121 $Q = CE$

$Q_2 = 2Q_1$

$C_2E_2 = 2C_1E_1$

$C_2 = 2C_1$



$$C = \Sigma_0 \Sigma_r \frac{A}{d}$$

$C_2 = 2C_1$

Substitute:

$$\Sigma_0\Sigma_{r2}\frac{A}{d} = 2\left(\Sigma_0\Sigma_{r1}\frac{A}{d}\right)$$

$$\Sigma_{r2} = 2$$

Note: Supply voltage, area of plates and distances between plates in both conditions are constant. Also in the first condition, the dielectric is air, thus $\Sigma_{r1} = 1$

122

$$C = \frac{\Sigma_0\Sigma_r A}{d} = \frac{(8.854 \times 10^{-12})(1)(100\ \text{cm}^2 \times \left(\frac{1\text{m}}{100\ \text{cm}}\right)^2}{0.002}$$

$$= 0.4427 \times 10^{-12}\ \text{F}$$

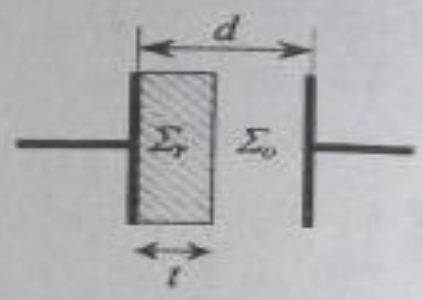
$$C \cong 0.443\ \mu\mu\text{F}$$

123

$$C = \frac{\Sigma_0 A}{d - \left(t - \frac{t}{\Sigma_r}\right)} = \frac{8.854 \times 10^{-12}(0.45 \times 0.45)}{0.01 - \left(0.005 - \frac{0.005}{4}\right)}$$

$$= 286.87 \times 10^{-12}\ \text{F}$$

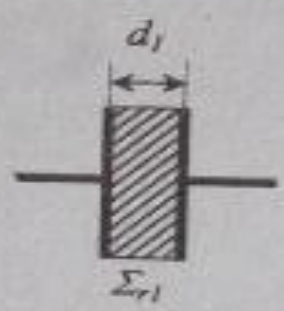
$$C \cong 287\ \mu\mu\text{F}$$



124

$$C_1 = \frac{\Sigma_0\Sigma_r A}{d_1}$$

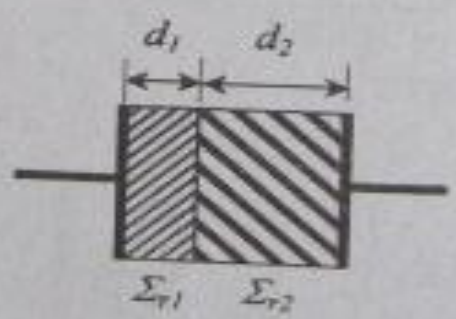
$$C_2 = \frac{\Sigma_0 A}{\frac{d_1}{\Sigma_{r1}} + \frac{d_2}{\Sigma_{r2}}}$$



$$C_2 = \frac{C_1}{2}$$

$$2C_2 = C_1$$

$$2\left(\frac{\Sigma_0 A}{\frac{d_1}{\Sigma_{r1}} + \frac{d_2}{\Sigma_{r2}}}\right) = \frac{\Sigma_0\Sigma_{r1}A}{d_1}$$



$$\left(\frac{2}{\frac{0.003}{4} + \frac{0.006}{\Sigma_{r2}}}\right) = \frac{4}{0.003}$$

$$\Sigma_{r2} = 8$$

125 $C = \frac{\Sigma_0 \Sigma_r A}{d}(n-1)$

$n = \frac{Cd}{\Sigma_0 \Sigma_r A} + 1$

$n = \frac{(850 \times 10^{-12})(0.0002)}{(8.854 \times 10^{-12})(8)\left[6\ cm^2 \times \left(\frac{1\ m}{100\ cm}\right)^2\right]} + 1$

$n = 5$

126

$C = \frac{\Sigma_0 \Sigma_r A}{d}(n-1) = \frac{(8.854 \times 10^{-12})(2.5)(15 \times 30)\left(\frac{1\ cm}{100\ cm}\right)}{0.00001}(350-1)$

$C = 34.76\ \mu F$

$C \cong 35\ \mu F$

127 $W = \frac{1}{2}CE^2$

$C = \frac{2W}{E^2} = \frac{2(0.23)}{48^2} = 199.65\ \mu F$

$C \cong 200\ \mu F$

128 $W = \frac{1}{2}CE^2 = \frac{(20 \times 10^{-6})(12)^2}{2}$

$W = 1.44 \times 10^{-3}\ J$

129 Notes: 1. The capacitance of a capacitor is directly proportional to the dielectric constant of the dielectric material being used.
2. With voltage constant, the energy stored is directly proportional to the capacitance of the capacitor.

$C = k E_r$

$W = k' C$

$\frac{W_2}{W_1} = \frac{C_2}{C_1} = \frac{E_{r2}}{E_{r1}}$

$W_2 = \frac{W_1 E_{r2}}{E_{r1}} = \frac{(W)(3)}{1}$

$W_2 = 3W$

Where: k and k' respectively are proportionality constant symbols

130

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{(8.854 \times 10^{-12})(1)(20 \times 3)\left(\frac{1\,m}{100\,cm}\right)^2}{0.001} = 53.124 \times 10^{-12}\ F$$

$$Q = CE = (53.124 \times 10^{-12})(12) = 6.37 \times 10^{-10} C$$

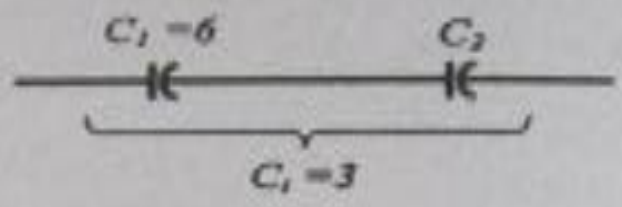
$$Q \cong 6.4 \times 10^{-10} C$$

131

$$\frac{1}{C_1} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{3} = \frac{1}{6} + \frac{1}{C_2}$$

$$C_2 = 6\ \mu F$$



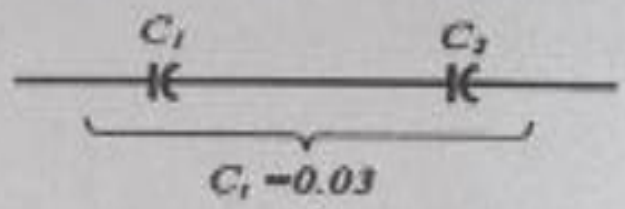
132

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_t = \frac{C_1 C_2}{C_1 + C_2} = 0.03 \Rightarrow ❶$$

$$C_t = C_1 + C_2 = 0.16$$

$$C_2 = 0.16 - C_1 \Rightarrow ❷$$

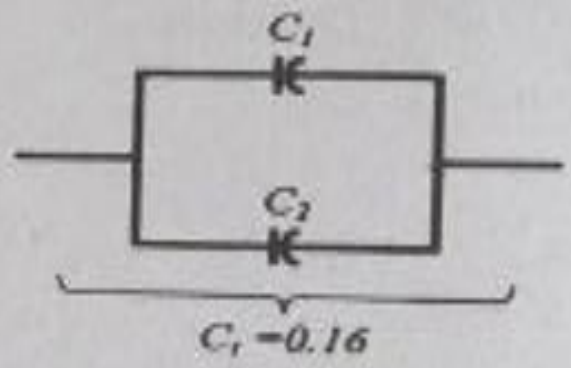


Substitute Eq.2 to Eq.1:

$$0.03(C_1 + C_2) = C_1 C_2$$

$$0.03(0.16) = C_1(0.16 - C_1)$$

$$C_1^2 - 0.16C_1 + 0.0048 = 0$$



Using the quadratic formula:

$$C_1 = \frac{0.16 \pm \sqrt{0.16^2 - 4(1)(0.0048)}}{2} = \frac{0.16 \pm 0.08}{2}$$

$$C_1^{(+)} = 0.12\ \mu F$$

$$C_2 = 0.16 - 0.12 = 0.04\ \mu F$$

$$C_2' = 2(0.04) = 0.08\ \mu F$$

$$\frac{1}{C_t'} = \frac{1}{C_1} + \frac{1}{C_2'} = \frac{1}{0.12} + \frac{1}{0.08}$$

$$C_t' = 0.048\ \mu F$$

133 Note: In a series circuit, the smallest capacitance in the combination shall always carry the higher voltage.

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{16} + \frac{1}{15} + \frac{1}{12} = 0.2125$$

$C_t = 4.705\ \mu F$

$Q_3 = C_3E_3 = (12 \times 10^{-6})(100) = 0.0012\ C$

Since in series, $Q_1 = Q_2 = Q_3 = Q_t$

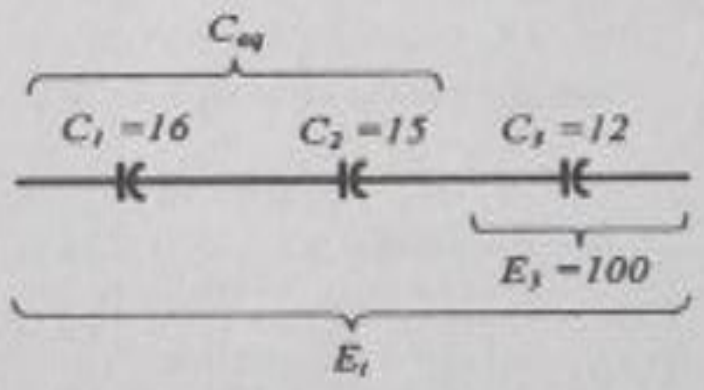
$$E_1 = \frac{Q_t}{C_1} = \frac{0.0012}{4.705 \times 10^{-6}}$$

$E_1 = 255.04$ volts

a Another solution.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{16} + \frac{1}{15}$$

$C_{eq} = 7.742\ \mu F$



Using VDT:

$$E_3 = \frac{E_tC_{eq}}{C_{eq} + C_3}$$

$$E_t = \frac{E_3(C_{eq} + C_3)}{C_{eq}} = \frac{100(7.742 + 12)}{7.742}$$

$E_t = 255$ volts

134 $$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{20} + \frac{1}{10} + \frac{1}{15} = 0.21666$$

$$C_t = \frac{1}{0.21666} = 4.615\ \mu F$$

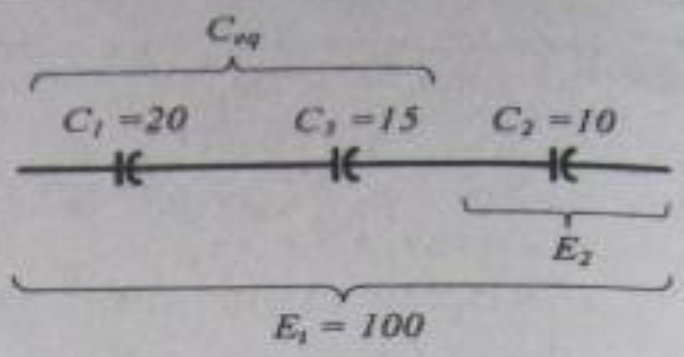
$Q_t = C_tE_t = (4.615)(100) = 461.5\ \mu C$

Since in series, $Q_1 = Q_2 = Q_3 = Q_t$

$$E_2 = \frac{Q_t}{C_2} = \frac{461.5}{10}$$

$E_2 = 46.15\ V$

Another solution.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_3} = \frac{1}{20} + \frac{1}{15}$$

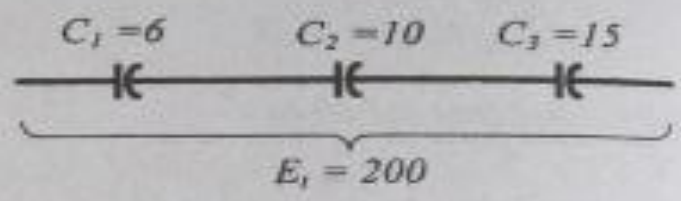
$C_{eq} = 8.5714\ \mu F$

Using VDT:

$$E_2 = \frac{E_1 C_{eq}}{C_{eq} + C_2} = \frac{100(8.5714)}{8.5714 + 10}$$

$E_2 = 46.15$ volts

135 $\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$



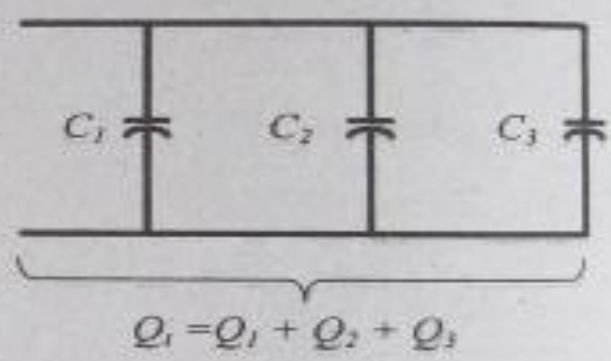
$C_t = 3\ \mu F$

$Q_t = C_t E_t = (3)(200) = 600\ \mu C$

Since in series, $Q_1 = Q_2 = Q_3 = Q_t$

$C_t = C_1 + C_2 + C_3 = 6 + 10 + 15 = 31\ \mu F$

Since in parallel, $Q_t = Q_1 + Q_2 + Q_3$



$$W = \frac{1}{2}\left(\frac{Q_t^2}{C_t}\right) = \frac{[3(600 \times 10^{-6})]^2}{2(31 \times 10^{-6})}$$

$W = 0.052$ J

136 $\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{4}$

$C_t = 2.4\ \mu F$

$Q_t = C_t E_t = (2.4)(100) = 240\ \mu C$

$C_t' = C_1 + C_2 = 6 + 4 = 10\ \mu F$

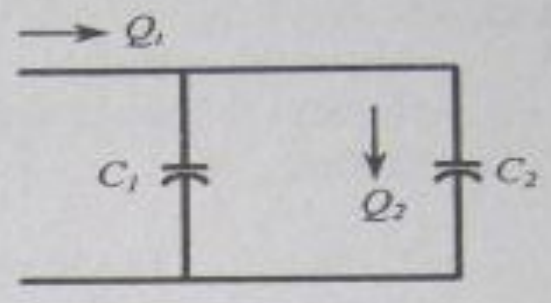
Since in parallel, $Q_t = Q_1 + Q_2$

$$E_1' = \frac{Q_t'}{C_t'} = \frac{(2)(240)}{10} = 48 \text{ volts}$$

$Q_2 = C_2 E_1' = (4)(48)$

$Q_2 = 192\ \mu C$

Another solution.



$Q_t = 240 + 240 = 480\ \mu C$

Using CDT :

$$Q_2 = \frac{Q_t C_2}{C_1 + C_2} = \frac{(480)(4)}{6+4}$$

$Q_2 = 192\ \mu C$

137 $$W = \frac{1}{2}\left(\frac{Q^2}{C}\right)$$

$Q = \sqrt{2WC} = \sqrt{2(0.2)(40 \times 10^{-6})} = 0.004\ C$

$C_t = C_1 + C_2 = 40 + 60 = 100\ \mu F$

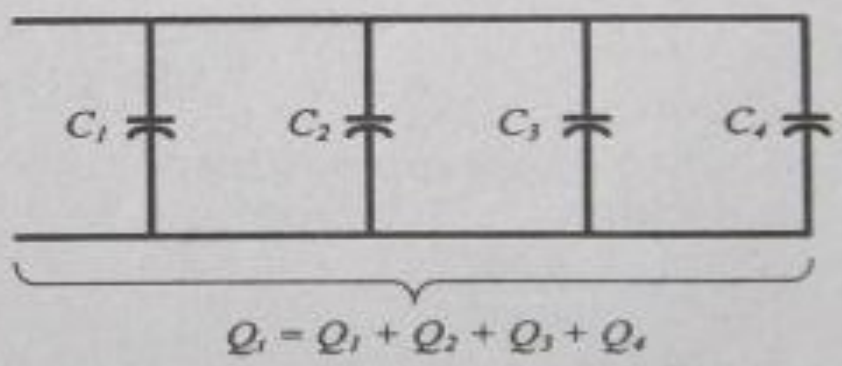
$$W' = \frac{1}{2}\left(\frac{Q^2}{C}\right) = \frac{(0.004)^2}{2(100 \times 10^{-6})} = 0.08\ J$$

$W_{loss} = W - W' = 0.2 - 0.08$

$W_{loss} = 0.12\ J$

138 $Q_t = C_t E_t = (35)(150)$

$Q_t = 5250\ \mu C$



Since in parallel,

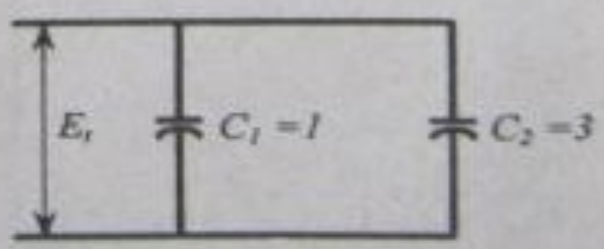
$Q_t = Q_1 + Q_2 + Q_3 + Q_4$

$Q_4 = Q_t - (Q_1 + Q_2 + Q_3)$

$Q_4 = 5250 - (2000 + 750 + 1500)$

$Q_4 = 1000\ \mu C$

139 $Q_t = C_t E_t = (1)(100) = 100\ \mu C$



Since in parallel, $C_t = C_1 + C_2$

$C_t = C_1 + C_2 = 1 + 3 = 4\ \mu F$

$$E_t = \frac{Q_t}{C_t} = \frac{100}{4}$$

$E_t = 25$ volts

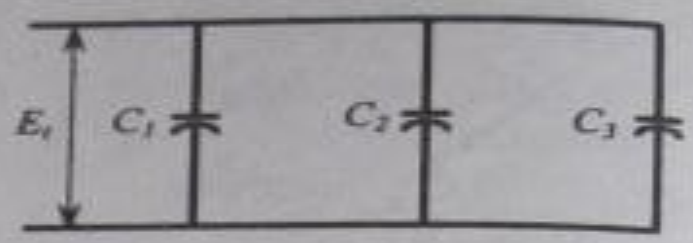
140

$Q_t = Q_1 + Q_2 + Q_3$

$= C_1E_1 + C_2E_2 + C_3E_3$

$Q_t = 10(100) + 15(150) + 25(200)$

$Q_t = 8250\ \mu C$



$C_t = C_1 + C_2 + C_3 = 10 + 15 + 25$

$C_t = 50\ \mu F$

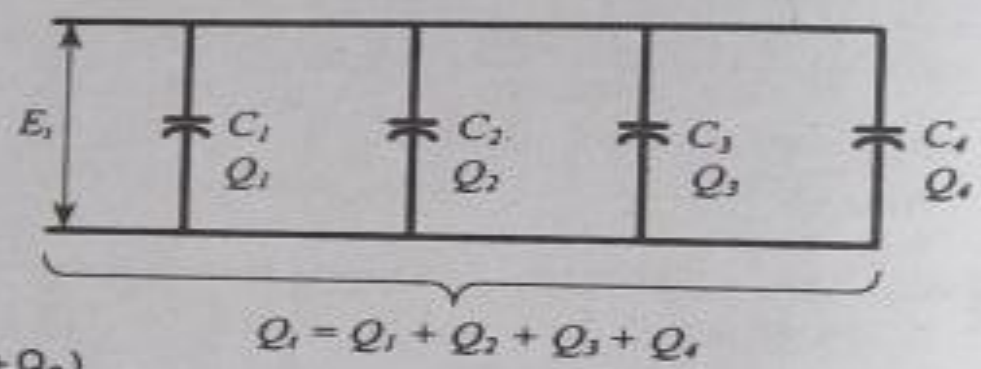
$E_t = \dfrac{Q_t}{C_t} = \dfrac{8250}{50}$

$E_t = 165$ volts

141

$Q_t = C_tE_t = 100 \times 10^{-6}(200)$

$Q_t = 0.02\ C$



$Q_1 = Q_t - (Q_2 + Q_2 + Q_3)$

$= 0.02 - (0.004 + 0.005 + 0.006)$

$Q_1 = 0.005\ C$

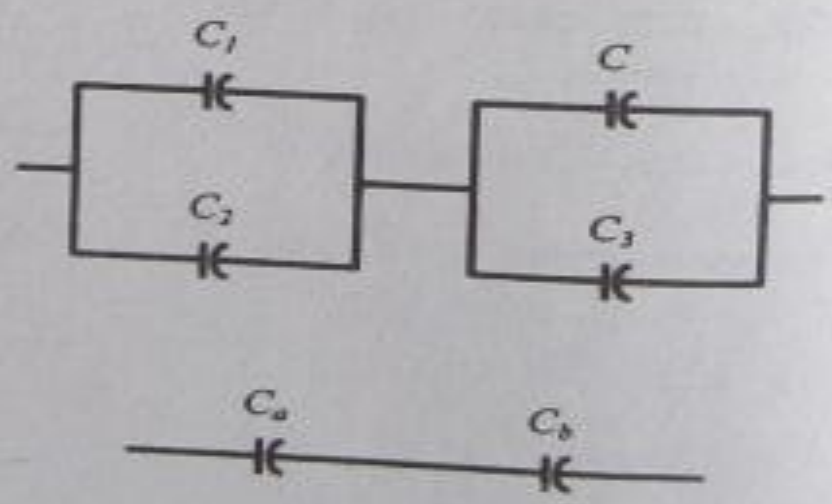
$E_1 = \dfrac{Q_1}{C_1} = \dfrac{0.005}{100 \times 10^{-6}}$

$E_1 = 50$ volts

142

$C_a = C_1 + C_2 = 200 + 800$

$C_a = 1000$



$C_b = C + C_3$

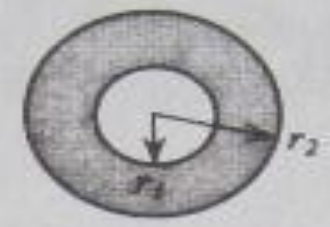
$C_b = C + 600$

$\dfrac{1}{C_t} = \dfrac{1}{C_a} + \dfrac{1}{C_b}$

$\dfrac{1}{500} = \dfrac{1}{1000} + \dfrac{1}{C + 600}$

$C = 400\ nF$

143 $C = \dfrac{2\pi\Sigma_0\Sigma_r}{\ln\dfrac{r_2}{r_1}}$



$r_2 = \dfrac{20}{2} = 10\text{ mm}$

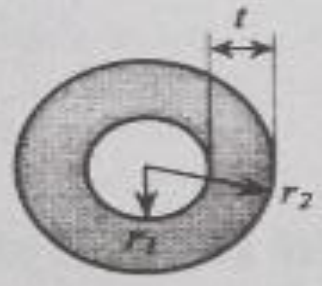
$r_1 = \dfrac{10}{2} = 5\text{ mm}$

$$C = \frac{2\pi(8.854 \times 10^{-12})(2)}{\ln\dfrac{10}{5}} = 1.605 \times 10^{-10}\text{F/m}$$

$$C = 1.605 \times 10^{-10}\ \frac{\text{F}}{\text{m}} \times \frac{1 \times 10^6\ \mu\text{F}}{1\text{F}} \times \frac{1000\text{ m}}{1\text{km}}$$

$C = 0.1605\ \mu\text{F/km}$

144 $C = \dfrac{2\pi\Sigma_0\Sigma_r}{\ln\dfrac{r_2}{r_1}}$



$r_1 = \dfrac{0.35}{2} = 0.175\text{ inch}$

$r_2 = r_1 + t = 0.175 + 0.2$

$r_2 = 0.375\text{ inch}$

$$C = \frac{2\pi(8.854 \times 10^{-12})(4.3)}{\ln\dfrac{0.375}{0.175}} = 3.138 \times 10^{-10}\text{F/m}$$

$$C = 3.138 \times 10^{-10}\ \frac{\text{F}}{\text{m}} \times \frac{1 \times 10^6\ \mu\text{F}}{1\text{F}} \times \frac{\text{m}}{3.281\text{ ft}} \times \frac{5280\text{ ft}}{1\text{mi}}$$

$C = 0.5049\ \mu\text{F/km}$

Magnetic Circuits

FORCE ON A CONDUCTOR CARRYING A CURRENT IN A MAGNETIC FIELD

$$F = \beta I L$$

where : F = force (newton)
β = flux density (tesla)
I = current (ampere)
L = length of conductor (meter)

Note: This formula presupposes that the direction of the axis of the conductor is at right angles to the direction of the field. If the directions of I and β form an angle of α , the expression must be multiplied by sin α .

FORCE BETWEEN TWO STRAIGHT WIRES WITH A CURRENT

$$F = (2 \times 10^{-7})(I_1)(I_2)\left(\frac{L}{d}\right)$$

where: F = force (newton)
I_1 , I_2 = currents in the two wires (ampere)
d = distance between the two wires (meter)
L = length of the wires (meter)

MAGNETOMOTIVE FORCE OR MMF

A force that sets up or tends to set up magnetic flux in a magnetic circuit. By passing an electric current through a number of turns of a wire produces it.

$$\mathfrak{F} = NI$$

where: $\mathfrak{F}$ = mmf (ampere-turn)
N = number of turns
I = current carried (ampere)

- **Gilbert** – cgs unit of magnetomotive force. Named after the English **physician** and physicist, **William Gilbert** (1540 – 1603).

$$\mathfrak{F} = 0.4\pi NI$$

where: $\mathfrak{F}$ = mmf (gilbert)
N = number of turns
I = current carried (ampere)

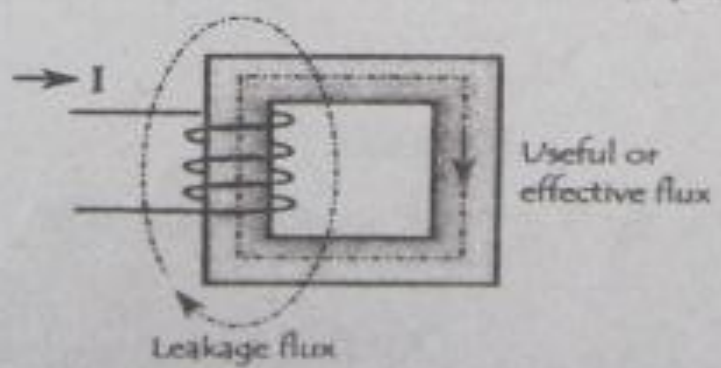
MAGNETIC FLUX

Magnetic flux - number of magnetic lines of forces in a magnetic field.

- **Maxwell** – unit of magnetic flux equal to **one line** of force. Named after the Scottish physicist, **James Clerk Maxwell** (1831 – 1879).

- **Weber** – SI unit of magnetic flux equal to **10^8 lines or maxwells**. Named after the German physicist, **Wilhelm Weber** (1804 – 1891). One weber is equal to **1×10^8** maxwells.

Leakage flux – the flux that does not follow the intended path in a magnetic circuit.



MAGNETIC FLUX DENSITY

Magnetic flux density - number of lines of flux per unit area through any substance in a plane at right angles to the lines of flux.

$$\beta = \frac{\text{flux passing}}{\text{area}} = \frac{\Phi}{A}$$

where : β = flux density (tesla)
Φ = flux (weber)
A = surface area perpendicular to the field (square meter)

- **Tesla** – SI unit of magnetic flux density equal to **webers per square meter**. Named after the Croatian-American engineer, **Nikola Tesla** (1856 – 1943)
- **Gauss** – cgs unit of magnetic flux density equal to **maxwells per square centimeter**. Named after the German mathematician **Johann Karl Freidrich Gauss** (1777 – 1855)

MAGNETIC FIELD INTENSITY OR STRENGTH

Magnetic field intensity - mmf per unit length of path of the magnetic flux. It is also called as the magnetizing force or the **magnetic potential** gradient.

$$H = \frac{NI}{\ell} \qquad H = \frac{\mathfrak{I}}{\ell}$$

where: H = magnetic field intensity (ampere-turn per meter)
ℓ = mean length of flux lines (meter)
$\mathfrak{I}$ = magnetomotive force (ampere-turn)

- **Oersted** – cgs unit of magnetic field strength equal to **gilbert per centimeter**. Named after the Danish physicist, H.C. **Oersted** (1777 – 1851).

PERMEABILITY

Permeability - the ability of a material to conduct magnetic flux through it

$$\mu = \frac{B}{H}$$

where: μ = permeability of the material (henry per meter)

- **Relative permeability** – ratio of the permeability of material to the permeability of air or vacuum.

$$\mu_r = \frac{\mu}{\mu_o}$$

where: μ_o = free space permeability ($4\pi \times 10^{-7}$ henry in meter)

MAGNETIC FIELD OF AN INFINITELY LONG STRAIGHT WIRE

$$\beta = \frac{\mu_o \mu_r I}{2\pi r} \qquad H = \frac{I}{2\pi r}$$

where: I = current carried by the wire (ampere)
r = perpendicular distance from the point considered to the surface of the wire (meter)
H = magnetic field intensity or field strength (ampere-turn per meter)
β = magetic flux density (tesla)
μ_o = free space permeability
μ_r = relative permeability of the core

RELUCTANCE

Reluctance - property of a material that **opposes flux flow**. It is equal to the ratio of the magnetomotive force in a magnetic circuit to the magnetic flux through any cross section of the magnetic circuit.

$$\Re = \frac{\ell}{\mu_o \mu_r A} \qquad \Re = \nu \frac{\ell}{A}$$

where: $\Re$ = reluctance (reciprocal henry)
ℓ = mean length of the magnetic path (meter)
μ_o = free space permeability
= $4\pi \times 10^{-7}$ (SI units)
= 1 (cgs units)
μ_r = relative permeability of the core
A = cross sectional area of the magnetic path (square meter)
ν = reluctivity

- **Reluctivity** – reciprocal of permeability

PERMEANCE

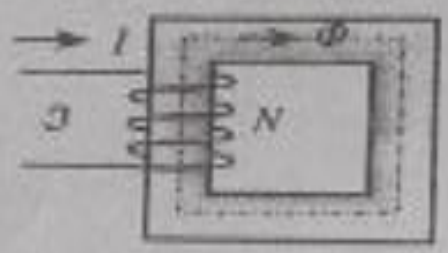
Permeance - reciprocal of reluctance.

$$P = \frac{1}{\mathfrak{R}} \qquad P = \frac{\mu_o \mu_r A}{\ell}$$

where: P = permeance

MAGNETIC CIRCUIT

Magnetic circuit - a closed path in which magnetic induction or flux flows.



$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}} = \frac{NI}{\mathfrak{R}}$$

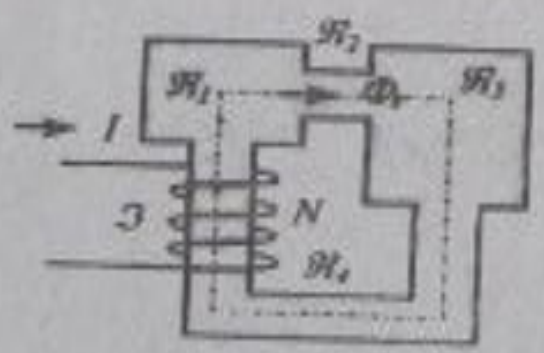
where: Φ = flux (weber)
ℑ = mmf (ampere-turn)
ℜ = reluctance (reciprocal henry)

- In cgs units

$$\Phi = \frac{0.4\pi NI}{\mathfrak{R}}$$

where: Φ = flux (maxwell)
N = number of turns in the coil
I = current flowing in the coil (ampere)
ℜ = reluctance (gilbert per maxwell)

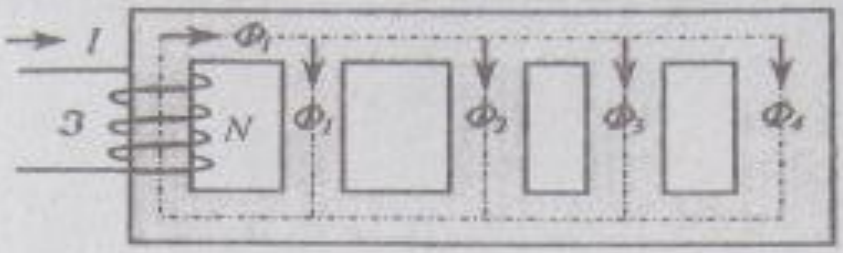
RELUCTANCE IN SERIES



$$\mathfrak{R}_t = \mathfrak{R}_1 + \mathfrak{R}_2 + \mathfrak{R}_3 + \mathfrak{R}_4$$

$$\Phi_t = \Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$$

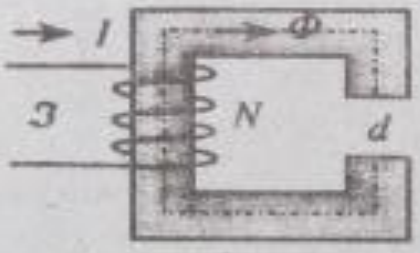
RELUCTANCE IN PARALLEL



$$\Re_t = \frac{1}{\frac{1}{\Re_1} + \frac{1}{\Re_2} + \frac{1}{\Re_3} + \frac{1}{\Re_4}}$$

$$\Phi_t = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4$$

AMPERE-TURNS FOR AN AIR GAP



$$\Im_{air} = \frac{d\beta}{\mu_o}$$

where : $\Im_{air}$ = mmf in the air gap (amperes-turn)
d = length of air gap (meter)
β = flux density in the air gap (tesla)
μ_o = free space permeability

ENERGY STORED IN A MAGNETIC CIRCUIT

$$W_m = \frac{1}{2}\Re\Phi^2 \qquad W_m = \frac{1}{2}(\ell A)\left(\frac{\beta^2}{\mu}\right)$$

where : W_m = energy stored (joule)
μ = permeability of core

TRACTIVE OR LIFTING FORCE OF A MAGNET

$$F = \frac{1}{2}\left(\frac{AB^2}{\mu_o}\right)$$

where: F = force (newton)
β = flux density in the air gap (tesla)
A = total area of the contact between the armature and the core (m^2)
μ_o = free space permeability

STEINMETZ'S FORMULA

According to experiments by Charles Proteus Steinmetz, (1865 – 1923) the heat energy due to hysteresis released per cycle per unit volume of iron is approximately:

$$W_h = \eta f \beta_m^{1.6}$$

where: W_h = heat energy due to hysteresis released per unit volume of iron (J/m^3)
β_m = maximum flux density (tesla)
f = frequency (hertz)
η = hysteresis coefficient

Test 5

Problem 145: EE Board October 1994

The force acting on a pole of 3 Wb is 12 N. The magnetic intensity of the magnetic field is

A. 4 N per Wb
B. 12 N per Wb
C. 36 N per Wb
D. none of these

Problem 146:

Two straight parallel wires 2 m long and 3 mm apart carries a current of 8.0 A in opposite direction. Calculate the force between these conductors.

A. 8.33×10^{-3} N
B. 8.25×10^{-3} N
C. 8.53×10^{-3} N
D. 8.75×10^{-3} N

Problem 147:

A wire 12 cm long and carrying a current of 30 A is placed in between the pole face of a magnet whose magnetic flux density is 0.9 tesla. If the wire is inclined at an angle 60 degrees from the plane of the magnetic field, what is the force exerted on the wire?

A. 2.8 N
B. 2.5 N
C. 3.1 N
D. 2.2 N

Problem 148: EE Board October 1992

The reluctance of a non-magnetic circuit is 12 units. How much flux will be set up if surrounded by a coil 600 turns carrying a current of 3 A?

A. 150
B. 16.67
C. 50
D. 200

Problem 149:

The coil in a magnetic contactor requires 0.5 A to provide a magnetizing force of 500 AT. How many turns are necessary?

A. 250
B. 500
C. 1000
D. 2000

Problem 150:

A solenoid 30-cm long is wound with 300 turns, what is the value of its field strength inside the solenoid, when the coil is carrying a current of 2 amperes?

A. 1500 AT/m
B. 2000 AT/m
C. 1800 AT/m
D. None of these

Problem 151:

Calculate the magnetic field intensity to produce a flux density of 10×10^{-3} tesla at the center of a long straight solenoid.

A. 7958 AT/m
B. 6423 AT/m
C. 8323 AT/m
D. 7869 AT/m

Problem 152: EE Board October 1996

If a current of 5 A flows through a long wire of radius 0.004 meter, what is the flux density (H) produced 0.02 meter away from the surface of the wire.

A. 19.70 AT/m
B. 39.80 AT/m
C. 66.34 AT/m
D. 33.17 AT/m

Problem 153: EE Board October 1997, EE Board October 1998

The relative permeability of a certain silicon steel is 4500. A certain magnetic loop consists of a silicon steel of 10 cm square, 20 cm long and an air gap of 1/4 cm. What is the reluctance of the magnetic circuit?

A. 2.54×10^{-2}
B. 6.44×10^{-3}
C. 6.44×10^{-4}
D. 2.54×10^{-3}

Problem 154: EE Board March 1998

A certain laminated steel core has a permeability of 3000. The length is 5 cm and the cross section is 2 sq. cm. What is its reluctance?

A. 7500
B. 8.33×10^{-4}
C. 1200
D. 2.5

Problem 155:

A coil with 900 turns is wound over a magnetic core with a reluctance of 10,000 AT/Wb. If a current of 0.5 A is pass through the coil, how much is the magnetic flux that the coil generates?

A. 0.062 Wb
B. 0.058 Wb
C. 0.032 Wb
D. 0.045 Wb

Problem 156:

An air-core circular coil has 5000 turns. The inner and outer diameters are 20 and 24 cm respectively. If a current of 2 A is passed through the coil, determine the flux density inside the coil.

A. 0.018 T
B. 0.021 T
C. 0.013 T
D. 0.015 T

Problem 157:

A given magnetic circuit has a magnetic field intensity of 400 AT/m. If the length of the magnetic path is doubled maintaining the same magnetomotive force, how much is the new magnetic field intensity?

A. 800 AT/m
B. 400 AT/m
C. 200 AT/m
D. 600 AT/m

Problem 158: EE Board October 1998

A magnetic circuit consists of silicon steel of 3000 permeability, of 10 cm length and a cross section of 1.5 sq. cm and an air gap of the same cross section and of 2 cm length. A 1/2-ampere current flows through the 5000-turn coil. What is the field intensity at the air gap?

A. 250 oersted
B. 795 oersted
C. 2262 oersted
D. 1567 oersted

Problem 159: EE Board March 1998

A magnetomotive force is supplied by a current of one ampere through 100 turns. The magnetic circuit consists of a steel core of 1000 permeability, 10 cm long and 4 sq. cm area and an air gap one cm long. What is the field intensity at the air gap?

A. 12.25 oersted
B. 497.66 oersted
C. 100.53 oersted
D. 124.4 oersted

Problem 160:

A circular ring of iron having a cross sectional area of 5 cm^2, an average diameter of 30 cm is wound with a coil of 1000 turns. If a current of 3 A is pass through the coil, determine the flux generated? Assume the relative permittivity of iron is 250.

A. 5×10^{-4} Wb
B. 6×10^{-4} Wb
C. 6.5×10^{-4} Wb
D. 4.75×10^{-4} Wb

Problem 161:

A non-magnetic ring having a cross sectional area of 10 cm^2 is uniformly wound with 300 turns of a given wire. If a current of 1 A is passed through the coil, 2.4 μWb of flux is generated inside the ring. Determine the average diameter of the ring.

A. 50 cm
B. 42 cm
C. 48 cm
D. 54 cm

Problem 162:

A coil with 150 turns is wound over a 200 cm a cylindrical iron core whose relative permittivity is 250. If a current of 2 A is pass through the coil, determine the flux density in the core.

A. 0.04 T
B. 0.07 T
C. 0.12 T
D. 0.10 T

Problem 163: EE Board March 1998

A magnetic circuit consists of silicon steel of 3000 permeability and an air gap. The length of the steel core is 10 cm and the air gap is 2 cm. Both have the same cross section of 1.5 sq. cm. A current of 1/2 ampere flows through the windings to produce 2351 maxwells flux. How many turns are there in the coil?

A. 4120 turns
B. 5000 turns
C. 2500 turns
D. 1250 turns

Problem 164:

A toroidal core with a mean circumference of 100 cm and a cross sectional area of 10 cm^2 is wound with 500 turns of wire. What current would be required to generate a flux of 1 mWb in the core. Assume the core has a relative permeability of 800.

A. 1.0 A
B. 2.0 A
C. 1.5 A
D. 2.5 A

Problem 165:

A magnetic ring (relative permittivity = 800) has a mean radius of 10 cm and a cross sectional area of 5 cm^2. An air gap measuring 1.5 mm is cut in the ring. Determine the required mmf in order to produce a flux of 0.25 mWb in the air gap.

A. 890
B. 920
C. 909
D. 972

Problem 166:

A magnetic ring with a mean diameter of 25 cm and a cross sectional area of 5 cm^2 is wound with a coil of 600 turns. An air gap of 4 mm is made by cutting a section of the ring. A current of 10 A is passed through the coil. Determine the energy stored in the air gap. Assume relative permittivity of the ring to be 1000.

A. 2.08 J
B. 1.96 J
C. 1.57 J
D. 1.66 J

Problem 167: EE Board April 1995

A permanent magnet loudspeaker is equipped with a 10 ohm 10 turn voice coil wound on a tube 1 inch in diameter. The flux density in the air gap is 10,000 lines per square inch. If the impedance of the voice coil is a pure resistance, what will be the maximum thrust delivered to the core if the impressed signal is 20 dB above 0.005 watt?

A. 0.0336 N
B. 0.0116 N
C. 0.0276 N
D. 0.0532 N

Problem 168: EE Board October 1997

A small single-phase transformer has 10.2 watts at no load. The core has a volume of 750 cubic cm. The maximum flux density is 10,000 gauss and the hysterisis constant of the core is 5×10^{-4}. Using the Steinmetz law to find the hysterisis, determine the eddy current loss.

A. 4.55 W
B. 5.55 W
C. 3.55 W
D. 2.55 W

ANSWER KEY

145. A	150. B	155. D	160. A	165. C
146. C	151. A	156. A	161. A	166. B
147. A	152. B	157. C	162. B	167. C
148. A	153. D	158. D	163. B	168. A
149. C	154. B	159. D	164. B	

☑ RATING

21-24 - Topnotcher
16-20 - Passer
11-15 - Conditional
0-10 - Failed

Solutions to Test 5

145 Note: By considering the units concerned, this particular problem is solved.
Let: H = magnetic field strength in Newtons per Weber.

$$H = \frac{12\,N}{3\,Wb}$$

$$H = 4\,N/Wb$$

146 $$F = 2 \times 10^{-7}\left(\frac{I_1 I_2 L}{d}\right) = 2 \times 10^{-7}\left(\frac{(8)(8)(2)}{0.003}\right)$$

$$F = 8.53 \times 10^{-3} N$$

147 $$F = \beta IL \sin\alpha = (0.9)(30)(0.12)\sin 60^\circ$$

$$F = 2.8\,N$$

148 $$\Phi = \frac{\mathfrak{I}}{\mathfrak{R}} = \frac{NI}{\mathfrak{R}} = \frac{3(600)}{12}$$

$$\Phi = 150\,Wb$$

149 $$N = \frac{\mathfrak{I}}{I} = \frac{500}{0.5}$$

$$N = 1000 \text{ turns}$$

150 $$H = \frac{NI}{\ell} = \frac{2(300)}{0.3}$$

$$H = 2000\,AT/m$$

151 Note: Assume the medium is air, $\mu_r = 1$.

$$\mu = \frac{\beta}{H}$$

$$H = \frac{\beta}{\mu} = \frac{\beta}{\mu_o \mu_r} = \frac{10 \times 10^{-3}}{(4\pi \times 10^{-7})(1)} = 7957.74$$

$$H \cong 7958\,AT/m$$

152 $$H = \frac{I}{2\pi d} = \frac{5}{2\pi(0.02)} = 39.788$$

$$H \cong 39.8\,AT/m$$

153 $$\mathfrak{R} = \mathfrak{R}_{core} + \mathfrak{R}_{air} = \frac{\ell_{core}}{\mu_r \mu_o A_{core}} + \frac{\ell_{air}}{\mu_o A_{air}}$$

$$\Re = \frac{20}{4500(1)(10)(10)} + \frac{0.25}{1(10)(10)}$$

$\Re = 2.54 \times 10^{-3}$ cgs units (gilbert per maxwell)

Note: Use "cgs system" since with SI, the answer is not found in the choices.

154 $$\Re = \frac{\ell}{\mu_r \mu_o A} = \frac{5}{3000(1)(2)}$$

$\Re = 8.33 \times 10^{-4}$ cgs units

155 $$\Phi = \frac{\Im}{\Re} = \frac{NI}{\Re} = \frac{(0.5)(900)}{10{,}000}$$

$\Phi = 0.045$ Wb

156 $$\mu = \frac{\beta}{H} \quad H = \frac{NI}{\ell}$$

$$d_{mean} = \frac{20 + 24}{2} = 22 \text{ cm}$$

$$\beta = \frac{\mu NI}{\ell} = \frac{\mu_o \mu_r NI}{\ell} = \frac{(4\pi \times 10^{-7})(1)(2)(5000)}{(\pi)(0.22)}$$

$\beta = 0.018$ T

157 $$H = \frac{NI}{\ell} = \frac{\Im}{\ell}$$

Note: With $\Im$ constant, the magnetic field intensity (H) of a given magnetic circuit is inversely proportional to the mean length (ℓ) of the magnetic path.

$$\frac{H_2}{H_1} = \frac{\ell_1}{\ell_2}$$

$$H_2 = \frac{H_1 \ell_1}{\ell_2} = \frac{400(\ell_1)}{2\ell_1}$$

$H_2 = 200$ AT/m

158 $$\Re = \Re_{core} + \Re_{air} = \frac{\ell_{core}}{\mu_r \mu_o A_{core}} + \frac{\ell_{air}}{\mu_o A_{air}}$$

$$\Re = \frac{10}{3000(1)(1.5)} + \frac{2}{1(1.5)}$$

$\Re = 1.3355$ cgs units

$$\Phi = \frac{0.4\pi NI}{\Re} = \frac{0.4\pi(5000)(0.5)}{1.336} = 2351.5 \text{ maxwells}$$

$$\beta = \frac{\Phi}{A} = \frac{2351.5}{1.5} = 1567.66 \text{ gauss}$$

$$\mu = \frac{\beta}{H}$$

$$H = \frac{\beta}{\mu} = \frac{1567.66}{1} = 1567.66 \text{ oersted}$$

159 $$\Re = \Re_{core} + \Re_{air} = \frac{\ell_{core}}{\mu_r \mu_o A_{core}} + \frac{\ell_{air}}{\mu_o A_{air}}$$

$$\Re = \frac{10}{1000(1)(4)} + \frac{1}{1(4)}$$

$$\Re = 0.2525 \text{ cgs units}$$

$$\Phi = \frac{0.4\pi NI}{\Re} = \frac{0.4\pi(100)(1)}{0.2525} = 497.68 \text{ maxwells}$$

$$\beta = \frac{\Phi}{A} = \frac{497.68}{4} = 124.42 \text{ gauss}$$

$$\mu = \frac{\beta}{H}$$

$$H = \frac{\beta}{\mu} = \frac{124.42}{1} = 124.42 \text{ oersted}$$

160 $$\mu = \frac{\beta}{H} \qquad H = \frac{NI}{\ell}$$

$$\beta = \frac{\mu NI}{\ell} = \frac{(4\pi \times 10^{-7})(250)(1000)(3)}{\pi(0.3)} = 1\,T$$

$$\Phi = \beta A = (1)(5\text{ cm}^2)\left(\frac{1\text{m}}{100\text{ cm}}\right)^2 = 5 \times 10^{-4} \text{ Wb}$$

161 $$\beta = \frac{\Phi}{A} = \frac{2.4 \times 10^{-6}}{10\text{ cm}^2\left(\frac{1\text{m}}{100\text{ cm}}\right)^2} = 0.0024\text{ T}$$

$$\beta = \frac{\mu NI}{\ell}$$

$$\ell = \frac{\mu NI}{\beta} = \frac{(4\pi \times 10^{-7})(1)(300)}{0.0024} = 0.157\text{ m}$$

$\ell = \pi d$

$d = \frac{0.157}{\pi} = 0.05\ m$

$d = 50\ cm$

162 $\beta = \frac{\mu NI}{\ell} = \frac{\mu_o \mu_r NI}{\ell}$

$\beta = \frac{(4\pi \times 10^{-7})(250)(150)(3)}{2}$

$\beta = 0.07\ T$

Note: The mean length of the magnetic path is equivalent to the height of the cylindrical core.

163 $\Re = \Re_{core} + \Re_{air} = \frac{\ell_{core}}{\mu_r \mu_o A_{core}} + \frac{\ell_{air}}{\mu_o A_{air}}$

$\Re = \frac{10}{3000(1)(1.5)} + \frac{2}{1(1.5)}$

$\Re = 1.3355$ cgs units

$\Phi = \frac{0.4\pi NI}{\Re}$

$N = \frac{\Phi \Re}{0.4\pi I} = \frac{2351(1.3355)}{0.4\pi(0.5)} = 4997.08$

$N \simeq 5000$ turns

164 $\beta = \frac{\Phi}{A} = \frac{0.001}{10\ cm^2 \times \left(\frac{1m}{100\ cm}\right)^2} = 1\ T$

$H = \frac{\beta}{\mu} = \frac{\beta}{\mu_o \mu_r} = \frac{1}{(4\pi \times 10^{-7})(800)} = 994.718\ AT/m$

$H = \frac{NI}{\ell}$

$I = \frac{H\ell}{N} = \frac{(994.718)(1)}{500} = 1.989 \approx 2\ A$

165 $\beta = \frac{\Phi}{A} = \frac{0.25 \times 10^{-3}}{5\ cm^2 \times \left(\frac{1m}{100\ cm}\right)^2} = 0.5\ T$

$$\mu = \frac{\beta}{H}; \qquad H = \frac{\Im}{\ell}$$

$$\Im = \frac{\beta\ell}{\mu}$$

For the air gap :

$$\Im_{air} = \frac{\beta\ell}{\mu} = \frac{(0.5)(0.0015)}{4\pi \times 10^{-7}} = 596.83 \text{ AT}$$

For the core :

$$\Im_{core} = \frac{\beta\ell}{\mu} = \frac{(0.5)(2\pi)(0.1)}{(4\pi \times 10^{-7})(800)} = 312.5 \text{ AT}$$

$$\Im_{total} = \Im_{air} + \Im_{core} = 596.83 + 312.5 = 909.33$$

$$\Im_{total} \cong 909 \text{ AT}$$

166 $$\Im = \frac{\beta\ell}{\mu}$$

For the air gap :

$$\Im_{air} = \frac{\beta\ell}{\mu} = \frac{\beta(0.004)}{4\pi \times 10^{-7}} = 3183.098\,\beta$$

For the core :

$$\Im_{air} = \frac{\beta\ell}{\mu} = \frac{\beta(\pi)(0.25)}{(4\pi \times 10^{-7})(1000)} = 625\,\beta$$

$$\Im_{total} = \Im_{air} + \Im_{core} = 3183.098\,\beta + 625\,\beta = 3808.098\,\beta$$

$$\Im_{total} = NI = 600(10) = 6000 \text{ AT}$$

$$\beta = \frac{6000}{3808.098} = 1.57 \text{ T}$$

$$W = \frac{\beta^2 V}{2\mu_o} = \frac{(1.57)^2\left((5 \text{ cm}^2)\left(\frac{1\text{m}}{100 \text{ cm}}\right)^2(0.004)\right)}{2(4\pi \times 10^{-7})}$$

$$W = 1.96 \text{ J}$$

167 $$F_{per\ turn} = \beta IL$$

$$F_{total} = \beta ILN$$

$$dB = 10 \log \frac{P}{P_{ref.}}$$

$$20 = 10 \log \frac{P}{0.005}$$

$$P = 0.5 \text{ watt}$$

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.5}{10}} = 0.2236 \text{ A}$$

$$L = \pi d = \pi \left(1 \text{inch} \times \frac{0.0254 \text{ m}}{1 \text{inch}} \right) = 0.0798 \text{ m}$$

$$\beta = 10{,}000 \frac{\text{lines}}{\text{in}^2} \times \frac{1 \text{Wb}}{1 \times 10^8 \text{lines}} \times \left(\frac{1 \text{inch}}{0.0254 \text{ m}} \right)^2 = 0.155 \text{ T}$$

Substitute :

$$F_{total} = \beta ILN$$

$$= (0.155)(0.2236)(0.0798)(10)$$

$$F_{total} = 0.0276 \text{ Newton}$$

Note: The active length (L) of the wire is equivalent to the length per turn (circumference of the cylindrical core).

168 $P_{core} = P_h + P_e = 10.2 \text{ W}$

From Steinmetz formula:

$$W_h = \eta \beta_m^{1.6} fV$$

$$= (5 \times 10^{-4})(10{,}000)^{1.6}(60)(750)$$

$$W_h = 56.5174 \times 10^6 \text{ erg} \times \frac{1 \text{J}}{1 \times 10^7 \text{ erg}}$$

$$W_h = 5.65 \text{ J}$$

Note: Since time is not specified in the problem, assume the constant (η) in the formula is in per unit second, thus making the answer (W_h) in unit joules per second or watt.

$$P_h = 5.65 \text{ W}$$

$$P_e = P_{core} - P_h$$

$$= 10.2 - 5.65$$

$$P_e = 4.55 \text{ W}$$

Electromagnetic Induction

FARADAY'S LAW

- **Faraday's first law of electromagnetic induction:** Whenever the flux linking a coil or current changes, an emf is induced in it.
- **Faraday's second law of electromagnetic induction:** The magnitude of the induced emf is proportional to the rate of change of flux linkages.

Flux linkages = flux X number of turns

$$e = N\frac{d\Phi}{dt}$$

where : e = induced emf (volt)
N = number of turns
$d\Phi/dt$ = rate of change of flux (weber per second)

INDUCED EMF IN A SHORT LENGTH OF WIRE

A voltage can be developed in a wire by moving the wire across a magnetic field so that flux cutting results (Faraday's principle).

$$e = \beta L v \qquad e = \frac{\Phi}{t}$$

where : e = induced emf (volt)
Φ = flux cut by the conductor (weber)
t = time of flux cutting (second)
β = flux density at the location of the conductor (tesla)
L = length of the conductor (meter)
v = relative velocity (meter per second)

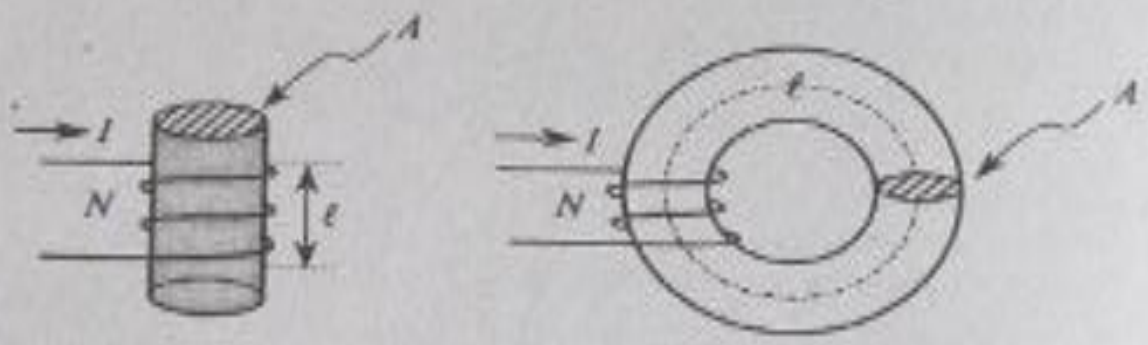
SELF-INDUCTANCE OR COEFFICIENT OF SELF INDUCTION

It is a measure of how much counter emf is generated in a circuit or component for a change in current through that circuit or component.

- **Inductor or choke coil** - a two terminal device that consist of a coiled wire wound in common core or in free air.
- **Henry** (H) - unit of inductance. Named after the American physicist, **Joseph Henry** (1791 – 1878)

$$e = L\frac{di}{dt}$$

where: e = self-induced voltage (volt)
L = self inductance (henry)
di/dt = rate of change of current (ampere per second)



$$L = \frac{N\Phi}{I} = \frac{\lambda}{I} \qquad L = \frac{N^2}{\Re} \qquad L = \frac{\mu_o \mu_r A N^2}{\ell}$$

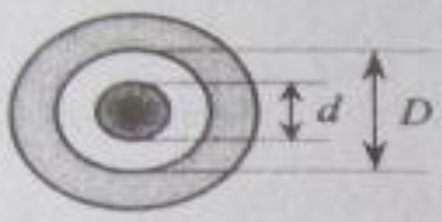
where: L = inductance (henry)
μ_o = permeability of free space
= $4\pi \times 10^{-7}$ **henry per meter**
μ_r = relative permeability of the core used
A = cross sectional area of the magnetic path (square meter)
N = number of turns
Φ = flux (weber)
I = current (ampere)
ℓ = mean length of the magnetic path (meter)
$\Re$ = reluctance of the magnetic path (reciprocal henry)
$N\Phi$ or λ = flux linkage (weber)

- **Inductance with varying reluctance**

$$L = N\frac{d\Phi/dt}{di/dt}$$

where: $d\Phi/dt$ = rate of change in flux (webers per second)
di/dt = rate of change in current (ampere per second)

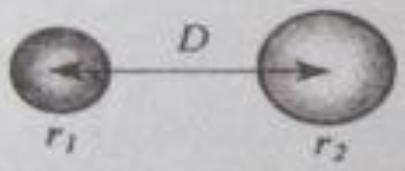
- **Inductance of a coaxial cable**



$$L = 2\times10^{-7}\ln\frac{D}{d}$$

where : L = inductance (henry per meter)
D = diameter of the outer conductor
d = diameter of the inner conductor

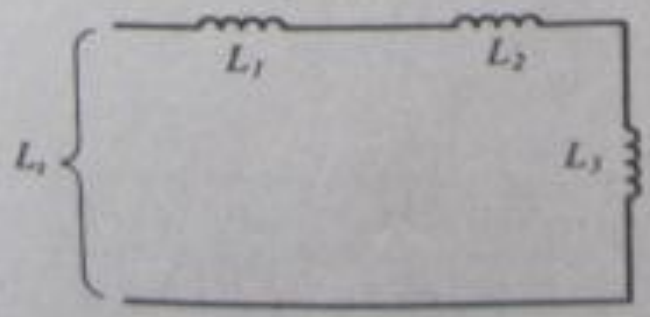
- **Inductance of two long cylindrical conductors, parallel and external to each other**



$$L = 4\times10^{-7}\ln\frac{D}{\sqrt{r_1 r_2}}$$

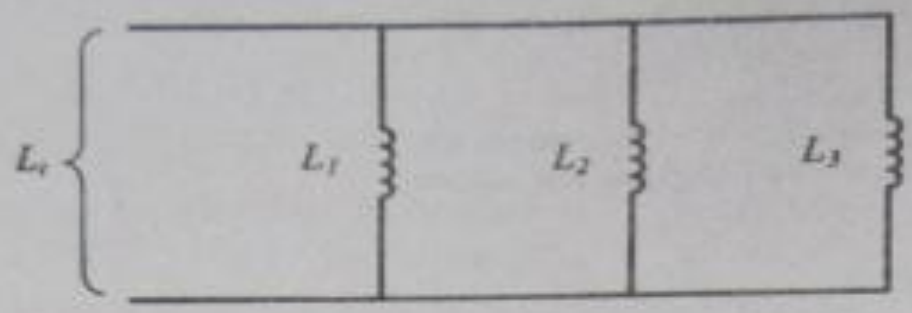
where: L = inductance (henry)
D = distance between the two wires (meter)
r_1, r_2 = radius of the two wires respectively (meter)

SERIES-CONNECTED INDUCTORS



$$L_t = L_1 + L_2 + L_3$$

PARALLEL-CONNECTED INDUCTORS



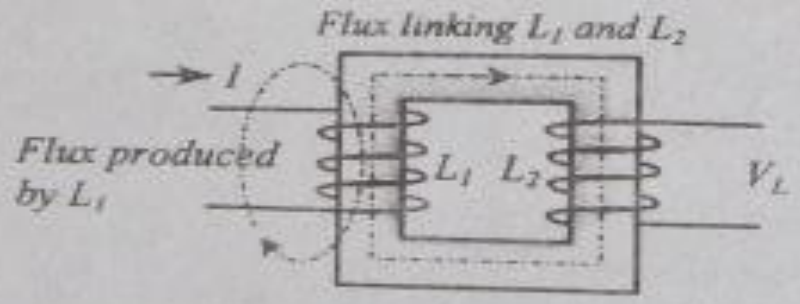
$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

Note: For series-parallel and parallel-series connections, use the basic principles in series and parallel connections to simplify the circuit and answer the question.

MUTUAL INDUCTION

It is the condition of inducing an emf in a coil or conductor by magnetic flux lines generated in another coil or conductors.

- **Mutual inductance** – the amount or degree of mutual induction that exists between two coils or windings.



$$M = k\sqrt{L_1 L_2} \qquad M = \frac{N_1 N_2}{\mathcal{R}} \qquad M = \frac{\mu_o \mu_r A N_1 N_2}{\ell}$$

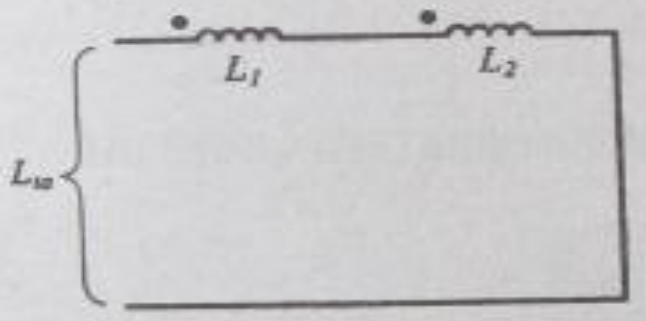
$$k = \frac{\text{flux linking } L_1 \text{ and } L_2}{\text{flux produced by } L_1}$$

Note: Two coils have a mutual inductance of 1 H when a current change of 1 A/s in one coil induces 1 V in the other coil.

where: M = mutual inductance (henry)
L_1 = self inductance of coil 1 (henry)
L_2 = self inductance of coil 2 (henry)
k = coefficient of coupling
μ_o = permeability of free space ($4\pi \times 10^{-7}$ **henry per meter**)
μ_r = relative permeability of the core used
A = cross sectional area of the magnetic path (square meter)
N_1 = number of turns in the primary winding
N_2 = number of turns in the secondary winding
ℓ = mean length of the magnetic path (meter)
$\Re$ = reluctance of the magnetic path (reciprocal henry)

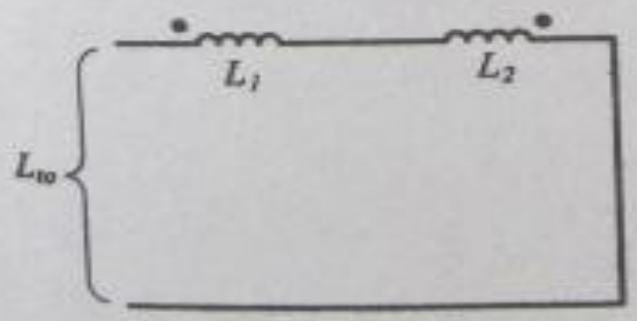
SERIES COILS WITH MUTUAL INDUCTANCE

Series aiding – the common current produces the same direction of magnetic fields.



$$L_{ia} = L_1 + L_2 + 2M$$

Series opposing – connections result in opposite fields.



$$L_{io} = L_1 + L_2 - 2M$$

$$M = \frac{L_{ia} - L_{io}}{4}$$

where : L_{ia} = total inductance, series aiding (henry)
L_{io} = total inductance, series opposing (henry)
M = mutual inductance between the two coils (henry)

PARALLEL COILS WITH MUTUAL INDUCTANCE

Parallel aiding

$$L_{ia} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Parallel opposing

$$L_{io} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

ENERGY STORED IN AN INDUCTOR CARRYING A CURRENT

$$W = \frac{1}{2} L I^2$$

where: W = energy stored (joule)
L = inductance of the coil (henry)
I = current flowing in the coil (ampere)

Test 6

Problem 169: EE Board October 1998

A magnetic coil produces 100,000 maxwells with 2,000 turns and with a current of 2 A. The current is cut-off and the flux collapses in 0.01 sec. What is the average voltage that will appear across the coil?

A. 20 kV
B. 200 V
C. 2,000 kV
D. 2 kV

Problem 170: EE Board October 1998

A magnet, which generates 800 maxwells of flux, is inserted through a coil of 1000 turns. It is then withdrawn within 0.15 sec. What is the voltage that appears across the coil?

A. 0.22 V
B. 53.3 V
C. 533 V
D. 0.0533 V

Problem 171:

The armature coil of a generator with 50 conductors on it rotates in a magnetic field with a magnetic flux density 1.5 T. The effective area of the coil is 1000 square cm. If this coil is move across the pole in 0.5 sec, determine the average emf induced in the coil.

A. 20 V
B. 15 V
C. 10 V
D. 12 V

Problem 172: EE Board April 1997

A conductor, 50 mm long, moves at a velocity of 2.5 meters per second across a magnetic field of 0.90 Wb/m^2. What is the voltage generated?

A. 0.125 V
B. 0.0648 V
C. 0.1125 V
D. 0.072 V

Problem 173:

An airplane travels at a speed of 1000 kph in a region where the earth's magnetic field is 5.0×10^{-5} T and is nearly vertical. What is the potential difference induced between the wing tips of the airplane, which is 20 m apart?

A. 0.28 V
B. 0.23 V
C. 0.36 V
D. 0.15 V

Problem 174: EE Board October 1998

A piece of conductor 10 cm long moves across a magnetic field of 10,000 gauss at a velocity of 120 cm/sec. What voltage appears across the conductor?

A. 0.24 V
B. 0.12 V
C. 60 V
D. 120 V

Problem 175:

A conductor 50 cm in length moves at an angle of 30° to the direction of a magnetic field with a velocity of 100 cm/s. If the field has a flux density of 2 Wb/m^2, calculate the emf induced in it.

A. 0.5 V
B. 2.0 V
C. 1.0 V
D. 1.5 V

Problem 176:

A metal disk of 30 cm radius rotates at an angular velocity of 1200 rpm across a uniform vertical magnetic field of flux density 0.5 Wb/m^2. Calculate the voltage induced between the rim and the center of the disk.

A. 4.7 V
B. 5.8 V
C. 5.1 V
D. 4.4 V

Problem 177:

A conducting disc 20-cm in diameter pivoted by a vertical spindle which is lying in uniform vertical magnetic field of flux density 0.3 T. A fixed brush is connected at the edge of the disc. When a 4 Ω resistance is connected across the brush and the spindle, a current of 15 mA was observed to flow through the resistor. Determine the velocity in which the disc is rotating.

A. 394 rpm
B. 382 rpm

C. 302 rpm
D. 228 rpm

Problem 178:
How much is the inductance of a coil that induces 500 V when the current changes at the rate of 5 mA in 2 μs?

A. 12.5 μH
B. 0.2 H
C. 0.1H
D. 2 mH

Problem 179:
The field winding of a DC generator is wound with 900 turns and has a resistance of 50 ohms. If the winding is excited from a 240 V dc source, the magnetic flux linking the coil is 4 mWb. Determine the self-inductance of the coil.

A. 0.75 H
B. 0.83 H
C. 0.92 H
D. 0.56 H

Problem 180:
A coil of 1000 turns is uniformly wound on a ring of non-magnetic material, the mean diameter being 20 cm. The cross sectional area of the coil is 4 square cm. Determine the self-inductance of the coil.

A. 0.7 mH
B. 0.9 mH
C. 0.6 mH
D. 0.8 mH

Problem 181:
An iron core 0.4 m long and 5 square cm. in cross section, is wound with 300 turns. When a current of 0.5 A flows in the coil, how much is the inductance of the coil. Assume the core has a permeability of 2500.

A. 0.35 H
B. 0.42 H
C. 0.26 H
D. 0.31 H

Problem 182: EE Board October 1990
What is the coefficient of coupling of two coils whose mutual inductance is 1 H and whose self-inductance are 1.2 and 2 H?

A. 1.549
B. 2.400

C. 0.645
D. 0.912

Problem 183:

A coil of inductance 8 mH, produces 80 μWb of magnetic flux. Of this total flux, 60 μWb are linked to a second coil of inductance 2 mH, how much is the mutual inductance between coils?

A. 3 mH
B. 4 mH
C. 5 mH
D. 2 mH

Problem 184: EE Board April 1992

The total inductance of two coils, A and B when connected in series aiding is 0.5 H and 0.2 H when connected in series opposing. Coil A when isolated from coil B has a self-inductance of 0.2 H. Calculate the self-inductance of coil B.

A. 0.16 H
B. 0.12 H
C. 0.20 H
D. 0.15 H

Problem 185:

Two identical coils with self-inductance of 250 μH each are connected in series. When connected series aiding, the equivalent inductance is 550 μH and 450 μH when connected series opposing. How much is the coupling coefficient of two coils?

A. 0.2
B. 0.3
C. 0.1
D. 0.4

Problem 186:

A cylinder whose radius is 2 cm and altitude of 100 cm. is uniformly wound with 2500 turns of wires in a single layer. A second layer of 100 turns and of much finer wires is wound over the first near its center. Calculate the mutual inductance of the two coils.

A. 0.40 mH
B. 0.38 mH
C. 0.52 mH
D. 0.63 mH

Problem 187:

Two coils having 200 and 300 turns respectively are wound side by side on a common closed core whose cross section is 100 cm^2 and mean length of 200 cm. What will be the voltage induced in the second coil if the current in the first coil changes from 0 to 5 A in 0.05 second. Assume relative permittivity of the core to be 1500.

A. 52.3 V
B. 67.8 V
C. 62.6 V
D. 58.6 V

Problem 188: EE Board October 1996

The energy (Wo) stored in a coil is dependent in the inductance (L) of the coil and the current flowing. If the inductance were doubled with the same current flowing, what would be the resulting stored energy?

A. 2.0 Wo
B. 0.5 Wo
C. 1.0 Wo
D. 4.0 Wo

Problem 189: EE Board October 1990

A 6.0 H coil whose resistance is 12 ohms is connected in series with a 24 ohms resistor and to a 144 V battery and a switch. The switch is closed at t = 0. Determine the energy stored in the magnetic field at steady state.

A. 50 J
B. 48 J
C. 52 J
D. 46 J

Problem 190:

Two coils having self-inductance and mutual inductance of 3H, 2H and 1H respectively are connected in series. If a current of 4 A is passed through the coil, evaluate the ratio of the energy stored in the magnetic field when the fluxes (self and mutual) are in the same direction to the energy stored when the said fluxes are against each other.

A. 3.24
B. 2.33
C. 1.73
D. 2.67

Problem 191: EE Board April 1995

Two coils of inductances 4 and 6 henry are connected in parallel. If their mutual inductance is 3 henry, calculate the equivalent inductance of the combination if mutual inductance assist and if mutual inductance opposes the self-inductance respectively.

A. 2.4 H & 1.4 H
B. 3.75 H & 0.94 H
C. 6.25 H & 2.75 H
D. 16 H & 4 H

Problem 192:

A 50-mH inductor is charged to store 1 J of energy. If an uncharged 30-mH inductor is then connected across its terminals, how much energy is transferred to the second inductor?

A. 0.154 J
B. 0.262 J
C. 0.234 J
D. 0.141 J

Problem 193:

A solenoid 500 cm long and 10 cm in diameter is wound with 1500 turns. Find the energy stored in the magnetic field when a current of 5 A flows.

A. 0.45 J
B. 0.55 J
C. 0.46 J
D. 0.72 J

Problem 194:

A coil of 250 turns is wound on a closed iron ring whose cross sectional area is 20 cm^2 and mean length of 100 cm. Determine the energy stored in the magnetic field if a current of 2 A is passed through it. Assume the permeability of iron is 900.

A. 2.83 J
B. 3.12 J
C. 2.03 J
D. 1.88 J

ANSWER KEY

169.	B	175.	A	181.	A	187.	B	193.	B
170.	D	176.	A	182.	C	188.	A	194.	A
171.	B	177.	B	183.	A	189.	B		
172.	C	178.	B	184.	D	190.	B		
173.	A	179.	A	185.	C	191.	B		
174.	B	180.	D	186.	A	192.	C		

☑ RATING

23-26 - Topnotcher

18-22 - Passer

13-17 - Conditional

0-12 - Failed

Did you know that... the first battery was invented in 1796 and was considered the greatest of all electrical discoveries? It revolutionized the use of electricity and gave the world one of its greatest benefits, the electric current.

Alessandro Volta, an Italian professor of Physics at the University of Padua made the greatest electrical discoveries of all time. In 1796, Volta produced a continuous electric current for the first time on earth, through his voltaic pile (battery). It was the first source of direct current.

Volta was born in the Italian City of Como, then a part of the Austrian Empire and at the age of 18 he was performing electrical experiments with well-known European electrical investigators. In 1872, he became professor of Physics at the University of Padua, where he became involved in controversy with another electrical pioneer, an Italian physicist Luigi Galvani whose experiments led him to believe that electricity was in the muscles or nerves of animals. Volta on the other hand, maintained that current electricity was metallic electricity.

Volta made a stack of zinc disk and a copper disk, with paper disk soaked in a salt acid solution in between them. He built up a high stack alternating the zinc, paper and the copper disks. The pile was self-charging. The discharges could be repeated again & again. Volta believed that the contact between unlike metals, via the conducting water produced electricity.

Volta was showered with honors during his lifetime. Napoleon made him a senator and later a count in the French Empire. After Napoleon's defeat, the Austrians allowed Volta to return to his Italian estate as a citizen in good stead. Volta was rewarded 54 years after his death, when the unit of electromotive force was officially named "volt".

Solutions to Test 6

169 $e = N\left(\frac{\Phi}{t}\right) x\, 10^{-8} = \frac{(2000)(100{,}000)}{0.01} x\, 10^{-8}$

$e = 200\ V$

170 $e = N\left(\frac{\Phi}{t}\right) x\, 10^{-8} = \frac{(1000)(800)}{0.15} x\, 10^{-8}$

$e = 0.0533\ V$

171

$$e = N\left(\frac{\Phi}{t}\right) = \frac{N\beta A}{t} = \frac{(50)(1.5)\left[1000\ cm^2 \times \left(\frac{1\,m}{100\,cm}\right)^2\right]}{0.5}$$

$e = 15\ V$

172 $e = \beta L v = (0.90)(0.05)(2.5)$

$e = 0.1125\ V$

173 $e = \beta L v = (5 \times 10^{-5})(20)\left(1000\,\frac{km}{hr} \times \frac{1000\,m}{1\,km} \times \frac{1\,hr}{3600\,sec}\right)$

$e = 0.28\ V$

174 $e = \beta L v \times 10^{-8} = (10{,}000)(10)(120) \times 10^{-8}$

$e = 0.12\ V$

175 $e = \beta L v \sin\alpha = (2)(0.5)(1)\sin 30°$

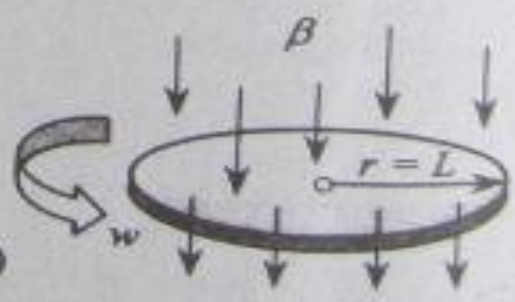
$e = 0.5\ V$

176 Note: The disk maybe viewed as made up of a number of metal rods all connected in parallel. The induced emf is the same in each of them.

$e = \beta L v_{ave}$

$v = r\omega = 0.5\left(1200\,\frac{rev}{min} \times \frac{2\pi\,rad}{1\,rev} \times \frac{1\,min}{60\,sec}\right)$

$v = 62.83\ m/s$



where: v = peripheral velocity of the disk (m/s)
w = angular velocity (rad/s)

$$v_{ave} = \frac{V_{rim} + V_{center}}{2} = \frac{62.83 + 0}{2} = 31.415 \text{ m/s}$$

$$e = \beta L v_{ave} = 0.5(0.3)(31.415)$$

$$e = 4.7 \text{ V}$$

177 Using Ohm's law :

$$e = IR = (0.015)(4) = 0.06 \text{ V}$$

$$v_{ave} = \frac{e}{\beta L} = \frac{0.06}{0.3(0.10)} = 2 \text{ m/s}$$

$$v_{ave} = \frac{v_{rim} + v_{center}}{2} = \frac{V_{rim} + 0}{2}$$

$$v_{rim} = 2\, v_{ave} = 2(2) = 4 \text{ m/s}$$

$$v_{rim} = r\omega$$

$$\varpi = \frac{v_{rim}}{r} = \frac{4}{0.10} = 40 \frac{\text{rad}}{\text{sec}} \times \frac{1 \text{rev}}{2\pi \text{ rad}} \times \frac{60 \text{ sec}}{1 \text{min}} = 381.97$$

$$\varpi \cong 382 \text{ rpm}$$

178 $$e = L\frac{di}{dt}$$

$$L = \frac{e}{di/dt} = \frac{500}{0.005/2 \times 10^{-6}}$$

$$L = 0.2 \text{ H}$$

179 $$L = \frac{N\Phi}{I}$$

Using Ohm's law :

$$I = \frac{E}{R} = \frac{240}{50} = 4.8 \text{ A}$$

$$L = \frac{(900)(4 \times 10^{-3})}{4.8}$$

$$L = 0.75 \text{ H}$$

180

$$L = \frac{\mu_0 \mu_r N^2 A}{\ell} = \frac{(4\pi \times 10^{-7})(1)(1000)^2 \left[4 \text{ cm}^2 \times \left(\frac{1 \text{m}}{100 \text{ cm}}\right)^2\right]}{\pi(0.2)}$$

$$L = 0.8 \text{ mH}$$

181

$$L = \frac{\mu_0 \mu_r N^2 A}{\ell} = \frac{(4\pi \times 10^{-7})(2500)(300)^2 \left[5\ \text{cm}^2 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2\right]}{0.4}$$

$L = 0.35\ \text{H}$

182 $M = k\sqrt{L_1 L_2}$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{1}{\sqrt{(1.2)(2)}}$$

$k = 0.645$

183 $$k = \frac{\text{flux linking } L_1 \text{ and } L_2}{\text{flux produced by } L_1} = \frac{60\ \mu\text{Wb}}{80\ \mu\text{Wb}} = 0.75$$

$M = k\sqrt{L_1 L_2} = 0.75\sqrt{(8)(2)}$

$M = 3\ \text{H}$

184 $$M = \frac{L_{ta} - L_{to}}{4} = \frac{0.5 - 0.2}{4} = 0.075\ \text{H}$$

$L_{ta} = L_1 + L_2 + 2M$

$L_2 = L_{ta} - 2M - L_1$

$L_2 = 0.5 - 2(0.075) - 0.2$

$L_2 = 0.15\ \text{H}$

185 $$M = \frac{L_{ta} - L_{to}}{4} = \frac{550 - 450}{4} = 25\ \mu\text{H}$$

$M = k\sqrt{L_1 L_2}$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{25}{\sqrt{(250)(250)}}$$

$k = 0.1$

186 Note: Since the second coil is wound closed to the first coil, assume that the flux produced by the first coil is completely linked to the other coil, thus making the coefficient of coupling between the two coils equal to unity ($k = 1$).

$$M = \frac{\mu_0 \mu_r A N_1 N_2}{\ell}$$

$A = \pi r^2 = \pi (0.02)^2 = 0.00125\ \text{m}^2$

$$M = \frac{(4\pi \times 10^{-7})(1)(0.00125)(2500)(100)}{1} = 0.392\ \text{mH}$$

$$M \cong 0.4\ \text{mH}$$

187 Note: Since the two coils are linked on a common close core, then it will be assumed that the total flux produced by one coil is completely linked to the other coil, thus k = 1.

$$M = k\frac{\mu_o \mu_r A N_1 N_2}{\ell}$$

$$M = \frac{(1)(4\pi \times 10^{-7})(1500)\left[100\ \text{cm}^2 \times \left(\frac{1\text{m}}{100\ \text{cm}}\right)^2\right](200)(300)}{2}$$

$$M = 0.565\ \text{H}$$

$$e_2 = M\left(\frac{di_1}{dt}\right) = \frac{(0.565)(6)}{0.05}$$

$$e_2 = 67.8\ \text{V}$$

188 $W = \frac{1}{2}LI^2$

Note: From the formula, if current (I) is constant, energy therefore is directly proportional to the inductance (L).

$$W = kL$$

$$\frac{W_1}{W_2} = \frac{L_1}{L_2}$$

$$W_2 = W_1\left(\frac{L_2}{L_1}\right) = W_o\left(\frac{2L_1}{L_1}\right)$$

$$W_2 = 2W_o$$

189 $I = \frac{E}{R} = \frac{144}{12+24} = 4\ \text{A}$

$$W = \frac{1}{2}LI^2 = \frac{(6)(4)^2}{2}$$

$$W = 48\ \text{J}$$

190 $L_{te} = L_1 + L_2 + 2M = 3 + 2 + 2(1)$

$$L_{te} = 7\ \text{H}$$

$L_{to} = L_1 + L_2 - 2M = 3 + 2 - 2(1)$

$L_{to} = 3\,H$

$W_{ta} = \frac{1}{2}LI^2 = \frac{1}{2}(7)(4)^2 = 56J$

$W_{to} = \frac{1}{2}LI^2 = \frac{1}{2}(3)(4)^2 = 24\,J$

$\text{Ratio} = \frac{W_{ta}}{W_{to}} = \frac{56}{24} = 2.33$

191 $L_{ta} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{(4)(6) - (3)^2}{4 + 6 - 2(3)}$

$L_{ta} = 3.75\,H$

$L_{to} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{(4)(6) - (3)^2}{4 + 6 + 2(3)} = 0.9375$

$L_{to} \cong 0.94\,H$

192 Note: Assume flux linkage (λ) remains constant.

$W = \frac{1}{2}LI^2 \qquad L = \frac{\lambda}{I}$

$W = \frac{\lambda^2}{2L}$

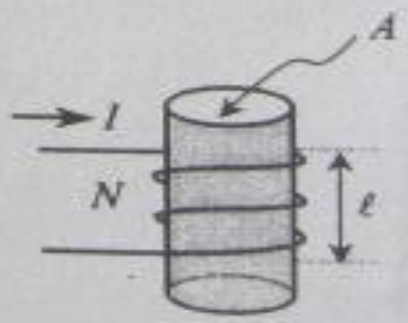
$\lambda = \sqrt{2WL} = \sqrt{2(1)(0.05)} = 0.316\ Wb$

$I = \frac{\lambda}{L_1 + L_2} = \frac{0.316}{0.05 + 0.03} = 3.95\,A$

$W = \frac{1}{2}LI^2 = \frac{(0.03)(3.95)^2}{2}$

$W = 0.234\,J$

193 $L = \mu_o\mu_r N^2 \frac{A}{\ell}$



$A = \frac{\pi d^2}{4} = \frac{\pi(0.1)^2}{4} = 0.00785\,m^2$

$L = \frac{(4\pi \times 10^{-7})(1)(1500)^2(0.00785)}{0.5}$

$L = 0.044\,H$

$$W = \frac{1}{2}LI^2 = \frac{(0.044)(5)^2}{2}$$

$$W = 0.55\ J$$

194 $L = \mu_o \mu_r N^2 \frac{A}{\ell}$

$$A = 20\ cm^2 \times \left(\frac{1m}{100\ cm}\right)^2 = 0.002\ m^2$$

$$L = \frac{(4\pi \times 10^{-7})(900)(250)^2(0.002)}{0.1} = 1.4137\ H$$

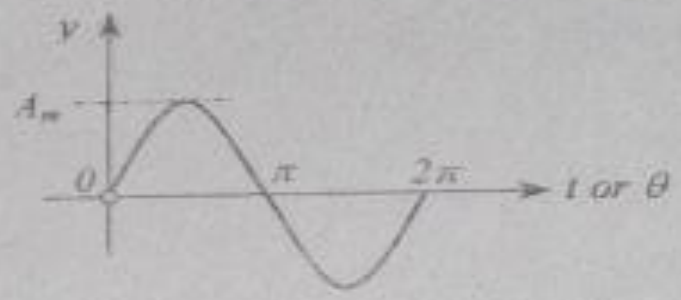
$$W = \frac{1}{2}LI^2 = \frac{(1.4137)(2)^2}{2}$$

$$W = 2.83\ J$$

1-phase AC Circuits: Part 1

THE SINUSOIDAL WAVE

Note: Sinusoidal wave is the most popular alternating wave.



$y(\theta) = A_m \sin\theta$ $\quad$ $y(t) = A_m \sin wt$ $\quad$ $w = 2\pi f$

where: A_m = amplitude or maximum value
w = angular velocity (radians per second)
t = time (second)
θ = angle of rotation (degrees)

- **Frequency** (f) - number of cycles produced per second or **Hertz** (Hz). Named after the German physicist, **Heinrich Rudolf Hertz** (1886 – 1888)
- **Period** (T) - time needed in seconds to produced one cycle
- **Wavelength** (λ) - length of one complete cycle

$$T = \frac{1}{f} \qquad \lambda = \frac{V_c}{f}$$

where: V_C = velocity of propagation (meter per second)
= **3 x 10⁸ m/sec** (for radio waves)
= **344 m/sec** (for sound waves)
f = frequency (Hertz)

- **Alternation** – equal to one-half of a cycle.
- **Instantaneous value** - magnitude of the wave at any instant of time or angle of rotation.

- **RMS (root-mean square) or effective value of a sinusoidal wave** - the r.m.s value of a sinusoidal wave is that which when applied to a given circuit for a given time, produces the same expenditure of energy when DC is applied to the same circuit for the same interval of time.

$$\text{RMS value} = \frac{A_m}{\sqrt{2}}$$

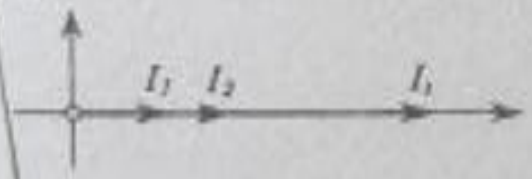
- **Average value of a sinusoidal wave** - the mean of all the instantaneous values of one-half cycle.

$$\text{Average value} = 0.636\ A_m$$

- **Form factor** - ratio of rms value to the average value of an alternating quantity.
- **Peak factor** - ratio of maximum value to the rms value of an alternating quantity.

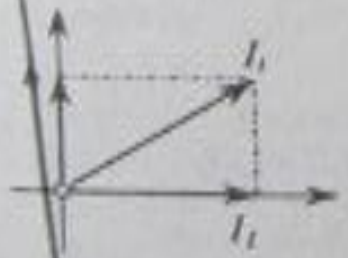
SUM OF TWO ALTERNATING WAVES

If in-phase,



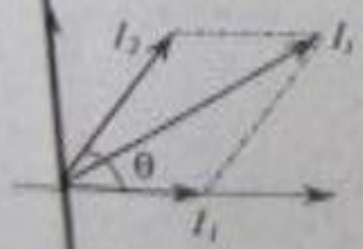
$$I_t = I_1 + I_2$$

If out-of-phase by 90°,



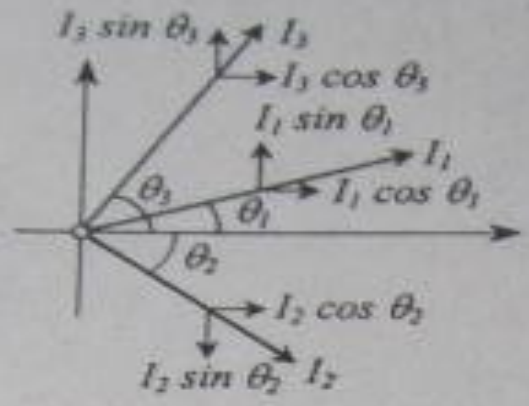
$$I_t = \sqrt{(I_1)^2 + (I_2)^2}$$

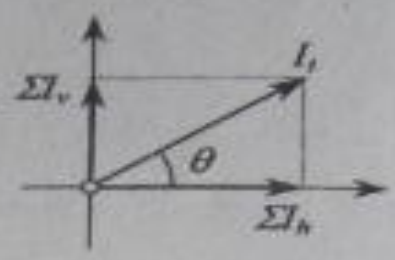
If out-of-phase by an acute angle θ,



$$I_t = \sqrt{(I_1)^2 + (I_2)^2 - 2I_1I_2\cos(180 - \theta)}$$

SUM OF MORE THAN TWO OUT OF PHASE ALTERNATING WAVES





$$\Sigma I_h = I_1 \cos\theta_1 + I_2 \cos\theta_2 + I_3 \cos\theta_3$$

$$I_t = \sqrt{(\Sigma I_h)^2 + (\Sigma I_v)^2}$$

$$\Sigma I_v = I_1 \sin\theta_1 - I_2 \sin\theta_2 + I_3 \sin\theta_3$$

$$\theta = \tan^{-1} \frac{\Sigma I_v}{\Sigma I_h}$$

Note: Standard vector sign conventions:
Vector directed to the right (→), positive sign
Vector directed to the left (←), negative sign
Vector directed upward (↑), positive sign
Vector directed downward (↓), negative sign

REACTANCE & IMPEDANCE

- **Reactance** – property of an inductor or a capacitor to **oppose** current low in a given circuit.

$$X_L = wL = 2\pi fL \qquad X_C = \frac{1}{wC} = \frac{1}{2\pi fC}$$

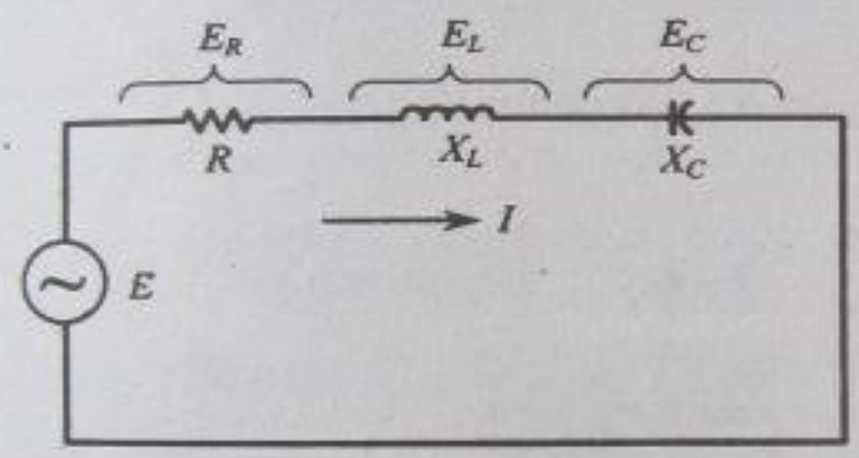
where: X_L = inductive reactance (ohm)
X_C = capacitive reactance (ohm)
C = capacitance (farad)
L = inductance (henry)
w = angular velocity (radians per second)
f = operating frequency (Hertz)

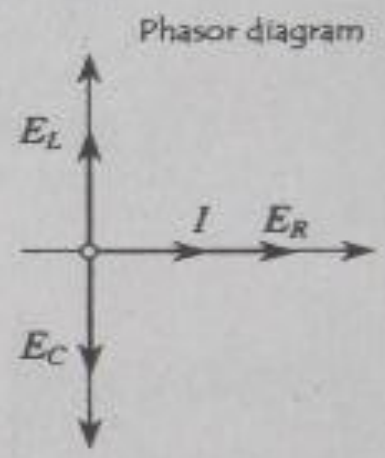
- **Impedance** (Z) – the joint effect of combining resistance and reactance in an AC circuit.

BEHAVIORS OF R, L & C IN AC CIRCUITS

- **Resistor** – takes a current that is **in phase** with the voltage across it.
- **Ideal Inductor** – takes a current that **lags** the voltage across it by **90** electrical degrees.
- **Ideal Capacitor** – takes a current that **leads** the voltage across it by **90** electrical degrees.

SERIES AC CIRCUITS





$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad E = \sqrt{E_R^2 + (E_L - E_C)^2}$$

$$I = \frac{E}{Z} \qquad E_R = IR \qquad E_L = IX_L \qquad E_C = IX_C$$

where: Z = impedance of the circuit (ohm)
I = current drawn (ampere)
E = supply voltage (volt)
E_R = voltage drop across the resistor (volt)
E_L = voltage drop across the inductor (volt)
E_C = voltage drop across the capacitor (volt)
R = resistance of the resistor (ohm)
X_L = inductive reactance (ohm)
X_C = capacitive reactance (ohm)

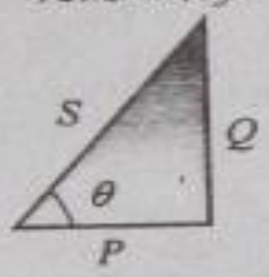
COMPLEX IMPEDANCE EQUIVALENTS OF R, L & C

Circuit elements	Impedance Equivalents
R	R
L	jX_L
C	$-jX_C$
Series RL	$R + jX_L$
Series RC	$R - jX_C$
Series LC	$j(X_L - X_C)$
Series RLC	$R + j(X_L - X_C)$

Note: j = symbol for imaginary component

ACTIVE, REACTIVE & APPARENT POWERS

Power triangle



$P = I^2R$ $\quad$ $P = EI\cos\theta$

$$P = \frac{E^2\cos\theta}{Z}$$

$Q = I^2X$ $\quad$ $Q = EI\sin\theta$

$S = EI$ $\quad$ $S^2 = P^2 + Q^2$

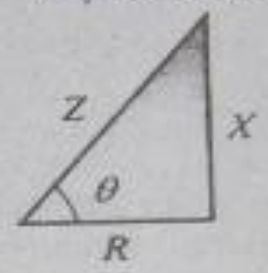
where: P = active or real or true power (watt)
Q = reactive power (var)
S = apparent power (volt-ampere)
θ = power factor angle
E = supply voltage (volt)
I = current drawn (ampere)
R = equivalent resistance of the circuit (ohm)
X = equivalent reactance of the circuit (ohm)
Z = impedance of the circuit (ohm)

POWER FACTOR

It is defined as the ratio of true power and apparent power.

$$pf = \frac{P}{S} = \cos\theta \qquad \cos\theta = \frac{R}{Z}$$

Impedance triangle



where: pf = power factor (unitless)

TYPES OF POWER FACTOR

- **Unity pf** – the voltage and current are in **phase**.
- **Lagging pf** - the current **lags** the voltage by an acute angle θ.
- **Leading pf** - the current **leads** the voltage by an acute angle θ.
- **Zero pf** - if the voltage and current are out of phase by exactly 90°

ACTIVE POWER FORMULA GIVEN EQUATIONS OF VOLTAGE & CURRENT

$e = E_m \sin(wt \pm \alpha)$ volts.
$i = I_m \sin(wt \pm \beta)$ amps

$$P = \frac{E_m I_m}{2}\cos(\alpha - \beta)$$

where: E_m = maximum value of the voltage
I_m = maximum value of current
α = phase angle of the voltage
β = phase angle of the current

Test 7

Problem 195: EE Board October 1996

What is the wavelength of a carrier wave with a frequency of 100 megahertz?

A. 3.0 m
B. 7.5 m
C. 1.5 m
D. 6.0 m

Problem 196: EE Board April 1997

A chart speed of a recording instrument is 25 mm/sec. One cycle of the signal being recorded extends over 5 mm. What is the frequency of the signal?

A. 20 cps
B. 2 cps
C. 50 cps
D. 5 cps

Problem 197: EE Board April 1990

A 240-V, 25 Hz sinusoidal generator is connected to a 20 ohms resistor. Determine the instantaneous current when elapsed time is 0.01 second.

A. 15.43 A
B. 16.30 A
C. 16.97 A
D. 12.00 A

Problem 198: EE Board April 1997

A wire carries a current, $i = 3 \cos 314t$ amperes. What is the average current over 6 seconds?

A. 0 A
B. 1.5 A
C. 3.0 A
D. 0.523 A

Problem 199:

Two current sources deliver current to a common load. The first source delivers a current whose equation is $25 \sin 100\pi t$ amperes while the second delivers a current whose equation is $15 \cos 100\pi t$ amperes. What is the rms value of the current in the load?

A. 29.15 A
B. 40 A
C. 20.6 A
D. 10 A

Problem 200:

Two alternators A and B delivers 100 and 150 A, respectively to a load. If these currents are out of phase by 30 electrical degrees, determine the total current drawn by the load.

A. 201.5 A
B. 250.0 A
C. 215.4 A
D. 241.8 A

Problem 201: EE Board June 1990

Three loads, units A, B and C are connected in parallel and take currents that are respectively 12, 10 and 15 A respectively. Assuming I_a to be the reference phasor. I_b leads I_a by 30° and I_c lags behind I_a by 65°, calculate the total (resultant) current.

A. 28.33 A
B. 30.21 A
C. 26.46 A
D. 32.10 A

Problem 202: EE Board October 1984

An industrial coil has a resistance of 32 ohms and a reactance of 24 ohms and rated 440 volts at 60 Hz. A factory will connect the coil to a 440 V, 50 Hz supply. Solve for the value of a series resistor needed to avoid overcurrent condition.

A. 2.07 Ω
B. 2.64 Ω
C. 2.44 Ω
D. 2.25 Ω

Problem 203: EE Board April 1997

Across a 230 V, 60 Hz powers supply is a 15-ohm non-inductive resistor. What is the equation of the resulting current?

A. 21.68 sin 377t
B. 26.55 sin 377t
C. 15.33 sin 377t
D. 28.16 sin 377t

Problem 204: EE Board October 1998

Two relays each with 20 ohms resistance and 0.16 henry inductance are connected in series. What is the equivalent impedance?

A. 20 + j 102.2 ohms
B. 20 + j 95.32 ohms
C. 40 + j120.63 ohms
D. 40 + j 25.32 ohms

Problem 205: EE Board October 1990

An inductive coil takes a current of 2 A and consumes 160 W when connected to a 240 V AC supply. A second coil when connected across the same supply takes 3 A and 500 W. Find the total power when the two coils are connected in series to this supply.

A. 144.56 W
B. 134.31 W
C. 150.22 W
D. 128.35 W

Problem 206:

A ½ hp, 110-V, 60 Hz, single-phase induction motor has an efficiency of 88% and a power factor of 0.707 lagging at rated load. This motor is to be connected temporarily on a 220 V, 60 Hz line. Determine the resistance required to be placed in series with the motor in order to prevent the machine from experiencing overcurrent?

A. 25.2 ohms
B. 23.5 ohms
C. 19.5 ohms
D. 27.6 ohms

Problem 207:

Two coils A and B known to have the same resistance are connected in series across a 110-V, 60-cycle line. The current and power delivered by the source are respectively 4.1 A and 300 W. If the voltage across coil A is twice that across coil B, calculate the inductance of coil B.

A. 8.63 mH
B. 7.36 mH
C. 9.02 mH
D. 4.49 mH

Problem 208: EE Board October 1985

A coil draws 1875 watts when connected to a 150 V DC source. It consumes 3072 watts when used on 240 V, 60 Hz AC source. Find the inductance of the coil.

A. 0.0255 H
B. 0.0341 H
C. 0.0153 H
D. 0.0240 H

Problem 209: EE Board October 1994

A current of 10 A and a power factor of 0.8 lagging is taken from a single phase 250 volt supply. The reactive power of the system is

A. 1500
B. 2000
C. 2500
D. None of these

Problem 210: EE Board October 1996

A resistor of 6 ohms and an unknown impedance coil in series draws 12 A from a 120-V, 60–Hz line. If the real power taken from the line is 1152 watts, what is the coil inductance?

A. 15.9 mH
B. 10.0 mH
C. 20.0 mH
D. 1.59 mH

Problem 211: EE Board April 1997

Determine the power angle in the series circuit which consists of R = 25 ohms, L = 0.2 H across a power supply of 200 volts, 30 Hz?

A. 36.4°
B. 52.4°
C. 46.4°
D. 56.4°

Problem 212:

When a 30 V DC source is applied to a given coil, the coil draws 150 W. When 230 V AC is applied to the same coil, it draws 3174 W. Determine the operating power factor of the coil.

A. 0.7
B. 0.6
C. 0.9
D. 0.8

Problem 213: EE Board April 1993

The impedance coil absorbs 250 watts when connected across 220 V, 60 Hz mains. It is then connected across 110 V, 25 Hz mains and also absorbs 250 watts. What is the inductance of the coil?

A. 0.125 H
B. 0.149 H
C. 0.154 H
D. 0.163 H

Problem 214: EE Board April 1995

An impedance coil takes 10 A and absorbs 250 W when connected across a 220 V, 60 Hz source. What power will it absorb when connected across 110 V, 25 Hz mains?

A. 539 W
B. 239 W
C. 439 W
D. 339 W

Problem 215: EE Board October 1984

An industrial coil has a resistance of 32 ohms and a reactance of 24 ohms and rated 440 volts at 60 Hz. A factory will connect the coil to a 440 V, 50 Hz supply. How much percentage overcurrent will the coil suffer?

A. 5%
B. 10%
C. 6%
D. 8%

Problem 216:

The open circuit voltage across the output terminals of an AC power supply is 15.2 V. When these terminals are short-circuited, a current of 3.2 A flows. When a 5-ohm resistance is placed across these terminals, the current through it is 1.8 A. Calculate the series internal resistance of the power supply.

A. 2.374 ohms
B. 2.240 ohms
C. 2.125 ohms
D. 2.045 ohms

Problem 217:

A single-phase load takes 55 kW at 70% pf lagging from a 240 V, 50 Hz supply. If the supply is made 60 Hz, with the voltage remaining the same, what will be the kW load at 60 Hz?

A. 42 kW
B. 48 kW
C. 40 kW
D. 45 kW

Problem 218: EE Board March 1998

A 25-ohm resistor connected in series with a coil of 50 ohm resistance and 150 mH inductance. What is the power factor of the circuit?

A. 85%
B. 80%
C. 90%
D. 75%

Problem 219: EE Board October 1998

The ohmic resistance of a large magnetic contractor is measured to be 20 ohms. A 230 V is impressed on the contractor and the current is taken as 3.2 A. Neglecting core loss, determine the inductance of the contactor in mH?

A. 261
B. 315
C. 183
D. 251

Problem 220:

An 8-ohm resistor is connected in series with an inductive coil and the combination is connected across a 120 V, 60 Hz source. A voltmeter connected across the resistor reads 32 V and 104 V when connected across the coil. Determine the resistance of the coil.

A. 8 Ω
B. 9 Ω
C. 12 Ω
D. 10 Ω

Problem 221:

When connected to a 220-V, 50 Hz supply, an impedance coil circuit takes 5 A, but this current falls to 4.4 A when a 10-ohm resistor is added in series. Find the inductance of the coil.

A. 0.20 H
B. 0.15 H
C. 0.23 H
D. 0.12 H

Problem 222:

An impedance coil, in series with a resistor takes a current of 5 A when connected to a 100-V AC source. Using a voltmeter, the voltage across the coil was found out equal to 80 V and that across the resistor equal to 30 V. Find the operating power factor of the coil.

A. 0.56
B. 0.83
C. 0.73
D. 0.62

Problem 223: EE Board March 1998

A load of 20 + j35 ohms is connected across a 220 volts source. Determine the power factor and the VARS.

A. 49.6%, 1042 VARS
B. 85.3%, 975 VARS

C. 52.2%, 1023 VARS
D. 42.3%, 1087 VARS

Problem 224:

A load is rated at 115 V, 60 Hz, 10 A, and 1150 W. A coil is to be wound with a ratio of X_L to R of 5, such that when connected in series with the load on a 230-V, 60 Hz line, the load may operate normally. Solve for R.

A. 2.450 ohms
B. 3.489 ohms
C. 2.215 ohms
D. 3.826 ohms

Problem 225: EE Board October 1990

Find the total impedance in rectangular for the following three series impedances: $12\angle 10°$, $25\angle 15°$, $34\angle 26°$

A. 66.52 + j 23.46
B. 68.34 + j 20.54
C. 74.31 + j 21.56
D. 67.70 + j 22.04

Problem 226: EE Board October 1997

An impedance draws a current i = 10 cos (wt – 30°) from a voltage, v = 220 sin (wt + 30°). What is the impedance?

A. 15.6 – j 15.6
B. 15.6 + j 15.6
C. 19.1 – j 11.1
D. 11.0 + j 19.1

Problem 227: EE Board April 1990

A series resistance-capacitance (R-C) circuit is connected to a 230 volt 60 cycle source. If the power taken by the circuit is 4,800 watts and the voltage drop across the resistor is 115 volts, calculate the capacitance of the capacitor.

A. 540 μF
B. 530 μF
C. 556 μF
D. 503 μF

Problem 228: EE Board March 1998

A 50 and 100-microfarad capacitors are connected in series and across a 100 sin (wt + 30°) voltage. Write the equation of the current.

A. 1.26 sin (wt + 120°)
B. 1.26 sin (wt + 90°)

C. 5.65 sin (wt + 120°)
D. 5.56 sin (wt + 90°)

Problem 229: EE Board October 1996

A series circuit composed of 100-ohm resistor and a 20-microfarad capacitor connected across a 240-V, 60 Hz line. Which of the following answers is WRONG?

A. The impedance of the circuit is 167 ohms
B. Angle between the current and the voltage vectors is 53.1 degrees
C. The resulting current is 0.723 ampere
D. The voltage across the resistance is 144.6 volts

Problem 230: EE Board April 1994

A capacitance is connected to a 115-V, 25-Hz mains and takes 5 A. What current will it take when the capacitance and the frequency are both doubled?

A. 2.5 A
B. 5 A
C. 20 A
D. 10 A

Problem 231:

An ideal inductor of 50 mH is connected in series with a 175-μF capacitor. The combination is then connected across a 120-V variable frequency AC source. At what frequency will this circuit take a leading current of 5 A?

A. 25 Hz
B. 30 Hz
C. 28 Hz
D. 22 Hz

Problem 232:

A series circuit consisting of a variable resistor in series with a capacitance of 80 μF is connected across a 120 V, 50-Hz supply. To what value should R is adjusted so that the power absorbed by the series circuit will be100 W?

A. 20 ohms
B. 18 ohms
C. 15 ohms
D. 12 ohms

Problem 233: EE Board October 1996

A capacitor is rated 100 kVAR, 380 volts, 50 Hz. What will its rating be at 60 Hz, 220 volts?

A. 50 kVAR
B. 40 kVAR
C. 90.9 kVAR
D. 57.7 kVAR

Problem 234: EE Board October 1992

A resistor and a capacitor are connected in series across a supply of 250 V. When the supply frequency is 50 Hz the current in the circuit is 5 A. When the supply frequency is 60 Hz, the current is 5.8 A. Find the value of the capacitance.

A. 58.3 μF
B. 60.2 μF
C. 69.1 μF
D. 70.2 μF

Problem 235: EE Board October 1993

A series circuit composed of a 0.2 henry inductor and a 74-microfarad capacitor is connected to a 60 V variable frequency source. At what frequency is the current be 4 amperes with a lagging power factor?

A. 50 Hz
B. 51 Hz
C. 48 Hz
D. 49 Hz

Problem 236:

A single phase lagging power factor load takes 300 W and 5 A at 120 V. Find the reactance of a pure capacitor that maybe placed in series with this load so that it will operate normally from a 240-V source.

A. 62.32 ohms
B. 67.25 ohms
C. 68.15 ohms
D. 64.48 ohms

Problem 237:

A coil (RL) is connected in series with a capacitor across a 220 V, 60 Hz AC supply. The circuit is designed such that the voltage across the coil and the capacitor are numerically equal. If this circuit operates at 0.87 leading pf, determine the magnitude of this voltage.

A. 212 V
B. 228 V
C. 205 V
D. 223 V

Problem 238:

A series circuit connected across a 100-V AC source consists of a 10-Ω resistor, a 5-Ω capacitor, a resistance R that takes 50 W and a reactance X taking 100 inductive VARS. Determine the minimum value of current required satisfying the above conditions.

A. 1.20 A

B. 0.92 A
C. 1.04 A
D. 1.13 A

Problem 239: EE Board October 1998

The maximum instantaneous voltage and current outputs of an alternator are 300 V and 20 A, respectively. What is the power output in watts if the voltage leads the current by 30°?

A. 2598
B. 6000
C. 5196
D. 3000

Problem 240: EE Board October 1998

A 50-microfarad is connected in series with a coil having 50 ohms resistance and 150-mH inductance. The source voltage is 100 sin (wt - 30°) V. What is the maximum power?

A. 199 watts
B. 147 watts
C. 212 watts
D. 165 watts

Problem 241: EE Board October 1997

An impedance draws a current i = 10 cos (wt – 30°) from a voltage v = 220 sin wt. What is the maximum power?

A. 2200 watts
B. 1100 watts
C. 190.5 watts
D. 1320 watts

Problem 242: EE Board April 1995

An incandescent lamp load generally considered to be made up of resistors take 4.8 kW from a 120-V AC source. The instantaneous maximum value of power is

A. 4800 W
B. 2400 W
C. 480 W
D. 9600 W

ANSWER KEY

195. A	206. B	217. D	228. A	239. A
196. D	207. A	218. B	229. C	240. A
197. C	208. D	219. C	230. C	241. B
198. A	209. A	220. D	231. C	242. D
199. C	210. A	221. D	232. D	
200. D	211. D	222. A	233. B	
201. A	212. B	223. A	234. C	
202. B	213. B	224. B	235. C	
203 A	214. D	225. A	236. B	
204. C	215. C	226. C	237. D	
205. A	216. A	227. C	238. D	

☑ RATING

43-48 - Topnotcher

33-42 - Passer

24-32 - Conditional

0-25 - Failed

Did you know that... when Ohm's law was discovered, it was not believed due to the fact that it was so simple? Due to criticism, the man who developed it was even forced to resign from his professorship and live in obscurity until he was recognized 14 years after his discovery.

The most basic and most widely used of all the laws of electricity, Ohm's law was published in 1827 by the German physicist Georg Simon Ohm.

Ohm was born in Erlangen, Bavaria, and the oldest of seven children in a middle class to poor family. He was an early dropout at the University of Erlangen but returned in 1811 and earned his doctorate and the first of several modest, low paying mathematics-teaching positions. To improve his lot, he threw himself into his electrical research at every opportunity allowed by his heavy teaching duties, and his efforts culminated in his famous law.

Although he discovered one of the most fundamental laws of current electricity, he was virtually ignored for most of his life by scientists in his own country. The law was so simple that it was not believed. He was forced to resign from his professorship and live in obscurity until he was recognized 14 years after his discovery.

Despite the misplaced criticisms of his work during his lifetime, Ohm received the fame that was due him. The Royal Society of London awarded him the Copley Medal in 1841, and the University of Munich gave him its Professor of Physics chair in 1849.

Solutions to Test 7

195 $\lambda = \frac{V_c}{f} = \frac{3 \times 10^8}{100 \times 10^6}$

$\lambda = 3\,m$

196 $f = \frac{V}{\lambda} = \frac{25\,mm/s}{5\,mm} = 5\,Hz$

$f = 5\,cps$

197 $V_{rms} = \frac{V_m}{\sqrt{2}}$

$I_m = \frac{V_m}{R} = \frac{\sqrt{2}V_{rms}}{R} = \frac{\sqrt{2}(240)}{(20)} = 16.97\,A$

$i = I_m \sin wt = I_m \sin(2\pi f)t = 16.97 \sin\left[(2\pi)(25)(0.01)\left(\frac{180°}{\pi}\right)\right]$

$i = 16.97\,A$

198 $I_{ave} = \frac{1}{T}\int_0^T i(t)\,dt$

$I_{ave} = \frac{1}{6}\int_0^6 3\cos 314t\,dt = \frac{1}{2(314)}\int_0^6 \cos 314t\,(314)dt$

$I_{ave} = \left.\frac{\sin 314t}{628}\right|_0^6 = \frac{\sin\left[(314)(6)\left(\frac{180°}{\pi}\right)\right] - \sin 0°}{626} = -0.0013$

$I_{ave} \cong 0\,A$

199 $i_1 = 25 \sin 100\pi t$

$i_2 = 15 \cos 100\pi t = 15 \sin(100\pi t + 90°)$

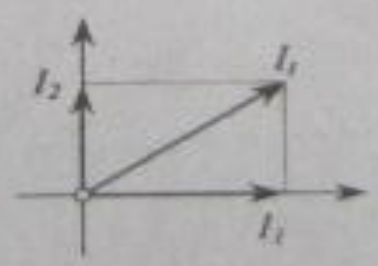
Note: These are vector currents out of phase with each other by 90°

$I_t = \sqrt{(I_1)^2 + (I_2)^2}$

$I_t = \sqrt{(25)^2 + (15)^2} = 29.15\,A$

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{29.15}{\sqrt{2}}$

$I_{rms} = 20.612\,A$



200 $I_t = \sqrt{I_1^2 + I_2^2 - 2I_1I_2\cos(180-\theta)}$

$I_t = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos(180-30)}$

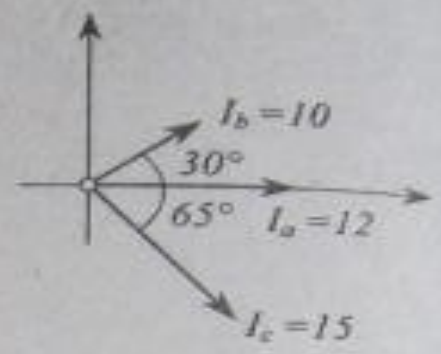
$I_t = 241.827\ A$

201 $\sum I_h = 12 + 10\cos 30° + 15\cos 65° = 27$

$\sum I_v = 10\sin 30° - 15\sin 65° = -8.59$

$I_t = \sqrt{(\sum I_h)^2 + (\sum I_v)^2} = \sqrt{(27)^2 + (-8.59)^2}$

$I_t = 28.33\ A$

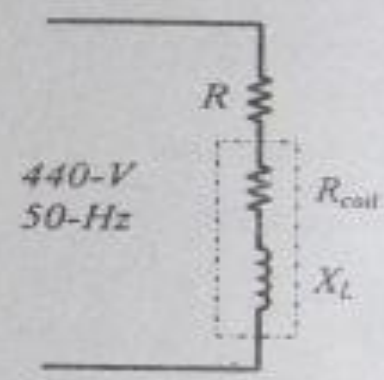


202 At 60 Hz:

$Z = \sqrt{R_{coil}^2 + X_L^2} = \sqrt{(32)^2 + (24)^2} = 40\ \Omega$

$I = \frac{E}{Z} = \frac{440}{40} = 11\ A$

$L = \frac{X_L}{2\pi f} = \frac{24}{2\pi(60)} = 0.06366\ H$



At 50 Hz:

$X_L = 2\pi fL = 2\pi(50)(0.06366)$

$X_L = 20\ \Omega$

Note: To prevent overcurrent, total current should be equal to 11 A, thus Z = 40.

$Z^2 = (R_{coil} + R)^2 + X_L^2$

$40^2 = (32 + R)^2 + (20)^2$

$R = 2.64\ \Omega$

203 $E_m = \sqrt{2}(E_{rms}) = \sqrt{2}(230) = 325.269\ V$

$I_m = \frac{E_m}{R} = \frac{352.269}{15} = 21.68\ A$

$w = 2\pi f = 2\pi(60) = 377$

Assume: $e = E_m \sin 377t$

$i = I_m \sin wt$

$i = 21.68 \sin 377t$

Note: For purely resistive load, the current is in phase with the voltage

204

$R_t = 2(20) = 40\,\Omega$

$L_t = L_1 + L_2 = 2(0.16) = 0.32\,H$

$X_{Lt} = 2\pi f L_t = 2\pi(60)(0.32) = 120.636\,\Omega$

$Z = R_t + jX_{Lt}$

$Z = 40 + j120.636\,\Omega$

Note: Since not given, assume a standard electrical frequency of 60 Hz.

205

$$R_a = \frac{P_a}{I_a^2} = \frac{160}{2^2} = 40\,\Omega$$

$$Z_a = \frac{E}{I_a} = \frac{240}{2} = 120\,\Omega$$

$$X_a = \sqrt{Z_a^2 - R_a} = \sqrt{120^2 - 40^2} = 113.137\,\Omega$$

$$R_b = \frac{P_b}{I_b^2} = \frac{500}{3^2} = 55.555\,\Omega$$

$$Z_b = \frac{E}{I_b} = \frac{240}{3} = 80\,\Omega$$

$$X_b = \sqrt{Z_b^2 - R_b} = \sqrt{80^2 - 55.555^2} = 57.564\,\Omega$$

$$Z_t = \sqrt{(R_a + R_b)^2 + (X_a + X_b)^2}$$

$$Z_t = \sqrt{(40 + 55.555)^2 + (113.137 + 57.564)^2}$$

$$Z_t = 195.626\,\Omega$$

$$I_t = \frac{E}{Z_t} = \frac{240}{195.626} = 1.23\,A$$

$$P_t = I_t^2 R_t = (1.23)^2(40 + 55.555)$$

$$P_t = 144.56\,W$$

206

$$P_{in} = \frac{P_{out}}{\eta} = \frac{0.5(746)}{0.88} = 423.86\,W$$

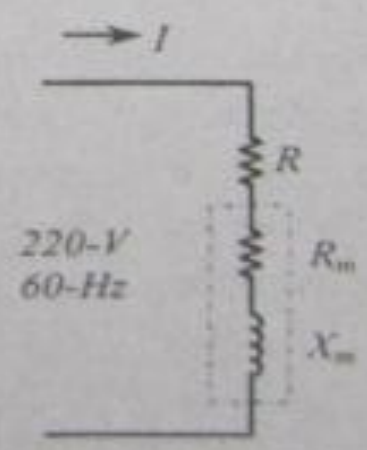
$$I = \frac{P_{in}}{Epf} = \frac{423.86}{(110)(0.707)} = 5.45\,A$$

$$\theta = \cos^{-1} 0.707 = 45°$$

$$Z = \frac{E}{I} = \frac{110}{5.45} = 20.183\,\Omega$$

$$R_m = Z\cos\theta = 20.183(0.707) = 14.27\,\Omega$$

$$X_m = Z\sin\theta = 20.183(\sin 45°) = 14.27\,\Omega$$

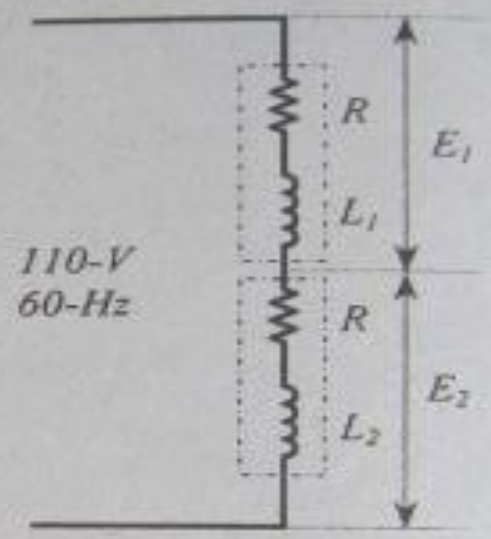


Note: In order for the motor not to experience overcurrent when the supply voltage is 220 V, its current must not exceed its rating which is equal to 5.45 A.

$$Z = \frac{E}{I} = \frac{220}{5.45} = 40.367\ \Omega$$
$$Z^2 = (R_m + R)^2 + (X_m)^2$$
$$(40.367)^2 = (14.27 + R)^2 + (14.27)^2$$
$$R = 23.5\ \Omega$$

207

$$R_t = \frac{P}{I^2} = \frac{300}{(4.1)^2} = 17.846\ \Omega$$
$$R = \frac{R_t}{2} = \frac{17.846}{2} = 8.923\ \Omega$$



$$E_1 = 2E_2$$
$$IZ_1 = 2(IZ_2)$$
$$\sqrt{R^2 + (wL_1)^2} = 2\sqrt{R^2 + (wL_2)^2}$$

Squaring both sides:

$$(8.923)^2 + (377)^2 L_1^2 = 4(8.923)^2 + 4(377)^2 L_2^2$$
$$L_1^2 = 0.00168 + 4L_2^2 \Rightarrow ❶$$

$$Z = \frac{E}{I} = \frac{110}{4.1} = 26.83\ \Omega$$
$$Z^2 = R_t^2 + X_{Lt}^2$$
$$(26.83)^2 = (17.846)^2 + [377(L_1 + L_2)]^2$$
$$0.05314 = L_1 + L_2 \Rightarrow ❷$$

Substitute Eq.2 to Eq.1:

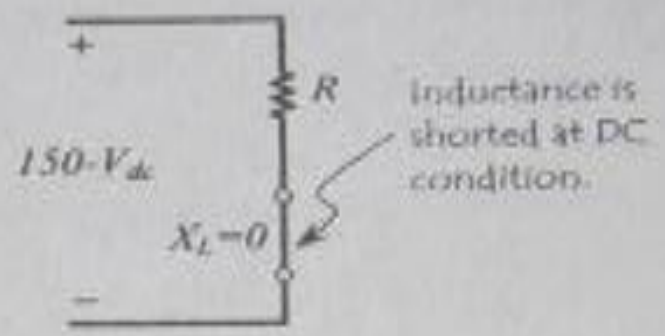
$$(0.05314 - L_2)^2 = 0.00168 + 4L_2^2$$
$$0.00282 - 0.10628L_2 + L_2^2 = 0.00168 + 4L_2^2$$
$$L_2^2 + 0.0354L_2 - 0.00038 = 0$$

Using the quadratic formula:

$$L_2 = \frac{-0.0354 \pm \sqrt{(0.0354)^2 - 4(1)(-0.00038)}}{2(1)} = \frac{-0.0354 \pm 0.05266}{2}$$
$$L_2^{(+)} = \frac{-0.0354 + 0.05266}{2} = 0.00863\ H = 8.63\ mH$$

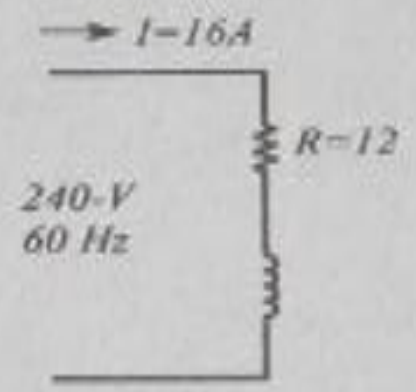
208 When supplied with a DC source :

$$R = \frac{E_{dc}^2}{P} = \frac{150^2}{1875} = 12\,\Omega$$



When supplied with an AC source :

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{3072}{12}} = 16\text{ A}$$



$$Z = \frac{E}{I} = \frac{240}{16} = 15\,\Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{15^2 - 12^2} = 9\,\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{9}{2\pi(60)}$$

$$L = 0.024\text{ H}$$

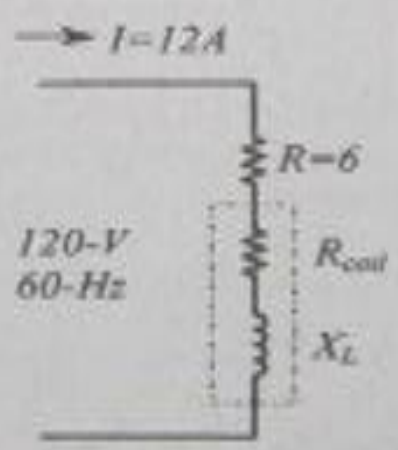
209 $\theta = \cos^{-1} 0.8 = 36.869°$

$Q = EI\sin\theta$

$Q = (250)(10)\sin 36.869°$

$Q = 1500$ VARS

210 $P = I^2(R + R_{coil})$



$$R_{coil} = \frac{P}{I^2} - R = \frac{1152}{(12)^2} - 6 = 2\,\Omega$$

$$Z = \frac{E}{I} = \frac{120}{12} = 10\,\Omega$$

$$Z^2 = (R + R_{coil})^2 + (X_L)^2$$

$$X_L = \sqrt{Z^2 - (R + R_{coil})^2}$$

$$X_L = \sqrt{(10)^2 - (6+2)^2} = 6\,\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{6}{2\pi(60)} = 0.0159\text{ H}$$

$$L = 15.9\text{ mH}$$

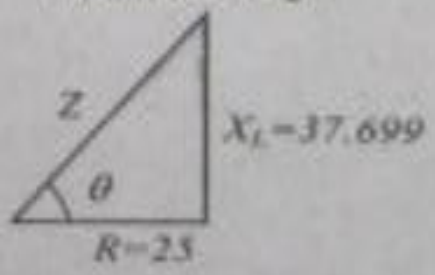
211 $X_L = 2\pi f L = 2\pi(30)(0.2) = 37.699\,\Omega$

$$\tan\theta = \frac{X_L}{R}$$

$$\theta = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{37.699}{25}$$

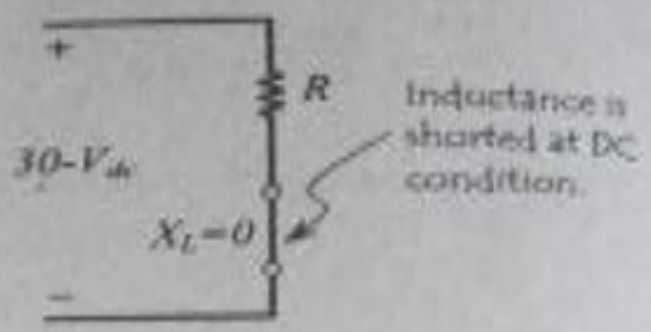
$$\theta = 56.4°$$

Impedance triangle



212 When supplied with a DC source :

$$R = \frac{E_{dc}^2}{P} = \frac{30^2}{150} = 6\,\Omega$$



When supplied with an AC source :

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{3174}{6}} = 23\text{ A}$$

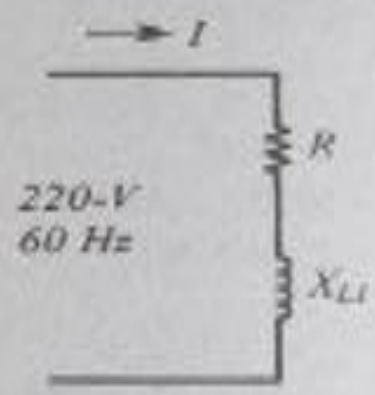
$$Z = \frac{E}{I} = \frac{230}{23} = 10\,\Omega$$

$$pf = \frac{R}{Z} = \frac{6}{10}$$

$$pf = 0.6$$

213 At f = 60 Hz, w = 377 :

$$I = \frac{E}{Z} = \frac{220}{\sqrt{R^2 + (377L)^2}} \Rightarrow ❶$$



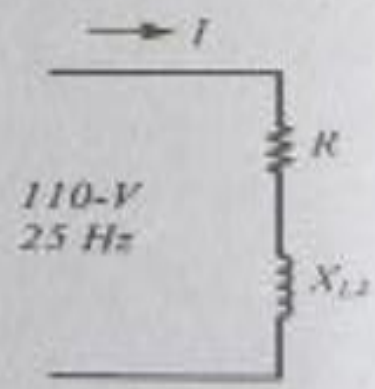
At f = 25 Hz, w = 157 :

$$I = \frac{E}{Z} = \frac{110}{\sqrt{R^2 + (154L)^2}} \Rightarrow ❷$$

Note: Since R is constant and power drawn in both conditions are equal then currents drawn respectively are also equal.

Equate Eq. 1 to Eq. 2 :

$$\frac{220}{\sqrt{R^2 + (377L)^2}} = \frac{110}{\sqrt{R^2 + (154L)^2}}$$



$$\left(\frac{220}{110}\right)^2 = \frac{R^2 + (377L)^2}{R^2 + (157L)^2}$$

$$3R^2 = 43533L^2$$

$$R = 120.461L \Rightarrow ❸$$

$$P = I^2R$$

$$250 = \left(\frac{220^2}{R^2 + 377^2L^2}\right)R \Rightarrow ❹$$

Substitute Eq. 3 in Eq. 4 :

$$250 = \frac{220^2(120.46\,L)}{(120.46L)^2 + (377)^2L^2}$$

$$250(120.46)^2L^2 + 250(377)^2L^2 = (220)^2(120.46)L$$

$$L = \frac{220^2(120.46)}{250(120.46^2 + 377^2)}$$

$$L = 0.149\,H$$

214 At 60 Hz :

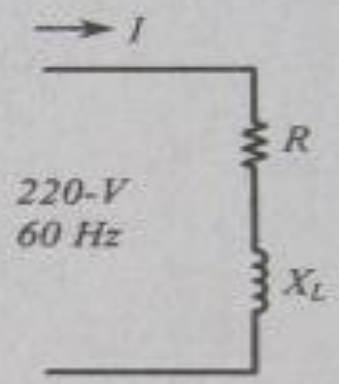
$$R = \frac{P}{I^2} = \frac{250}{10^2} = 2.5\,\Omega$$

$$Z = \frac{E}{I} = \frac{220}{10} = 22\,\Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(22)^2 - (2.5)^2}$$

$$X_L = 21.857\,\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{21.857}{2\pi(60)} = 0.0579\,H$$



At 25 Hz :

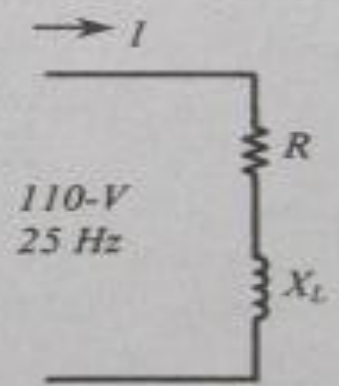
$$X_L = 2\pi fL = 2\pi(25)(0.0579) = 9.095\,\Omega$$

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{2.5^2 + 9.095^2} = 9.43\,\Omega$$

$$I = \frac{E}{Z} = \frac{110}{9.43} = 11.66\,A$$

$$P = I^2R = (11.66)^2(2.5)$$

$$P = 339.889\,W$$



215 At 60 Hz :

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{32^2 + 24^2} = 40\,\Omega$$

$$I = \frac{E}{Z} = \frac{440}{40} = 11\,A$$

$$L = \frac{X_L}{2\pi f} = \frac{24}{2\pi(60)} = 0.06366$$

At 50 Hz :

$$X = 2\pi fL = 2\pi(50)(0.06366) = 20\,\Omega$$

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{32^2 + 20^2} = 37.736\,\Omega$$

$$I = \frac{E}{Z} = \frac{440}{37.736} = 11.66\,A$$

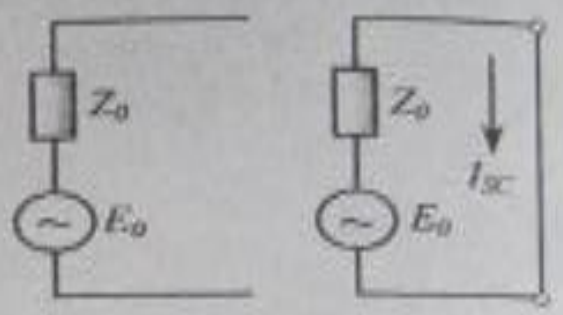
$$\% \text{ Overcurrent} = \frac{I_{60} - I_{60}}{I_{60}} = \frac{11.66 - 11}{11} \times 100\%$$

$$\%\text{Overcurrent} = 6\%$$

216

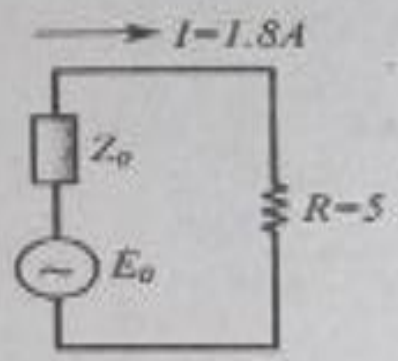
$$Z_0 = \frac{E_0}{I_{SC}} = \frac{15.2}{3.2} = 4.75\,\Omega$$

$$X_0^2 = Z_0^2 - R_0^2 = (4.75)^2 - R_0^2 \Rightarrow ❶$$



$$Z_L = \frac{E_0}{I_L} = \frac{15.2}{1.8} = 8.444\,\Omega$$

$$Z_L^2 = (R_0 + 5)^2 + X_0^2 \Rightarrow ❷$$



Substitute Eq.1 in Eq.2:

$$(8.444)^2 = (R_0 + 5)^2 + (4.75)^2 - R_0^2$$

$$48.739 = R_0^2 + 10R_0 + 25 - R_0^2$$

$$R_0 = \frac{48.739 - 25}{10}$$

$$R_0 = 2.374\,\Omega$$

217

At 50 Hz:

$$Z = \frac{E^2 \cos\theta}{P} = \frac{(240)^2(0.7)}{55000} = 0.733\,\Omega$$

$$\theta = \cos^{-1} 0.7 = 45.57^\circ$$

$$R = Z\cos\theta = 0.733(0.7) = 0.513\,\Omega$$

$$X_L = Z\sin\theta = 0.733(\sin 45.57^\circ) = 0.523$$

$$L = \frac{X_L}{2\pi f} = \frac{0.523}{2\pi(50)} = 1.66\,\text{mH}$$

At 60 Hz:

$$X_L = 2\pi fL = 2\pi(60)(1.666 \times 10^{-3}) = 0.628\,\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(0.513)^2 + (0.628)^2} = 0.81\,\Omega$$

$$\cos\theta = \frac{R}{Z} = \frac{0.513}{0.81} = 0.633$$

$$P = \frac{E^2\cos\theta}{Z} = \frac{(240)^2(0.633)}{0.81} = 45013.33\,\text{W}$$

$$P \approx 45\,\text{kW}$$

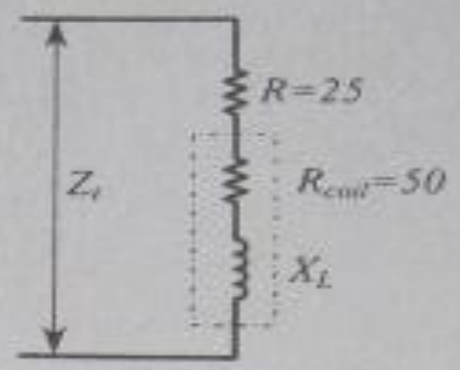
218 $X_L = 2\pi fL = 2\pi(60)(150 \times 10^{-3}) = 56.548\ \Omega$

$R_t = 25 + 50 = 75\ \Omega$

$Z_t = \sqrt{R_t^2 + X_L^2} = \sqrt{75^2 + 56.548^2}$

$Z_t = 93.93\ \Omega$

$pf = \dfrac{R_t}{Z_t} = \dfrac{75}{93.93} = 0.8 = 80\%$



Note: Since not given, assume a standard electrical frequency of 60 Hz

219 $Z = \dfrac{E}{I} = \dfrac{230}{3.2} = 71.875\ \Omega$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{(71.875)^2 - (20)^2} = 69.036\ \Omega$

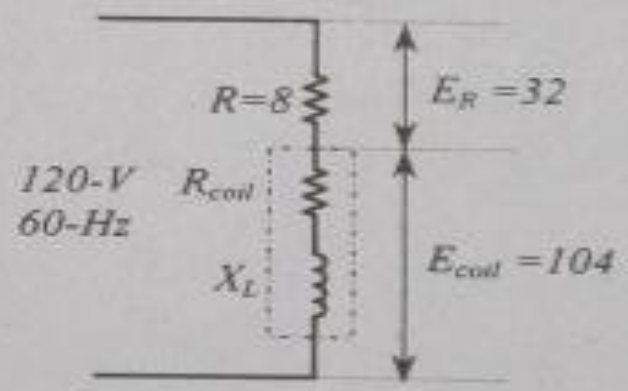
$L = \dfrac{X_L}{w} = \dfrac{69.036}{377} = 183\ mH$

220 $I = \dfrac{E_R}{R} = \dfrac{32}{8} = 4\ A$

$Z_{coil} = \dfrac{E_{coil}}{I} = \dfrac{104}{4} = 26\ \Omega$

$Z_{coil}^2 = R_{coil}^2 + X_L^2$

$26^2 = R_{coil}^2 + X_L^2 \Rightarrow$ ❶



$Z_t = \dfrac{E_t}{I} = \dfrac{120}{4} = 30\ \Omega$

$Z_t^2 = (8 + R_{coil})^2 + X_L^2$

$30^2 = (8 + R_{coil})^2 + X_L^2 \Rightarrow$ ❷

Substitute Eq.1 in Eq.2:

$30^2 = (8 + R_{coil})^2 + 26^2 - R_{coil}^2$

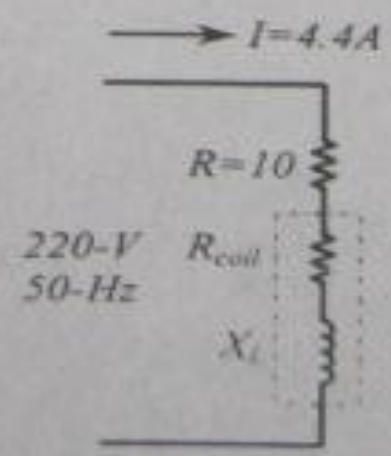
$224 = 64 + 16R_{coil} + R_{coil}^2 - R_{coil}^2$

$R_{coil} = 10\ \Omega$

221 $Z_{coil} = \dfrac{E}{I} = \dfrac{220}{5} = 44\ \Omega$

$Z_{coil}^2 = R_{coil}^2 + X_L^2$

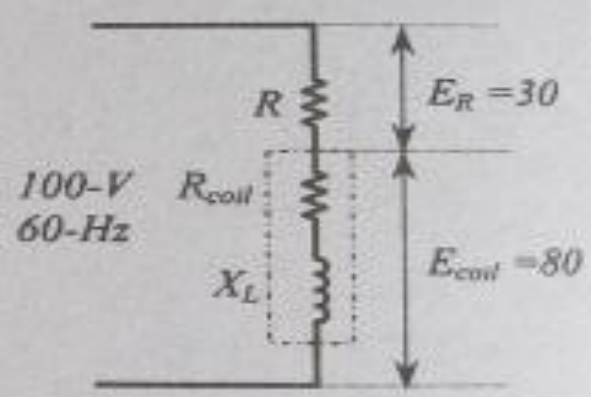
$44^2 = R_{coil}^2 + X_L^2 \Rightarrow$ ❶



$$Z_t = \frac{E}{I} = \frac{220}{4.4} = 50\,\Omega$$
$$Z_t^2 = (10 + R_{coil})^2 + X_L^2$$
$$50^2 = (10 + R_{coil})^2 + X_L^2 \Rightarrow ❷$$
Substitute Eq.1in Eq.2:
$$50^2 = (10 + R_{coil})^2 + 44^2 - R_{coil}^2$$
$$564 = 100 + 20R_{coil} + R_{coil}^2 - R_{coil}^2$$
$$R_{coil} = 23.2\,\Omega$$

$$X_L = \sqrt{44^2 - 23.2^2} = 37.386\,\Omega$$
$$L = \frac{X_L}{2\pi f} = \frac{37.386}{2\pi(50)} = 0.119$$
$$L \cong 0.12\,H$$

222 $$R = \frac{E_R}{I} = \frac{30}{5} = 6\,\Omega$$
$$Z_{coil} = \frac{E_{coil}}{I} = \frac{80}{5} = 16\,\Omega$$
$$Z_{coil}^2 = R_{coil}^2 + X_L^2$$
$$16^2 = R_{coil}^2 + X_L^2 \Rightarrow ❶$$



$$Z_t = \frac{E_t}{I} = \frac{100}{5} = 20\,\Omega$$
$$Z_t^2 = (6 + R_{coil})^2 + X_L^2$$
$$20^2 = (6 + R_{coil})^2 + X_L^2 \Rightarrow ❷$$

Substitute Eq.1in Eq.2:
$$20^2 = (6 + R_{coil})^2 + 16^2 - R_{coil}^2$$
$$144 = 36 + 12R_{coil} + R_{coil}^2 - R_{coil}^2$$
$$R_{coil} = 9\,\Omega$$

$$pf_{coil} = \frac{R_{coil}}{Z_{coil}} = \frac{9}{16}$$
$$pf_{coil} = 0.56$$

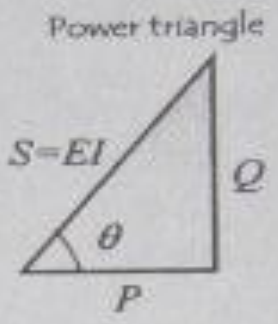
223 $Z = 20 + j35 = 40.31\angle 60.255°$

$pf = \cos\theta = \cos 60.255° = 0.496$

$pf = 49.6\%$

$Q = EI\sin\theta = \dfrac{E^2}{Z}\sin\theta = \dfrac{220^2}{40.31}\sin 60.255°$

$Q = 1042.25 VARS$



224 $R_L = \dfrac{P}{I^2} = \dfrac{1150}{(10)^2} = 11.5\,\Omega$

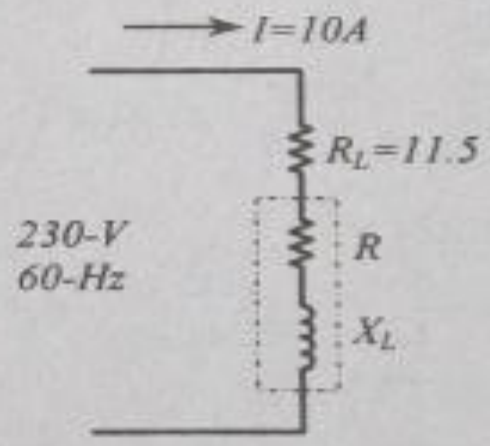
Notes: 1. By inspection the load is resistive (pf = 1) since when rated voltage and current are multiplied, the result is exactly the rated active power.
2. When the supply voltage is 230 V, the current drawn by the circuit should be 10 A also, in order for the said load to operate normally.

$Z = \dfrac{E}{I} = \dfrac{230}{10} = 23\,\Omega$

$Z^2 = (R_L + R)^2 + X_L^2$

$23^2 = (11.5 + R)^2 + X_L^2 \Rightarrow$ ❶

$X_L = 5R \Rightarrow$ ❷



Substitute Eq.2 in Eq.1:

$23^2 = (11.5 + R)^2 + (5R)^2$

$529 = 132.25 + 23R + R^2 + 25R^2$

$26R^2 + 23R - 396.75 = 0$

Using the quadratic formula :

$$R = \frac{-23 \pm \sqrt{23^2 - 4(26)(-396.75)}}{2(26)} = \frac{-23 \pm 204.428}{52}$$

$$R^{(+)} = \frac{-23 + 204.428}{52}$$

$R = 3.489\,\Omega$

225 $Z_t = Z_1 + Z_2 + Z_3$

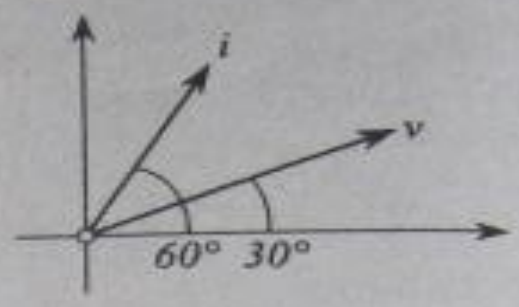
$= 12\angle 10° + 25\angle 15° + 34\angle 26°$

$= (11.817 + j2.083) + (24.148 + j6.47) + (30.558 + j14.904)$

$Z_t = 66.52 + j23.46\,\Omega$

226

$i = 10\cos(wt - 30°)$
$= 10\sin(wt - 30° + 90°)$
$i = 10\sin(wt + 60°)$



$v = 220\sin(wt + 30°)$

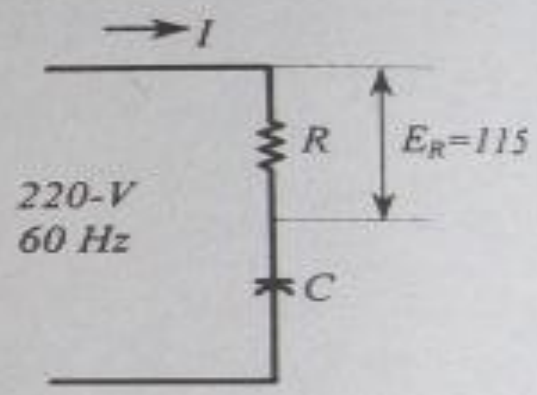
$$Z = \frac{V_m}{I_m} = \frac{220\angle 30°}{10\angle 60°}$$

$Z = 22\angle -30°\ \Omega$

Convert Z to rectangular form:
$Z = 19.05 - j11\Omega$

Note: cos wt = sin wt + 90°

227



$$R = \frac{E_R^2}{P} = \frac{(115)^2}{4800} = 2.755\ \Omega$$

$P = I^2R$

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{4800}{2.755}} = 41.74\ A$$

$P = EI\cos\theta$

$$\cos\theta = \frac{P}{EI} = \frac{4800}{(230)(41.74)} = 0.5$$

$\theta = \cos^{-1} 0.5 = 60°$

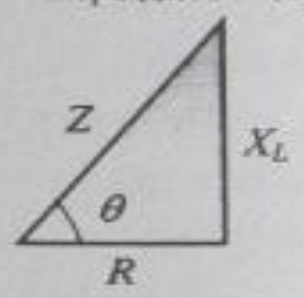
$$\tan\theta = \frac{X_C}{R}$$

$X_C = R\tan\theta = 2.755\tan 60° = 4.77\ \Omega$

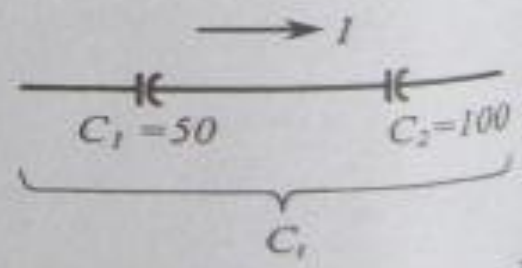
$$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60)(4.77)}$$

$C = 556.096\ \mu F$

Impedance triangle



228



$$C_t = \frac{C_1C_2}{C_1 + C_2} = \frac{50(100)}{50 + 100} = 33.33\ \mu F$$

Note: Since not given, assume a standard electrical frequency of 60 Hz or w = 377

$$X_{C_t} = \frac{1}{wC_t} = \frac{1}{377(33.33 \times 10^{-6})} = 79.583\ \Omega$$

$$I = \frac{E}{X_{C_1}} = \frac{100\angle 30^\circ}{79.583\angle -90^\circ} = 1.26\angle 120^\circ \text{ A}$$

$$i = 1.26 \sin(wt + 120^\circ) \text{ A}$$

229 $$X_c = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(60)(20 \times 10^{-6})} = 132.629\,\Omega$$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{100^2 + 132.629^2} = 166.1\,\Omega$$

$$I = \frac{E}{Z} = \frac{240}{166.1} = 1.44\,5\text{A}$$

$$\theta = \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{100}{166.1} = 52.98^\circ$$

$$E_R = IR = 1.445(100) = 144.5 \text{ V}$$

Note: Although from the choices, there are slight differences on the values of the impedance, voltage drop across the resistor and the pf angle, but the value of the resulting current is far beyond the computed value.

230 $$Z = X_C = \frac{1}{2\pi fC}$$

$$I = \frac{E}{Z} = E(2\pi fC)$$

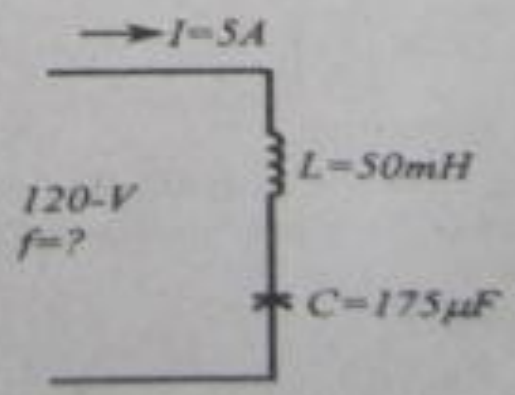
Note: With constant voltage, current is directly proportional to the frequency of the supply voltage and that of the capacitance.

$$I = k\,fC$$

$$\frac{I_2}{I_1} = \frac{f_2C_2}{f_1C_1}$$

$$I_2 = I_1\left(\frac{f_2\,C_2}{f_1\,C_1}\right) = 5\left(\frac{2f_1\,2C_1}{f_1\,C_1}\right) = 20 \text{ A}$$

231 $$Z = \frac{E}{I} = \frac{120}{5} = 24\,\Omega$$



$$Z = X_C - X_L = \frac{1}{2\pi fC} - 2\pi fL$$

$$24 = \frac{1}{2\pi f(175 \times 10^{-6})} - 2\pi f(0.05)$$

$$24 = \frac{909.45}{f} - 0.314f$$

$$76.433f = 2896.337 - f^2$$

$$f^2 + 76.433f - 2896.337 = 0$$

Using quadratic formula:

$$f = \frac{-76.433 \pm \sqrt{76.433^2 - 4(1)(-2896.337)}}{2(1)} = \frac{-76.433 \pm 132.01}{2}$$

$$f^{(+)} = \frac{-76.433 + 132.01}{2}$$

$$f = 28\ \text{Hz}$$

232 $$X_c = \frac{1}{wC} = \frac{1}{2\pi(50)(80 \times 10^{-6})} = 39.788\ \Omega$$

$$I = \frac{E}{Z} = \frac{120}{Z}$$

$$P = I^2R$$

$$100 = \left(\frac{120}{Z}\right)^2 R \Rightarrow ❶$$

$$Z^2 = R^2 + X_c^2 = R^2 + 39.788^2$$

$$Z^2 = R^2 + 1583.08 \Rightarrow ❷$$

Substitute Eq.2 in Eq.1:

$$100 = \frac{14400\,R}{R^2 + 1583.08}$$

$$R^2 + 1583.08 = 144\,R$$

$$R^2 - 144R + 1583.08 = 0$$

$$R = \frac{144 \pm \sqrt{144^2 - 4(1)(1583.08)}}{2(1)} = \frac{144 \pm 120.015}{2}$$

$$R^{(-)} = \frac{144 - 120}{2}$$

$R = 12\ \Omega$ (minimum value)

233 $$Q = EI = E\left(\frac{E}{X}\right) = \frac{E^2}{X}; \qquad X = \frac{1}{2\pi fC}$$

$$Q = E^2(2\pi fC)$$

Note: With the capacitance constant, reactive power is directly proportional to the square of the supply voltage and to the frequency of the supply voltage.

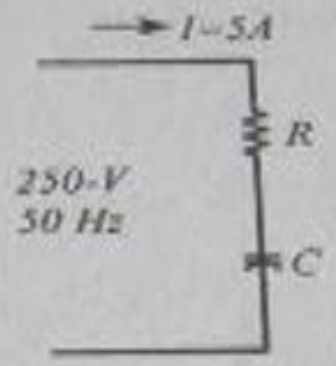
$$Q = kE^2f$$

$$\frac{Q_2}{Q_1} = \frac{E_2^2 f_2}{E_1^2 f_1}$$

$$Q_2 = Q_1\left(\frac{E_2^2 f_2}{E_1^2 f_1}\right) = 100\left(\frac{(220)^2(60)}{(380)^2(50)}\right)$$

$$Q_2 = 40.22\ \text{kVAR}$$

234 At 50 Hz (w = 314):



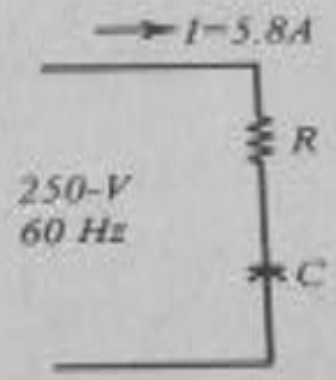
$$Z = \frac{E}{I} = \frac{250}{5} = 50\,\Omega$$

$$X_c = \frac{1}{wC}$$

$$Z^2 = R^2 + X_c^2$$

$$(50)^2 = R^2 + \frac{1}{(314)^2 C^2} \Rightarrow ❶$$

At 60 Hz (w = 377):



$$Z = \frac{E}{I} = \frac{250}{5.8} = 43\,\Omega$$

$$Z^2 = R^2 + X_c^2$$

$$(43)^2 = R^2 + \frac{1}{(377)^2 C^2} \Rightarrow ❷$$

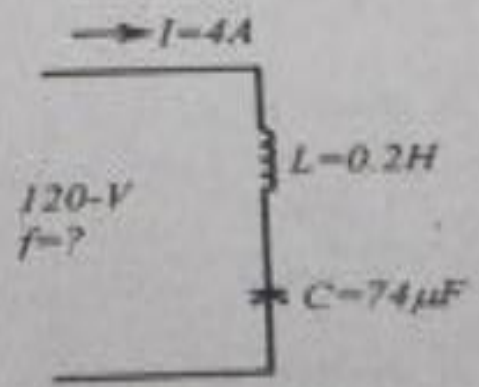
Substitute Eq.1 in Eq.2:

$$(43)^2 = (50)^2 - \frac{1}{(314)^2 C^2} + \frac{1}{(377)^2 C^2}$$

$$\frac{3.106 \times 10^{-6}}{C^2} = 651$$

$$C = 69.07\ \mu F$$

235



$$Z = \frac{E}{I} = \frac{60}{4} = 15\,\Omega$$

$$Z = X_L - X_C = 2\pi fL - \frac{1}{2\pi fC}$$

$$15 = 2\pi f(0.2) - \frac{1}{2\pi f(74 \times 10^{-6})}$$

$$15 = 1.256f - \frac{2150.74}{f}$$

$$15f = 1.256f^2 - 2150.74$$

$$f^2 - 11.94f - 1712.37 = 0$$

Using the quadratic formula :

$$f = \frac{11.94 \pm \sqrt{11.94^2 - 4(1)(-1712.37)}}{2(1)} = \frac{11.94 \pm 83.618}{2}$$

$$f^{(+)} = \frac{11.94 + 83.618}{2}$$

$$f \cong 48\ \text{Hz}$$

236 $R_{coil} = \frac{P}{I^2} = \frac{300}{5^2} = 12\,\Omega$

$$Z_{coil} = \frac{E}{I} = \frac{120}{5} = 24\,\Omega$$

$$X_{coil} = \sqrt{Z_{coil}^2 - R_{coil}^2} = \sqrt{24^2 - 12^2} = 20.78\,\Omega$$

Note: When the supply voltage is 240 V, the current drawn should be the same so that the coil may operate normally.

$$Z = \frac{E}{I} = \frac{240}{5} = 48\,\Omega$$

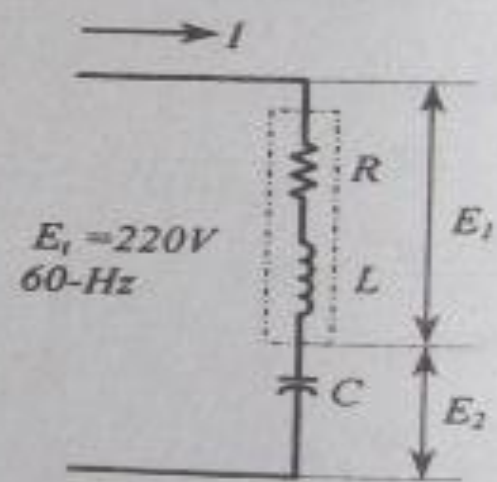
$$Z^2 = R_{coil}^2 + (X_c - X_{coil})^2$$

$$48^2 = 12^2 + (X_c - 20.78)^2$$

$$X_c = 67.25\,\Omega$$

237 $\theta = \cos^{-1} 0.87 = 29.54°$

$$\alpha = 90° - 29.54° = 60.46°$$



By cosine law :

$$E_1^2 = E_2^2 + E_t^2 - 2E_2E_t \cos\alpha$$

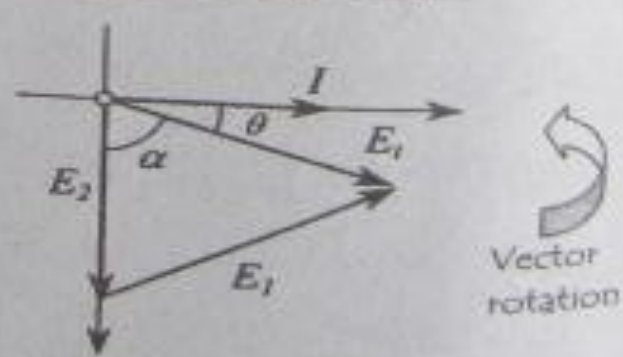
As given : $E_1 = E_2 = E$

$$E^2 = E^2 + E_t^2 - 2EE_t \cos\alpha$$

$$E = \frac{E_t^2}{2E_t \cos\alpha} = \frac{E_t}{2\cos\alpha}$$

$$E = \frac{220}{2\cos 60.46°}$$

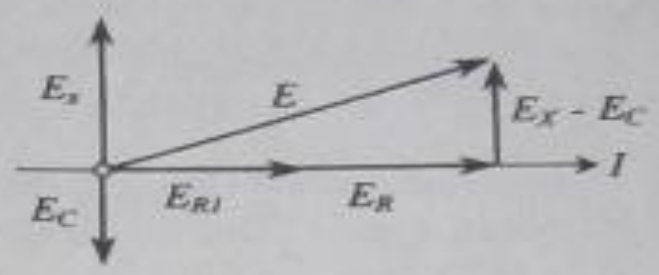
$$E = 223.109\ \text{volts}$$



Note: The voltage across the capacitor lags behind the current by 90° while that across the coil leads the current by a certain acute angle.

238 $E_R = \frac{P_R}{I} = \frac{50}{I} \Rightarrow$ ❶

$E_X = \frac{Q_X}{I} = \frac{100}{I} \Rightarrow$ ❷



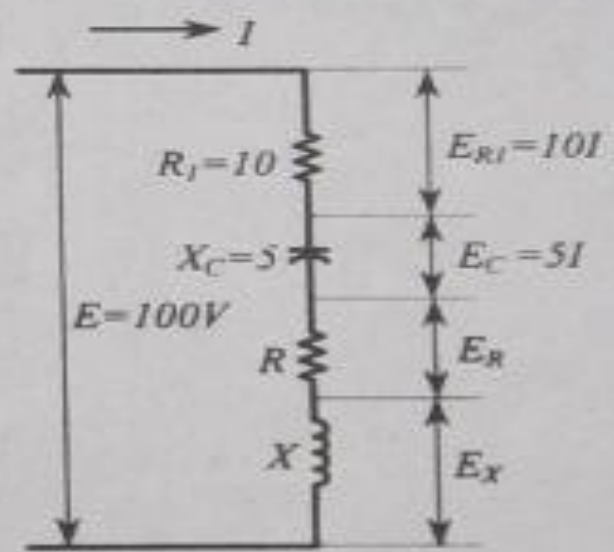
Refer to the vector diagram :

$E^2 = (E_{R1} + E_R)^2 + (E_X - E_C)^2$

$100^2 = \left(10I + \frac{50}{I}\right)^2 + \left(\frac{100}{I} - 5I\right)$

$100^2 = 100I^2 + 1000 + \frac{2500}{I^2} + \frac{10{,}000}{I^2} - 1000 + 25I^2$

$\left(10{,}000 = 125I^2 + \frac{12{,}500}{I^2}\right)\frac{I^2}{125}$



$I^4 - 80I^2 + 100 = 0$

Using the quadratic formula :

$I^2 = \frac{80 \pm \sqrt{80^2 - 4(1)(100)}}{2}$

$I^2 = \frac{80 \pm 77.46}{2}$

Take (-) sign for minimum current :

$I = \sqrt{\frac{80 - 77.46}{2}} = 1.127$

$I \cong 1.13\ A$

239 $P_{ave} = \frac{E_m I_m}{2}\cos\theta = \frac{(300)(20)}{2}\cos 30°$

$P_{ave} = 2598.076\ W$

240 $X_L = wL = 377(0.150) = 56.55\ \Omega$

$X_C = \frac{1}{wC} = \frac{1}{377(50 \times 10^{-6})} = 53.05\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (56.55 - 53.05)^2}$

$Z = 50.122\ \Omega$

$\theta_L = \cos^{-1}\frac{R}{Z} = \cos^{-1}\left(\frac{50}{50.122}\right) = 3.998°$

$\theta_L° \cong 4°$

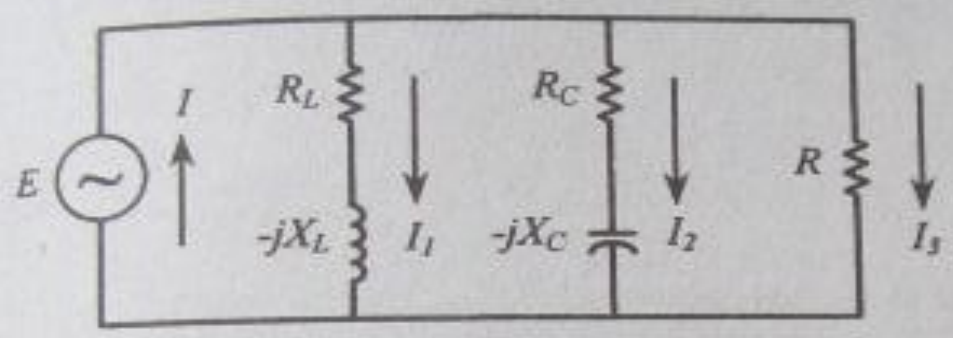
$$I_m = \frac{E_m}{Z} = \frac{100\angle -30^\circ}{50.122\angle 4^\circ} = 1.995\angle -34^\circ$$
$$P_m = E_m I_m \cos\theta = (100)(1.995)\cos 4^\circ$$
$$P_m = 199\ \text{W}$$

241 $i = 10\cos(wt - 30^\circ) = 10\sin(wt - 30^\circ + 90^\circ) = 10\sin(wt + 60^\circ)$
$$P_m = E_m I_m \cos\theta = (220)(10)\cos 60^\circ$$
$$P_m = 1100\ \text{W}$$

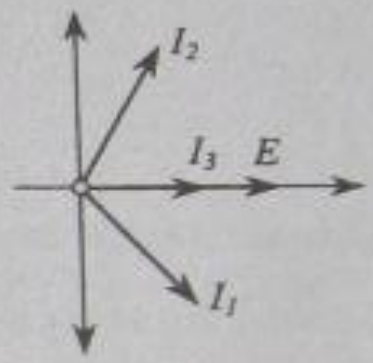
242 Note: Since resistive, pf = 1.
$$P_{ave} = \frac{E_m I_m}{2};\quad P_m = E_m I_m$$
$$P_{ave} = \frac{P_m}{2};\quad P_m = 2P_{ave}$$
$$P_{ave} = 2(4800) = 9600\ \text{W}$$

1-phase AC Circuits: Part 2

PARALLEL AC CIRCUITS



Phasor diagram



$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \qquad I = \frac{E}{Z_t} \qquad I = I_1 + I_2 + I_3$$

$$I_1 = \frac{E}{Z_1} \qquad I_2 = \frac{E}{Z_2} \qquad I_3 = \frac{E}{Z_3}$$

where: E = supply voltage
Z_1, Z_2, Z_3 = impedances of the first, second and third branch respectively
I_1, I_2, I_3 = current drawn by each branch
I = total current drawn by the parallel combination

Notes:

1. All operations (add, subtract, multiply and divide) must be done under the rules of complex mathematical operations.

2. For combinational circuits (series-parallel and parallel-series), the basic principles in series and parallel circuits will be used in analyzing these circuits.

Test 8

Problem 243: EE Board October 1981

A circuit consists of X_a = j5 ohms, X_c = - j5 ohms and R = 5 ohms all are connected in parallel. Find the equivalent impedance.

A. 5.5 Ω
B. 5.0 Ω
C. 4.8 Ω
D. 5.2 Ω

Problem 244: EE Board October 1985

Given: Z_1 = - j2.5 ohms; Z_2 = j4 ohms; Z_3 = 5 ohms; Z_4 = 1 + j5 ohms. If the four impedances are connected in parallel, find the equivalent impedance in ohms.

A. 4.1 + j0.72
B. 4.3 + j0.45
C. 4.2 + j0.35
D. 4.0 + j0.97

Problem 245: EE Board April 1984, EE Board April 1987

Three impedances Z_a = 3 + j4 ohms, Z_c = 4 – j4 ohms and Z_r = 0 + j3 ohms are connected in parallel. Solve for the pf of the combination.

A. 0.653 leading
B. 0.554 lagging
C. 0.503 leading
D. 0.620 lagging

Problem 246: EE Board October 1983

A pure capacitance of 530.515 x 10^{-6} farad and an inductance of 530.515 x 10^{-4} henry are connected in parallel across an AC power source. Solve for their resultant impedance assuming that the frequency is 30 Hz.

A. 10 Ω
B. Infinite
C. Zero
D. Undefined

Problem 247: EE Board March 1998

A coil of a 50-ohm resistance and of 150 mH inductance is connected in parallel with a 50 μF capacitor. What is the power factor of the circuit?

A. 80%
B. 50%
C. 70%
D. 60%

Problem 248: EE Board April 1982

Three impedances Z_a, Z_c, Z_r are connected in parallel. If at 60 Hz, $Z_a = 0 + j8$, $Z_c = 0 - j2$ and $Z_r = 5 + j0$ ohms. Solve for the resultant power factor.

A. 0.471 lagging
B. 0.471 leading
C. 0.573 lagging
D. 0.573 leading

Problem 249: EE Board October 1997

A resistor of 50 ohms and an impedance of 100 + j 50 ohms are connected in parallel across a 220 volts supply. What is the power factor of the load?

A. 96%
B. 99%
C. 98%
D. 95%

Problem 250: EE Board April 1990

A capacitor, an electric resistance heater, and an impedance are connected in parallel to a 120 V, 60 Hz system. The capacitor draws 50 VAR, the heater draws 100 W and the impedance coil draws 269 VA at a pf of 0.74 lagging. Determine the system power factor.

A. 0.933 lagging
B. 0.928 lagging
C. 0.916 lagging
D. 0.911 lagging

Problem 251:

A given load takes 40 kVA at 50% lagging power factor, while another load connected in parallel across the same source takes 80 kVA at 86.7% lagging power factor. Find the apparent power.

A. 114.5 kVA
B. 117.2 kVA
C. 116.4 kVA
D. 115.3 kVA

Problem 252

A 440 V, 60 Hz generator, supplies two parallel-connected loads A and B. Load A draws an apparent power of 100 kVA at 0.80 pf lagging and load B draws an apparent power of 70 kVA at unity pf. Determine the kVAR supplied by the generator

A. 109.5 kVAR
B. 120.2 kVAR
C. 104.3 kVAR
D. 115.7 kVAR

Problem 253: EE Board April 1992

Two single-phase motors are connected in parallel across a 120-volt, 60-cycle source of supply. Motor A is a split-phase induction type and motor B is a capacitor type:

Motor	HP output	Efficiency	Pf
A	1/4	0.60	0.7 lag.
B	1/2	0.70	0.95 lag

Determine total power factor.

A. 0.886 lag
B. 0.864 lag
C. 0.817 lag
D. 0.825 lag

Problem 254:

A 250-kVA, 0.5 lagging power factor load is connected in parallel to a 180-W, 0.8 leading power factor load and to a 300-VA, 100 VAR inductive load. Determine the total apparent power in kVA.

A. 628
B. 548
C. 615
D. 704

Problem 255: EE Board April 1992

A 250 V, 30 Hz generator supplies power to a parallel circuit consisting of a 20 hp motor whose efficiency is 90% at 0.80 pf lagging and a second load that draws an apparent power of 7 kVA at unity pf. Determine the system power factor.

A. 0.828 lagging
B. 0.831 lagging
C. 0.802 lagging
D. 0.884 lagging

Problem 256: EE Board April 1985

A resistance of 5 ohms is connected in series with a capacitor of 442.1 μF. The combination is then connected in parallel with an inductance of 21.22 mH. Solve for the resultant current if the circuit is connected across a 120 V, 60 Hz AC source.

A. 9.44 A
B. 10.68 A
C. 11.29 A
D. 10.34 A

Problem 257:

Two impedances $Z_1 = 3 + j\,4$ and $Z_2 = 5 - j\,8.66$ ohms respectively are connected in parallel. If the combination is connected across a 240 V AC source, how much is the total current?

A. 44.4 A
B. 42.1 A
C. 40.6 A
D. 39.9 A

Problem 258:

A coil having a resistance of 30 ohms and an inductance L is connected in parallel with a resistor having a resistance of 100-ohm. The combination is connected across a 100-V, 60 Hz source. If the power delivered by the source is 400 W, find the value of L.

A. 0.204 H
B. 0.011 H
C. 0.026 H
D. 0.107 H

Problem 259: EE Board April 1993

An inductor L_1 is connected in series with a parallel combination of inductor L_2 and capacitor C. The impedance of the circuit w = 400 ra/sec is j100 ohms. The circuit is to yield infinite impedance at w = 1,000 rad/sec and zero impedance at w = 2,000 rad/sec. Determine the value of C.

A. 1.26 μF
B. 1.67 μF
C. 2.06 μF
D. 1.32 μF

Problem 260:

A resistor and an inductor are connected in parallel across a 120-V, 60-Hz source. If the total current and power are respectively 15 A and 1600 watts, solve for the inductance of the inductor.

A. 50.2 mH

B. 48.3 mH
C. 43.2 mH
D. 46.3 mH

Problem 261:

An inductive coil with a resistance R and inductance L is connected in parallel with a 30-ohm resistor. The combination is then connected across a 60 Hz AC source. If the currents in the coil, resistor and the total are 6, 4 and 8 A, respectively, determine the inductance L of the coil.

A. 51.36 mH
B. 48.32 mH
C. 60.35 mH
D. 53.62 mH

Problem 262: EE Board April 1992

A sinusoidal current source, 10 cos 1000t, is in parallel both with a 20-ohm resistor and the series combination of a 10-ohm resistor and a 10-mH inductor. Find the equation of the voltage across the 10-ohm resistor.

A. 63.25 cos (1000t – 18.43°)
B. 61.32 cos (1000t – 20.34°)
C. 59.36 cos (1000t – 17.45°)
D. 60.12 cos (1000t – 19.38°)

Problem 263: EE Board April 1993

A 1-hp, 220 V, 60 Hz capacitor-start motor has main and auxiliary winding impedance at starting of 3.5 + j2.5 ohms and 8.6 + j2.5 ohms, respectively. Determine the value of the starting capacitance that will place the main and auxiliary winding currents 90° apart at starting.

A. 186.75 μF
B. 174.35 μF
C. 182.43 μF
D. 170.67 μF

Problem 264: EE Board October 1990

Two impedances A and B are connected in parallel across a 120 V AC supply. The total current and the current in each impedance are adjusted to 20 A. The power drawn by A is doubled that of B and the power factor is lagging. Determine power factor of A.

A. 0.650 lagging
B. 0.704 lagging
C. 0.841 lagging
D. 0.677 lagging

Problem 265: EE Board March 1998

A coil of 50-ohm resistance and of 150-mH inductance is connected in parallel with a 50-μF capacitor. The source voltage is 100 sin (wt + 30°). What is the equation of the line current?

A. 1.91 sin (wt + 52.5°)
B. 1.25 sin (wt + 75.5°)
C. 1.82 sin (wt – 62°)
D. 1.32 sin (wt – 75.5°)

Problem 266: EE Board October 1984

A resistor R is connected in parallel with a 10-ohm inductive reactance. The combination is then connected in series with a 4-ohm capacitive reactance. The whole combination is connected across a 100-volt, 60 Hz supply. How much is R if the angle between the supply voltage and the total current is 45 degrees?

A. 12 Ω
B. 25 Ω
C. 16 Ω
D. 20 Ω

Problem 267:

A resistance of 20 ohms and an unknown capacitance are connected in parallel across a 110 V, variable frequency AC source. When the frequency is 60 Hz, the current drawn by the circuit is 6 A. At what frequency will the current drawn falls to 5.8 A?

A. 42.33 Hz
B. 50.12 Hz
C. 46.02 Hz
D. 44.18 Hz

Problem 268:

A single-phase capacitor-start squirrel-cage induction motor take 2.5 A from a 220 V line. The current in the starting winding is 1.3 A and the current in the main winding is 1.45 A. The total power input is 550 W. What is the resistance of the main winding? Assume the auxiliary winding takes a leading current while the main winding takes a lagging current.

A. 139.43 Ω
B. 125.68 Ω
C. 142.45 Ω
D. 151.97 Ω

Problem 269:

An inductive reactance of 10 ohms is connected in parallel with a capacitive reactance of 30 ohms. If the combination is connected in series with a 10-ohn resistance, solve for the equivalent power factor of the whole combination.

A. 0.555 lagging
B. 0.586 lagging
C. 0.603 lagging
D. 0.623 lagging

Problem 270:

A resistor R and a variable capacitor C are connected in parallel. The combination is then connected in series with an inductive reactance of 25 Ω. The circuit is energized by 110-V, 60 Hz source. If the capacitance is varied until the voltage across the resistor and that across the inductor are equal to the supply voltage, determine the real power supplied by the source.

A. 537 W
B. 419 W
C. 620 W
D. 402 W

Problem 271: EE Board April 1980

Three impedances $Z_1 = 1 - j4$, $Z_2 = - j6$ and $Z_3 = 4 + j3$ ohms respectively are connected in series parallel. Z_1 is connected in series with the parallel combination of Z_2 and Z_3. Determine the equivalent impedance of the combination.

A. 4.32 – 1.21
B. 2.23 – j3.32
C. 6.76 – j5.68
D. 5.42 –j 7.21

Problem 272:

An inductive reactance of 3 ohms is connected in parallel with a capacitive reactance of 4 ohms. If the combination is connected in series with a 4-ohm resistance, solve for the power factor of the whole combination.

A. 0.333
B. 0.409
C. 0.567
D. 0.316

Problem 273:

An inductive reactance of 10 ohms is connected in parallel with a capacitive reactance of 30 ohms. If the combination is connected in series with a 10-ohm resistance, solve for total real power if the whole circuit is connected across a 240 V, 60 Hz supply.

A. 2.05 kW
B. 2.87 kW
C. 1.77 kW
D. 1.25 kW

Problem 274: EE Board October 1984

A 5-ohm resistor is connected in parallel with a 10-ohm inductive reactance. The combination is then connected in series with a 4-ohm capacitive reactance. The whole combination is connected across a 100-volt, 60 Hz supply. How much is the total current drawn by the circuit?

A. 22.36 A
B. 20.45 A
C. 23.16 A
D. 19.89 A

Problem 275: EE Board April 1983

A non-inductive resistor R is connected in parallel with an inductive reactance of 10 ohms. The combination is then connected in series with a capacitive reactance of 5 ohms. The whole combination is connected across a 100-V, 60 Hz AC source. If R is equal to 5 ohms, solve for the voltage across the parallel combination.

A. 87.53 V
B. 88.34 V
C. 89.44 V
D. 91.87 V

Problem 276: EE Board April 1980

Three impedances $Z_1 = 1 - j4$, $Z_2 = -j6$ and $Z_3 = 4 + j3$ ohms respectively are connected in series parallel. Z_1 is connected in series with the parallel combination of Z_2 and Z_3. If this circuit is connected across a 230 V, 60 Hz source, determine the voltage across the parallel combination of Z_2 and Z_3.

A. 156.3 V
B. 146.8 V
C. 135.7 V
D. 163.2 V

Problem 277:

A 30-ohm resistor is connected in parallel with a 0.106-H inductor. The parallel combination is then connected in series with a 98-F capacitor. If the whole circuit is connected across a 125 V, 60 Hz, find the current through the inductor.

A. 4.51 A
B. 3.26 A
C. 2.72 A
D. 2.34 A

Problem 278: EE Board October 1980

Given three impedances: $Z_1 = 10 + j0$, $Z_2 = 3 + j4$ and $Z_3 = 8 - j6$. Impedance Z_2 and Z_3 are connected in parallel and the combination is connected in series with impedance Z_1 across 120 V single-phase 60 Hz source. Find the total power drawn by the impedances.

A. 1008 W
B. 1204 W
C. 1038 W
D. 1103 W

ANSWER KEY

243. B	251. C	259. D	267. C	275. C
244. A	252. A	260. D	268. A	276. A
245. B	253. B	261. A	269. A	277. B
246. D	254. C	262. A	270. B	278. A
247. C	255. D	263. C	271. C	
248. B	256 D	264. A	272. D	
249. B	257. A	265. B	273. C	
250. C	258. C	266. D	274. A	

☑ RATING

32-36 - Topnotcher
25-31 - Passer
18-24 - Conditional
0-17 - Failed

Solutions to Test 8

243 $\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$

$\frac{1}{Z_t} = \frac{1}{j5} + \frac{1}{-j5} + \frac{1}{5} = -j0.2 + j0.2 + 0.2 = 0.2$

$Z_t = \frac{1}{0.2} = 5\,\Omega$

244 $\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4}$

$\frac{1}{Z_t} = \frac{1}{-j2.5} + \frac{1}{j4} + \frac{1}{5} + \frac{1}{1+j5}$

$= j0.4 - j0.25 + 0.2 + \frac{1}{5.1\angle 78.69^\circ}$

$= 0.2 + j0.15 + 0.196\angle -78.69^\circ$

$= 0.2 + j0.15 + 0.0384 - j0.1922$

$\frac{1}{Z_t} = 0.2384 - j0.0422 = 0.2421\angle -10.038^\circ$

$Z_t = \frac{1}{0.2421\angle -10.038^\circ} = 4.13\angle 10.038^\circ = 4.066 + j0.719\,\Omega$

$Z_t \cong 4.1 + j0.72\,\Omega$

245 $\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{3+j4} + \frac{1}{4-j4} + \frac{1}{j3}$

$= \frac{1}{5\angle 53.13^\circ} + \frac{1}{5.656\angle -45^\circ} - j0.333$

$= 0.2\angle -53.13^\circ + 0.1768\angle 45^\circ - j0.333$

$= 0.12 - j0.16 + 0.125 + j0.125 - j0.333$

$= 0.245 - j0.368 = 0.442\angle -56.34^\circ$

$Z_t = \frac{1}{0.442\angle -56.34^\circ} = 2.262\angle 56.34^\circ$

pf = cos 56.34°

pf = 0.554 lagging

246 $X_c = -j\frac{1}{2\pi fC} = \frac{-j}{2\pi(30)(530.515 \times 10^{-6})} = -j10$

$X_L = j2\pi fL = j2\pi(30)(530.515 \times 10^{-4}) = j10$

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{-j10} + \frac{1}{j10} = 0$$

$$Z_t = \frac{1}{0}$$

Z_t = indeterminate

Note: Indeterminate means undefined.

247

$$X_L = wL = 377(0.15) = 56.55\,\Omega$$

$$X_C = \frac{1}{wC} = \frac{1}{377(50 \times 10^{-6})} = 53.05\,\Omega$$

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{50 + j56.55} + \frac{1}{-j53.05}$$

$$= \frac{1}{75.484\angle 48.517^\circ} + j0.01885$$

$$= 0.0132\angle -48.517^\circ + j0.01885 = 0.00874 - j0.00988 + j0.01885$$

$$= 0.00874 + j0.00897 = 0.0125\angle 45.744^\circ$$

$$Z_t = \frac{1}{0.0125\angle 45.744^\circ} = 80\angle -45.744^\circ$$

$$pf = \cos(-45.744^\circ)$$

$$pf \cong 0.7$$

248

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\frac{1}{Z_t} = \frac{1}{j8} + \frac{1}{-j2} + \frac{1}{5} = -j0.125 + j0.5 + 0.2 = 0.2 + j0.375 = 0.425\angle 61.927^\circ$$

$$Z_t = \frac{1}{0.425\angle 61.927^\circ} = 2.353\angle -61.927\,\Omega$$

$$pf = \cos(-61.927)$$

$$pf = 0.471 \text{ leading}$$

249

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{50} + \frac{1}{100 + j50} = 0.02 + \frac{1}{111.803\angle 26.565^\circ}$$

$$= 0.02 + 0.00894\angle -26.565^\circ = 0.02 + 0.008 - j0.004$$

$$= 0.028 - j0.004 = 0.0283\angle 8.13^\circ$$

$$Z_t = \frac{1}{0.0283\angle 8.13^\circ} = 35.335\angle -8.13^\circ$$

$$pf = \cos(-8.13^\circ) = 0.9899$$

$$pf \cong 99\%$$

250 $S_1 = j50$

$S_2 = 100$

$\theta_3 = \cos^{-1} 0.74 = 42.268°$

$S_3 = 269\angle -42.268° = 199.06 - j180.93$

$S_t = S_1 + S_2 + S_3 = j50 + 100 + 199.06 - j180.93$

$S_t = 299.06 - j130.93 = 326.46\angle -23.64°$ VA

$pf_t = \cos(-23.64°)$

$pf_t = 0.916$ lagging

251 $\theta_1 = \cos^{-1} 0.5 = 60°$

$\theta_2 = \cos^{-1} 0.867 = 29.888 \cong 30°$

$S_t = S_1 + S_2 = 40\angle -60° + 80\angle -30°$

$= 20 - j34.641 + 69.282 - j40$

$= 89.282 - j74.641$

$S_t = 116.37\angle -39.896°$ kVA

252 $\theta_1 = \cos^{-1} 0.8 = 36.869°$

$\theta_2 = \cos^{-1} 0.707 = 45°$

$S_t = S_1 + S_2 = 100\angle -36.869° + 70\angle -45°$

$= 80 - j60 + 49.5 - j49.5$

$S_t = 129.5 - j109.5$

By inspection $Q_t = 109.5$ kVAR

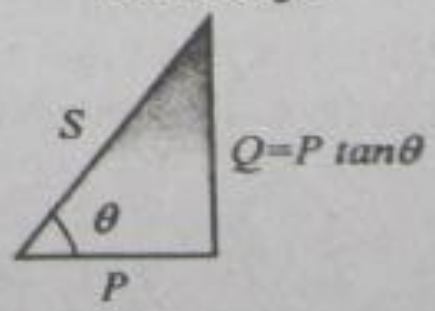
253 $\theta_a = \cos^{-1} 0.7 = 45.57°$

$\theta_b = \cos^{-1} 0.95 = 18.19°$

$P_a = \dfrac{(1/4)(746)}{0.6} = 310.83$ W

$P_b = \dfrac{(1/2)(746)}{0.7} = 532.85$ W

Power triangle



$S_t = S_a + S_b$

$S_t = (P_a - jP_a \tan\theta_a) + (P_b - jP_b \tan\theta_b)$

$= 310.83 - j310.83 \tan 45.57° + 532.85 - j532.85 \tan 18.19°$

$S_t = 843.68 - j492.166 = 976.74\angle -30.257°$

$pf_t = \cos(-30.257°)$

$pf_t = 0.864$ lagging

254

$\theta_1 = \cos^{-1} 0.5 = 60°$

$\theta_2 = \cos^{-1} 0.8 = 36.869°$

$P_3 = \sqrt{S_3^2 - Q_3^3} = \sqrt{300^2 - 100^2} = 282.84\,\text{kW}$

$S_t = S_1 + S_2 + S_3$

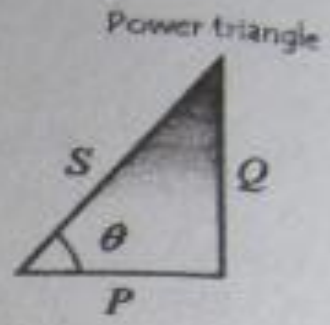
$= 250\angle -60° + (180 + j180\tan 36.869°)$

$+ (282.84 - j100)$

$= 125 - j216.506 + 180 + j135 + 282.84 - j100$

$= 587.84 - j181.506$

$S_t = 615.22\angle -17.158\,\text{kVA}$



255

$P_1 = \dfrac{P_{out}}{\eta} = \dfrac{20(746)}{0.9} = 16.58\,\text{kW}$

$S_1 = \dfrac{P_1}{pf} = \dfrac{16.58}{0.8} = 20.725\,\text{kVA}$

$\theta_1 = \cos^{-1} 0.8 = 36.869°$

$S_t = S_1 + S_2$

$= 20.725\angle -36.869° + 7\angle 0°$

$= 16.58 - j12.434 + 7 = 23.58 - j12.434$

$S_t = 26.657\angle -27.803°$

$pf_t = \cos 27.803°$

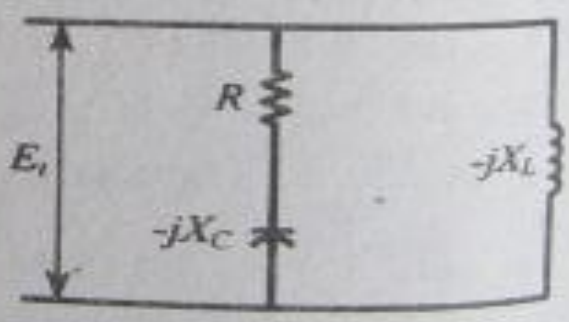
$pf_t = 0.884$ lagging

256

$X_c = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60)(442.1\times 10^{-6})} = 6\,\Omega$

$Z_1 = R - jX_c = 5 - j6 = 7.81\angle -50.19°$

$Z_2 = j2\pi fL = j(2\pi)(60)(0.02122) = j8$



$\dfrac{1}{Z_t} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2} = \dfrac{1}{7.81\angle -50.19°} + \dfrac{1}{j8}$

$= 0.128\angle 50.19° - j0.125$

$= 0.08195 + j0.0983 - j0.125 = 0.08195 - j0.0267$

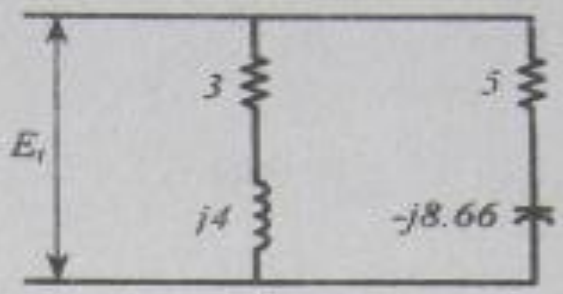
$= 0.08618\angle 18.046°$

$Z_t = \dfrac{1}{0.08618\angle 18.046°} = 11.6\angle -18.046°$

$I_t = \dfrac{E_t}{Z_t} = \dfrac{120}{11.6} = 10.34\,\text{A}$

257 $Z_1 = 3 + j4 = 5\angle 53.13°$

$Z_2 = 5 - j8.66 = 10\angle -60°$



$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{5\angle 53.13°} + \frac{1}{10\angle -60°}$$

$$= 0.2\angle -53.13° + 0.1\angle 60°$$

$$\frac{1}{Z_t} = 0.12 - j0.16 + 0.05 + j0.0866$$

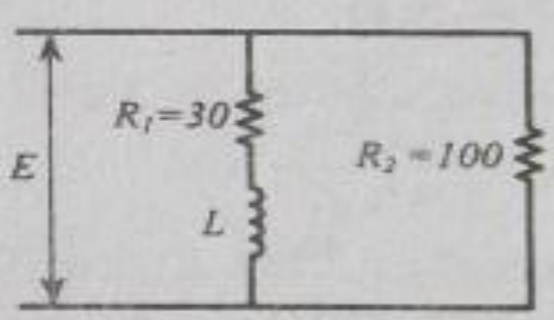
$$= 0.17 - j0.0734 = 0.185\angle -23.35°$$

$$Z_t = \frac{1}{0.185\angle -23.35°} = 5.405\angle 23.35°$$

$$I_t = \frac{E}{Z_t} = \frac{240}{5.405}$$

$$I_t = 44.4\ A$$

258 $P_t = P_1 + P_2 = I_1^2 R_1 + I_2^2 R_2$



$$P_t = \left(\frac{E}{Z_1}\right)^2 R_1 + \left(\frac{E}{Z_2}\right)^2 R_2$$

$$400 = \frac{(100)^2(30)}{Z_1^2} + \frac{(100)^2(100)}{(100)^2}$$

$$Z_1^2 = 1000$$

$$Z_1^2 = R_1^2 + X_L^2$$

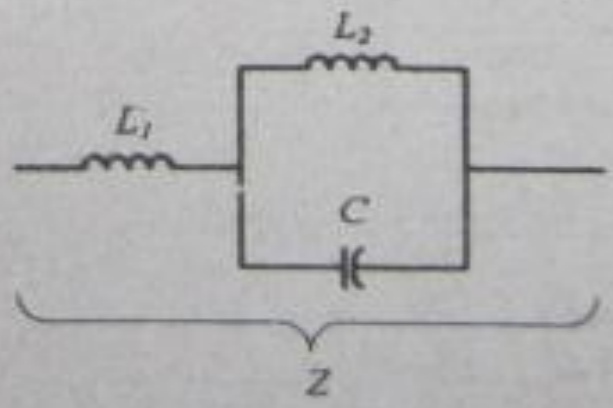
$$X_L^2 = 1000 - (30)^2 = 100$$

$$X_L = 10\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{10}{2\pi(60)}$$

$$L = 0.026\ H$$

259 $Z = jwL_1 + \dfrac{(jwL_2)(-j1/wC)}{jwL_2 - j(1/wC)}$



$$= jwL_1 + \frac{L_2/C}{-j(1/wC - wL_2)}$$

$$= jwL_1 + \frac{j(L_2/C)}{\dfrac{1 - w^2 L_2 C}{wC}}$$

$$Z = jwL_1 + \frac{jwL_2}{1 - w^2 L_2 C}$$

Note: When w = 1000, Z = ∝ and the quantity "$1-w^2L_2C$" must be equal to zero

$$1 - w^2L_2C = 0$$

$$1 = (1000)^2 L_2C$$

$$L_2C = \frac{1}{1000^2} \Rightarrow ❶$$

Note: When w = 2000, Z = 0

$$wL_1 + \frac{wL_2}{1 - w^2L_2C} = 0$$

$$L_1 + \frac{L_2}{1-(2000)^2\left(\frac{1}{1000^2}\right)} = 0$$

$$L_1 - \frac{L_2}{3} = 0$$

$$L_2 = 3L_1 \Rightarrow ❷$$

Note: When w = 400, Z = j100

$$wL_1 + \frac{wL_2}{1 - w^2L_2C} = 100$$

$$400L_1 + \frac{400L_2}{1 - 400^2\left(\frac{1}{1000^2}\right)} = 100$$

$$400L_1 - 1.19L_2 = 100 \Rightarrow ❸$$

Substitute Eq.2 in Eq.3 :

$$400L_1 - 1.19(3L_1) = 100$$

$$L_1 = 0.252\ H$$

$$L_2 = 3(0.252) = 0.756\ H$$

$$C = \frac{1}{1000^2 L_2} = \frac{1}{1000^2(0756)}$$

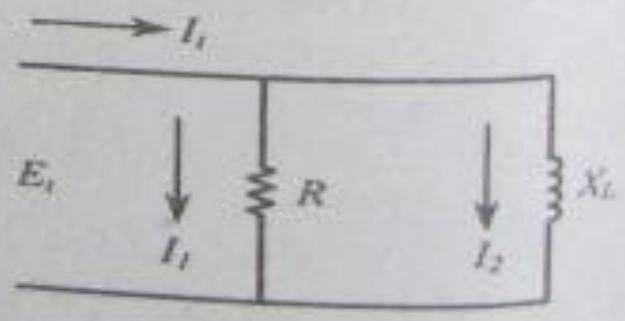
$$C = 1.32\ \mu F$$

260 $R = \frac{E_1^2}{P} = \frac{120^2}{1600} = 9\ \Omega$

$$I_1 = \frac{E_1}{R} = \frac{120}{9} = 13.33\ A$$

$$I_2 = \sqrt{I_1^2 - I_1^2} = \sqrt{15^2 - 13.33^2}$$

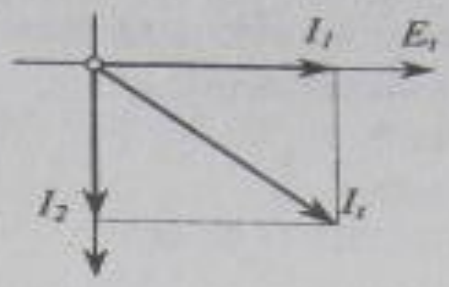
$$I_2 = 6.878\ A$$



$$X_L = \frac{E_1}{I_2} = \frac{120}{6.878} = 17.45\,\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{17.45}{2\pi(60)} = 0.0463\,H$$

$$L = 46.3\,mH$$



261

$$E_1 = I_2R_2 = (4)(30) = 120\,\Omega$$

$$Z_1 = \frac{E_1}{I_1} = \frac{120}{6} = 20\,\Omega$$

By cosine law :

$$I_t^2 = I_1^2 + I_2^2 - 2I_1I_2\cos\alpha$$

$$8^2 = 6^2 + 4^2 - 2(6)(4)\cos\alpha$$

$$\alpha = 104.47°$$

$$\theta_1 = 180° - \alpha = 180 - 104.47$$

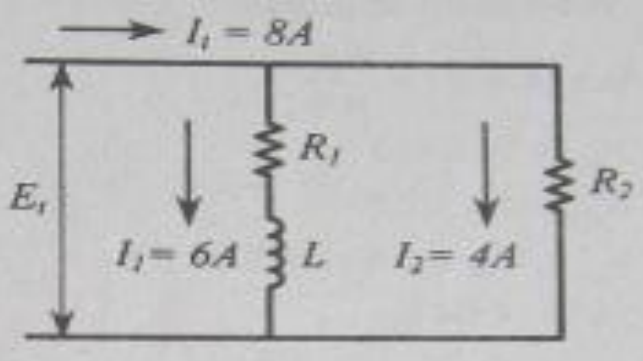
$$= 75.53°$$

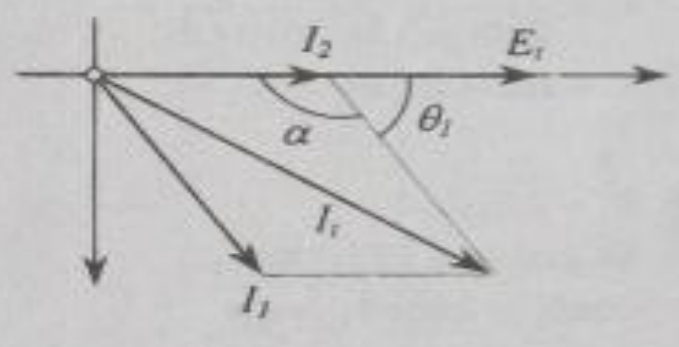
$$X_L = Z_1\sin\theta_1 = 20\sin 75.53°$$

$$= 19.365\,\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{19.365}{2\pi(60)} = 0.05136\,H$$

$$L = 51.36\,mH$$





262

$$X_L = wL = 1000(10 \times 10^{-3}) = 10\,\Omega$$

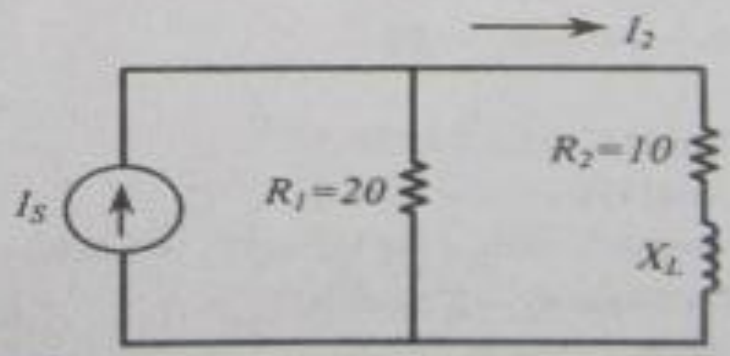
by CDT :

$$I_2 = \frac{I_sZ_1}{Z_1 + Z_2}$$

$$Z_1 + Z_2 = 20 + 10 + j10$$

$$= 30 + j10$$

$$Z_1 + Z_2 = 31.623\angle 18.43°$$



Substitute :

$$I_2 = \frac{(10\angle 0°)(20\angle 0°)}{31.623\angle 18.43°} = 6.32\angle -18.43°$$

$$V_R = I_2R_2 = (6.32\angle -18.43°)(10\angle 0°)$$

$$= 63.2\angle -18.43°$$

$$V_R = 63.2\cos(1000t - 18.43°)\,V$$

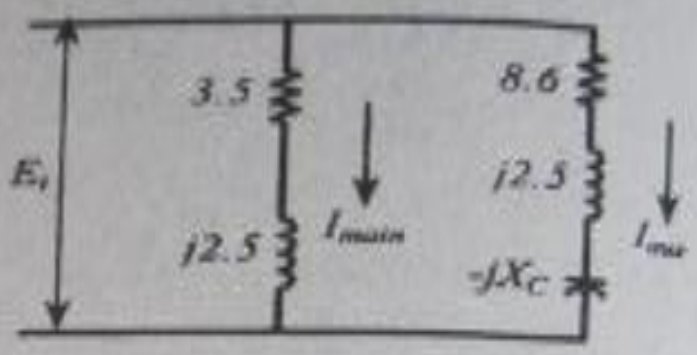
263 $Z_{main} = 3.5 + j2.5 = 4.301\angle 35.537°$

Refer to the vector diagram:

$\theta_{main} = 35.537°$

$\theta_{main} + \theta_{aux} = 90°$

$\theta_{aux} = 90° - 35.537° = 54.463°$



Refer to the impedance triangle:

$\tan 54.463° = \dfrac{X_{eq}}{8.6}$

$X_{eq} = 12.04\ \Omega$

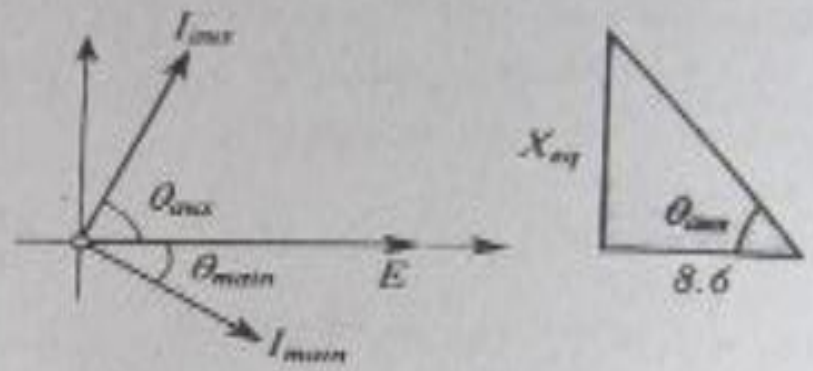
$X_{eq} = X_C - 2.5$

$X_c = X_{eq} + 2.5 = 12.04 + 2.5$

$X_c = 14.54\ \Omega$

$C = \dfrac{1}{2\pi f X_c} = \dfrac{1}{2\pi(60)(14.54)}$

$C = 182.433\ \mu F$



264 $P_1 = 2P_2$

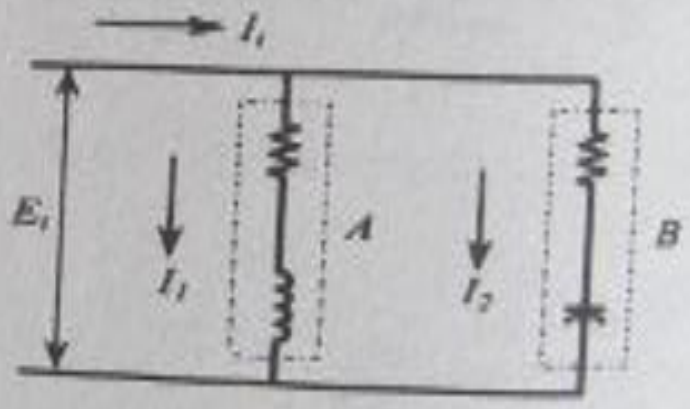
$EI_1\cos\theta_1 = 2EI_2\cos\theta_2$

$\cos\theta_1 = 2\cos\theta_2 \Rightarrow$ ❶

Refer to the vector diagram:

$\theta_1 + \theta_2 + 60 = 180$

$\theta_1 = 120 - \theta_2 \Rightarrow$ ❷



Substitute Eq.2 to Eq.1:

$\cos(120 - \theta_2) = 2\cos\theta_2$

$\cos 120\cos\theta_2 + \sin 120\sin\theta_2 = 2\cos\theta_2$

$0.866\sin\theta_2 = 2.5\cos\theta_2$

$\dfrac{\sin\theta_2}{\cos\theta_2} = \dfrac{2.5}{0.866}$

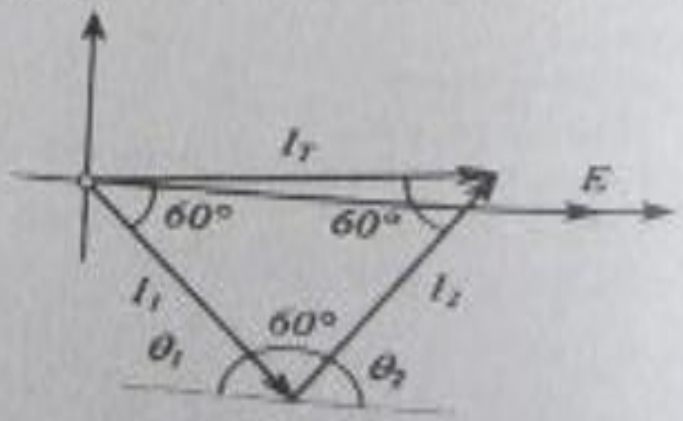
$\tan\theta_2 = 2.8868$

$\theta_2 = \tan^{-1} 2.8868 = 70.89°$

$\theta_1 = 120 - 70.89 = 49.11°$

$pf_1 = \cos\theta_1 = \cos 49.11°$

$pf_1 = 0.65$ lagging



265 Note: Since not specified, assume a standard electrical frequency of 60 Hz or w = 377.

$$Z_1 = 50 + j377(150 \times 10^{-3}) = 50 + j56.55 = 75.484\angle 48.518^\circ$$

$$I_1 = \frac{E}{Z_1} = \frac{100\angle 30^\circ}{75.484\angle 48.518^\circ} = 1.325\angle -18.518^\circ = 1.256 - j0.42 \text{ A}$$

$$Z_2 = \frac{-j}{377(50 \times 10^{-6})} = -j53.05 = 53.05\angle -90^\circ$$

$$I_2 = \frac{E}{Z_2} = \frac{100\angle 30^\circ}{53.05\angle -90^\circ} = 1.885\angle 120^\circ = -0.9425 + j1.632 \text{ A}$$

$$I_t = I_1 + I_2 = 1.256 - j0.42 + (-0.9425 + j1.632) = 0.3135 + j1.212$$

$$I_t = 1.25\angle 75.5^\circ$$

$$I_t = 1.25 \sin(wt + 75.5^\circ) \text{ A}$$

266

$$Z_{eq} = \frac{R(j10)}{R + j10}$$

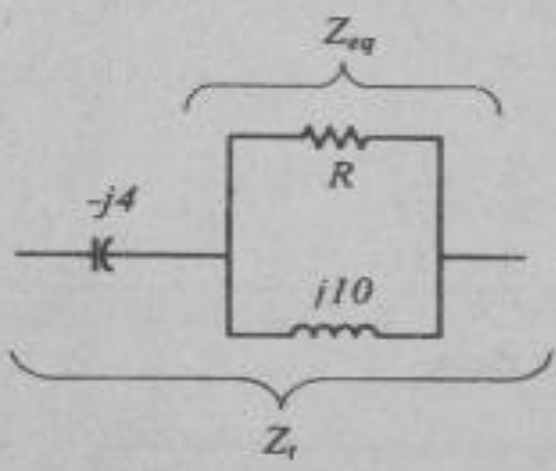
$$Z_{eq} = \frac{R(j10)}{R + j10} \cdot \frac{(R - j10)}{(R - j10)}$$

$$= \frac{j10R^2 - j^2 100R}{R^2 + 100}$$

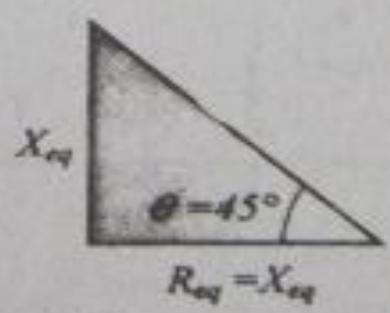
$$Z_{eq} = \frac{100R}{R^2 + 100} + j\frac{10R^2}{R^2 + 100}$$

$$Z_t = Z_{eq} - j4$$

$$Z_t = \frac{100R}{R^2 + 100} + j\left(\frac{10R^2}{R^2 + 100} - 4\right)$$



Note: if θ = 45°, equivalent resistance must be equal to equivalent reactance.



$$\frac{100R}{R^2 + 100} = \frac{10R^2}{R^2 + 100} - 4$$

$$100R = 10R^2 - 4(R^2 + 100)$$

$$100R = 6R^2 - 400$$

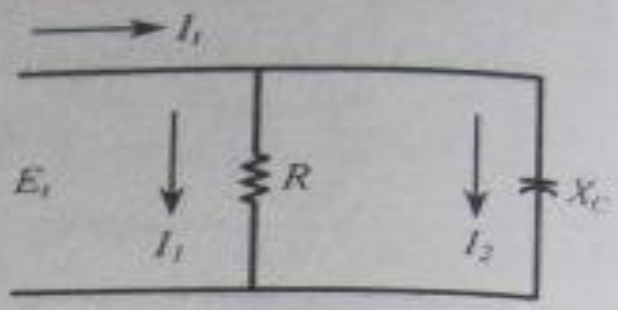
$$6R^2 - 100R - 400 = 0$$

Using the quadratic formula :

$$R = \frac{100 \pm \sqrt{(-100)^2 - 4(6)(-400)}}{2(6)} = \frac{100 \pm 140}{12}$$

$$R = 20\,\Omega$$

267 At 60 Hz (refer to the vector diagram):



$$I_1 = \frac{E_1}{R} = \frac{110}{20} = 5.5\text{ A}$$

$$I_2 = \sqrt{I_t^2 - I_1^2} = \sqrt{(6)^2 - (5.5)^2}$$

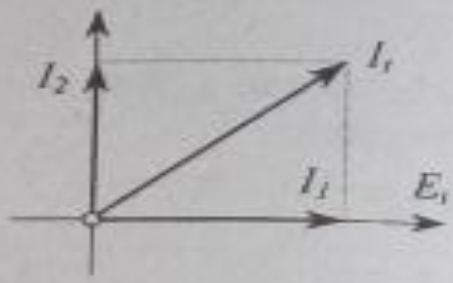
$$I_2 = 2.398\text{ A}$$

$$X_c = \frac{E_2}{I_2} = \frac{110}{2.398}$$

$$X_c = 45.87\ \Omega$$

$$C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi(60)(45.87)}$$

$$C = 57.828\ \mu\text{F}$$



When current drawn is 5.8 A:

$$I_2 = \sqrt{I_t^2 - I_1^2} = \sqrt{(5.8)^2 - (5.5)^2}$$

$$I_2 = 1.84\text{ A}$$

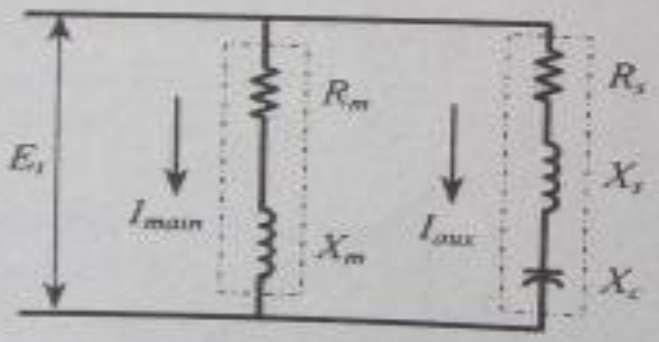
$$X_c = \frac{E_2}{I_2} = \frac{110}{1.84}$$

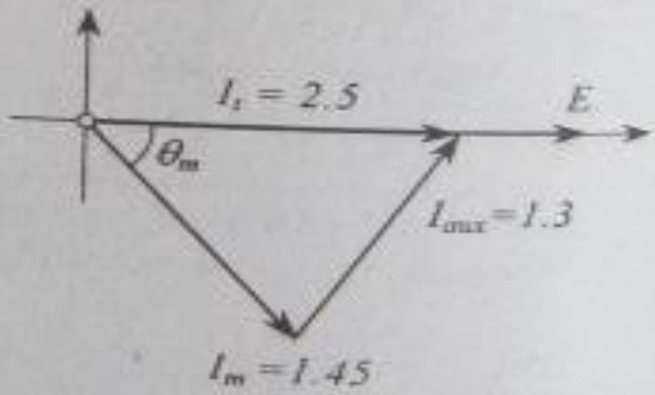
$$X_c = 59.783\ \Omega$$

$$f = \frac{1}{2\pi C X_c} = \frac{1}{2\pi(57.828 \times 10^{-6})(59.783)}$$

$$f = 46.04\text{ Hz}$$

268





$$P_t = E_t I_t \cos\theta_t$$

$$\theta_t = \cos^{-1}\frac{P_t}{E_t I_t} = \cos^{-1}\frac{550}{220(2.5)}$$

$$\theta_t = 0°$$

Note: Refer to the vector diagram. Since the angle θ_t is zero, this means that the total voltage must be coincident or in-phase with the total voltage.

By cosine law :

$(1.3)^2 = (1.45)^2 + (2.5)^2 - 2(1.45)(2.5)\cos\theta_m$

$\theta_m = 23.22°$

$P_m = EI_m \cos\theta_m$

$= (220)(1.45)(\cos 23.22°)$

$P_m = 293.16\ W$

$P_m = I_m^2 R_m$

$R_m = \dfrac{P_m}{I_m^2} = \dfrac{293.16}{(1.45)^2}$

$R_m = 139.43\ \Omega$

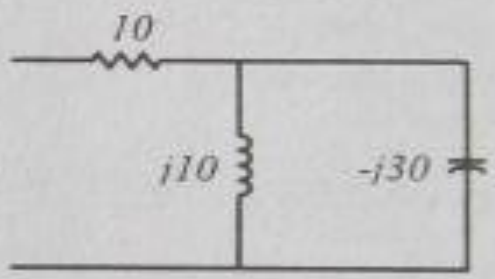
269 $Z_t = 10 + \dfrac{(j10)(-j30)}{j10 - j30}$

$Z_t = 10 + \dfrac{-j^2 300}{-j20} = 10 + j15$

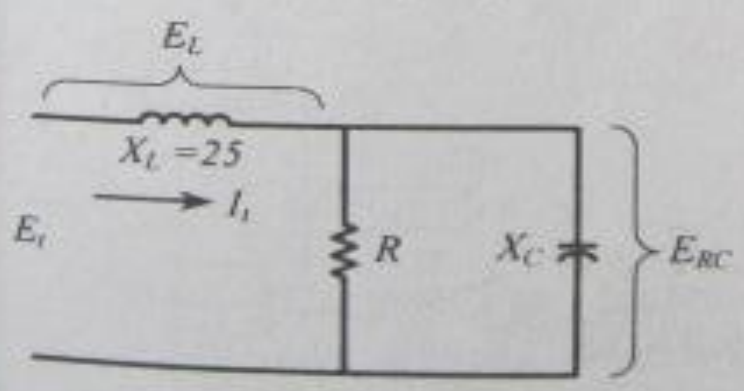
$Z_t = 18.027\angle 56.31°$

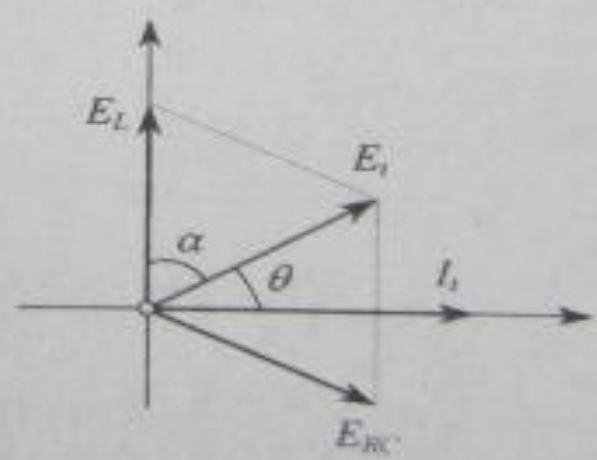
pf = cos 56.31°

pf = 0.555 lagging



270 Note: Refer to the vector diagram, since E_L, E_{RC} and E_t are equal, then the triangle formed by adding these vectors is an equilateral triangle with each interior angle (α) equal to 60°. And by inspection $\theta = 30°$.





$I_t = \dfrac{E_L}{X_L} = \dfrac{110}{25} = 4.4\ A$

$P = E_t I_t \cos\theta$

$= (110)(4.4)\cos 30°$

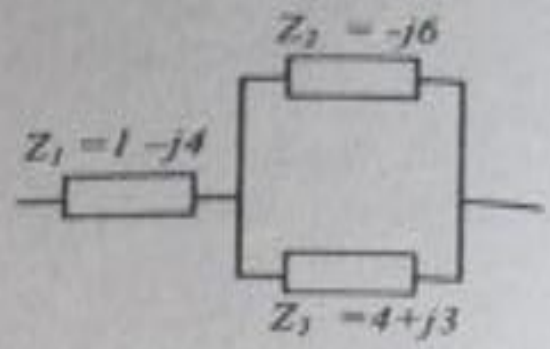
$P = 419.15\ W$

271

$Z_1 = 1 - j4 = 4.123\angle -75.963^\circ\ \Omega$

$Z_2 = -j6 = 6\angle -90^\circ\ \Omega$

$Z_3 = 4 + j3 = 5\angle 36.869^\circ\ \Omega$



$$Z_t = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= (1 - j4) + \frac{(6\angle -90^\circ)(5\angle 36.869^\circ)}{-j6 + 4 + j3}$$

$$= (1 - j4) + \frac{30\angle -53.13^\circ}{5\angle -36.869^\circ} = (1 - j4) + 6\angle -16.261^\circ$$

$$= (1 - j4) + (5.76 - j1.68)$$

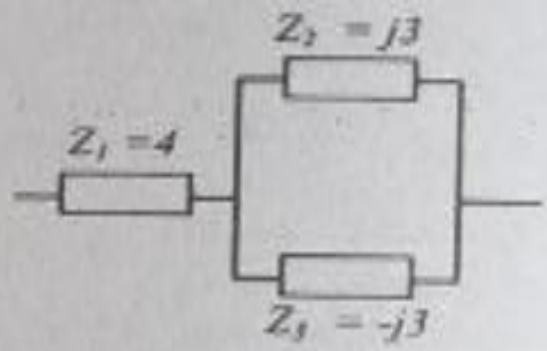
$Z_t = 6.76 - j5.68\ \Omega$

272

$Z_1 = 4 + j0$

$Z_2 = j3 = 3\angle 90^\circ$

$Z_3 = -j4 = 4\angle -90^\circ$



$$Z_t = Z_1 + \frac{Z_2 Z_3}{Z_2 Z_3} = 4 + \frac{(3\angle 90^\circ)(4\angle -90^\circ)}{j3 - j4}$$

$$= 4 + \frac{12\angle 0}{1\angle -90^\circ} = 4 + 12\angle 90^\circ = 4 + j12$$

$Z_t = 12.65\angle 71.565^\circ\ \Omega$

$pf_t = \cos(71.565^\circ)$

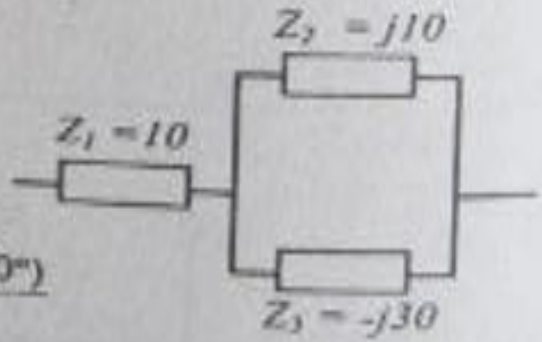
$pf_t = 0.316$ lagging

273

$Z_1 = 10 + j0$

$Z_2 = j10 = 10\angle 90^\circ$

$Z_3 = -j30 = 30\angle -90^\circ$



$$Z_t = Z_1 + \frac{Z_2 Z_3}{Z_2 Z_3} = 10 + \frac{(10\angle 90^\circ)(30\angle -90^\circ)}{j10 - j30}$$

$$= 10 + \frac{300\angle 0}{20\angle -90^\circ} = 10 + 15\angle 90^\circ = 10 + j15$$

$Z_t = 18.027\angle 56.31^\circ\ \Omega$

$$I_t = \frac{E_t}{Z_t} = \frac{240}{18.027} = 13.31\text{ A}$$

$P = E_t I_t pf_t = (240)(13.31)\cos(56.31^\circ) = 1771.93$ W

$P = 1.77$ kW

274 $Z_t = Z_1 + \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

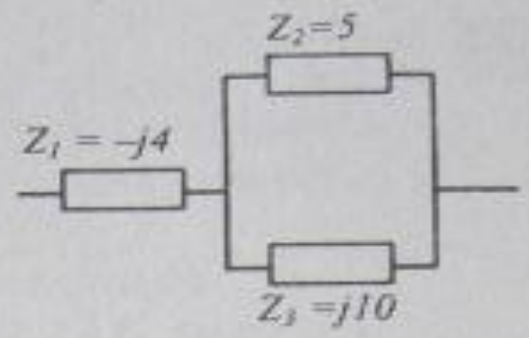
$= -j4 + \dfrac{(5)(j10)}{5 + j10} = -j4 + \dfrac{50\angle 90^\circ}{11.18\angle 63.43^\circ}$

$= -j4 + 4.472\angle 26.57^\circ = -j4 + 4 + j2$

$Z_t = 4 - j2 = 4.472\angle -26.56^\circ$

$I_t = \dfrac{E_t}{Z_t} = \dfrac{100}{4.472}$

$I_t = 22.36\ \text{A}$



275 $Z_{eq} = \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

$= \dfrac{(5)(j10)}{5 + j10} = \dfrac{50\angle 90^\circ}{11.18\angle 63.43^\circ}$

$Z_{eq} = 4.472\angle 26.57^\circ = 4 + j2$

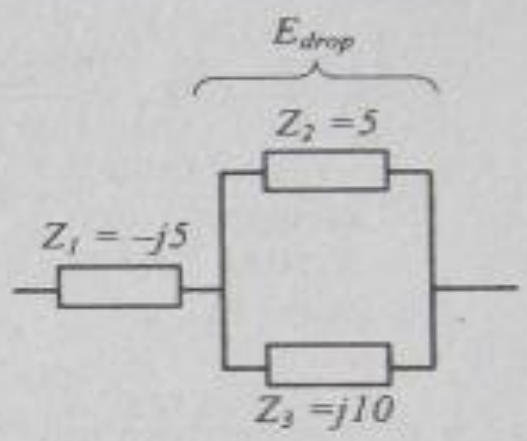
$Z_t = Z_{eq} + Z_1 = 4 + j2 + -j5 = 4 - j3$

$Z_t = 5\angle -36.869^\circ\Omega$

$I_t = \dfrac{E_t}{Z_t} = \dfrac{100}{5\angle -36.869^\circ} = 20\angle 36.869^\circ\ \text{A}$

$E_{drop} = I_t Z_{eq} = (20)(4.472)$

$E_{drop} = 89.44\ \text{V}$



276 $Z_{eq} = \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

$= \dfrac{(-j6)(4 + j3)}{-j6 + 4 + j3}$

$= \dfrac{(6\angle -90^\circ)(5\angle 36.869^\circ)}{5\angle -36.869}$

$= 6\angle -16.26^\circ$

$Z_{eq} = 5.76 - j1.68$

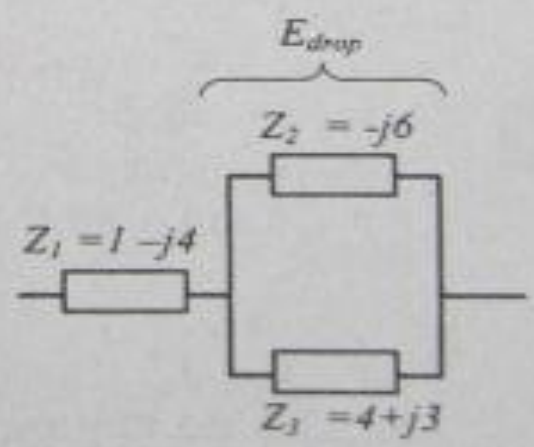
$Z_t = Z_{eq} + Z_1$

$= (5.76 - j1.68) + (1 - j4) = 6.76 - j5.68 = 8.83\angle -40.038^\circ$

$I_t = \dfrac{E_t}{Z_t} = \dfrac{230}{8.83\angle -40.038^\circ} = 26.047\angle 40.038^\circ\ \text{A}$

$E_{drop} = I_t Z_{eq} = 26.047(6)$

$E_{drop} = 156.3\ \text{V}$



277

$Z_1 = 30 + j0$

$X_L = wL = (2\pi)(60)(0.106)$

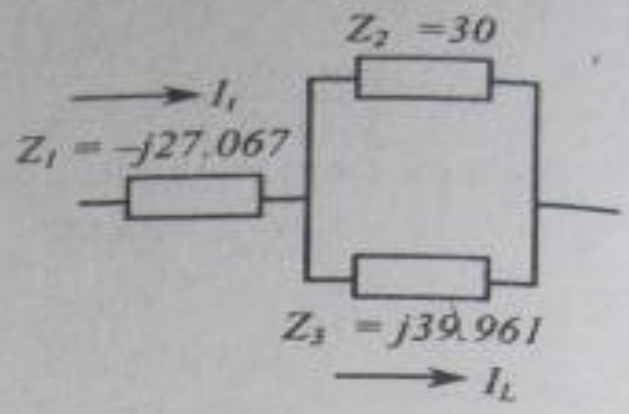
$X_L = 39.961\Omega$

$Z_2 = j39.961 = 39.961\angle 90°$

$$X_C = \frac{1}{wC} = \frac{1}{2\pi(60)(98 \times 10^{-6})}$$

$= 27.067\,\Omega$

$Z_3 = -j27.067$



$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3 = \frac{(30\angle 0°)(39.961\angle 90°)}{30 + j39.961} - j27.067$$

$$= \frac{1198.83\angle 90°}{49.968\angle 53.103°} - j27.067 = 24\angle 36.897° - j27.067$$

$= 19.193 + j14.409 - j27.067$

$Z_t = 23\angle -33.405°$

$$I_t = \frac{E_t}{Z_t} = \frac{125}{23\angle -33.405°} = 5.43\angle 33.405°\text{ A}$$

By CDT :

$$I_L = \frac{I_t Z_1}{Z_1 + Z_2} = \frac{(5.43\angle 33.405°)(30\angle 0°)}{49.968\angle 53.103°} = 3.26\angle -19.698°$$

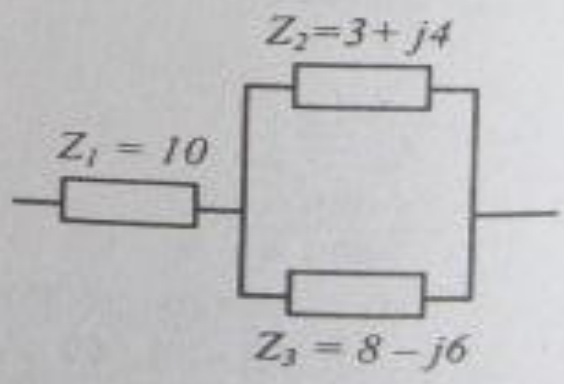
$I_L = 3.26\text{ A}$

278

$Z_1 = 10 + j0 = 10\angle 0°$

$Z_2 = 3 + j4 = 5\angle 53.13°$

$Z_3 = 8 - j6 = 10\angle -36.869°$



$$Z_t = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= 10 + \frac{(5\angle 53.13°)(10\angle -36.869°)}{3 + j4 + 8 - j6}$$

$$= 10 + \frac{50\angle 16.261°}{11.18\angle -10.304°} = 10 + 4.472\angle 26.565°$$

$Z_t = 10 + 4 + j2 = 14.142\angle 8.13°$

$$P = \frac{E_t^2}{Z_t}\cos\theta = \frac{120^2}{14.142}\cos(8.13°)$$

$P = 1008$ watts

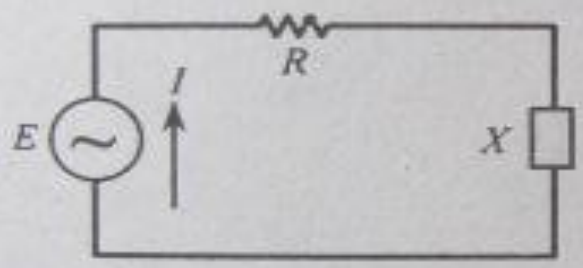
1-phase AC Circuits: Part 3

ADMITTANCE, CONDUCTANCE, SUSCEPTANCE

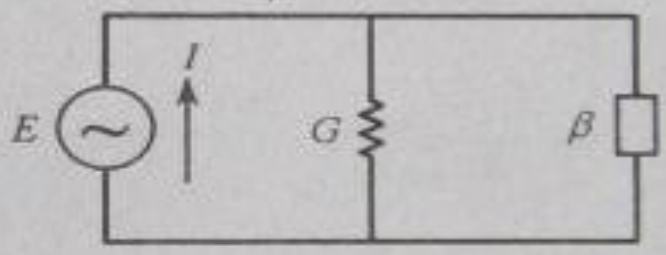
- **Admittance** – defined as the reciprocal of impedance
- **Siemens** (formerly **mho**) – unit of admittance. Named after the German engineer, **Ernst Werner von Siemens** (1816-1892)

$$Y = \frac{1}{Z}$$

- **Series circuit**



Equivalent circuit

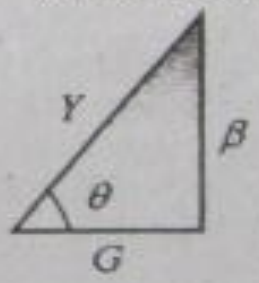


Note: X is reactance, either inductive or capacitive

$$G = \frac{R}{R^2 + X^2} \qquad \beta = \frac{X}{R^2 + X^2}$$

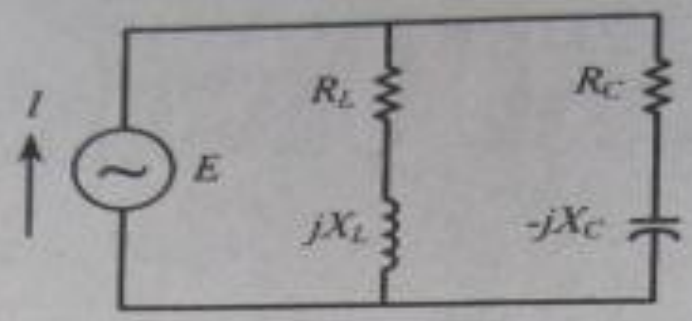
$$Y^2 = G^2 + \beta^2 \qquad Y = G \pm j\beta \qquad I = EY$$

Admittance triangle

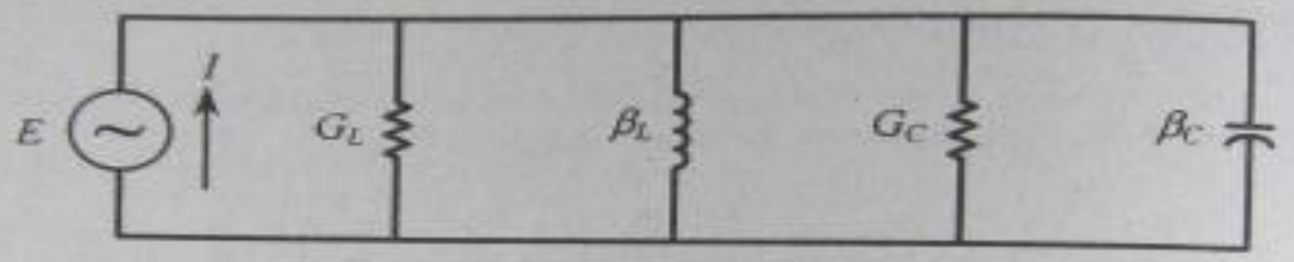


where: G = conductance (siemens)
β = susceptance (siemens)
Y = admittance (siemens)
Z = impedance (ohm)
I = current (ampere)
E = supply voltage (volt)
R = resistance (ohm)
X = reactance (ohm)
+ jβ = if X is a capacitive reactance
- jβ = if X is an inductive reactance

- **Parallel circuit:**



Equivalent circuit



$$G_L = \frac{R_L}{R_L^2 + X_L^2} \qquad G_C = \frac{R_C}{R_C^2 + X_C^2} \qquad G_t = G_L + G_C$$

$$\beta_t = \beta_C - \beta_L$$

$$\beta_L = \frac{X_L}{R_L^2 + X_L^2} \qquad \beta_C = \frac{X_C}{R_C^2 + X_C^2}$$

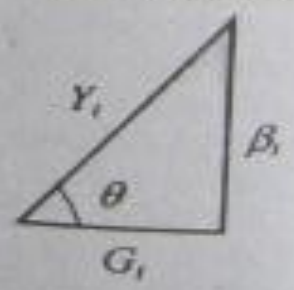
$$I = EY_t$$

$$Y_t = G_t \pm j\beta_t \qquad Y_t^2 = G_t^2 + \beta_t^2$$

where: G_t = total conductance (siemens)
Y_t = total admittance (siemens)
β_t = total susceptance (siemens)
$+ j\beta_t$ = if $\beta_C > \beta_L$
$- j\beta_t$ = if $\beta_C < \beta_L$

$$\theta = \tan^{-1}\frac{\beta_t}{G_t} \qquad pf = \frac{G_t}{Y_t}$$

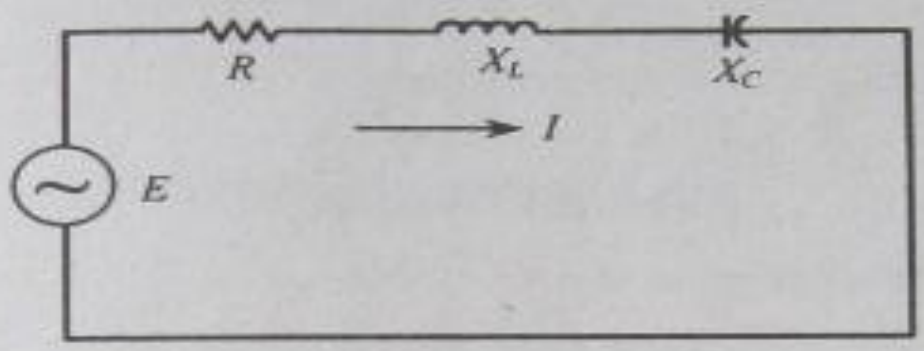
Admittance triangle



SERIES RESONANCE

- **Resonance** – a circuit phenomenon wherein the circuit power factor becomes unity.

Series resonance characteristics:
- total impedance is minimum
- total current is maximum

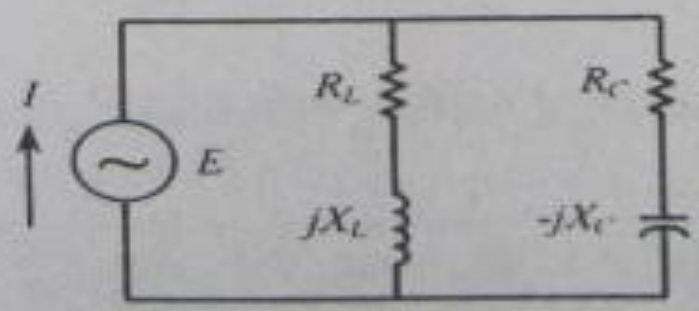


$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad I = \frac{E}{R}$$

$$Q - \text{factor} = \frac{\text{voltage drop across L or C}}{\text{total voltage}} \qquad X_L = X_C$$

PARALLEL RESONANCE OR ANTI-RESONANCE

Parallel resonance characteristics:
- total impedance is maximum
- total current is minimum



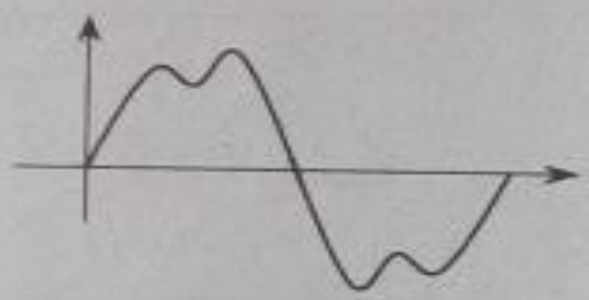
$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2C - L}{R_C^2C - L}} \qquad I = E\left[\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}\right]$$

$$Q - \text{factor} = \frac{\text{current in L or C}}{\text{total current}} \qquad \beta_L = \beta_C$$

where : f_r = resonant frequency (hertz)
Q-factor = selectivity of the resonant circuit

NON-SINUSOIDAL WAVE

It is a distorted (not perfect) sinusoidal waveform.



Note: A non-sinusoidal wave can be expressed as a function represented by several waves, a constant (DC component) plus sinusoidal waves of different frequencies or harmonics.

Examples of non-sinusoidal voltage and current equations:

$$e = E_{dc} + E_{m1}\sin(wt \pm \alpha_1) + E_{m2}\sin(2wt \pm \alpha_2) + \ldots\ldots + E_{mn}\sin(nwt \pm \alpha_n)$$
$$i = I_{dc} + I_{m1}\sin(wt \pm \beta_1) + I_{m2}\sin(2wt \pm \beta_2) + \ldots\ldots + I_{mn}\sin(nwt \pm \beta_n)$$

This general expression was first imposed by the French mathematician, **Jean Baptiste Joseph Fourier** (1768 – 1830)

RMS VALUE OF A NON-SINUSOIDAL WAVE

$$I_{rms} = \sqrt{(I_{dc})^2 + \frac{1}{2}\left[(I_{m1})^2 + (I_{m2})^2 + \ldots + (I_{mn})^2\right]}$$

$$I_{rms} = \sqrt{(I_{dc})^2 + (I_1)^2 + (I_2)^2 + \ldots + (I_n)^2}$$

$$E_{rms} = \sqrt{(E_{dc})^2 + \frac{1}{2}\left[(E_{m1})^2 + (E_{m2})^2 + \ldots + (E_{mn})^2\right]}$$

$$E_{rms} = \sqrt{(E_{dc})^2 + (E_1)^2 + (E_2)^2 + \ldots + (E_n)^2}$$

where: E_{dc}, I_{dc} = DC voltage and current
E_1, E_2, I_1, I_2 = rms values of voltages and currents
E_{m1}, E_{m2}, I_{m1}, I_{m2} = maximum values of voltages and currents

Note: Subscript 1, means first harmonic component or fundamental component
Subscript 2, means second harmonic component
....etc

ACTIVE POWER DRAWN BY THE CIRCUIT

$$P = E_{dc}I_{dc} + \frac{E_{m1}I_{m1}}{2}\cos(\alpha_1 - \beta_1) + \frac{E_{m2}I_{m2}}{2}\cos(\alpha_2 - \beta_2) + \ldots\ldots$$

$$P = E_{dc}I_{dc} + E_1I_1\cos(\alpha_1 - \beta_1) + E_2I_2\cos(\alpha_2 - \beta_2) + \ldots\ldots$$

$$P = I_{rms}^2R_t \qquad P = I_{rms}E_{rms}\,pf$$

where: P = active power (watt)
R_t = equivalent resistance of the circuit (ohm)
pf = power factor

Note: In dealing with circuits supplied by non-sinusoidal source(s), use Superposition Theorem to solve the problem. As if, analyzed the circuit with the different harmonics including the DC component as the only source that supplies energy to the circuit.

Test 9

Problem 279: EE Board October 1993

If admittance Y = 0.06 – j 0.08 mho, then conductance G equals,

A. – 0.06
B. 0.06
C. 0.08
D. – 0.08

Problem 280: EE Board October 1986, EE Board April 1993

A parallel circuit consists of a resistor having a conductance of 4 mhos, an inductive reactor having a susceptance of 8 mhos and a capacitive reactor having a susceptance of 5 mhos. What is the impedance of the circuit?

A. 0.11 + j0.13
B. 0.13 + j0.11
C. 0.12 + j0.16
D. 0.16 + j0.12

Problem 281:

Two parallel branches have admittances 0.3 + j0.4 and 0.2 – j0.25 , respectively. If the current in the first branch is 10 A, determine the total current supplied to the parallel combination.

A. 10.44 A
B. 12.10 A
C. 15.32 A
D. 11.24 A

Problem 282: EE Board October 1994

A capacitor branch having a ratio of X_C to R of 5 is paralleled with an impedance consisting of a 4 Ω resistance and a 3 Ω inductive reactance. The power factor of the resulting circuit is 0.8 leading. Find the size of the capacitor in microfarads if the frequency is 60 Hz.

A. 879.9
B. 1078.9
C. 978.9
D. 778.9

Problem 283:

Two impedances $Z_1 = 2 + j\,4$ and $Z_2 = R + j0$ are connected in parallel. Determine R so that the power factor of the circuit is 0.9 lagging.

A. 3.2 ohms
B. 2.1 ohms
C. 2.8 ohms
D. 2.5 ohms

Problem 284:

What value of resistance should be placed in parallel with a 50 μF capacitor in order to have a total power factor of 0.8 on a 60-cycle AC system?

A. 50 Ω
B. 40 Ω
C. 42 Ω
D. 47 Ω

Problem 285: EE Board April 1993

Capacitor of 30-microfarad capacitance is in series with a coil across an 8,000-cycle supply. What inductance is required for resonance?

A. 14.34 μH
B. 10.45 μH
C. 13.19 μH
D. 12.55 μH

Problem 286:

A series RLC circuit is connected across a 110-V, variable frequency source. When the frequency is varied, the maximum effective current is 5.5 A and occurs at a frequency of 45 cps. At this frequency, the voltage across the capacitor is 200.8 V. Find the pf of the circuit at 60 Hz.

A. 0.686 lagging
B. 0.770 lagging
C. 0.673 lagging
D. 0.602 lagging

Problem 287: EE Board October 1998

One leg of a radio-tuned circuit has a capacitance of one times ten to the minus nine farad. It is tuned at 200 kHz, what is the inductance of the other leg in henry?

A. 6.33×10^{-4}
B. 8.25×10^{-5}
C. 20×10^{-3}
D. 120×10^{-3}

Problem 288: EE Board April 1988

A loud speaker whose inductance is 1.15 henries is coupled to a power tube through a condenser of 2 μF capacity. To what frequency will the combination be resonant?

A. 110 Hz
B. 108 Hz
C. 105 Hz
D. 100 Hz

Problem 289: EE Board April 1995

What capacitance must be placed in series with an inductance of 0.05 H so that at 100 Hz, the impedance becomes equal to the ohmic resistance?

A. 50.7 μF
B. 35.5 μF
C. 70.7 μF
D. 87.0 μF

Problem 290: EE Board April 1989

A coil has a resistance of 50 ohms and a reactance of 100 ohms, is shunted by a capacitor, which has practically no losses in order that the voltage across the coil be in phase with the total current supplied to the parallel combination. What is the impedance of the parallel combination under the given condition?

A. 250 ohms
B. 200 ohms
C. 230 ohms
D. 220 ohms

Problem 291: EE Board April 1983

A non-inductive resistor R is connected in parallel with an inductive reactance of 10 ohms. The combination is then connected in series with a capacitive reactance of 5 ohms. Solve for R at which the power factor of the given circuit would be unity.

A. 10 Ω
B. 12 Ω
C. 13 Ω
D. 11 Ω

Problem 292:

A 10-ohm resistor is connected in parallel with an inductive reactance of 10 ohms. The combination is then connected in series with a capacitive reactance of 6 ohms. Solve for the frequency at resonance. The indicated values have been derived at 50 Hz.

A. 68.08 Hz
B. 61.22 Hz

C. 67.51 Hz
D. 64.23 Hz

Problem 293:

A 300-ohm inductive reactance is connected across a series combination of a resistor R and a capacitive reactance X_C. Solve for R if the equivalent impedance of the combination is a pure resistance of 100 ohms.

A. 92 ohms
B. 88 ohms
C. 78 ohms
D. 90 ohms

Problem 294: EE Board October 1982

Two impedances $Z_1 = 15 + j20$ and $Z_2 = 5 - j X_C$ are connected in parallel. Solve for the value of X_C so that the total current drawn by the combination will be in phase with any supply voltage V.

A. 28.54 Ω
B. 30.43 Ω
C. 33.12 Ω
D. 29.55 Ω

Problem 295:

A coil whose resistance is 10 Ω and inductance of 50 mH is connected in parallel with a series combination of 5 Ω resistance and a variable capacitor C. The circuit is supplied by a 220-V, 60 Hz frequency. Calculate the power (minimum value for X_C) delivered by the source at resonance.

A. 10.31 kW
B. 15.35 kW
C. 14.65 kW
D. 11.07 kW

Problem 296: EE Board April 1985

A resistance of 5 ohms is connected in series with a capacitor of 442.1 μF. The combination is then connected in parallel with an inductance of 21.22 mH. Solve for the frequency of the impressed voltage with which the inductive reactance is equal to the capacitive reactance in magnitude.

A. 50 Hz
B. 51 Hz
C. 52 Hz
D. None of these

Problem 297:

An impedance of 4 – j3 ohms is connected in parallel with an ideal inductor L across a 110 V, 60 Hz mains. Determine L if the total voltage supplied to the circuit will be in phase with the total current drawn.

A. 16.6 mH
B. 19.2 mH
C. 22.1 mH
D. 26.8 mH

Problem 298: EE Board April 1989

A coil has a resistance of 50 ohms and a reactance of 100 ohms, is shunted by a capacitor, which has practically no losses. What must be the reactance of the capacitor in order that the voltage across the coil be in phase with the total current supplied to the parallel combination?

A. 120 ohms
B. 127 ohms
C. 125 ohms
D. 132 ohms

Problem 299: EE Board April 1982

Three impedances Z_a, Z_c, Z_r are connected in parallel. If at 60 Hz, $Z_a = 0 + j8$, $Z_c = 0 - j2$ and $Z_r = 5 + j0$ ohms. Solve for the frequency at resonance.

A. 30 Hz
B. 34 Hz
C. 36 Hz
D. 28 Hz

Problem 300:

A resistance R is connected in parallel with a variable inductive reactance X_L. The combination is connected in series with impedance of 5 – j2 ohms. Solve for R such so there is only one condition of resonance as X_L as varied.

A. 2 ohms
B. 5 ohms
C. 3 ohms
D. 4 ohms

Problem 301: EE Board April 1981

A resistor R is connected in parallel with a 20-ohm inductive reactance. The combination is then connected in series with 5-ohm capacitive reactance. Solve for the value of R at which the power factor of the resultant impedance is unity.

A. 10.05 Ω
B. 9.15 Ω
C. 11.55 Ω

D. 10.73 Ω

Problem 302:

A coil consumes 2,000 watts, while drawing a current of 25 A from a 110 V, 100-Hz AC source. Determine the capacitance of a capacitor to be connected in parallel with the coil in order to make the total current be in phase with the supply voltage.

A. 236 μF
B. 240 μF
C. 248 μF
D. 205 μF

Problem 303:

A coil having an impedance of 10 + j5 ohms is connected in parallel with a series combination of a variable resistance R and a capacitor having a reactance of 8 ohms. The combination is connected across a 24-V AC source. Determine the total current drawn if R is adjusted in order to bring the total pf of the circuit to unity.

A. 3.32 A
B. 4.54 A
C. 2.28 A
D. 4.07 A

Problem 304: EE Board October 1998

A coil has a resistance of 50 ohms and a reactance of 70 ohms. A capacitor is connected in parallel to produce resonance. The source voltage is 120 V. What is the power drawn by the circuit?

A. 162 W
B. 97 W
C. 132 W
D. 52 W

Problem 305:

A coil whose resistance is 100 Ω and inductance of L_1 is connected in parallel with a series combination of inductance L_2 and capacitance C. When $w = 1 \times 10^5$, the impedance of the whole combination is 300 ohms, purely resistive. When $w = 2 \times 10^5$ no current flows towards the coil. Find the value of C.

A. 38.23 nF
B. 34.02 nF
C. 35.36 nF
D. 32.77 nF

Problem 306:

A series RLC circuit has a Q of 5.0 at its resonance frequency of 100 kHz. Assuming the power dissipation of the circuit is 100 W when drawing a current of 0.8 A, determine the capacitance C of the circuit.

A. 2.04 nF
B. 2.32 nF
C. 3.02 nF
D. 2.54 nF

Problem 307:

An impedance coil draws an apparent power of 50 volt-amperes and an active power of 40 watts. Solve for the Q-factor of the coil.

A. 0.6
B. 1.25
C. 0.8
D. 0.75

Problem 308: EE Board April 1995

A coil is supplied with 200 volts and takes a current (rms) of 2 amperes at 0.707 lagging. The quality factor (Q) of the coil is

A. 25
B. 1
C. 10
D. 100

Problem 309:

A non-inductive resistor of 10 ohms requires a current of 8 A and is to be feed from a 200 V, 50 Hz supply. If a choking coil of effective resistance 1.2 ohms is used to cut down the voltage, find the required Q-factor of the coil.

A. 18.6
B. 14.2
C. 20.3
D. 16.7

Problem 310:

The total current in a given circuit has the equation:
$i = 10 + 16 \sin wt - 8 \sin 2wt + 5 \cos 3wt$. Calculate the effective value of the current.

A. 18.3 A
B. 12.2 A
C. 16.5 A
D. None of these

Problem 311: EE Board April 1992

Determine rms value of the current drawn by a 2 μF condenser, which is connected across a source of potential. The potential has a third and fifth harmonic components, which are 30% and 20% respectively of the fundamental. The fundamental sinusoidal component has a peak value of 1000 volts and 60 Hz frequency.

A. 0.89 A
B. 0.75 A
C. 0.91 A
D. 0.84 A

Problem 312: EE Board October 1992

A capacitor of 3.18 microfarads is connected in parallel with a resistance of 2,000 ohms. The combination is further connected in series with an inductance of 795 mH and resistance of 100 ohms across a supply given by e = 400 sinwt + 80 sin (3wt + 60°). Assume w = 314 radians/sec. Determine the rms value of the total current.

A. 0.40 A
B. 0.33 A
C. 0.56 A
D. 0.45 A

Problem 313: EE Board April 1991

Determine the effective value of the circuit current of an emf of 151 sin 377t is connected in series with a DC emf of 110 volts. Both supply a load of 10 + j8 ohms.

A. 10.3 A
B. 12.5 A
C. 13.8 A
D. 11.4 A

Problem 314: EE Board April 1994

An alternating current and a direct current flows simultaneously in the same conductor. If the effective value of the alternating current is 5 A and the direct current is 10 A, what will an AC ammeter read when connected in the circuit?

A. 7.5 A
B. 15 A
C. 11.18 A
D. None of these

Problem 315: EE Board April 1993

A 100∠0° V 120 Hz generator and a 80∠0° V 60 Hz generator are connected in series with a 60 V battery and a coil. The resistance and inductance of the coil are 3 Ω and 2.65 mH, respectively. Determine the rms current of the coil.

A. 42.54 A
B. 44.24 A
C. 43.55 A
D. 40.44 A

Problem 316: EE Board April 1997

If e = 100 sin (wt +30°) – 50 cos 3wt + 25 sin (5wt +150°) and i = 20 sin (wt + 40°) + 10 sin (3wt +30°) – 5 sin (5wt – 50°). Calculate the power in watts.

A. 1177
B. 918.5
C. 1043
D. 922.4

Problem 317:

Find the average power in a resistance R = 10 ohms, if the current in the Fourier- series form is i = 12 sinwt + 8 sin3wt + 3 sin 5wt amperes.

A. 1085 W
B. 1203 W
C. 1150 W
D. 1027 W

Problem 318: EE Board October 1992

A capacitor of 3.18 microfarads is connected in parallel with a resistance of 2,000 ohms. The combination is further connected in series with an inductance of 795 mH and resistance of 100 ohms across a supply given by e = 400 sinwt + 80 sin (3wt + 60°). Assume w = 314 radians/sec. Determine the power dissipated.

A. 74.66 W
B. 78.05 W
C. 80.28 W
D. 75.66 W

Problem 319:

A series RL circuit in which R = 5 Ω and L = 20 mH has an applied voltage e = 100 + 50 sin wt + 25 sin 3wt, with w = 500 radians per sec. Determine the power dissipated in the resistor of the circuit.

A. 2510 W
B. 2234 W
C. 2052 W
D. 2302 W

Problem 320:

Three sinusoidal generators and a battery are connected in series with a coil whose resistance and inductance are 8 Ω and 26.53 mH, respectively. The frequency and rms voltages of the respective generators are 15 V, 20 Hz; 30 V, 50 Hz and 40 V, 100 Hz. The open circuit of the battery is 6 V. Neglect internal resistance of the battery. Find the apparent power delivered by the circuit.

A. 194.4 VA
B. 178.5 VA

C. 198.3 VA
D. 182.7 VA

Problem 321:

A series circuit containing a 295 μF capacitor and a coil whose resistance and inductance are 3 Ω and 4.42 mH, respectively are supplied by the following series connected generators: 35 V at 60 Hz, 10 V at 180 Hz and 8 V at 240 Hz. Determine the power factor of the circuit.

A. 0.486
B. 0.418
C. 0.465
D. 0.437

Problem 322: EE Board October 1992

A capacitor of 3.18 microfarads is connected in parallel with a resistance of 2,000 ohms. The combination is further connected in series with an inductance of 795 mH and resistance of 100 ohms across a supply given by e = 400 sinwt + 80 sin (3wt + 60°). Assume w = 314 radians per sec. Determine the circuit power factor.

A. 0.702
B. 0.650
C. 0.633
D. 0.612

ANSWER KEY

279. B	288. C	297. C	306. A	315. A
280. D	289. A	298. C	307. D	316. B
281. A	290. A	299. A	308. B	317. A
282. D	291. A	300. D	309. A	318. A
283. A	292. B	301. C	310. C	319. C
284. B	293. D	302. C	311. A	320. A
285. C	294. B	303. A	312. A	321. D
286. A	295. A	304. B	313. C	322. B
287. A	296. C	305. C	314. C	

☑ RATING

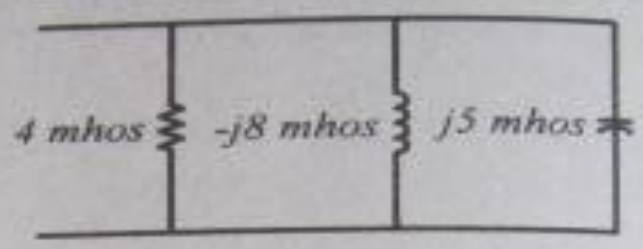
39-44 - Topnotcher
30-38 - Passer
22-29 - Conditional
0-21 - Failed

Solutions to Test 9

279 By inspection, conductance (G) is equal to 0.06 mho while the inductive susceptance (β) is equal to 0.08 mho.

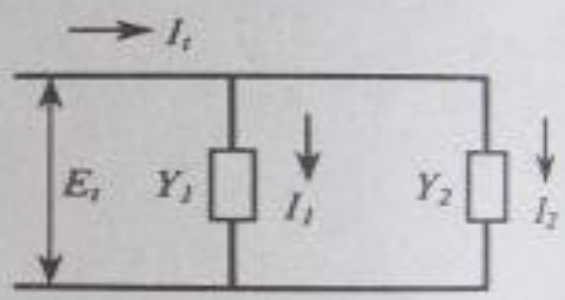
280

$$Y_t = G_t \pm j\beta_t$$
$$= 4 - j8 + j5$$
$$Y_t = 4 - j3 = 5\angle -36.869^\circ$$
$$Z_t = \frac{1}{Y_t} = \frac{1}{5\angle -36.869^\circ}$$
$$= 0.2\angle 36.869^\circ$$
$$Z_t = 0.16 + j0.12 \text{ ohm}$$



281 Note: Since in parallel, the voltage drop across each branch is the same. Assume the voltage as reference.

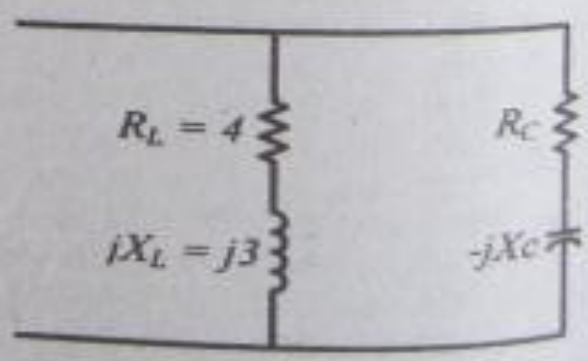
$$Y_1 = 0.03 + j0.04 = 0.05\angle 53.13^\circ$$
$$Y_2 = 0.02 - j0.025 = 0.032\angle -51.34^\circ$$
$$I = EY; \quad E = \frac{I}{Y}$$
$$E_t = \frac{I_1}{Y_1} = \frac{10}{0.05} = 200 \text{ V}$$
$$I_2 = E_t Y_2 = (200)(0.032) = 6.4 \text{ A}$$



$$I_t = I_1 + I_2 = 10\angle 53.13^\circ + 6.4\angle -51.34^\circ$$
$$= 6 + j8 + 4 - j5$$
$$I_t = 10 + j3 = 10.44\angle 16.7^\circ$$
$$I_t = 10.44 \text{ A}$$

282 Note: $X_C = 5R_C$

$$\beta_t = \beta_C - \beta_L$$
$$= \frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}$$
$$= \frac{5R_C}{R_C^2 + (5R_C)^2} - \frac{3}{3^2 + 4^2}$$
$$\beta_t = \frac{5R_c}{R_C^2 + (5R_C)^2} - 0.12$$



$$\beta_t = \frac{5R_c - 3.12R_c^2}{26R_c^2} \Rightarrow ❶$$

$$G_t = G_C + G_L = \frac{R_C}{R_C^2 + X_C^2} + \frac{R_L}{R_L^2 + X_L^2}$$

$$= \frac{R_C}{R_C^2 + (5R_C)^2} + \frac{4}{3^2 + 4^2}$$

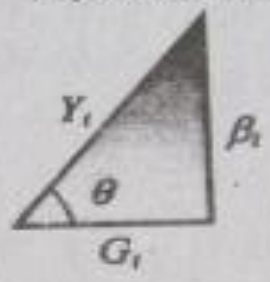
$$= \frac{R_c}{R_C^2 + (5R_C)^2} + 0.16$$

$$G_t = \frac{R_c + 4.16R_c^2}{26R_c^2} \Rightarrow ❷$$

$$\theta = \cos^{-1} 0.8 = 36.869°$$

$$\tan\theta = \frac{\beta_t}{G_t} \Rightarrow ❸$$

Admittance triangle



Substitute Eq.1 & Eq.2 in Eq.3 :

$$\tan 36.869° = \frac{5R_c - 3.12R_c^2}{R_c + 4.16R_c^2}$$

$$0.75 = \frac{5R_c - 3.12R_c^2}{R_c + 4.16R_c^2}$$

$$0.75R_C + 3.12R_C^2 = 5R_C - 3.12R_C^2$$

$$R_C = 0.681\,\Omega$$

$$X_C = 5R_C = 5(0.681) = 3.405\,\Omega$$

At 60 Hz, w = 377

$$C = \frac{1}{w X_C} = \frac{1}{377(3.405)} = 779\,\mu F$$

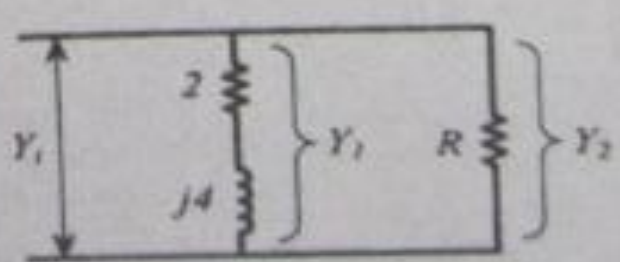
263 $$Y_1 = \frac{1}{2 + j4} = \frac{1}{4.472\angle 63.43°} = 0.2236\angle -63.43°$$

$$Y_1 = 0.1 - j0.2$$

$$Y_2 = \frac{1}{R}$$

$$Y_t = G_t \pm j\beta_t = \left(0.1 + \frac{1}{R}\right) - j0.2$$

$$\theta = \cos^{-1} 0.9 = 25.84°$$



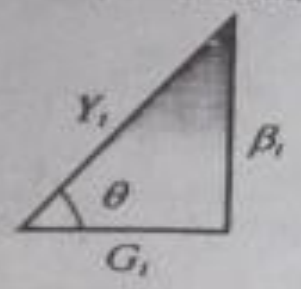
$$\tan\theta = \frac{\beta_t}{G_t}$$

$$\tan 25.84° = \frac{0.2}{0.1 + \frac{1}{R}}$$

$$\frac{1}{R} = \frac{0.2}{\tan 25.84°} - 0.1$$

$$R = 3.2\,\Omega$$

Admittance triangle



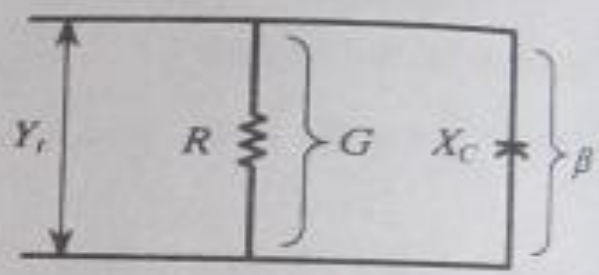
284 $$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60)(50 \times 10^{-6})} = 53.05\,\Omega$$

$$\beta_C = \frac{1}{X_C} = \frac{1}{53.05} = 0.0188 \text{ mho}$$

$$G = \frac{1}{R}$$

$$Y_t = G_t + j\beta_t = \left(\frac{1}{R}\right) + j0.0188$$

$$\theta = \cos^{-1} 0.8 = 36.869°$$



$$\tan\theta = \frac{\beta_t}{G_t}$$

$$\tan 36.869° = \frac{0.0188}{\frac{1}{R}}$$

$$\frac{1}{R} = \frac{0.0188}{\tan 36.869°}$$

$$R = 39.89 \cong 40\,\Omega$$

285 $$f = \frac{1}{2\pi\sqrt{LC}}$$

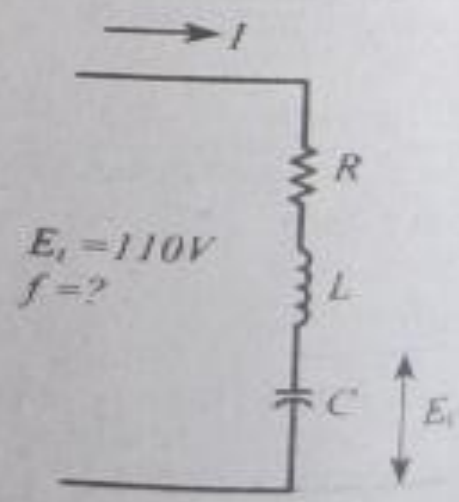
$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{(4\pi^2)(8000)^2(30 \times 10^{-6})}$$

$$L = 13.19\,\mu H$$

286 $$E_C = IX_C; \quad X_C = \frac{1}{2\pi fC}$$

$$C = \frac{I}{2\pi f E_C} = \frac{5.5}{(2\pi)(45)(200)} = 97.26\,\mu F$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$



$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{(4\pi^2)(45)^2(97.26 \times 10^{-6})}$$

$L = 0.1286\ H$

Note: At series resonance, $I = E/R$

$$R = \frac{E}{I} = \frac{110}{5.5} = 20\ \Omega$$

At 60 Hz:

$X_L = 2\pi(60)(0.1286) = 48.48\ \Omega$

$$X_C = \frac{1}{2\pi(60)(97.26 \times 10^{-6})} = 27.27\ \Omega$$

$Z = R + j(X_L - X_C) = 20 + j(48.48 - 27.27)$

$= 20 + j21.21$

$Z = 29.152\angle 46.68°$

$pf = \cos\theta = \cos 46.68°$

$pf = 0.686$ lagging

287 $$f = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{(2\pi f)^2 C} = \frac{1}{(4\pi^2)(200{,}000)^2(1 \times 10^{-9})}$$

$L = 6.33 \times 10^{-4}\ H$

288 $$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.15)(2 \times 10^{-6})}}$$

$= 104.9$

$f \cong 105\ Hz$

289 $$f = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(4\pi^2)(100)^2(0.05)}$$

$= 50.66 \times 10^{-6}$

$C \cong 50.7\ \mu F$

290 Note: Since the given condition is at resonance, the admittance of the circuit is equivalent to the total conductance.

$$Y_t = G_t = \frac{R_L}{R_L^2 + X_L^2} = \frac{50}{50^2 + 100^2} = 0.004 \text{ mho}$$

$$Z_t = \frac{1}{G_t} = \frac{1}{0.004}$$

$$Z_t = 250\ \Omega$$

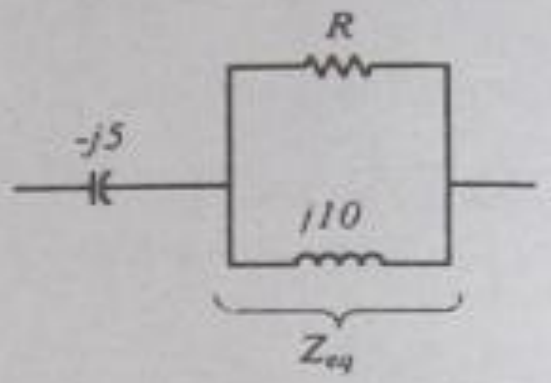
291

$$Z_{eq} = \frac{R(j10)}{R + j10}$$

$$= \frac{R(j10)}{R + j10} \bullet \frac{R - j10}{R - j10}$$

$$= \frac{j10R^2 - j^2 100R}{R^2 + 100}$$

$$Z_{eq} = \frac{100R}{R^2 + 100} + j\frac{10R^2}{R^2 + 100}$$



Note: Since the given condition is at resonance, the equivalent reactance of the parallel branch is equivalent to the series capacitive reactance.

$$\frac{10R^2}{R^2 + 100} = 5$$

$$10R^2 = 5R^2 + 500$$

$$R = 10\ \Omega$$

292

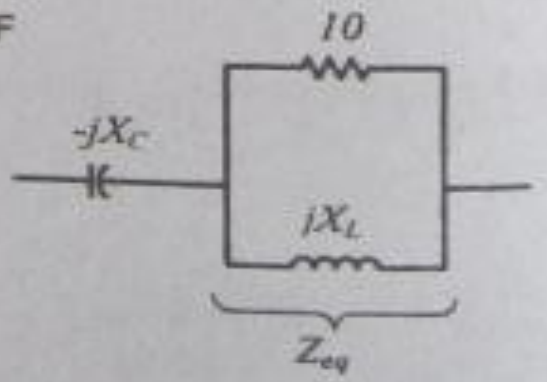
$$L = \frac{X_L}{2\pi f} = \frac{10}{2\pi(50)} = 0.0318 \text{ H}$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(6)} = 530.516\ \mu\text{F}$$

$$Z_{eq} = \frac{10(jX_L)}{10 + jX_L} \bullet \frac{10 - jX_L}{10 - jX_L}$$

$$= \frac{j100X_L - j^2 10X_L^2}{100 + X_L^2}$$

$$Z_{eq} = \frac{10X_L^2}{100 + X_L^2} + j\frac{100X_L}{100 + X_L^2}$$



Note: Since the given condition is at resonance, the equivalent reactance of the parallel branch is equivalent to the series capacitive reactance.

$$\frac{100X_L}{100 + X_L^2} = X_C$$

$$\frac{100wL}{100 + w^2L^2} = \frac{1}{wC}$$

$$100w^2LC = 100 + w^2L^2$$

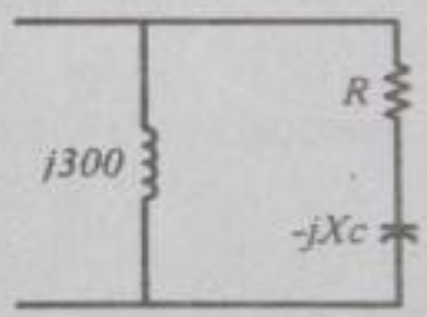
$$w = \sqrt{\frac{100}{100LC - L^2}} = \sqrt{\frac{100}{100(0.0318)(530.516 \times 10^{-6}) - (0.0318)^2}}$$

$$w = 384.672 \text{ rad/sec}$$

$$f = \frac{w}{2\pi} = \frac{384.672}{2\pi}$$

$$f = 61.22 \text{ Hz}$$

293. Note: For parallel resonance, the total conductance is equivalent to the reciprocal of the total impedance of the circuit.



$$G_t = G_C$$

$$\frac{1}{Z_t} = \frac{R}{R^2 + X_C^2}$$

$$\frac{1}{100} = \frac{R}{R^2 + X_C^2}$$

$$X_C^2 = 100R - R^2 \Rightarrow (1)$$

$$\beta_L = \beta_C$$

$$\frac{1}{X_L} = \frac{X_C}{R^2 + X_C^2}$$

$$\frac{1}{300} = \frac{X_C}{R^2 + X_C^2}$$

$$R^2 + X_C^2 = 300X_C \Rightarrow (2)$$

Substitute Eq.1 in Eq.2 :

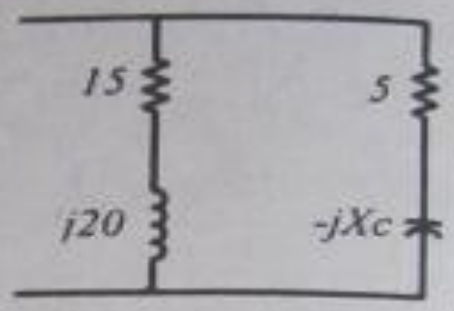
$$R^2 + 100R - R^2 = 300\sqrt{100R - R^2}$$

$$100^2R^2 = 300^2(100R - R^2)$$

$$R^2 = 900R - 9R^2$$

$$R = 90\,\Omega$$

294 $\beta_L = \beta_C$



$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\frac{20}{15^2 + 20^2} = \frac{X_C}{5^2 + X_C^2}$$

$$0.032 = \frac{X_C}{25 + X_C^2}$$

$$0.8 + 0.032X_C^2 = X_C$$

$$X_C^2 - 31.25X_C + 25 = 0$$

Using the quadratic formula :

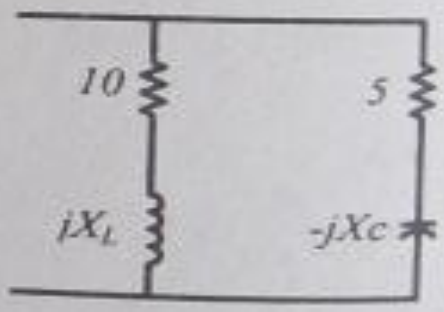
$$X_C = \frac{31.25 \pm \sqrt{(-31.25)^2 - 4(1)(25)}}{2(1)} = \frac{31.25 \pm 29.606}{2}$$

$$X_C^{(+)} = \frac{31.25 + 29.606}{2} = 30.43\,\Omega$$

$$X_C^{(-)} = \frac{31.25 - 29.606}{2} = 0.822\,\Omega$$

295 $X_L = 2\pi(60)(50 \times 10^{-3}) = 18.85\,\Omega$

$\beta_L = \beta_C$



$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\frac{18.85}{10^2 + 18.85^2} = \frac{X_C}{5^2 + X_C^2}$$

$$0.0414\,(25 + X_C^2) = X_C$$

$$X_C^2 - 24.15X_C + 25 = 0$$

Using the quadratic formula :

$$X_C = \frac{24.15 \pm \sqrt{(-24.15)^2 - 4(1)(25)}}{2(1)} = \frac{24.15 \pm 21.98}{2}$$

$$X_C^{(-)} = \frac{24.15 - 21.98}{2} = 1.085\,\Omega$$

$$G_1 = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}$$

$$G_t = \frac{10}{10^2 + 18.85^2} + \frac{5}{5^2 + 1.085^2}$$

$G_t = 0.213$ mho

$P_t = E_t^2 Y_t = (220)^2(0.213) = 10{,}309.2\text{W}$

$P_t \cong 10.31\text{kW}$

296 Note: The question of the problem is referring to the condition wherein the magnitudes of the "inductive reactance" and the "capacitive reactance" are equal. This is a concept of series resonance ($X_L = X_C$).

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(21.22 \times 10^{-3})(442.1 \times 10^{-6})}}$$

$= 51.96$

$f \cong 52$ Hz

297 $\beta_L = \beta_C$

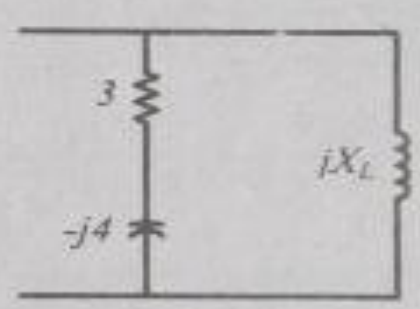
$$\frac{1}{X_L} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\frac{1}{X_L} = \frac{3}{4^2 + 3^2}$$

$X_L = 8.33\ \Omega$

$$L = \frac{X_L}{2\pi f} = \frac{8.33}{2\pi(60)}$$

$L = 22.1$mH

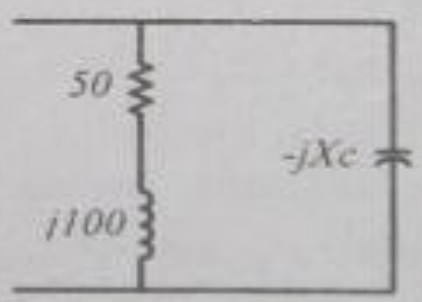


298 $\beta_L = \beta_C$

$$\frac{X_L}{R_L^2 + X_L^2} = \frac{1}{X_C}$$

$$\frac{100}{50^2 + 100^2} = \frac{1}{X_C}$$

$X_C = 125\ \Omega$

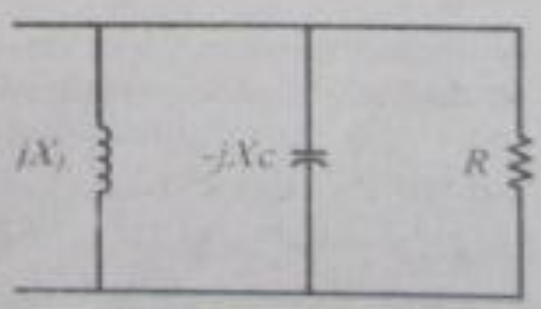


299 $$L = \frac{X_L}{2\pi f} = \frac{8}{2\pi(60)}$$

$L = 0.0212$ H

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)2}$$

$C = 1326.29\ \mu$F

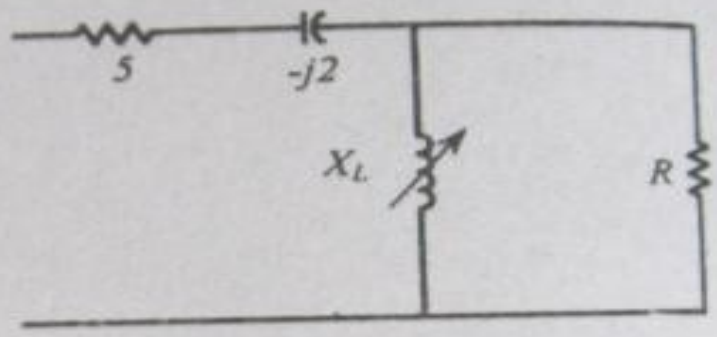


Note: At resonance, equivalent susceptance is zero. Only the first two branches constitute a susceptance since the third branch is a resistor.

$$B_L = B_C$$
$$\frac{1}{X_L} = \frac{1}{X_C}$$
$$X_L = X_C$$
$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.0212)(1326.29 \times 10^{-6})}}$$
$$f = 30\ \text{Hz}$$

300

$$Z_{eq} = \frac{R(jX_L)}{R + jX_L}$$
$$= \frac{R(jX_L)}{R + jX_L} \cdot \frac{R - jX_L}{R - jX_L}$$
$$= \frac{jX_L R^2 - j^2 R X_L^2}{R^2 + X_L^2}$$
$$Z_{eq} = \frac{RX_L^2}{R^2 + X_L^2} + j\frac{R^2 X_L}{R^2 + X_L^2}$$



Note: At resonance, the equivalent reactance of the parallel branch is equivalent to the reactance of the series impedance

$$\frac{R^2 X_L}{R^2 + X_L^2} = 2$$
$$R^2 X_L = 2R^2 + 2X_L^2$$
$$X_L^2 - 0.5R^2 X_L + R^2 = 0$$

Using the quadratic formula :

$$X_L = \frac{0.5R^2 \pm \sqrt{(-0.5R^2)^2 - 4(1)(R^2)}}{2(1)}$$

Note: So that there is only one root of the quadratic equation (one condition of resonance), the discriminant of the quadratic equation must be zero.

$$(-0.5R^2)^2 - 4(1)(R^2) = 0$$
$$0.25R^4 - 4R^2 = 0$$
$$R = 4\,\Omega$$

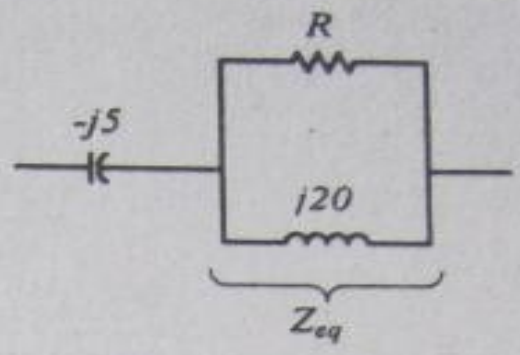
301

$$Z_{eq} = \frac{R(j20)}{R + j20}$$

$$= \frac{R(j20)}{R + j20} \bullet \frac{R - j20}{R - j20}$$

$$= \frac{j20R^2 - j^2\,400R}{R^2 + 400}$$

$$Z_{eq} = \frac{400R}{R^2 + 400} + j\frac{20R^2}{R^2 + 400}$$



Note: At resonance, the equivalent reactance of the parallel branch is equivalent to the series capacitive reactance.

$$\frac{20R^2}{R^2 + 400} = 5$$

$$20R^2 = 5R^2 + 2000$$

$$R = 11.55\,\Omega$$

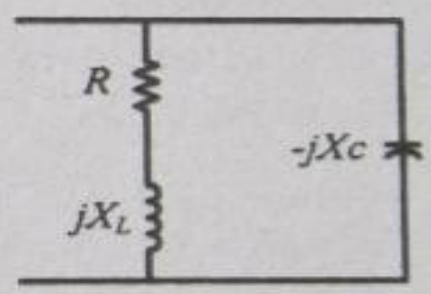
302

$$\cos\theta = \frac{P}{EI} = \frac{2000}{110(25)} = 0.727$$

$$\theta = \cos^{-1} 0.727 = 43.36°$$

$$R = \frac{P}{I^2} = \frac{2000}{(25)^2} = 3.2\,\Omega$$

$$X_L = R\tan\theta = 3.2\tan 43.36° = 3.02\,\Omega$$



$$\beta_L = \beta_C$$

$$\frac{X_L}{R^2 + X_L^{\,2}} = \frac{1}{X_C}$$

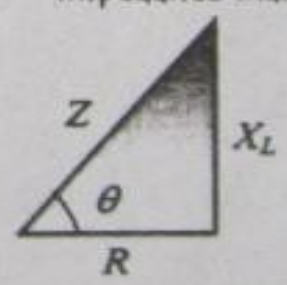
$$\frac{3.02}{3.2^2 + 3.02^2} = \frac{1}{X_C}$$

$$X_C = 6.41\,\Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(100)(6.41)}$$

$$C = 248.3\,\mu F$$

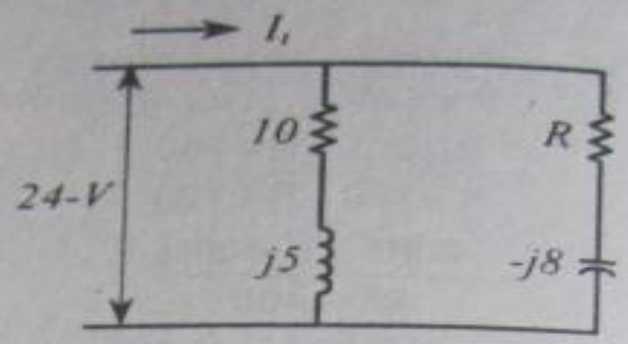
Impedance triangle



303 Note: At parallel resonance, total admittance is equivalent to the total conductance and total susceptance is equal to zero.

$$\beta_L = \beta_C$$

$$\frac{X_L}{R_L^{\,2} + X_L^{\,2}} = \frac{X_C}{R^2 + X_C^{\,2}}$$



$$\frac{5}{10^2+5^2}=\frac{8}{R^2+8^2}$$

$$\frac{5}{125}=\frac{8}{R^2+64}$$

$R = 11.66\,\Omega$

$I_t = E_t Y_t$

$$I_t = E_t\left[\frac{R_L}{R_L{}^2+X_L{}^2}+\frac{R_C}{R_C{}^2+X_C{}^2}\right]$$

$$I_t = 24\left[\frac{10}{10^2+5^2}+\frac{11.66}{11.66^2+8^2}\right]$$

$I_t = 3.32\text{ A}$

304 $Y_t = G_t$

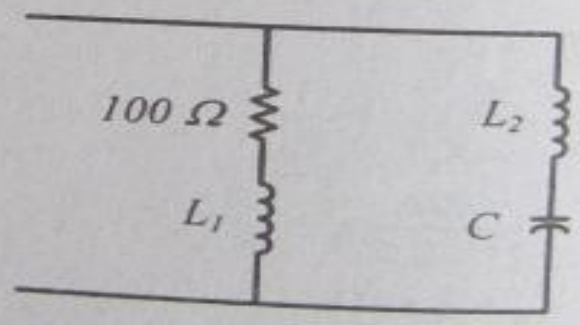
$$G_t = G_L = \frac{R_L}{R_L{}^2+X_L{}^2}=\frac{50}{50^2+70^2}$$

$G_t = 0.00675\text{ mho}$

$P_t = E_t{}^2 Y_t = (120)^2(0.00675)$

$P_t = 96.48\text{ W}$

305 Note: When $w = 2\times10^5$, since no current will flow towards coil L_1, then coil L_2 branch must be short-circuited so that all current will flow towards it.



$X_L = X_C$

$$wL_2 = \frac{1}{wC}$$

$$\frac{1}{C} = (2\times10^5)^2 L_2$$

$$\frac{1}{C} = 4\times10^{10} L_2 \Rightarrow ❶$$

Note: When $w = 1\times10^5$, the circuit is at resonance.

$$Y_t = G_t = \frac{R_L}{R_L{}^2+X_{L_1}{}^2}$$

$$\frac{1}{300}=\frac{100}{100^2+(1\times10^5)^2 L_1{}^2}$$

$L_1 = 1.414\text{ mH}$

$\beta_L = \beta_C$

$$\frac{X_{L1}}{R_L^2 + X_{L_1}^2} = \frac{1}{X_C - X_{L_2}}$$

$$X_C - X_{L_2} = \frac{R_L^2 + X_{L_1}^2}{X_{L_1}}$$

$$\frac{1}{w}\left(\frac{1}{C}\right) - wL_2 = \frac{R_L^2 + X_{L_1}^2}{X_{L_1}} \Rightarrow ❷$$

Substitute Eq.1 and X_{L1} in Eq.2 :

$$\frac{4 \times 10^{10} L_2}{1 \times 10^5} - (1 \times 10^5)L_2 = \frac{100^2 + (1 \times 10^5)^2 (1.414 \times 10^{-3})^2}{(1 \times 10^5)(1.414 \times 10^{-3})}$$

$L_2 = 0.707$ mH

$$C = \frac{1}{(4 \times 10^{10})(0.707 \times 10^{-3})}$$

$C = 35.36$ nF

306. $R = \frac{P}{I^2} = \frac{100}{0.8^2} = 156.25\ \Omega$

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

$$L = \frac{QR}{2\pi f_r} = \frac{5(156.25)}{2\pi(100{,}000)} = 1.24 \text{ mH}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{4\pi^2 f_r^2 L} = \frac{1}{4\pi^2 (100{,}000)^2 (1.24 \times 10^{-3})}$$

$C = 2.04276 \times 10^{-9}$ F

$C = 2.04$ nF

307 $Q = \frac{X_L}{R} = \frac{X_L}{R} \bullet \frac{I^2}{I^2} = \frac{Q_p}{P}$

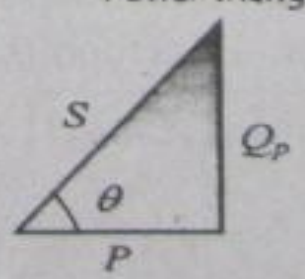
$Q_p = \sqrt{S^2 - P^2} = \sqrt{50^2 - 40^2}$

$Q_p = 30$

Substitute :

$Q = \frac{30}{40} = 0.75$

Power triangle



308 $\theta = \cos^{-1} 0.707 = 45°$

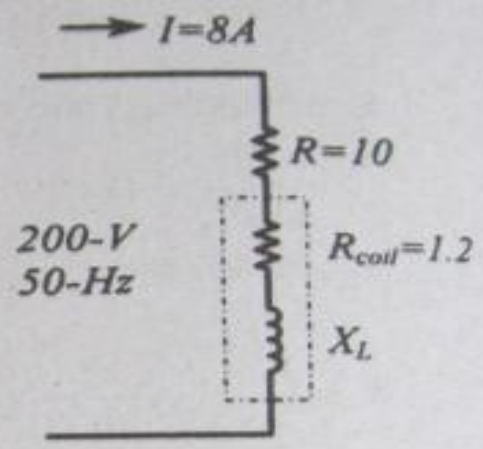
$$Z = \frac{E}{I} = \frac{200}{2} = 100\,\Omega$$

$$R = Z\cos\theta = 100(0.707) = 70.7\,\Omega$$

$$X_L = Z\sin\theta = 100\sin 45° = 70.7\,\Omega$$

$$Q = \frac{X_L}{R} = \frac{70.7}{70.7} = 1$$

309



$$Z = \frac{E}{I} = \frac{200}{8} = 25\,\Omega$$

$$Z^2 = (R + R_{coil})^2 + (X_L)^2$$

$$X_L = \sqrt{Z^2 - (R + R_{coil})^2}$$

$$= \sqrt{(25)^2 - (10 + 1.2)^2}$$

$$X_L = 22.35\,\Omega$$

$$Q = \frac{X_L}{R_{coil}} = \frac{22.35}{1.2}$$

$$Q = 18.625$$

310

$$I_{rms} = \sqrt{I_{dc}^2 + \frac{1}{2}\left(I_{m1}^2 + I_{m2}^2 + I_{m3}^2\right)}$$

$$= \sqrt{(10)^2 + \frac{1}{2}\left(16^2 + 8^2 + 5^2\right)}$$

$$I_{rms} = 16.5\text{ A}$$

311 Note: Use Nodal node analysis: Let the circuit be energized by the different voltage sources one at a time.

$$w = 2\pi f = 2\pi(60) = 377$$

$$I_m = \frac{E_m}{X_C} = \frac{E_m}{1/wC} = E_m wC$$

$$I_{m1} = E_m wC = 1000(377)(2 \times 10^{-6}) = 0.754\text{ A}$$

$$I_{m3} = E_{m3} 3wC = 0.3(1000)(3)(377)(2 \times 10^{-6}) = 0.6786\text{ A}$$

$$I_{m5} = E_{m5} 5wC = 0.2(1000)(5)(377)(2 \times 10^{-6}) = 0.754\text{ A}$$

$$I_{rms} = \sqrt{\frac{1}{2}\left[I_{m1}^2 + I_{m3}^2 + I_{m5}^2\right]} = \sqrt{\frac{1}{2}\left[0.754^2 + 0.6786^2 + 0.754^2\right]}$$

$$I_{rms} = 0.89\text{ A}$$

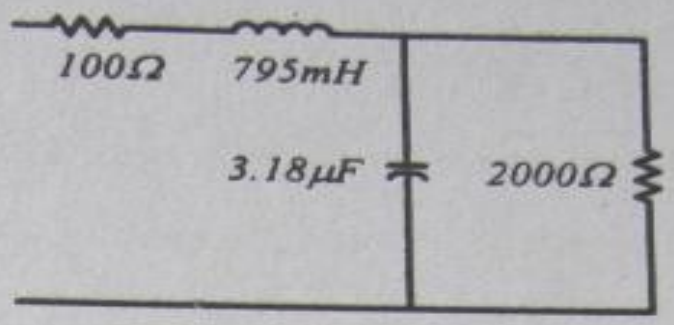
312 Using the fundamental wave :

$X_{L1} = jwL = 314(795 \times 10^{-3})$

$= j249.63\ \Omega$

$X_{C1} = -j\frac{1}{wC} = \frac{1}{314(3.18 \times 10^{-6})}$

$= -j1001.482$



Let : Z_p = impedance of the parallel branch

Z_t = total impedance of the circuit

$$Z_{p1} = \frac{2000(1001.482\angle -90°)}{2000 - j1001.482} = \frac{2000(1001.482)\angle -90°}{2236.731\angle -26.599°}$$

$$Z_{p1} = 895.487\angle -63.401° = 400.948 - j800.71\ \Omega$$

$$Z_{t1} = 100 + j249.63 + 400.948 - j800.71 = 500.948 - j551.08$$

$$Z_{t1} = 744.74\angle -47.728°$$

$$I_{m1} = \frac{E_{m1}}{Z_{t1}} = \frac{400\angle 0°}{744.74\angle -47.728°} = 0.537\angle 47.728°\ A$$

Using the 3rd harmonics :

$$X_{L3} = 3X_{L1} = 3(249.63) = j748.89\ \Omega$$

$$X_{C3} = \frac{X_{C1}}{3} = \frac{1001.482}{3} = -j333.827\ \Omega$$

$$Z_{p3} = \frac{2000(333.827\angle -90°)}{2000 - j333.827} = \frac{2000(333.827)\angle -90°}{2027.668\angle -9.476°}$$

$$Z_{p3} = 329.272\angle -80.524° = 54.209 - j324.778\ \Omega$$

$$Z_{t3} = 100 + j748.89 + 54.209 - j324.778 = 154.209 + j424.112$$

$$Z_{t3} = 451.277\angle 70.018°$$

$$I_{m3} = \frac{E_{m3}}{Z_{t3}} = \frac{80\angle 60°}{451.277\angle 70.018°} = 0.177\angle -10.018°\ A$$

$$I_{rms} = \sqrt{\frac{1}{2}\left[I_{m1}^2 + I_{m3}^2\right]} = \sqrt{\frac{1}{2}\left[0.537^2 + 0.177^2\right]} = 0.399$$

$$I_{rms} \cong 0.4\ A$$

313 $I_{dc} = \frac{E_{dc}}{R} = \frac{110}{10} = 11\,A$

Note: At steady state, an inductor is equivalent to a short-circuited wire.

$Z = 10 + j8 = 12.806\angle 38.66°$

$I_m = \frac{E_m}{Z} = \frac{151}{12.806} = 11.79\,A$

$I_{rms} = \sqrt{I_{dc}^2 + \frac{I_m^2}{2}} = \sqrt{11^2 + \frac{11.79^2}{2}}$

$I_{rms} = 13.8\,A$

314 Note: An AC ammeter will always register rms or effective value of the current.

$I_{rms} = \sqrt{I_{dc}^2 + I_{ac}^2} = \sqrt{(10)^2 + (5)^2}$

$I_{rms} = 11.18\,A$

315 When supplied with 60 V DC source :

$I_{dc} = \frac{E_{dc}}{R} = \frac{60}{3} = 20\,A$

When supplied with 80 V, 60 Hz (w = 377) AC source :

$I_1 = \frac{E_1}{Z_1} = \frac{80}{\sqrt{3^2 + [377(2.65 \times 10^{-3}]^2}} = 25.3\,A$

When supplied with 100 V, 120 Hz (w = 754) AC source :

$I_2 = \frac{E_2}{Z_2} = \frac{100}{\sqrt{3^2 + [754(2.65 \times 10^{-3}]^2}} = 27.74\,A$

$I_{rms} = \sqrt{I_{dc}^2 + I_1^2 + I_2^2} = \sqrt{20^2 + 25.3^2 + 27.74^2}$

$I_{rms} = 42.54\,A$

316 $e_3 = 50\cos 3wt = 50\sin(3wt + 90°)$

Thus :

$e = 100\sin(wt + 30°) - 50\sin(3wt + 90°) + 25\sin(5wt + 150°)$

$i = 20\sin(wt + 40°) + 10\sin(3wt + 30°) - 5\sin(5wt - 50°)$

$$P = \frac{E_m I_m}{2}\cos(\alpha - \beta)$$
$$= \frac{100(20)}{2}\cos(30° - 40°) + \frac{(-50)(10)}{2}\cos(90° - 30°)$$
$$+ \frac{25(-5)}{2}\cos[(150° - (-50°)]$$
$$P = 918.5 \text{ watts}$$

317 $$I_{rms} = \sqrt{\frac{1}{2}\left(I_{m1}^2 + I_{m3}^2 + I_{m5}^2\right)} = \sqrt{\frac{1}{2}\left(12^2 + 8^2 + 3^2\right)} = 10.416 \text{ A}$$
$$P = I_{rms}^2 R = (10.416)^2(10) = 1084.9$$
$$P \cong 1085 \text{ watts}$$

318 Note: Refer to problem #312, for the solution to solve for the values of the currents I_{m1} and I_{m3}.

$$I_{m1} = 0.537\angle 47.728° \text{ A}$$
$$E_{m1} = 400\angle 0° \text{ V}$$
$$P_1 = \frac{E_{m1}I_{m1}}{2}\cos(\alpha_1 - \beta_1) = \frac{400(0.537)}{2}\cos(0° - 47.728°)$$
$$P_1 = 72.24 \text{ watts}$$

$$I_{m3} = 0.177\angle -10.018° \text{ A}$$
$$E_{m3} = 80\angle 60°$$
$$P_3 = \frac{E_{m3}I_{m3}}{2}\cos(\alpha_3 - \beta_3) = \frac{80(0.177)}{2}\cos(60° + 10.018°)$$
$$P_3 = 2.42 \text{ watts}$$

$$P_t = P_1 + P_3 = 72.24 + 2.42$$
$$P_t = 74.66 \text{ W}$$

319 When supplied with 100 V DC source :

$$I_{dc} = \frac{E_{dc}}{R} = \frac{100}{5} = 20 \text{ A}$$

When supplied with 50 V, w = 500 AC source :

$$I_{m1} = \frac{E_{m1}}{Z_1} = \frac{50}{\sqrt{5^2 + [500(0.02)]^2}}$$
$$I_{m1} = 4.472 \text{ A}$$

When supplied with 25 V, w = 1500 AC source :

$$I_{m3} = \frac{E_{m3}}{Z_3} = \frac{25}{\sqrt{5^2 + [1500(0.02)]^2}}$$

$$I_{m3} = 0.822 \text{ A}$$

$$I_{rms} = \sqrt{I_{dc}^2 + \frac{1}{2}\left(I_{m1}^2 + I_{m3}^2\right)} = \sqrt{(20)^2 + \frac{1}{2}\left(4.472^2 + 0.822^2\right)}$$

$$I_{rms} = 20.256 \text{ A}$$

$$P = I_{rms}^2 R = (20.256)^2(5) = 2051.52$$

$$P \cong 2052 \text{ W}$$

320 When supplied with a battery :

$$I_{dc} = \frac{E_{dc}}{R} = \frac{6}{8} = 0.75 \text{ A}$$

When supplied with 15 V, 20 Hz AC source ($w_1 = 125.66$) :

$$X_{L1} = wL = (125.66)(26.53 \times 10^{-3}) = 3.333\ \Omega$$

$$Z_1 = \sqrt{R^2 + X_{L1}^2} = \sqrt{(8)^2 + (3.333)^2} = 8.666\ \Omega$$

$$I_1 = \frac{E_1}{Z_1} = \frac{15}{8.666} = 1.73 \text{ A}$$

When supplied with 30 V, 60 Hz AC source (third harmonics) :

$$X_{L3} = 3X_{L1} = 3(3.333) = 9.999\ \Omega$$

$$Z_3 = \sqrt{R^2 + X_{L3}^2} = \sqrt{(8)^2 + (9.999)^2} = 12.8\ \Omega$$

$$I_3 = \frac{E_3}{Z_3} = \frac{30}{12.8} = 2.34 \text{ A}$$

When supplied with 40 V, 100 Hz AC source (fifth harmonics) :

$$X_{L5} = 5X_{L1} = 5(3.333) = 16.665\ \Omega$$

$$Z_5 = \sqrt{R^2 + X_{L5}^2} = \sqrt{(8)^2 + (16.665)^2} = 18.485\ \Omega$$

$$I_5 = \frac{E_5}{Z_5} = \frac{40}{18.485} = 2.16 \text{ A}$$

$$I_{rms} = \sqrt{I_{dc}^2 + I_1^2 + I_3^2 + I_5^2} = \sqrt{(0.75)^2 + (1.73)^2 + (2.34)^2 + (2.16)^2}$$

$$I_{rms} = 3.7 \text{ A}$$

$E_{rms} = \sqrt{E_{dc}^2 + E_1^2 + E_3^2 + E_5^2} = \sqrt{(6)^2 + (15)^2 + (30)^2 + (40)^2}$

$E_{rms} = 52.54\ V$

$S = E_{rms} I_{rms} = (52.54)(3.7)$

$S \cong 194.4\ VA$

321 When supplied with 35 V, 60 Hz AC source ($w_1 = 377$):

$X_{L1} = wL = (377)(4.42 \times 10^{-3}) = 1.667\ \Omega$

$X_{C1} = \dfrac{1}{wC} = \dfrac{1}{377(295 \times 10^{-6})} = 9\ \Omega$

$Z_1 = \sqrt{R^2 + (X_{C1} - X_{L1})^2} = \sqrt{(3)^2 + (9 - 1.667)^2} = 7.92\ \Omega$

$I_1 = \dfrac{E_1}{Z_1} = \dfrac{35}{7.92} = 4.42\ A$

When supplied with 10 V, 180 Hz AC source (third harmonics):

$X_{L3} = 3X_{L1} = 3(1.667) = 5\ \Omega$

$X_{C3} = \dfrac{X_{c1}}{3} = \dfrac{9}{3} = 3\ \Omega$

$Z_3 = \sqrt{R^2 + (X_{L3} - X_{C3})^2} = \sqrt{(3)^2 + (5 - 3)^2} = 3.605\ \Omega$

$I_3 = \dfrac{E_3}{Z_3} = \dfrac{10}{3.605} = 2.77\ A$

When supplied with 8 V, 240 Hz AC source (fourth harmonics):

$X_{L4} = 4X_{L1} = 4(1.667) = 6.668\ \Omega$

$X_{C4} = \dfrac{X_{C1}}{4} = \dfrac{9}{4} = 2.25\ \Omega$

$Z_4 = \sqrt{R^2 + (X_{L4} - X_{C4})^2} = \sqrt{(3)^2 + (6.668 - 2.25)^2} = 5.34\ \Omega$

$I_4 = \dfrac{E_4}{Z_4} = \dfrac{8}{5.34} = 1.498\ A$

$I_{rms} = \sqrt{I_1^2 + I_3^2 + I_4^2} = \sqrt{(4.42)^2 + (2.77)^2 + (1.498)^2} = 5.427\ A$

$E_{rms} = \sqrt{E_1^2 + E_3^2 + E_4^2} = \sqrt{(35)^2 + (10)^2 + (8)^2} = 37.27\ V$

$P = I_{rms}^2 R \qquad P = E_{rms} I_{rms} pf$

$pf = \dfrac{I_{rms}^2 R}{E_{rms} I_{rms}} = \dfrac{I_{rms} R}{F_{rms}} = \dfrac{(5.427)(3)}{37.27}$

$pf \cong 0.437$

322 Note: Refer to problem #312, for the solution to solve for the rms value of the total current and refer to problem #318, for the solution to solve for total real power consumed.

$P_t = 74.66$ watts

$I_{rms} = 0.4$ A

$$E_{rms} = \sqrt{\frac{1}{2}\left(E_{m1}^{\ 2} + E_{m3}^{\ 2}\right)} = \sqrt{\frac{1}{2}\left(400^2 + 80^2\right)} = 288.44 \text{ V}$$

$$pf = \frac{P_t}{E_{rms} I_{rms}} = \frac{74.66}{(288.44)(0.4)} = 0.648$$

$pf \cong 0.65$

3-phase AC System: Part 1

POLYPHASE SYSTEM

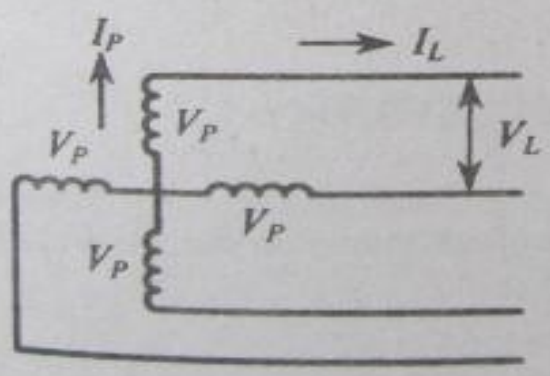
It consists of an arrangement with **two or more** similar single-phase systems in which the maximum values of the independent voltages **do not occur** at the same time.

Advantages of polyphase system over single-phase system:

a. Polyphase machines like motors, generators and transformers are simpler, cheaper, and more efficient.
b. Polyphase transmission lines can deliver more power for a given weight and cost.
c. Voltage regulation of polyphase lines is inherently better

n-PHASE STAR CONNECTED SYSTEM

One terminal of each phase (windings) are connected together while the other terminals represent the line terminals.

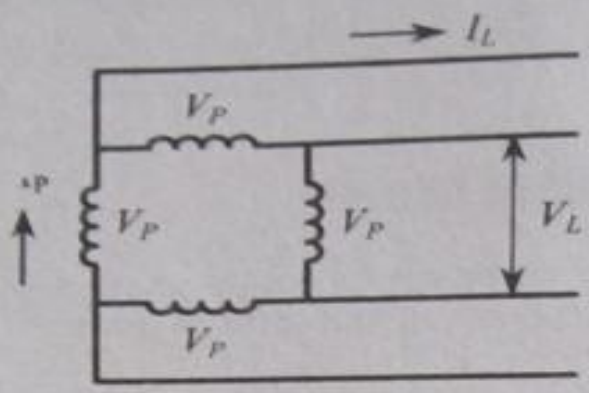


$$V_L = 2V_P \sin\frac{180^\circ}{n}$$

$$I_L = I_P$$

n-PHASE MESH CONNECTED SYSTEM

Each phase (windings) is connected end to end thus forming a closed geometric figure. The line terminals are tapped at each phase terminals.



$$I_L = 2I_P \sin\frac{180^\circ}{n}$$

$$V_L = V_P$$

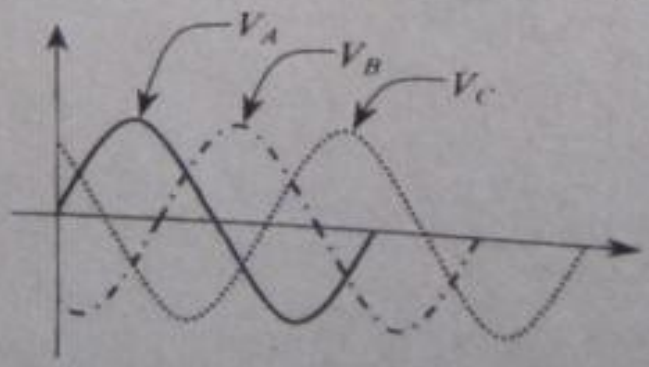
n-PHASE ACTIVE POWER FORMULA

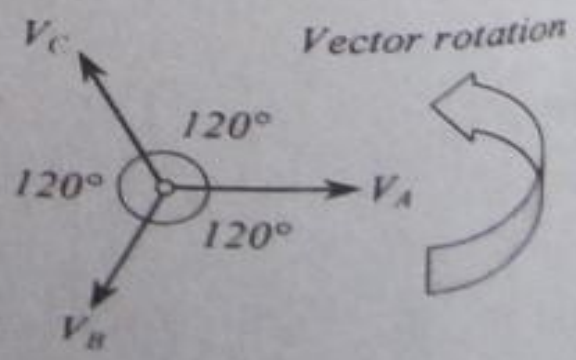
$$P = nV_PI_P\cos\theta$$

where: n = number of phases
V_L = line to line voltage
V_P = phase voltage
I_L = line current
I_P = phase current
$\cos\theta$ = load power factor

THREE-PHASE SYSTEM

Most commonly used polyphase system. It is an interconnection of **three single phase systems**, whose voltage waves, though similar in amplitude and frequency are displaced **120 electrical degrees** apart.





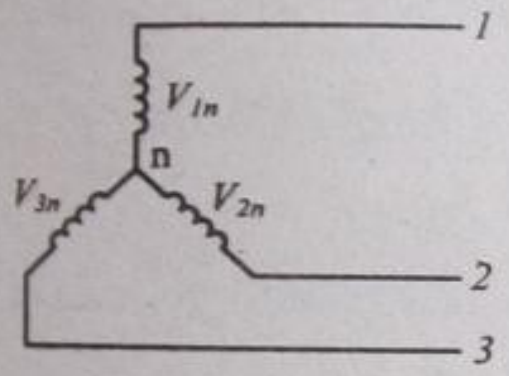
PHASE SEQUENCE

The **order of rotation** of the coil voltage in a balanced polyphase system.

Example: If phase sequence is a-b-c, this means that "vector A" comes first, followed by "vector B" and then "vector C". (Refer to the vector diagram)

Note: Vector A is the reference vector according to the given sequence. Vector rotation unless otherwise specified is assumed to rotate in a counterclockwise direction.

WYE OR 3-PHASE STAR-CONNECTED BALANCED SYSTEM



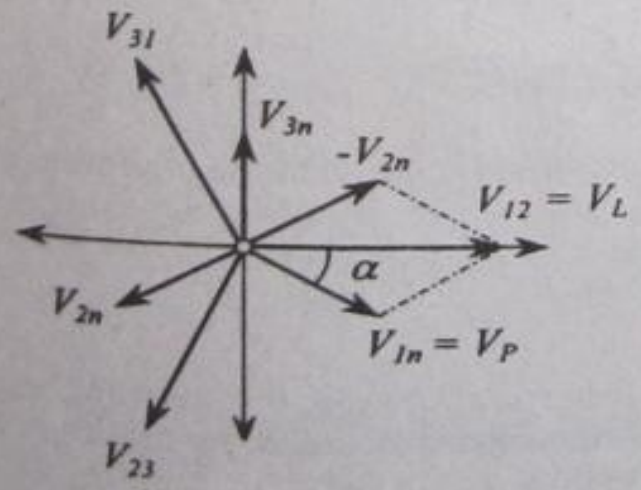
$$V_{1n} = V_{2n} = V_{3n} = V_P$$
$$V_{12} = V_{23} = V_{31} = V_L$$

$$V_{12} = V_{1n} + (-V_{2n})$$
$$V_{23} = V_{2n} + (-V_{3n})$$
$$V_{31} = V_{3n} + (-V_{1n})$$

$$\alpha = 30°$$

$$V_L = \sqrt{3}\, V_P$$

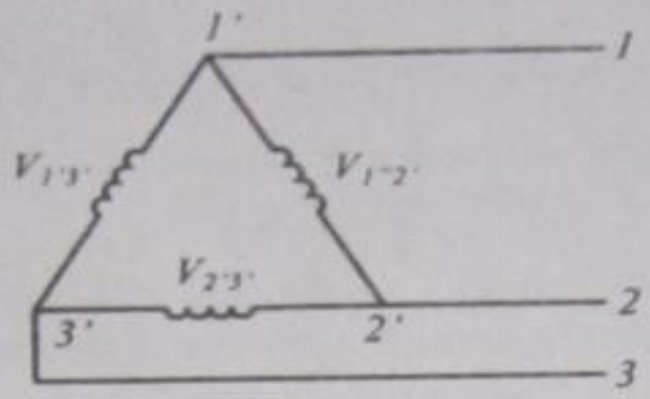
Phasor diagram with V_{12} as reference:



$$V_{1n} = V_p\angle -30°$$
$$V_{2n} = V_p\angle -150°$$
$$V_{3n} = V_p\angle 90°$$

$$V_{12} = V_L\angle 0°$$
$$V_{23} = V_L\angle -120°$$
$$V_{31} = V_L\angle 120°$$

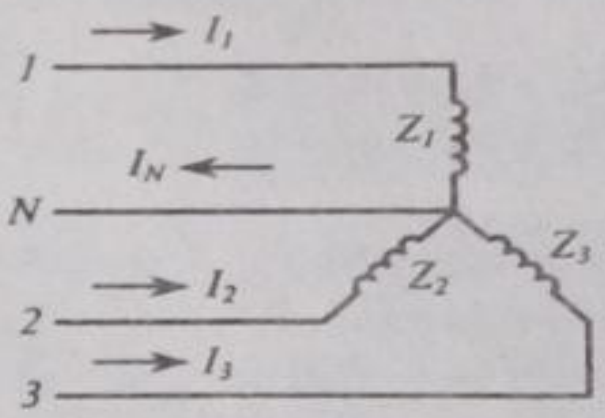
DELTA OR 3-PHASE MESH-CONNECTED BALANCED SYSTEM



$$V_{12} = V_{23} = V_{31} = V_P$$
$$V_{12} = V_{23} = V_{31} = V_L$$

$$V_L = V_P$$

WYE-CONNECTED BALANCED LOADS



$$Z_1 = Z_2 = Z_3 = Z$$

$$I_1 = I_2 = I_3 = \frac{V_P}{Z}$$

Note: If loads are unbalanced, transform the wye-connected loads into an equivalent delta connected load and analyzed it using delta connected load principles.

NEUTRAL CURRENT

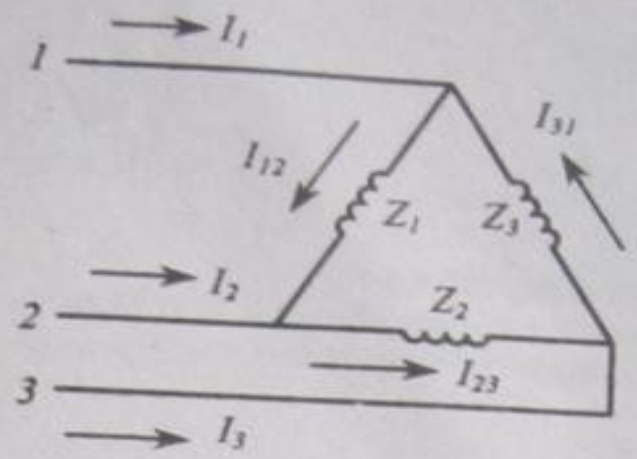
Current that flows in the neutral conductor, which is equal to the **algebraic sum** of the three line currents.

$$I_N = I_1 + I_2 + I_3$$

From the concept of conservation of current and charge, the algebraic sum of the currents in a given node is **zero**. Hence, the algebraic sum of the three line currents vanishes when **no neutral connection exists**.

$$I_1 + I_2 + I_3 = 0$$

DELTA -CONNECTED BALANCED OR UNBALANCED LOADS



- If **balanced loads:** (Refer to the circuit diagram)

$Z_1 = Z_2 = Z_3 = Z$
$I_{12} = I_{23} = I_{31} = I_P$
$I_1 = I_2 = I_3 = I_L$

$$I_P = \frac{V_L}{Z} \qquad I_L = \sqrt{3}\, I_P$$

- If **unbalanced loads:** (Refer to the circuit diagram)

$Z_1 \neq Z_2 \neq Z_3$

$$I_{12} = \frac{V_{12}}{Z_1} \qquad I_{23} = \frac{V_{23}}{Z_2} \qquad I_{31} = \frac{V_{31}}{Z_3}$$

$$I_1 = I_{12} + (-I_{31}) \qquad I_2 = I_{23} + (-I_{12}) \qquad I_3 = I_{31} + (-I_{23})$$

Note: All quantities (voltages, impedances and currents) must be in complex form and all mathematical operations must be done under the rules in complex operations

THREE-PHASE POWER FORMULAS

$$P = 3V_P I_P \cos\theta \qquad P = \sqrt{3} V_L I_L \cos\theta$$

$$S = 3V_P I_P \qquad S = \sqrt{3} V_L I_L$$

Test 10

Problem 323: EE Board April 1997

A 170 kV, 3-phase electric source delivers 200 MVA to a balanced load, which has a power factor of 90% lagging. What is the line current?

A. 257 A
B. 502 A
C. 402 A
D. 679 A

Problem 324: EE Board October 1997

A three-phase motor is rated 50 hp, 440 volts and 85% power factor. What is its rated current?

A. 61.5 A
B. 57.5 A
C. 55 A
D. 59 A

Problem 325: EE Board April 1985

A balanced 3-phase load draws 120 amperes line current at 230 volts line to line, 0.848 pf lagging current. Solve for the real power.

A. 40.54 kW
B. 42.35 kW
C. 41.45 kW
D. 43.15 kW

Problem 326: EE Board March 1998

A generator supplies three-phase power to balanced load. The voltage is 230 volts, the current is 18 A and the power factor is 85%. What is the power?

A. 3.6 kW
B. 1.6 kW
C. 6.1 kW
D. 1.4 kW

Problem 327: EE Board April 1984

A balanced 3-phase load draws 75 amperes line current at 230 V line to line and 0.848 lagging power factor. Solve for the reactive power being drawn.

A. 15.83 kVAR
B. 15.26 kVAR
C. 15.35 kVAR
D. 15.94 kVAR

Problem 328: EE Board April 1990

The input power factor to a three-phase, 6-poles, 460 volts, 60 Hz, 50-hp induction motor is 0.62 as 20 A is drawn by the motor. Find the power input to the motor.

A. 9,880 W
B. 9,675 W
C. 9,895 W
D. 9,478 W

Problem 329: EE Board April 1992

A 460 volt, three-phase motor draws 208 A with a power factor of 0.91 lagging. Calculate the kW input to the motor.

A. 150.8
B. 156.3
C. 152.4
D. 160.3

Problem 330:

Determine the ratio of the line current drawn by three identical impedances connected in wye to the line current drawn by the same impedances connected in delta. Both connections being impressed with the same balanced 3-ϕ voltages.

A. 1:2
B. 3:4
C. 2:3
D. 1:3

Problem 331: EE Board April 1993

A wye-connected load has a $5\angle 20°$ ohm impedance per phase and is connected across a 120-V three-phase source. Calculate the line current.

A. 24 A
B. 13.85 A
C. 41.56 A
D. 15.45 A

Problem 332:

A balanced delta connected load having impedance per phase of $20\angle 36.87°$ ohms is supplied from a balanced 3-phase, 240 V source. Determine the total real power.

A. 6824 W
B. 6912 W
C. 7015 W
D. 6740 W

Problem 333: EE Board April 1993

Three condensers, each having a capacity of 75 microfarads are connected in star to a 440 volts, 3-phase, 50 cycles supply. Calculate the capacitance of each of the three condensers so that when they are connected in delta to the same supply the line current remains the same.

A. 20 μF
B. 28 μF
C. 25 μF
D. 30 μF

Problem 334: EE Board March 1998

Three impedances, -j10, j10 and 10 ohms are wye connected. Determine the impedance of an equivalent delta.

A. 12.5, j12.5, -12.5 ohms
B. 10, j10, -j10 ohms
C. j8.5, -j12.5, 8 ohms
D. 5, j5, -j5 ohms

Problem 335:

A system consisting of three equal resistors connected in star is fed from a three-phase supply. By how much is the power reduced if one of the three resistors is disconnected?

A. 1/3
B. 3/4
C. 1/2
D. 2/3

Problem 336: EE Board April 1993

A balanced three-phase load is wye-connected and has an impedance $Z_Y = 4 - j$ 3 ohms per phase. Find the line current if this load is connected across a 220 V three-phase source.

A. 25.4 A
B. 22.3 A
C. 20.5 A
D. 26.7 A

Problem 337: EE Board October 1998

Three 10-ohm resistances are connected delta on a balanced three-phase source. If the equation of the phase V_{an} =120 sin wt. What is the equation of the line current in line a?

A. 20.78 sin (wt + 30°)
B. 13.15 sin (wt – 30°)
C. 12 sin (wt – 56.56°)
D. 36 sin wt

Problem 338: EE Board April 1982

Given a balanced 3-wire, three-phase system serving the following loads:

$V_{ab} = 200\angle 0°$ V $\quad Z_{ab} = 10\angle 53.13°\ \Omega$

$V_{bc} = 200\angle 240°$ V $\quad Z_{bc} = 10\angle 0°\ \Omega$

$V_{ca} = 200\angle 120°$ V $\quad Z_{ca} = 10\angle 30°\ \Omega$

Determine line current on line b.

A. 20.34 A
B. 22.04 A
C. 24.36 A
D. 21.57 A

Problem 339: EE Board April 1982

Given the following line voltages and two load impedances:

$V_{ab} = 117\angle 0°$ V $\quad Z_{ac} = 5 + j12$ ohms

$V_{bc} = 117\angle 240°$ V $\quad Z_{bc} = 13 + j0$ ohms

$V_{ca} = 117\angle 120°$ V

Solve for the line current in line c.

A. 17.41 A
B. 17.95 A
C. 16.62 A
D. 18.46 A

Problem 340: EE Board April 1988

Three unequal single-phase loads so connected across the lines of a balanced, 3-phase, 230 volts circuit. The first takes 106 A at 0.78 pf lagging and is connected across lines 1 & 2. The second takes 142 A, at 0.82 pf lagging and is connected across lines 2 & 3. And the third takes 28.4 kW at 0.77 pf lagging and is connected across lines 3 & 1. Find the three line currents.

A. 254.40 A, 211.38 A, 252 A
B. 231.26 A, 215.20 A, 268 A
C. 254.40 A, 215.20 A, 252 A
D. 231.26 A, 211.38 A, 268 A

Problem 341: EE Board October 1992

A 120-V per phase, three-phase Y connected source delivers power to the following delta-connected load:

Phase 1 = $40\angle 0°$ Phase 2 = $20\angle -60°$ Phase 3 = $15\angle 45°$

Determine the three line currents.

A. 12.45 A, 9 A, 22.45 A
B. 13.49 A, 9 A, 22.45 A
C. 13.49 A, 10 A, 20.22 A
D. 12.45 A, 10 A, 20.22 A

Problem 342: EE Board October 1985

Given:

$V_{ab} = 240\angle 0°$ V $Z_{ab} = 6 + j8$ ohms
$V_{bc} = 240\angle 240°$ V $Z_{bc} = 5 + j8.66$ ohms
$V_{ca} = 240\angle 120°$ V $Z_{ca} = 10 + j0$ ohms

Solve for the three line currents I_a, I_b and I_c.

A. $I_a = 45$ A, $I_b = 43$ A, $I_c = 20$ A
B. $I_a = 48$ A, $I_b = 42$ A, $I_c = 24$ A
C. $I_a = 45$ A, $I_b = 42$ A, $I_c = 20$ A
D. $I_a = 48$ A, $I_b = 43$ A, $I_c = 24$ A

Problem 343: EE Board April 1985

A three-phase 230-V circuit serves two single phase loads, A and B. Load A is an induction motor rated 8 hp, 230 V, 0.70 pf, 0.90 efficiency, which is connected across lines a and b. Load B draws 5 kW at 1.0 pf and is connected across lines b and c. Assume a sequence of a-b-c, solve for the line current on line b.

A. 42.19 A
B. 27.74 A
C. 41.08 A
D. 34.46 A

Problem 344:

A four wire 208-V three-phase system is used to supply power to a three-phase 5-hp induction motor and a single phase 6 kW heater connected between line c and the neutral line. The operating efficiency and power factor of the motor are 81 and 71 percent, respectively. Calculate the current in line c. Assume a phase sequence of a-b-c.

A. 71 A
B. 75 A
C. 64 A
D. 69 A

Problem 345: EE Board April 1980

A factory is supplied by a three-phase, 3-wire system with the following characteristics:

$V_{ab} = 230\angle 0°$ V $\quad I_a = 110\angle -36.87°$ A

$V_{bc} = 230\angle 240°$ V $\quad I_c = 125\angle 53.13°$ A

Find line current I_b.

A. 145.3 A
B. 163.3 A
C. 184.6 A
D. 166.5 A

Problem 346:

Given a balanced 3-wire, three-phase system serving an unbalanced wye-connected load. Determine the line current in line a.

$V_{ab} = 210\angle 0°$ V $\quad Z_{an} = 10$

$V_{bc} = 210\angle 120°$ V $\quad Z_{bn} = 10 + j10$

$V_{ca} = 210\angle 240°$ V $\quad Z_{cn} = 10$

A. 15 A
B. 12 A
C. 14 A
D. 16 A

Problem 347: EE Board October 1994

Three resistors 10, 15 and 30 ohmic values are connected in wye-configuration to a balanced 208-volt three-phase supply. Calculate the total power of the system.

A. 2644 W
B. 2880 W
C. 3080 W
D. 3280 W

Problem 348: EE Board March 1998

Three impedances each 10 + j5 ohms are connected in delta on a balance three-phase source. If the equation of the phase voltage is V_{an} =120 sin wt. What is the equation of the current through the impedance connected across phase A and B?

A. 20.02 sin (wt –22°)
B. 18.59 sin (wt + 3.44°)
C. 16.21 sin (wt + 56.56°)
D. 21.32 sin (wt – 8.15°)

Problem 349: EE Board June 1990

Two parallel connected loads A and B are supplied by a 440 V, 3-phase, 60 Hz generator. Load A draws an apparent power of 100 kVA at 0.80 pf lagging and load B draws an apparent power of 70 kVA at unity pf. Determine the feeder current.

A. 208 A
B. 212 A
C. 214 A
D. 202 A

Problem 350:

Two different loads A and B are connected across a balanced 230 V, 3-ϕ lines. Load A is a three-phase load that draws 10 kVA at 70% lagging pf. Load B is a single-phase load connected across lines a and b drawing 5 kVA at unity pf. Determine line current in line a. Assume a phase sequence of a-b-c.

A. 37 A
B. 42 A
C. 31 A
D. 46 A

Problem 351: EE Board April 1990

A three-phase motor takes 10 kVA at 0.6 pf lagging from a source of 230 volts. It is in parallel with a balanced delta load having 16 ohms resistance and 12 ohms capacitive reactance in series in each phase. Determine the total power factor.

A. 0.966 lagging
B. 0.896 lagging
C. 0.917 lagging
D. 0.967 lagging

Problem 352:

A balanced delta load whose impedance is $45\angle 70°$ ohms per branch, a three-phase motor that draws a total of 10 kVA at 0.65 pf lagging and a wye connected load whose impedance is 10 ohms (resistance) per branch are supplied from a three-phase, three-wire, 208 V, 60 Hz feeder. Determine overall power factor of the feeder loads.

A. 0.753
B. 0.706
C. 0.826
D. 0.762

Problem 353: EE Board April 1985

A three-phase 230-V circuit serves two single phase loads, A and B. Load A is an induction motor rated 8 hp, 230 V, 0.70 pf, 0.90 efficiency, which is connected across lines a and b. Load B draws 5 kW at 1.0 pf and is connected across lines b and c. Assume a sequence of a-b-c, solve for the total power factor of the load.

A. 0.907
B. 0.704
C. 0.864
D. 0.886

Problem 354:

A 440-V three-phase, 3-wire wire, 60 Hz feeder supplies power to two 30-hp induction motors. One motor is operating at full load, 85% efficiency and has a power factor of 90%. The other motor is operating at one-half of its rated horsepower, 89% efficiency and has a power factor of 78%. What is the overall power factor of the system of motors?

A. 0.862 lagging
B. 0.826 lagging
C. 0.838 lagging
D. 0.801 lagging

Problem 355: EE Board April 1988

Three unequal single-phase loads so connected across the lines of a balanced, 3-phase, 230 volts circuit. The first takes 106 A at 0.78 pf lagging and is connected across lines 1 & 2. The second takes 142 A, at 0.82 pf lagging and is connected across 2 & 3. And the third takes 28.4 kW at 0.77 pf lagging. Determine total apparent power.

A. 94 kVA
B. 83 kVA
C. 78 kVA
D. 101 kVA

Problem 356:

Two balanced Y-connected loads A and B are connected across a three-phase 3-wire balanced system. Load A draws 6 kW at 80% power factor lagging while load B draws 12 kW at 83.5% power factor leading. If the current in the three lines are each equal to 8 A, determine the current drawn by load A.

A. 4.45 A
B. 3.58 A
C. 4.15 A
D. 3.28 A

Problem 357: EE Board March 1998

The phase b line voltage and the phase a line current of a balanced three-phase system are $V = 220 \sin (wt + 210°)$ and $I = 10 \sin (wt - 30°)$, respectively. What is the power of the system.

A. 1905 W
B. 3300 W
C. 5716 W
D. 3810 W

Problem 358: EE Board October 1986

The following information is given for a delta-connected load of three numerically equal impedances that differ in power factor: Line voltage = 120 volts $Z_{ab} = 15\angle 30°$; $Z_{bc} = 15\angle 0°$; $Z_{ca} = 15\angle -30°$ phase sequence of voltages is a-b-c. Using the phase sequence as a guide, calculate the total power drawn by the load.

A. 2,624 W
B. 2,472 W
C. 2,564 W
D. 2,731 W

Problem 359: EE Board April 1990

A three-phase motor takes 10 kVA at 0.6 pf lagging from a source of 230 volts. It is in parallel with a balanced delta load having 16 ohms resistance and 12 ohms capacitive reactance in series in each phase. Determine the total line current.

A. 30.05 A
B. 34.55 A
C. 32.04 A
D. 31.24 A

Problem 360: EE Board April 1993

In AC circuits, find the total power in kW in an unbalanced three-phase circuit loaded as follows: Phase I = 120 V, 100 A, unity pf. Phase II = 100 V, 230 A, 80% pf and phase III = 110 V, 85 A, 77% pf.

A. 37.6 kW
B. 35.3 kW
C. 32.8 kW
D. 38.2 kW

Problem 361: EE Board April 1983

Two single-phase transformers are connected in V (open delta) and serving a delta connected impedance load, each impedance is equal to $16\angle 36.87°$ ohms. If the transformer voltages impressed on the impedances are:

$$V_{ab} = 240\angle 0°,\ V_{ab} = 240\angle 240°,\ V_{ab} = 240\angle 120° \text{ volts}$$

Solve for the total kVA drawn by the load.

A. 6.23
B. 8.31
C. 10.8
D. 11.3

Problem 362: EE Board October 1980, EE Board October 1982

Three impedances, $Z_{an} = 20 + j0$, $Z_{bn} = 16 + j12$, $Z_{cn} = 5 - j15$ ohms, are connected in wye across a 230 V (line to line), 3-phase, 4-wire source. The phase sequence is a-b-c, counterclockwise. Determine the current passing thru the neutral.

A. 7.54 A
B. 9.12 A
C. 8.81 A
D. 8.02 A

Problem 363: EE Board April 1981

A wye-connected transformer with neutral connection has balanced voltages of 265 V between lines and neutral. The transformer is serving two single-phase motors. Motor A (rated 4 hp, 0.90 efficiency, 0.80 power factor lagging) is connected across line a and neutral. Motor B (rated 3 hp, 0.85 efficiency, 0.85 power factor lagging) is connected across line b and neutral. Solve for the neutral current, using V_{an} as reference vector.

A. 20.42 A
B. 25.37 A
C. 22.45 A
D. 23.14 A

Problem 364:

Two single-phase motors are connected between the lines and neutral terminals of a 3-phase, 4-wire, 230 V secondary side of a three-phase transformer. Load A is connected across line a and neutral while B is across line b and neutral. The currents I_a and I_b are 40 A, 0.707 pf lagging, and 25 A, unity pf, respectively. Determine neutral current, I_n.

A. 54.25 A
B. 50.34 A
C. 51.86 A
D. 52.37 A

Problem 365:

Two single-phase motors are connected to a 3-phase, 4-wire wye feeder having 220 V between phase and the neutral. One motor rated 5 HP, 0.866 pf lagging and 90% eff is connected across phase A and neutral. The second motor is connected across phase C and neutral, and rated 3 HP, 0.707 pf lagging and 90% eff. Solve for current through the neutral.

A. 20.56 A
B. 29.54 A
C. 21.67 A
D. 23.43 A

Problem 366:

A 230-V, three-phase, 4-wire balanced system supplies power to group of lamp loads. If the line currents are respectively 60 A, 86 A and 40 A respectively, solve for the current in the neutral wire. Assume the power factor of the lamps to be unity.

A. 40 A
B. 14 A

C. 66 A
D. 36 A

Problem 367: EE Board October 1998

The loads of a wye-connected transformer are:

$I_a = 10 \text{ cis } (-30°)$

$I_b = 12 \text{ cis } 215°$

$I_c = 15 \text{ cis } 82°$

What is the neutral current?

A. 1.04 cis 72.8°
B. 2.21 cis (-30°)
C. 0.92 cis 62.5°
D. 3.11 cis 72.8°

ANSWER KEY

323. D	332. B	341. B	350. A	359. C
324. B	333. C	342. D	351. D	360. A
325. A	334. B	343. C	352. A	361. C
326. C	335. C	344. C	353. C	362. C
327. A	336. A	345. D	354. A	363. D
328. A	337. D	346. C	355. A	364. D
329. A	338. B	347. A	356. D	365. D
330. D	339. B	348. B	357. B	366. A
331. B	340. D	349. B	358. A	367. D

☑ RATING

40-45 - Topnotcher
31-39 - Passer
22-30 - Conditional
0-21 - Failed

Solutions to Test 10

323 $I = \frac{S}{\sqrt{3}E} = \frac{200 \times 10^6}{\sqrt{3}(170{,}000)}$

$I = 679.23\ A$

324 Note: Since the efficiency of the motor is not given, assume the output rating is equivalent to the power drawn at the input.

$I = \frac{P}{\sqrt{3}Epf} = \frac{50(746)}{\sqrt{3}(440)(0.85)}$

$I = 57.58\ A.$

325 $P = \sqrt{3}Elpf = \sqrt{3}(230)(120)(0.848)$

$P = 40.54\ kW$

326 $P = \sqrt{3}Elpf = \sqrt{3}(230)(18)(0.85)$

$P = 6.1\ kW$

327 $\theta = \cos^{-1} 0.848 = 32°$

$Q = \sqrt{3}El \sin\theta = \sqrt{3}(230)(75) \sin 32° = 15{,}832.86\ VAR$

$Q \cong 15.83\ kVAR$

328 $P = \sqrt{3}Elpf = \sqrt{3}(460)(20)(0.62) = 9879.6$

$P \cong 9880\ W$

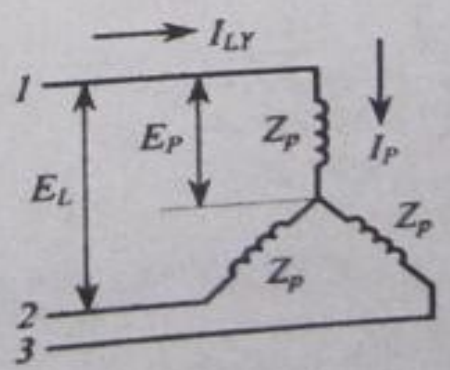
329 $P = \sqrt{3}Elpf = \sqrt{3}(460)(208)(0.91) = 150{,}807.58\ W$

$P = 150.8\ k\ W$

330 When wye - connected :

$I_p = \frac{E_P}{Z_P} = \frac{E_L / \sqrt{3}}{Z_P} = \frac{E_L}{\sqrt{3}Z_P}$

$I_{LY} = I_p = \frac{E_L}{\sqrt{3}Z_P}$

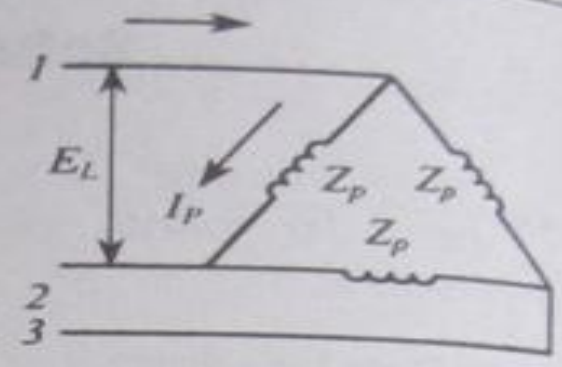


When delta - connected :

$I_p = \frac{E_L}{Z_P}$

$$I_{L\Delta} = \sqrt{3}I_P = \frac{\sqrt{3}E_L}{Z_P}$$

$$\text{Ratio} = \frac{I_{LY}}{I_{L\Delta}} = \frac{\dfrac{E_L}{\sqrt{3}Z_P}}{\dfrac{\sqrt{3}E_L}{Z_P}} = \frac{1}{3}$$



331 $I_L = I_P = \dfrac{E_P/\sqrt{3}}{Z} = \dfrac{120/\sqrt{3}}{5}$

$I_L = 13.85\ A$

332 $I_P = \dfrac{E_P}{Z} = \dfrac{240}{20} = 12\ A$

$Z_P = 20\angle 36.87° = 16 + j12$

$P_t = 3I_P^2R_P = 3(12)^2(16)$

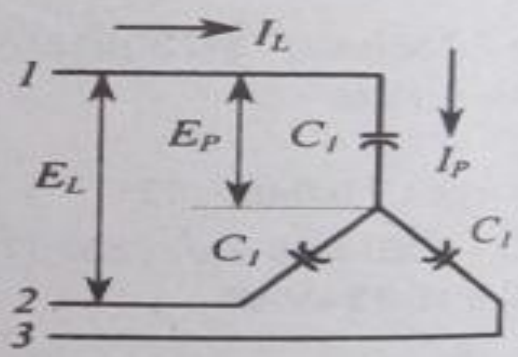
$P_t = 6912\ W$

333 If wye - connected :

$X_{C1} = \dfrac{1}{wC_1} = \dfrac{1}{(2\pi)(50)(75 \times 10^{-6})}$

$= 42.44\ \Omega$

$I_P = \dfrac{V_P}{X_C} = \dfrac{400/\sqrt{3}}{42.44} = 5.44\ A$



If delta connected :

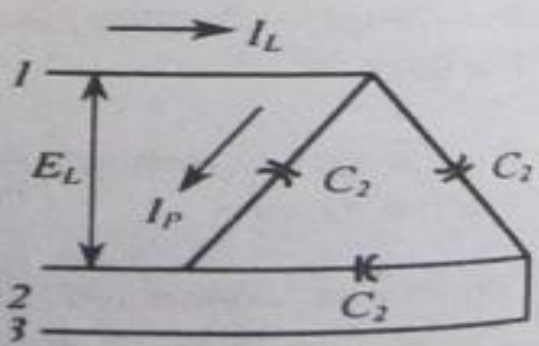
$I_P = \dfrac{I_L}{\sqrt{3}} = \dfrac{5.44}{\sqrt{3}} = 3.14\ A$

$X_{C2} = \dfrac{V_L}{I_P} = \dfrac{400}{3.14} = 127.388\ \Omega$

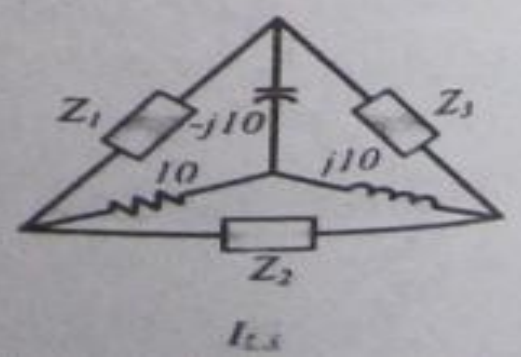
$C_2 = \dfrac{1}{w\,X_{C2}} = \dfrac{1}{(2\pi)(50)(127.388)}$

$= 24.987\ \mu F$

$C_2 \cong 25\ \mu F$



334 $Z_1 = \dfrac{j10(10) + 10(-j10) + (-j10)(j10)}{j10}$



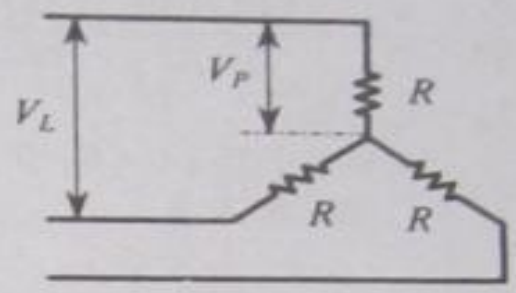
$$Z_1 = \frac{100}{j10} = -j10\,\Omega$$

$$Z_2 = \frac{100}{-j10} = j10\,\Omega$$

$$Z_3 = \frac{100}{10} = 10\,\Omega$$

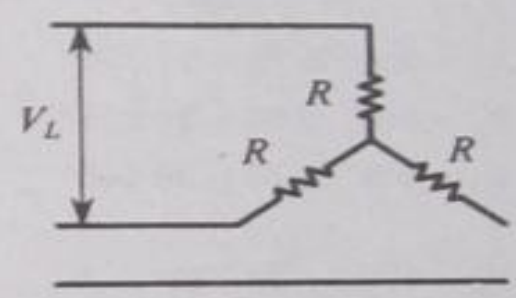
335 $$P_1 = 3\left(\frac{V_p^2}{R}\right) = \frac{3(V_L/\sqrt{3})^2}{R} = \frac{3V_L^2}{3R}$$

$$P_1 = \frac{V_L^2}{R}$$



$$P_2 = \frac{V_L^2}{2R} = \left(\frac{1}{2}\right)\left(\frac{V_L^2}{R}\right)$$

$$P_2 = \frac{P_1}{2}$$



336 Note: Since Y-connected, $I_L = I_P$.

$$Z_p = 4 - j3 = 5\angle -36.869^\circ$$

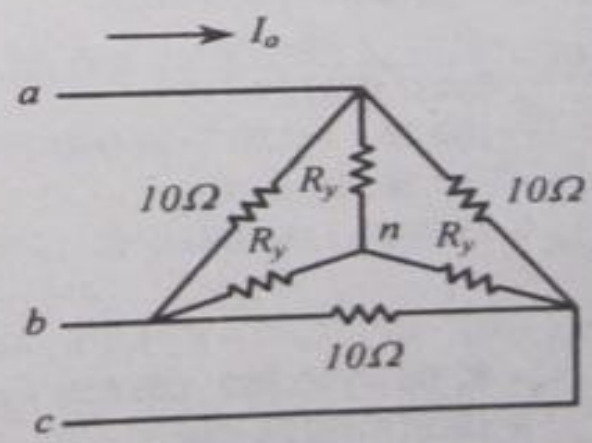
$$I_p = \frac{E_p}{Z_p} = \frac{220/\sqrt{3}}{5} = 25.4\text{ A}$$

$$I_L = 25.4\text{ A}$$

337 $$R_y = \frac{R_\Delta}{3} = \frac{10}{3} = 3.333\,\Omega$$

$$I_a = \frac{V_{an}}{R_y} = \frac{120}{3.333} = 36\text{ A}$$

$$I_a = 36\sin wt$$



Note: Since load is resistive, I_a is in phase with V_{an}.

338 $$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{200\angle 0^\circ}{10\angle 53.13^\circ} = 20\angle -53.13^\circ = 12 - j16\text{ A}$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{200\angle 240^\circ}{10\angle 0^\circ} = 20\angle 240^\circ = -10 - j17.32\text{ A}$$

$I_{bb} = I_{bc} - I_{ab}$

$= -10 - j17.32 - (12 - j16) = -22 - j1.32$

$I_{bb} = 22.04\angle -176.566°$ A

339 $I_{bc} = \dfrac{V_{bc}}{Z_{bc}} = \dfrac{117\angle 240°}{13\angle 0°} = 9\angle 240° = -4.5 - j7.79$ A

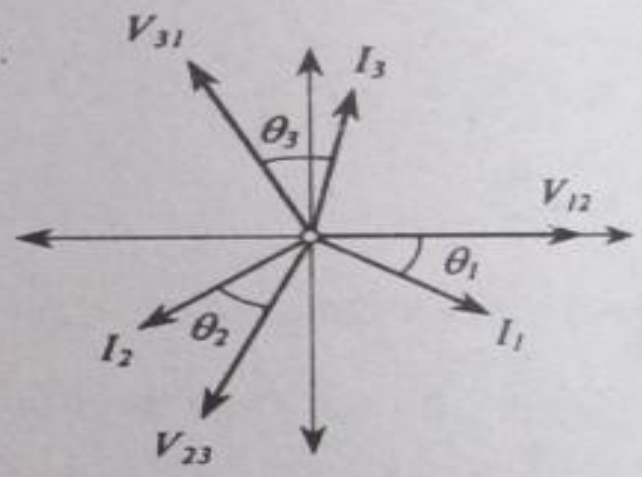
$I_{ca} = \dfrac{V_{ca}}{Z_{ca}} = \dfrac{117\angle 120°}{5 + j12} = \dfrac{117\angle 120°}{13\angle 67.38°} = 9\angle 52.62° = 5.46 + j7.15$ A

$I_{cc} = I_{ca} - I_{bc}$

$= 5.46 + j7.15 - (-4.5 - j7.79) = 9.96 + j14.94$

$I_{cc} = 17.95\angle 56.3°$ A

340 Note: Assume a phase sequence of 1-2-3.



$\theta_1 = \cos^{-1} 0.78 = 38.739°$

$\theta_2 = \cos^{-1} 0.82 = 34.915°$

$\theta_3 = \cos^{-1} 0.77 = 39.646°$

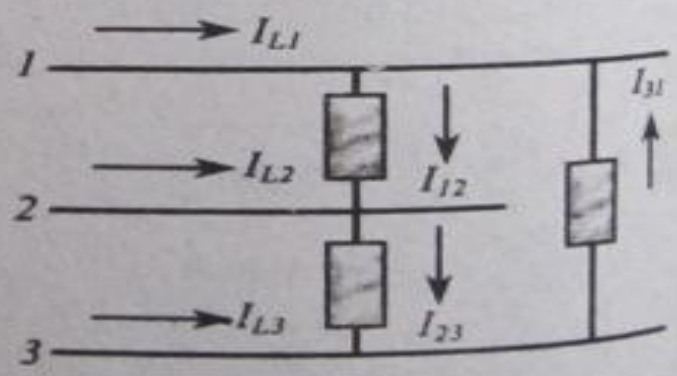
$I_{12} = 106\angle -38.739°$

$= 82.68 - j66.332$

$I_{23} = 142\angle -(120° + 34.915°)$

$= 142\angle -154.915°$

$= -128.606 - j60.202$



$I_{31} = \dfrac{P}{Epf} = \dfrac{28{,}400}{(230)(0.77)} = 160.36$ A

$= 160.36\angle +(120° - 39.646°)$

$= 160.36\angle 80.354°$

$I_{31} = 26.869 + j158.092$

$I_{L1} = I_{12} - I_{31}$

$= 82.68 - j66.332 - (26.869 + j158.092)$

$= 55.811 - j224.424$

$I_{L1} = 231.26\angle -76.03°$ A

$I_{L2} = I_{23} - I_{12}$

$= -128.606 - j60.202 - (82.68 - j66.332)$

$= -211.286 + j6.13$

$I_{L2} = 211.375\angle 178.338°$ A

$I_{L3} = I_{31} - I_{23}$
$= 26.869 + j158.092 - (-128.606 - j60.202)$
$= 155.475 + j218.294$
$I_{L3} = 268\angle 54.54°$ A

341 Note: Assume a phase sequence of 1-2-3.

$V_L = \sqrt{3}V_P = \sqrt{3}(120) = 208$ V

$I_{12} = \dfrac{V_{12}}{Z_1} = \dfrac{208\angle 0°}{40\angle 0°} = 5.2\angle 0° = 5.2 + j0$ A

$I_{23} = \dfrac{V_{23}}{Z_2} = \dfrac{208\angle -120°}{20\angle -60°} = 10.4\angle -60° = 5.2 - j9$ A

$I_{31} = \dfrac{V_{31}}{Z_3} = \dfrac{208\angle 120°}{15\angle 45°} = 13.867\angle 75° = 3.589 + j13.4$ A

$I_{L1} = I_{12} - I_{31} = (5.2 + j0) - (3.589 + j13.4) = 1.61 - j13.4$
$= 13.5\angle -83.15°$ A
$I_{L2} = I_{23} - I_{12} = (5.2 - j9) - (5.2 + j0) = -j9$
$= 9\angle -90°$ A
$I_{L3} = I_{31} - I_{23} = (3.589 + j13.4) - (5.2 - j9) = 1.61 + j22.4$
$= 22.45\angle 85.88°$ A

342 $Z_{ab} = 6 + j8 = 10\angle 53.13°$
$Z_{bc} = 5 + j8.66 = 10\angle 60°$
$Z_{ca} = 10 + j0 = 10\angle 0°$

$I_{ab} = \dfrac{V_{ab}}{Z_{ab}} = \dfrac{240\angle 0°}{10\angle 53.13°} = 24\angle -53.13° = 14.4 - j19.19$ A

$I_{bc} = \dfrac{V_{bc}}{Z_{bc}} = \dfrac{240\angle -120°}{10\angle 60°} = 24\angle -180° = -24$ A

$I_{ca} = \dfrac{V_{ca}}{Z_{ca}} = \dfrac{240\angle 120°}{10\angle 0°} = 24\angle 120° = -12 + j20.78$ A

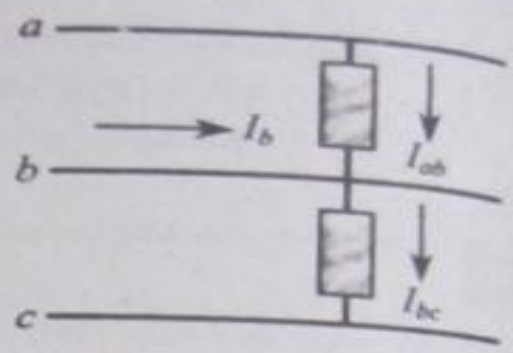
$I_{aa} = I_{ab} - I_{ca} = (14.4 - j19.19) - (-12 + j20.78) = 26.4 - j39.97$
$= 47.9\angle -56.55°$ A
$I_{bb} = I_{bc} - I_{ab} = (-24) - (14.4 - j19.19) = -38.4 + j19.19$
$= 42.93\angle 153.44°$ A
$I_{cc} = I_{ca} - I_{bc} = (-12 + j20.78) - (-24) = 12 + j20.78$
$= 23.99\angle 60°$ A

343 Consider the load connected across ab:

$$I_{ab} = \frac{P_{out}}{E_{ab} pf_a \eta_a} = \frac{8(746)}{230(0.7)(0.9)} = 41.187 \text{ A}$$

$$\theta_{ab} = \cos^{-1} 0.7 = 45.57°$$

$$I_{ab} = 41.187\angle -45.57° = 28.83 - j29.41 \text{ A}$$



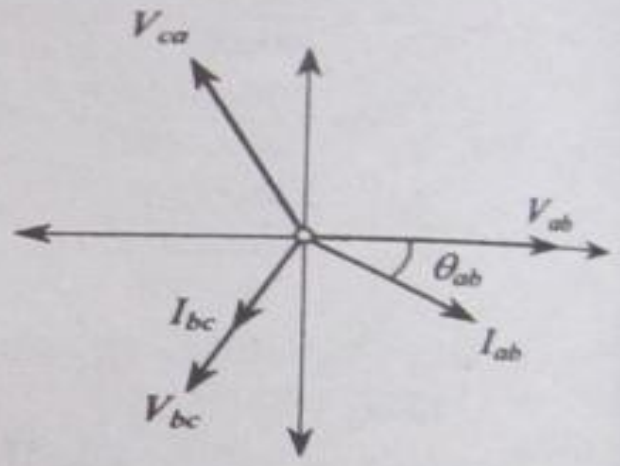
Consider the load connected across bc:

$$I_{bc} = \frac{P_b}{E_{bc} pf_b} = \frac{5000}{230(1.0)} = 21.739 \text{ A}$$

$$\theta_{bc} = \cos^{-1} 1.0 = 0°$$

$$I_{bc} = 21.739\angle -120°$$

$$= -10.869 - j18.826 \text{ A}$$



$$I_b = I_{bc} - I_{ab}$$

$$= -10.869 - j18.826 - (28.83 - j29.41)$$

$$= -39.699 + j10.584$$

$$I_b = 41.085\angle 165.071° \text{ A}$$

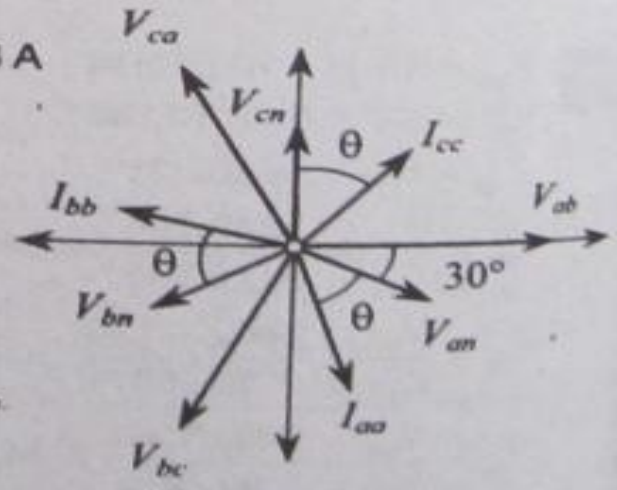
344 Note: Assume the motor load is wye-connected.

$$\theta = \cos^{-1} 0.71 = 44.76°$$

$$I_L = \frac{P_{out}}{\sqrt{3} V_L pf\, \eta} = \frac{5(746)}{\sqrt{3}(208)(0.71)(0.81)} = 18 \text{ A}$$

For the single-phase load:

$$I_{cn} = \frac{P}{V_{cn}} = \frac{6000}{208/\sqrt{3}} = 49.96 \text{ A}$$



Note: Current I_{cn} is in-phase with voltage V_{cn}.

$$I_c = I_{cc} + I_{cn}$$

$$= 18\angle(90° - 44.76°) + 49.96\angle 90°$$

$$= 12.674 + j12.781 + j49.96$$

$$I_c = 64\angle 78.58° \text{ A}$$

345 Note: In the absence of a neutral conductor, the sum of the three line currents is equal to zero.

$I_a + I_b + I_c = 0$

$I_b = -(I_a + I_b)$

$= -(110\angle -36.87° + 125\angle 53.13°) = -(88 - j66 + 75 + j100)$

$= -163 - j34$

$I_b = 166.508\angle -168.21°$ A

346 Note: Since the connection is unbalance wye, convert first the given wye-connected load to an equivalent delta-connected load.

$$Z_{ab} = \frac{10(10 + j10) + (10 + j10)(10) + (10)(10)}{10}$$

$$Z_{ab} = \frac{300 + j200}{10} = 30 + j20$$

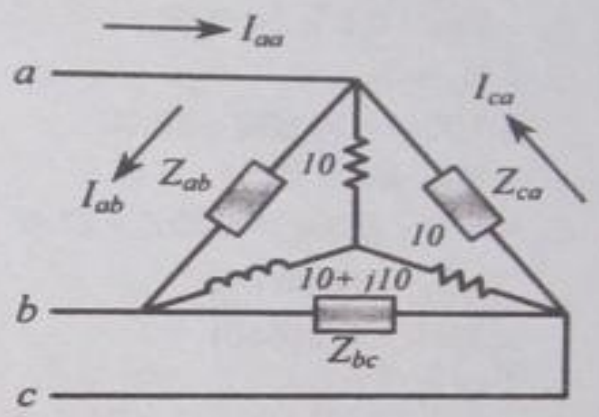
$$= 36.055\angle 33.69\ \Omega$$

$$Z_{bc} = \frac{300 + j200}{10} = 30 + j20$$

$$= 36.055\angle 33.69\ \Omega$$

$$Z_{ca} = \frac{300 + j200}{10 + j10} = \frac{360.55\angle 33.69°}{14.142\angle 45°}$$

$$\doteq 25.5\angle -11.31\Omega$$



$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{210\angle 0°}{36.055\angle 33.69°} = 5.824\angle -33.69° = 4.846 - j3.23\ \text{A}$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{210\angle 120°}{25.5\angle -11.31°} = 8.235\angle 131.31° = -5.436 + j6.186\ \text{A}$$

$I_{aa} = I_{ab} - I_{ca} = 4.846 - j3.23 - (-5.436 + j6.186)$

$= 10.282 - j9.416 = 13.94\angle -42.48°$ A

$I_{aa} \cong 14$ A

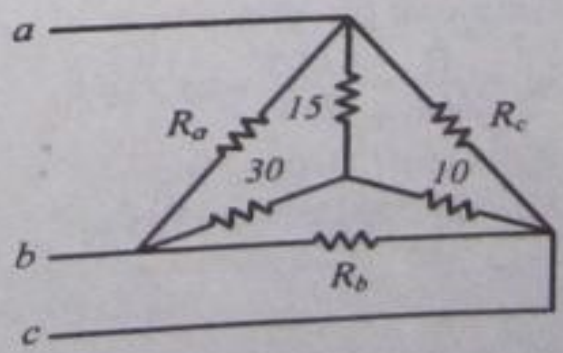
347 Note: Since the connection is unbalance wye, convert first the given wye-connected resistors to an equivalent delta-connected resistors.

$$R_a = \frac{15(30) + (30)(10) + 10(15)}{10}$$

$$= \frac{900}{10} = 90\ \Omega$$

$$R_b = \frac{900}{15} = 60\ \Omega$$

$$R_c = \frac{900}{30} = 30\ \Omega$$



$$P = \frac{E_L^2}{R_a} + \frac{E_L^2}{R_b} + \frac{E_L^2}{R_c} = \frac{208^2}{90} + \frac{208^2}{60} + \frac{208^2}{30}$$

$P = 2644\ W$

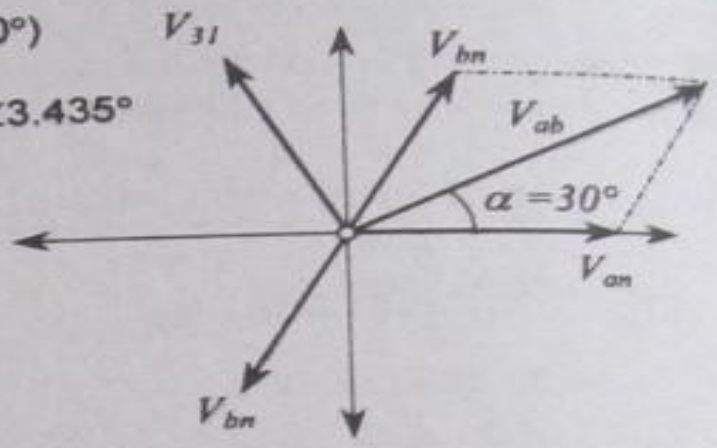
348

$Z = 10 + j5 = 11.18\angle 26.565°$

$V_{ab} = V_{an} - V_{bn} = \sqrt{3}(120)\sin(wt + 30°)$

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{\sqrt{3}(120)\angle 30°}{11.18\angle 26.565°} = 18.59\angle 3.435°$$

$I_{ab} = 18.59\sin(wt + 3.44°)\ A$



349

$\theta_a = \cos^{-1} 0.8 = 36.869°$

$S_t = S_a + S_b$

$= 100\angle -36.869 + 70\angle 0°$

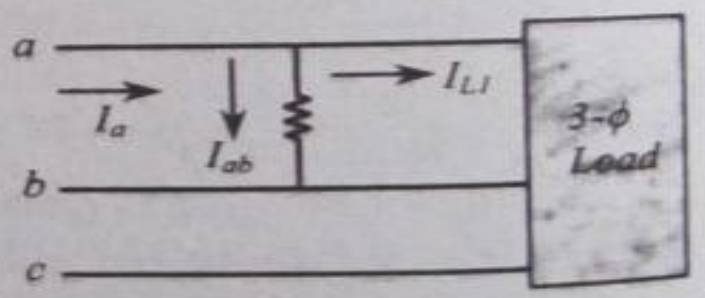
$= 80 - j60 + 70$

$S_t = 150 - j60 = 161.55\angle -21.8°$

$$I = \frac{S_t}{\sqrt{3}E} = \frac{161.55 \times 10^3}{\sqrt{3}(440)}$$

$I = 212\ A$

350 Note: Assume the 3-phase load is wye-connected (If load is delta-connected it has also an equivalent wye connection).



$$I_L = \frac{S}{\sqrt{3}V_L} = \frac{10{,}000}{\sqrt{3}(230)}$$

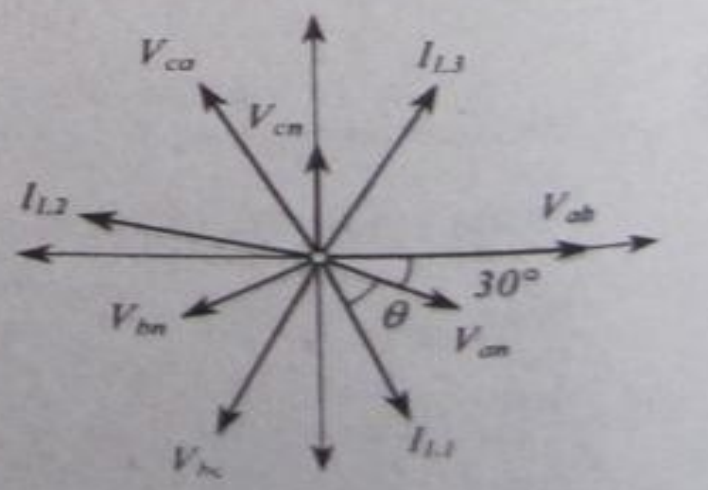
$= 25.102\ A$

$\theta = \cos^{-1} 0.7 = 45.57°$

Refer to the vector diagram :

$I_{L1} = 25.102\angle -75.57°$

$I_{L1} = 6.2553 - j24.31\ A$



For the single - phase load :

$$I_{ab} = \frac{S}{V_{ab}} = \frac{5{,}000}{230} = 21.739\ A$$

$\theta_{ab} = \cos^{-1} 1 = 0°$

$I_{ab} = 21.739\angle 0° = 21.739\ A$

Solving for line current on line a :

$I_a = I_{L1} + I_{ab} = 6.2553 - j24.31 + 21.739$

$I_a = 28 - j24.31$

$I_a = 37.08\angle 40.965°$ A

351

$Z = 16 - j12 = 20\angle 36.869°$

$I_P = \dfrac{E_P}{Z} = \dfrac{230}{20} = 11.5$ A

$I_L = \sqrt{3} I_P = \sqrt{3}(11.5) = 19.918$ A

$S_b = \sqrt{3} E_L I_L = \sqrt{3}(230)(19.918) = 7.934$ kVA

$\theta_a = \cos^{-1} 0.6 = 53.13°$

$S_t = S_a + S_b$

$= 10\angle -53.13° + 7.934\angle 36.869°$

$= 6 - j8 + 6.35 + j4.76$

$S_t = 12.35 - j3.24 = 12.768\angle -14.7°$

$pf_t = \cos 14.7°$

$pf_t = 0.967$ lagging

352

For the delta connected load :

$I_{p\Delta} = \dfrac{V_L}{Z_p} = \dfrac{208}{45} = 4.622$ A

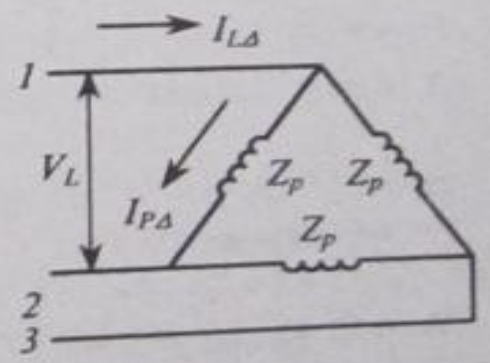
$I_{L\Delta} = \sqrt{3} I_{p\Delta} = \sqrt{3}(4.622) = 8$ A

$P_\Delta = \sqrt{3}\, V_L I_{L\Delta} pf = \sqrt{3}(208)(8)\cos 70°$

$P_\Delta = 985.75$ W

$Q_\Delta = \sqrt{3}\, V_L I_{L\Delta} \sin\theta = \sqrt{3}(208)(8)\sin 70°$

$Q_\Delta = 2708.32$ VAR



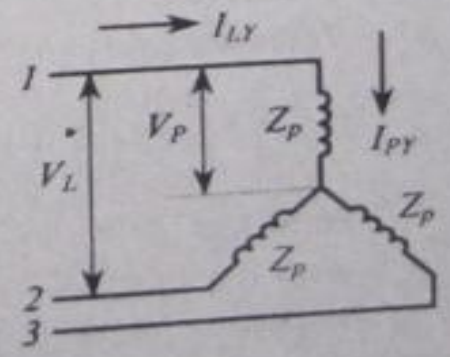
For the Y - connected load :

$I_{LY} = \dfrac{V_P}{Z_p} = \dfrac{208}{\sqrt{3}\,(10)} = 12$ A

$P_Y = \sqrt{3}\, V_L I_{L\Delta} pf = \sqrt{3}(208)(12)\cos 0°$

$P_Y = 4323.198$ W

$Q_Y = 0$



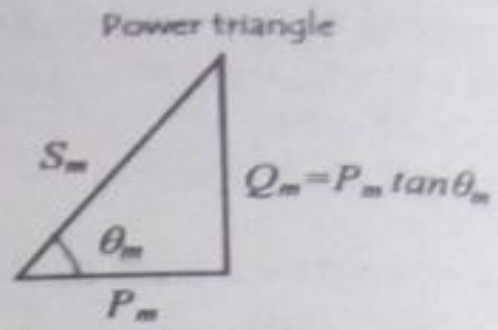
For the three - phase motor :

$P_m = S_m pf_m = 10{,}000(0.65) = 6500$ W

$\theta_m = \cos^{-1} 0.65 = 49.458°$

$Q_m = P_m \tan\theta_m = 650(\tan 49.458°)$

$Q_m = 7599.23$ VAR



$S_t = P_t - jQ_t$

$= (985.75 + 4323.198 + 6500) - j(2708.32 + 0 + 7599.23)$

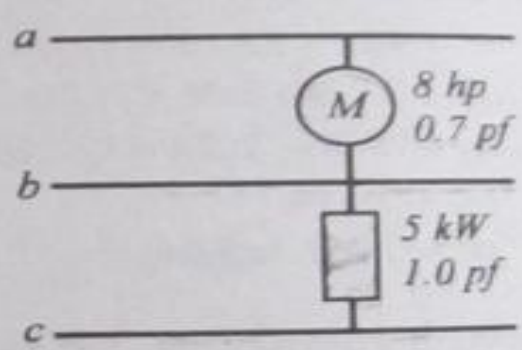
$= 11808.948 - j10307.55$

$S_t = 15{,}647.72\angle -41.116°$

$pf_t = \cos 41.116°$

$pf_t = 0.753$ lagging

353 Consider the load connected across ab :



$P_a = \dfrac{P_{out}}{\eta} = \dfrac{8(0.746)}{0.9} = 6.631$ kW

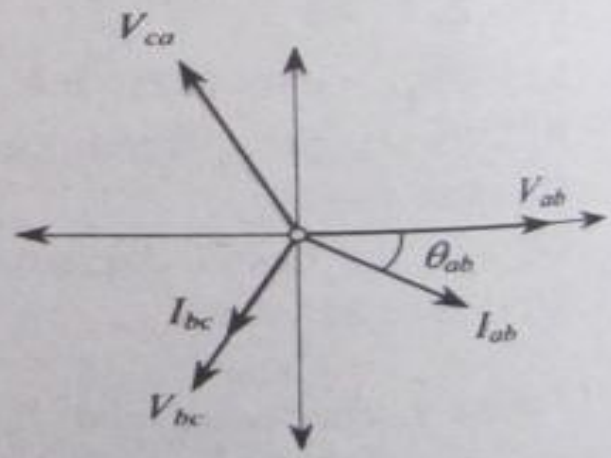
$S_a = \dfrac{P_a}{pf_a} = \dfrac{6.631}{0.7} = 9.473$ kVA

$\theta_b = \cos^{-1} 0.7 = 45.57°$

Consider the load connected across bc :

$P_b = 5$ kW

$\theta_b = \cos^{-1} 1.0 = 0°$



$S_t = S_a + S_b = 9.473\angle -45.57° + 5\angle 0°$

$= 6.631 - j6.764 + 5$

$= 11.631 - j6.764$

$S_t = 13.45\angle 30.18°$

$pf_t = \cos 30.18°$

$pf_t = 0.864$ lagging

354 $P_1 = \dfrac{P_{out}}{\eta} = \dfrac{30(0.746)}{0.85}$

$P_1 = 26.33$ kW

$\theta_1 = \cos^{-1} 0.9 = 25.84°$

$Q_1 = P_1 \tan\theta_1 = 26.33 \tan 25.84°$

$Q_1 = 12.75$ kVA

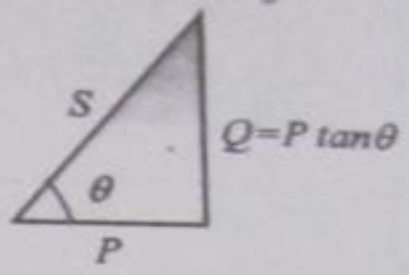
$$P_2 = \frac{0.5P_{out}}{\eta} = \frac{0.5(30)(0.746)}{0.89} = 12.57\text{ kW}$$

$\theta_2 = \cos^{-1} 0.78 = 38.74°$

$Q_2 = P_2 \tan\theta_2 = 12.57 \tan 38.74°$

$Q_2 = 10.08\text{ kVAR}$

Power triangle



$S_t = S_1 + S_2$

$= 26.33 - j12.75 + 12.57 - j10.08$

$= 38.9 - j22.83 = 45.104\angle 30.408°$

$pf_t = \cos 30.408°$

$pf_t = 0.862$ lagging

355 $S_1 = E_L I_1 = 230(106) = 24.38\text{ kVA}$

$\theta_1 = \cos^{-1} 0.78 = 38.739°$

$S_2 = E_L I_2 = 230(142) = 32.66\text{ kVA}$

$\theta_2 = \cos^{-1} 0.82 = 34.915°$

$$S_3 = \frac{P_3}{pf_3} = \frac{28.4}{0.77} = 36.88\text{ kVA}$$

$\theta_3 = \cos^{-1} 0.77 = 39.646°$

$S_t = S_1 + S_2 + S_3$

$= 24.38\angle -38.739° + 32.66\angle -34.915° + 36.88\angle -39.646°$

$= 19.016 - j15.256 + 26.781 - j18.693 + 28.397 - j23.531$

$S_t = 74.194 - j57.48$

$= 93.85\angle 37.76°$

$S_t \cong 94\text{ kVA}$

356 $\theta_a = \cos^{-1} 0.8 = 36.869°$

$Q_a = P_a \tan\theta_a = 6 \tan 36.869°$

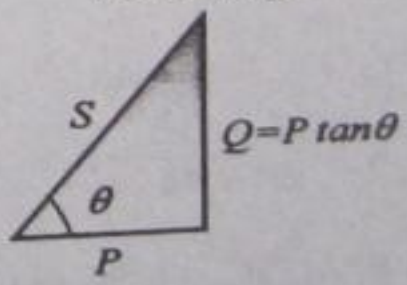
$Q_a = 4.5\text{ kVAR}$

$\theta_b = \cos^{-1} 0.835 = 33.384°$

$Q_b = P_b \tan\theta_b = 12 \tan 33.384°$

$Q_b = 7.907\text{ kVAR}$

Power triangle



$S_t = S_a + S_b = 6 - j4.5 + 12 + j7.907$

$= 18 + j3.407$

$S_t = 18.32\angle 10.718°$

$$S_t = \sqrt{3}E_L I_L$$

$$E_L = \frac{S_t}{\sqrt{3}I_L} = \frac{18.32}{\sqrt{3}(8)} = 1.322\,\text{kV}$$

$$I_1 = \frac{P_a}{\sqrt{3}E_L pf_a} = \frac{6}{\sqrt{3}(1.322)(0.8)}$$

$$I_1 \cong 3.28\ \text{A}$$

357 Assume the following:

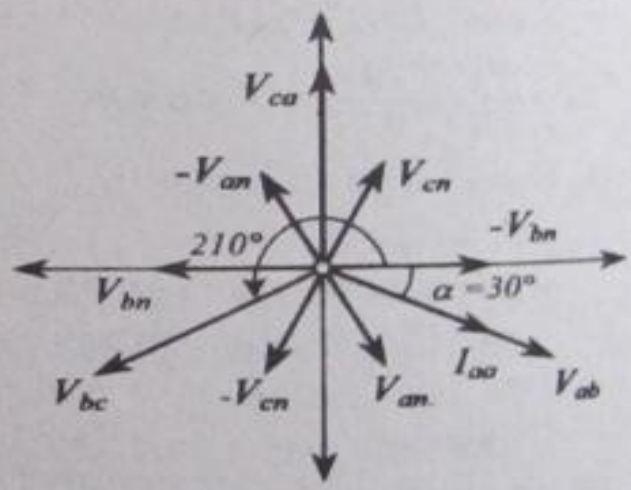
1. Load is wye connected for simplicity in the analysis.
2. System is balanced such the three voltages will be out of phase by 120°
3. Phase b line voltage refers to V_{bc}
4. Phase a line current refers to I_{aa} that is also equal to the phase current since load is assumed wye-connected.

Note: Refer to the vector diagram, by inspection the power factor angle (angle between the phase voltage and the phase current) is equal to 30°.

$$P_m = \sqrt{3}V_L I_L pf$$

$$= \sqrt{3}(220)(10)(\cos 30°)$$

$$P_m = 3300\ \text{W}$$



358

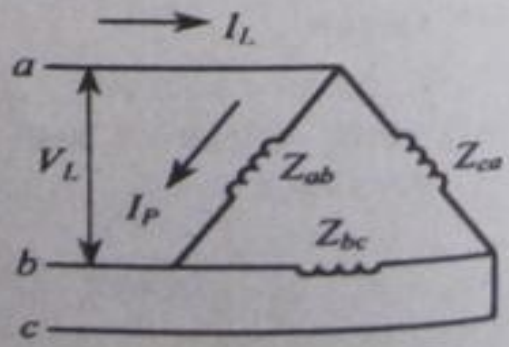
$$Z_{ab} = 15\angle 30° = 13 + j7.5$$

$$Z_{bc} = 15\angle 0° = 15$$

$$Z_{ca} = 15\angle -30° = 13 - j7.5$$

$$I_{ab} = I_{bc} = I_{ca} = I_p$$

$$I_p = \frac{V_p}{Z} = \frac{120}{15} = 8\ \text{A}$$



$$P_t = I_p^2 R_{ab} + I_p^2 R_{bc} + I_p^2 R_{ca}$$

$$= I_p^2 (R_{ab} + R_{bc} + R_{ca})$$

$$P_t = (8)^2 (13 + 15 + 13)$$

$$P_t = 2624\ \text{W}$$

359 For the balanced delta-connected load:

$Z_\Delta = 16 - j12 = 20\angle -36.869°$

$I_\Delta = \dfrac{V_L}{Z_\Delta} = \dfrac{230}{20} = 11.5\text{ A}$

$P_\Delta = 3(I_\Delta)^2 R_\Delta = 3(11.5)^2(16) = 6.348\text{ kW}$

$S_\Delta = \dfrac{P_\Delta}{pf} = \dfrac{6.348}{\cos 36.869} = 7.935\text{ kVA}$

For the motor load:

$\theta_m = \cos^{-1} 0.6 = 53.13°$

$S_t = S_m + S_\Delta = 10\angle -53.13° + 7.935\angle 36.869°$

$= (6 - j8) + (6.348 + j4.761)$

$S_t = 12.348 - j3.239 = 12.765\angle -14.69°\text{ kVA}$

$I_L = \dfrac{S_t}{\sqrt{3}V_L} = \dfrac{12{,}765}{\sqrt{3}(230)}$

$I_L = 32.04\text{ A}$

360 $P = EIpf$

$P_t = P_1 + P_2 + P_3$

$= 120(100)(1) + 100(230)(0.8) + 110(85)(0.77)$

$P_t = 37{,}599.5$

$P_t \cong 37.6\text{ kW}$

361 $I_P = \dfrac{V_P}{Z_P} = \dfrac{240}{16} = 15\text{ A}$

$I_L = \sqrt{3}I_p = \sqrt{3}(15) = 25.98\text{ A}$

$S = \sqrt{3}V_L I_L = \sqrt{3}(240)(25.98)$

$S = 10.8\text{ kVA}$

362 $V_{an} = \dfrac{230}{\sqrt{3}} = 132.79\angle 0°$

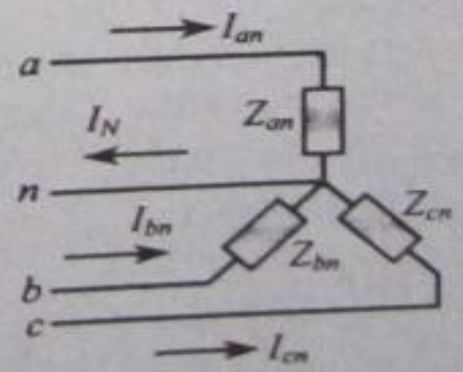
$V_{bn} = 132.79\angle -120°$

$V_{cn} = 132.79\angle 120°$

$Z_{an} = 20 + j0 = 20\angle 0°$

$Z_{bn} = 16 + j12 = 20\angle 36.869°$

$Z_{cn} = 5 - j15 = 15.811\angle -71.565°$



$$I_{an} = \frac{V_{an}}{Z_{an}} = \frac{132.79\angle 0^\circ}{20\angle 0^\circ} = 6.639\angle 0^\circ$$

$$i_{an} = 6.639 + j0$$

$$I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{132.79\angle -120^\circ}{20\angle 36.869^\circ} = 6.639\angle -156.869^\circ$$

$$I_{bn} = -6.105 - j2.608$$

$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{132.79\angle 120^\circ}{15.811\angle -71.565^\circ} = 8.398\angle 191.565^\circ$$

$$I_{cn} = -8.227 - j1.684$$

$$I_N = I_{an} + I_{bn} + I_{cn}$$
$$= 6.639 + (-6.105 - j2.608) + (-8.227 - j1.684)$$
$$= -7.693 - j4.292 = 8.809\angle -150.84^\circ$$

$$I_N \cong 8.81\,A$$

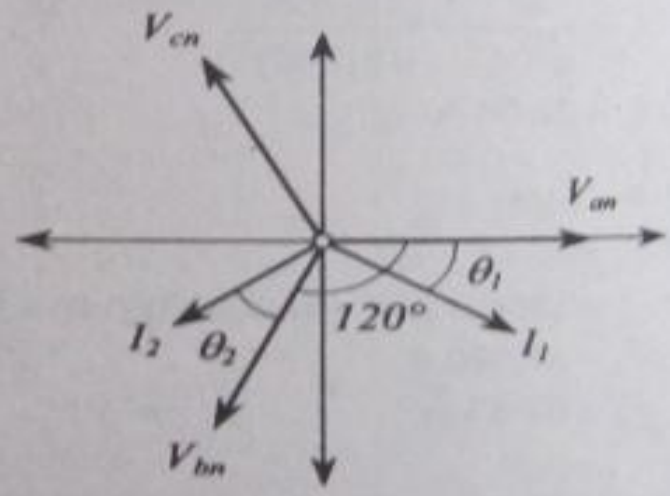
363 For motor A :

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{EIpf}$$

$$I_1 = \frac{P_{out}}{Epf\eta} = \frac{4(746)}{265(0.8)(0.9)}$$

$$I_1 = 15.64\ A$$

$$\theta_1 = \cos^{-1} 0.8 = 36.869^\circ$$



For motor B :

$$I_2 = \frac{P_{out}}{Epf\eta} = \frac{3(746)}{265(0.85)(0.85)}$$

$$I_2 = 11.69\ A$$

$$\theta_2 = \cos^{-1} 0.85 = 31.788^\circ$$

Refer to vector diagram :

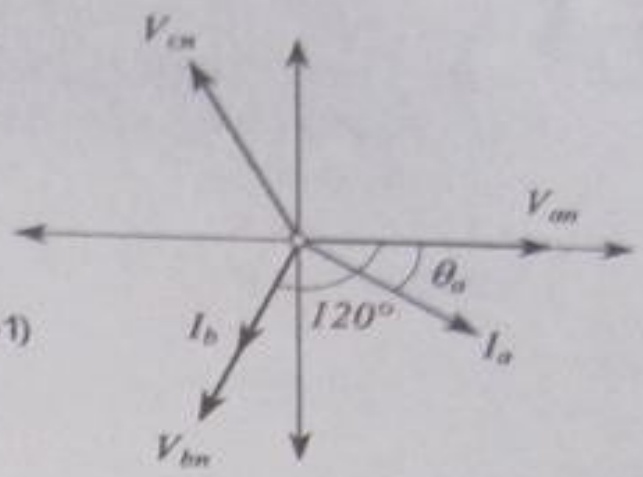
$$I_N = I_1 + I_2$$
$$= 15.64\angle -36.869^\circ + 11.69\angle -(120^\circ + 31.788^\circ)$$
$$= 12.512 - j9.384 - (-10.3 - j5.526) = 22.812 - j3.858$$

$$I_N = 23.136\angle -9.6^\circ$$

$$I_N \cong 23.14\ A$$

364 $\theta_a = \cos^{-1} 0.707 = 45°$

$\theta_b = \cos^{-1} 1 = 0°$



Refer to the vector diagram :

$I_N = I_a + I_b$

$= 40\angle -45° + 25\angle -120°$

$= 28.284 - j28.284 + (-12.5 - j21.651)$

$= 15.784 - j49.935$

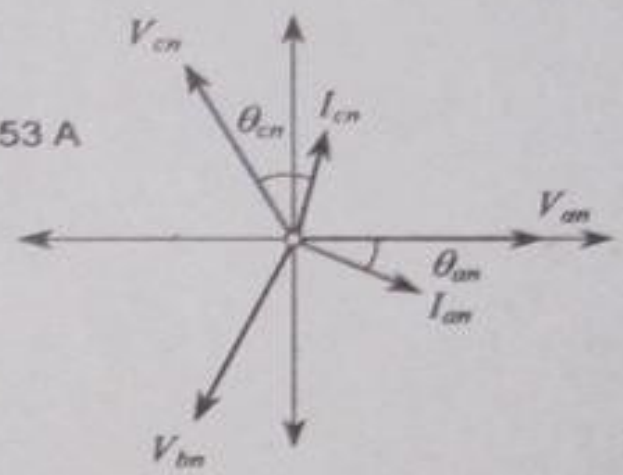
$I_N = 52.37\angle 72.458°$ A

365 For the 5 hp motor :

$$P_{in} = \frac{P_{out}}{\eta} = V_{an} I_{an} pf$$

$$I_{an} = \frac{P_{out}}{\eta V_{an} pf} = \frac{5(746)}{0.9(220)(0.866)} = 21.753 \text{ A}$$

$\theta_{an} = \cos^{-1} 0.866 = 30°$



For the 3 hp motor :

$$P_{in} = \frac{P_{out}}{\eta} = V_{cn} I_{cn} pf$$

$$I_{cn} = \frac{P_{out}}{\eta V_{cn} pf} = \frac{3(746)}{0.9(220)(0.707)} = 15.987 \text{ A}$$

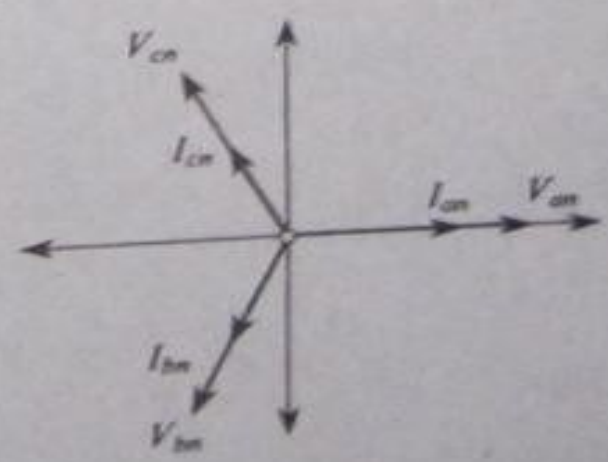
$\theta_{cn} = \cos^{-1} 0.707 = 45°$

$I_N = I_{an} + I_{cn} = 21.753\angle -30° + 15.987\angle(120° - 45°)$

$= 18.838 - j10.876 + 4.138 + j15.442 = 22.976 + j4.566$

$I_N = 23.425\angle 11.24°$ A

366 Note: Assume a phase sequence of a-b-c



Refer to the vector diagram :

$I_N = I_a + I_b + I_c$

$= 60\angle 0° + 86\angle -120° + 40\angle 120°$

$= 60 + j0 + (-43 - j74.478)$

$+ (-20 + j34.641)$

$= -3 - 39.838$

$I_N = 39.95\angle -94.306°$ A

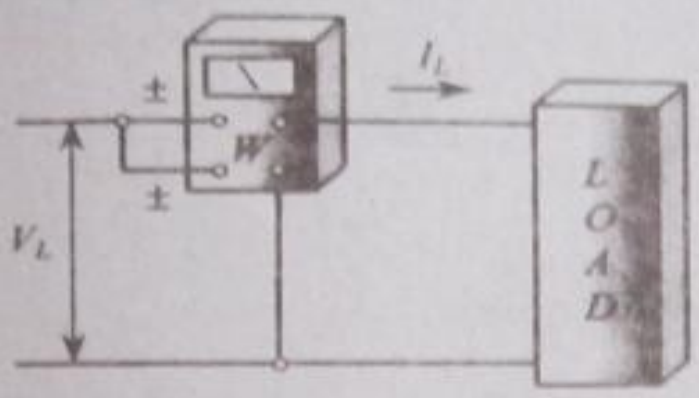
$I_N \cong 40$ A

367

$$I_N = I_a + I_b + I_c$$
$$= 10\angle -30^\circ + 12\angle 215^\circ + 15\angle 82^\circ$$
$$= 8.66 - j5 + (-9.829 - j6.883) + 2.087 + j14.854)$$
$$= 0.918 - j2.971$$
$$I_N = 3.11\angle 72.83^\circ \text{ A}$$
$$I_N = 3.11 \text{ cis } 72.83^\circ \text{ A}$$

3-phase AC System: Part 2

POWER MEASUREMENT USING A WATTMETER

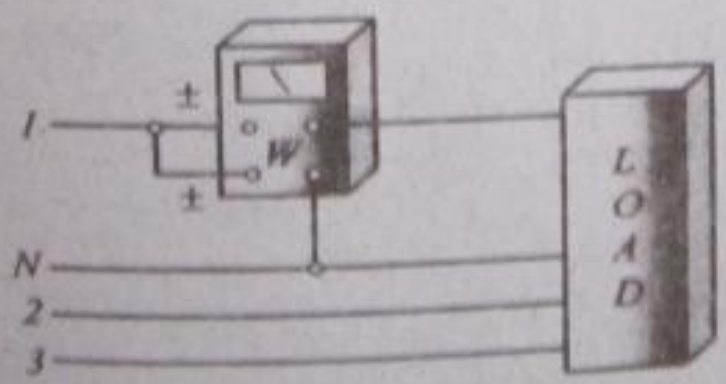


$$W = V_L I_L \cos\theta$$

where: θ = angle between V_L and I_L
$\cos\theta$ = load power factor

3-PHASE POWER MEASUREMENT WITH BALANCED LOAD USING ONLY A SINGLE WATTMETER

- If neutral point is available, then the single-phase wattmeter is connected to measure phase 3-power as shown.

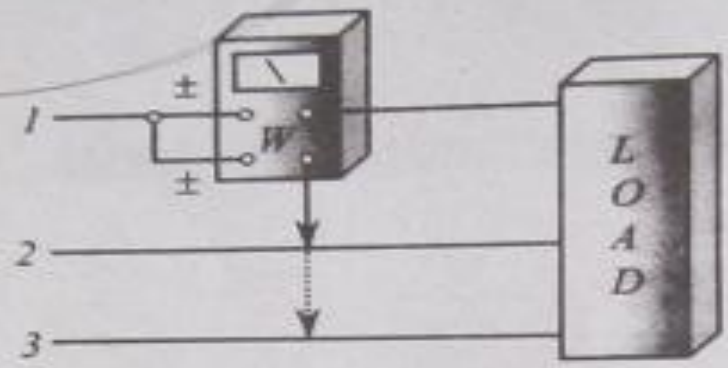


$$P_{total} = 3W$$

where: W = wattmeter reading

❖ Potential-lead shift method

In this scheme the current coil of the wattmeter is inserted in one line, say line 1. One of the potential coil terminals is connected to the same line while the other is shifted or touched to line 2 for one reading and to line 3 for another reading.



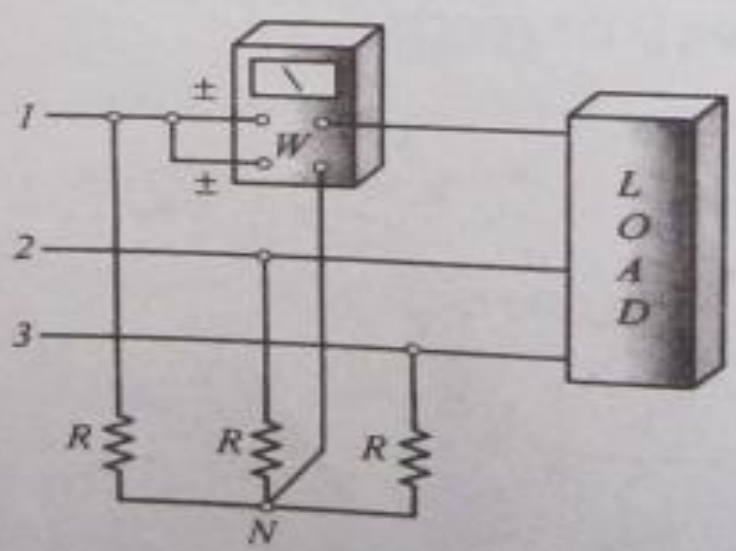
$$P_{total} = W' + W''$$

where:

W' = wattmeter reading on first position
W'' = wattmeter reading on second position

❖ Artificial neutral method

If three equal resistances are connected in star across the lines of a three-phase source, its neutral point will at every instant of time be exactly the same electrical potential as the neutral point of the load

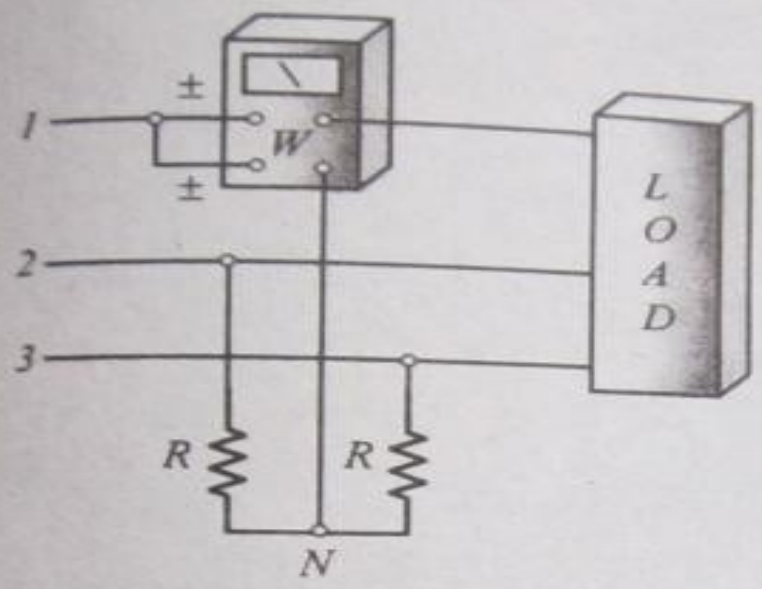


$$P_{total} = 3W$$

where: W = wattmeter reading

T – method

In this method, the current coil is inserted in one line say line 1 and one of the potential coil terminals is connected to the same line. The other terminal is fastened to the junction between a pair of equal resistance, the free ends of which is connected to the other two lines.

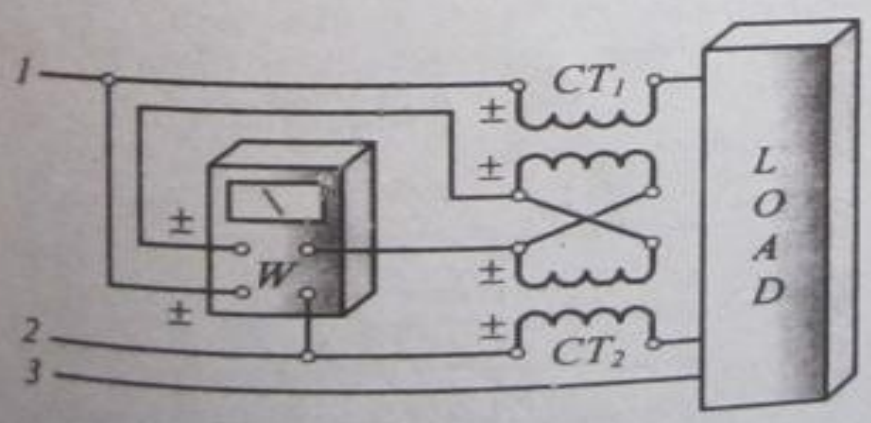


$P_{total} = 2W$

where: W = wattmeter reading

Current transformer (CT) method

This scheme gives direct measurement of the total power. It employs a pair of current transformers and a single wattmeter. The current coil of the wattmeter is connected to carry a current, which is the geometric difference of the currents of the secondaries of the CTs.

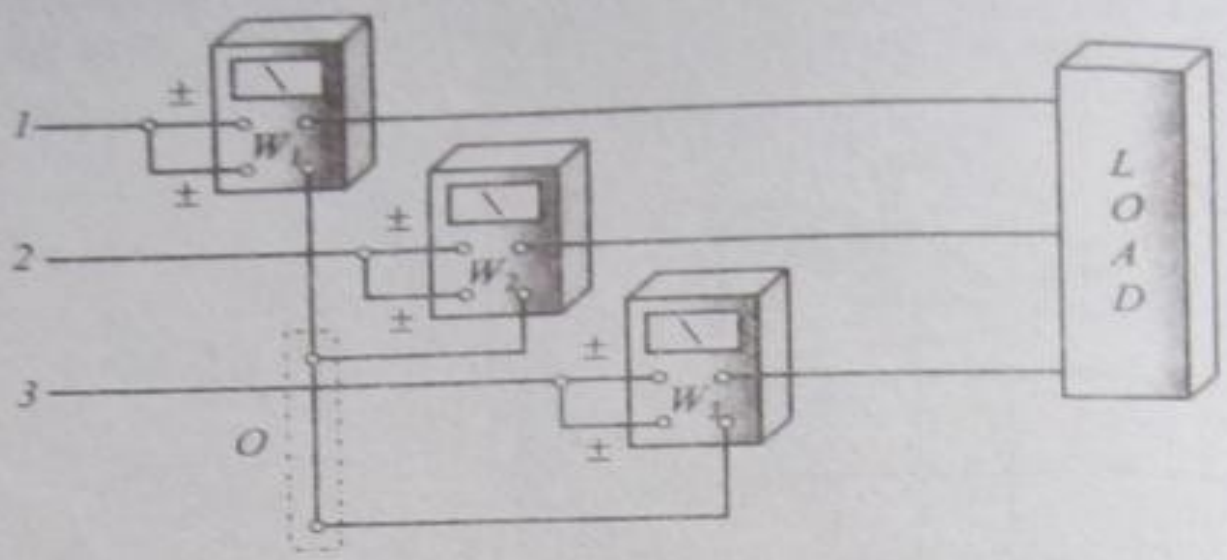


$P_{total} = W$

where: W = wattmeter reading

n-PHASE POWER MEASUREMENT USING "n" NUMBER OF WATTMETERS

Note: The load may be either balanced or unbalanced.



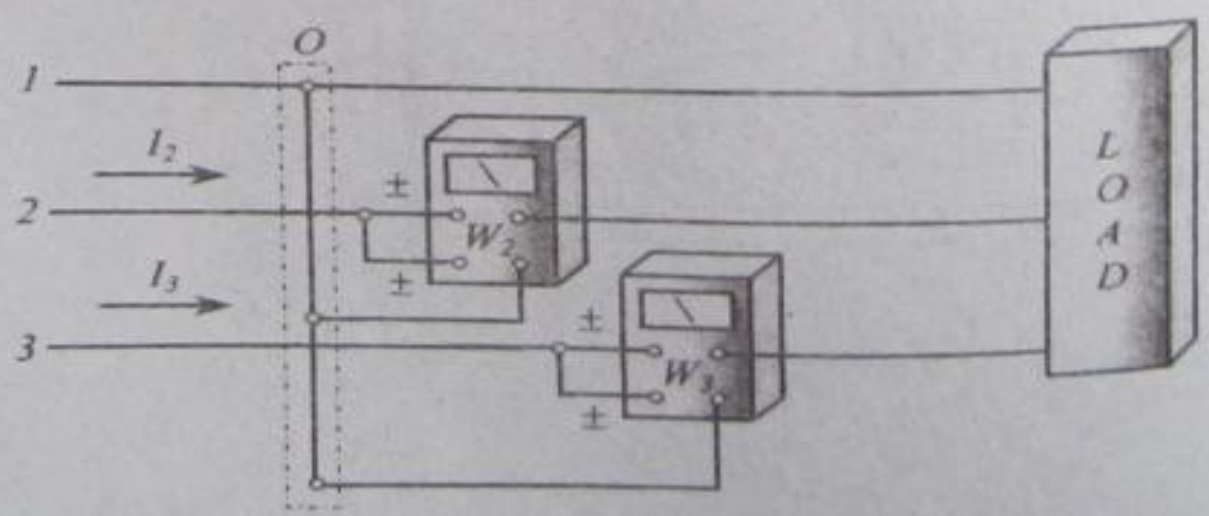
$$P_{total} = W_1 + W_2 + W_3$$

Note: Point O is an arbitrary common junction and no restriction is placed upon the location of this point.

n-PHASE POWER MEASUREMENT USING "n-1" NUMBER OF WATTMETERS

Any one of the "n" wattmeters maybe eliminated by transferring the point O from its arbitrary position in space to one of the "n" lines. This causes the voltage across the potential coil to one of the "n" wattmeters to vanish, and reduces its reading to zero. Thus, the wattmeter whose reading is now zero can be removed from the circuit and the total power can be measured by means of **"n - 1" wattmeters**.

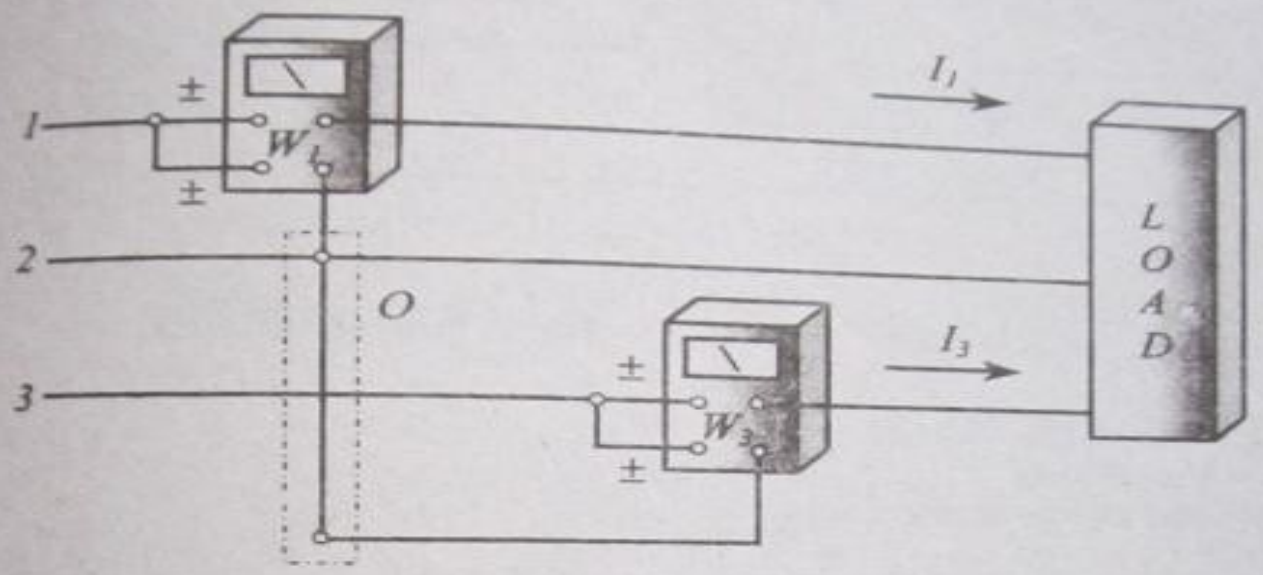
- **If point O is transferred to line 1, $W_1 = 0$.**



$$W_2 = V_{21} I_2 \cos\theta_{21} \qquad W_3 = V_{31} I_3 \cos\theta_{31}$$

where: θ_{21} = angle between V_{21} and I_2
θ_{31} = angle between V_{31} and I_3

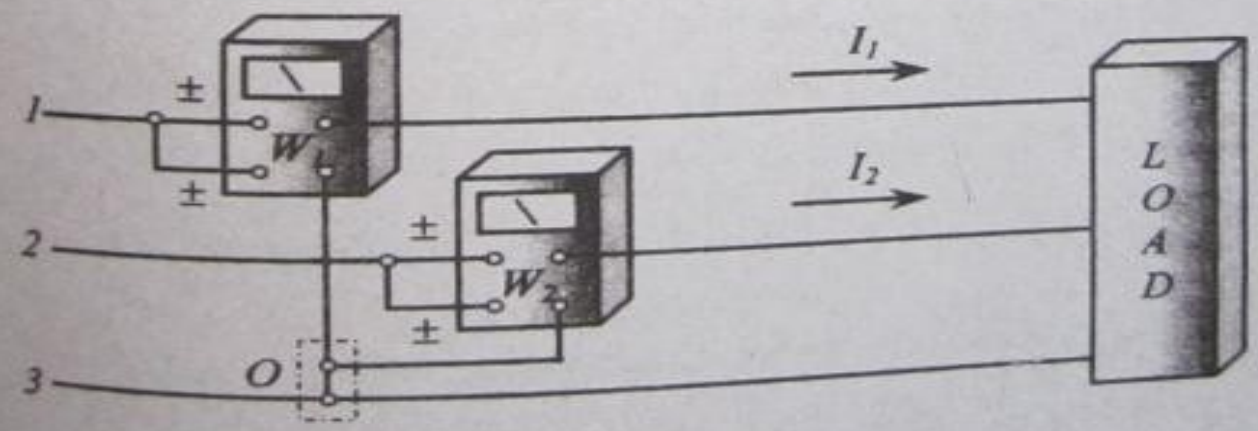
- If point O is transferred to line 2, $W_2 = 0$.



$$W_1 = V_{12} I_1 \cos\theta_{12} \qquad W_3 = V_{32} I_3 \cos\theta_{32}$$

where: θ_{12} = angle between V_{12} and I_1
θ_{32} = angle between V_{32} and I_3

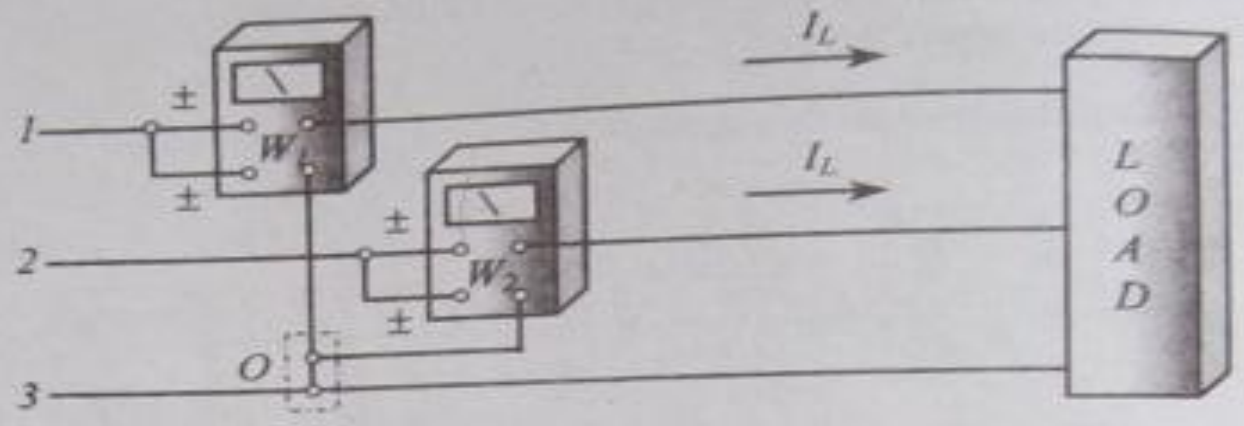
- If point O is transferred to line 3, $W_3 = 0$.



$$W_1 = V_{13} I_1 \cos\theta_{13} \qquad W_2 = V_{23} I_2 \cos\theta_{23}$$

where: θ_{13} = angle between V_{13} and I_1
θ_{23} = angle between V_{23} and I_2

TWO-WATTMETER READINGS WHEN LOAD IS BALANCED



$$W_1 = V_L I_L \cos(30° - \theta) \qquad W_2 = V_L I_L \cos(30° + \theta)$$

where: V_L = line to line voltage
I_L = line current
θ = load power factor angle

POWER FACTOR COMPUTATION USINGTHE TWO-WATTMETER READINGS

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \qquad pf = \cos\theta$$

Note: This formula is only applicable if load is balanced.

Test 11

Problem 368: EE Board April 1980

A factory is supplied by a three-phase, 3-wire system with the following characteristics:

$$V_{ab} = 230\angle 0^\circ \text{ V} \qquad I_a = 110\angle -36.87^\circ \text{ A}$$
$$V_{bc} = 230\angle 240^\circ \text{ V} \qquad I_c = 125\angle 53.13^\circ \text{ A}$$

Determine the power consumed by the load.

A. 42.76 kW
B. 48.78 kW
C. 40.23 kW
D. 45.12 kW

Problem 369:

A 3-phase, 3-wire feeder has the following line currents and line voltages:

$$V_{ab} = 220\angle 0^\circ \text{ V} \qquad I_{aa} = 30 - j30 \text{ A}$$
$$V_{bc} = 220\angle 240^\circ \text{ V} \qquad I_{cc} = 29 + j39.8 \text{ A}$$

Determine the reactive power supplied by the feeder.

A. 7.75 kVAR
B. 7.34 kVAR
C. 8.32 kVAR
D. 8.45 kVAR

Problem 370:

Given a balanced three-phase system with the following loads:

$$V_{ab} = 230\angle 0^\circ \text{ V} \qquad Z_{ab} = 60$$
$$V_{bc} = 230\angle 120^\circ \text{ V} \qquad Z_{bc} = 30 + j50$$
$$V_{ca} = 230\angle 240^\circ \text{ V} \qquad Z_{ca} = -j80$$

Determine the reading of the two wattmeters properly connected using line c as the common point.

A. 440.9 W, 907.5 W
B. 394.1 W, 951.2 W
C. 440.9 W, 951.2 W
D. 394.1 W, 907.5 W

Problem 371:

Given a balanced 3-wire, three-phase system serving the following loads:

$V_{ab} = 220\angle 0^\circ$ V $\quad Z_{ab} = 11\angle 0^\circ\ \Omega$

$V_{bc} = 220\angle 240^\circ$ V $\quad Z_{bc} = 11\angle 45^\circ\ \Omega$

$V_{ca} = 220\angle 120^\circ$ V $\quad Z_{ca} = 11\angle 30^\circ\ \Omega$

Determine the reading of the two wattmeters with line b as common.

A. 6921.5 W, 4820 W
B. 6921.5 W, 4400 W
C. 6342.5 W, 4650 W
D. 6342.5 W, 4400 W

Problem 372: EE Board April 1981

The following voltages and line currents were measured to a 3-phase, 3-wire feeder serving a commercial building:

$V_{ab} = 2{,}400\angle 0^\circ$ V $\quad I_a = 85\angle 330^\circ$ A

$V_{bc} = 2{,}400\angle 240^\circ$ V $\quad I_c = 100\angle 80^\circ$ A

Solve for the real power in kW drawn by the commercial building.

A. 402.2
B. 404.5
C. 419.3
D. 421.5

Problem 373: EE Board October 1981

A 3-phase, 3-wire load draws the following line currents:

$I_a = 60\angle 330^\circ$ A $\quad I_b = 78.4\angle 214^\circ$ A $\quad I_c = 75\angle 80^\circ$ A

If the voltages impressed on the load are balanced 3-phase, having a magnitude of 4140 volts line to line, solve for the total power in kW.

A. 556.15
B. 506.85
C. 536.54
D. 520.18

Problem 374:

Given the following voltages and load impedances:

$V_{ab} = 220\angle 0^\circ$ V $\quad Z_{ab} = 10\angle 36.87^\circ\ \Omega$

$V_{bc} = 220\angle 240^\circ$ V $\quad Z_{bc} = 10\angle 0^\circ\ \Omega$

$V_{ca} = 220\angle 120^\circ$ V $\quad Z_{ca} = 10\angle 45^\circ\ \Omega$

Determine the reading of a kW meter connected with its current coil connected in line b while the potential coil connected across lines b and c.

A. 5.12 kW
B. 4.26 kW

C. 4.08 kW
D. 5.45 kW

Problem 375: EE Board October 1983

Given the following load impedances in delta and the impressed voltages as follows:

$$V_{ab} = 220\angle 0^\circ \text{ V} \qquad Z_{ab} = 8 + j6$$
$$V_{bc} = 220\angle 240^\circ \text{ V} \qquad Z_{bc} = 8.66 - j5$$
$$V_{ca} = 220\angle 120^\circ \text{ V} \qquad Z_{ca} = 10 + j0$$

What will be the reading of the two wattmeters connected to measure total power. Use line a as the common potential point.

A. 3.869 kW, 9.031 kW
B. 2.546 kW, 8.357 kW
C. 3.125 kW, 6.778 kW
D. 4.055 kW, 9.848 kW

Problem 376: EE Board October 1984

The 3-phase power supply to a factory has the following measurements:

$$V_{ab} = 240\angle 0^\circ \text{ V} \qquad I_a = 120\angle 330^\circ \text{ A}$$
$$V_{bc} = 240\angle 240^\circ \text{ V} \qquad I_b = 157\angle 214^\circ \text{ A}$$
$$V_{ca} = 240\angle 120^\circ \text{ V} \qquad I_c = 150\angle 80^\circ \text{ A}$$

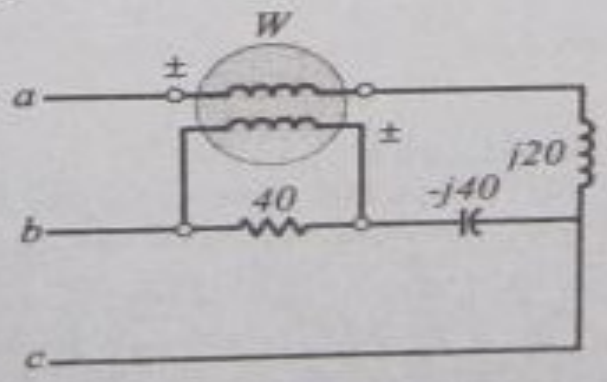
Solve for the total power drawn.

A. 60.2 kW
B. 56.5 kW
C. 58.8 kW
D. 62.4 kW

Problem 377:

Given $V_{ab} = 220\angle 0^\circ$ V $V_{bc} = 220\angle -120^\circ$ V $V_{ca} = 220\angle -240^\circ$ V

Determine the reading of the wattmeter as shown.



A. 405.45 W
B. 486.32 W
C. 442.88 W
D. 467.54 W

Problem 378: EE Board April 1985

A balanced 3-phase load draws 120 amperes line current at 230 volts line to line, 0.848 pf lagging current. Solve for the readings of the two wattmeters used to measure the 3-phase power.

A. 25.543 kW, 15.087 kW
B. 28.155 kW, 12.385 kW
C. 24.365 kW, 16.175 kW
D. 27.583 kW, 12.957 kW

Problem 379:

A number of incandescent lamps are connected in delta across the lines of a balanced 230 V, 3-phase, 3-wire system. A single-phase wattmeter is connected in the circuit. The current coil of the meter is connected in line 1 while the potential coil is connected across lines 1 and 3. This wattmeter registers 8.4 kW. If the potential terminal of the meter, which was connected in line 3, is transferred to line 2, the wattmeter reads 7.5 kW. If each lamp is rated 100 W, how many lamps are connected across lines 1 and 2. Assume lamp pf to be unity.

A. 42
B. 45
C. 43
D. 44

Problem 380: EE Board April 1988

MERALCO used two wattmeters to measure the balanced 3-phase dynatron elevator motor drive. The current coils of the wattmeters are connected to the current transformers, which are in lines 1 and 2 respectively. The potential coils are connected to potential transformers, which are across lines 2 & 3 and lines 3 & 1, respectively. The line potentials are 230 V and the line currents are each 150 A. The wattmeters each indicate 19.6 kW. Assume load is wye-connected. What is the total power supplied?

A. 49.175 kW
B. 48.236 kW
C. 45.461 kW
D. 47.350 kW

Problem 381: EE Board April 1992

A 460 volt, three-phase motor draws 208 A with a power factor of 0.91 lagging. Calculate the indications of W_1 and W_2 for the given condition.

A. 75.40 kW, 75.40 kW
B. 91.23 kW, 59.58 kW
C. 89.56 kW, 61.25 kW
D. 95.24 kW, 55.57 kW

Problem 382:

Two wattmeters are used to measure the power delivered to a balanced delta connected load. Wattmeter 1 reads 1000 W and wattmeter 2 reads 500 W after the current coil terminals are reversed. If the line to line voltage is 208 V, solve for the line current drawn by the load.

A. 8.23 A
B. 7.57 A
C. 8.08 A
D. 7.34 A

Problem 383:

The power drawn by a three-phase balanced load is measured using two wattmeters properly connected. At what power factor will the reading of one of the wattmeters be zero?

A. 0.866
B. 1.0
C. 0.5
D. 0.707

Problem 384:

Two wattmeters connected to a three-phase 440 V squirrel cage induction motor indicate a total of 12 kW. If the operating power factor of the motor is 0.4, determine the individual reading of the wattmeters.

A. 10.46 kW, 1.60 kW
B. 9.24 kW, 2.76 kW
C. 13.94 kW, -1.94 kW
D. None of these

Problem 385:

A 3-phase induction motor is connected across a 220-V three-phase source. A wattmeter reads 600 W when its current coil is connected in line 1 and its potential coil across line 1 and 2. If the potential coil of the wattmeter is transferred to lines 2 and 3, the reading of the wattmeter is the same. Calculate the power factor of the motor.

A. 0.766
B. 0.829
C. 0.866
D. 0.707

Problem 386:

Two wattmeters used to measure the total power drawn by a three-phase circuit and are connected properly. The first wattmeter reads4800 watts while the other reads 400 watts after reversing its current coil. Solve for the load power factor.

A. 0.564

B. 0.532
C. 0.646
D. 0.439

Problem 387: EE Board June 1990

Two-wattmeter method is used to test a 25 HP, 230 volt, 1800 rpm, 60 cycle, 3-phase induction motor. When the line voltages are 230 volts, one wattmeter reads +13,400 watts and the other +7,400 watts. Determine motor power factor.

A. 0.961
B. 0.886
C. 0.894
D. 0.807

Problem 388: EE Board October 1987

A wound rotor motor, 7.5 HP, 230 volts,3-phase takes a line current of 18.4 amperes, when operating at rated output at an efficiency of 88%. Calculate the indication on the wattmeter when this is inserted to measure power by the T-method.

A. 3.179 kW
B. 4.401 kW
C. 3.361 kW
D. 4.042 kW

Problem 389: EE Board October 1994

A wattmeter with its current coil in line 2 and potential coil across lines 2 and 3 is connected to a balanced 3-phase system. The only load supplied is a single phase one connected to lines 1 and 2. This load is known to be inductive. If the wattmeter reads zero watts, determine the power factor of the single-phase load.

A. 0.707
B. 0.866
C. 0.800
D. 0.900

Problem 390:

Three unequal single-phase motor loads are connected across the lines of a balanced 230 V, 60 Hz, three-phase system. Motor #1 is connected across lines 1 & 2 and draws 106 A at 80% pf. Motor #2 is connected across lines 2 & 3 and takes 142 A at 86.6% pf. And motor #3 is connected across lines 3 & 1 takes 164 A at 75% pf. Using two wattmeters properly connected with the common potential point in line 2, what are the readings of these instruments?

A. 62.42 kW, 13.66 kW
B. 50.71 kW, 25.37 kW
C. 48.21 kW, 27.87 kW
D. 46.25 kW, 29.83 kW

Problem 391: EE Board April 1984

A balanced 3-phase load draws 75 amperes line current at 230 volts line to line, and 0.848 lagging power factor. If the two-wattmeter method is used, solve for the readings of the wattmeters.

A. 15.32 kW, 10.02 kW
B. 17.86 kW, 7.48 kW
C. 16.42 kW, 8.92 kW
D. 17.24 kW, 8.10 kW

Problem 392: EE Board October 1980

Three equal impedances, each having a resistance of 8 ohms and an inductive reactance of 7 ohms are connected in delta to lines a, b, and c of a 240 V, 3-phase,3-wire line, phase sequence: a-b-c. What is the reading of a single-phase wattmeter connected with its current coil in line a and potential coil across lines b and c?

A. 6,180 W
B. 6.324 W
C. 6,561 W
D. 6,004 W

Problem 393:

A balanced three-phase load draws 20 kW at 0.447-pf lagging from a 230-V, 60 Hz three-phase transmission line. Find the readings of the two wattmeters properly connected to measure power.

A. 18.45 kW, 1.55 kW
B. 14.25 kW, 5.75 kW
C. 21.55 kW, -1.55 kW
D. 25.75 kW, - 5.75 kW

Problem 394:

Determine the operating power factor of a three phase balanced load if the readings of the two wattmeters properly connected to measure power is 3:1.

A. 0.667
B. 0.655
C. 0.756
D. 0.681

Problem 395: EE Board October 1992

A 3-phase feeder carries two lagging balanced loads. The power observed by each is measured by two wattmeters method, giving the following readings:

First load: $W_1 = 160$ kW $W_2 = 96$ kW

Second load: $W_1 = 90$ kW $W_2 = 48$ kW

What is the combined kVA load on the feeder?

A. 434.68
B. 462.35
C. 504.24
D. 420.12

Problem 396:

A single-phase wattmeter is used to determine the power taken by a three-phase squirrel cage induction motor. The current coil is placed in line 1 while the potential coil is shifted from lines 1 and 2, then from lines 1 and 3 respectively. If the wattmeter registers 5760 and 3380 watts respectively, what is the line current drawn by the motor. Line to line voltage is 230 V.

A. 25.17 A
B. 23.46 A
C. 34.61 A
D. 36.83 A

Problem 397: EE Board October 1992

National Power Corporation used to wattmeters to measure 3-phase power of a balanced Y-connected lagging power factor motor loads. Each wattmeter indicate 15.4 kW. The voltage coils are connected across line 2 and 3, and across lines 1 and 3, respectively. The line to line voltages are 230 volts with V_{12} leading V_{23} and the line currents are each 120 A. Calculate the total power supplied.

A. 37.44 kW
B. 30.72 kW
C. 39.67 kW
D. 34.88 kW

Problem 398:

A 3-phase, 600 V balanced system supplies energy to an unbalanced delta connected load. The three line currents I_a, I_b and I_c are 141.4 A, 100 A and 100 A respectively. A wattmeter with its current coil in line a and potential coil across a and b indicates 84.84 kW. Solve for the total power drawn by the load. Assume a sequence of a-b-c.

A. 102.45 kW
B. 100.37 kW
C. 107.25 kW
D. 110.12 kW

Problem 399:

Three equal non-inductive resistances are connected in star across a balanced 400 V, 60 Hz system. A single-phase wattmeter is connected on the system with its current coil in line 1 and potential coil in lines 1 and 3. This wattmeter registers 900 W. If line 2 is accidentally open circuited, what will be the wattmeter reading under this condition?

A. 900 W

B. 1200 W
C. 600 W
D. 300 W

Problem 400:

A balanced three-phase load having a resistance of 10 ohms and a reactance of 10 ohms are connected in star across the lines of a balanced 230 V three-phase supply. Find the reading on each two wattmeters properly connected to measure the power drawn.

A. 2055 W, 589 W
B. 2085 W, 559 W
C. 2165 W, 479 W
D. 2105 W, 539 W

Problem 401:

An unbalanced delta-connected load is connected across the lines of a 4-wire balanced three-phase supply with neutral connection. Two wattmeters W_1, W_2 are connected in lines 1 and 3 with the common potential connection in the neutral wire. These wattmeters read 1796.5 and 3000 watts, respectively. The current in lines 1 and 3 are 20 and 25 A, respectively. If the common potential connection is transferred to line 2, what will be the reading of the two wattmeters? The system voltage per phase is 208 V. Assume a phase sequence of 1-2-3 with V_{1n} as reference.

A. W_1 = 4013 watts, W_2 = 4560 watts
B. W_1 = 4043 watts, W_2 = 4530 watts
C. W_1 = 4063 watts, W_2 = 4510 watts
D. W_1 = 4073 watts, W_2 = 4500 watts

Problem 402:

Two wattmeters are used to measure the total power drawn by a balanced delta connected load. If the wattmeters read 1200 W and 800 W respectively, and the line to line voltage is 208 V, solve for the impedance per phase of the load.

A. 56.31 + j 23.67 ohms
B. 57.94 + j 20.07 ohms
C. 50.31 + j 31.72 ohms
D. 52.47 + j 23.82 ohms

Problem 403:

Two wattmeters used to measure the total power drawn by a three-phase circuit and are connected properly. One of the wattmeters read 10 kW when the power factor of the load is unity. What does each wattmeter read when the power factor falls to 0.866 lagging, Assume the total three-phase power remains the same.

A. 12.45 kW, 7.55 kW
B. 13.33 kW, 6.67 kW

C. 15.25 kW, 4.75 kW
D. 10.67 kW, 9.33 kW

Problem 404: EE Board April 1980

A factory is supplied by a three-phase, 3-wire system with the following characteristics. Determine the average power factor of the load.

$V_{ab} = 230\angle 0°$ V $\quad I_a = 110\angle -36.87°$ A

$V_{bc} = 230\angle 240°$ V $\quad I_c = 125\angle 53.13°$ A

A. 0.934 lagging
B. 0.908 lagging
C. 0.892 lagging
D. 0.866 lagging

Problem 405: EE Board October 1981

A 3-phase, 3-wire load draws the following line currents:

$I_a = 60\angle 330°$ A $\quad I_b = 78.4\angle 214°$ A $\quad I_c = 75\angle 80°$ A

If the voltages impressed on the load are balanced 3-phase, having a magnitude of 4140 volts line to line, solve for the power factor of the load.

A. 0.976
B. 0.999
C. 0.982
D. 0.906

ANSWER KEY

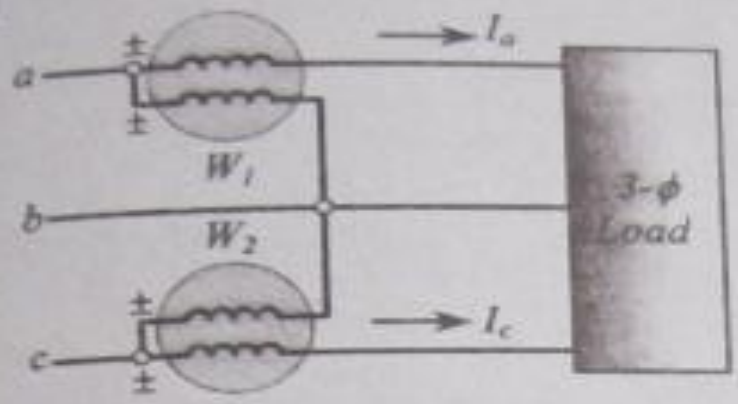
368. B	376. C	384. C	392. A	400. B
369. A	377. C	385. C	393. C	401. D
370. A	378. D	386. D	394. C	402. B
371. B	379. D	387. C	395. A	403. B
372. A	380. A	388. A	396. A	404. A
373. B	381. D	389. B	397. C	405. B
374. B	382. D	390. B	398. B	
375. A	383. C	391. D	399. A	

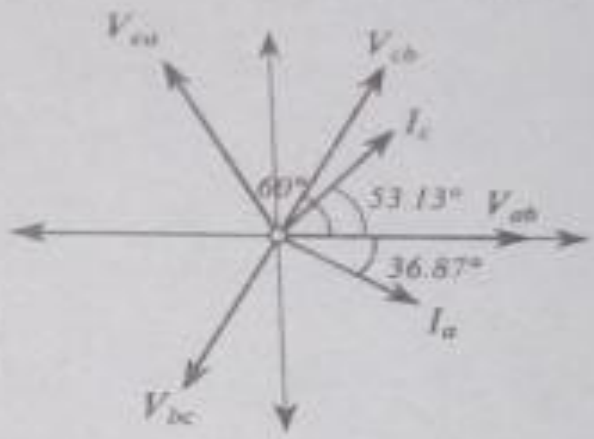
☑ RATING

34-38 - Topnotcher
26-33 - Passer
19-25 - Conditional
0-18 - Failed

Solutions to Test 11

368 Note: Use line b as the common potential terminal for the two wattmeters.





$$W_1 = V_{ab} I_a \cos\theta \Big|_{I_a}^{V_{ab}}$$

$$= (230)(110)\cos 36.87°$$

$$W_1 = 20.24 \text{ kW}$$

$$W_2 = V_{cb} I_c \cos\theta \Big|_{I_c}^{V_{cb}}$$

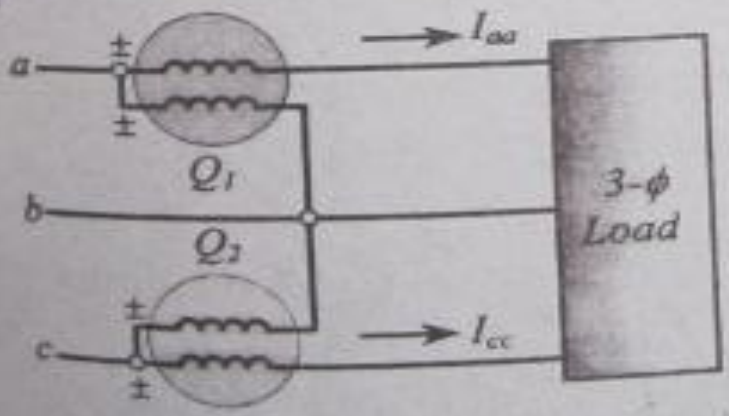
$$= (230)(125)\cos(60° - 53.13°)$$

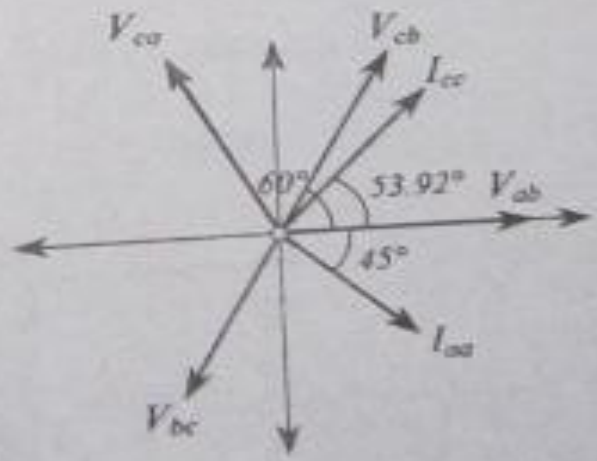
$$W_2 = 28.54 \text{ kW}$$

$$W_t = W_1 + W_2 = 20.24 + 28.54$$

$$W_t = 48.78 \text{ kW}$$

369 Note: Use line b as the common potential terminal for the two varmeters.





$$I_{aa} = 30 - j30 = 42.426\angle -45°$$

$$I_{cc} = 29 + j39.8 = 49.244\angle 53.92°$$

$$Q_1 = V_{ab} I_{aa} \sin\theta \Big|_{I_{aa}}^{V_{ab}}$$

$$= (220)(42.426)\sin 45°$$

$$Q_1 = 6.6 \text{ kVAR}$$

$$Q_2 = V_{cb} I_{cc} \sin\theta \Big|_{I_{cc}}^{V_{cb}}$$
$$= (220)(49.244)\sin(60° - 53.92°)$$
$$Q_2 = 1.15\,\text{kVAR}$$

$$Q_t = Q_1 + Q_2 = 6.6 + 1.15$$
$$Q_t = 7.75\,\text{kVAR}$$

Note: Since the two line currents are lagging with respect to their respective line voltages, the two readings of the varmeters are added arithmetically.

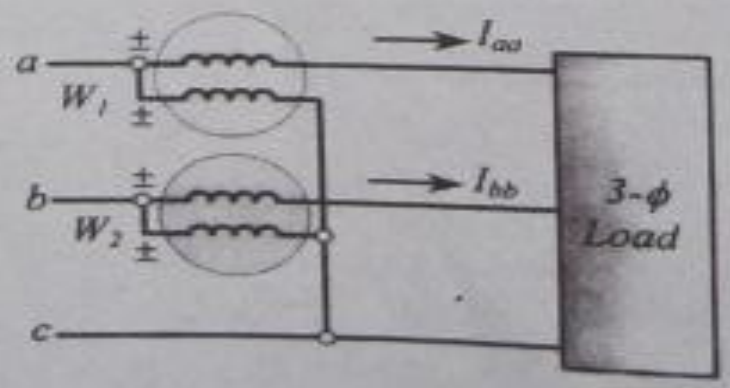
370 $$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{230\angle 0°}{60\angle 0°} = 3.833\angle 0° = 3.833 + j0$$

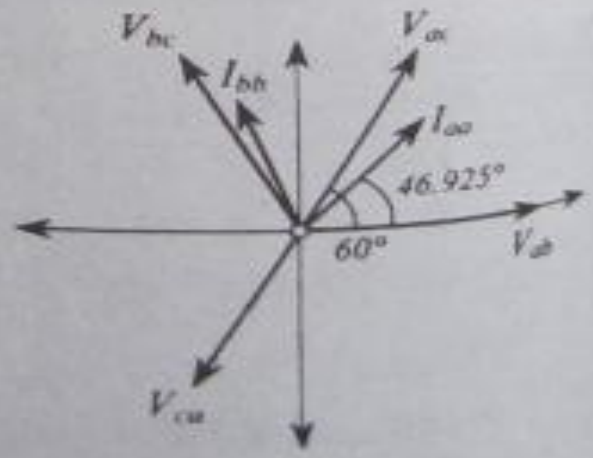
$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{230\angle 120°}{30 + j50} = \frac{230\angle 120°}{58.309\angle 59.036°} = 3.944\angle 60.964°$$
$$= 1.914 + j3.448$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{230\angle 240°}{80\angle -90} = 2.875\angle 330° = 2.489 - j1.4375$$

$$I_{aa} = I_{ab} - I_{ca}$$
$$= 3.833 - (2.489 - j1.4375)$$
$$= 1.344 + j1.4375$$
$$I_{aa} = 1.968\angle 46.925°$$

$$I_{bb} = I_{bc} - I_{ab}$$
$$= 1.914 + j3.448 - (3.833)$$
$$= -1.919 + j3.448$$
$$I_{bb} = 3.946\angle 119.098°$$





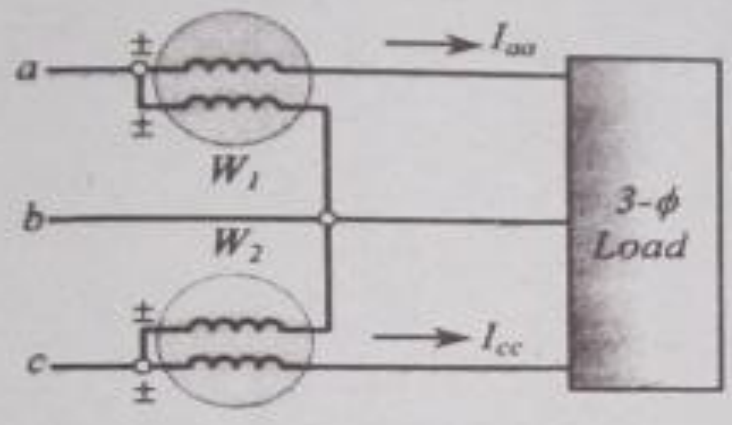
$$W_1 = V_{ac} I_{aa} \cos\theta \Big|_{I_{aa}}^{V_{ac}} = (230)(1.968)\cos(60° - 46.925°)$$

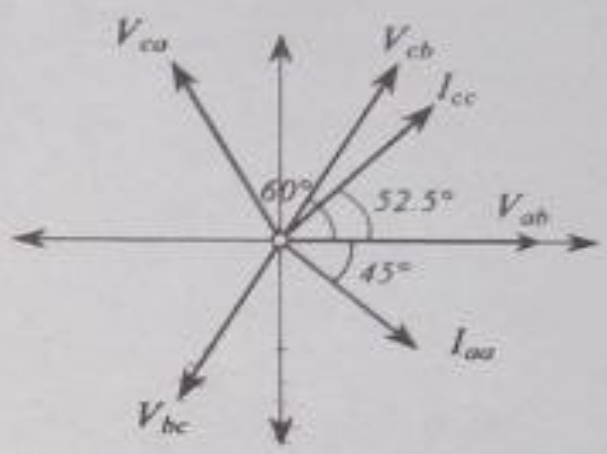
$$W_1 = 440.905 \text{ W}$$

$$W_2 = V_{bc} I_{bb} \cos\theta \Big|_{I_{bb}}^{V_{bc}} = (230)(3.946)\cos(120° - 119.098°)$$

$$W_2 = 907.467 \text{ W}$$

371 Note: Use line b as the common potential terminal for the two wattmeters





$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{220\angle 0°}{11\angle 0°} = 20\angle 0° = 20 \text{ A}$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{220\angle 240°}{11\angle 45°} = 20\angle 195°$$

$$= -19.318 - j5.176 \text{ A}$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{220\angle 120°}{11\angle 30°} = 20\angle 90° = j20 \text{ A}$$

$$I_{aa} = I_{ab} - I_{ca} = 20 - j20 = 28.284\angle -45°$$

$$I_{cc} = I_{ca} - I_{bc} = j20 - (-19.318 - j5.176)$$

$$= 19.318 + j25.176 = 31.733\angle 52.5°$$

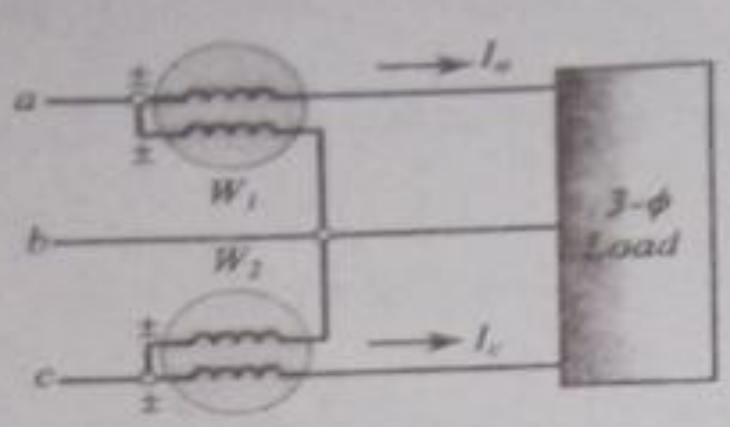
$$W_1 = V_{ab} I_{aa} \cos\theta \Big|_{I_{aa}}^{V_{ab}} = (220)(28.284)\cos 45°$$

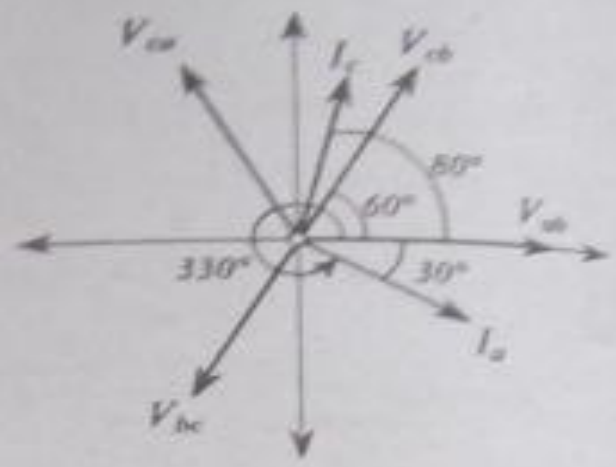
$$W_1 = 4{,}400 \text{ W}$$

$$W_2 = V_{cb} I_{cc} \cos\theta \Big|_{I_{cc}}^{V_{cb}} = (220)(31.733)\cos(60° - 52.5°)$$

$$W_2 = 6{,}921.5 \text{ W}$$

372 Note: Use line b as the common potential terminal for the two wattmeters.



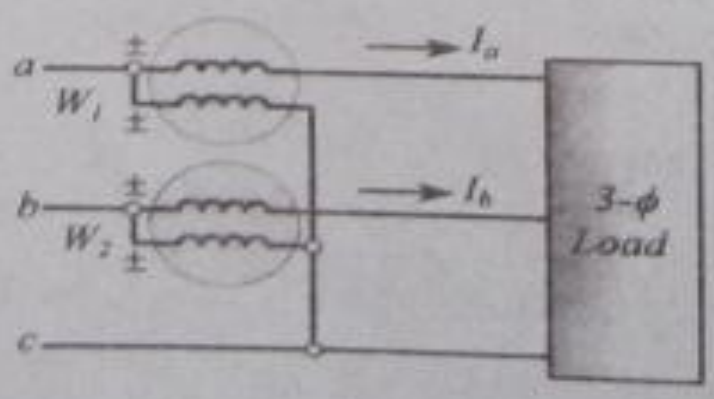


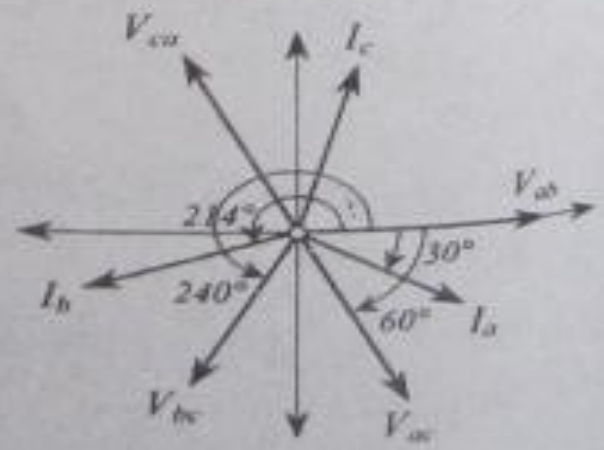
$$W_1 = V_{ab} I_a \cos\theta \Big|_{I_a}^{V_{ab}}$$
$$= (2400)(85)\cos 30°$$
$$W_1 = 176.67\ \text{kW}$$

$$W_2 = V_{cb} I_c \cos\theta \Big|_{I_c}^{V_{cb}}$$
$$= (2400)(100)\cos(80° - 60°)$$
$$W_2 = 225.526\ \text{kW}$$

$$W_t = W_1 + W_2$$
$$= 176.67 + 225.526$$
$$W_t = 402.196\ \text{kW}$$

373 Note: Use line c as the common potential terminal for the two wattmeters. Since not given, assume a phase sequence of a–b–c.





$$W_1 = V_{ac} I_a \cos\theta \Big|_{I_a}^{V_{ac}}$$
$$= (4140)(60)\cos(60° - 30°)$$
$$W_1 = 215.12\ \text{kW}$$

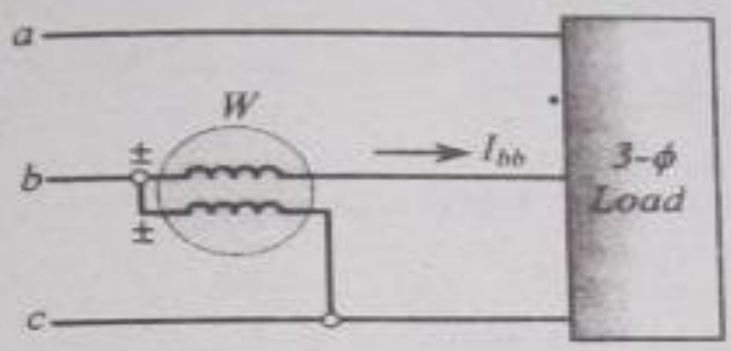
$$W_2 = V_{bc} I_b \cos\theta\Big|_{I_b}^{V_{bc}} = (4140)(78.4)\cos(240° - 214°)$$

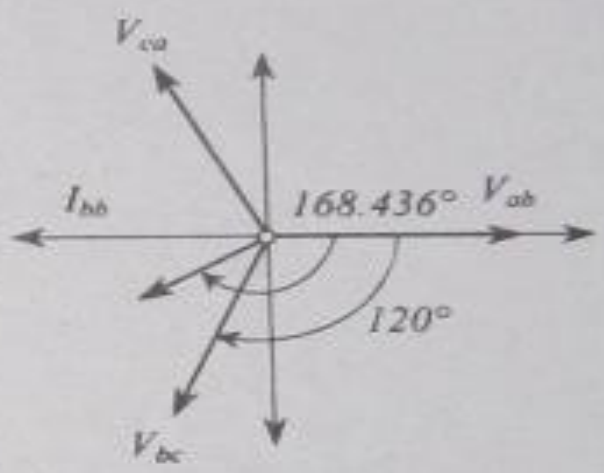
$$W_2 = 291.727\ \text{kW}$$

$$W_1 = W_1 + W_2 = 215.12 + 291.727$$

$$W_1 = 506.85\ \text{kW}$$

374





$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{220\angle 0°}{10\angle 36.87°}$$

$$= 22\angle -36.87° = 17.6 - j13.2$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{220\angle 240°}{10\angle 0°}$$

$$= 22\angle 240° = -11 - j19.052$$

$$I_{bb} = I_{bc} - I_{ab}$$

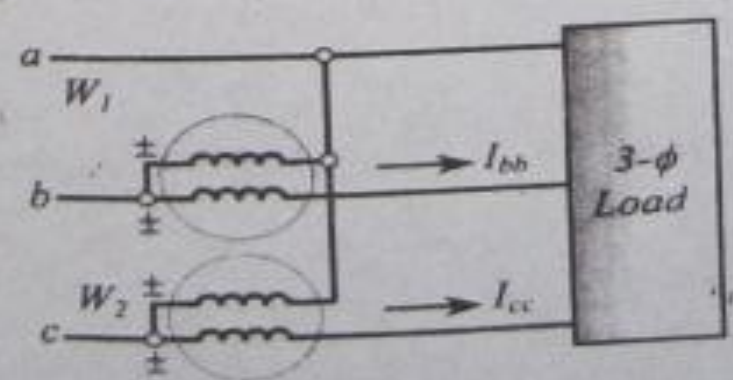
$$= 17.6 - j13.2 - (-11 - j19.052) = -28.6 - j5.852$$

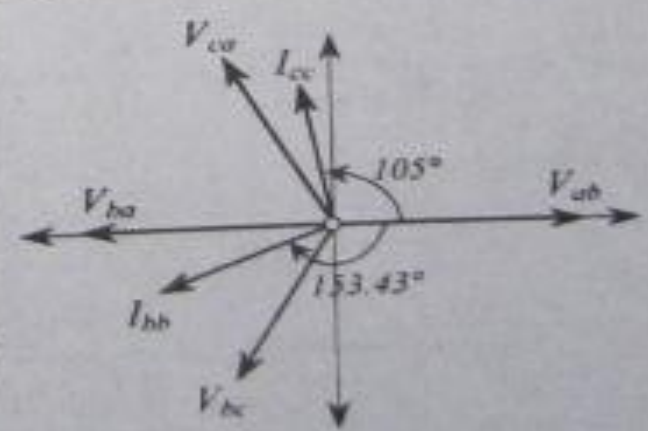
$$I_{bb} = 29.192\angle -168.436°$$

$$W = V_{bc} I_{bb} \cos\theta\Big|_{I_{bb}}^{V_{bc}} = 220(29.192)\cos(168.436° - 120°)$$

$$W = 4.26\ \text{kW}$$

375 Note: Use line a as the common potential terminal for the two wattmeters.





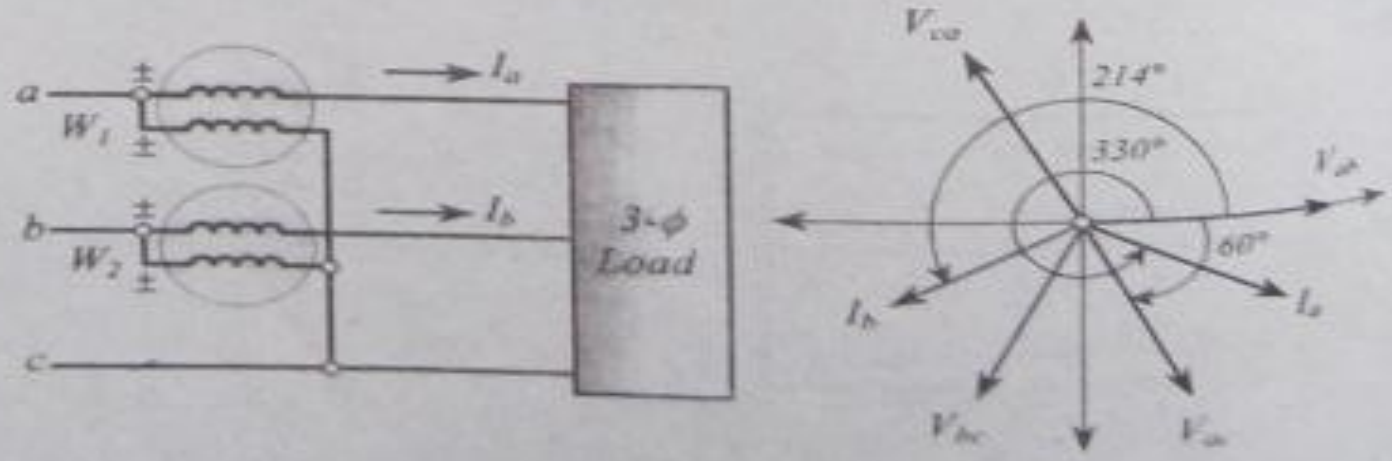
$Z_{ab} = 8 + j6 = 10\angle 36.869°$
$Z_{bc} = 8.66 - j5 = 10\angle -30°$
$Z_{ca} = 10 + j0 = 10\angle 0°$

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{220\angle 0°}{10\angle 36.869°} = 22\angle -36.869°$$
$$= 17.6 - j13.2$$
$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{220\angle 240°}{10\angle -30°} = 22\angle 270°$$
$$= -j22$$
$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{220\angle 120°}{10\angle 0°} = 22\angle 120°$$
$$= -11 + j19.05$$

$$I_{bb} = I_{bc} - I_{ab} = -j22 - (17.6 - j13.2)$$
$$= -17.6 - j8.8 = 19.667\angle -153.43°$$
$$I_{cc} = I_{ca} - I_{bc} = -11 + j19.05 - (-j22)$$
$$= -11 + j41.052 = 42.5\angle 105°$$

$$W_1 = V_{ba} I_{bb} \cos\theta\Big|_{I_{bb}}^{V_{ba}} = (220)(19.667)\cos(180° - 153.43°)$$
$$W_1 = 3.869\,\text{kW}$$
$$W_2 = V_{ca} I_{cc} \cos\theta\Big|_{I_{cc}}^{V_{ca}} = (220)(42.5)\cos(120 - 105°)$$
$$W_2 = 9.031\,\text{kW}$$

376 Note: Use line c as the common potential terminal for the two wattmeters. Refer to the vector diagram for the angles.



$$W_1 = V_{ac} I_a \cos\theta\Big|_{I_a}^{V_{ac}} = (240)(120)\cos(30°)$$

$$W_1 = 24.942\text{ kW}$$

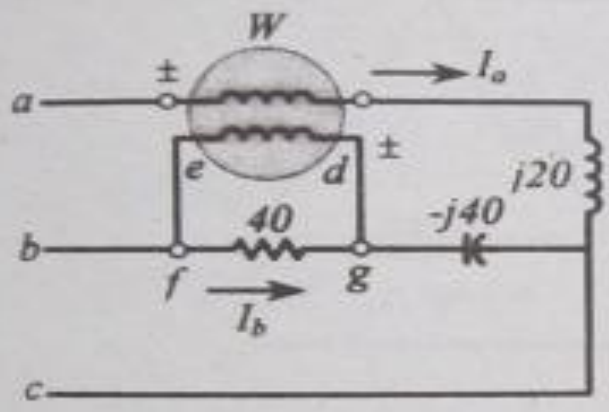
$$W_2 = V_{bc} I_b \cos\theta\Big|_{I_b}^{V_{bc}} = (240)(157)\cos(240° - 214°)$$

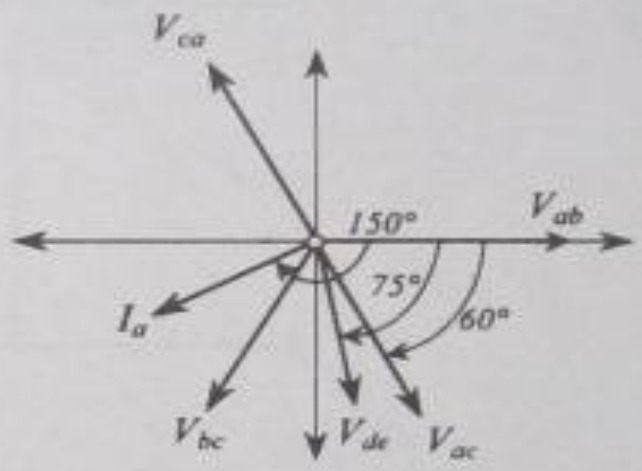
$$W_2 = 33.866\text{ kW}$$

$$W_t = W_1 + W_2 = 24.942 + 33.866$$

$$W_t = 58.808\text{ kW}$$

377 Note: Refer to given circuit connection diagram, the potential coil should measure the vector voltage measured from points "d to e".





$$Z_{ac} = j20 = 20\angle 90°$$

$$Z_{bc} = 40 - j40 = 56.568\angle -45°$$

$$I_a = \frac{V_{ac}}{Z_{ac}} = \frac{220\angle -60°}{20\angle 90°} = 11\angle -150°$$

$$I_b = \frac{V_{bc}}{Z_{bc}} = \frac{220\angle -120°}{56.568\angle -45°} = 3.889\angle -75°$$

By kVL on loop "defgd":

$$V_{de} - V_{fg} = 0$$

$$V_{de} = V_{fg} = I_b Z_{fg}$$

$$= (3.889\angle -75°)(40)$$

$$V_{de} = 155.56\angle -75°$$

$$W = V_{de} I_a \cos\theta\Big|_{I_a}^{V_{de}} = (155.56)(11)\cos(150° - 75°)$$

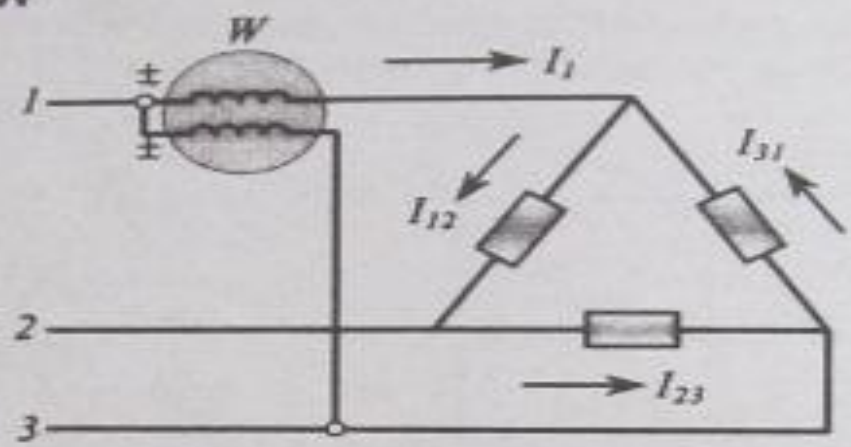
$$W = 442.88\text{ watts}$$

378 $\theta = \cos^{-1} 0.848 = 32°$

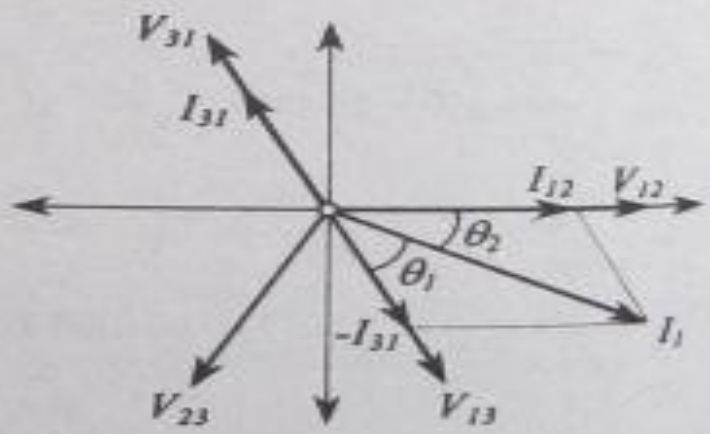
Since load is balanced, the reading of the wattmeters shall be :

$$W_1 = VI\cos(30° - \theta)$$
$$= (230)(120)\cos(30° - 32°)$$
$$W_1 = 27.583\text{ kW}$$
$$W_2 = VI\cos(30° + \theta)$$
$$= (230)(120)\cos(30° + 32°)$$
$$W_2 = 12.957\text{ kW}$$

379



$$W = V_{13}I_1\cos\theta_1$$
$$I_1 = \frac{W}{V_{13}\cos\theta_1} = \frac{8400}{(230)\cos\theta_1}$$
$$I_1 = \frac{36.522}{\cos\theta_1} \Rightarrow ❶$$



$$W' = V_{12}I_1\cos\theta_2$$
$$I_1 = \frac{W'}{V_{12}\cos\theta_2} = \frac{7500}{(230)\cos\theta_2}$$
$$I_1 = \frac{32.609}{\cos\theta_2} \Rightarrow ❷$$

Equate Eq.1 to Eq.2 :

$$\frac{36.522}{\cos\theta_1} = \frac{32.609}{\cos\theta_2}$$
$$\cos\theta_1 = 1.12\cos\theta_2 \Rightarrow ❸$$

Refer to the vector diagram :

$$\theta_1 + \theta_2 = 60°$$

Substitute in Eq.3 :

$\cos(60-\theta_2) = 1.12\cos\theta_2$

$\cos 60\cos\theta_2 + \sin 60\sin\theta_2 = 1.12\cos\theta_2$

$0.62\cos\theta_2 = 0.866\sin\theta_2$

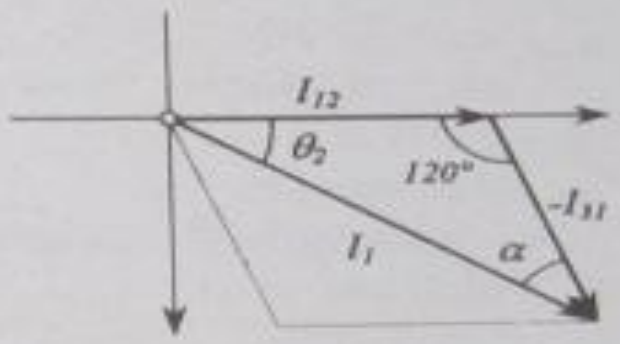
$\dfrac{\sin\theta_2}{\cos\theta_2} = 0.716$

$\theta_2 = \tan^{-1} 0.716$

$\theta_2 = 35.6°$

$I_1 = \dfrac{32.609}{\cos 35.6°} = 40.104$ A

Refer to the vector diagram :



$\alpha = 180 - (35.6 + 120) = 24.4°$

By sine law :

$\dfrac{I_{12}}{\sin\alpha} = \dfrac{I_1}{\sin 120°}$

$\dfrac{I_{12}}{\sin 24.4°} = \dfrac{40.104}{\sin 120°}$

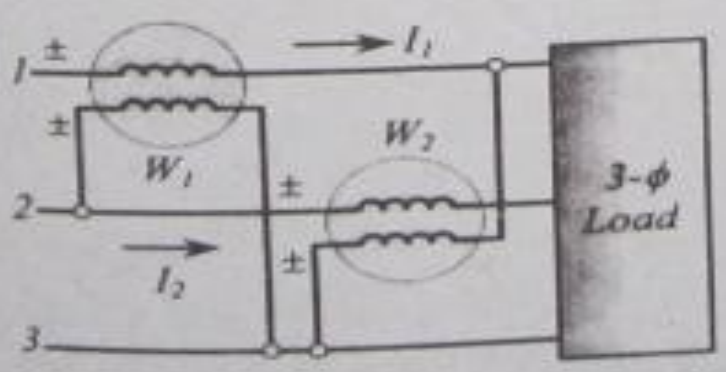
$I_{12} = 19.13$ A

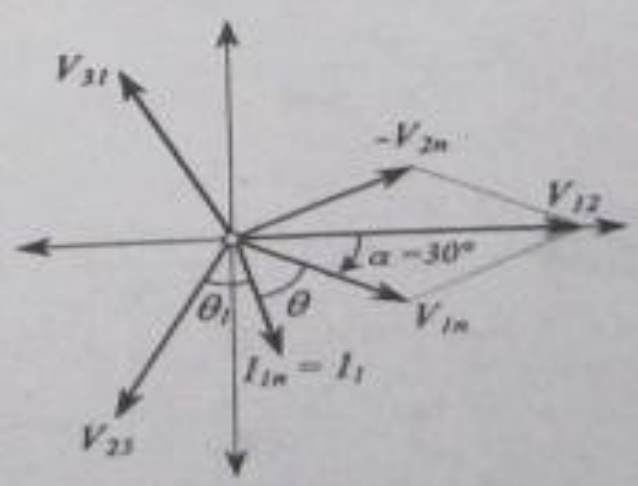
$P_{12} = V_{12}I_{12}pf = (230)(19.13)(1) = 4400$ W

$N = \dfrac{P_{12}}{100} = \dfrac{4400}{100}$

N = 44 lamps

380





$W_1 = V_{23}I_1\cos\theta_1$

$\theta_1 = \cos^{-1}\left(\dfrac{W}{V_{23}I_1}\right) = \cos^{-1}\left(\dfrac{19600}{(230)(150)}\right)$

$\theta_1 = 55.38°$

Refer to vector diagram :

$\theta_1 + \theta + 30° = 120°$

$\theta = 34.62°$

$P = \sqrt{3} V_L I_L \cos\theta$

$P = \sqrt{3}(230)(150)\cos 34.62°$

$P = 49.175\ kW$

381 $\theta = \cos^{-1} 0.91 = 24.49°$

Since load is balanced, the reading of the wattmeters shall be :

$W_1 = VI\cos(30° - \theta)$

$= (460)(208)\cos(30° - 24.49°)$

$W_1 = 95.24\ kW$

$W_2 = VI\cos(30° + \theta)$

$= (460)(208)\cos(30° + 24.49°)$

$W_2 = 55.57\ kW$

382 Note: Since the reading on one of the wattmeter is positive after reversing its current coil, then its reading during the proper two-wattmeter method must be negative.

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{1000 - (-500)}{1000 + (-500)}\right) = 79.106°$$

$P = W_1 + W_2 = 1000 + (-500) = 500$ watts

$$I = \frac{P}{\sqrt{3}E\cos\theta} = \frac{500}{\sqrt{3}(208)(\cos 79.106°)}$$

$I = 7.34\ A$

383 $\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

Assume $W_2 = 0$.

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - 0}{W_1 + 0}\right) = \tan^{-1}\sqrt{3} = 60°$$

$pf = \cos\theta = \cos 60°$

$pf = 0.5$

384 Note: Since the power factor of the load is below 0.5, one of the wattmeters will read negative.

$W_1 + W_2 = 12 \Rightarrow ❶$

$\theta = \cos^{-1} 0.4 = 66.42°$

$$\theta = \tan^{-1} \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\frac{\tan\theta}{\sqrt{3}} = \frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 - W_2 = \frac{(W_1 + W_2)\tan\theta}{\sqrt{3}} = \frac{12\tan 66.42°}{\sqrt{3}}$$

$W_1 - W_2 = 15.873 \Rightarrow ❷$

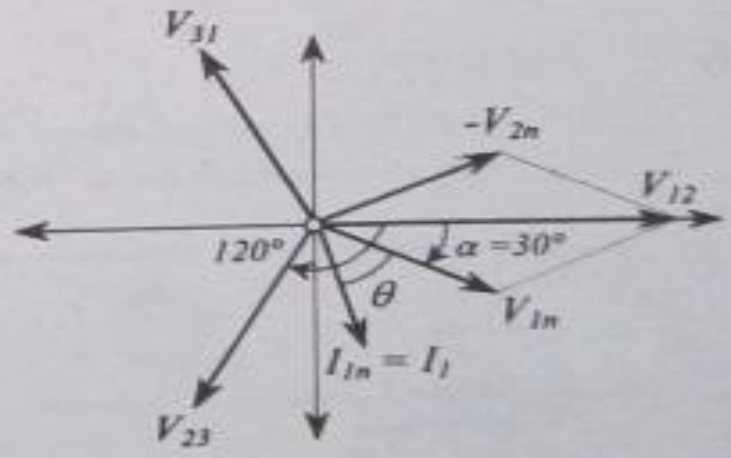
Substitute Eq.1 in Eq.2:

$(12 - W_2) - W_2 = 15.873$

$W_2 = -1.94$ watts

$W_1 = 12 - (-1.94) = 13.94$ watts

385 Note: Assume the load is wye-connected.



$$W = V_{12} I_1 \cos\theta\Big|_{I_1}^{V_{12}}$$

$$600 = (220) I_1 \cos\theta\Big|_{I_1}^{V_{12}} \Rightarrow ❶$$

$$W' = V_{23} I_1 \cos\theta\Big|_{I_1}^{V_{23}}$$

$$600 = (220) I_1 \cos\theta\Big|_{I_1}^{V_{23}} \Rightarrow ❷$$

Equate Eq.1 to Eq.2:

$$\cos\theta\Big|_{I_1}^{V_{12}} = \cos\theta\Big|_{I_1}^{V_{23}}$$

Refer to the vector diagram:

$\cos(30 + \theta) = \cos(90 - \theta)$

$30 + \theta = 90 - \theta$

$\theta = 30°$

$pf = \cos 30° = 0.866$

386 Note: Since the reading on one of the wattmeter is positive after reversing its current coil, then its reading during the proper two-wattmeter method must be negative.

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{4800-(-400)}{4800+(-400)}\right) = 63.963°$$

$$pf = \cos\theta = \cos 63.963°$$

$$pf = 0.439$$

387 $$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{13400-7400}{13400+7400}\right) = 26.548°$$

$$pf = \cos\theta = \cos 26.548°$$

$$pf = 0.894$$

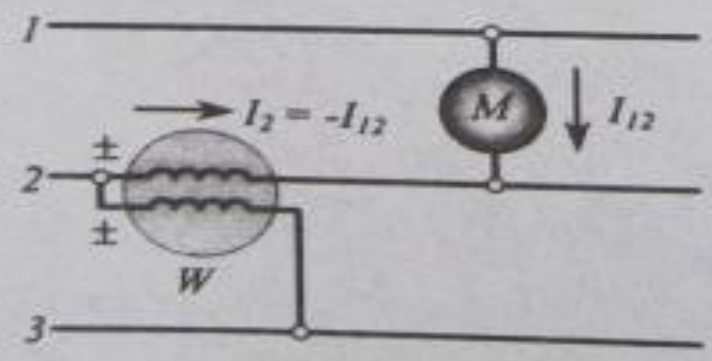
388 Note: In the T-method of measuring three-phase power, the wattmeter will register one-half of the total power drawn. Refer to the theory section.

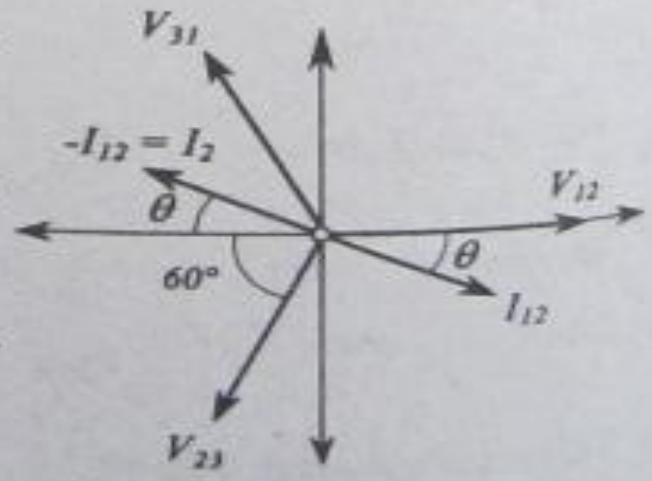
$$P_{in} = \frac{P_{out}}{\eta} = \frac{7.5(0.746)}{0.88} = 6.358\text{ kW}$$

$$W_{reading} = \frac{P_{in}}{2} = \frac{6.358}{2} = 3.179\text{ kW}$$

389 $$W = V_{23}I_2\cos\alpha\Big|_{I_2}^{V_{23}}$$

Note: Since V_{23} and I_2 are not zero, the only way the reading of the wattmeter will be zero is that when $\cos\alpha$ is zero. Thus $\alpha = 90°$ (angle in between V_{23} and I_2).





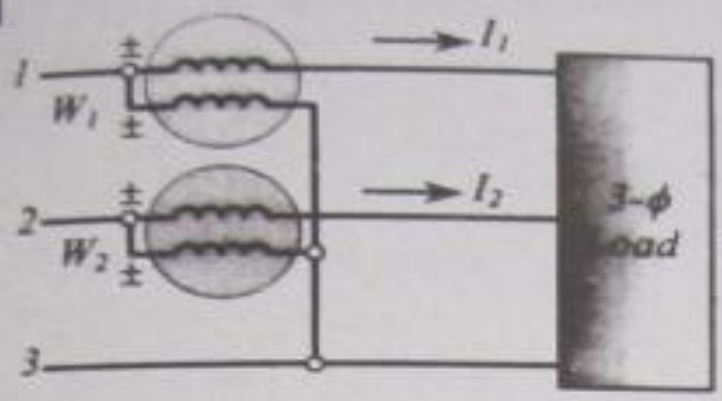
Refer to the vector diagram:

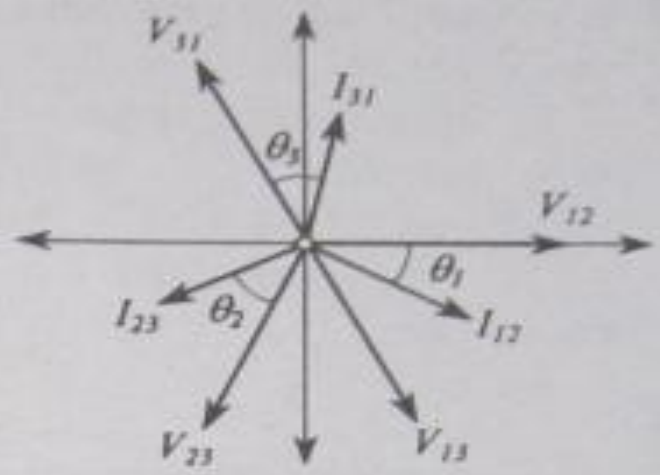
$$\alpha = \theta + 60°$$

$$90° = \theta + 60°$$

$$\theta = 30°$$

390 Note: Assume a phase sequence of 1-2-3.

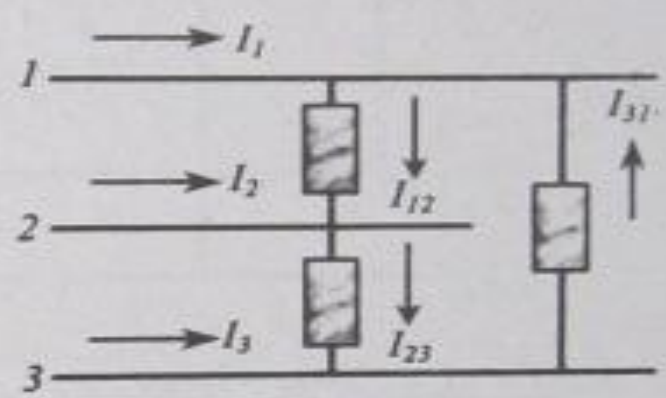




$\theta_1 = \cos^{-1} 0.8 = 36.87°$

$\theta_2 = \cos^{-1} 0.866 = 30°$

$\theta_3 = \cos^{-1} 0.75 = 41.41°$



Refer to the vector diagram :

$I_{12} = 106\angle -36.87°$

$= 84.8 - j63.6$

$I_{23} = 142\angle -(120° + 30°)$

$= -122.975 - j71$

$I_{31} = 164\angle(120° - 41.41°)$

$= 32.44 + j160.759$

$I_1 = I_{12} - I_{31}$

$= 84.8 - j63.6 - (32.44 + j160.759)$

$= 52.36 - j224.359$

$= 230.387\angle -76.863°$ A

$I_2 = I_{23} - I_{12}$

$= -122.975 - j71 - (84.8 - j63.6)$

$= -207.775 - j7.4$

$= 207.906\angle -177.96°$ A

$W_1 = V_{13} I_1 \cos\theta\Big|_{I_1}^{V_{13}} = (230)(230.387)\cos(76.863 - 60°)$

$W_1 = 50.71$ kW

$W_2 = V_{23} I_2 \cos\theta\Big|_{I_2}^{V_{23}} = (230)(207.906)\cos(177.96 - 120°)$

$W_2 = 25.368$ kW

391 $\theta = \cos^{-1} 0.848 = 32°$

Since load is balanced, the reading of the wattmeters shall be :

$W_1 = EI\cos(30° - \theta)$

$= 230(75)\cos(30° - 32°)$

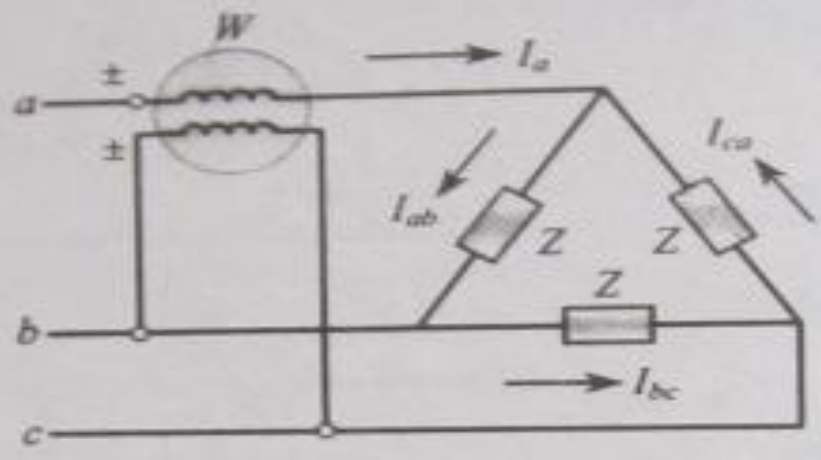
$W_1 = 17.24\ kW$

$W_2 = EI\cos(30° + \theta)$

$= 230(75)\cos(30° + 32°)$

$W_2 = 8.1 kW$

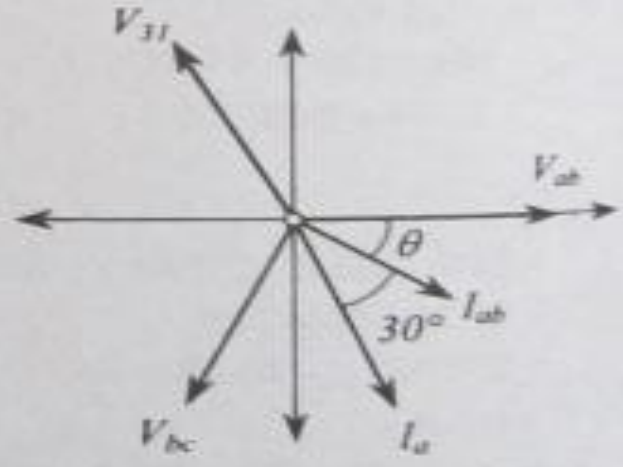
392



$Z = 8 + j7 = 10.63\angle 41.186°$

$\theta = 41.186°$

$I_{ab} = \dfrac{V_{ab}}{Z} = \dfrac{240}{10.63} = 22.577\ A$



Since balanced, $I_a = \sqrt{3} I_{ab}$:

$I_a = \sqrt{3}(22.577) = 39.104\ A$

Refer to the vector diagram :

$W = V_{bc} I_a \cos\theta \Big|_{I_a}^{V_{bc}}$

$= 240(39.104)\cos[120° - (41.186° + 30°)]$

$W = 6{,}180.048$ watts

393 Note: Since the power factor of the load is below 0.5, one of the wattmeters will read negative.

$W_1 + W_2 = 20 \Rightarrow$ ❶

$\theta = \cos^{-1} 0.447 = 63.448°$

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\frac{\tan\theta}{\sqrt{3}} = \frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 - W_2 = \frac{(W_1 + W_2)\tan\theta}{\sqrt{3}} = \frac{20\tan 63.448°}{\sqrt{3}}$$

$$W_1 - W_2 = 23.107 \Rightarrow ❷$$

Substitute Eq.1 in Eq.2:

$(20 - W_2) - W_2 = 23.107$

$W_2 = -1.55$ watts

$W_1 = 20 - (-1.55)$

$W_1 = 21.55$ watts

394 $\frac{W_1}{W_2} = \frac{3}{1}$

$W_1 = 3W_2$

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{3W_2 - W_2}{3W_2 + W_2}\right)$$

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{2}{4}\right) = 40.893°$$

$pf = \cos\theta = \cos 40.893°$

$pf = 0.756$

395 $P_1 = 160 + 96 = 256$ kW

$$\theta_1 = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{160 - 96}{160 + 96}\right)$$

$\theta_1 = 23.413°$

$P_2 = 90 + 48 = 138$ kW

$$\theta_1 = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{90 - 48}{90 + 48}\right)$$

$\theta_2 = 27.795°$

$$\begin{aligned} S_1 = S_1 + S_2 &= P_1 - jQ_1 + P_2 - jQ_2 \\ &= 256 - j256\tan 23.413° + 138 - j138\tan 27.795° \\ &= 394 - j183.594 \end{aligned}$$

$S_1 = 434.68\angle 24.984°$ kVA

396 Note: This method of wattmeter measurement is known as the "potential lead shift method"

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$= \tan^{-1}\sqrt{3}\left(\frac{5760 - 3380}{5760 + 3380}\right)$$

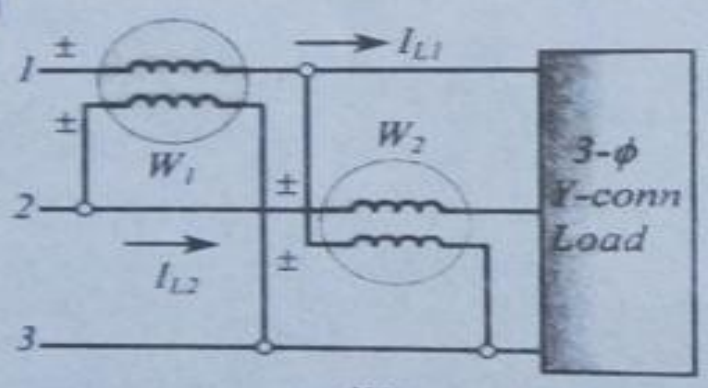
$$\theta = 24.276°$$

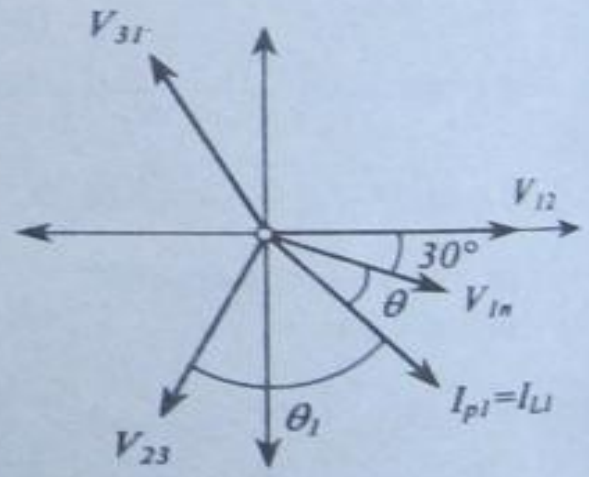
$$I_L = \frac{P_t}{\sqrt{3}V_L\cos\theta}$$

$$= \frac{5760 + 3380}{\sqrt{3}(230)(\cos 24.276°)}$$

$$I_L = 25.168\ \text{A}$$

397 Note: Since load is Y-connected, phase current is equivalent to line current.





$$W_1 = V_{23}I_{L1}\cos\theta_1\Big|_{I_{L1}}^{V_{23}}$$

$$15{,}400 = 230(120)\cos\theta_1$$

$$\theta_1 = 56.084°$$

Refer to the vector diagram :

$$\theta_1 + \theta + 30° = 120°$$

$$56.084° + \theta + 30° = 120°$$

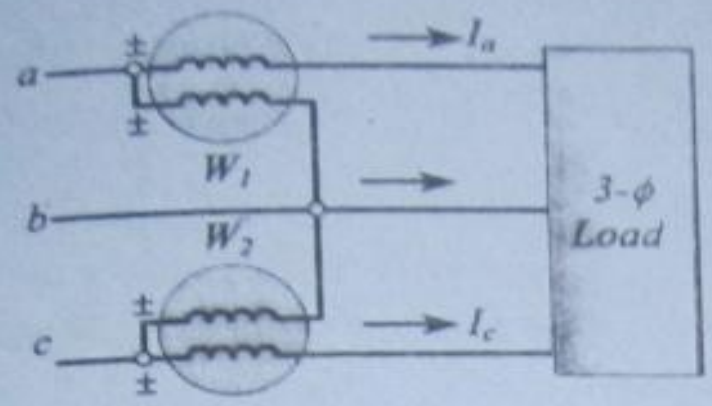
$$\theta = 33.9168°$$

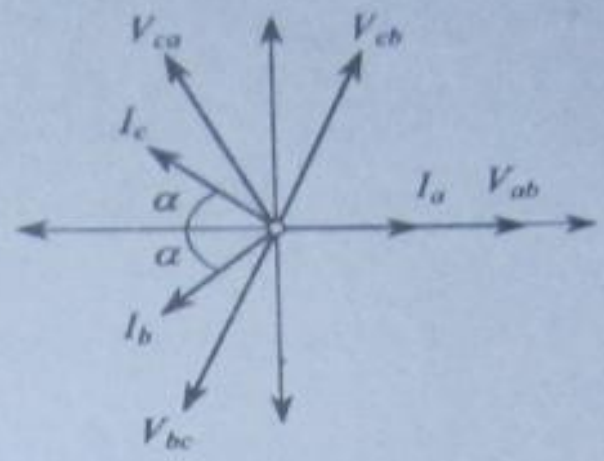
$$P_t = \sqrt{3}V_L I_L\cos\theta$$

$$= \sqrt{3}(230)(120)\cos 33.918°$$

$$P_t = 39.67\ \text{kW}$$

398

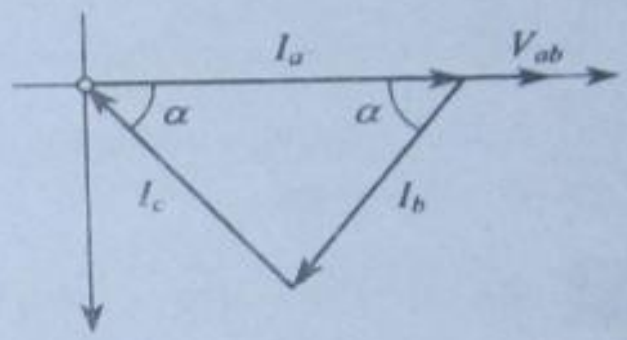




$$W_1 = V_{ab} I_a \cos\theta \Big|_{I_a}^{V_{ab}}$$

$$\cos\theta = \frac{84840}{600(141.4)} = 1.0$$

$$\theta = \cos^{-1} 1.0 = 0°$$



Note : $I_a + I_b + I_c = 0$

By cosine law :

$$_c^2 = I_a^2 + I_b^2 - 2I_aI_b \cos\alpha$$

$$100^2 = 141.4^2 + 100^2 - 2(141.4)(100)\cos\alpha$$

$$\alpha = 45°$$

Using another wattmeter with its current coil in line c and potential coil across lines c and b.

$$W_2 = V_{cb} I_c \cos\theta \Big|_{I_c}^{V_{cb}}$$

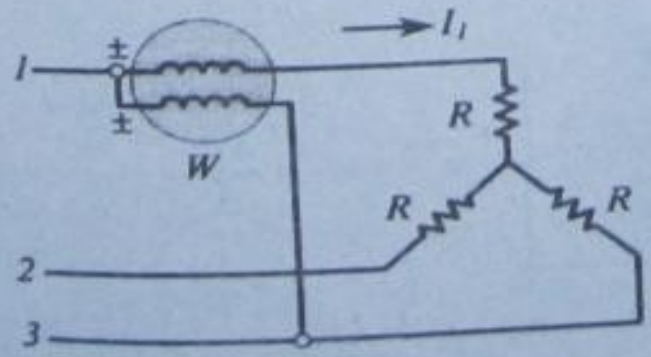
$$W_2 = 600(100)\cos(45 + 30)°$$

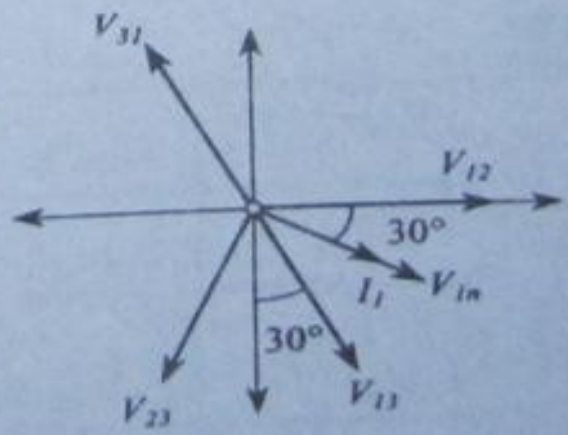
$$= 15.53 \text{ kW}$$

$$P_t = W_1 + W_2 = 84.84 + 15.53$$

$$P_t = 100.37 \text{ kW}$$

399





$$W = V_{13} I_1 \cos\theta_1$$

$$I_1 = \frac{W}{V_{13}\cos\theta_1} = \frac{900}{400\cos 30^\circ} = 2.6\text{ A}$$

$$R = \frac{V_p}{I_1} = \frac{400}{\sqrt{3}(2.6)} = 88.8\,\Omega$$

Note: If line 2 is open, the system becomes a single-phase line at the load side.

$$I_{13} = \frac{V_{13}}{Z_{13}} = \frac{V_{13}}{2R} = \frac{400}{2(88.8)} = 2.25\text{ A}$$

Note: The phase angle between V_{13} and I_{13} is zero (resistive load).

$$W = V_{13} I_1 \cos\theta_2 = 400(2.25)(1)$$
$$W = 900\text{ watts}$$

400

$$Z = \sqrt{R^2 + X^2} = \sqrt{10^2 + 10^2} = 14.142\,\Omega$$

$$\theta = \cos^{-1}\left(\frac{R}{Z}\right) = \cos^{-1}\left(\frac{10}{14.142}\right) = 45^\circ$$

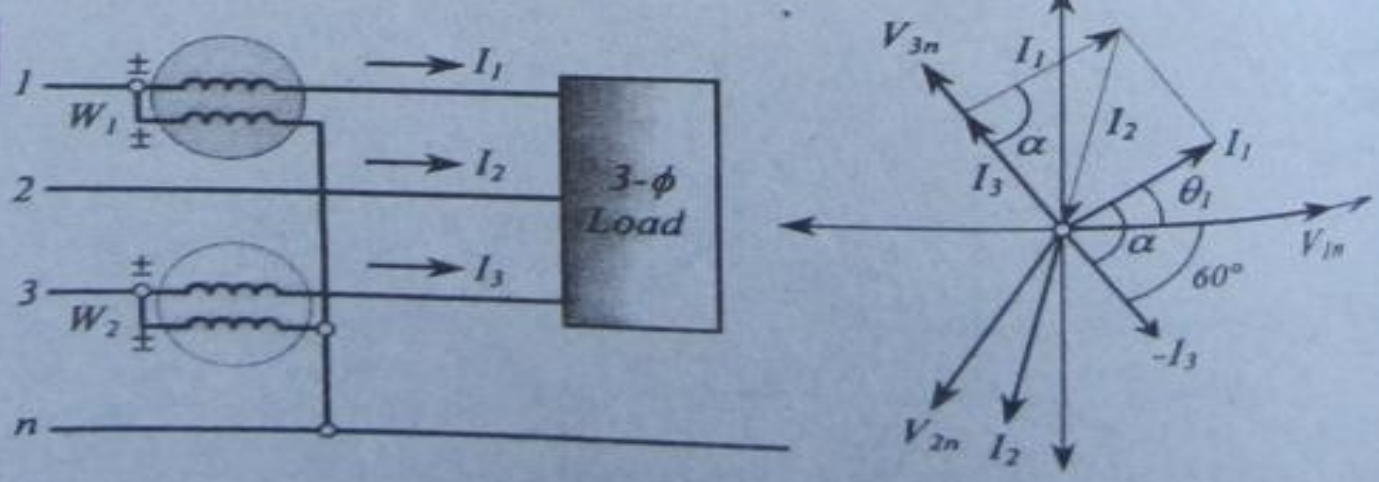
$$I_L = \frac{V_p}{Z} = \frac{230}{\sqrt{3}(14.142)} = 9.389\text{ A}$$

Since the load is balanced, the wattmeters will read:

$$W_1 = V_L I_L \cos(\theta - 30^\circ) = (230)(9.389)\cos(45^\circ - 30^\circ)$$
$$W_1 = 2085.88 \cong 2085\text{ W}$$

$$W_2 = V_L I_L \cos(\theta + 30^\circ) = (230)(9.389)\cos(45^\circ + 30^\circ)$$
$$W_2 = 558.91 \cong 559\text{ W}$$

401



$W_1 = V_{1n} I_1 \cos\theta_1$

$1796.5 = 120(20)\cos\theta_1$

$\theta_1 = 41.536°$

$W_2 = V_{3n} I_3 \cos\theta_2$

$3000 = (120)(25)\cos\theta_2$

$\theta_2 = 0°$

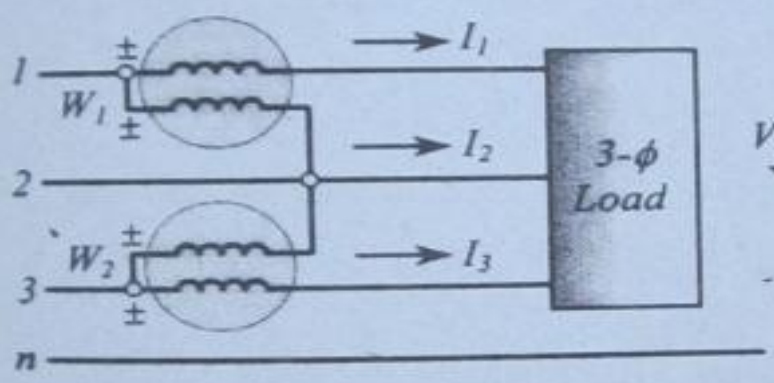
By cos[illegible]w:

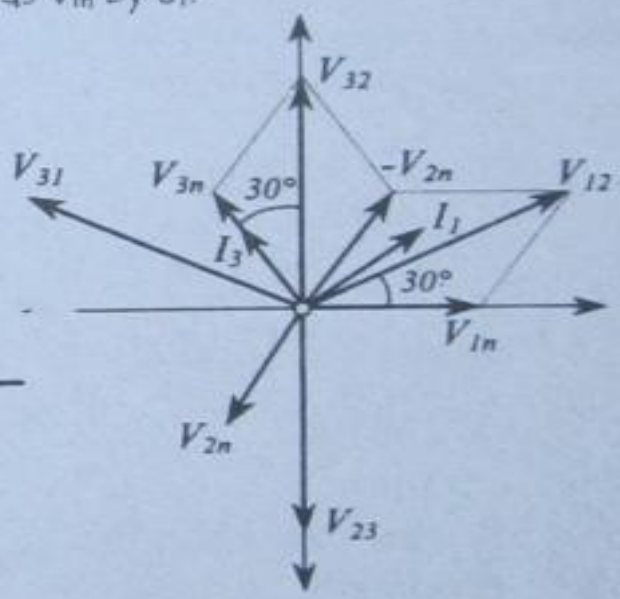
$I_2^2 = I_1^2 + I_3^2 - 2I_1 I_3 \cos\alpha$

$35^2 = 20^2 + 25^2 - 2(20)(25)\cos\alpha$

$\alpha = 101.536°$

Note: Since α is greater than 60°, then I_1 leads V_{1n} by θ_1.





$W_1 = V_{12} I_1 \cos\theta\Big|_{I_1}^{V_{12}}$

$= \sqrt{3}(120)(20)\cos(41.536° - 30°)$

$W_1 = 4073$ watts

$W_2 = V_{32} I_3 \cos\theta\Big|_{I_3}^{V_{32}} = \sqrt{3}(120)(25)\cos(30°)$

$W_2 = 4500$ watts

402 Note: Since load is balanced, power per phase is equivalent to one-third of the total power drawn.

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{1200 - 800}{1200 + 800}\right)$$

$\theta = 19.106°$

$P_t = W_1 + W_2 = 1200 + 800 = 2000$ watts

$P_t = W_1 + W_2 = 1200 + 800$

$P_t = 2000$ watts

$$P_{phase} = \frac{P_t}{3} = \frac{2000}{3} = 666.667 \text{ watts}$$

$$P = \frac{E^2}{Z}\cos\theta$$

$$Z_p = \frac{E_p{}^2 \cos\theta}{P_{phase}} = \frac{(208)^2 \cos 19.106°}{666.667} = 61.32\,\Omega$$

$$= 61.32\angle 19.106°$$

$$Z_p = 57.94 + j20.07\,\Omega$$

403 Note: Since load is balanced, power per phase is equivalent to one-third of the total power drawn. When pf of the load is unity, both wattmeters will register the same magnitude of power.

When $pf = 1.0$, $\theta = 0°$.

$$\tan\theta = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\tan 0° = \sqrt{3}\left(\frac{10 - W_2}{10 + W_2}\right)$$

$W_2 = 10$ kW

$P_t = W_1 + W_2 = 10 + 10$

$P_t = 20$ kW

When $pf = 0.866$:

$\theta = \cos^{-1} 0.866 = 30°$

$$\tan\theta = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

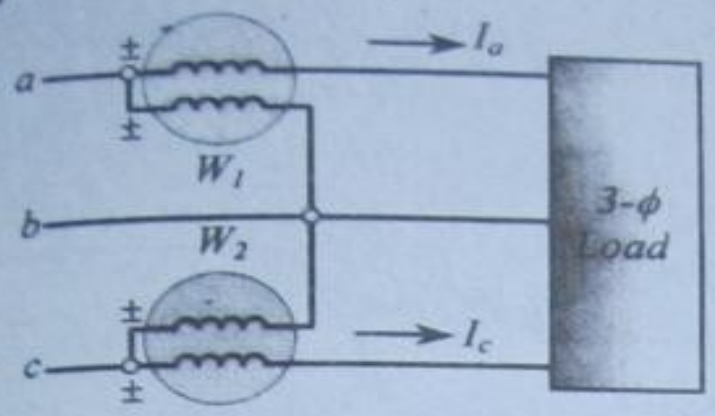
Substitute:

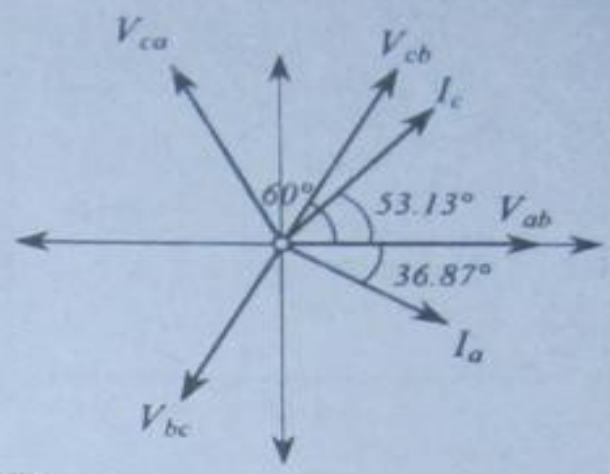
$$\tan 30° = \sqrt{3}\left(\frac{W_1 - (20 - W_1)}{20}\right)$$

$W_1 = 13.33$ kW

$W_2 = 20 - 13.33 = 6.67$ kW

404 Note: Use line b as the common potential terminal for the two wattmeters.





$$W_1 = V_{ab} I_a \cos\theta\Big|_{I_a}^{V_{ab}} = (230)(110)\cos 36.87°$$

$$W_1 = 20.24\ \text{kW}$$

$$W_2 = V_{cb} I_c \cos\theta\Big|_{I_c}^{V_{cb}} = (230)(125)\cos(60° - 53.13°)$$

$$W_2 = 28.54\ \text{kW}$$

$$W_t = W_1 + W_2 = 20.24 + 28.54$$

$$W_t = 48.78\ \text{kW}$$

Note: Replace the wattmeters with varmeters, to measure the total kVAR.

$$Q_1 = V_{ab} I_a \sin\theta\Big|_{I_a}^{V_{ab}} = (230)(110)\sin 36.87°$$

$$Q_1 = 15.18\ \text{kVAR}$$

$$Q_2 = V_{cb} I_c \sin\theta\Big|_{I_c}^{V_{cb}} = (230)(125)\sin(60° - 53.13°)$$

$$Q_2 = 3.438\ \text{kVAR}$$

Note: Since the currents I_a and I_c are respectively lagging with respect to voltage V_{ab} and V_{cb} respectively, the reading of the varmeters are simply added arithmetically to get the total kVAR.

$$Q_t = Q_1 + Q_2 = 15.18 + 3.438 = 18.618\ \text{kVAR}$$

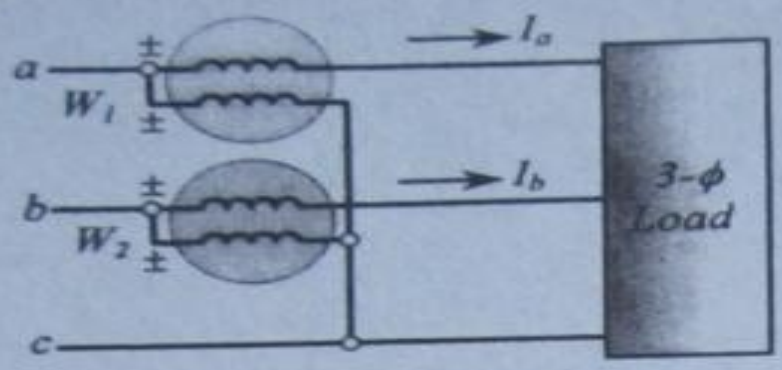
$$S_t = P_t - jQ_t$$

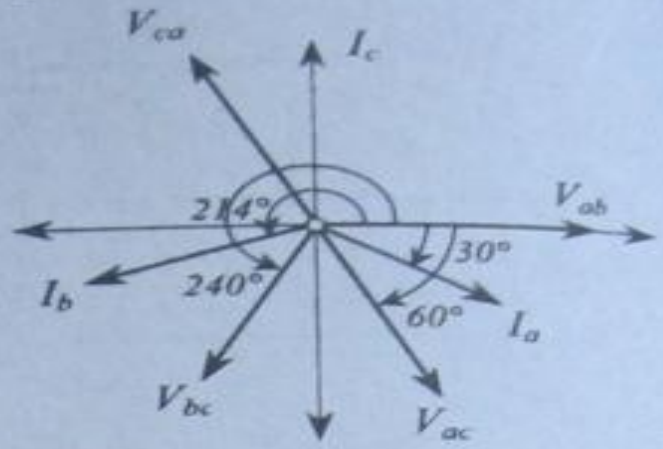
$$S_t = 48.78 - j18.618 = 52.212\angle 20.89°\ \text{kVA}$$

$$pf_t = \cos\theta_t = \cos 20.89°$$

$$pf_t = 0.934\ \text{lagging}$$

400 Note: Use line c as the common potential terminal for the two wattmeters. Sin not given, assume a phase sequence of a-b-c.





$$W_1 = V_{ac} I_a \cos\theta\Big|_{I_a}^{V_{ac}} = (4140)(60)\cos(60° - 30°)$$

$$W_1 = 215.12\,\text{kW}$$

$$W_2 = V_{bc} I_b \cos\theta\Big|_{I_b}^{V_{bc}} = (4140)(78.4)\cos(240° - 214°)$$

$$W_2 = 291.727\,\text{kW}$$

$$W_t = W_1 + W_2 = 215.12 + 291.727 = 506.85\,\text{kW}$$

Note: Replace the wattmeters with varmeters, to measure the total kVAR.

$$Q_1 = V_{ac} I_a \sin\theta\Big|_{I_a}^{V_{ac}} = (4140)(60)\sin(60° - 30°)$$

$$Q_1 = 124.2\,\text{kVAR}$$

$$Q_2 = V_{bc} I_b \cos\theta\Big|_{I_b}^{V_{bc}} = (4140)(78.4)\sin(240° - 214°)$$

$$Q_2 = 142.28\,\text{kVAR}$$

Note: Since the current I_a leads the voltage V_{ac} then, varmeter #1 will regist leading kVAR while since I_b lags behind V_{bc}, varmeter #2, will register a lag kVAR. Total kVAR is equivalent to the geometric sum of the two readings.

$$Q_t = Q_2 - Q_1 = 142.28 - 124.2 = 18.08\,\text{kVAR}$$

$$S_t = P_t - jQ_t = 506.85 - j18.08 = 507.172\angle 2.043°\,\text{kVA}$$

$$pf_t = \cos\theta_t = \cos 2.043°$$

$$pf_t = 0.999 \text{ lagging}$$

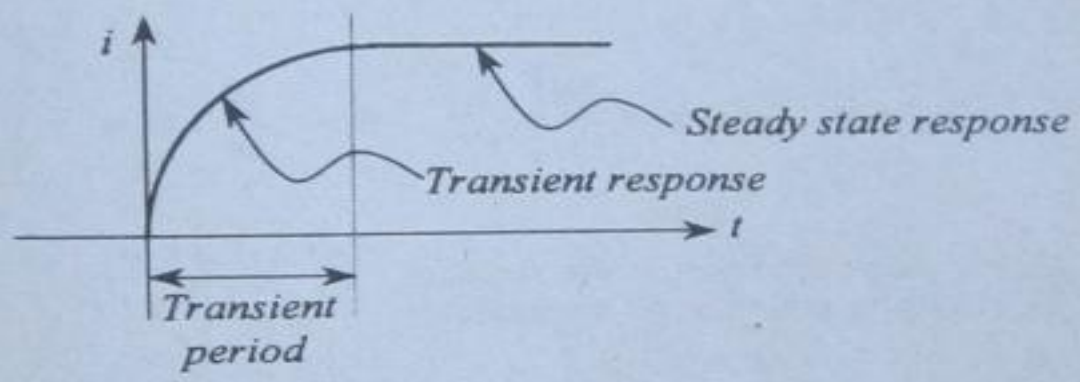
Note: The overall pf is lagging, since the lagging kVAR reading of varmeter bigger compared to the leading kVAR reading of varmeter #1.

Electrical Transients

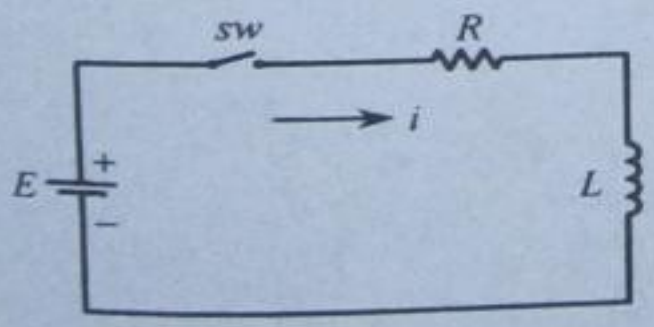
TRANSIENT PERIOD

When a circuit is to be switch-on it, it will not directly attain its steady state response but instead **passes through a transition period** wherein the currents or voltages adjust themselves to their **steady state response**. This transition period is called the **transient period**.

Example: A graph of the current growth in a given RL dc circuit..



RL SERIES DC CIRCUIT



$$I = \frac{E}{R} + ke^{-\frac{R}{L}t}$$

$$I = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

Note: "k" is an arbitrary constant whose value is found by substituting the initial conditions of the circuit.

If the initial value of the current (t = O) in the circuit is zero, **k = - E/R**

where: i = instantaneous current (ampere)
E= supply voltage (volt)
R = resistance of the resistor (ohm)
L = inductance of the coil (henry)
t = time (second)

- **Time constant (TC)** - time when the exponent of "e" in the formula for current is **unity** or equal to 1.

$$TC = \frac{L}{R}$$

where: R = resistance (ohm)
L = inductance (henry)
t = time (second)

- **Instantaneous voltage drops (volt):**

$$E_R = R(i) = E\left[1 - e^{-\frac{R}{L}t}\right] \qquad E_L = L\frac{di}{dt} = E\left[e^{-\frac{R}{L}t}\right]$$

where: E_R = voltage drop across the resistor
E_L = voltage drop across the inductor

- **Instantaneous powers (watt):**

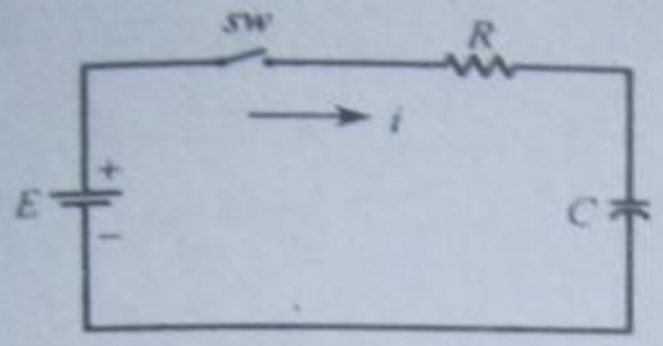
$$P_R = E_R(i) = \frac{E^2}{R}\left[1 - e^{-\frac{R}{L}t}\right]^2$$

$$P_L = E_L(i) = \frac{E^2}{R}\left[e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}\right]^2$$

$$P_t = P_R + P_L = E(i) = \frac{E^2}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

where: P_R = power dissipated by the resistor
P_L = power dissipated by the inductor
P_t = total power dissipated in the circuit

RC SERIES DC CIRCUIT



$$i = ke^{-\frac{t}{RC}}$$

Note: "k" is arbitrary constant whose value is found by substituting the initial conditions of the circuit.

- **Instantaneous currents (ampere):**

- Capacitor has no initial charge: $$i = \frac{E}{R}e^{-\frac{t}{RC}}$$

- Capacitor has an initial charge of Q_0: $$i = \left(\frac{E \pm \frac{Q_0}{C}}{R}\right)e^{-\frac{t}{RC}}$$

where: + (sign) = if capacitor is discharging
- (sign) = if capacitor is charging
i = instantaneous current (ampere)
E = supply voltage
R = resistance of the resistor (ohm)
C = capacitance of the capacitor (farad)
Q_0 = initial charge of the capacitor (coulomb)

- **Time constant (TC)** - time when the exponent of "e" in the formula for current is unity or equal to 1.

$$TC = RC$$

where: R = resistance (ohm)
C = capacitance (farad)
t = time (second)

- **Instantaneous voltage drops: (with $Q_0 = 0$)**

$$E_R = R(i) = E\,e^{-\frac{t}{RC}} \qquad E_C = \frac{1}{C}\int i(t)\,dt = E\left[1 - e^{-\frac{t}{RC}}\right]$$

where: E_R = voltage across the resistor (volt)
E_C = voltage across the capacitor (volt)

- **Instantaneous powers (watt):**

$$P_R = E_R(i) = \frac{E^2}{R}\left[e^{-\frac{2t}{RC}}\right]$$

$$P_C = E_C(i) = \frac{E^2}{R}\left[e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right]$$

$$P_t = P_R + P_L = E(i) = \frac{E^2}{R}\left[e^{-\frac{t}{RC}}\right]$$

where: P_R = power dissipated by the resistor
P_C = power dissipated by the capacitor
P_t = total power dissipated in the circuit

- **Instantaneous charge (coulomb) on the capacitor:**

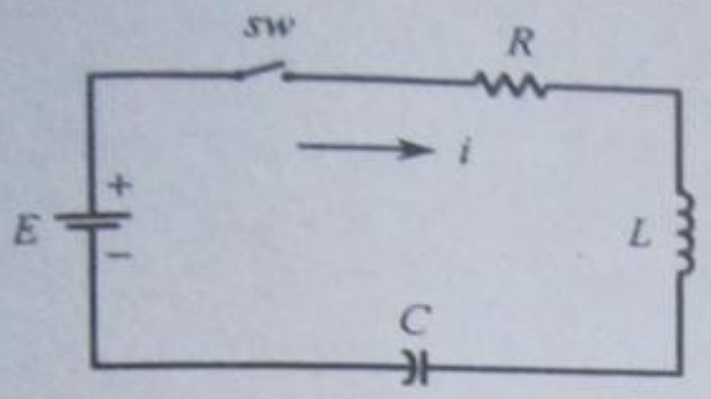
- Capacitor has no initial charge:

$$Q = CE\left[1 - e^{-\frac{t}{RC}}\right]$$

- Capacitor has an initial charge of Q_0:

$$Q = CE + (Q_0 - CE)\,e^{-\frac{t}{RC}}$$

RLC SERIES DC CIRCUIT



$$m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$$

Auxiliary equation

- **Overdamped case** – when the roots of the auxiliary equation are real and distinct.

$$\left[\frac{R}{L}\right]^2 - \frac{4}{LC} > 0 \qquad i = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

where: m_1 and m_2 are the roots of the auxiliary equation

- **Critically damped case** – when the roots of the auxiliary equation are repeated

$$\left[\frac{R}{L}\right]^2 - \frac{4}{LC} = 0 \qquad m = -\frac{R}{2L} \qquad i = (C_1 + C_2 t)e^{mt}$$

where: m is the repeated root

- **Underdamped case** – when the roots of the auxiliary equation contain a real and an imaginary part.

$$\left[\frac{R}{L}\right]^2 - \frac{4}{LC} < 0 \qquad \alpha = -\frac{R}{2L} \qquad \beta = \frac{\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$i = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$$

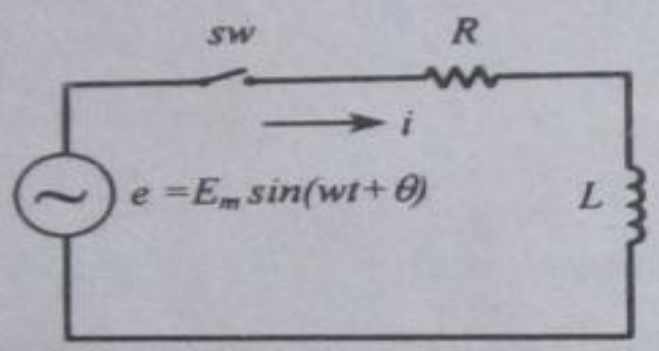
Note: C_1 and C_2 can be obtained using the initial conditions of the circuits.

General equation of the rate of change of the current: (At t = 0, i = 0)

$$\frac{di}{dt} = \frac{E \pm \frac{Q_o}{C}}{L}$$

where: Q_o = initial charge stored in the capacitor
$+Q_o$ = if discharging
$-Q_o$ = if charging

RL SERIES AC CIRCUIT

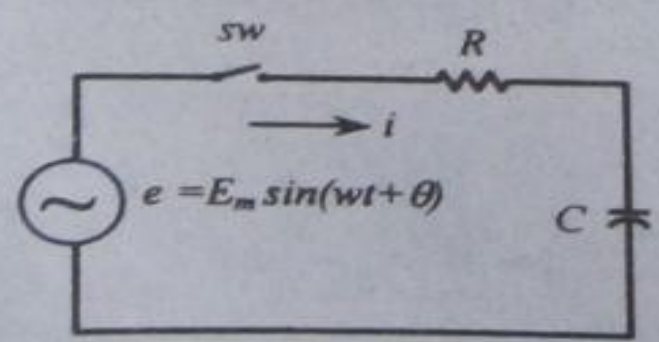


$$Z = \sqrt{R^2 + (wL)^2}$$

$$\delta = \tan^{-1}\frac{wL}{R} \qquad I_m = \frac{E_m}{Z}$$

$$i = I_m \sin(wt + \theta - \delta) - I_m \sin(\theta - \delta)\, e^{-\frac{R}{L}t}$$

RC SERIES AC CIRCUIT

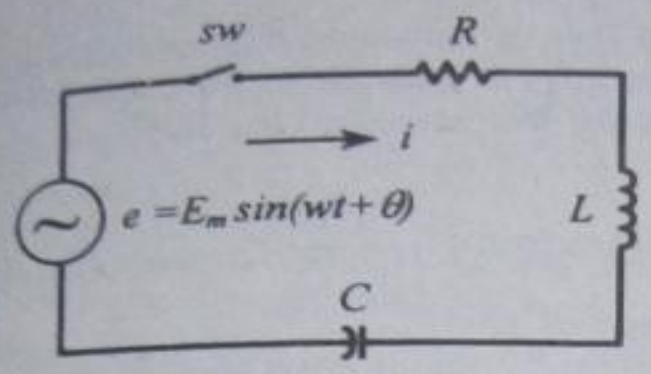


$$Z = \sqrt{R^2 + \left(\frac{1}{wRC}\right)^2}$$

$$\delta = \tan^{-1}\frac{1}{wRC} \qquad I_m = \frac{E_m}{Z}$$

$$i = I_m \sin(wt + \theta + \delta) + \left[\frac{E_m \sin\theta}{R} - I_m \sin(\theta + \delta)\right] e^{-\frac{t}{RC}}$$

RLC SERIES AC CIRCUIT



$$Z = \sqrt{R^2 + \left(\frac{1}{wRC} - wL\right)^2}$$

$$\delta = \tan^{-1}\left[\frac{\frac{1}{wC} - wL}{R}\right] \qquad I_m = \frac{E_m}{Z}$$

- **Overdamped case** – when the roots of the auxiliary equation are real and distinct.

$$i = C_1 e^{m_1 t} + C_2 e^{m_2 t} + I_m \sin(wt + \theta + \delta)$$

where: m_1 and m_2 are the roots of the auxiliary equation

- **Critically damped case** – when the roots of the auxiliary equation are repeated

$$i = (C_1 + C_2 t)e^{mt} + I_m \sin(wt + \theta + \delta)$$

where: m is the repeated root

- **Underdamped case** – when the roots of the auxiliary equation contain a real and an imaginary part.

$$i = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t) + I_m \sin(wt + \theta + \delta)$$

Note: To solve for the values of C_1, C_2, m, m_1, m_2, α and β, refer to RLC series DC transients.

General equation of the rate of change of the current: (At t = 0, i = 0)

$$\frac{di}{dt} = \frac{E_m \sin\theta \pm \frac{Q_o}{C}}{L}$$

where: Q_o = initial charge stored in the capacitor
$+Q_o$ = if discharging
$-Q_o$ = if charging

TRANSIENT ANALYSIS BY LAPLACE TRANSFORM METHOD

Laplace transform is a powerful mathematical tool in solving simultaneous linear differential equations formulated using Kirchhoff's laws in a given electric circuit. This method simply eliminates the effort of determining the values of the constants for integration.

Steps used in circuit analysis using laplace transform:

- Formulate necessary equations using function of "t".
- Transform the formulated equations (function of "t") to functions of "s".
- Simply the equations transformed and re-write the simplified equation to the standard formulas in inverse laplace functions.
- Obtain the inverse laplace function of each term in the simplified equation and solve the question asked by the problem.

LAPLACE TRANSFORM FORMULAS APPLICABLE TO ELECTRIC CIRCUIT ANALYSIS

f(t)	f(s)
k (constant)	$\frac{k}{s}$
kt^n	$\frac{kn!}{s^{n+1}}$
$ke^{\pm at}$	$\frac{k}{s \mp a}$
$kt^n e^{\pm at}$	$\frac{k}{(s \mp a)^{n+1}}$
$k \sin wt$	$\frac{kw}{s^2 + w^2}$
$k \cos wt$	$\frac{ks}{s^2 + w^2}$
$\sin(wt + \theta)$	$\frac{s \sin\theta + w \cos\theta}{s^2 + w^2}$
$\cos(wt + \theta)$	$\frac{s \sin\theta - w \cos\theta}{s^2 + w^2}$
$E_R = Ri_{(t)}$	$RI_{(s)}$
$E_L = L\frac{di}{dt}$	$L(sI_{(s)} - I_{(0)})$
$E_C = \frac{1}{C}\int i_{(t)}dt$	$\frac{1}{C}\left(\frac{I_{(s)}}{s} \pm \frac{Q_0}{s}\right)$

Test 12

Problem 406: EE Board April 1979, EE Board October 1982

In an RL circuit, Kirchhoff's Law gives the following relation: $E = L\,di/dt + Ri$

where: E = supply voltage (200 volts)
R = resistance (20 ohms)
L = inductance (1 henry)
t = time in seconds
i = current in amperes

If i = 0 when t = 0, find i when t = 0.02 second.

A. 3.3 A
B. 3.1 A
C. 3.2 A
D. 3.0 A

Problem 407: EE Board October 1980

In an RL circuit, Kirchhoff's Law gives the following relation: $E = L\,di/dt + Ri$

where: E = supply voltage (200 volts)
R = resistance (20 ohms)
L = inductance (1 henry)
t = time in seconds
i = current in amperes

If i = 0 when t = 0, find i after a long time.

A. 10 A
B. 11.2 A
C. 0
D. infinite

Problem 408: EE Board October 1990

A 6.0 H coil whose resistance is 12 ohms is connected in series with a 24 ohms resistor and to a 144 V battery and a switch. The switch is closed at t = 0. Determine the time constant of the circuit.

A. 0.36 sec
B. 0.45 sec
C. 0.66 sec
D. 0.50 sec

Problem 409:

The current in a coil decays as follows: when t = 0.002 sec, the current is 10 mA; when t = 0.006 s, the current drops to 4 mA. The time constant of the coil is ____

A. 4.50 ms
B. 3.28 ms
C. 4.36 ms
D. 2.15 ms

Problem 410: EE Board April 1990

A time of 10 milliseconds is required for the current on a series RL dc circuit to reach 90% of its final steady state value. Assume at t = 0, i(0) = 0. What is the time constant in seconds for the circuit?

A. 4.25 ms
B. 3.86 ms
C. 3.39 ms
D. 4.34 ms

Problem 411:

A coil has a resistance of 10 ohms and a time constant of 1 second. If the coil is connected across a steady potential of 24 V, how much is the current after 2 ms?

A. 4.07 mA
B. 4.79 mA
C. 4.91 mA
D. 4.04 mA

Problem 412: EE Board April 1995

The shunt winding of a machine has a resistance of 80 ohms and an inductance of 4 H is suddenly switched on to a 220-V supply. Find the time taken for the current to rise to half its steady state value.

A. 0.05 sec.
B. 0.0346 sec
C. 0.0251 sec
D. 0.0172 sec

Problem 413:

A dc voltage E is applied across an RL circuit at t = 0. At what time after, is the voltage across the resistor equal to that across the inductor?

A. 0.693 TC
B. 0.673 TC
C. 0.707 TC
D. 0.779 TC

Problem 414:

A 500-ohm relay coil has an inductance of 10 mH. This relay requires a current of 40 mA in order to trigger. How long after the switch is closed will the relay trigger if a 24 V DC source is applied across its terminals. Assume at t = 0, i = 0.

A. 23.54 μsec
B. 30.36 μsec
C. 35.83 μsec
D. 37.22 μsec

Problem 415: EE Board October 1990

A 6.0 H coil whose resistance is 12 ohms is connected in series with a 24 ohms resistor and to a 144 V battery and a switch. The switch is closed at t = 0. Determine the current at t = 0.1 second.

A. 1.538 A
B. 1.750 A
C. 1.805 A
D. 1.624 A

Problem 416:

A coil having a resistance of 10 Ω and inductance of 10 H is connected in series with a 15 Ω resistor. A 200-V DC source is connected to the circuit at t = 0. Determine the voltage across the coil at t = 0.05 sec.

A. 190.86 V
B. 188.51 V
C. 178.62 V
D. 185.90 V

Problem 417:

A series RL circuit is connected across a dc source through a switch. Three (3) milliseconds after the switch is closed, the voltage across the inductance is 20 V and drops to 5 V after 20 ms. If R = 100 ohms, find L.. Assume at t = 0, I = 0.

A. 1.30 H
B. 1.22 H
C. 1.59 H
D. 1.25 H

Problem 418:

A 1Ω resistor is connected in series with a 2-mH inductor. If a 12-V dc source is suddenly connected across its terminals, how long will it take, such that the voltage across the resistor will be twice as that of the inductor.

A. 2.5 ms
B. 1.7 ms
C. 2.2 ms

D. 1.9 ms

Problem 419:

A resistance of 100 kΩ is connected in series with a 100 μF capacitor. If the combination is suddenly connected across a 125 V DC source, determine the current one second after the switch is closed.

A. 1.13 mA
B. 1.25 mA
C. 1.05 mA
D. 1.12 mA

Problem 420: EE Board October 1992

An uncharged condenser in series with a 120-volt voltmeter of 10,000 ohms resistance is suddenly connected to a 100 V battery. One second later, the voltmeter reads 60 volt. Determine the capacitance of the condenser.

A. 187.54 μF
B. 190.62 μF
C. 195.76 μF
D. 192.23 μF

Problem 421:

A 120 μF capacitor has an initial charge of 600 μC. It is discharged through a 40 Ω resistor. How long will it take in order for the capacitor to discharge to 150 μC.

A. 6.65 ms
B. 6.82 ms
C. 4.66 ms
D. 5.85 ms

Problem 422:

A 50 μF capacitor is connected across a 400 V dc source. It is then discharged through a voltmeter whose resistance is 100-kΩ. How long will it take for the capacitor to discharge to 1.8 J of stored energy?

A. 2.3 s
B. 1.9 s
C. 2.0 s
D. 1.6 s

Problem 423: EE Board April 1999

A 20-ohm resistance R and a 0.001 farad capacitance C are in series. A direct current voltage E of 100 volts is applied across the series circuit at t = 0 and the initial current i(0) = 5 A. The applicable differential equation is:

$$R\left(\frac{di}{dt}\right)+\frac{i}{C}=0$$

Solve the differential equation and determine the resulting current i(t) at t = 0.01 second.

A. 3.34 A
B. 3.67 A
C. 2.78 A
D. 3.03 A

Problem 424: EE Board October 1991

A series RC circuit consist of R = 2 MΩ and an uncharged capacitor C = 5 μF. The circuit is connected across a 100 VDC source at t = 0. Determine the voltage across the resistor 5 seconds later.

A. 63.31 V
B. 60.65 V
C. 66.24 V
D. 69.22 V

Problem 425: EE Board October 1992

An uncharged condenser in series with a 120-volt voltmeter of 10,000 ohms resistance is suddenly connected to a 100 V battery. One second later, the voltmeter reads 60 volt. Determine the rate at which the voltage across the condenser is charging.

A. $51\,e^{-0.55t}$
B. $51\,e^{-0.51t}$
C. $55\,e^{-0.55t}$
D. $55\,e^{-0.51t}$

Problem 426: EE Board October 1981

In a circuit consisting of a resistance, R ohms, in series with a capacitance, C farad, and connected to a DC source with emf E volts, the electrical charge, Q coulombs and the current i, amperes, maybe determined by solving the simultaneous equations:

$$Ri+\frac{Q}{C}=E \text{ and } I=\frac{dQ}{dt}$$

If R = 20 ohms, C = 250 x 10^{-6} farad and E = 100 volts, find i after a long time.

A. 1 A
B. 0 A

C. infinity
D. 5 A

Problem 427:

An 80 μF capacitor in series with a voltmeter of 10,000-ohm resistance is connected suddenly across a 100-V supply. How long will it take for the voltmeter to register 15 V?

A. 1.84 sec
B. 1.52 sec
C. 1.63 sec
D. 1.25 sec

Problem 428: EE Board April 1993

A 100 μF capacitor initially charged to 24 V is discharge across a series combination of a 1 kΩ resistor and a 200 μF capacitor. Find the current after 1 sec.

A. 7.34 nA
B. 7.24 nA
C. 8.43 nA
D. 8.84 nA

Problem 429:

An 80 μF capacitor in series with a 1000-ohm resistor is connected suddenly across a 110 V DC supply. Find the value of the current after one time constant.

A. 0.0405 A
B. 0.0652 A
C. 0.0543 A
D. 0.0322 A

Problem 430: EE Board October 1991

A series RC circuit consist of R = 2 MΩ and an uncharged capacitor C = 5 μF. The circuit is connected across a 100 VDC source at t = 0. What is the initial rate of change of voltage across the resistor?

A. –10.0 V/s
B. 10.0 V/s
C. –12.4 V/s
D. None of these

Problem 431:

A series RLC circuit with R = 1 kΩ, L = 1 H and C = 6.25 μF is suddenly connected across a 24 V dc source. At t = 0, 1 = 0 and q = 0. Determine the current after 0.01 sec.

A. 3.45 mA
B. 4.61 mA

C. 5.40 mA
D. 5.05 mA

Problem 432:
A series RLC circuit has R = 200 Ω, L = 0.1 H and a capacitor C = 10 μF. If a 100 V dc source is connected across the end terminals of the series circuit at t = 0, determine the current after 1 millisecond. Assume zero initial conditions.

A. 0.353 A
B. 0.229 A
C. 0.253 A
D. 0.368 A

Problem 433:
A series RLC circuit has R = 40 ohms, L =100 mH and C = 50 μF. The circuit is connected across a 100 V dc source at t = 0 through a switch. Determine the current 0.02 second after the switch is closed. Assume all initial condition to be zero.

A. 6.40 mA
B. 5.31 mA
C. 6.22 mA
D. 4.85 mA

Problem 434: EE Board March 1998
A 10-ohm resistance R and a 1.0-henry inductance L are connected in series. An AC voltage e(t) = 100 sin 377t is applied across the series circuit. The applicable differential equation is:

$$Ri + L\left(\frac{di}{dt}\right) = e(t)$$

Solve for the particular solution (without the complimentary solution) and determine the amplitude of the resulting sinusoidal current i(t).

A. 0.321 A
B. 0.292 A
C. 0.241 A
D. 0.265 A

Problem 435: EE Board April 1991
A certain electric welder has a basic circuit equivalent to a series RL with R = 0.1 Ω and L = 1 mH. It is connected to an AC source "e" through a switch "s" operated by an automatic timer, which closes the circuit at any desired point on the 60 cycle, sinusoidal wave "e". Calculate the magnitude of the transient current resulting when "s" closes as "e" is passing through its peak value of 100 volts.

A. 256.41 A
B. 65.74 A
C. 80.54 A
D. 76.32 A

Problem 436: EE Board October 1999

Given the differential equation:

$$L\frac{di}{dt} + Ri = 100\sin(377t)$$

where L = 0.1 henry, R = 10 ohms and i(0) = 0. Determine the current at t = 0.01 second.

A. 2.784
B. 2.301
C. 2.531
D. 3.062

Problem 437:

A series RC circuit with R = 30 ohms and C = 250 μF is connected across an AC voltage source of emf 100 sin 100t through a switch. The switch is closed at t = 0. Determine the current when t = 0.01 second. Assume at t = 0, voltage across the capacitor is zero.

A. 1.45 A
B. 1.24 A
C. 0.93 A
D. 2.05 A

Problem 438: EE Board April 1999

A series circuit has R = 10 ohms, L = 0.1 henry and C = 0.0001 farad. An AC voltage e = 100 sin 377t is applied across the series circuit and the applicable differential equation is:

$$L\left(\frac{d^2i}{dt^2}\right) + R\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)i = \frac{de}{dt}.$$

Solve for the particular solution (without the complimentary solution) and determine the amplitude of the resulting sinusoidal current i(t).

A. 5.51 A
B. 6.06 A
C. 6.67 A
D. 7.34 A

Problem 439: EE Board October 1999

A 10-ohm resistance R and a 0.001 farad capacitance C are in series. An AC voltage e = 100 sin 377t is applied across the series circuit. The applicable differential equation is:

$$R\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)i = \frac{de}{dt}$$

Solve for the particular solution (without the complimentary solution) and determine the amplitude of the resulting sinusoidal current i(t).

A. 9.67 A
B. 10.63 A
C. 8.79 A
D. 11.70 A

Problem 440:

A series RL circuit has R = 10 ohms and L = 1 henry. The circuit is connected across a voltage source $e(t) = 10\,e^{-10t}$ through a switch that is initially open. The switch is closed at t = 0. Determine the current after 0.01 second. Assume initial current to be zero.

A. 86.8 mA
B. 90.5 mA
C. 89.3 mA
D. 92.2 mA

Problem 441: EE Board October 1999

The differential equation of a series RC circuit with DC applied voltage is:

$$Ri + 1/C \int i\,dt = E.$$

If R = 50 ohms, C = 0.0001 farad, E = 100 volts and i(0) = 2.0 amperes, determine the Laplace transform expression for I(s).

A. $I(s) = \dfrac{2}{s+200}$

B. $I(s) = \dfrac{2}{s(s+2)}$

C. $I(s) = \dfrac{2}{s+50}$

D. $I(s) = \dfrac{2}{s+2}$

Problem 442:

A series RC circuit has R = 1kΩ and C = 100 μF with an initial charge of 2000 μC. The combination is connected across a 100 V dc source and the switch is closed at t = 0. If the capacitor is connected in the circuit such that it will be discharging, determine the Laplace transform expression of the current drawn.

A. $i_{(s)} = \dfrac{0.12}{s+10}$

B. $i_{(s)} = \dfrac{0.12}{s+12}$

C. $i_{(s)} = \dfrac{0.10}{s+10}$

D. $i_{(s)} = \dfrac{0.10}{s+12}$

Problem 443: EE Board March 1998

Determine the inverse Laplace transform of $i_{(s)} = \dfrac{200}{s^2 + 50s + 10625}$

A. $i_{(t)} = 2\,e^{-25t}\sin 100t$

B. $i_{(t)} = 2t\,e^{-25t}\sin 100t$

C. $i_{(t)} = 2\,e^{-25t}\cos 100t$

D. $i_{(t)} = 2t\,e^{-25t}\cos 100t$

ANSWER KEY

406. A	414. C	422. C	430. A	438. C
407. A	415. C	423. D	431. C	439. A
408. D	416. D	424. B	432. D	440. B
409. C	417. C	425. B	433. A	441. A
410. D	418. C	426. B	434. D	442. A
411. B	419. A	427. B	435. B	443. A
412. B	420. C	428. A	436. C	
413. A	421. A	429. A	437. A	

☑ RATING

34-38 - Topnotcher

26-33 - Passer

19-25 - Conditional

0-18 - Failed

Solutions to Test 12

406 $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{200}{20}\left(1 - e^{-\frac{20}{1}(0.02)}\right) = 3.296$

$i \cong 3.3\ A$

407 Note: After a "long time" means time $t = \infty$. It simply means also that the circuit is already at "steady state". And at steady state, the inductor is equivalent to a short-circuited wire, thus leaving only the resistance R to oppose the current flow.

$$i = \frac{E}{R} = \frac{200}{20} = 10\ A$$

408 Note: Time constant (TC) of RL circuit means the time in seconds required for the current to reach 63.2% of the steady state value.

$$TC = \frac{L}{R} = \frac{6}{12} = 0.5\ \text{second}$$

409 Note: "Current decay" means the circuit is source free, as if after being initialized the coil is short-circuited.

$$i = I_0\, e^{-\frac{R}{L}t} \qquad TC = \frac{L}{R}$$

$$i = I_0\, e^{-\frac{t}{TC}}$$

Note: I_0 = initial value of the current before it is made source free.

When $t = 0.002\ s$, $i = 0.01\ A$

$$0.01 = I_0\, e^{-\frac{0.002}{TC}}$$

$$I_0 = \frac{0.01}{e^{-\frac{0.002}{TC}}} \Rightarrow ❶$$

When $t = 0.006\ s$, $i = 0.004\ A$

$$0.004 = I_0\, e^{-\frac{0.006}{TC}}$$

$$I_0 = \frac{0.004}{e^{-\frac{0.006}{TC}}} \Rightarrow ❷$$

Equate Eq.1 to Eq.2 :

$$\frac{0.01}{e^{-\frac{0.002}{TC}}} = \frac{0.004}{e^{-\frac{0.006}{TC}}}$$

$$0.01e^{-\frac{0.006}{TC}} = 0.004\,e^{-\frac{0.002}{TC}}$$

$$e^{-\frac{0.006}{TC}} = 0.4\,e^{-\frac{0.002}{TC}}$$

Take ln on both sides :

$$\ln e^{-\frac{0.006}{TC}} = \ln 0.4\,e^{-\frac{0.002}{TC}}$$

$$\frac{-0.006}{TC} = \ln 0.4 - \frac{0.002}{TC}$$

$$TC = \frac{-0.004}{\ln 0.4}$$

$$TC = 4.36\text{ ms}$$

410 Note: Final steady state value of current is equal to E/R.

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$0.9\left(\frac{E}{R}\right) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}(0.01)}\right)$$

$$0.9 = 1 - e^{-\frac{R}{L}(0.01)}$$

$$e^{-\frac{R}{L}(0.01)} = 0.1$$

Take ln on both sides :

$$-\frac{R}{L}(0.01) = \ln(0.1)$$

$$\frac{R}{L} = 230.258$$

$$TC = \frac{L}{R} = \frac{1}{230.258}$$

$$TC = 4.34\text{ ms}$$

411 $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$ $\quad TC = \frac{L}{R}$

$$i = \frac{E}{R}\left(1 - e^{-\frac{t}{TC}}\right) = \frac{24}{10}\left(1 - e^{-\frac{0.002}{t}}\right)$$

$i = 4.8\ mA$

412 Note: Since not specified, assume initial current of the circuit to be zero. Final steady state value of current is equal to E/R

$$I = \frac{E}{R} = \frac{220}{80} = 2.75\ A$$

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\frac{1}{2}(2.75) = 2.75\left(1 - e^{-\frac{80}{4}t}\right)$$

$0.5 = 1 - e^{-20t}$

$e^{-20t} = 0.5$

Take ln on both sides :

$\ln e^{-20t} = \ln 0.5$

$$t = \frac{\ln 0.5}{-20}$$

$t = 0.0346$ sec.

413 $E_R = E\left(1 - e^{-\frac{t}{TC}}\right) \Rightarrow$ ❶

$E_L = Ee^{-\frac{t}{TC}} \Rightarrow$ ❷

Equate Eq.1 to Eq.2 :

$1 - e^{-\frac{t}{TC}} = e^{-\frac{t}{TC}}$

$2e^{-\frac{t}{TC}} = 1$

$e^{-\frac{t}{TC}} = 0.5$

Take ln on both sides :

$\ln e^{-\frac{t}{TC}} = \ln 0.5$

$t = 0.693\ TC$

414

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$0.04 = \frac{24}{500}\left(1 - e^{-\frac{500}{0.01}t}\right)$$

$$0.8333 = 1 - e^{-50{,}000t}$$

$$e^{-50{,}000t} = 0.1667$$

Take ln on both sides :

$$\ln e^{-50{,}000t} = \ln 0.1667$$

$$t = \frac{\ln 0.1667}{-50{,}000}$$

$$t = 35.83\ \mu s$$

415

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$R_r = 12 + 24 = 36\ \Omega$$

$$i = \frac{144}{36}\left(1 - e^{-\frac{36}{6}(0.1)}\right)$$

$$i = 1.805\ A$$

416

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$R_r = 10 + 15 = 25\ \Omega$$

$$i = \frac{200}{25}\left(1 - e^{-\frac{25}{10}(0.05)}\right) = 0.94\ A$$

$$E_{150} = iR_{150}$$
$$= (0.94)(15)$$
$$E_{100} = 14.1\ \text{volts}$$
$$E_{coil} = E - E_{150}$$
$$= 200 - 14.1$$
$$E_{coil} = 185.9\ \text{volts}$$

417 $E_L = Ee^{-\frac{R}{L}t}$

$$20 = Ee^{-\frac{100}{L}(0.003)} \Rightarrow ❶$$

$$5 = Ee^{-\frac{100}{L}(0.025)} \Rightarrow ❷$$

Divide Eq.1 by Eq.2 :

$$\frac{20}{5} = \frac{e^{-\frac{0.3}{L}}}{e^{-\frac{2.5}{L}}}$$

$$4e^{-\frac{2.5}{L}} = e^{-\frac{0.3}{L}}$$

Take ln on both sides :

$$\ln 4 + \ln e^{-\frac{2.5}{L}} = \ln e^{-\frac{0.3}{L}}$$

$$\ln 4 - \frac{2.5}{L} = -\frac{0.3}{L}$$

$$L = \frac{2.5 - 0.3}{\ln 4}$$

$$L = 1.586\,H$$

418 $E_R = E\left(1 - e^{-\frac{R}{L}t}\right)$

$$E_R = 12\left(1 - e^{-\frac{1}{0.002}t}\right)$$

$$E_R = 12\left(1 - e^{-500t}\right) \Rightarrow ❶$$

$$E_L = Ee^{-\frac{R}{L}t}$$

$$E_L = 12e^{-\frac{1}{0.002}t}$$

$$E_L = 12e^{-500t} \Rightarrow ❷$$

$$E_R = 2E_L$$

$$12\left(1 - e^{-500t}\right) = 2\left(12e^{-500t}\right)$$

$$3e^{-5000t} = 1$$

$$e^{-5000t} = 0.333$$

Take ln on both sides :

$$t = \frac{\ln 0.333}{-500}$$

$$t = 2.2\,\text{ms}$$

419

$$i = \frac{E}{R}\left(e^{-\frac{t}{RC}}\right)$$

Substitute :

$$i = \frac{125}{100{,}000}\left(e^{-\frac{1}{(100{,}000)(100\times10^{-6})}}\right)$$

$$i = 1.13\,\text{mA}$$

420

$$E_R = Ee^{-\frac{t}{RC}}$$

$$60 = 100\left(e^{-\frac{1}{(10{,}000)C}}\right)$$

$$0.6 = e^{-\frac{1}{(10{,}000)C}}$$

Take ln on both sides :

$$-\frac{1}{10{,}000C} = \ln 0.6$$

$$C = 195.76\,\mu\text{F}$$

421 Note: This is a problem about a source free RC circuit. The word "to discharge to" means the charges that remain in the capacitor after a given time.

$$q = Q_0\, e^{-\frac{t}{RC}}$$

$$150\times10^{-6} = 600\times10^{-6}\, e^{-\frac{t}{(40)(120\times10^{-6})}}$$

$$0.25 = e^{-\frac{t}{0.0048}}$$

Take ln on both sides :

$$\ln 0.25 = -\frac{t}{0.0048}$$

$$t = 6.65\,\text{ms}$$

422 Note: This is a problem about a source free RC circuit.

$$W_C = W_0\, e^{-\frac{2t}{RC}}$$

$$W_0 = \frac{1}{2}CV_0^{\,2} = \frac{1}{2}(50 \times 10^{-6})(400)^2 = 4\text{ J}$$

Substitute :

$$1.8 = 4e^{-\frac{2t}{(100{,}000)(50 \times 10^{-6})}}$$

$$0.45 = e^{-0.4t}$$

Take ln on both sides :

$$\ln 0.45 = -0.4t$$

$$t = 2\text{ s}$$

423 Note: Based on the given differential equation, the problem is a source free RC circuit.

$$i = I_0\, e^{-\frac{t}{RC}}$$

Substitute :

$$i = 5\, e^{-\frac{0.01}{(20)(0.001)}}$$

$$i = 3.03\text{ A}$$

424 $E_R = E\, e^{-\frac{t}{RC}}$

Subsititute : t = 5

$$E_R = 100\, e^{-\frac{5}{(2 \times 10^6)(5 \times 10^{-6})}}$$

$$E_R = 60.65\text{ volts}$$

425 $E_R = Ee^{-\frac{t}{RC}}$

$$60 = 100\left(e^{-\frac{1}{(10{,}000)C}}\right)$$

$$0.6 = e^{-\frac{1}{(10{,}000)C}}$$

Take ln on both sides:

$$-\frac{1}{10{,}000C} = \ln 0.6$$

$$C = 195.76\ \mu F$$

$$E_C = E\left(1 - e^{-\frac{t}{RC}}\right) = E - Ee^{-\frac{t}{RC}}$$

$$= 100 - 100e^{-\frac{t}{(10{,}000)(195.76 \times 10^{-6})}}$$

$$E_C = 100 - 100e^{-0.51t}$$

$$\frac{dE_C}{dt} = 0 - 100e^{-0.51t}(-0.51)$$

$$\frac{dE_C}{dt} = 51e^{-0.51t} \text{ volts/second}$$

426 Note: Based on the differential equation given, the circuit is a basic RC circuit. After a "long time" means, time is "infinity". When $t = \infty$, the capacitor is open circuited, since the voltage across its terminals is equivalent to the supply voltage, thus no current will flow in the circuit under this condition.

$$i = \frac{E}{R}\left(e^{-\frac{t}{RC}}\right)$$

When $t = \infty$, $e^{-\infty} = 0$

Thus, $i = 0$

427 $E_R = E\,e^{-\frac{t}{RC}}$

$$15 = 100\,e^{-\frac{t}{10{,}000(80 \times 10^{-6})}}$$

$$0.15 = e^{-1.25t}$$

Take ln on both sides:

$$t = \frac{\ln 0.15}{-1.25} = 1.517$$

$$t \cong 1.52\ s$$

428 $i = \frac{E}{R}\left(e^{-\frac{t}{RC}}\right)$

Since the capacitors are in series :

$$i = \frac{C_1C_2}{C_1 + C_2} = \frac{100(200)}{100 + 200} = 66.667\ \mu F$$

Substitute : t = 1

$$i = \frac{24}{1000}\left(e^{-\frac{1}{(1000)(66.667 \times 10^{-6})}}\right) = 7.342 \times 10^{-9}$$

$i \cong 7.34$ nA

429 $i = \frac{E}{R}e^{-\frac{t}{RC}}$

For an RC series circuit, time constant, (TC) = RC

$$i = \frac{110}{1000}e^{-\frac{RC}{RC}} = 0.11e^{-1}$$

$i = 0.0405$ A

430 Note: "Initial condition" means when time (t) is zero.

$$E_R = Ee^{-\frac{t}{RC}}$$

$$\frac{dE_R}{dt} = Ee^{-\frac{t}{RC}}\left(-\frac{1}{RC}\right)$$

Substitute : t = 0

$$\frac{dE_R}{dt} = 100\left(e^{-\frac{-0}{(2 \times 10^6)(5 \times 10^{-6})}}\right)\left(-\frac{1}{(2 \times 10^6)(5 \times 10^{-6})}\right)$$

$$\frac{dE_R}{dt} = -10\ V/s$$

Note: The negative answer simply means that the voltage across the resistor is "decreasing" 10 volts every second after.

431 Solving for the roots :

$$m = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \frac{-\frac{1000}{1} \pm \sqrt{\left(\frac{1000}{1}\right)^2 - \frac{4}{(1)(6.25 \times 10^{-6})}}}{2}$$

$$m = \frac{-1000 \pm 600}{2} \quad m_1 = \frac{-1000 + 600}{2} = -200; \quad m_2 = \frac{-1000 - 600}{2} = -800$$

Note: Since the roots m_1 and m_2 are real and unequal, the circuit is an "Overdamped case".

$$i = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$
$$i = C_1 e^{-200t} + C_2 e^{-800t} \Rightarrow ❶$$
$$\frac{di}{dt} = -200C_1 e^{-200t} - 800C_2 e^{-800t} \Rightarrow ❷$$

At $t = 0, i = 0$, substitute in Eq.1:

$$0 = C_1 e^{-200(0)} + C_2 e^{-800(0)}$$
$$C_1 = -C_2 \Rightarrow ❸$$

At $t = 0$, $q = 0$:

$$\frac{di}{dt} = \frac{E}{L} = \frac{24}{1} = 24 \text{ A/s}$$

Substitute $di/dt = 24, t = 0$ and Eq.3 in Eq.2:

$$24 = -200(-C_2)e^{-200(0)} - 800C_2 e^{-800(0)}$$
$$C_2 = -0.04$$
$$C_1 = 0.04$$
$$i = C_1 e^{-200t} + C_2 e^{-800t}$$
$$i = 0.04\, e^{-200(0.01)} - 0.04\, e^{-800(0.01)}$$
$$i = 5.4 \text{ mA}$$

432 Solving for the roots:

$$m = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \frac{-\frac{200}{0.1} \pm \sqrt{\left(\frac{200}{0.1}\right)^2 - \frac{4}{(0.1)(10 \times 10^{-6})}}}{2}$$
$$m = \frac{-2{,}000 \pm 0}{2} = -1000$$

Note: Since there is only one real root, the circuit is a "Critically damped case".

$$i = (C_1 + C_2 t)e^{mt}$$
$$i = (C_1 + C_2 t)e^{-1000t} \Rightarrow ❶$$

Substitute $t = 0$ and $i = 0$ in Eq.1:

$$0 = C_1 e^{-1000(0)} + C_2(0)e^{-1000(0)}$$
$$C_1 = 0$$

Subsitute $C_1 = 0$ in Eq.1:

$i = C_2 t e^{-1000t} \Rightarrow$ ❷

$$\frac{di}{dt} = C_2\left(e^{-1000t}(1) + te^{-1000t}(-1000)\right)$$

At $t = 0$, $q = 0$:

$$\frac{di}{dt} = \frac{E}{L} = \frac{100}{0.1} = 1000 \text{ A/s}$$

$$1000 = C_2\left(e^{-1000(0)} + (0)\,e^{-1000(0)}(-1000)\right)$$

$C_2 = 1000$

Subsitute : $t = 0.001$

$i = C_2 t e^{-1000t} = (1000)(0.01)\,e^{-1000(0.001)}$

$i = 0.368$ A

433 Solving for the roots:

$$m = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \frac{-\frac{50}{0.1} \pm \sqrt{\left(\frac{50}{0.1}\right)^2 - \frac{4}{(0.1)(50 \times 10^{-6})}}}{2}$$

Note: Since the quantity inside the radical sign is less than zero, the circuit is an "Under damped case".

$i = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$

$$\alpha = -\frac{R}{2L} = -\frac{40}{2(0.1)} = -200$$

$$|\beta| = \frac{\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \frac{\sqrt{\left(\frac{40}{0.1}\right)^2 - \frac{4}{(0.1)(50 \times 10^{-6})}}}{2} = 400$$

$i = e^{-200t}(C_1 \cos 400t + C_2 \sin 400t) \Rightarrow$ ❶

Substitute $t = 0$ and $i = 0$ in Eq.1:

$0 = e^{-200(0)}(C_1 \cos 400(0) + C_2 \sin 400(0))$

$C_1 = 0$

$i = e^{-200t}(C_2 \sin 400t)$

$$\frac{di}{dt} = C_2\left(\sin 400t\, e^{-200t}(-200) + e^{-200t}\cos 400t(400)\right) \Rightarrow ❷$$

$$\frac{di}{dt} = \frac{E}{L} = \frac{100}{0.1} = 1000$$

Substitute di/dt and t = 0 in Eq.2 :

$$1000 = C_2\left(\sin 400(0)\, e^{-200(0)}(-200) + e^{-200(0)}(400)\cos 400(0)\right)$$

$$C_2 = 2.5$$

Substitute : t = 0.02

$$i = 2.5e^{-200t}\sin 400t = e^{-200(0.02)}(2.5)\sin 400(0.02)$$

$$i = 6.4\,mA$$

434 Note: As given by the differential equation, the circuit is a series RL supplied by an AC source. And the problem is asking only for the maximum magnitude of the resulting current.

$$I_m = \frac{E_m}{Z}$$

$$Z = R + jX_L$$

$$X_L = wL = 377(1) = j377$$

$$Z = 10 + j377 = 377.133\angle 88.48°$$

$$I_m = \frac{100}{377.133}$$

$$I_m = 0.265\ A$$

435 $i_{tran} = I_m \sin(\theta - \delta)\, e^{-\frac{R}{L}t}$

where :

θ = angle at the instant when the switch is closed

δ = pf angle

$$Z = R + jX_L$$

$$X_L = wL = 2(\pi)(60)(0.001) = 0.377$$

$$Z = 0.1 + j0.377 = 0.39\angle 75.144°$$

Note: At the instant the switch is closed, t = 0.

$$i_{tran} = \frac{100}{0.39}\sin(90° - 75.144°)\, e^{-\frac{0.1(0)}{0.001}}$$

$$i_{tran} = 65.74\ A$$

436 Note: As given by the differential equation, the circuit is a series RL supplied by an AC source.

$$i = I_m \sin(wt + \theta - \delta) - I_m \sin(\theta - \delta)\, e^{-\frac{R}{L}t}$$

$$Z = R + jX_L$$

$$X_L = wL = 377(0.1) = 37.7\,\Omega$$

$$Z = 10 + j37.7 = 39\angle 75.144^\circ$$

$$I_m = \frac{E_m}{Z} = \frac{100}{39} = 2.564\text{ A}$$

$$i = 2.564 \sin\left[377(0.01)(180^\circ/\pi) + 0^\circ - 75.144^\circ\right]$$

$$- 2.564 \sin(0^\circ - 75.144^\circ)\, e^{-\frac{10}{0.1}(0.01)}$$

$$i = 2.53\text{ A}$$

437 $$i = I_m \sin(wt + \theta + \delta) + \left[\frac{E_m \sin\theta}{R} - I_m \sin(\theta + \delta)\right] e^{-\frac{1}{RC}}$$

$$Z = R - jX_C$$

$$X_C = \frac{1}{wC} = \frac{1}{100(250 \times 10^{-6})} = 40\,\Omega$$

$$Z = 30 + j40 = 50\angle 53.13^\circ$$

$$I_m = \frac{E_m}{Z} = \frac{100}{50} = 2\text{ A}$$

$$i = 2\sin\left[100(0.01)(180^\circ/\pi) + 0^\circ + 53.13^\circ\right]$$

$$+ \left[\frac{100 \sin 0^\circ}{30} - 2\sin(0^\circ + 53.13^\circ)\right] e^{-\frac{0.01}{30(250 \times 10^{-6})}}$$

$$i = 1.45\text{ A}$$

438 Note: As given by the differential equation, the circuit is a series RLC supplied by an AC source. And the problem is asking only for the maximum magnitude of resulting current.

$$I_m = \frac{E_m}{Z}$$

$$X_L = wL = 377(0.1) = 37.7$$

$$X_C = \frac{1}{wC} = \frac{1}{377(0.0001)} = 26.525\,\Omega$$

$$Z = 10 + j37.7 - j26.525 = 10 + j11.175$$

$$Z = 15\angle 48.176^\circ$$

$$I_m = \frac{100}{15}$$

$$I_m = 6.67\ A$$

439 Note: As given by the differential equation, the circuit is a series RL supplied by an AC source. And the problem is asking only for the maximum magnitude of the resulting current..

$$I_m = \frac{E_m}{Z}$$

$$X_C = \frac{1}{wC} = \frac{1}{377(0.001)} = 2.652\ \Omega$$

$$Z = 10 + j37.7 - j26.525 = 10 + j2.652$$

$$Z = 10.345\angle 14.85°$$

$$I_m = \frac{100}{10.345}$$

$$I_m = 9.67\ A$$

440 Note: Use Laplace functions to solve the problem for a much easier solution. Refer to the Laplace function table for the equivalents.

$$Ri + L\frac{di}{dt} = 10\,e^{-10t}$$

$$R\,i_{(s)} + L\left(s\,i_{(s)} - i_{(0)}\right) = \frac{10}{s+10}$$

$$10\,i_{(s)} + 1\left(s\,i_{(s)} - 0\right) = \frac{10}{s+10}$$

$$i_{(s)}(s+10) = \frac{10}{s+10}$$

$$i_{(s)} = \frac{10}{(s+10)^2}$$

Note: Transform the function of "s" equation to its equivalent function of "t" using the Laplace function table.

$$i_{(t)} = 10\,t\,e^{-10t}$$

At $t = 0.01$

$$i = 10(0.01)e^{-10(0.01)} = 0.09048\ A$$

$$i \cong 90.5\ mA$$

441 $Ri + \frac{1}{C}\int i\,dt = e$

$$R\,i_{(s)} + \frac{1}{C}\left(\frac{i_{(s)}}{s} - \frac{Q_{(0)}}{s}\right) = \frac{e}{s}$$

$$50\,i_{(s)} + \frac{1}{0.0001}\left(\frac{i_{(s)}}{s} - 0\right) = \frac{100}{s}$$

$$50\,i_{(s)} + \frac{10{,}000\,i_{(s)}}{s} = \frac{100}{s}$$

$$\left(50\,i_{(s)} + \frac{10{,}000\,i_{(s)}}{s} = \frac{100}{s}\right)\frac{s}{50}$$

$$si_{(s)} + 200\,i_{(s)} = 2$$

$$i_{(s)} = \frac{2}{s+200}$$

Another solution:

Note: The current can also be solved using function of "t" analysis and convert the answer to function of "s" later using the Laplace function table.

$$i_{(t)} = I_0\,e^{-\frac{t}{RC}}$$

$$i_{(t)} = 2\,e^{-\frac{t}{50(0.0001)}} = 2\,e^{-200t}$$

$$i_{(s)} = \frac{2}{s+200}$$

442 $Ri + \frac{1}{C}\int i\,dt = e$

$$R\,i_{(s)} + \frac{1}{C}\left(\frac{i_{(s)}}{s} - \frac{Q_{(0)}}{s}\right) = \frac{e}{s}$$

$$1000\,i_{(s)} + \frac{1}{100 \times 10^{-6}}\left(\frac{i_{(s)}}{s} - \frac{2000 \times 10^{-6}}{s}\right) = \frac{100}{s}$$

$$1000\,i_{(s)} + \frac{10{,}000\,i_{(s)}}{s} = \frac{120}{s}$$

$$\left(1000\,i_{(s)} + \frac{10{,}000\,i_{(s)}}{s} = \frac{120}{s}\right)\frac{S}{1000}$$

$$si_{(s)} + 10\,i_{(s)} = 0.12$$

$$i_{(s)} = \frac{0.12}{s+10}$$

Another solution:

Note: The current can also be solved using function of "t" analysis and convert the answer to function of "s" later using the Laplace function table.

$$i_{(t)} = \left(\frac{E \pm \frac{Q_0}{C}}{R}\right) e^{-\frac{1}{RC}} = \left(\frac{100 + \frac{2000}{100}}{1000}\right) e^{-\frac{t}{(1000)(100 \times 10^{-6})}}$$

$$i_{(t)} = 0.12e^{-10t}$$

Note: Convert the equation to function of "s" using the Laplace function table.

$$\frac{k}{s \mp a} = ke^{\pm at} \Rightarrow \text{Formula}$$

Thus:

$$i_{(s)} = \frac{0.12}{s+10}$$

443 $$i_{(s)} = \frac{200}{s^2 + 50s + 10625} = 2\left(\frac{100}{(s+25)^2 + (100)^2}\right)$$

Note: Refer to the Laplace function table for the equivalent function of "t".

$$\frac{k}{(s+a)^2 + k^2} = e^{-at} \sin kt \Rightarrow \text{Formula}$$

Thus :

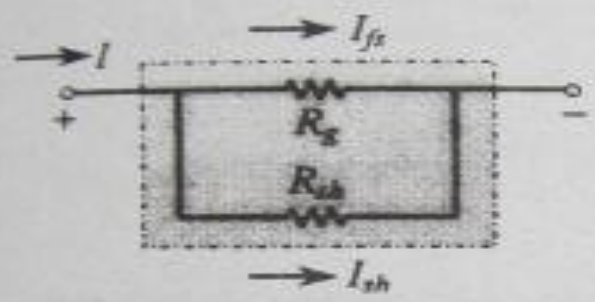
$$i_{(s)} = 2\left(\frac{100}{(s+25)^2 + (100)^2}\right)$$

$$i_{(t)} = 2\,e^{-25t} \sin 100t$$

Instruments & Measurements

AMMETER

- **Ammeter shunt (shunt resistance)** - used to increase the range of an ammeter.



$$I = I_{sh} + I_{fs}$$

$$I = I_{fs} \times \text{Multiplier}$$

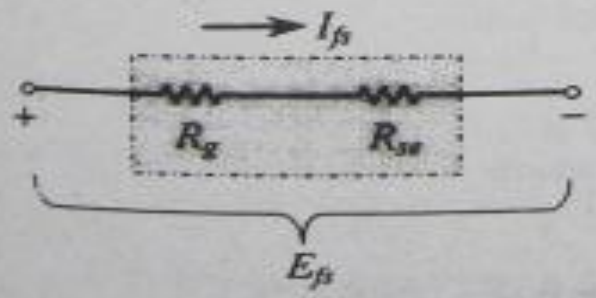
$$R_{sh} = \frac{I_{fs} R_g}{I_{sh}}$$

$$\text{Multiplier} = \frac{R_g + R_{sh}}{R_{sh}}$$

where: I = maximum current that the meter can measure (ampere)
I_{fs} = full scale current of the meter (current flowing in the galvanometer coil)
I_{sh} = current flowing in the shunt resistance (ampere)
R_g = resistance of the galvanometer coil (ohm)
R_{sh} = shunt resistance (ohm)

VOLTMETER

- **Potential divider (series resistance)** - used to increase the range of a voltmeter.



$$E_{fs} = I_{fs}(R_g + R_{se})$$

$$E_g = I_{fs} R_g$$

$$E_{fs} = E_g \times \text{Multiplie}$$

$$\text{Multiplier} = \frac{R_g + R_{se}}{R_g}$$

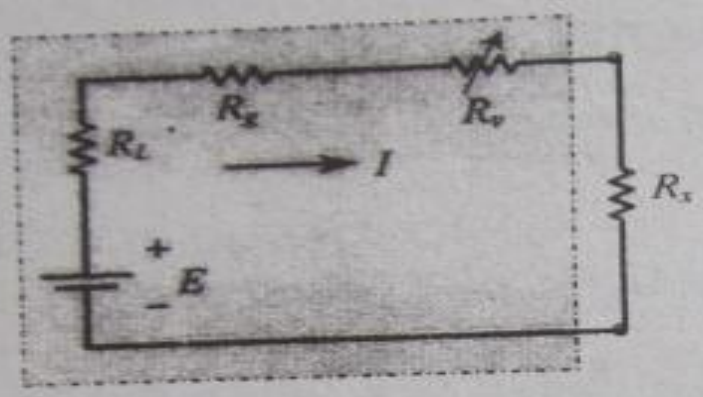
$$\text{Sensitivity} = \frac{1}{I_{fs}}$$

where: E_{fs} = maximum voltage (full-scale) that the meter can measure (volt)
E_g = voltage across the galvanometer coil (volt)

I_g = current flowing in the galvanometer coil (ampere)
R_g = resistance of the galvanometer coil (ohm)
R_{se} = series (potential divider) resistance (ohm)

Note: Meter sensitivity ($1/I_G$) has a unit ohm per volt.

OHMMETER



$$I_{max} = \frac{E}{R_L + R_v + R_g}$$

$$R_L = \frac{E}{I_{max}} - (R_g + R_v)$$

$$I = \frac{E}{R_L + R_v + R_g + R_x}$$

where: R_L = current limiting resistor
R_v = resistance used to bring the pointer to the zero reading when $R_x = 0$
R_x = resistance to be measured
R_g = resistance of the galvanometer coil
E = supply voltage of the meter
I_{max} = galvanometer current when R_x is zero

WATT-HOUR METER

$$W = kN \qquad W = Pt$$

where: W = reading of the watt-hour meter (watt-hour)
P = power drawn (watt)
t = time of usage (hour)
N = number of revolutions made by the disk
k = proportionality constant (meter constant)

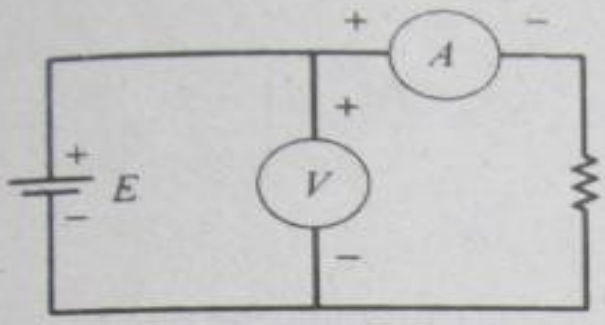
WATTMETER READING USING A "PT" AND A "CT"

$$P_{actual} = W_{reading} \times \text{Multiplier} \qquad \text{Multiplier} = \text{CT ratio} \times \text{PT ratio}$$

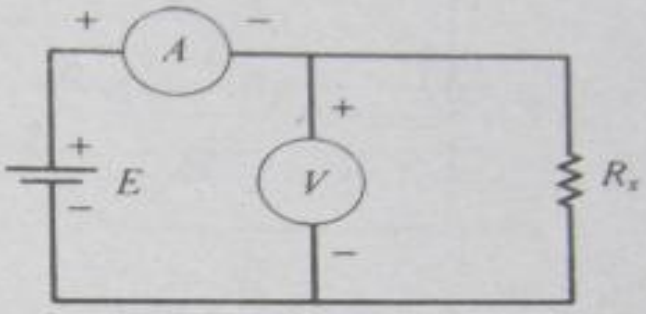
where: P_{actual} = actual power consumption (watt)

$W_{reading}$ = wattmeter reading (watt)
CT ratio = current ratio of the current transformer
PT ratio = voltage ratio of the potential transformer

RESISTANCE MEASUREMENT USING A VOLTMETER AND AN AMMETER



High resistance load

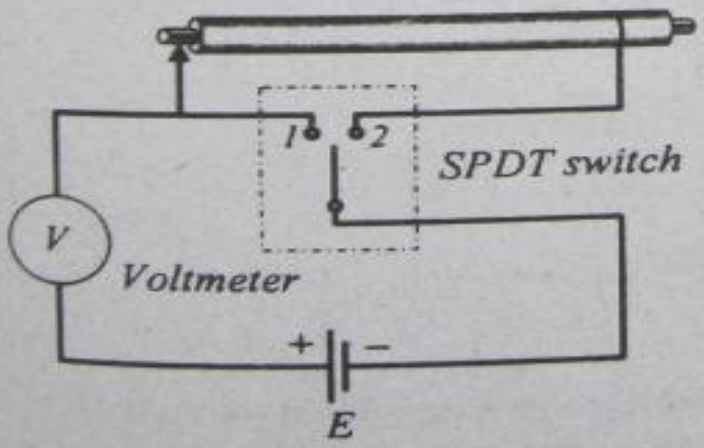


Low resistance load

$$R_x = \frac{V}{I}$$

where: R_x = unknown resistance (ohm)
V = voltmeter reading (volt)
I = ammeter reading (ampere)

INSULATION RESISTANCE OF CABLES

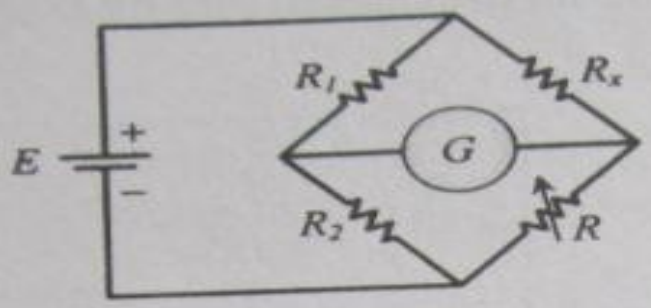


$$R = R_v\left(\frac{E}{E_v} - 1\right)$$

where: R = resistance of the cable (ohm)
R_v = resistance of the voltmeter (ohm)
E = supply voltage (volt)
= voltmeter reading at position #1
E_v = voltmeter reading at position #2 (volt)

WHEATSTONE BRIDGE

It is used to determine the ohmic value of an unknown resistance.



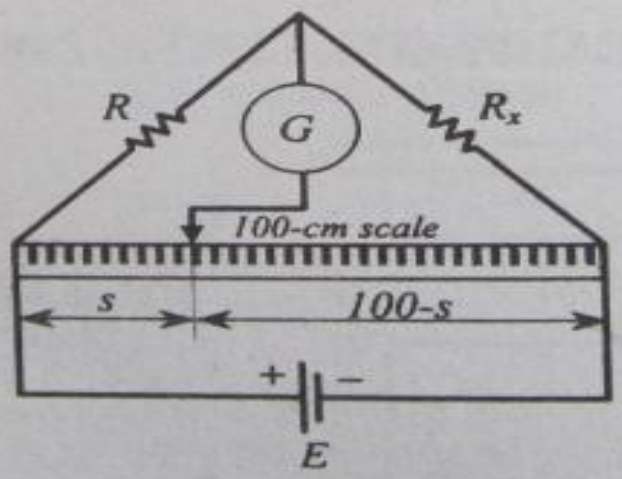
$$R_x = R\left(\frac{R_1}{R_2}\right)$$

Note: Resistor R is adjusted, until the bridge is balanced (galvanometer reading is zero).

where: R_x = unknown resistance (ohm)
R = called the balanced or rheostat arm (ohm)
R_1/R_2 = called the ratio arms

SLIDE-WIRE BRIDGE

Slide wire bridge is a simplified Wheatstone bridge.



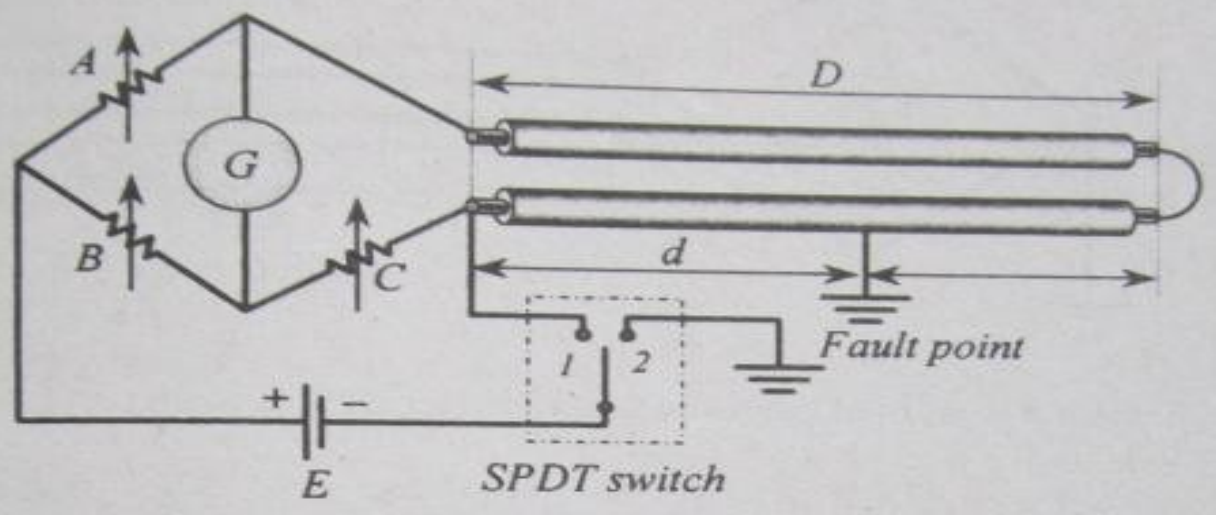
Note: The movable terminal of the galvanometer is transferred within the wire until the bridge is balanced (galvanometer reading is zero).

$$R_x = R\left(\frac{100}{s} - 1\right)$$

where: s = distance in unit length (based on the 100-scale)
R_x = unknown resistance (ohm)

VARLEY LOOP TEST

This is used to locate accidental ground point in cables.



Note: At position #1, resistors A, B and C are adjusted until the galvanometer reading is zero.

$$R = \frac{AC}{B} \qquad r = \frac{R}{2D}$$

where: R = total resistance of the cable (both lengths connected in series)
r = resistance per unit length of the cable
D = length of the transmission line

Note: At position #2, resistors A, B and C are adjusted until the galvanometer reading is zero:

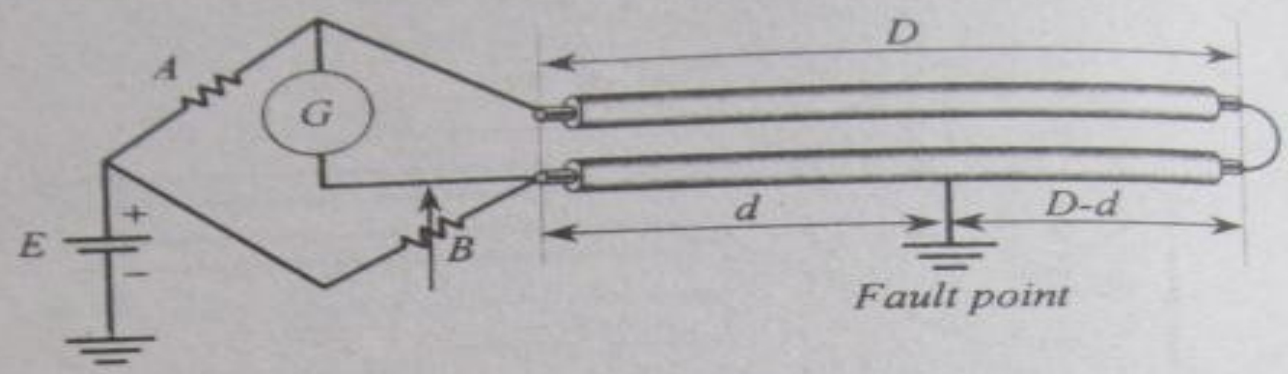
$$\frac{A}{B} = \frac{rD + r(D-d)}{C + rd} \qquad d = \frac{2D}{R}\left(\frac{BR - AC}{A + B}\right)$$

where: d = distance of the fault point

Note: A, B and C might not be equal when the switch is at position #1 and at position #2.

MURRAY LOOP TEST

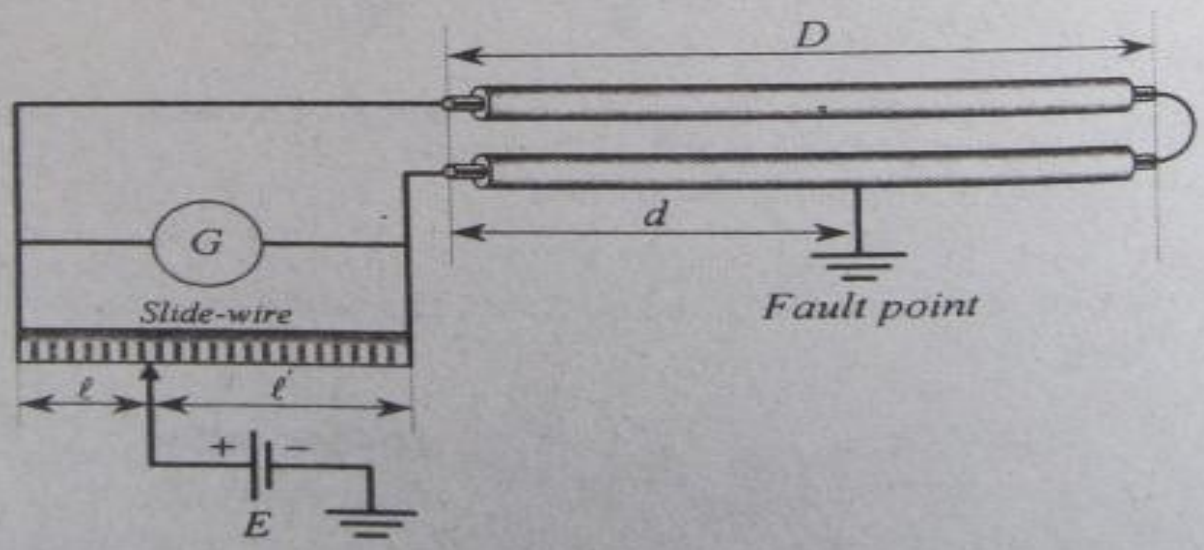
This is used to locate faults with very low resistance loop.



Note: Resistor B is adjusted, until the galvanometer reading is zero.

$$d = 2D\left(\frac{A}{A+B}\right)$$

SLIDE-WIRE MURRAY LOOP TEST



Note: The movable terminal of the battery is transferred within the wire until the bridge is balanced (galvanometer reading is zero).

$$d = 2D\left(\frac{\ell'}{\ell+\ell'}\right)$$

$$E = \frac{PNZ\Phi}{60a} = \frac{PNZ\Phi}{60(mP)} = \frac{NZ\Phi}{60m}$$

$$N = \frac{60Em}{Z\Phi} = \frac{60(114.286)}{728(25 \times 10^{-3})} = 376.767$$

$$N \cong 377\text{rpm}$$

485

$$I_v = \frac{V}{R_v} = \frac{10}{2000} = 0.005\text{ A}$$

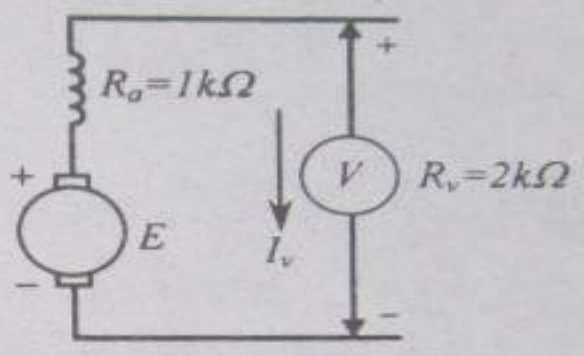
$$E = V + I_v R_a = 10 + 0.005(1000)$$

$$E = 15\text{ V}$$

$$E = \frac{PNZ\Phi}{60a} \times 10^{-8}$$

$$N = \frac{60\,Ea}{PZ\Phi} = \frac{60(15)(2)}{2(1200)(40{,}000) \times 10^{-8}}$$

$$N = 1562.5\text{ rpm}$$



486

$$I_L = \frac{P_L}{V_L} = \frac{5{,}500}{110} = 50\text{ A}$$

$$V_{feeder} = I_L R_{feeder}$$
$$= 50(0.04) = 2\text{ V}$$

$$V_{sh} = V_L + V_{feeder}$$
$$= 110 + 2 = 112\text{ V}$$

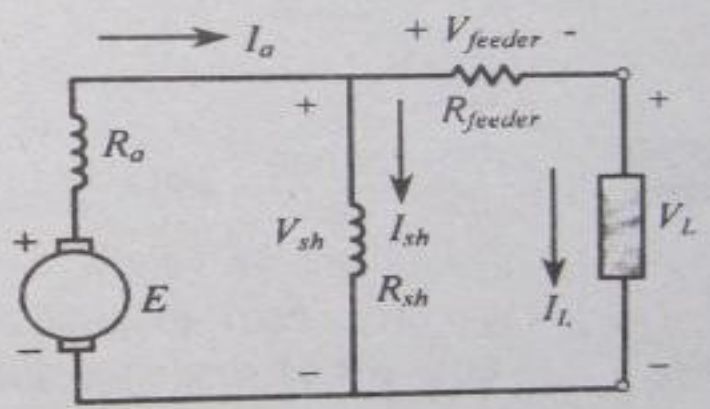
$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{112}{50} = 2.24\text{ A}$$

$$I_a = I_L + I_{sh} = 50 + 2.24$$

$$I_a = 52.24\text{ A}$$

$$E = V_{sh} + I_a R_a = 112 + 52.24(0.15)$$

$$E = 119.84 \cong 120\text{ V}$$



487

$$I_L = \frac{P_L}{V_L} = \frac{50{,}000}{220} = 227.27\text{ A}$$

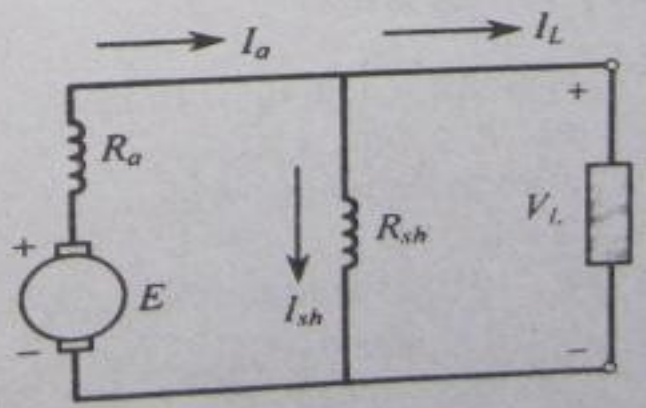
$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{220}{52} = 4.23\text{ A}$$

$$I_a = I_L + I_{sh} = 227.27 + 4.23$$

$$I_a = 231.5\text{ A}$$

$$E = V_L + I_a R_a = 220 + 231.5(0.026)$$

$$E = 226.02\text{ V}$$



488

$$I_L = \frac{P_L}{V_L} = \frac{180{,}000}{250} = 720\text{ A}$$

$$I_a = I_L + I_{sh} = 720 + 6 = 726\text{ A}$$

$$E = V_L + I_aR_a = 250 + 726(0.02) = 264.52\text{ V}$$

$$P_g = EI_a = 264.52(726) = 192.04\text{ kW}$$

$$\eta = \frac{P_L}{P_g} = \frac{180}{192.04} \times 100\%$$

$$\eta = 93.73\%$$

489

$$I_L = \frac{P_L}{V_L} = \frac{200{,}000}{230} = 869.56\text{ A}$$

$$I_a = I_L + I_{sh} = 869.56 + 8 = 877.56\text{ A}$$

$$E = V_L + I_aR_a = 230 + 877.56(0.03) = 256.32\text{ V}$$

$$P_g = EI_a = 256.32(877.56) = 224.936\text{ kW}$$

$$\eta = \frac{P_L}{P_g} = \frac{200}{224.936} \times 100\%$$

$$\eta = 88.91\%$$

490

$$I_L = \frac{P_L}{V_L} = \frac{100{,}000}{250} = 400\text{ A}$$

$$I_a = I_L + I_{sh} = 400 + 5 = 405\text{ A}$$

$$E = V_L + I_aR_a = 250 + 405(0.01) = 254.05\text{ V}$$

$$P_g = EI_a = 254.05(405) = 102.89\text{ kW}$$

$$\eta = \frac{P_L}{P_g} = \frac{100}{102.89} \times 100\% = 97.2\%$$

491 Note: Since the data of the field windings are not given, assume the generator is separately excited.

$$P_L = V_LI_L = 100(40) = 4{,}000\text{ W}$$

$$E = V_L + I_L(R_a + R_b)$$

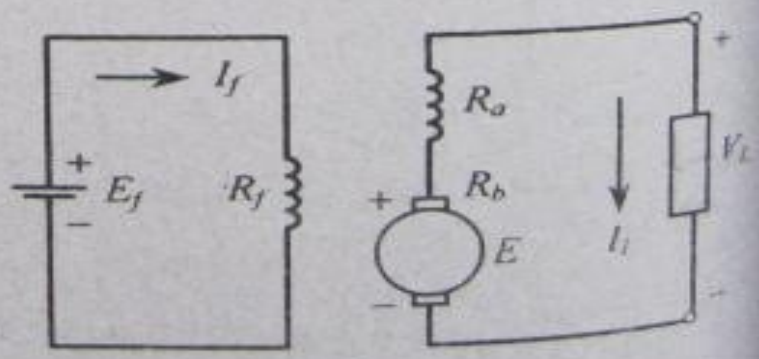
$$= 100 + 40(0.05 + 0.02)$$

$$E = 102.8\text{ V}$$

$$P_g = EI_L = 102.8(40) = 4{,}112\text{ W}$$

$$\eta = \frac{P_L}{P_g} = \frac{4{,}000}{4{,}112} \times 100\%$$

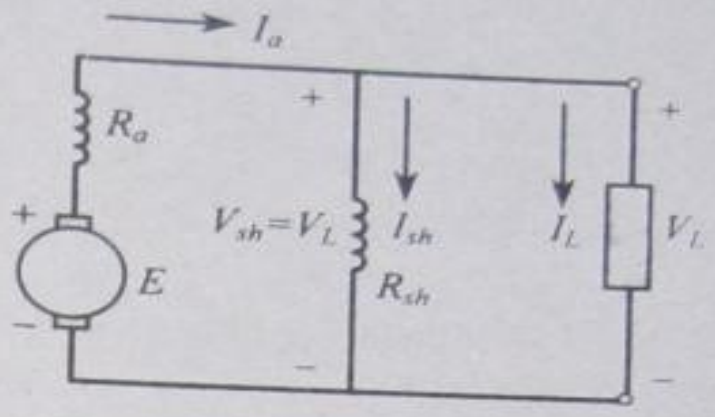
$$\eta = 97.3\%$$



492 $I_L = \dfrac{P_L}{V_L} = \dfrac{28{,}800}{V_L}$

$I_{sh} = \dfrac{V_L}{R_{sh}} = \dfrac{V_L}{48}$

$I_a = I_L + I_{sh} = \dfrac{28{,}800}{V_L} + \dfrac{V_L}{48} \Rightarrow ❶$



$E = V_L + I_a R_a \Rightarrow ❷$

Substitute Eq.1 in Eq.2 :

$E = V_L + \left(\dfrac{28{,}800}{V_L} + \dfrac{V_L}{48}\right)(0.04)$

$E = 1.000833\ V_L + \dfrac{1152}{V_L} \Rightarrow ❸$

$P_g = EI_a \Rightarrow ❹$

Substitute Eq.1 and Eq. 3 in Eq.4 :

$1.063(28800) = \left(1.000833\ V_L + \dfrac{1152}{V_L}\right)\left(\dfrac{28{,}800}{V_L} + \dfrac{V_L}{48}\right)$

$30614.4 = 28824 + 0.02085V_L^{\ 2} + \dfrac{33177600}{V_L^{\ 2}} + 24$

$V_L^{\ 4} - 84719.42V_L + 1591251799 = 0$

Using the quadratic formula :

$V_L^{\ 2} = \dfrac{84719.42 \pm \sqrt{(84719.42)^2 - 4(1)(1591251799)}}{2(1)}$

$V_L^{\ 2} = \dfrac{84719.42 \pm 28502.156}{2}$

$V_L^{(+)} = \sqrt{\dfrac{84719.42 + 28502.156}{2}} = 237.93$

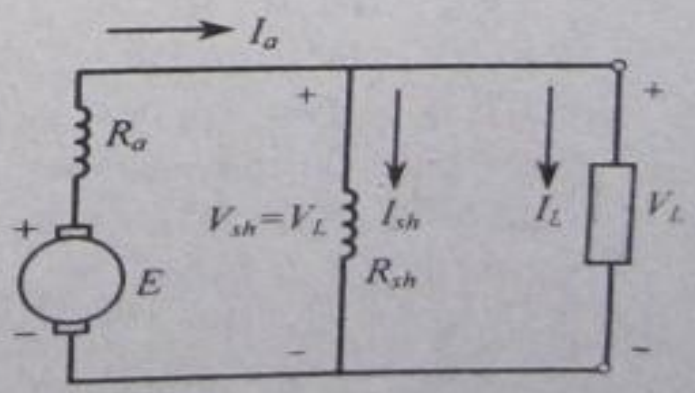
$V_L \cong 238\ V$

493 $P_{stray} = V_L I_a - I_a^{\ 2} R_a$

$P_{stray} = (254)(2) - (2)^2(0.25)$

$= 507$ watts

$I_L = \dfrac{P_L}{V_L} = \dfrac{(0.5)(15{,}000)}{230} = 32.6\ A$



$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{230}{100} = 2.3\text{ A}$$

$$P_{sh} = V_L I_{sh} = 230(2.3) = 529\text{ W}$$

$$I_a = I_L + I_{sh} = 32.6 + 2.3 = 34.9\text{ A}$$

$$P_a = I_a^2 R_a = (34.9)^2(0.25) = 304.5\text{ W}$$

$$\eta = \frac{P_L}{P_L + P_{stray} + P_a + P_{sh}} = \frac{(0.5)(15000) \times 100\%}{(0.5)(15000) + 507 + 304.5 + 529}$$

$$\eta = 84.83\%$$

494 Note: Refer to the circuit diagram of problem#493.

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{200}{100} = 2\text{ A}$$

$$I_a = I_L + I_{sh} = 100 + 2 = 102\text{ A}$$

$$P_a = I_a^2 R_a = (102)^2(0.025) = 260.1\text{ W}$$

$$P_{sh} = V_L I_{sh} = (200)(2) = 400\text{ W}$$

$$P_{copper} = P_a + P_{sh} = 260.1 + 400 = 660.1\text{ W}$$

$$P_L = V_L I_L = 200(100) = 20000\text{ W}$$

$$P_{stray} = P_{copper}$$

$$P_{input} = P_L + P_{copper} + P_{stray} = 20000 + 660.1 + 660.1$$

$$P_{input} = 21320.2\text{ W}$$

$$P_{brake} = P_{input} = 21320.2\text{ W} \times \frac{1\text{ hp.}}{746\text{ W}}$$

$$P_{brake} = 28.58\text{ hp}$$

Note: The input power to the generator is equivalent to the output power (brake horsepower) of the driving engine.

495 $$I_L = \frac{P_L}{V_L} = \frac{20000}{200} = 100\text{ A}$$

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{200}{50} = 4\text{ A}$$

$$I_a = I_L + I_{sh} = 100 + 4 = 104\text{ A}$$

$$P_a = I_a^2 R_a = (104)^2(0.0.05) = 540.8\text{ W}$$

$$P_{sh} = V_L I_{sh} = (200)(4) = 800\text{ W}$$

$$P_{copper} = P_a + P_{sh} = 540.8 + 800 = 1340.8\text{ W}$$

$$P_{input} = 30\text{ hp} \times \frac{746\text{ watts}}{1\text{ hp}} = 22{,}380\text{ W}$$

$P_{stray} = P_{input} - (P_L + P_{copper}) = 22380 - (20{,}000 + 1340.8)$

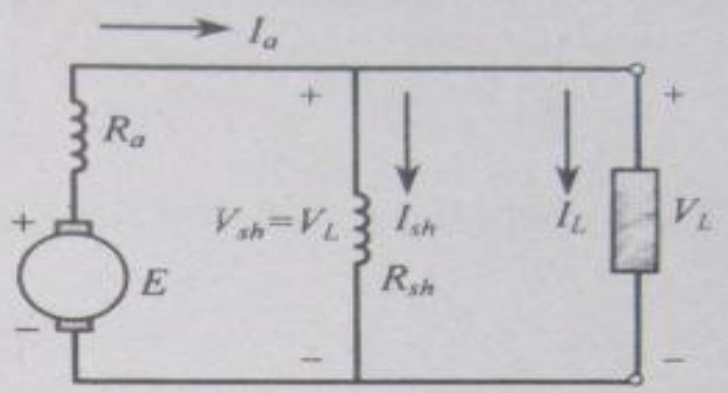
$P_{input} = 1039.2\ W$

496 $P_{input} = 62.25\ hp \times \dfrac{746\ W}{hp}$

$= 46438.5\ W$

$I_{sh} = \dfrac{V_L}{R_{sh}} = \dfrac{220}{44} = 5\ A$

$I_a = I_L + I_{sh} = I_L + 5$



$P_{input} = P_L + P_a + P_{sh} + P_{stray}$

$P_{input} = V_L I_L + I_a^2 R_a + V_L I_{sh} + P_{stray}$

$46438.5 = 220 I_L + (I_L + 5)^2 (0.02) + 220(5) + 1600$

$43738.5 = 220 I_L + 0.02 I_L^2 + 0.2 I_L + 0.5$

$I_L^2 + 11010 I_L - 2186900 = 0$

Using the quadratic formula :

$$I_L = \frac{-11010 \pm \sqrt{(11010)^2 - 4(1)(2186900)}}{2(1)} = \frac{-11010 \pm 11400}{2}$$

$I_L^{(+)} = 195\ A$

497 Note: Since the data of the field circuit is not available, assume the generator to be separately excited.

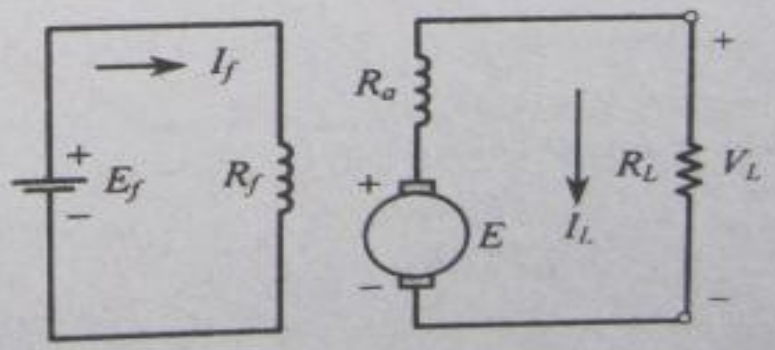
$R_L = \dfrac{V_L^2}{P} = \dfrac{(115)^2}{2000} = 6.6\ \Omega$

$I_L = \dfrac{E}{R_a + R_L} = \dfrac{120}{0.95 + 6.6}$

$I_L = 15.894\ A$

$P_L = I_L^2 R_L = (15.894)^2 (6.6)$

$P_L = 1667.286\ W$



498 $E = \dfrac{PNZ\Phi}{60a} = \dfrac{PNZ\Phi}{60mP} = \dfrac{6(1000)(534)(30 \times 10^{-3})}{60(1)(6)} = 267\ V$

$E = V_L + V_a + V_b$

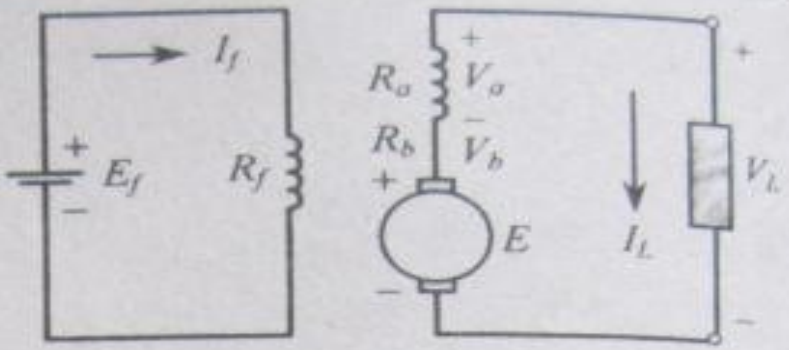
$V_a = E - (V_L + V_b) = 267 - (250 + 2) = 15\ V$

$P_a = V_a I_a$

$I_a = \frac{P_a}{V_a} = \frac{640}{15} = 42.667\ A$

$P_L = V_L I_L = 250(42.667)$

$P_L = 10.67\ kW$



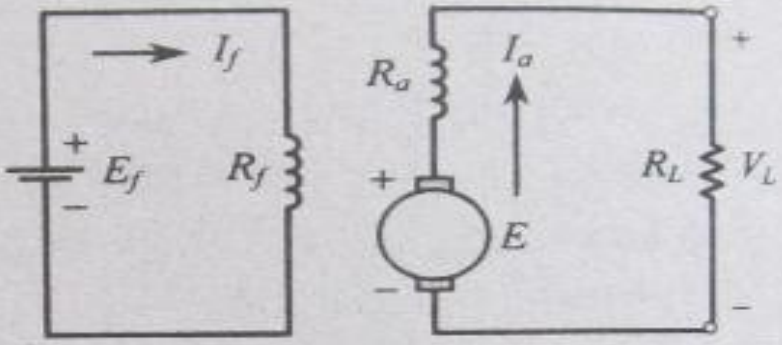
499 $E_1 = V_{L1} + I_{a1}R_a = 125 + 200(0.1) = 145\ V$

$E = kN$

$\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$E_2 = \frac{E_1 N_2}{N_1} = \frac{145(1000)}{1200}$

$E_2 = 120.83\ V$



Solving for load resistance :

$R_L = \frac{V_L}{I_{a1}} = \frac{125}{200} = 0.625\ \Omega$

$I_{a2} = \frac{E_2}{R_a + R_L} = \frac{120.83}{0.1 + 0.625} = 166.66\ A$

$V_{L2} = I_{a2}R_L = 166.66(0.625)$

$V_{L2} = 104.16\ V$

500 Let : R_{equiv} = equivalentresistance of the series field branch

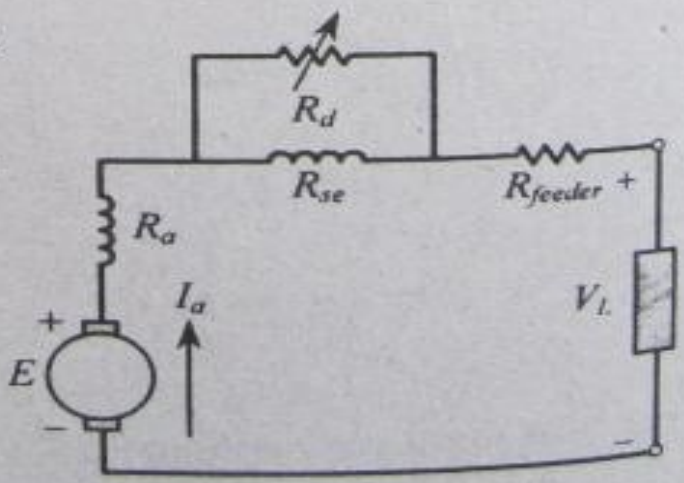
$R_{equiv} = \frac{R_{se}R_d}{R_{se} + R_d} = \frac{15(30)}{15 + 30} = 10\ \Omega$

$E = V_L + I_a(R_a + R_{equiv} + R_{feeder})$

$E = 2500 + 5(18 + 10 + 1) = 2645\ V$

$P_g = EI_a = 2645(5)$

$P_g = 132.25\ kW$



501 $\eta = \frac{P_{out}}{P_{out} + P_{loss}}$

$P_{lossFL} = \frac{P_{outFL}}{\eta_{FL}} - P_{outFL}$

$P_{lossFL} = \frac{30{,}000}{0.88} - 30{,}000 = 4090.91\ W$

$P_{strayFL} = 0.15\,P_{lossFL} = 0.15\,(4090.91) = 613.636\text{ W}$

$P_{cuFL} = 0.85\,P_{lossFL} = 0.85\,(4090.91) = 3477.273\text{ W}$

Note: For a series generator, at maximum efficiency, the copper loss is equivalent to the stray power loss.

$P_{cum} = 613.636\text{ W}$

$P_{cum} = x^2 P_{cuFL}$

$$x = \sqrt{\frac{P_{cum}}{P_{cuFL}}} = \sqrt{\frac{613.636}{3477.273}} = 0.42$$

$P_{outm} = xP_{outFL} = 0.42(30{,}000) = 12600\text{ W}$

$$\eta_m = \frac{P_{outm}}{P_{outm} + 2P_{cum}} = \frac{12600}{12600 + 2(613.636)} = 0.9112$$

$\eta_m = 91.12\%$

502 $P_h = k_h N$

$$\frac{P_{h2}}{P_{h1}} = \frac{N_2}{N_1}$$

$$P_{h2} = P_{h1}\left(\frac{N_2}{N_1}\right) = \frac{250N_2}{1000} = 0.25N_2 \Rightarrow ❶$$

$P_e = k_e N^2$

$$\frac{P_{e2}}{P_{e1}} = \left(\frac{N_2}{N_1}\right)^2$$

$$P_{e2} = P_{e1}\left(\frac{N_2}{N_1}\right)^2 = 100\left(\frac{N_2}{1000}\right)^2 = 0.0001N_2^{\,2} \Rightarrow ❷$$

$$P_{h2} + P_{e2} = \frac{1}{2}\left(P_{h1} + P_{e1}\right) \Rightarrow ❸$$

Substitute Eq.1 and Eq.2 in Eq.3 :

$0.25N_2 + 0.0001N_2^{\,2} = 0.5(250 + 100)$

$N_2^{\,2} + 2500\,N_2 - 1750000 = 0$

Using the quadratic formula :

$$N_2 = \frac{-2500 \pm \sqrt{(2500)^2 - 4(1)(-1750000)}}{2(1)} = \frac{-2500 \pm 3640}{2}$$

$N_2^{(+)} = 570\text{ rpm}$

503 $P_e = k(N\beta_m)^2$

$$\frac{P_{e2}}{P_{e1}} = \left(\frac{N_2\beta_{m2}}{N_1\beta_{m1}}\right)^2$$

$$P_{e2} = P_{e1}\left(\frac{N_2\beta_{m2}}{N_1\beta_{m1}}\right)^2 = 600\left(\frac{1200(2{,}500{,}000)}{1000(2{,}000{,}000)}\right)^2$$

$P_{e2} = 1350$ W

504 $P_{core} = P_e + P_h$

$P_{core} = k_e N^2 + k_h N$

At 1000 rpm:

$10 = k_e(1000)^2 + k_h(1000)$

$k_h = 0.01 - 1000k_e \Rightarrow$ ❶

At 750 rpm:

$6 = k_e(750)^2 + k_h(750)$

$k_h = 0.008 - 750k_e \Rightarrow$ ❷

Equate Eq.1 to Eq.2:

$0.01 - 1000k_e = 0.008 - 750k_e$

$k_e = 8 \times 10^{-6}$

$k_h = 0.01 - 1000(8 \times 10^{-6}) = 2 \times 10^{-3}$

At 500 rpm:

$P_{core} = k_e N^2 + k_h N$

$= 8 \times 10^{-6}(500)^2 + 2 \times 10^{-3}(500)$

$P_{core} = 3$ kW

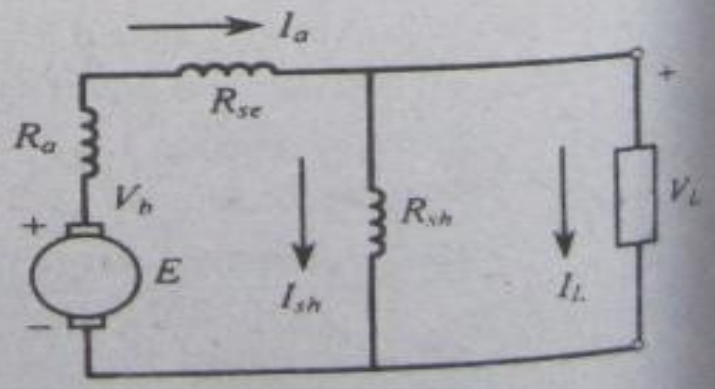
505 $I_L = \frac{P_L}{V_L} = \frac{20{,}000}{400} = 50$ A

$V_{sh} = V_L + I_L R_f = 400 + 50(0.1)$

$V_{sh} = 405$ V

$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{405}{85} = 4.764$ A

$I_a = I_L + I_{sh} = 50 + 4.764 = 54.764$ A



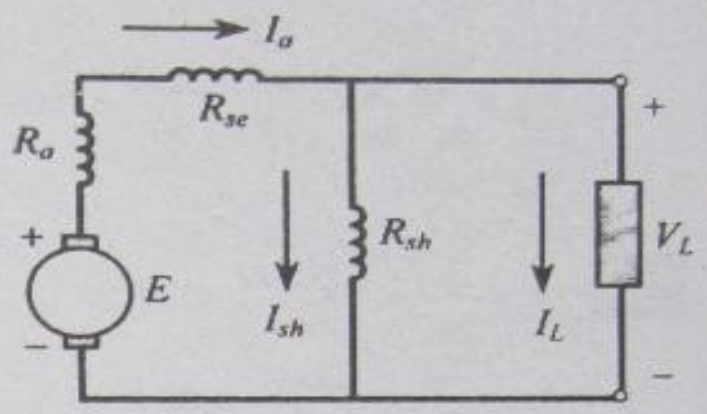
$$E = V_{sh} + I_a(R_a + R_{se}) + V_b$$
$$= 405 + 54.764(0.1 + 0.06) + 2$$
$$E = 415.76\ V$$

506 $I_L = \dfrac{P_L}{V_L} = \dfrac{10000}{V_L}$

$I_{sh} = \dfrac{V_L}{R_{sh}} = \dfrac{V_L}{120}$

$I_a = I_L + I_{sh} = \dfrac{10000}{V_L} + \dfrac{V_L}{120} \Rightarrow$ ❶

$E = V_L + I_a(R_a + R_{se}) \Rightarrow$ ❷



Substitute Eq.1 in Eq.2 :

$$264 = V_L + \left(\frac{10000}{V_L} + \frac{V_L}{120}\right)(0.1 + 0.05)$$

$$\left(264 = V_L + \frac{1500}{V_L} + 0.00125V_L\right)V_L$$

$$1.00125V_L^2 + 1500 - 264V_L = 0$$

$$V_L^2 - 263.67V_L + 1498.127 = 0$$

Using the quadratic formula :

$$V_L = \frac{263.67 \pm \sqrt{(263.67)^2 - 4(1)(1498.127)}}{2(1)}$$

$$V_L = \frac{263.67 \pm 252.05}{2}$$

$$V_L^{(+)} = 257.86 \text{ volts}$$

507 Let : Φ_{add} = additionalnumber of ampere - turns required to make the generator flat compounded.

$$\Phi_{add} = N_{sh}I_{sh} = (1000)(0.2) = 200\ AT$$

Note: This additional flux shall be supplied by the series field windings when the generator is connected as a compound machine.

$$\Phi_{series} = N_{se}I_{se}$$

$$I_{se} = \frac{\Phi_{series}}{N_{se}} = \frac{200}{4} = 50\ A$$

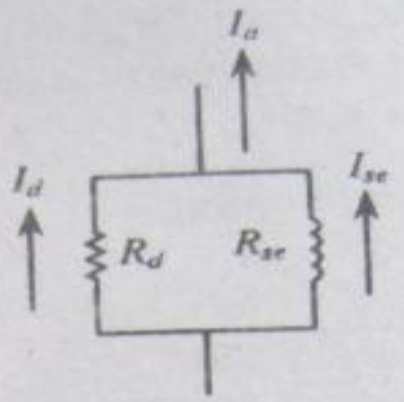
By CDT :

$$I_{se} = \frac{I_a R_d}{R_d + R_{se}}$$

$$R_d(I_a - I_{se}) = I_{se} R_{se}$$

$$R_d = \frac{I_{se} R_{se}}{I_a - I_{se}} = \frac{50(0.025)}{95 - 50} = 0.0277$$

$$R_d = 0.028\ \Omega$$



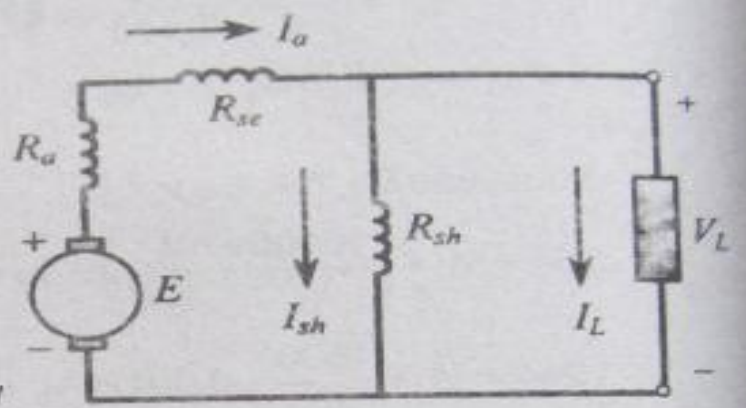
508

$$I_L = \frac{P_L}{V_L} = \frac{125{,}000}{250} = 500\ \text{A}$$

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{250}{30} = 8.33\ \text{A}$$

$$I_a = I_L + I_{sh} = 500 + 8.33$$

$$I_a = 508.33\ \text{A}$$



$$P_{sh} = V_L I_{sh} = 250(8.33) = 2082.5\ \text{W}$$

$$P_a = I_a^2 R_a = (508.33)^2(0.025) = 6459.98\ \text{W}$$

$$P_{se} = I_a^2 R_{se} = (508.33)^2(0.01) = 2583.99\ \text{W}$$

$$P_{in} = P_{FL} + P_a + P_{sh} + P_{se} + P_{stray} + P_{stray\ load}$$

$$= 125{,}000 + 6459.98 + 2082.5 + 2583.99 + 5000 + 0.01(125{,}000)$$

$$P_{in} = 142{,}376.47\ \text{W}$$

$$P_{brake} = 142376.47\ \text{W} \times \frac{1\ \text{hp}}{746\ \text{W}} = 190.85\ \text{hp}$$

Note: The input of the generator is equivalent to the brake horsepower rating of the motor.

509

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{250}{100} = 2.5\ \text{A}$$

$$I_a = I_L + I_{sh} = 100 + 2.5 = 102.5\ \text{A}$$

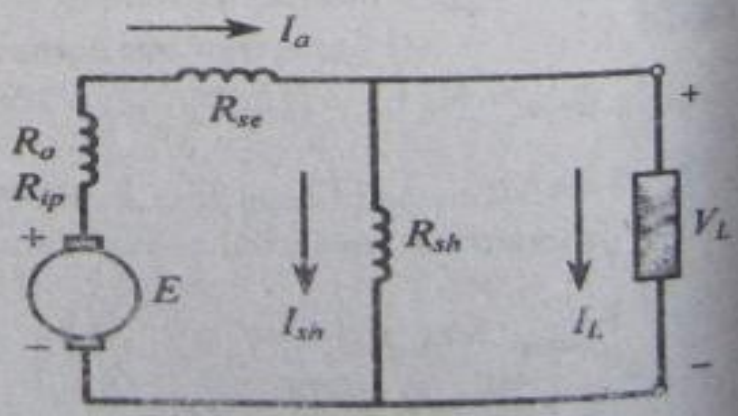
$$P_L = V_L I_L = 250(100) = 25\ \text{kW}$$

$$P_a = I_a^2 R_a = (102.5)^2(0.1)$$

$$P_a = 1.05\ \text{kW}$$

$$P_{ip} = I_a^2 R_{ip} = (102.5)^2(0.02)$$

$$P_{ip} = 0.21\ \text{kW}$$



$P_{sh} = V_L I_{sh} = 250(2.5) = 0.625\ kW$

$P_{se} = I_a^2 R_{se} = (102.5)^2 (0.025) = 0.263\ kW$

$$\eta = \frac{P_L}{P_L + P_a + P_{ip} + P_{sh} + P_{se} + P_{stray}}$$

$$\eta = \frac{25 \times 100\%}{25 + 1.05 + 0.21 + 0.625 + 0.263 + 1.8}$$

$\eta = 86.36\%$

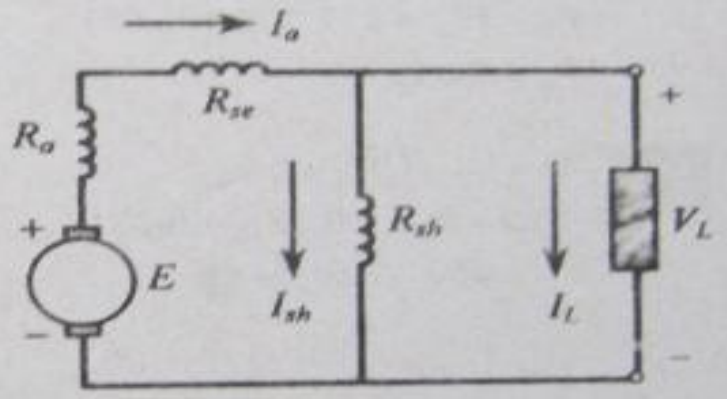
Note: Interpole windings (R_{ip}) are always connected in series with the armature windings.

510 As a shunt motor at no - load :

$P_{stray} = V_L I_a - I_a^2 R_a$

$P_{stray} = (135)(2.5) - (2.5)^2 (0.2) = 336.25\ W$

As a long shunt compound motor :



$$I_L = \frac{P_L}{V_L} = \frac{5{,}500}{125} = 44\ A$$

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{125}{50} = 2.5\ A$$

$I_a = I_L + I_{sh} = 44 + 2.5 = 46.5\ A$

$P_{sh} = V_L I_{sh} = 125(2.5) = 312.5\ W$

$P_a = I_a^2 R_a = (46.5)^2 (0.2) = 432.45\ W$

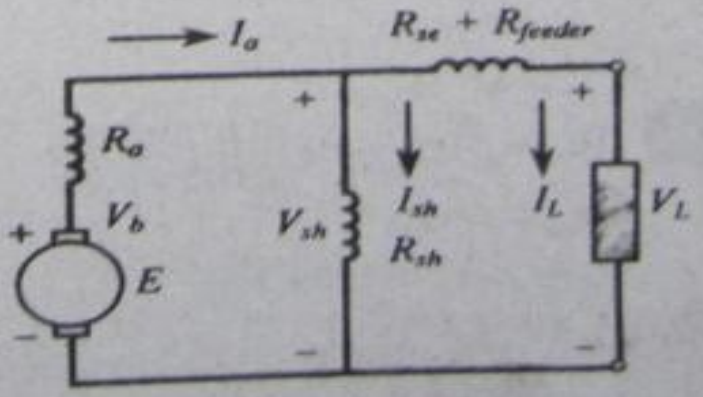
$P_{se} = I_a^2 R_{se} = (46.5)^2 (0.25) = 540.56\ W$

$$\eta = \frac{P_L}{P_L + P_a + P_{sh} + P_{se} + P_{stray}}$$

$$\eta = \frac{5500 \times 100\%}{5500 + 432.45 + 312.5 + 336.25}$$

$\eta = 83.57\%$

511



$$I_L = \frac{P_L}{V_L} = \frac{20{,}000}{400} = 50\ A$$

$V_{sh} = V_L + I_L (R_{se} + R_f)$

$= 400 + 50(0.06 + 0.1)$

$V_{sh} = 408\ V$

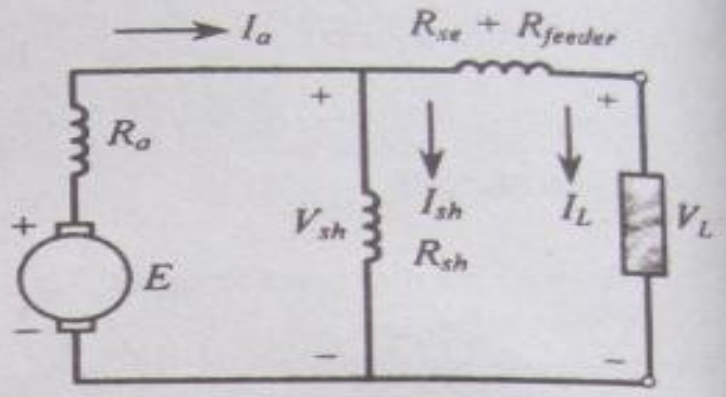
$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{408}{85} = 4.8\ A$$

$I_a = I_L + I_{sh} = 50 + 4.8 = 54.8\ A$

$E = V_{sh} + I_a R_a + V_b = 408 + 54.8(0.1) + 2$

$E = 415.48\ V$

512



$I_L = \dfrac{P_L}{V_L} = \dfrac{20{,}000}{200} = 100\ A$

$V_{sh} = V_L + I_L(R_{se} + R_f)$

$= 200 + 100(0.03 + 0.1)$

$V_{sh} = 213$ volts

$I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{213}{80} = 2.66\ A$

$I_a = I_L + I_{sh} = 100 + 2.66$

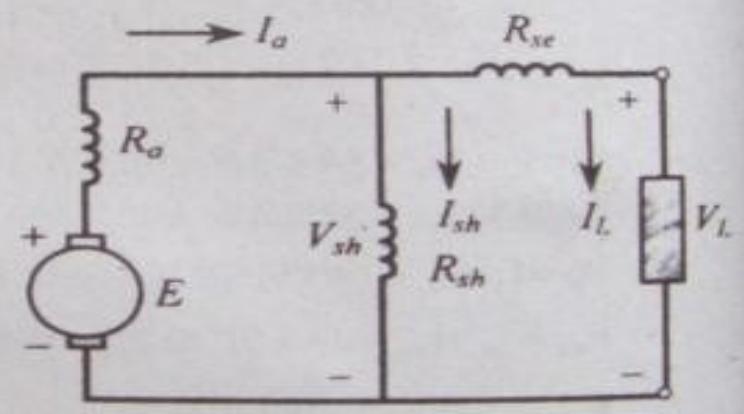
$I_a = 102.66\ A$

$E = V_{sh} + I_a R_a = 213 + 102.66(0.04) = 217.106$ volts

$P_g = EI_a = 217.106(102.66)$

$P_g = 22.29\ kW$

513



$E = V_L + I_a R_a + I_L R_{se}$

$510 = 500 + 0.02 I_a + 0.03 I_L$

$I_a = 500 - 1.5 I_L \Rightarrow$ ❶

$I_a = I_L + I_{sh}$

$I_a = I_L + \dfrac{V_L + I_L R_{se}}{R_{sh}}$

$I_a = I_L + \dfrac{500 + 0.03 I_L}{80}$

$I_a = 6.25 + 1.000375 I_L \Rightarrow$ ❷

Equate Eq.1 to Eq.2 :

$500 - 1.5 I_L = 6.25 + 1.000375 I_L$

$I_L = 197.47\ A$

514

$R_{eq} = \dfrac{R_{se} R_d}{R_{se} + R_d} = \dfrac{(0.015)(0.03)}{0.015 + 0.03}$

$R_{eq} = 0.01\Omega$

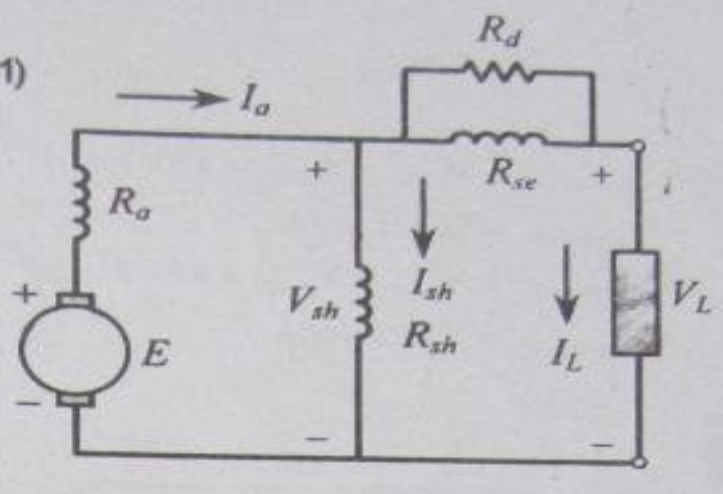
$I_a = I_L + I_{sh} = 150 + 2.5$

$I_a = 152.5\ A$

$E = V_L + I_a R_a + I_L R_{eq}$

$= 230 + (152.5)(0.032) + (150)(0.01)$

$E = 236.38\ V$



$$\eta = \frac{P_L}{P_g} = \frac{V_L I_L}{E I_a}$$

$$= \frac{(230)(150)}{(236.38)(152.5)} \times 100\%$$

$\eta = 95.7\%$

Note: A diverter resistor is always connected across or in parallel with the series field winding, the purpose of which is to control the current flow in the series field winding.

515 At 110 V operating voltage :

$$I_1 = \frac{100{,}000}{110} = 909.1\ A$$

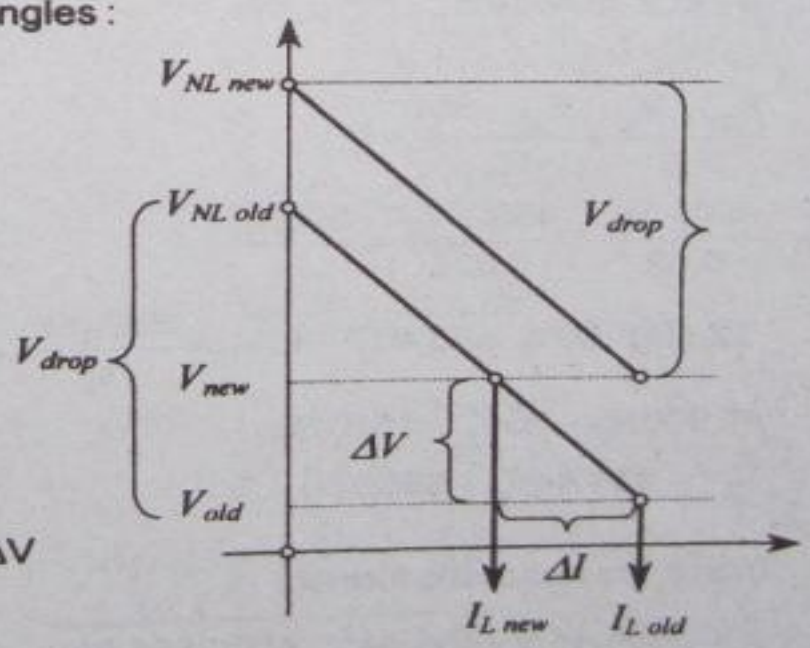
$$I_2 = \frac{150{,}000}{110} = 1363.64\ A$$

$I_t = I_1 + I_2 = 909.1 + 1363.64 = 2272.74\ A$

Using their characteristic triangles :

$$\frac{\Delta V}{\Delta I} = \frac{V_{drop}}{I_L}$$

$$\Delta I = \frac{I_L \Delta V}{V_{drop}}$$



For generator G1 :

$$\Delta I_1 = \frac{909.1\,\Delta V}{10} = 90.91\,\Delta V$$

For generator G2 :

$$\Delta I_2 = \frac{1363.64\,\Delta V}{20} = 68.182\,\Delta V$$

$\Delta I_1 + \Delta I_2 = \Delta I$

$90.91\,\Delta V + 68.182\,\Delta V = 2272.74 - 2000$

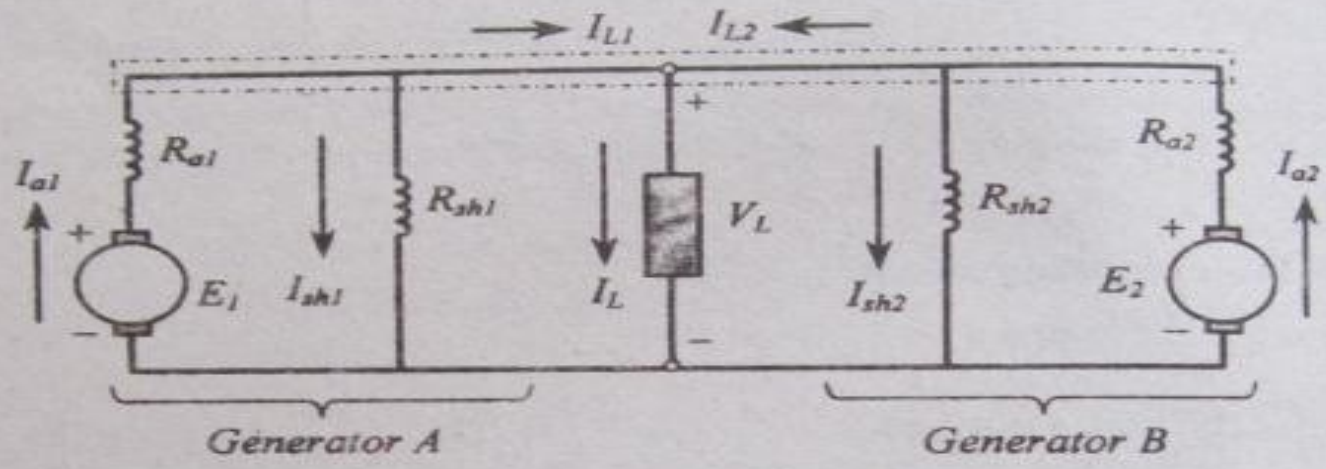
$\Delta V = 1.714\ V$

Note: Since the load decreases, the new operating voltage will increase.

$V_{new} = V_{old} + \Delta V = 110 + 1.7$
$V_{new} = 111.7$ volts

$V_{no\ load\ G1} = V_{new} + V_{drop} = 111.7 + 10$
$V_{no\ load\ G1} = 121.7$ volts
$V_{no\ load\ G2} = V_{new} + V_{drop} = 111.7 + 20$
$V_{no\ load\ G2} = 131.7$ volts

516



Note: As given the current in the field windings are negligible.

By KCL at node a :

$I_{a1} + I_{a2} = I_L$

$$\frac{E_{g1} - V_L}{R_{a1}} + \frac{E_{g2} - V_L}{R_{a2}} = I_L$$

$$\frac{450 - V_L}{0.02} + \frac{460 - V_L}{0.025} = \frac{320{,}000}{V_L}$$

$$22{,}500 - 50V_L + 18{,}400 - 40V_L = \frac{320{,}000}{V_L}$$

$$40{,}900\,V_L - 90V_L^2 = 320{,}000$$

$$V_L^2 - 454.44V_L + 3555.55 = 0$$

Using the quadratic formula :

$$V_L = \frac{454.44 \pm \sqrt{454.44^2 - 4(1)(3555.55)}}{2(1)} = \frac{454.44 \pm 438.51}{2}$$

$$V_L^{(+)} = \frac{454.44 + 438.51}{2} = 446.475$$

$V_L \cong 446.5$ V

517 Note: Refer to the circuit diagram of problem #517.

By KCL at node a :

$I_{a1} + I_{a2} = I_L + I_{sh1} + I_{sh2}$

$$\frac{E_{g1} - V_L}{R_{a1}} + \frac{E_{g2} - V_L}{R_{a2}} = I_L + \frac{V_L}{R_{sh1}} + \frac{V_L}{R_{sh2}}$$

$$\frac{230 - V_L}{0.01} + \frac{235 - V_L}{0.01} = 2500 + \frac{V_L}{50} + \frac{V_L}{50}$$

$-200\,V_L + 46{,}500 = 2500 + 0.04\,V_L$

$V_L = 219.95$

$V_L \cong 220\ V$

518 Note: The concept used in problems # 516 & 517 are still applicable when there are three or more generators in parallel.

By KCL at node a :

$I_{a1} + I_{a2} + I_{a3} = I_L + I_{sh1} + I_{sh2} + I_{sh3}$

$$\frac{E_{g1} - V_L}{R_{a1}} + \frac{E_{g2} - V_L}{R_{a2}} + \frac{E_{g3} - V_L}{R_{a3}} = I_L + \frac{E_{g1} - V_L}{R_{a1}} + \frac{V_L}{R_{sh2}} + \frac{V_L}{R_{sh3}}$$

$$\frac{240 - V_L}{0.02} + \frac{242 - V_L}{0.02} + \frac{245 - V_L}{0.02} = 2000 + \frac{V_L}{40} + \frac{V_L}{40} + \frac{V_L}{40}$$

$-150\,V_L + 36350 = 2000 + 0.075\,V_L$

$V_L = 228.88\ V$

$$I_{L_1} = I_{a1} - I_{sh1} = \frac{E_{g1} - V_L}{R_{a1}} - \frac{V_L}{R_{sh1}} = \frac{240 - 228.88}{0.02} - \frac{228.88}{40} = 550\ A$$

$$I_{L_2} = I_{a2} - I_{sh2} = \frac{E_{g2} - V_L}{R_{a2}} - \frac{V_L}{R_{sh2}} = \frac{242 - 228.88}{0.02} - \frac{228.88}{40} = 650\ A$$

$$I_{L_3} = I_{a3} - I_{sh3} = \frac{E_{g3} - V_L}{R_{a3}} - \frac{V_L}{R_{sh3}} = \frac{245 - 228.88}{0.02} - \frac{228.88}{40} = 800\ A$$

519 Note: Refer to the circuit diagram of problem #516.

By KCL at node a :

$I_{a1} + I_{a2} = I_L + I_{sh1} + I_{sh2}$

$$\frac{E_{g1} - V_L}{R_{a1}} + \frac{E_{g2} - V_L}{R_{a2}} = I_L + \frac{V_L}{R_{sh1}} + \frac{V_L}{R_{sh2}}$$

$$\frac{450 - V_L}{0.01} + \frac{460 - V_L}{0.01} = 4000 + \frac{V_L}{60} + \frac{V_L}{60}$$

$$-200\,V_L + 91000 = 4000 + 0.033\,V_L$$

$$V_L = 434.93\text{ V}$$

$$I_{L_1} = I_{a1} - I_{sh1} = \frac{E_{g1} - V_L}{R_{a1}} - \frac{V_L}{R_{sh1}} = \frac{450 - 434.93}{0.01} - \frac{434.93}{60}$$

$$I_{L_1} = 1499.75 \cong 1500\text{ A}$$

$$I_{L_2} = I_{a2} - I_{sh2} = \frac{E_{g2} - V_L}{R_{a2}} - \frac{V_L}{R_{sh2}} = \frac{460 - 434.93}{0.01} - \frac{434.93}{60}$$

$$I_{L_2} = 2499.75 \cong 2500\text{ A}$$

$$P_A = V_L I_{L_1} = (434.93)(1500) = 652.4\text{ kW}$$

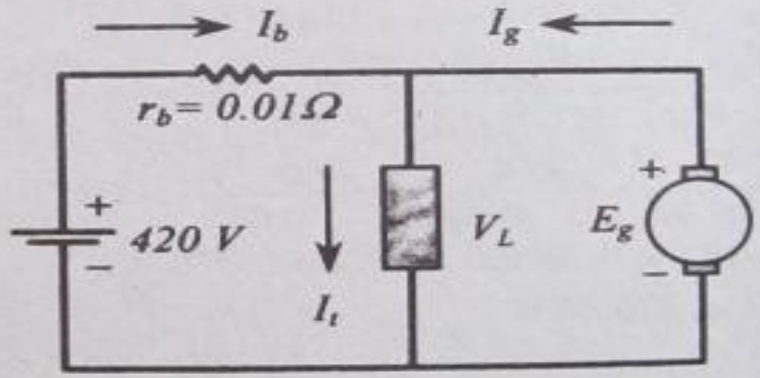
$$P_B = V_L I_{L2} = (434.93)(2500) = 1087.3\text{ kW}$$

520 By KVL:

$$420 - I_b r_b - V_L = 0$$

$$420 - 0.01 I_b - V_L = 0$$

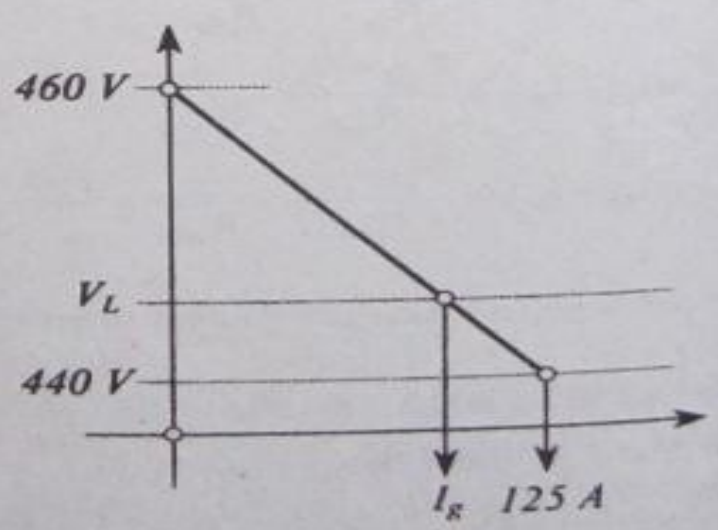
$$I_b = \frac{420 - V_L}{0.01}$$



By ratio and proportion:

$$\frac{460 - 440}{125} = \frac{460 - V_L}{I_g}$$

$$I_g = \frac{460 - V_L}{0.16}$$



$$I_t = I_b + I_g$$

$$500 = \frac{420 - V_L}{0.01} + \frac{460 - V_L}{0.16}$$

$$V_L = 417.65\text{ volts}$$

$$I_b = \frac{420 - V_L}{0.01}$$

$$I_b = \frac{420 - 417.65}{0.01} = 235\text{ A}$$

$$\% \text{ Share} = \frac{I_b}{I_t} = \frac{235}{500} \times 100\%$$

$$\% \text{ Share} = 47\%$$

521

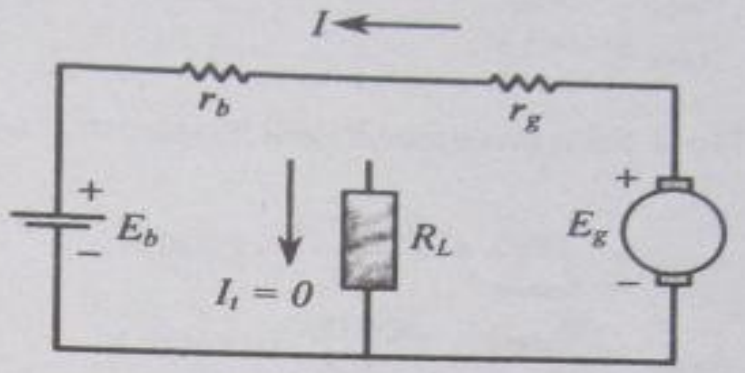
$$\%\text{Reg} = \frac{V_{drop}}{V_{FL}}$$

$$V_{drop} = \%\text{Reg}(V_{FL})$$

$$= (0.10)(220) = 22 \text{ V}$$

$$E_g = 220 + 20 = 240 \text{ V}$$

$$E_b = (115)(2) = 230 \text{ V}$$



$$r_b = (115)(0.01) = 1.15\ \Omega$$

$$I_g = \frac{P_L}{V_{FL}} = \frac{50{,}000}{220} = 227.27 \text{ A}$$

$$r_g = \frac{V_{drop}}{I_{FL}} = \frac{22}{227.27} = 0.0968\ \Omega$$

By KVL :

$$E_g - I(r_g + r_b) - E_b = 0$$

$$I = \frac{E_g - E_b}{r_g + r_b} = \frac{240 - 230}{0.0968 + 1.15}$$

$$I = 8.02 \text{ A}$$

522

$$I_{FL_1} = I_{FL_2} = \frac{P_{FL}}{V_{FL}} = \frac{600{,}000}{230} = 2608.69 \text{ A}$$

$$I_{load} = \frac{P_{load}}{V_L} = \frac{800{,}000}{250} = 3200 \text{ A}$$

$$I_{L_1} = I_{L_2} = \frac{3200}{2} = 1600 \text{ A}$$

$$\frac{\Delta V}{\Delta I} = \frac{\%\text{VR}(V_L)}{I_{FL}}$$

$$\Delta I_1 = \frac{(\Delta V)(I_{FL_1})}{\%\text{VR}(V_{FL})} = \frac{2608.69\Delta V}{(0.05)(230)}$$

$$\Delta I_1 = 226.84\ \Delta V$$

Note: Assume generator #2 has tripped off from the line. The change in the busload therefore is due to the load given off by this tripped machine.

$\Delta I_1 = \Delta I_{bus}$

$226.84\ \Delta V = 1600$

$\Delta V = 7.05$ volts

$V_{L_{new}} = V_{L_{old}} - \Delta V = 250 - 7.05 = 242.95$

$V_{L_{new}} \cong 243\ V$

Note: Since generator #1 will increase its load, its terminal voltage will drop.

523

$$I_{rated1} = \frac{P_{rated1}}{V_{rated1}} = \frac{500{,}000}{250} = 2000\ A$$

$$I_{rated2} = \frac{P_{rated2}}{V_{rated2}} = \frac{400{,}000}{250} = 1600\ A$$

$$\frac{\Delta V}{\Delta I_1} = \frac{\%VR_1(V_{rated1})}{I_{rated1}} = \frac{0.02(250)}{2000}$$

$\Delta I_1 = 400\ \Delta V \Rightarrow$ ❶

$$\frac{\Delta V}{\Delta I_2} = \frac{\%VR_2(V_{rated2})}{I_{rated2}} = \frac{0.04(250)}{1600}$$

$\Delta I_2 = 160\ \Delta V \Rightarrow$ ❷

$\Delta I_1 + \Delta I_2 = \Delta I \Rightarrow$ ❸

Substitute Eq.1 and Eq.2 in Eq.3 :

$400\Delta V + 160\Delta V = 3000 - 1500$

$\Delta V = 2.678$

$V_{L_{new}} = V_{L_{old}} + \Delta V = 250 + 2.678$

$= 252.678$

$V_{L_{new}} \cong 253\ V$

524 Note: Since the busload decreases, terminal voltage increases.

$$I_{rated_1} = \frac{P_{rated_1}}{V_{rated_1}} = \frac{500{,}000}{250} = 2000\ A$$

$$I_{rated_2} = \frac{P_{rated_2}}{V_{rated_2}} = \frac{600{,}000}{250} = 2400\ A$$

$$\frac{\Delta V}{\Delta I_1} = \frac{\%VR_1(V_{rated_1})}{I_{rated_1}} = \frac{0.03(250)}{2000}$$

$$\Delta I_1 = 266.67\ \Delta V \Rightarrow (1)$$

$$\frac{\Delta V}{\Delta I_2} = \frac{\%VR_2(V_{rated_2})}{I_{rated_2}} = \frac{0.04(250)}{2400}$$

$$\Delta I_2 = 240\ \Delta V \Rightarrow (2)$$

$$\Delta I_1 + \Delta I_2 = \Delta I \Rightarrow (3)$$

Substitute Eq. 1 and Eq. 2 in Eq. 3 :

$$266.67\Delta V + 240\ \Delta V = 600$$

$$\Delta V = 1.184$$

$$I_1 = I_2 = \frac{2400}{2} = 1200\ A$$

$$I_{1_{new}} = 1200 + \Delta I_1 = 1200 + 266.67(1.184)$$

$$I_{1_{new}} = 1515.73\ A$$

$$I_{2_{new}} = 1200 + \Delta I_2 = 1200 + 240(1.184)$$

$$I_{2_{new}} = 1484.16\ A$$

Note: Since there is an increase in the busload, the current delivered by each generator will also increase.

DC Motors

SPEED CHARACTERISTICS OF A DC MOTOR

$$N = \frac{60(a)(E_b)}{PZ\Phi} \qquad N = k\frac{E_b}{\Phi}$$

where : N = speed (rpm)
a = number of armature current paths
P = number of poles
Z = number of conductors
Φ = flux per pole (weber)
E_b = back emf or counter emf (volt)
k = proportionality constant

TORQUE DEVELOPED IN THE ARMATURE

$$T = 9.55\left(\frac{P_d}{N}\right) \qquad T = \frac{PZ\Phi I_a}{6.283a}$$

$$P_d = E_b I_a \qquad T = k I_a \Phi$$

where: T = torque developed (newton-meter)
I_a = armature current (ampere)
N = speed of armature rotation (rpm)
P_d = power developed in the armature (watt)
a = number of armature current paths
P = number of poles
Z = number of conductors
Φ = flux per pole (weber)
k = proportionality constant

MECHNICAL POWER OUTPUT

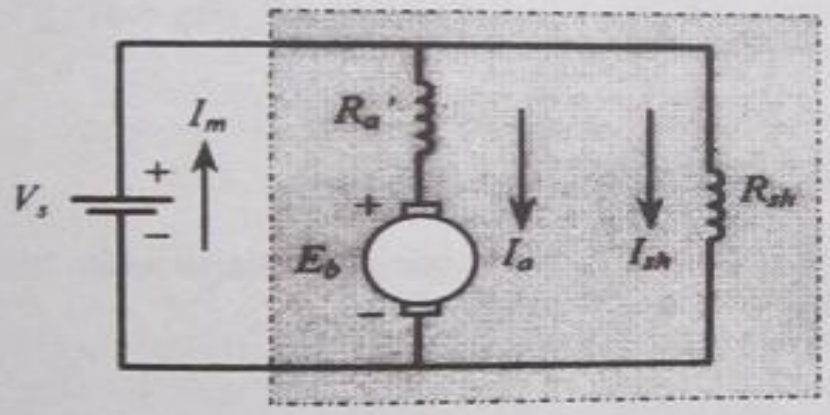
$$Hp = \frac{2\pi NT}{33{,}000} \qquad Hp = \frac{2\pi NT}{44760}$$

here: Hp = mechanical power output (horsepower)
N = speed of armature rotation (rpm)
T = torque developed

Note: If the given torque is in pound-foot use the constant 33,000 and if it is in newton-meter, use the constant 44760.

SHUNT MOTOR

The armature coils and the shunt field coils are connected in parallel.



$$I_{sh} = \frac{V_s}{R_{sh}}$$

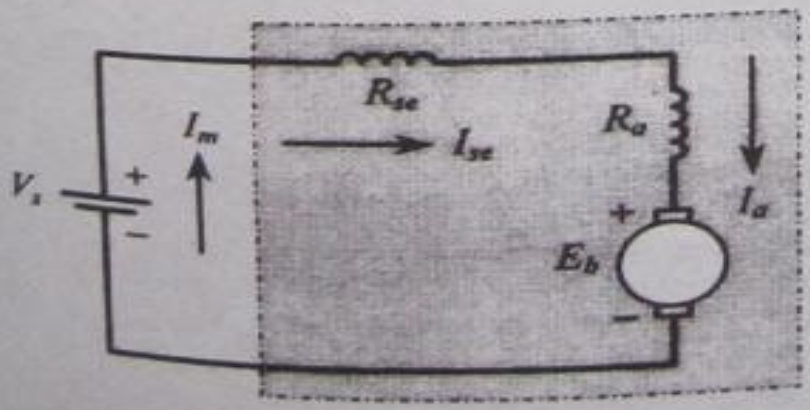
$$I_m = I_a + I_{sh}$$

$$E_b = V_s - I_a R_a'$$

Note: In a shunt motor, unless otherwise specified, the flux is assumed to be constant.

SERIES MOTOR

The armature coils and the shunt field coils are connected in series.



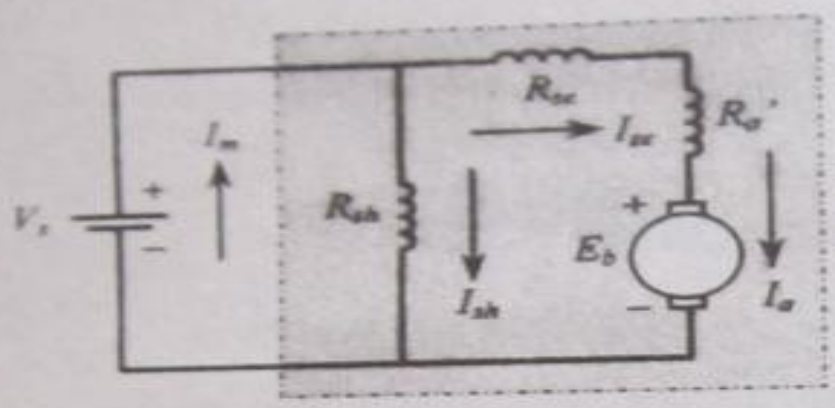
$$I_a = I_{se} = I_m$$

$$E_b = V_s - I_a(R_a' + R_{se})$$

Note: In a series motor, flux is directly proportional to the armature current.

LONG SHUNT COMPOUND MOTOR

The series field coils are connected in series with the armature coils while the shunt field coils are connected across the series combination.



$$I_{se} = I_a$$

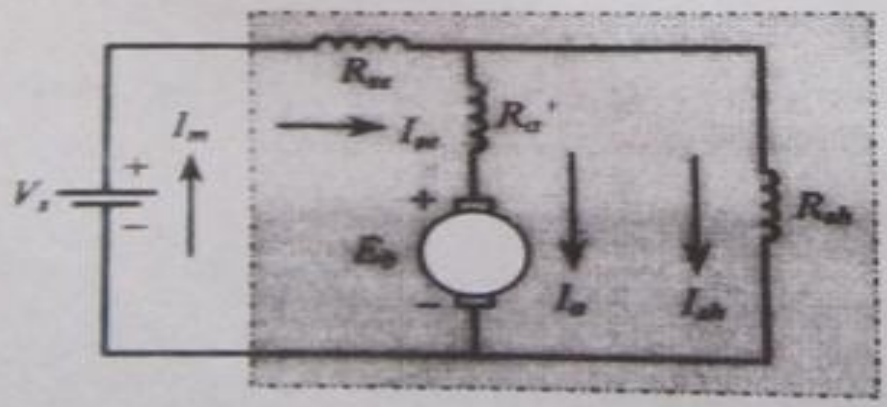
$$I_m = I_a + I_{sh}$$

$$I_{sh} = \frac{V_s}{R_{sh}}$$

$$E_b = V_s - I_a(R_a' + R_{se})$$

SHORT SHUNT COMPOUND MOTOR

The series field coils are connected in series with the supply voltage while the shunt field coils and the armature coils are connected in parallel.



$$I_{se} = I_m$$

$$I_m = I_a + I_{sh}$$

$$I_{sh} = \frac{V_s - I_{se}R_{se}}{R_{sh}}$$

$$E_b = V_s - I_{se}R_{se} - I_aR_a'$$

where : V_s = supply voltage (volt)
E_b = back emf (volt)
R_a' = armature equivalent resistance (ohm)
R_{se} = series field winding resistance (ohm)
R_{sh} = shunt field winding resistance (ohm)
I_m = current drawn from supply (ampere)
I_a = armature current (ampere)
I_{se} = series field current (ampere)
I_{sh} = shunt field current (ampere)

MOTOR STARTING

Note: At starting, the armature draws a high current due to the back emf of the motor at the instant of starting is equal to zero ($E_b = 0$).

- **Armature starting current without a starting rheostat**

$$I_{as} = \frac{V_s}{R_a'}$$

- **Armature starting current with a starting rheostat** inserted in the armature

$$I_{as} = \frac{V_s}{R_a' + R} \qquad R = \frac{V_s}{I_{as}} - R_a'$$

where: I_{as} = armature starting current (ampere)
R_a' = equivalent resistance of the armature circuit (ohm)
V_s = supply voltage (volt)
R = resistance of the starting rheostat (ohm)

SPEED REGULATION

- **Speed relation** is the percentage rise in speed when the mechanical load of the motor is removed.

$$\%NR = \frac{N_{NL} - N_{FL}}{N_{FL}} \times 100\%$$

where: N_{NL} = no-load speed of the motor
N_{FL} = full-load speed of the motor

POWER LOSSES AND EFFICIENCY

$$\eta = \frac{P_{output}}{P_{input}} \qquad \eta = \frac{P_{input} - P_{losses}}{P_{input}} \qquad P_{input} = V_s I_m$$

Note: For power losses, refer to the formulas in DC generators.

TEST TO DETERMINE THE CONSTANT LOSSES OF A DC MACHINE (GENERATOR OR A MOTOR)

Run the machine as a motor at no load with the same operating speed.

$$P_k = V_{sNL} I_{aNL} - (I_{aNL})^2 R_a'$$

where: P_k = rotational or constant losses (watt)
V_{sNL} = supply voltage during the test (volt)
I_{aNL} = armature current during the test (ampere)
R_a' = armature equivalent resistance (ohm)

PONY BRAKE TEST OF A MOTOR

This test is used to determine the output horsepower of the machine.

T = (scale reading - dead weight) x length of arm

$$Hp = \frac{2\pi NT}{k}$$

where: T = torque exerted by the motor during the test
N = speed of the motor shaft during the test (rpm)
Hp = output horsepower of the motor
k = constant for conversion
= 33,000 if torque (T) is in pound-foot (lb-ft)
= 44760 if torque (T) is in Newton-meter (N-m)

SPEED REVERSION OF A SELF-EXCITED DC MOTOR

The direction of rotation of a self-excited DC motor can be reversed using any of the ff. techniques:

- by interchanging the armature terminals
- by interchanging the field terminals

Note: If both of them were interchanged, the direction of rotation will not change.

SPEED CONTROL OF A SELF-EXCITED DC MOTOR

The speed of a self-excited DC motor can be varied using any of the ff. techniques:

- by inserting a rheostat in the armature circuit
- by inserting a rheostat in the field circuit
- by using a potentiometer to vary the supply voltage applied to the motor

Test 15

Problem 525:

A simplex lap wound armature has 580 conductors and carries a current of 125 amperes per armature current path. If the flux per pole is 20 mWb. Calculate the electromagnetic torque developed by the armature?

A. 210.64 N-m
B. 252.72 N-m
C. 230.77 N-m
D. 207.63 N-m

Problem 526:

A 6-pole lap wound shunt motor takes 300 A when the speed is 500 rpm. The flux per pole is 0.06 Wb and the armature has 870 turns. Neglecting the shunt field current, calculate the brake horsepower of the motor. Assume a constant loss of 4%.

A. 175
B. 168
C. 172
D. 165

Problem 527:

A 220 V shunt motor driving its normal load draws an armature current of 50 A from a 220 V dc source. The armature resistance of this motor including brushes is 0.25 ohm. How much armature current (minimum) will this motor draw from a 200 V dc source when driving the same load with the field adjusted to maintain the same speed.

A. 55.75 A
B. 58.24 A
C. 60.12 A
D. 63.48 A

Problem 528: EE Board October 1990

A DC shunt motor has a full load rating of 15 hp, 230 volts, 57.1 amperes, 1400 rpm. The armature circuit resistance is 0.13 ohm and the field circuit resistance is 115 ohms. Neglecting the effect of armature reaction, determine the no-load line current.

A. 6.74 A
B. 5.62 A
C. 7.45 A

D. 8.64 A

Problem 529:

At rated load, a shunt motor draws an armature current of 50 A from a 230 V dc mains while running at 1000 rpm. At no load the armature current drops to 5 A. If the effect of armature reaction has weaken the flux by 2% from no-load to full load, determine speed of the motor at no-load. Assume the armature resistance to be 0.15 ohm.

A. 1202 rpm
B. 1221 rpm
C. 1122 rpm
D. 1010 rpm

Problem 530: EE Board October 1990

The nameplate rating of a shunt motor is 150 hp, 600 volts, 205 A, 1700 rpm. The resistance of the shunt field circuit is 240 ohms and the total armature circuit resistance is 0.15 ohm. Calculate the speed regulation of the motor.

A. 4.07%
B. 4.89%
C. 5.25%
D. 5.18%

Problem 531:

A DC shunt motor runs at 600 rpm on a 240-volt supply while drawing a line current of 30 A. Its armature and field resistances are 0.5 Ω and 120 Ω respectively. What resistance should be placed in series with the armature circuit in order to reduce the speed to 400 rpm. Assume no changes in the armature or field current.

A. 2.69 Ω
B. 2.41 Ω
C. 2.07 Ω
D. 2.83 Ω

Problem 532:

A 120-V shunt motor has an armature equivalent of 0.5 Ω and a field resistance of 60 Ω. At full load, the motor takes 10 A and the speed is 1000 rpm. At what speed must this motor maybe driven as a generator to deliver 10 A to an external load at 120 V?

A. 1158 rpm
B. 1086 rpm
C. 1262 rpm
D. 1045 rpm

Problem 533: EE Board April 1990

A belt-driven 150 kW shunt wound DC generator is running at 450 rpm and is supplying full load to a bus bar at 240 V. At what speed will it run if the belt breaks and the machine continues to run taking 8.5 kW from the bus bar? The armature and field resistances are 0.05 ohm and 85 ohms respectively. The brush contact drop is 1.5 volt per brush. Neglect armature reaction.

A. 395 rpm
B. 388 rpm
C. 402 rpm
D. 358 rpm

Problem 534:

A shunt motor draws a current of 40 A from a 120 V source and runs at 1200 rpm at rated load. The armature and field circuit resistances are 0.1 Ω and 60 Ω, respectively. Determine the speed of the motor at half load. Assume a brush drop of 2 V at rated load and 1 V at half load.

A. 1180 rpm
B. 1238 rpm
C. 1230 rpm
D. 1026 rpm

Problem 535: EE Board October 1990

A DC shunt motor has a full load rating of 15 hp, 230 volts, 57.1 amperes, 1400 rpm. The armature circuit resistance is 0.13 ohm and the field circuit resistance is 115 ohms. Neglecting the effect of armature reaction, determine the no-load speed.

A. 1392 rpm
B. 1264 rpm
C. 1441 rpm
D. 1380 rpm

Problem 536:

A 220-V shunt motor delivers 40 hp on full load at 950 rpm and has an efficiency of 88%. The armature and field resistances are 0.2 ohm and 110 ohms, respectively. Determine the starting resistance such that the starting armature current does not exceed 1.6 times the full load current.

A. 0.704 ohm
B. 0.732 ohm
C. 0.712 ohm
D. 0.739 ohm

Problem 537:

A 120-V DC motor rated at 5-hp has a full load efficiency of 86%. The field and armature resistances are 60 ohms and 0.75 ohm, respectively. What starter resistance will be required to limit the armature current at starting to 200% of its rated armature current?

A. 1.02 ohms
B. 1.12 ohms
C. 1.00 ohm
D. 1.22 ohms

Problem 538:

A 10-hp 230 V DC motor of 85% full load efficiency is located 450 ft from the supply mains. If the motor's starting current is 1.75 times the full load current, what is the smallest cross-sectional area of copper wire required when the allowable voltage drop in the feeder at starting is limited to 24 volts?

A. 30 MCM
B. 28 MCM
C. 26 MCM
D. 27 MCM

Problem 539: EE Board October 1990

The nameplate rating of a shunt motor is 150 hp, 600 volts, 205 A, 1700 rpm. The resistance of the shunt field circuit is 240 ohms and the total armature circuit resistance is 0.15 ohm. If the motor is to be deliver full-load torque at 1,200 rpm, what value of resistance must be added to the armature circuit?

A. 0.833 ohm
B. 0.802 ohm
C. 0.827 ohm
D. 0.862 ohm

Problem 540:

A 120 V shunt motor draws a current of 77 A at rated load. The armature and shunt field resistances are 0.2 Ω and 60 ohms respectively. A multi-tap starter rheostat is inserted in the armature circuit to limit the starting current. If the rheostat has a resistance of 1.732 ohms, determine the percentage tap used such that the starting current will be limited to 150% of its full load value?

A. 60%
B. 50%
C. 55%
D. 45%

Problem 541: EE Board April 1985

A 10-hp power 220 V DC shunt motor has an armature and field resistance of 0.25 ohm and 100 ohms respectively. The full load efficiency is 83%. Determine the value of starting resistance in order that the starting current will not exceed 200 percent of full-load value.

A. 2.88 Ω
B. 2.59 Ω
C. 2.15 Ω
D. 2.45 Ω

Problem 542:

A 50 HP, 550 V shunt wound motor draws a line current of 4.5 A at no load. The shunt field resistance is 275 ohms and the armature resistance exclusive of brushes, is 0.3 ohm, the brush drop at full load is 2 V. At full load, the motor draws a line current of 84 A. Calculate the efficiency at full load.

A. 92.0%
B. 88.5%
C. 91.2%
D. 89.9%

Problem 543:

A 100-volt shunt motor is developing 6 hp while operating at an overall efficiency of 86%. The armature and shunt field resistances are 0.06 and 50 ohms respectively. Determine stray power losses.

A. 378 W
B. 367 W
C. 392 W
D. 364 W

Problem 544: EE Board October 1998

The shaft power of a shunt motor is 7.8 hp. It draws 50 A from 120 V. The field winding draws 1.2 A. What is the efficiency of the motor?

A. 94.7%
B. 93%
C. 96.9%
D. 98.2%

Problem 545:

A 500 V shunt motor running at 720 rpm takes an armature current of 50 A. The equivalent armature resistance is 0.4 ohm. What resistance must be placed in series with the armature to reduce the speed to 540 rpm while maintaining the same torqua?

A. 2.2 ohms

B. 2.6 ohms
C. 2.4 ohms
D. 2.8 ohms

Problem 546: EE Board March 1998

A certain shunt motor has an armature resistance of 0.05 ohm. It draws 50 A at a terminal voltage of 120 V. Assume other miscellaneous losses at 1%. Determine the output horsepower of the motor.

A. 8.5 hp
B. 7.1 hp
C. 6.8 hp
D. 7.8 hp

Problem 547:

The armature and field resistances of a shunt machine are 0.2 ohm and 110 ohms respectively. While running as a generator, the generated emf is 250 V at 1200 rpm. If the machine is run as a shunt motor, it takes 4 A at 220 V. At a certain load the motor takes 30 A at 220 V. However on load, the armature reaction weakens the field by 2 %. Find the motor speed at this load.

A. 1120 rpm
B. 1050 rpm
C. 1042 rpm
D. 1025 rpm

Problem 548: EE Board October 1990

A shunt motor, which has a field resistance of 220 ohms and an armature resistance of 0.8 ohm takes 26 A from a 260 V supply when running at 500 rpm on full load. In order to control the speed of the motor a 1.2-ohm resistor is connected in series with the armature. Calculate the speed at which the motor will run when supplying full load torque.

A. 438 rpm
B. 445 rpm
C. 424 rpm
D. 472 rpm

Problem 549: EE Board October 1992

The input and output powers of a 220 V, shunt motor are 30 and 28 kW respectively. The field and armature circuit resistances are 40 ohms and 0.0125 ohm respectively. Determine the efficiency of the motor if the output power is reduced to 10 kW.

A. 83.56%
B. 88.67%
C. 84.93%
D. 86.48%

Problem 550:

A 220 V series motor takes 15 A and runs at 700 rpm. The armature resistance including the resistance of the series winding is 0.8 ohm. At what speed will it run if a 2-ohm resistance is connected in series with supply voltage? Assume the motor is taking the same current.

A. 599 rpm
B. 601 rpm
C. 572 rpm
D. 583 rpm

Problem 551:

The equivalent armature resistance of a series motor is 0.1 ohm. When connected across a 110-V mains, the armature takes 20 A and its speed is 1000 rpm. Determine its speed when the armature takes 50 A from the same mains, with the field increased by 10%.

A. 809 rpm
B. 954 rpm
C. 856 rpm
D. 884 rpm

Problem 552:

A 400 V series motor has a field resistance of 0.2 ohm and an armature resistance of 0.1 ohm. The motor takes 30 A of current at 1000 rpm while developing full load torque. What is the motor speed when this motor develops 60% of full load torque.

A. 1302.4 rpm
B. 1256.2 rpm
C. 1297.6 rpm
D. 1135.5 rpm

Problem 553:

A 10 hp, 230-V, 1200 rpm series motor having rated load efficiency of 85.5%. The armature resistance including brushes is 0.3 ohm. The field resistance is 0.25 ohm. Assuming the flux varies directly as the armature current, what value of resistance should be placed in series with this motor when starting, in order that the starting current maybe limited to a value that will exert a starting torque equal to 150% of its rated torque?

A. 4.7 ohms
B. 4.4 ohms
C. 5.1 ohms
D. 3.8 ohms

Problem 554:

The field and armature resistances of a 220-V series motor are 0.2 Ω and 0.1Ω, respectively. The motor takes 30 A of current while running at 700 rpm. If the total iron and friction losses are 350 W, determine the motor efficiency.

A. 90.6%
B. 91.5%
C. 89.4%
D. 92.2%

Problem 555:

A 100-V series motor is used to drive a load through a pulley. This machine has an armature resistance of 0.2 ohm and a series field resistance of 0.25 ohm. When a torque of 25 N-m is applied to the pulley, the speed is 600 rpm. If stray power losses at this load is 300 W, calculate the armature current.

A. 20.62 A
B. 21.89 A
C. 22.72 A
D. 23.41 A

Problem 556:

A 400 V series motor, having an equivalent armature circuit resistance of 0.5 ohm, takes 44 A of current while running at 650 rpm. What is the motor speed for a line current of 36 A?

A. 803 rpm
B. 822 rpm
C. 812 rpm
D. 806 rpm

Problem 557:

The resistance of each of the two coils of a series motor is 0.04 ohm, armature resistance, 0.04 ohm. The motor takes 50 A at 100 V while running at 800 rpm and the coils are in series. What will it speed if the coils are re-connected in parallel while the load torque is doubled?

A. 800 rpm
B. 400 rpm
C. 200 rpm
D. 650 rpm

Problem 558:

On full load a 500 V series motor takes 100 A and runs at 820 rpm. The armature and series field resistances are 0.1 and 0.04 ohm respectively. What will be its speed when developing half-full load torque and with a 0.08-ohm diverter connected across the series field winding. Assume flux is proportional to the field current.

A. 1563 rpm
B. 1634 rpm
C. 1359 rpm
D. 1429 rpm

Problem 559:

A 400 V series motor working with unsaturated field is taking 60 A and running at 840 rpm. The total resistance of the motor is 0.1 ohm. At what speed will the motor run when developing half-full load torque.

A. 1193 rpm
B. 1202 rpm
C. 1167 rpm
D. 1352 rpm

Problem 560:

A long shunt compound motor takes a current of 42 A from a 230 V source. The generator parameters are: Ra = 0.1 Ω, Rse = 0.2 Ω and Rsh = 50 Ω. If the friction and windage losses amount to 400 W, determine the overall efficiency of the machine.

A. 82.24%
B. 80.56%
C. 81.22%
D. 79.53%

Problem 561:

A 230 V, long shunt machine has the following parameters: Ra = 2.0 Ω, Rsh = 460 Ω and Rse = 0.25 Ω. When the machine is run at no-load at its normal speed and rated voltage, the armature draws 0.6 A. Determine the armature current drawn if the machine delivers an output of 5 bhp.

A. 24.35 A
B. 21.22 A
C. 20.36 A
D. 22.83 A

Problem 562:

A 25 hp, 250 V long shunt compound motor takes 5 A when running light and runs at 1150 rpm. Armature resistance, 0.15 Ω, Shunt field resistance, 200 Ω and series field resistance, 0.05 Ω. At rated load the motor takes 86.5 A and runs at 720 rpm. Determine the ratio of torque developed from no-load to full load.

A. 1:26
B. 1:34
C. 1:22
D. 1:38

Problem 563:

A 200 V shunt motor has armature and shunt field resistances of 0.15 Ω and 125 Ω, respectively. When running light, it takes 6 A and runs at 1200 rpm. A series winding of 0.05 ohm is added to make it long shunt cumulatively compounded. This winding increases the flux per pole by 25% when the motor is taking its full load current of 100 A. Neglecting armature reaction, solve for the operating speed as a compound motor.

A. 887 rpm
B. 890 rpm
C. 868 rpm
D. 865 rpm

Problem 564:

A 220 V DC motor has an armature resistance of 0.4 ohm, a shunt field resistance of 175 ohms and a series field resistance of 4 ohms. This machine is connected as a short shunt compound motor to 220 V mains. Calculate the power developed by the armature if the armature current is 30 A. Neglect the brush drop.

A. 2254.6 W
B. 2572.8 W
C. 2652.6 W
D. 2150.5 W

Problem 565:

The input current to a 220 V, short shunt compound motor at no load is 6 A. The shunt field circuit resistance is 220 ohms; the armature resistance is 0.10 ohm and the series field resistance is 0.08 ohm. What is the stray power loss?

A. 1153.31 W
B. 1066.22 W
C. 1232.54 W
D. 1073.25 W

Problem 566: EE Board April 1988

The pony brake test of an elevator door drive shunt motor, the ammeter and voltmeter measuring the input read 34 A and 220 V. The speed of the motor is found to be 910 rpm and the balance on a 2 ft brake arm read 27.2 pounds. The tare weight of the arm is found to be positive 2.3 pounds. Determine the efficiency of the motor at this load.

A. 87.03%
B. 86.05%
C. 84.45%
D. 88 32%

Problem 567: EE Board October 1986

In a brake test of an elevator door motor, the ammeter and voltmeter measuring the input read 34 A at 220 V. The speed of the motor is found to be 910 rpm and the balanced on a 0.61 m brake arm reads 11.9 kg. The dead weight of the arm is found to be 1.09 kg. What is the output hp of the door motor?

A. 9.40 hp
B. 10.38 hp
C. 8.26 hp
D. 9.58 hp

Problem 568:

A shunt motor with an armature and field resistance of 0.055 and 32 ohms, respectively, is to be tested for its mechanical efficiency by means of a rope brake. When the motor is running at 1400 rpm, the longitudinal pull on the 6-inch diameter pulley is 57 pounds. The readings of the line ammeter and voltmeter are 35 and 105, respectively. Calculate the efficiency.

A. 77.10%
B. 75.32%
C. 78.28%
D. 79.12%

Problem 569:

A shunt motor was tested by means of a pony brake having a length arm of 3.5 feet and a tare weight of 5.7 lbs. The current drawn by the machine from a 240 V line was 50.9 A when the scale reading was 24 lbs and the speed of the motor was 1215 rpm. Calculate the rotational losses of the motor. The armature and shunt field resistances of the machine are 0.25 Ω and 120 Ω, respectively.

A. 674 W
B. 692 W
C. 620 W
D. 705 W

Problem 570: EE Board October 1993

Calculate the force that will be exerted on the scale in a pony brake test when a 20-hp, 1,400 rpm motor is operating at full load. The length of the brake arm is 3 ft. and the tare weight of the brake is 3.75 lb.

A. 30.45 lbs
B. 28.76 lbs
C. 21.54 lbs
D. 36.33 lbs

ANSWER KEY

525. C	535. C	545. C	555. A	565. D
526. B	536. A	546. D	556. A	566. B
527. A	537. C	547. B	557. A	567. C
528. A	538. C	548. A	558. D	568. A
529. D	539. C	549. C	559. A	569. A
530. D	540. B	550. A	560. B	570. B
531. A	541. B	551. D	561. B	
532. B	542. D	552. C	562. B	
533 A	543. A	553. B	563. C	
534. C	544. C	554. A	564. B	

☑ RATING

41-46 - Topnotcher

32-40 - Passer

23-31 - Conditional

0-22 - Failed

Did you know that.... the great English chemist Sir Humphry Davy known as the inventor of the miner's safety lamp once said "My greatest discovery was Michael Faraday"?

Michael Faraday, an English chemist and physicist made one of the most significant discoveries in the history of electricity, the "ELECTROMAGNETIC INDUCTION".

Faraday, one of 10 children of a blacksmith was born near London. He was first apprenticed to a bookbinder, but at the age of 22 realized his boyhood dream by becoming assistant at the Royal Institute to his idol, the great chemist Sir Humphry Davy.

It was Oersted's discovery of electromagnetism that prompted Faraday to turn from chemistry to concentrate on electricity. On August 29, 1831, he discovered electromagnetic induction, when he found out that moving a magnet through a coil of wire caused an electric current to flow in the wire. His discovery of the electromagnetic induction was the forerunner of the electric motor and generator principles.

Faraday remained working at the institution for 54 years, taking over Davy's position when Davy retired. He was perhaps the greatest experimentalist who ever lived, with achievements to his credit in nearly all the areas of physical sciences under investigation in his time.

Failures never discourage Faraday. Once he said " FAILURES ARE JUST AS IMPORTANT AS SUCCESSES, FAILURES ALSO TEACHES". He was hailed as one of Europe's top scientist.

Solutions to Test 15

525 Note: The meaning of "N" in the formula below is speed in "rpm"

$$P_d = E_b I_a = \frac{PNZ\Phi I_a}{60a} = \frac{PNZ\Phi I_a}{60mP} = \frac{NZ\Phi I_a}{60m} \Rightarrow ❶$$

$$P_d = \frac{2\pi NT}{60} \Rightarrow ❷$$

Equate Eq.1 to Eq.2 :

$$\left(\frac{NZ\Phi}{60m}\right) I_a = \frac{2\pi NT}{60}$$

$$T = \frac{Z\Phi I_a}{2\pi m} = \frac{(580)(20 \times 10^{-3})(125)}{2\pi(1)}$$

$$T = 230.77 \text{ N-m}$$

526 $$E_b = \frac{PNZ\Phi}{60a} = \frac{PNZ\Phi}{60mP} = \frac{NZ\Phi}{60m} = \frac{500(870)(0.06)}{60(1)} = 435 \text{ V}$$

$$P_d = \frac{E_b I_a}{746} = \frac{435(300)}{746} = 175 \text{ hp}$$

$$P_{brake} = (0.96)(175)$$

$$P_{brake} = 168 \text{ hp}$$

Note: P_{brake} = output power of the motor

527 Note: For the same load and speed, the power developed in the armature for both conditions are the same.

$$P_{d_1} = P_{d_2}$$

$$E_{b_1} I_{a_1} = E_{b_2} I_{a_2}$$

$$(V_s - I_{a_1} R_a) I_{a_1} = (V_s - I_{a_2} R_a) I_{a_2}$$

$$(220 - 50(0.25))50 = (200 - I_{a_2}(0.25)) I_{a_2} \quad 0.25 I_{a_2}^2 - 200 I_{a_2} = 10375$$

$$I_{a_2}^2 - 800 I_{a_2} + 41500 = 0$$

Using the quadratic formula :

$$I_{a_2} = \frac{800 \pm \sqrt{(800)^2 - 4(1)(41500)}}{2(1)} = \frac{800 \pm 688.47}{2}$$

$$I_{a_2}^{(-)} = 55.76 \text{ A}$$

528 $I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{230}{115} = 2\text{ A}$

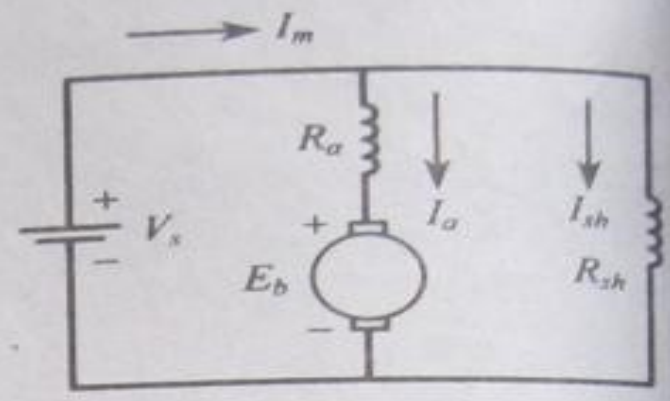
$I_a = I_m - I_{sh} = 57.1 - 2 = 55.1\text{ A}$

$P_a = I_a^2 R_a = (55.1)^2(0.13) = 394.68\text{ W}$

$P_{sh} = V_s I_{sh} = (230)(2) = 460\text{ W}$

$P_{in} = I_m V_s = 230(57.1) = 13{,}133\text{ W}$

$P_{out} = 15(746) = 11{,}190\text{ W}$



$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$

$13{,}133 = 11190 + 394.68 + 460 + P_{stray}$

$P_{stray} = 1{,}088.32\text{ W}$

$P_{stray} = V_{aNL} I_{aNL} - I_{aNL}^2 R_a$

$1088.32 = 230 I_{aNL} - I_{aNL}^2(0.13)$

$I_{aNL}^2 - 1769.23 I_{aNL} + 8371.692 = 0$

Using the quadratic formula :

$$I_{aNL} = \frac{1769.23 \pm \sqrt{(1769.23)^2 - 4(1)(8371.692)}}{2(1)}$$

$$= \frac{1769.23 \pm 1759.74}{2}$$

$I_{aNL}^{(-)} = 4.745\text{ A}$

$I_{mNL} = I_{aNL} + I_{shNL} = 4.745 + 2$

$I_{mNL} = 6.745\text{ A}$

529 $E_{b_1} = V_s - I_{a_1} R_a = 230 - 50(0.15) = 222.5\text{ V}$

$E_{b_2} = V_s - I_{a_2} R_a = 230 - 5(0.15) = 229.25\text{ V}$

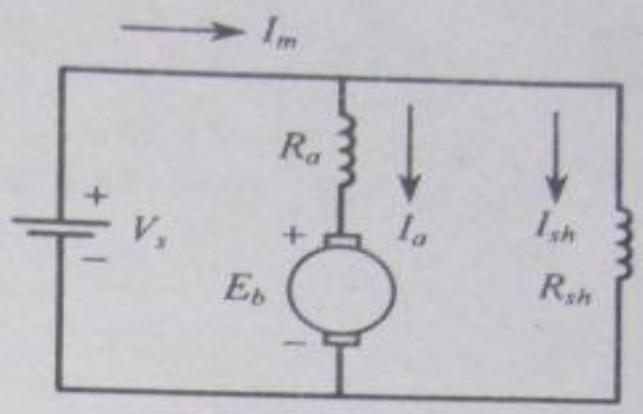
$N = k_s \dfrac{E}{\Phi}$

$\dfrac{N_2}{N_1} = \dfrac{E_{b_2}\Phi_1}{E_{b_1}\Phi_2}$

$N_2 = N_1\left(\dfrac{E_{b_2}\Phi_1}{E_{b_1}\Phi_2}\right) = 1000\left(\dfrac{229.25\,(0.98\Phi_2)}{222.5\Phi_2}\right) = 1009.73$

$N_2 \cong 1010\text{ rpm}$

530 $I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{600}{240} = 2.5\text{ A}$



$I_a = I_m - I_{sh} = 205 - 2.5 = 202.5\text{ A}$

$P_a = I_a^2 R_a = (202.5)^2(0.15)$

$= 6150.937\text{ W}$

$P_{sh} = V_s I_{sh} = (600)(2.5) = 1500\text{ W}$

$P_{in} = I_m V_s = 205(600) = 123{,}000\text{ W}$

$P_{out} = 150(746) = 111900\text{ W}$

$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$

$123{,}000 = 111900 + 6150.937 + 1500 + P_{stray}$

$P_{stray} = 3449\text{ W}$

$P_{stray} = V_{sNL} I_{aNL} - I_{aNL}^2 R_a$

$3449 = 600\, I_{aNL} - I_{aNL}^2(0.15)$

$I_{aNL}^2 - 4000\, I_{aNL} + 22993.33 = 0$

Using the quadratic formula :

$$I_{aNL} = \frac{4000 \pm \sqrt{(4000)^2 - 4(1)(22993.33)}}{2(1)} = \frac{4000 \pm 3988.486}{2}$$

$I_{aNL}^{(-)} = 5.757\text{ A}$

$E_{bNL} = V_s - I_{aNL} R_a = 600 - 5.757(0.15) = 599.136\text{ V}$

$E_{bFL} = V_s - I_{aFL} R_a = 600 - 202.5(0.15) = 569.625\text{ V}$

$N = k_s E$

$$\frac{N_{NL}}{N_{FL}} = \frac{E_{bNL}}{E_{bFL}}$$

$$N_{NL} = N_{FL}\left(\frac{E_{bNL}}{E_{bFL}}\right) = 1700\left(\frac{599.136}{569.625}\right) = 1788.07\text{ rpm}$$

$$\%NR = \frac{N_{NL} - N_{FL}}{N_{FL}} \times 100\% = \frac{1788.07 - 1700}{1700} \times 100\%$$

$\%NR = 5.18\%$

Note: Since not specified, flux is assumed constant for both conditions.

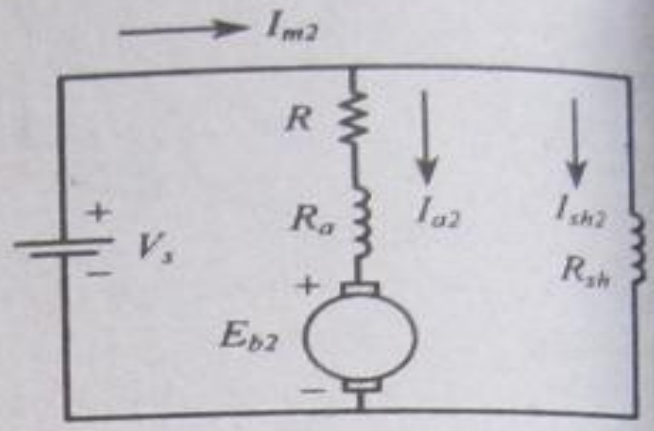
531

$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{240}{120} = 2\text{ A}$$

$$I_a = I_m - I_{sh} = 30 - 2 = 28\text{ A}$$

$$E_{b_1} = V_s - I_{a_1}R_a = 240 - 28(0.5)$$

$$E_{b_1} = 226\text{ V}$$



$$N = k_s E$$

$$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}}$$

Note: Since the field current as given is constant, the flux in the air gap will also become constant.

$$E_{b_2} = E_{b_1}\left(\frac{N_2}{N_1}\right) = 226\left(\frac{400}{600}\right) = 150.667\text{ V}$$

$$E_{b_2} = V_s - I_{a_2}(R_a + R)$$

$$150.667 = 240 - 28(0.5 + R)$$

$$R = 2.69\ \Omega$$

532

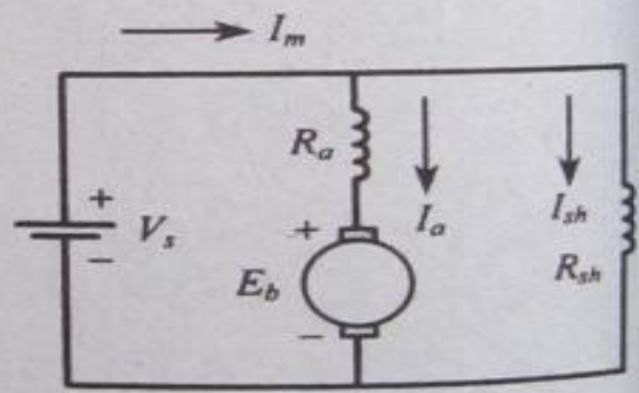
As a motor :

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{120}{60} = 2\text{ A}$$

$$I_a = I_m - I_{sh} = 10 - 2 = 8\text{ A}$$

$$E_b = V_s - I_aR_a = 120 - 8(0.5)$$

$$E_b = 116\text{ volts}$$

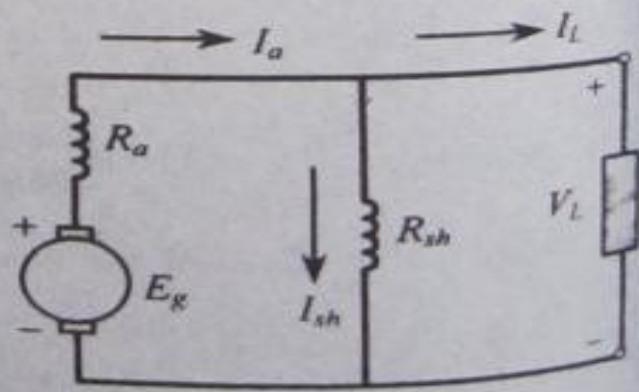


As a generator :

$$I_a = I_L + I_{sh} = 10 + 2 = 12\text{ A}$$

$$E_g = V_L + I_aR_a = 120 + 12(0.5)$$

$$E_g = 126\text{ volts}$$



$$E = kN$$

$$\frac{E_b}{E_g} = \frac{N_m}{N_g}$$

$$N_g = \frac{N_mE_g}{E_b} = \frac{1000(126)}{116} = 1086.2$$

$$N_g \cong 1086\text{ rpm}$$

533 As a generator :

$$I_L = \frac{P_L}{V_L} = \frac{150{,}000}{240} = 625 \text{ A}$$

$$I_{sh} = \frac{V_L}{R_{sh}} = \frac{240}{85} = 2.823 \text{ A}$$

$$I_a = I_L + I_{sh} = 625 + 2.823 = 627.823 \text{ A}$$

$$E_g = V_L + I_a R_a = 240 + 627.823(0.05) = 271.39 \text{ V}$$

As a motor :

$$I_m = \frac{P_m}{V_s} = \frac{8500}{240} = 35.416 \text{ A}$$

$$I_a = I_m - I_{sh} = 35.416 - 2.823 = 32.593 \text{ A}$$

$$E_b = V_s - I_a R_a = 240 - 32.593(0.05) = 238.37 \text{ V}$$

$$E = kN$$

$$\frac{E_g}{E_m} = \frac{N_g}{N_m}$$

$$N_m = \frac{N_g E_m}{E_g} = \frac{450(238.37)}{271.39} = 395.25$$

$$N_m \cong 395 \text{ rpm}$$

534 Note: At half load, the motor draws half the armature current at rated load.

$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{120}{60} = 2 \text{ A}$$

$$I_{a_1} = I_{m1} - I_{sh} = 40 - 2 = 38 \text{ A}$$

$$E_{b_1} = V_s - I_{a_1} R_a - V_{b1} = 120 - 38(0.1) - 2 = 114.2 \text{ V}$$

$$I_{a_2} = \frac{I_{a_1}}{2} = \frac{38}{2} = 19 \text{ A}$$

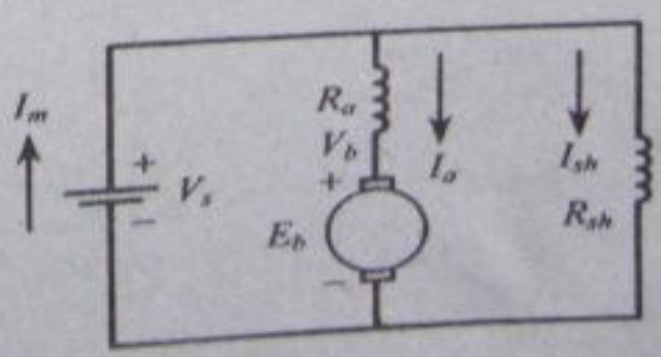
$$E_{b_2} = V_s - I_{a_2} R_a - V_{b2} = 120 - 19(0.1) - 1 = 117.1 \text{ V}$$

$$N = k_s E$$

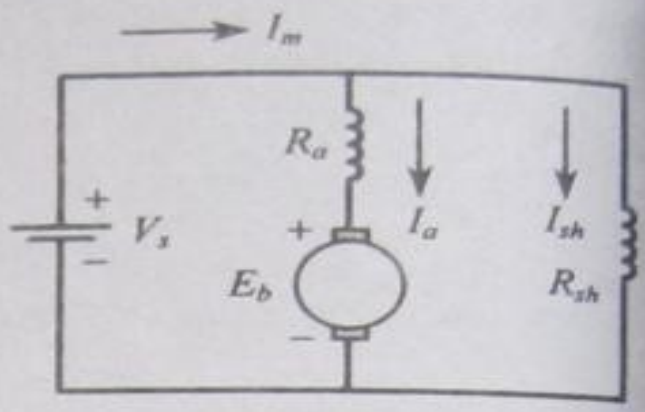
$$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}}$$

$$N_2 = N_1\left(\frac{E_{b_2}}{E_{b_1}}\right) = 1200\left(\frac{117.1}{114.2}\right)$$

$$N_2 \cong 1230 \text{ rpm}$$



535



$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{230}{115} = 2\text{ A}$$

$$I_{aFL} = I_{mFL} - I_{sh} = 57.1 - 2 = 55.1\text{ A}$$

$$P_{aFL} = I_{aFL}{}^2 R_a = (55.1)^2(0.13)$$
$$= 394.68\text{ W}$$

$$P_{sh} = V_s I_{sh} = (230)(2) = 460\text{ W}$$

$$P_{inFL} = I_{mFL} V_s = 230(57.1) = 13{,}133\text{ W}$$

$$P_{outFL} = 15(746) = 11{,}190\text{ W}$$

$$P_{inFL} = P_{outFL} + P_{aFL} + P_{sh} + P_{stray}$$
$$13{,}133 = 11190 + 394.68 + 460 + P_{stray}$$
$$P_{stray} = 1{,}088.32\text{ W}$$

$$P_{stray} = V_{sNL} I_{aNL} - I_{aNL}{}^2 R_a$$
$$1088.32 = 230 I_{aNL} - I_{aNL}{}^2(0.13)$$
$$I_{aNL}{}^2 - 1769.23 I_{aNL} + 8371.692 = 0$$

Using the quadratic formula :

$$I_{aNL} = \frac{1769.23 \pm \sqrt{(1769.23)^2 - 4(1)(8371.692)}}{2(1)}$$
$$= \frac{1769.23 \pm 1759.74}{2}$$
$$I_{aNL}{}^{(-)} = 4.745\text{ A}$$

$$E_{bNL} = V_s - I_{aNL} R_a = 230 - 4.745(0.13) = 229.383\text{ V}$$
$$E_{bFL} = V_s - I_{aFL} R_a = 230 - 55.1(0.13) = 222.837\text{ V}$$

$$N = k_a E$$
$$\frac{N_{NL}}{N_{FL}} = \frac{E_{bNL}}{E_{bFL}}$$
$$N_{NL} = N_{FL}\left(\frac{E_{bNL}}{E_{bFL}}\right) = 1400\left(\frac{229.383}{222.837}\right) = 1441.126$$
$$N_{NL} \cong 1441\text{ rpm}$$

536 $P_{in} = \frac{P_{out}}{\eta} = \frac{40(746)}{0.88} = 33{,}909.1\,W$

$I_m = \frac{P_{in}}{V_s} = \frac{33{,}909.1}{220} = 154.13\,A$

$I_{sh} = \frac{V_s}{R_{sh}} = \frac{220}{110} = 2\,A$

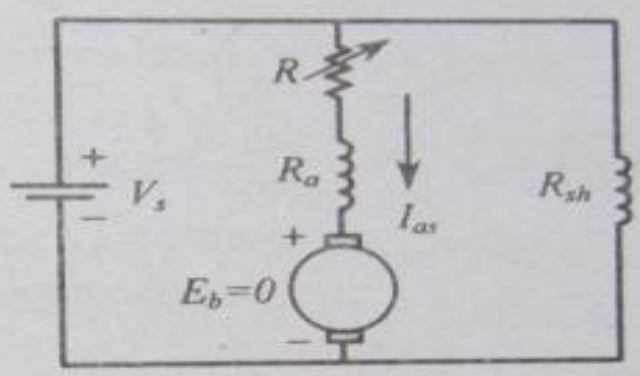
$I_a = I_m - I_{sh} = 154.13 - 2 = 152.13\,A$

Let: R = resistance of a resistor inserted in series with the armature windings at starting ($E_b = 0$)

$I_{as} = \frac{V_s}{R + R_a}$

$R = \frac{V_s}{I_{as}} - R_a = \frac{220}{1.6(152.13)} - 0.2$

$R = 0.704\,\Omega$



537 $P_{in} = \frac{P_{out}}{\eta} = \frac{5(746)}{0.86} = 4337.21\,W$

$I_m = \frac{P_{in}}{V_s} = \frac{4337.21}{120} = 36.14\,A$

$I_{sh} = \frac{V_s}{R_{sh}} = \frac{120}{60} = 2\,A$

$I_a = I_m - I_{sh} = 36.14 - 2 = 34.14\,A$

Let: R = resistance of a resistor inserted in series with the armature windings at starting ($E_b = 0$)

$I_{as} = \frac{V_s}{R + R_a}$

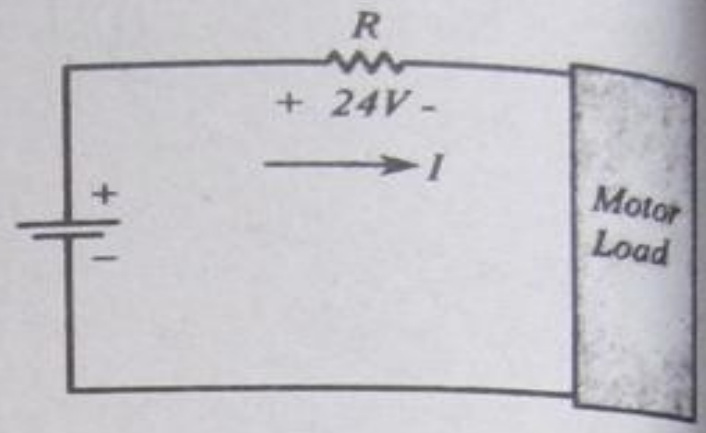
$R = \frac{V_s}{I_{as}} - R_a = \frac{120}{2(34.14)} - 0.75$

$R = 1\,\Omega$

538 $P_{in} = \frac{P_{out}}{\eta} = \frac{10(746)}{0.85} = 8776.47\,W$

$I_m = \frac{P_{out}}{V_s} = \frac{8776.47}{230} = 38.15\,A$

Let: I = current in the feeder at starting.
R = total resistance of the feeder



$I = 1.75 I_m$

$I = 1.75(38.15) = 66.76\ A$

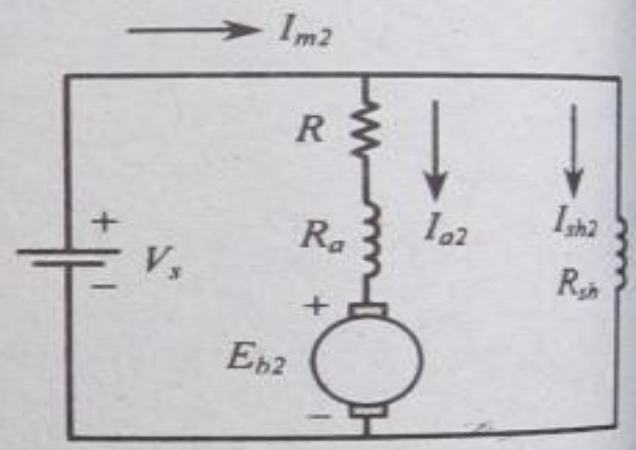
$$R = \frac{V_{drop}}{I} = \frac{24}{66.76} = 0.359\ \Omega$$

$$R = \rho\frac{L}{A}$$

$$A = \frac{\rho L}{R} = \frac{10.37(2 \times 450)}{0.359} = 25{,}997.2\ CM \cong 26\ MCM$$

Note: Resistivity (ρ) of copper conductor is 10.37 ohms-circular mils per foot.

539



$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{600}{240} = 2.5\ A$$

$I_{a1} = I_{m1} - I_{sh} = 205 - 2.5 = 202.5\ A$

$E_{b1} = V_s - I_{a1}R_a = 600 - 202.5(0.15)$

$E_{b1} = 569.625\ V$

$N = k_s E$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

$$E_{b2} = E_{b1}\left(\frac{N_2}{N_1}\right) = 569.625\left(\frac{1200}{1700}\right) = 402.088\ V$$

$E_{b2} = V_s - I_{a2}(R_a + R)$

$$R = \frac{V_s - E_{b2}}{I_{a2}} - R_a = \frac{600 - 402.088}{202.5} - 0.15$$

$R = 0.827\ \Omega$

Note: Since torque is the same, armature current is also the same.

540 Let: R = resistance required to limit the starting armature current to 150% of its full load value.

$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{120}{60} = 2\ A$$

$I_a = I_m - I_{sh} = 77 - 2 = 75\ A$

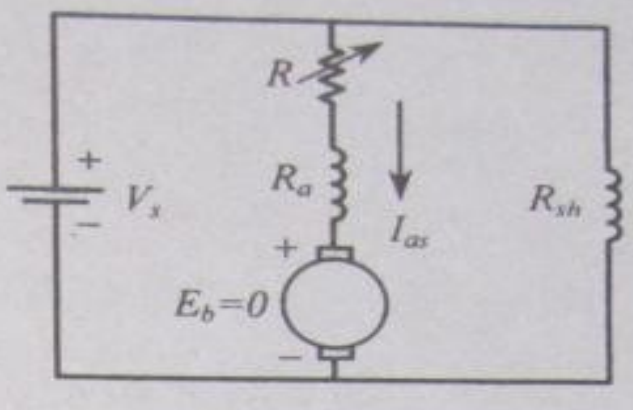
$$I_{as} = \frac{V_s}{R + R_a}$$

$$R = \frac{V_s}{I_{as}} - R_a = \frac{120}{1.5(75)} - 0.2$$

$$R = 0.866\ \Omega$$

$$\%Tap = \frac{R}{R_{rheostat}} = \frac{0.866}{1.732} \times 100\%$$

$$\%Tap = 50\%$$



541 $$P_{in} = \frac{P_{out}}{\eta} = \frac{10(746)}{0.83} = 8987.952\ W$$

$$I_m = \frac{P_{in}}{V_s} = \frac{8987.952\ W}{220} = 40.854\ A$$

$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{220}{100} = 2.2\ A$$

$$I_a = I_m - I_{sh} = 40.854 - 2.2 = 38.654\ A$$

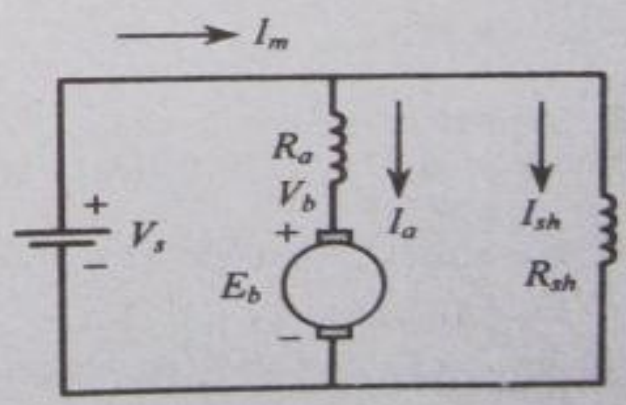
$$I_{as} = \frac{V_s}{R + R_a}$$

$$R = \frac{V_s}{I_{as}} - R_a = \frac{220}{2(38.654)} - 0.25$$

$$R = 2.59\ \Omega$$

542 $$I_{sh} = \frac{V_s}{R_{sh}} = \frac{550}{275} = 2\ A$$

$$I_{aNL} = I_{mNL} - I_{sh} = 4.5 - 2 = 2.5\ A$$



$$P_{stray} = V_s I_{aNL} - I_{aNL}^2 R_a$$

$$P_{stray} = 550(2.5) - (2.5)^2(0.3)$$

$$P_{stray} = 1373.125\ W$$

$$I_{aFL} = I_{mFL} - I_{sh} = 84 - 2 = 82\ A$$

$$P_{inFL} = V_s I_{mFL} = 550(84) = 46200\ W$$

$$P_{aFL} = I_{aFL}^2 R_a = (82)^2(0.3) = 2017.2\ W$$

$$P_{shFL} = V_s I_{sh} = 550(2) = 1100\ W$$

$$P_b = V_b I_{aFL} = 2(82) = 164\ W$$

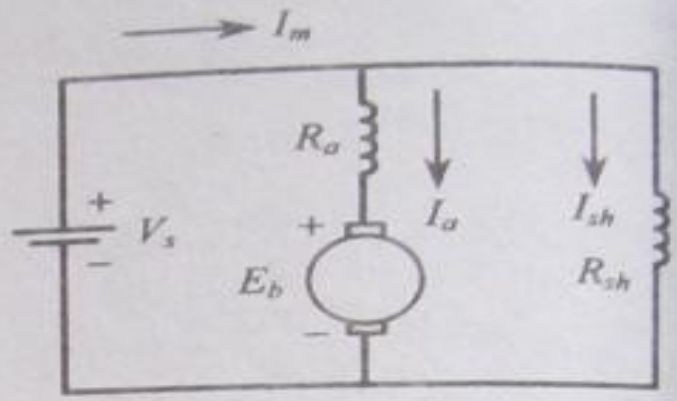
$$P_{lossFL} = P_{aFL} + P_{shFL} + P_{stray} + P_b$$

$$P_{lossFL} = 2017.2 + 1100 + 1373.125 + 164 = 4654.325\ W$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} = \frac{46200 - 4654.325}{46200} \times 100\%$$

$$\eta = 89.92\%$$

543



$$P_{out} = 6(746) = 4476 \text{ W}$$

$$P_{in} = \frac{P_{out}}{\eta} = \frac{4476}{0.86} = 5204.65 \text{ W}$$

$$I_m = \frac{P_{in}}{V_s} = \frac{5204.65}{100} = 52.04 \text{ A}$$

$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{100}{50} = 2 \text{ A}$$

$$I_a = I_m - I_{sh} = 52.04 - 2 = 50.04 \text{ A}$$

$$P_a = I_a^2 R_a = (50.04)^2(0.06) = 150.24 \text{ W}$$

$$P_{sh} = V_s I_{sh} = 100(2) = 200 \text{ W}$$

$$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$$

$$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$$

$$5204.65 = 4476 + 150.24 + 200 + P_{stray}$$

$$P_{stray} = 378.41 \text{ W}$$

544

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{V_s I_m} = \frac{7.8(746)}{120(50)} \times 100\%$$

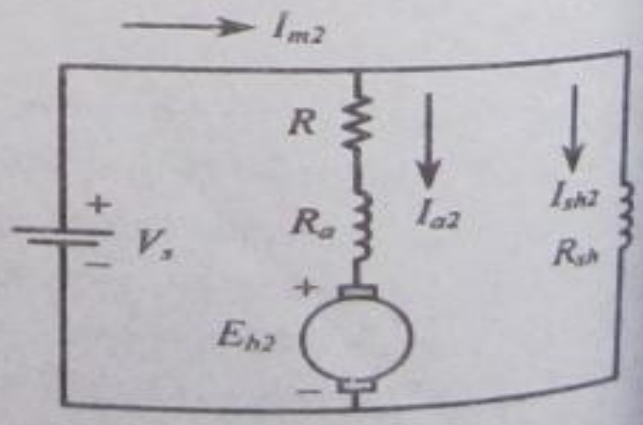
$$\eta = 96.98\%$$

545 Note: Since torque is constant, the armature current also for both conditions must be constant.

$$E_{b1} = V_s - I_{a_1} R_a = 500 - 50(0.4) = 480 \text{ V}$$

$$E_b = kN$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$



$$E_{b2} = E_{b1}\left(\frac{N_2}{N_1}\right) = \frac{480(540)}{720} = 360 \text{ V}$$

$$E_{b2} = V_s - I_{a2}(R_a + R)$$

$$R = \frac{V_s - E_{b2}}{I_{a2}} - R_a$$

$$R = \frac{500 - 360}{50} - 0.4$$

$$R = 2.4\ \Omega$$

546 Note: Since the shunt field resistance is not given, assume the motor is separately excited and therefore its losses are not charge to the motor.

$P_{in} = V_s I_m = 120(50) = 6000\ W$

$P_a = I_a^2 R_a = (50)^2(0.05) = 125\ W$

$P_{miscellaneous} = (0.01)(6000) = 60\ W$

$P_{out} = P_{in} - (P_a + P_{miscellaneous}) = 6000 - (125 + 60) = 5815\ W$

$P_{out} = \dfrac{5815}{746} = 7.79$

$P_{out} \cong 7.8\ hp$

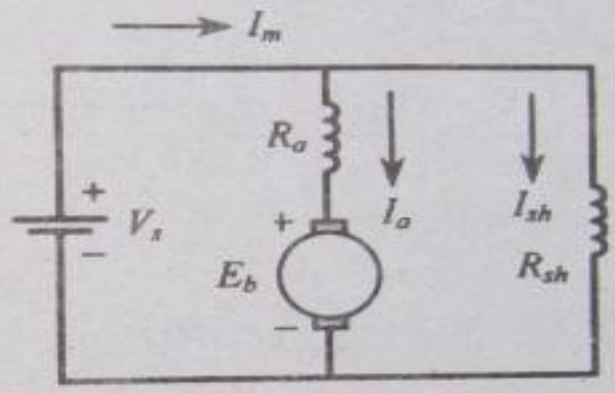
547 As a motor taking 4A from a 220 V source :

$I_{sh} = \dfrac{V_s}{R_{sh}} = \dfrac{220}{110} = 2\ A$

$I_{a1} = I_{m1} - I_{sh} = 4 - 2 = 2\ A$

$E_{b1} = V_s - I_{a1}R_a = 220 - 2(0.2)$

$E_{b1} = 219.6\ V$



$E_b = kN$

$\dfrac{E_{b1}}{E_g} = \dfrac{N_1}{N_g}$

$N_1 = N_g\left(\dfrac{E_{b1}}{E_g}\right) = \dfrac{1200(219.6)}{250} = 1054.08\ rpm$

As a motor taking 30 A from a 220 V source :

$I_{a2} = I_{m2} - I_{sh} = 30 - 2 = 28\ A$

$E_{b2} = V_s - I_{a2}R_a = 220 - 28(0.2) = 214.4\ V$

Note: In the last condition, the flux is no longer constant.

$E = kN\Phi$

$\dfrac{E_{b2}}{E_{b1}} = \dfrac{N_2\Phi_2}{N_1\Phi_1}$

$N_2 = N_1\left(\dfrac{E_{b2}\Phi_1}{E_{b1}\Phi_2}\right) = \dfrac{1054.08(214.4)(\Phi_1)}{219.6(0.98\ \Phi_1)}$

$N_2 \cong 1050\ rpm$

548 Condition 1:

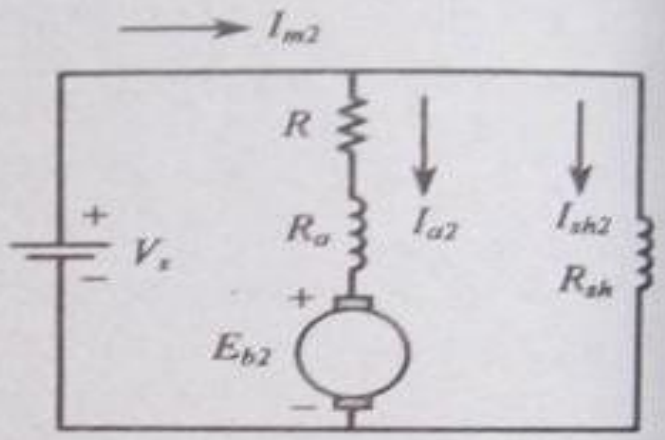
$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{260}{220} = 1.18\text{ A}$$

$$I_{a1} = I_{m1} - I_{sh} = 26 - 1.18 = 24.82\text{ A}$$

$$E_{b1} = V_s - I_{a1}R_a = 260 - 24.82(0.8) = 240.14\text{ V}$$

Note: Since torque is constant, the armature current is also constant.

$$E_{b2} = V_s - I_{a2}(R_a + R) = 200 - 24.82(0.8 + 1.2) = 210.36\text{ V}$$



$$E = kN$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$N_2 = N_1\left(\frac{E_{b2}}{E_{b1}}\right) = \frac{500(210.36)}{240.14}$$

$$N_2 \cong 438\text{ rpm}$$

549 $$I_m = \frac{P_{in}}{V_s} = \frac{30{,}000}{200} = 150\text{ A}$$

$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{200}{40} = 5\text{ A}$$

$$I_a = I_m - I_{sh} = 150 - 5 = 145\text{ A}$$

$$P_a = I_a^2R_a = (145)^2(0.0125) = 262.8125\text{ W}$$

$$P_{sh} = V_sI_{sh} = 200(5) = 1000\text{ W}$$

$$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$$

$$30{,}000 = 28{,}000 + 262.8125 + 1000 + P_{stray}$$

$$P_{stray} = 737.187\text{ W}$$

When output is 10 kW:

$$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$$

$$V_s(I_a + I_{sh}) = P_{out} + I_a^2R_a + I_{sh}V_{sh} + P_{stray}$$

$$V_sI_a + V_sI_{sh} = P_{out} + I_a^2R_a + I_{sh}V_{sh} + P_{stray}$$

$$200I_a = 10{,}000 + I_a^2(0.0125) + 737.187$$

$$I_a^2 - 16{,}000I_a + 858{,}974.96 = 0$$

Using the quadratic formula :

$$I_a = \frac{16{,}000 \pm \sqrt{16{,}000^2 - 4(1)(858{,}974.96)}}{2(1)} = \frac{16{,}000 \pm 15{,}892.26}{2}$$

$$I_a^{(-)} = 53.87 \text{ A}$$

$$I_m = I_a + I_{sh} = 53.87 + 5 = 58.87 \text{ A}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{V_s I_m} = \frac{10{,}000}{200(58.87)} \times 100\%$$

$$\eta = 84.93\%$$

550 Note: Since current drawn is the same, flux therefore is constant.

$$E_{b1} = V_s - I_{a1}R_{equiv} = 220 - 15(0.8) = 208 \text{ V}$$

$$E_{b2} = V_s - I_{a2}(R_{equiv} + R) = 220 - 15(0.8 + 2) = 178 \text{ V}$$

$$E = kN$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$N_2 = N_1\left(\frac{E_{b2}}{E_{b1}}\right) = \frac{700(178)}{208}$$

$$N_2 \cong 599 \text{ rpm}$$

551 Note: In a series motor if not specified, flux is directly proportional to the armature current.

$$E_{b1} = V_s - I_{a1}R_{equiv} = 110 - 20(0.1) = 108 \text{ V}$$

$$E_{b2} = V_s - I_{a2}R_{equiv} = 110 - 50(0.1) = 105 \text{ V}$$

$$E = kN\Phi$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2\Phi_2}{N_1\Phi_1}$$

$$N_2 = N_1\left(\frac{E_{b2}\Phi_1}{E_{b1}\Phi_2}\right) = \frac{1000(105)(\Phi_1)}{108(1.1\Phi_1)} = 883.83$$

$$N_2 \cong 884 \text{ rpm}$$

552 Note: In a series motor, the torque is directly proportional to the square of the armature current.

$$T = kI_a^2$$

$$\frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2$$

$$I_{a2} = I_{a1}\sqrt{\frac{T_2}{T_1}} = 30\sqrt{\frac{0.6T_1}{T_1}} = 23.24\text{ A}$$

$$E_{b1} = V_s - I_{a1}(R_a + R_{se}) = 400 - 30(0.1 + 0.2) = 391\text{ V}$$

$$E_{b2} = V_s - I_{a2}(R_a + R_{se}) = 400 - 23.24(0.1 + 0.2) = 393.028\text{ V}$$

Note: In a series motor, the flux is proportional to the armature current

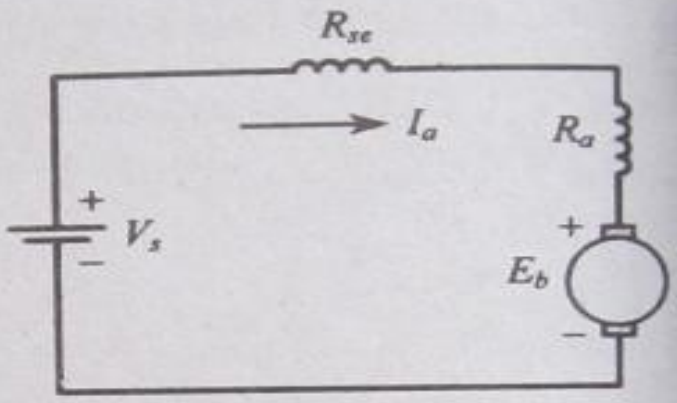
$$E = kN\Phi$$

$$\Phi = k'I_a$$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1 I_{a1}}{N_2 I_{a2}}$$

$$N_2 = \frac{N_1 E_{b2} I_{a1}}{E_{b1} I_{a2}} = \frac{1000(393.028)(30)}{391(23.24)}$$

$$N_2 = 1297.6\text{ rpm}$$



553 $$P_{in} = \frac{P_{out}}{\eta} = \frac{10(746)}{0.855} = 8725.146\text{ W}$$

$$I_m = I_a = \frac{P_{in}}{V_s} = \frac{8725.146}{230} = 37.93\text{ A}$$

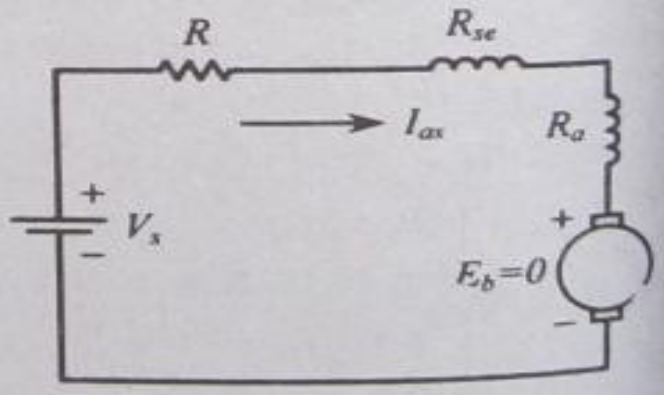
Note: In a series motor, torque is directly proportional to the square of the armature current.

$$T = kI_a^2$$

$$\frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2$$

$$I_{a2} = I_{a1}\sqrt{\frac{T_2}{T_1}} = 37.93\sqrt{\frac{1.5T_1}{T_1}}$$

$$I_{a2} = 46.45\text{ A}$$



Note: At starting, back emf is zero.

$$I_a = \frac{V_s}{R_a + R_{se} + R}$$

$$R = \frac{V_s}{I_a} - (R_a + R_{se}) = \frac{230}{46.45} - (0.3 + 0.25) = 4.4\ \Omega$$

554 $P_{in} = V_s I_m = 220(30) = 6600\ W$

$P_{cu} = I_a^2(R_a + R_{se}) = (30)^2(0.1 + 0.2) = 270\ W$

$P_{losses} = P_{cu} + P_{stray} = 270 + 350 = 620\ W$

$$\eta = \frac{P_{in} - P_{losses}}{P_{in}} = \frac{6600 - 620}{6600} \times 100\%$$

$\eta = 90.6\%$

555 $$P_{out} = \frac{2\pi NT}{60} = \frac{(2\pi)(600)(25)}{60} = 1570.796\ W$$

$P_d = P_{out} + P_{stray} = 1570.796 + 300 = 1870.796\ W$

$P_d = E_b I_a$

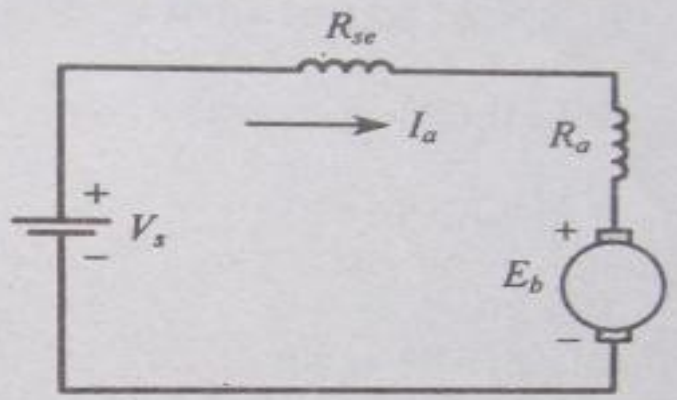
$P_d = [V_s - I_a(R_a + R_{se}]I_a$

$P_d = V_s I_a - I_a^2(R_a + R_{se})$

Substitute :

$1870.796 = 100 I_a - I_a^2(0.2 + 0.25)$

$I_a^2 - 222.22 I_a + 4157.32 = 0$



Using the quadratic formula :

$$I_a = \frac{222.22 \pm \sqrt{(222.22)^2 - 4(1)(4157.32)}}{2(1)} = \frac{222.22 \pm 180.976}{2}$$

$I_a^{(-)} = 20.62\ A$

556 $E_{b1} = V_s - I_{a1}R_a = 400 - 44(0.5) = 378\ V$

$E_{b2} = V_s - I_{a2}R_a = 400 - 36(0.5) = 382\ V$

Note: In a series motor, the flux is proportional to the armature current

$E = kN\Phi$

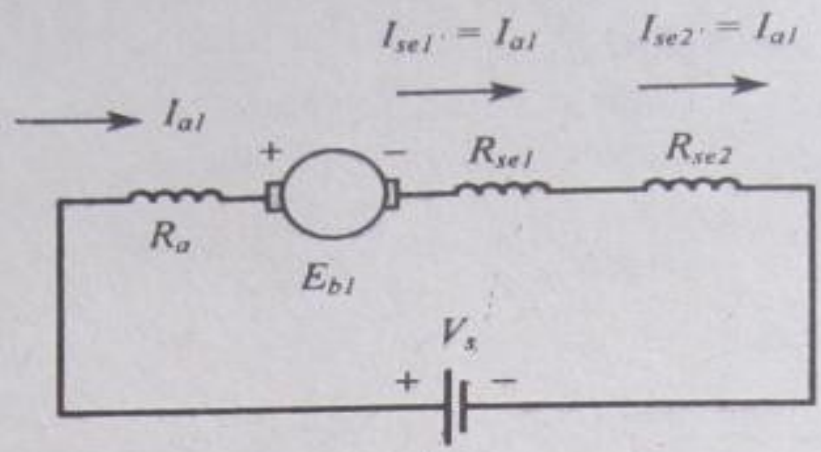
$\Phi = k' I_a$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1 I_{a1}}{N_2 I_{a2}}$$

$$N_2 = \frac{N_1 E_{b2} I_{a1}}{E_{b1} I_{a2}} = \frac{650(382)(44)}{378(36)}$$

$N_2 = 802.85\ rpm$

557

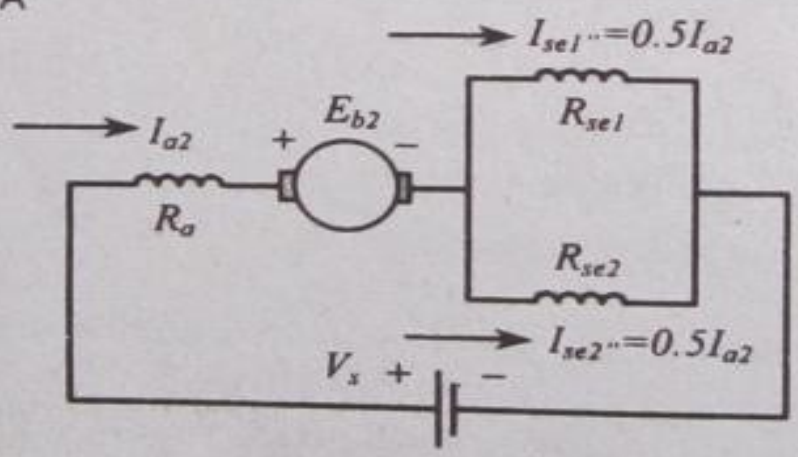


$E_{b1} = V_s - I_{a1}(R_a + R_{se1} + R_{se2})$
$E_{b1} = 100 - 50(0.04 + 0.04 + 0.04) = 94$ volts

Note: Total flux generated is proportional to the sum of the currents in the coils.

$\Phi = k(I_{se1} + I_{se2})$

$T_2 = 2T_1$
$k_t I_{a2} \Phi_2 = 2 k_t I_{a1} \Phi_1$
$I_{a2}[k(I_{se1}'' + I_{se2}'')] = 2I_{a1}[k(I_{se1}' + I_{se2}')]$
$I_{a2}(0.5I_{a2} + 0.5I_{a2}) = 2(50)(50 + 50)$
$I_{a2} = 100$ A



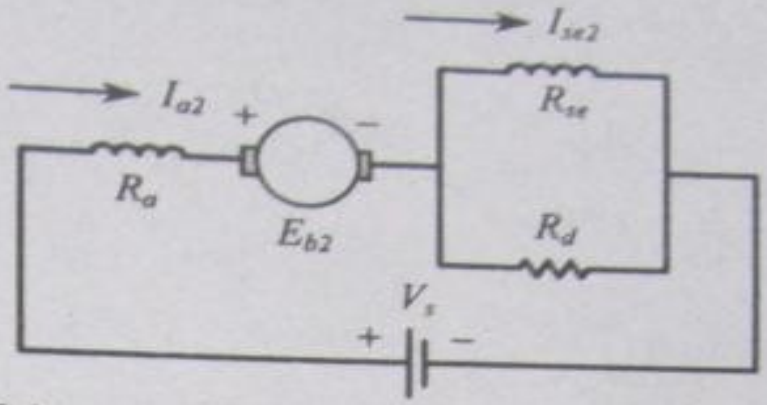
by KVL :
$E_{b2} = V_s - I_{a2}R_a - I_{se1}R_{se1} = 100 - 100(0.04) - 0.5(100)(0.04) = 94$ V

$$\frac{N_1}{N_2} = \frac{E_{b1}\Phi_2}{E_{b2}\Phi_1}$$
$\Phi = k'(I_{se_1} + I_{se_2})$
$$N_2 = \frac{N_1 E_{b_2}(I_{se_1}' + I_{se_2}')}{E_{b_1}(I_{se_1}'' + I_{se_2}'')} = \frac{800(94)(50 + 50)}{94(50 + 50)} = 800 \text{ rpm}$$

558 $E_{b1} = V_s - I_{a1}(R_a + R_{se}) = 500 - 100(0.1 + 0.04) = 486\ V$

$T = kI_a\Phi$

$$\frac{T_1}{T_2} = \frac{I_{a1}\Phi_1}{I_{a2}\Phi_2} = \frac{I_{a1}I_{se1}}{I_{a2}I_{se2}}$$



$$\frac{T_1}{0.5T_1} = \frac{100(100)}{I_{a2}\left(\frac{I_{a2}(0.08}{0.08 + 0.04}\right)}$$

$$I_{a2} = \sqrt{\frac{0.5(100)(100)}{0.6667}} = 86.6\ A$$

By CDT :

$$I_{se2} = \frac{86.6(0.08)}{0.08 + 0.04} = 57.733\ A$$

$$E_{b2} = V_s - I_{a2}\left(R_a + \frac{R_{se}R_d}{R_{se} + R_d}\right) = 500 - 86.6\left(0.1 + \frac{0.04(0.08)}{0.04 + 0.08}\right)$$

$$E_{b2} = 489\ V$$

$$\frac{N_1}{N_2} = \frac{E_{b1}\Phi_2}{E_{b2}\Phi_1} = \frac{E_{b1}I_{se2}}{E_{b2}I_{se1}}$$

$$N_2 = \frac{N_1E_{b2}I_{se1}}{E_{b1}I_{se2}} = \frac{820(489)(100)}{486(57.733)}$$

$$N_2 = 1429\ rpm$$

559 $E_{b1} = V_s - I_aR_a = 400 - 60(0.1) = 394\ V$

$T = kI_a^2$

$$\frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2$$

$$I_{a2} = \sqrt{\frac{T_2I_{a1}^2}{T_1}} = \sqrt{\frac{0.5T_1(60)^2}{T_1}} = 42.43\ A$$

$$E_{b2} = V_s - I_{a2}R_a = 400 - 42.43(0.1) = 395.76\ V$$

Note: In a series motor, flux is proportional to the armature current.

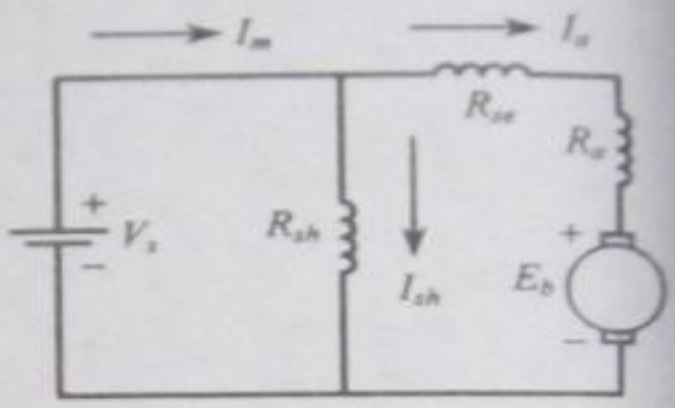
$E = kN\Phi$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1\Phi_1}{N_2\Phi_2}$$

$$N_2 = N_1\left(\frac{E_{b2}I_{a1}}{E_{b1}I_{a2}}\right) = 840\left(\frac{395.76(60)}{394(42.43)}\right)$$

$N_2 = 1193$ rpm

560



$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{230}{50} = 4.6\text{ A}$$

$I_a = I_m - I_{sh} = 42 - 4.6 = 37.4$ A

$P_a = I_a^2R_a = (37.4)^2(0.1)$
$= 139.876$ W

$P_{se} = I_a^2R_{se} = (37.4)^2(0.2)$
$= 279.752$ W

$P_{sh} = I_{sh}V_s = 4.6(230) = 1058$ W

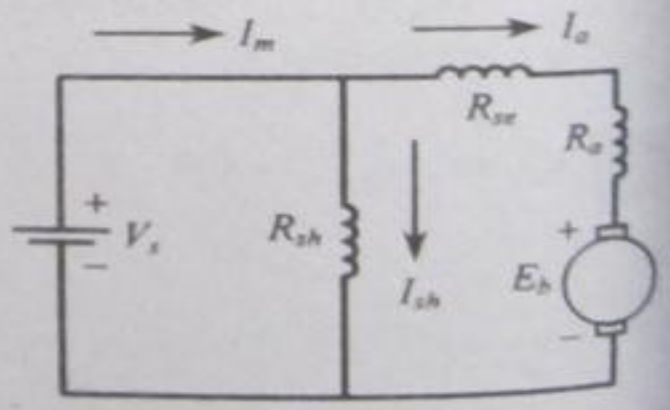
$P_{losses} = P_a + P_{se} + P_{sh} + P_{stray}$
$= 139.876 + 279.752 + 1058 + 400 = 1877.628$ W

$P_{in} = V_sI_m = 230(42) = 9660$ W

$$\eta = \frac{P_{in} - P_{losses}}{P_{in}} = \frac{9660 - 1877.628}{9660} \times 100\%$$

$\eta = 80.56\%$

561



$P_{stray} = V_sI_{aNL} - I_{aNL}^2(R_a + R_{se})$
$= 230(0.6) - (0.6)^2(2 + 0.25)$

$P_{stray} = 137.2$ W

At loaded condition:

$P_{out} = 5(746) = 3730$ W

$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{230}{460} = 0.5\text{ A}$$

$I_m = I_a + I_{sh} = I_a + 0.5$

$P_a = I_a^2R_a = I_a^2(2)$

$P_{se} = I_a^2R_{se} = I_a^2(0.25)$

$P_{sh} = V_sI_{sh} = 230(0.5) = 115$ W

$P_{in} = V_sI_m = 230(I_a + 0.5) = 230I_a + 115$

$$P_{in} = P_{out} + P_a + P_{se} + P_{sh} + P_{stray}$$
$$230 I_a + 115 = 3730 + 2 I_a^2 + 0.25 I_a^2 + 115 + 137.2$$
$$I_a^2 - 102.22 I_a + 1718.755 = 0$$

Using the quadratic formula :

$$I_a = \frac{102.22 \pm \sqrt{(102.22)^2 - 4(1)(1718.755)}}{2(1)} = \frac{102.22 \pm 59.782}{2}$$
$$I_a^{(-)} = 21.22 \text{ A}$$

562 $I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{250}{200} = 1.25 \text{ A}$

$$I_{aNL} = I_{mNL} - I_{sh} = 5 - 1.25 = 3.75 \text{ A}$$
$$I_{aFL} = I_{mFL} - I_{sh} = 86.5 - 1.25 = 85.25 \text{ A}$$

$$E_{bNL} = V_s - I_{aNL}(R_a + R_{se}) = 250 - 3.75(0.15 + 0.05) = 249.25 \text{ V}$$
$$E_{bFL} = V_s - I_{aFL}(R_a + R_{se}) = 250 - 85.25(0.15 + 0.05) = 232.95 \text{ V}$$

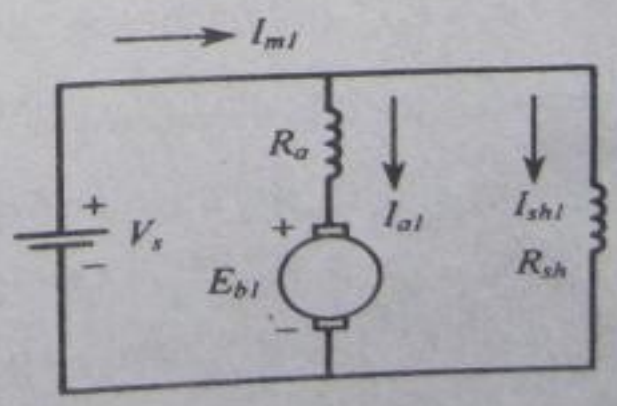
$$E = kN\Phi$$
$$\frac{E_{bFL}}{E_{bNL}} = \frac{N_{FL}\Phi_{FL}}{N_{NL}\Phi_{NL}}$$
$$\frac{\Phi_{FL}}{\Phi_{NL}} = \frac{E_{bFL} N_{NL}}{E_{bNL} N_{FL}} = \frac{232.95(1150)}{249.25(720)} = 1.49$$

$$T = k' I_a \Phi$$
$$\frac{T_{FL}}{T_{NL}} = \frac{I_{aFL}\Phi_{FL}}{I_{aNL}\Phi_{NL}} = \left(\frac{I_{aFL}}{I_{aNL}}\right)\left(\frac{\Phi_{FL}}{\Phi_{NL}}\right) = \left(\frac{85.25}{3.75}\right)(1.49) = 33.87$$
$$\frac{T_{FL}}{T_{NL}} \cong 34 \text{ times}$$

563 $I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{200}{125} = 1.6 \text{ A}$

As a shunt motor :

$$I_{a_1} = I_{m_1} - I_{sh} = 6 - 1.6 = 4.4 \text{ A}$$
$$E_{b_1} = V_s - I_{a_1} R_a = 200 - 4.4(0.15)$$
$$E_{b1} = 199.34 \text{ V}$$

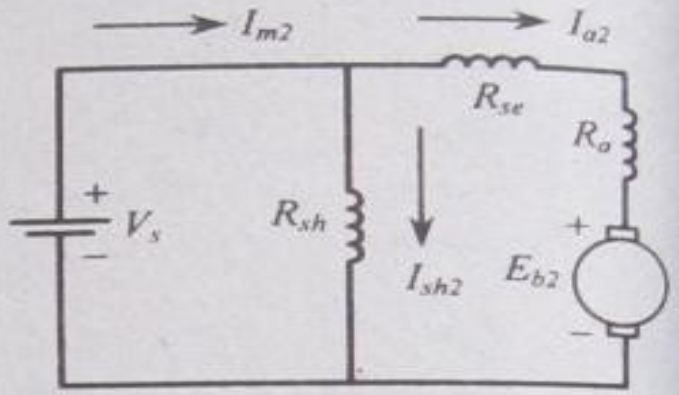


As a long shunt compound motor :

$I_{a2} = I_{m2} - I_{sh} = 100 - 1.6 = 98.4\ A$

$E_{b2} = V_s - I_{a2}(R_a + R_{se}) = 200 - 98.4(0.15 + 0.05)$

$E_{b2} = 180.32\ V$



$E = kN\Phi$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1\Phi_1}{N_2\Phi_2}$$

$$N_2 = N_1\left(\frac{E_{b2}\Phi_1}{E_{b1}\Phi_2}\right)$$

$$= 1200\left(\frac{180.32\Phi_1}{199.34\,(1.25)\Phi_1}\right)$$

$N_2 = 868.4$ rpm

564 $V_{sh} = E_b + I_aR_a$

$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{E_b + I_aR_a}{R_{sh}}$$

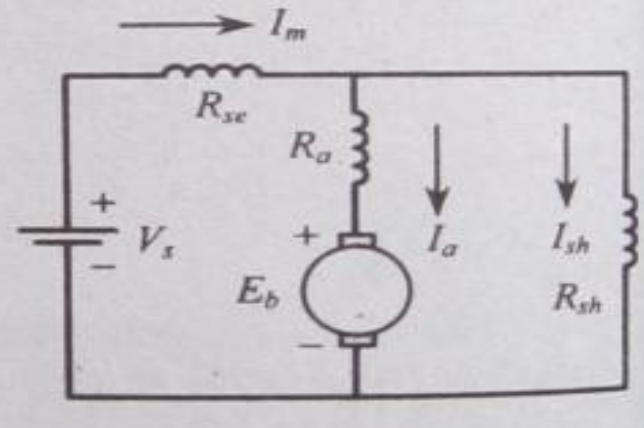
$$= \frac{E_b + 30(0.4)}{175}$$

$I_{sh} = 0.005714\,E_b + 0.06857$

$I_m = I_a + I_{sh}$

$= 30 + 0.005714\,E_b + 0.06857$

$I_m = 0.005714\,E_b + 30.06857$



$E_b = V_s - I_mR_{se} - I_aR_a$

$= 220 - (0.005714\,E_b + 30.06857)(4) - (30)(0.4)$

$E_b = 87.7257 - 0.02285\,E_b$

$E_b = 85.76$ volts

$P_d = E_bI_a = 85.76(30) = 2572.8\ W$

565 Note: Refer to the circuit diagram of problem #573.

$V_{sh} = V_s - I_mR_{se} = 220 - 6(0.08) = 219.52\ V$

$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{219.52}{200} = 1.1\,A$$

$I_a = I_m - I_{sh} = 6 - 1.0976 = 4.9\ A$

$P_a = I_a^2 R_a = (4.9)^2(0.10) = 2.4\ W$

$P_{se} = I_m^2 R_{se} = (6)^2(0.08) = 2.88\ W$

$P_{sh} = V_s I_{sh} = 219.52(1.1) = 241.47\ W$

$P_{in} = V_s I_m = 220(6) = 1320\ W$

$P_{in} = P_{out} + P_a + P_{se} + P_{sh} + P_{stray}$

$1320 = 0 + 2.4 + 2.88 + 241.47 + P_{stray}$

$P_{stray} = 1073.25\ W$

Note: At no load, the output power is zero.

566 T = (scale reading – dead weight) x length of arm

$T = (27.2 - 2.3) \times 2 = 49.8\ lb\text{-}ft$

$$HP_{out} = \frac{2\pi NT}{33000} = \frac{(2\pi)(910)(49.8)}{33000} = 8.628\ hp$$

$$HP_{in} = \frac{V_s I_m}{746} = \frac{220(34)}{746} = 10.026\ hp$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{8.628}{10.026} \times 100\%$$

$\eta = 86.056\%$

567 T = (scale reading – dead weight) x length of arm

$T = (11.9 - 1.09)(9.81) \times 0.61 = 64.688\ N\text{-}m$

$$HP_{out} = \frac{2\pi NT}{44760} = \frac{(2\pi)(910)(64.688)}{44760}$$

$HP_{out} = 8.26\ hp$

Note: To convert "kilogram" to "Newton", multiply "kg" by 9.81.

568 T = force on the rope x radius of pulley

$T = (57)(3/12) = 14.25\ lb\text{-}ft$

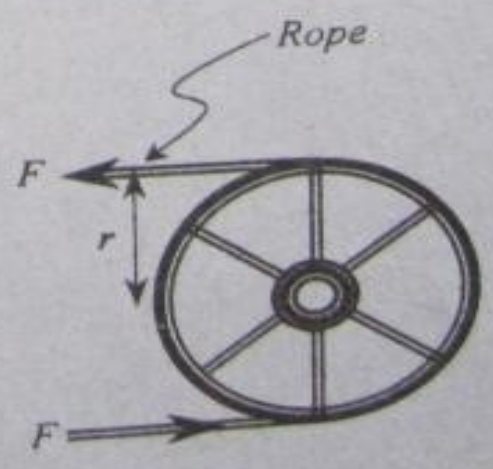
$$HP_{out} = \frac{2\pi NT}{33000} = \frac{(2\pi)(1400)(14.25)}{33000}$$

$= 3.798\ hp$

$$HP_{in} = \frac{V_s I_m}{746} = \frac{105(35)}{746} = 4.926\ hp$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{3.798}{4.926} \times 100\%$$

$\eta = 77.1\%$



569 T = (scale reading – dead weight) x length of arm

$T = (24 - 5.7) \times 3.5 = 64.05$ lb - ft

$$HP_{out} = \frac{2\pi NT}{33000} = \frac{(2\pi)(1215)(64.05)}{33000} = 14.817 \text{ hp}$$

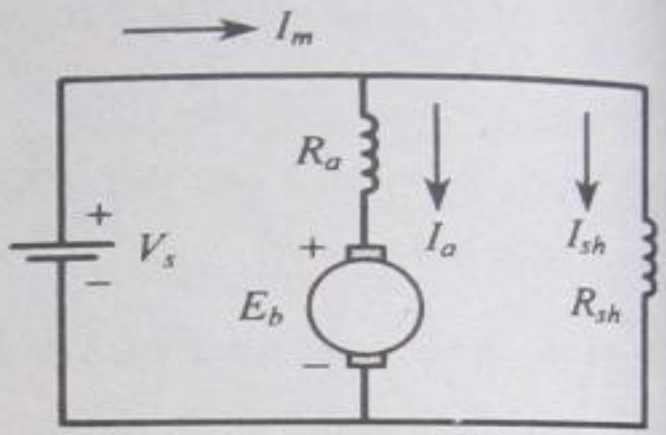
$$I_{sh} = \frac{V_s}{R_{sh}} = \frac{240}{240} = 1 \text{ A}$$

$I_a = I_m - I_{sh} = 50.9 - 1 = 49.9$ A

$P_a = I_a^2 R_a = (49.9)^2(0.1) = 249$ W

$P_{sh} = V_s I_{sh} = (240)() = 240$ W

$P_{in} = V_s I_m = 240(50.9) = 12216$ W



$P_{in} = P_{out} + P_a + P_{sh} + P_{stray}$

$12216 = 14.817(746) + 249 + 240 + P_{stray}$

$P_{stray} = 673.5$ W

570 T = (scale reading – dead weight) x length of arm

$T = (F - 3.75)(3)$

$$HP_{out} = \frac{2\pi NT}{33{,}000}$$

$$20 = \frac{2\pi(1400)(F - 3.75)(3)}{33{,}000}$$

$F = 28.76$ lbs.

Alternators

GENERATED EMF PER PHASE (DESIGN EQUATION)

$$E_p = 2.22\, k_p k_b f \Phi Z$$

$$k_b = \frac{\sin\left(\frac{n\delta}{2}\right)}{n \sin\left(\frac{\delta}{2}\right)}$$

$$k_p = \sin[(90°)(\text{pitch})]$$

$$\delta = \frac{180°}{\text{slots per pole}}$$

$$\text{pitch} = \frac{\text{coil span in number of slots}}{\text{slots per pole}}$$

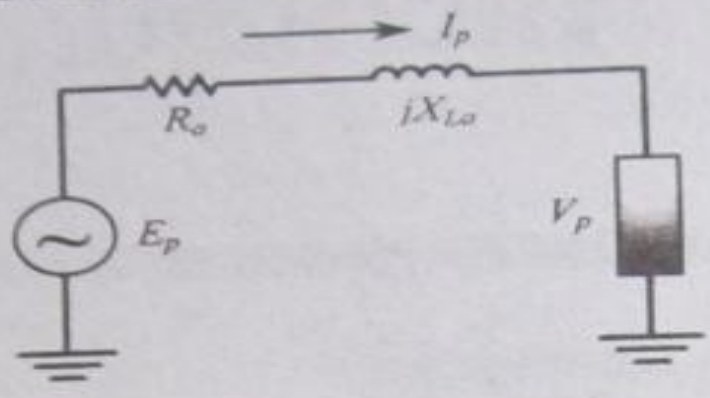
where: E_p = generated emf per phase (volt)
k_p = pitch factor or coil span factor
k_b = belt factor or distribution factor
f = frequency (Hertz)
Φ = flux per pole (weber)
Z = number of conductors in series per phase
n = number of slots per pole per phase

FREQUENCY OF THE GENERATED EMF

$$f = \frac{PN}{120}$$

where: f = frequency (Hertz)
P = number of poles
N = speed of rotor's rotation (rpm)

GENERATED EMF PER PHASE (ELECTRIC CIRCUIT RELATIONSHIP)



$$E_p = \sqrt{[V_p \cos\theta + I_p R_a]^2 + [V_p \sin\theta \pm I_p X_{La}]^2}$$

$$\alpha = \tan^{-1}\frac{[V_p \sin\theta \pm I_p X_{La}]}{V_p \cos\theta + I_p R_a}$$

where : E_p = generated emf per phase (volt)
V_p = load voltage per phase (volt)
I_p = armature current per phase (ampere)
R_a = armature winding resistance per phase (ohm)
X_{La} = inductive reactance of armature winding per phase (ohm)
$+I_pX_{La}$ = if pf is lagging
$-I_pX_{La}$ = if pf is leading
α = angle between E_p and I_p

- Using complex number format

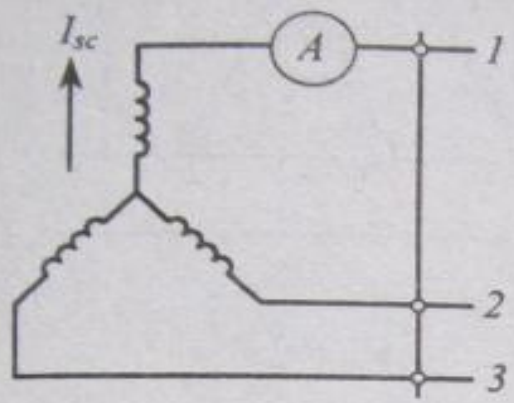
$$E_p\angle\pm\delta = V_p\angle 0^\circ + (I_p\angle\pm\theta)(R_a + jX_{La})$$

where: $+\theta$ = if pf is leading
$-\theta$ = if pf is lagging
δ = angle between E_p and V_p

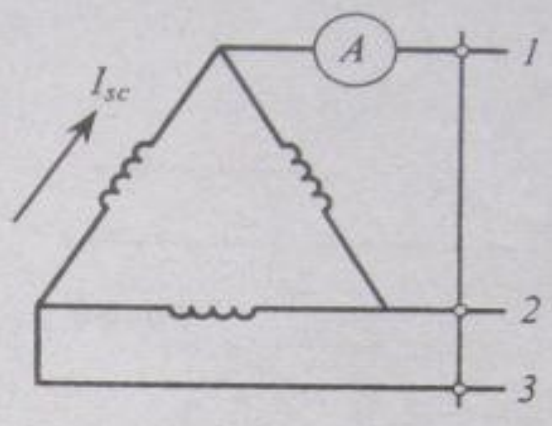
Note: The sign of δ is dependent after the result of simplifying the right side of the equation.

SHORT CIRCUIT TEST ON ALTERNATORS

The armature terminals are short circuited while a small field current is applied. Adjustments maybe made such that I_P is approximately equal to the rated load armature current per phase.



$$I_{sc} = A$$

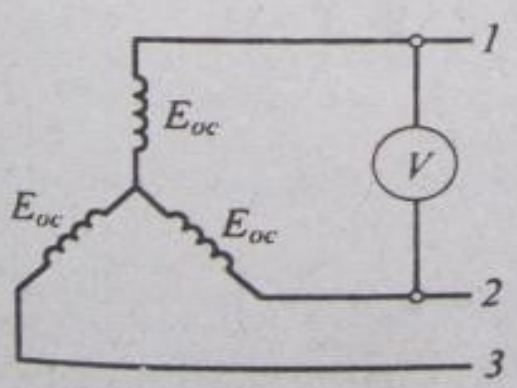


$$I_{sc} = \frac{A}{\sqrt{3}}$$

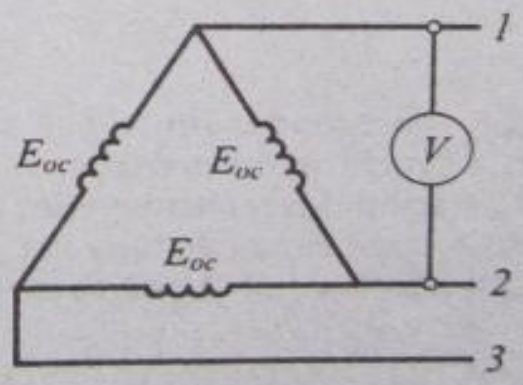
where: A = ammeter reading during the test

OPEN CIRCUIT TEST ON ALTERNATORS

The armature terminals are open circuited and the field current is made equal to the field current during the short circuit test.



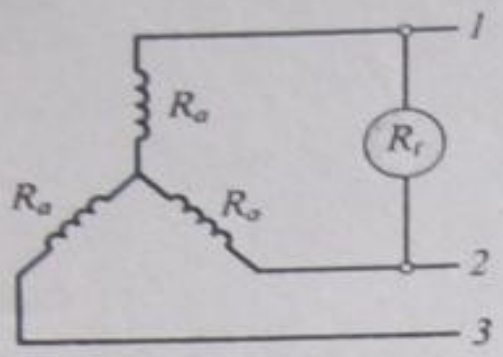
$$E_{oc} = \frac{V}{\sqrt{3}}$$

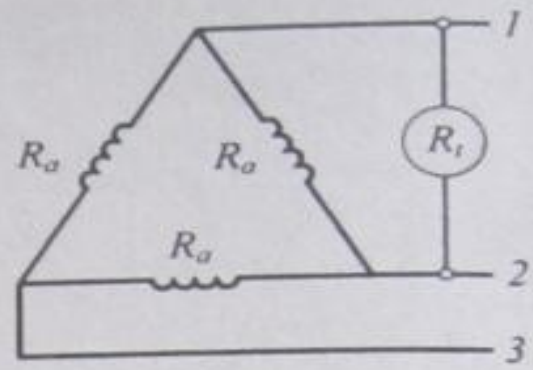


$$E_{oc} = V$$

where: V = voltmeter reading during the test

DC RESISTANCE TEST ON ALTERNATORS





$$R_a = \frac{1}{2}R_t$$

$$R_a = \frac{3}{2}R_t$$

where: R_t = DC resistance of the alternator between any two terminals.
= Ohmmeter reading during the test

Note: In practice, it is customary to multiply the DC resistance R_a by a factor of 1.25 to 1.75, to obtain a value more nearly equal to the armature resistance when it carries AC current.

ALTERNATOR PARAMETERS DERIVED FROM TESTS

$$Z_s = \frac{E_{oc}}{I_{sc}} \qquad X_s = \sqrt{(Z_s)^2 - (R_a)^2}$$

where : X_s = synchronous reactance per phase
Z_s = synchronous impedance per phase
R_a = armature resistance per phase
E_{oc} = open circuit voltage per phase
I_{sc} = short circuit armature current per phase

PER UNIT VALUES OF THE ALTERNATOR PARAMETERS

$$I_{base} = \frac{S_{base}}{V_{base}} \qquad Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

$$R_{pu} = \frac{R_a}{Z_{base}} \qquad X_{pu} = \frac{X_s}{Z_{base}} \qquad Z_{pu} = R_{pu} \pm jX_{pu}$$

where: S_{base} = rated apparent power per phase (volt-ampere)
V_{base} = rated voltage per phase (volt)
I_{base} = rated armature current per phase (ampere)
R_{pu}, X_{pu} and Z_{pu} = resistance, reactance and impedance in per unit values

VOLTAGE REGULATION

- **Voltage regulation** - percentage rise in the terminal voltage of the alternator when the alternator load is removed.

$$\%VR = \frac{E_p - V_p}{V_p}$$

$$\%VR = \sqrt{(\cos\theta + \%IR)^2 + (\sin\theta \pm \%IX)^2} - 1$$

where: +%IX = if pf is lagging
-%IX = if pf is leading

POWER DEVELOPED IN THE ARMATURE PER PHASE

$$P_d = \frac{E_p V_p}{X_s}\sin\delta \qquad P_{d_{max}} = \frac{E_p V_p}{X_s}$$

where : P_d = power developed per phase (watt)
P_{dmax} = maximum power developed (when, $\delta = 90°$)
E_p = generated emf per phase (volt)
V_p = terminal voltage per phase (volt)
X_s = synchronous reactance per phase (ohm)
δ = angle between E_p and V_p

POWER LOSSES AND EFFICIENCY

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} \qquad P_{losses} = P_{cu} + P_{hv} + P_{core} + P_f + P_v$$

$$P_{out(1\phi)} = V_L I_L \cos\theta \qquad P_{out(3\phi)} = \sqrt{3} V_L I_L \cos\theta$$

$$P_{cu(1\phi)} = I_p^2 R_a \qquad P_{cu(3\phi)} = 3 I_p^2 R_a$$

$$P_f = I_f^2 R_f = I_f V_f = \frac{V_f^2}{R_f}$$

where: V_L = line to line voltage
I_L = line current
I_p = phase current
$\cos\theta$ = load power factor
P_{cu} = armature winding copper losses
P_{fw} = friction and windage losses
= caused by mechanical friction and the fanning action of the rotor
P_{core} = core losses
= caused by eddy currents and hysteresis
P_v = ventilation losses
= the power required to circulate the cooling of air
P_f = field winding losses
I_f = field current (ampere)
R_f = field resistance (ohm)
R_a = armature resistance per phase
V_f = voltage across field winding (volt)

Note: Friction and windage loss, field loss, core loss and ventilation loss are constant losses, unless otherwise specified.

ARMATURE COPPER LOSSES AT ANY SIZE OF LOAD

$$P_{cu_x} = (x)^2 P_{cuFL}$$

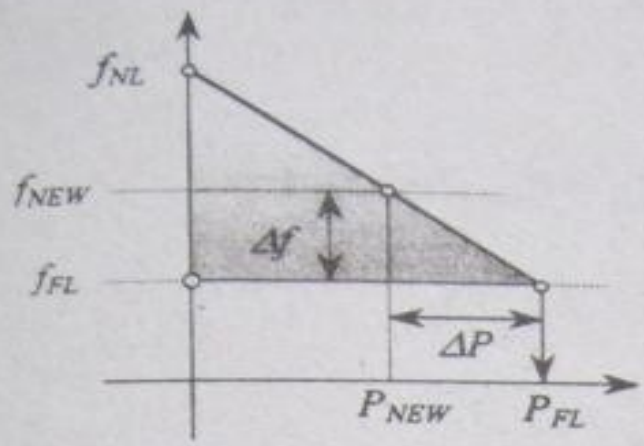
where: x = percentage loading on the alternator
P_{cu_x} = copper losses at x% load
P_{cuFL} = full load copper losses

GOVERNOR'S SPEED REGULATION (GSR)

$$GSR = \frac{N_{NL} - N_{FL}}{N_{FL}} \qquad GSR = \frac{f_{NL} - f_{FL}}{f_{FL}}$$

where: N_{NL} = no load speed
N_{FL} = full load speed
f_{NL} = no load frequency
f_{FL} = full load frequency

CHARACTERISTIC-TRIANGLE OF AN ALTERNATOR



$$\frac{\Delta P}{\Delta f} = \frac{P_{FL}}{f_{NL} - f_{FL}}$$

$$\frac{\Delta f}{\Delta P} = GD \qquad \frac{\Delta f}{\Delta P} = \frac{GSR \times f_{FL}}{P_{FL}}$$

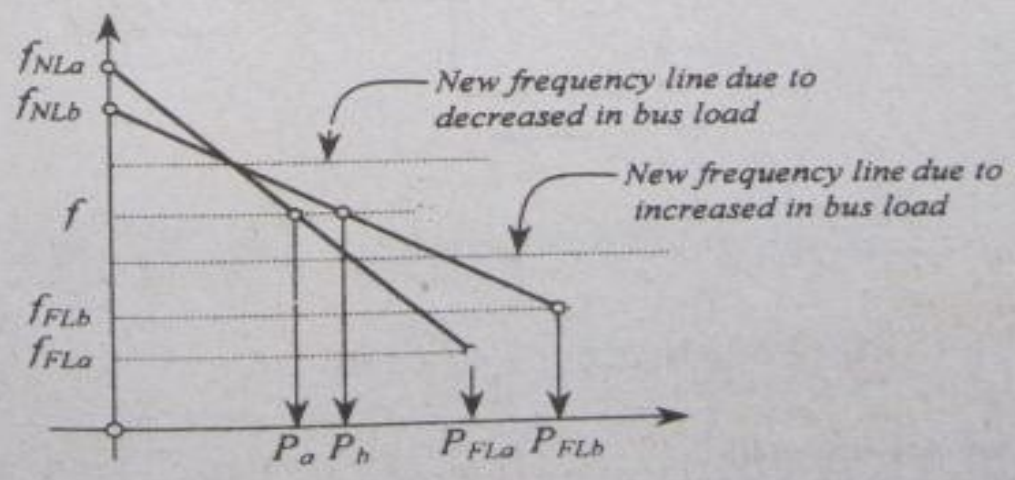
where: Δf = change in frequency due to change in load
P_{FL} = rating of the alternator
GD = governor's droop

PARALLEL OPERATION OF ALTERNATORS

Requirements:

- Equal magnitude of terminal voltage
- Equal operating frequencies
- The same phase sequence.

❖ **Case I: With external characteristics given (kW rating, speed regulations, bus load, etc.)**



$$\frac{\Delta f}{\Delta P_a} = \frac{GSR_a \times f_{FLa}}{P_{FLa}} \qquad \frac{\Delta f}{\Delta P_b} = \frac{GSR_b \times f_{FLb}}{P_{FLb}}$$

Before bus load changes,

$$P_t = P_a + P_b$$

After bus load changes,

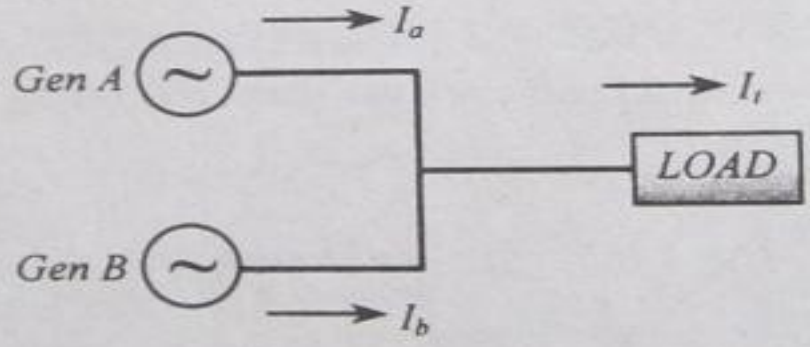
$$P_{tNEW} = P_t \pm \Delta P \qquad P_{tNEW} = P_{aNEW} + P_{bNEW}$$

$$P_{aNEW} = P_a \pm \Delta P_a \qquad \Delta P = \Delta P_a + \Delta P_b$$

$$P_{bNEW} = P_b \pm \Delta P_b \qquad f_{NEW} = f \pm \Delta f$$

where : $+\Delta P$ = if there is an increase in bus load
$-\Delta P$ = if there is a decrease in bus load
f = frequency prior to the change in bus load
f_{NEW} = frequency after the change in bus load
$+\Delta f$ = if there is a decrease in bus load
$-\Delta f$ = if there is a increase in bus load

- **CASE II. With load sharing conditions given (load carried by each alternator, operating power factor of each alternator, current delivered, etc).**



$$I_t\angle\pm\theta_t = I_a\angle\pm\theta_a + I_b\angle\pm\theta_b \qquad P_t = P_a + P_b$$

$$S_t\angle\pm\theta_t = S_a\angle\pm\theta_a + S_b\angle\pm\theta_b$$

where: P = real power (watt)
S = apparent power (volt-ampere)
$+\theta$ = if operating at a leading pf
$-\theta$ = if operating at a lagging pf

Note: The + sign or – sign for θ_t is dependent after the result of simplifying the right side of the equation.

Test 16

Problem 571: EE Board October 1980

What is the speed of a 100 kW, 230-volt, three-phase, four-pole, 60 Hz alternator?

A. 450 rpm
B. 900 rpm
C. 1200 rpm
D. 1800 rpm

Problem 572: EE Board April 1997

A six-pole, 3-phase AC generator has 72 slots, the coil span is 12. What is the pitch factor?

A. 1.000
B. 0.939
C. 0.985
D. 0.966

Problem 573:

A three-phase alternator has 6 slots per pole per phase. Determine the distribution factor.

A. 0.977
B. 0.956
C. 0.963
D. 0.948

Problem 574: EE Board October 1994

A 30 MVA, 15 kV three-phase alternator will have a per phase nominal impedance of _____

A. 7.5 ohms
B. 9 ohms
C. 15 ohms
D. None of these

Problem 575:

A two-pole three phase 50 Hz Y-connected AC generator has 9 slots per pole and 6 conductors per slot. What is the flux per pole if the voltage on open circuit is 1.1 kV. Assume a coil span of unity.

A. 0.1657 Wb
B. 0.1742 Wb
C. 0.1507 Wb
D. 0.1728 Wb

Problem 576: EE Board April 1990

A 6-pole, 3-phase, 60-cycle alternator has 12 slots per pole and four conductors per slot. The winding is 5/6 pitch. There are 2,500,000 maxwells (0.025 weber) entering the armature from each north pole, and this flux is sinusoidally distributed along the air gap. The armature coils are all connected in series. The winding is wye-connected. Determine the open circuit emf of the alternator.

A. 532.1 V
B. 504.2 V
C. 512.4 V
D. 572.4 V

Problem 577:

A four –pole, three-phase, 50 Hz, star connected AC generator has 24 stator slots. There are 6 conductors per slot and a sinusoidally distributed flux per pole of 63 mWb. Find the terminal voltage on open circuit, if the coil span factor is unity.

A. 546.05 V
B. 579.44 V
C. 568.48 V
D. 558.62 V

Problem 578:

A three-phase, 6-pole, 72-slot armature of a synchronous generator is wound using a two-layer lap coil having 20 conductors per coil with a pitch of 83.33%. The flux per pole is 4,800,000 maxwells. If the rotor of the generator is driven at a speed of 1200 rpm, calculate the induced emf per phase.

A. 5,680 V
B. 5,390 V
C. 5,564 V
D. 5,854 V

Problem 579:

A 1MVA, 11 kV, three-phase wye-connected synchronous generator has a synchronous reactance of 5 ohms and a negligible resistance per phase. At a certain field current the generator delivers rated load at 0.9 lagging power factor at 11 kV. For the same excitation, what is the terminal voltage at 0.9 leading power factor full-load?

A. 11.6 kV
B. 11.4 kV
C. 11.3 kV
D. 11.5 kV

Problem 580:

A three-phase star connected AC generator is rated at 2.5 MVA, 11 kV, 60 Hz. The generator has a negligible resistance and a synchronous reactance 20 ohms per phase. To what value will the terminal voltage rise when full load at 80% pf lagging is switched-off?

A. 14.2 kV
B. 13.5 kV
C. 15.3 kV
D. 12.7 kV

Problem 581: EE Board April 1995

A 1200 kVA, 6600 volt, 3-phase Y connected alternator has an effective resistance of 0.40 ohm and a reactance of 6 ohms per phase. It delivers full load current at 0.80 lagging power factor at rated voltage. What will be the terminal voltage for the same excitation and load current, if the power factor is 0.80 leading?

A. 4560 V
B. 9878 V
C. 7898 V
D. 4250 V

Problem 582:

A 2500 kVA, 6600 V synchronous turbo alternator is operating at full load and 0.8 pf lagging. This machine has a synchronous reactance per phase is 10.4 ohms and a negligible resistance. Calculate the percentage voltage regulation?

A. 48.56%
B. 42.35%
C. 45.84%
D. 43.96%

Problem 583:

A 50-kVA, 550-volt single-phase alternator draws a field current 10 A at rated load. With the same field current, the open circuit emf and short-circuit current are 300 V and 150 A. The ohmic resistance of the armature between terminals is 0.18 ohm. The ratio of effective to ohmic resistance maybe taken as 1.5. Determine the regulation at 0.8 power factor lagging.

A. 27.8%
B. 26.4%
C. 25.5%
D. 24.9%

Problem 584: EE Board March 1998

A generator is rated 100 MW, 13.8 kV and 90% power factor. The effective resistance is 1.5 times the ohmic resistance. The ohmic resistance is obtained by connecting two terminals to a DC source. The current and voltage are 87.6 A and 6 V respectively. What is the effective resistance per phase?

A. 0.0513 ohm
B. 0.0685 ohm
C. 0.342 ohm
D. 0.0617 ohm

Problem 585: EE Board October 1997

To get the armature resistance of a 100 MW, 13.8 kV and 90% power factor generator, two terminals are connected to a DC source. The measured current and voltage are 87.6 amperes and 6 volts respectively. What is the DC resistance per phase?

A. 0.0223 ohm
B. 0.0342 ohm
C. 0.0685 ohm
D. 0.0617 ohm

Problem 586:

A 2 MVA, 2.3-kV three-phase Y-connected alternator operates at rated kVA at a power factor of 80%. The dc armature winding resistance between terminals is 0.08 Ω. The field takes 70 A at 125 V from an exciter equipment. Friction and windage loss is 20 kW, iron losses 36 kW and stray load losses are 2.0 kW. Calculate the efficiency of the alternator. Assume the effective armature-winding resistance is 120% of the DC resistance.

A. 94.0%
B. 92.1%
C. 93.2 %
D. 89.5%

Problem 587: EE Board October 1991

Determine the efficiency of a 1500 kVA, 2300 V, 3-phase, Y-connected alternator, which operates at rated output with a power factor of 80%. The DC armature resistance at 70 °C between terminals is 0.08 ohm. The field takes 70 A at 120 V from the exciter equipment. Friction and windage losses are 15 kW, iron loss is 35 kW and stray load losses is 1.5 kW. Assume the effective armature winding resistance is 1.5 times the DC value.

A. 94.56%
B. 90.31%
C. 93.35%
D. 92.62%

Problem 588:

A 2,500 kVA, three-phase, 60 Hz, 6.6 kV wye-connected alternator has a field resistance of 0.45 Ω and an armature resistance of 0.05 Ω per phase. The field current at full load 0.85 pf, is 200 A. The stray power losses amount to 82 kW. Calculate the efficiency of the alternator at full load, 0.85 pf lagging.

A. 93.7%

B. 94.8%
C. 95.2%
D. 96.3%

Problem 589:

A 6.6 kV, Y-connected, 3-phase alternator has a synchronous reactance of 6 Ω per phase and a negligible resistance. At a certain field current the armature is known to induce 5.8 kV per phase. Calculate the maximum power that can be developed by the alternator at this excitation.

A. 12 MW
B. 11 MW
C. 9.8 MW
D. 10 MW

Problem 590:

An 11,000 volts three-phase wye-connected turbo alternator has a synchronous reactance of 6 ohms and a negligible resistance per phase. When the field current is 8 A, the open circuit voltage is 12,000 volts. Determine the armature current when alternator developed maximum power.

A. 1456 A
B. 1712 A
C. 1643 A
D. 1566 A

Problem 591: EE Board April 1994

A three-phase, 11-kV wye connected synchronous alternator has a synchronous reactance of 8 ohms per phase but negligible resistance. If the excitation is such that the open circuit voltage is 14 kV, determine the power factor at the maximum output.

A. 0.786
B. 0.772
C. 0.793
D. 0.708

Problem 592: EE Board April 1994

A three-phase AC generator is supplying power to a load of 3200 kW at 2300 volts and a power factor of 60%. Assume that the loss of the line, the generator armature and the load is equal to 10% of the load, what would be the savings in watts if the power factor were raised to 80%?

A. 100,000
B. 140,000
C. 80,000
D. 230,000

Problem 593:

A 3-phase 50 MVA, 13.8 kV, 3600 rpm, 60 Hz grid connected generator with a synchronous reactance of 3.8 ohms per phase operating with a lagging power factor of 0.88 at rated voltage and current. Determine new power factor if the rotor current is decreased by 5%.

A. 0.920
B. 0.935
C. 0.907
D. 0.977

Problem 594:

A 3-phase 50 MVA, 13.8 kV, 3600 rpm, 60 Hz grid connected generator with a synchronous reactance of 3.8 ohms per phase operating with a lagging pf of 0.88 at rated voltage and current. If the load decreases such that its prime mover power is decreased by 25% and the rotor current decreases by the same percentage, what will be the new pf ?

A. 0.928
B. 0.998
C. 0.987
D. 0.909

Problem 595:

A 3-phase 50 MVA, 13.8 kV, 3600 rpm, 60 Hz grid connected generator with a synchronous reactance of 3.8 ohms per phase operating with a lagging pf of 0.88 at rated voltage and current. If the machine is to provide a lagging reactive power of 40 MVAR and still operating at its rated current, what percent change in the rotor current is required?

A. 10.48%
B. 10.24%
C. 11.30%
D. 12.28%

Problem 596: EE Board April 1992

A three-phase turbo alternator has a reactance of 15 ohms and negligible resistance, the machine draws an armature current of 250 amperes at 0.8 pf lagging when running on 12,000 volts infinite busbars. If the steam admission is constant but the emf is raised by 20%, calculate the new operating pf.

A. 0.547
B. 0.586
C. 0.509
D. 0.575

Problem 597:

A three-phase, 20 kV, synchronous alternator has a reactance of 10 ohms and a negligible resistance. The machine draws an armature current of 150 A, 0.80 pf lagging. If the steam admission is constant and field current is adjusted raising the induced the emf by 25%, determine the value of the new armature current.

A. 442 A
B. 452 A
C. 421 A
D. 433 A

Problem 598: EE Board April 1980

A standby Diesel generator set will have the following loads: Inductive load drawing 50 kVA at 0.8 power factor: Lighting load drawing 20 kVA at 1.0 power factor. At what power factor will the generator operate?

A. 0.855 lagging
B. 0.872 lagging
C. 0.821 lagging
D. 0.894 lagging

Problem 599: EE Board April 1992

A 250 V, 30 Hz generator supplies power to a parallel circuit consisting of a 20 hp motor whose efficiency is 90% at 0.80 pf lagging and a second load that draws an apparent power of 7 kVA at unity pf. Determine the system reactive power.

A. 23.582 kVAR
B. 12.435 kVAR
C. 10.341 kVAR
D. 20.384 kVAR

Problem 600:

Two 3-ϕ, 4160 V, 60 Hz alternators are operated in parallel. The total load of the system is 1050 W at 75% lagging pf. If alternator A is carrying 700 kW at 80% pf lagging, determine kVAR of alternator B.

A. 401 kVAR
B. 405 kVAR
C. 411 kVAR
D. 408 kVAR

Problem 601: EE Board October 1980

Two alternators A and B are operating in parallel supplying a load drawing 1000 kVA at 0.80 power factor lagging. If alternator A contributes 500 kVA at 0.60 power factor lagging, determine the power factor of alternator B.

A. 0.928 lagging
B. 0.852 lagging

C. 0.901 lagging
D. 0.977 lagging

Problem 602:

Two identical three-phase Y-connected alternators A and B share equally a load of 10 MW at 33 kV and 80% lagging pf. The synchronous reactance of each machine is 4 ohms per phase and the armature resistance is negligible. Alternator A has its field excitation adjusted to carry 125 A lagging current, what is the current supplied by alternator B?

A. 96 A
B. 97 A
C. 98 A
D. 95 A

Problem 603:

Alternator A (100 kVA, 3-ϕ, 240 V, 60 Hz, 1800 rpm) is operating in parallel with alternator B (125 kVA, 3-ϕ, 240 V, 60 Hz, 1800 rpm). The load of alternator A is 60 kW at 90% pf lagging and the load of alternator B is 80 kW at 70% pf lagging. Determine pf of load.

A. 0.775 lagging
B. 0.784 lagging
C. 0.893 lagging
D. 0.866 lagging

Problem 604:

Two 3-phase, Y-connected turbo alternators supply a load of 3000 kW at 6.6 kV and at 0.8 pf lagging. The excitation of machine A is adjusted so that it delivers 150 A at a lagging power factor, and the governors are so set that the load is shared equally between the machines. Determine the operating power factor of machine B.

A. 0.726
B. 0.792
C. 0.704
D. 0.768

Problem 605: EE Board October 1981

Two alternators are operating in parallel supplying a common load of 4250 kW at 0.85 power factor lagging. Alternator A contributes 600 kVAR at 0.92 power factor lagging. Solve for the power factor of alternator B

A. 0.813 lagging
B. 0.834 lagging
C. 0.804 lagging
D. 0.820 lagging

Problem 606:

Two Y-connected alternators A and B running in parallel supply the following loads at 3.3 kV:

Load 1: 800 kW at unity pf
Load 2: 600 kW at 0.8 pf lagging
Load 3: 400 kW at 0.707 pf lagging

If alternator A is adjusted to carry an armature current of 150 A at 0.85 pf lagging, what is the armature current of alternator B?

A. 208 A
B. 180 A
C. 200 A
D. 190 A

Problem 607:

Two AC generators running in parallel supply the following loads: 400 kW at unity pf; 500 kW at 0.9 pf lagging; 600 kW at 0.866 pf lagging. One machine is loaded to 1000 kW at a pf of 0.92 lagging, what is the operating power factor of the second machine.

A. 0.964 lagging
B. 0.921 lagging
C. 0.932 lagging
D. 0.951 lagging

Problem 608:

Two alternators are connected in parallel. The total load is 4250 kW at 0.85 pf lagging. Alternator A operates with a load of 2125 kW at 0.707 pf lagging. Determine the kVA load of alternator B.

A. 2245
B. 2078
C. 2185
D. 2410

Problem 609: EE Board April 1988

Sipalay Mines, has two 3-phase, 60 Hz AC generators operating in parallel. The first unit has a capacity of 1000 kVA and the second unit has a capacity of 1500 kVA. The first is driven by a prime mover so adjusted that the frequency fall from 61 Hz at no-load to 59.6 Hz at full load. The second has a different speed-load characteristics, the frequency fall from 61.4 Hz at no-load to 59.2 Hz at full load. When these alternators are jointly delivering 2000 kW, what is the load of each generator?

A. 884 kW, 1116 kW
B. 867 kW, 1133 kW
C. 915 kW, 1085 kW
D. 804 kW, 1196 kW

Problem 610: EE Board March 1998

Two alternators are driven by shunt motors. The shunt motors have speed-load droop characteristics of 3% and 4% respectively. The alternators are in parallel and each carrying 50 kW. There is no automatic speed-load control. An additional 50 kW is switched on. What are the resulting loads of the alternators assuming that the speed-load control of each is not adjusted?

A. 78.57 kW / 71.43 kW
B. 82.51 kW / 67.49 kW
C. 75 kW / 75 kW
D. 80.31 kW / 69.69 kW

Problem 611: EE Board April 1989

There are two alternators, 100 kW, 3-phase in parallel are driven by shunt motors whose speed-load characteristics are as follows: Alternator No.1, no-load speed 600 rpm and the full-load speed 530 rpm, while alternator No. 2, no-load speed 590 rpm and full-load speed 550 rpm. For what load will the alternator divide the load equally?

A. 62.45 kW
B. 68.78 kW
C. 67.54 kW
D. 64.67 kW

Problem 612: EE Board April 1989

There are two alternators, 100 kW, 3-phase in parallel are driven by shunt motors whose speed-load characteristics are as follows: Alternator No.1, no-load speed 600 rpm and the full-load speed 530 rpm, while alternator No. 2, no-load speed 590 rpm and full-load speed 550 rpm. What is the greatest load that can be delivered without overloading either alternator?

A. 171.43 kW
B. 168.78 kW
C. 177.54 kW
D. 164.67 kW

ANSWER KEY

571. D	580. A	589. B	598. D	607. D
572. A	581. C	590. D	599. B	608. C
573. B	582. D	591. A	600. A	609. A
574. A	583. C	592. B	601. A	610. A
575. A	584. A	593. A	602. B	611. C
576. C	585. B	594. B	603. B	612. A
577. B	586. A	595. A	604. A	
578. A	587. C	596. D	605. A	
579. B	588. C	597. C	606. C	

☑ RATING

37-42 - Topnotcher

29-36 - Passer

21-28 - Conditional

0-20 - Failed

Solutions to Test 16

571 $N = \frac{120f}{P} = \frac{120(60)}{4}$

$N = 1800$ rpm

572 $\text{Pitch} = \frac{\text{Coil span}}{\text{Slots per pole}} = \frac{12}{72/6} = 1.0$

Pitch factor = sin (90° x pitch) = sin (90° x 1)

Pitch factor = 1.0

573 n = slots per pole per phase = 6

slot / pole = 6(3) = 18

$$\delta = \frac{180°}{\text{slots/pole}} = \frac{180°}{18} = 10°$$

$$k_d = \frac{\sin\frac{n\delta}{2}}{n\sin\frac{\delta}{2}} = \frac{\sin\frac{6(10°)}{2}}{6\sin\frac{10°}{2}}$$

$k_d = 0.956$

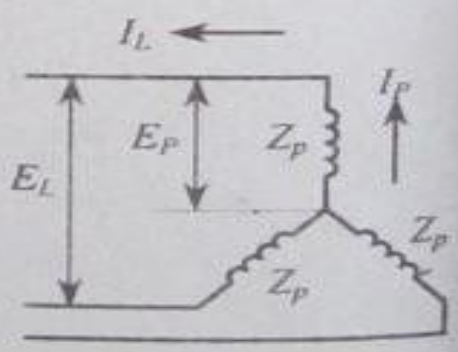
574 Note: Assume the load is wye-connected in the following derivation. The final formula is still the same if the load is delta-connected.

$Z_P = \frac{E_P}{I_P}$; $E_P = \frac{E_L}{\sqrt{3}}$; $I_L = \frac{S}{\sqrt{3}E_L}$; $I_P = I_L$

Combining the above relations:

$$Z_p = \frac{E_L^2}{S} = \frac{(15000)^2}{30 \times 10^6}$$

$Z_p = 7.5\,\Omega$



575 $\delta = \frac{180°}{\text{slots per pole}} = \frac{180°}{9} = 20°$

n = slots per pole per phase = 9/3 = 3

$$k_d = \frac{\sin\frac{n\delta}{2}}{n\sin\frac{\delta}{2}} = \frac{\sin\frac{3(20°)}{2}}{3\sin\frac{20°}{2}} = 0.959$$

$k_p = \sin(90° \times \text{pitch}) = \sin(90° \times 1) = 1$

$$Z = \frac{9 \text{ slots/pole} \times 2 \text{ poles} \times 6 \text{ conductors per slot}}{3} = 36$$

$E_P = 2.22\, k_p k_d f \Phi Z$

$$\Phi = \frac{E_P}{2.22\, k_p k_d f Z} = \frac{1100/\sqrt{3}}{2.22(1)(0.959)(50)(36)}$$

$\Phi = 0.1657$ Wb

576 $$n = \frac{\text{slots/pole}}{\text{phase}} = \frac{12}{3} = 4$$

$$\delta = \frac{180°}{\text{slots/pole}} = \frac{180°}{12} = 15°$$

$$k_d = \frac{\sin\frac{n\delta}{2}}{n\sin\frac{\delta}{2}} = \frac{\sin\frac{4(15°)}{2}}{4\sin\frac{15}{2}} = 0.958$$

$k_p = \sin(90° \times \text{pitch}) = \sin(90° \times 5/6) = 0.966$

$$Z = \frac{12 \text{ slots}}{\text{pole}} \times 6 \text{ poles} \times \frac{4 \text{ cond}}{\text{slot}} = 96$$

$E_p = 2.22\, k_p k_d f \Phi Z = 2.22(0.966)(0.958)(60)(0.025)(96)$

$= 295.84$ V

$E_L = \sqrt{3}(295.84) = 512.4$ V

577 $$n = \frac{\text{slots/pole}}{\text{phase}} = \frac{24/4}{3} = 2$$

$$\delta = \frac{180°}{\text{slots/pole}} = \frac{180°}{24/4} = 30°$$

$$k_d = \frac{\sin\frac{n\delta}{2}}{n\sin\frac{\delta}{2}} = \frac{\sin\frac{2(30°)}{2}}{2\sin\frac{30}{2}} = 0.966$$

$k_p = \sin(90° \times \text{pitch}) = \sin(90° \times 1) = 1.0$

$$Z = \frac{24 \text{ slots}}{3 \text{ phase}} \times \frac{6 \text{ cond}}{\text{slot}} = 48$$

$E_p = 2.22\, k_p k_d f \phi Z = 2.22(1.0)(0.966)(50)(65 \times 10^{-3})(48)$

$= 334.54$ V

$E_L = \sqrt{3}(334.54) = 579.44$ V

578 $\delta = \dfrac{180°}{\text{slots per pole}} = \dfrac{180°}{72/6} = 15°$

$n = \dfrac{\text{slots / pole}}{\text{phase}} = \dfrac{72/6}{3} = 4$

$k_d = \dfrac{\sin\frac{n\delta}{2}}{n\sin\frac{\delta}{2}} = \dfrac{\sin\frac{4(15°)}{2}}{4\sin\frac{15°}{2}} = 0.958$

$k_p = \sin[90° \times \text{pitch}] = \sin[90° \times 0.8333] = 0.966$

$Z = \dfrac{72\text{ slots}}{3\text{ phase}} \times \dfrac{20\text{ conductors}}{\text{coil}} \times \dfrac{2\text{ coils}}{\text{slot}} = 960$

$f = \dfrac{PN}{120} = \dfrac{6(1200)}{120} = 60\text{ Hz}$

$E_p = 2.22 k_p k_d f\Phi Z \times 10^{-8}$

$= (2.22)(0.966)(0.958)(60)(4{,}800{,}000)(960) \times 10^{-8}$

$E_p = 5680.14 \cong 5680\text{ V}$

579 $I = \dfrac{S}{\sqrt{3}E} = \dfrac{1{,}000{,}000}{\sqrt{3}(11{,}000)} = 52.48\text{ A}$

$\theta = \cos^{-1}0.9 = 25.84°$

$E_{p1} = V_p + I_{p1} X_p$

$= \dfrac{11{,}000}{\sqrt{3}}\angle 0° + (52.48\angle -25.84°)(5\angle 90°)$

$= 6350.853 + 262.4\angle 64.16° = 6350.853 + 114.37 + j236.16$

$= 6465.22 + j236.16$

$E_{p1} = 6469.53\angle 2.09°\text{ V}$

Note: Since excitation is the same in both conditions, induced emf in both conditions also are equal.

$E_{p2} = V_p + I_{p2} X_p$

$= V_p\angle 0° + (52.48\angle 25.84°)(5\angle 90°)$

$= V_p + 262.4\angle 115.84° = V_p - 114.37 + j236.16$

$6469.53\angle\delta = (V_p - 114.37) + j236.16$

By Pythagorean relations:

$(6469.53)^2 = (V_p - 114.37)^2 + (236.16)^2$

$V_p = 6579.588\ V$

$V_L = \sqrt{3}(6579.588) = 11396.18$

$V_L \cong 11.4\ kV$

580 Note: When the load is removed, the voltage between the terminals of the generator will be equal to the generated emf.

$$I = \frac{S}{\sqrt{3}E} = \frac{2{,}500{,}000}{\sqrt{3}(11{,}000)} = 131.2\ A$$

$\theta = \cos^{-1}0.8 = 36.87°$

$$E_p = V_p + I_p X_p$$

$$= \frac{11{,}000}{\sqrt{3}}\angle 0° + (131.2\angle -36.87°)(20\angle 90°)$$

$$= 6350.853 + 2624\angle 53.13° = 6350.853 + 1574.4 + j2099.197$$

$$= 7925.253 + j2099.197$$

$E_p = 8198.552\angle 14.835°\ V$

$E_L = \sqrt{3}E_p = \sqrt{3}(8198.552) = 14{,}200.308\ V$

$E_L \cong 14.2\ kV$

581 Note: Since excitation is constant, generated emf for both conditions must be the same.

$$I = \frac{S}{\sqrt{3}E} = \frac{1{,}200{,}000}{\sqrt{3}(6600)} = 104.97\ A$$

$\theta = \cos^{-1}0.8 = 36.87°$

$Z_p = 0.4 + j6 = 6.013\angle 86.186°$

$$E_p = V_p + I_p Z_p$$

$$= \frac{6600}{\sqrt{3}}\angle 0° + (104.97\angle -36.87°)(6.013\angle 86.186°)$$

$$= 3810.5 + 631.184\angle 49.316° = 3810.5 + 411.46 + j478.637$$

$$= 4221.96 + j478.637$$

$E_p = 4249\angle 6.468°\ V$

When load has a leading pf :

$E_p = V_p + I_p Z_p$

$4249\angle\delta = V_p\angle 0° + (104.97\angle 36.87°)(6.013\angle 86.186°)$

$= V_p + 631.184\angle 123.056°$

$4249\angle\delta = V_p - 344.284 + j529.02$

Using Pythagorean relations:

$(4249)^2 = (V_p - 344.284)^2 + (529.02)^2$

$V_p = 4560.22\ V$

$V_L = \sqrt{3}(4560) = 7898.53\ V$

582 $I = \dfrac{S}{\sqrt{3}E} = \dfrac{2{,}500{,}000}{\sqrt{3}(6{,}600)} = 218.69\ A$

$\theta = \cos^{-1}0.8 = 36.869°$

$E_p = V_p + I_p X_s$

$= \dfrac{6600}{\sqrt{3}}\angle 0° + (218.69\angle -36.869°)(10.4\angle 90°)$

$= 3810.5 + 2274.376\angle 53.13°$

$= 3810.5 + 1364.628 + j1819.49$

$= 5175.128 + j1819.49$

$E_p = 5485.66\angle 19.37°$

$\%VR = \dfrac{E_p - V_p}{V_p} = \dfrac{5485.66 - 3810.5}{3810.5} \times 100\%$

$\%VR = 43.96\%$

583 $I = \dfrac{S}{V_L} = \dfrac{50000}{550} = 91\ A$

$\theta = \cos^{-1}0.8 = 36.87°$

$R_a = R_{dc}(1.5) = 0.18(1.5) = 0.27\ \Omega$

$Z_s = \dfrac{E_o}{I_{sc}} = \dfrac{300}{150} = 2\ \Omega$

$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{2^2 - 0.27^2} = 1.98\ \Omega$

$Z_s = 0.27 + j1.98 = 2\angle 82.23°$

$E_p = V_p + I_p Z_s$

$E_p = 550\angle 0° + (91\angle -36.87°)(2\angle 82.23°)$

$E_p = 550 + 182\angle 45.36°$

$= 550 + 127.88 + j129.5$

$E_p = 690.13\angle 10.81°\ V$

$$\%VR = \frac{E_p - V_P}{V_p} = \frac{690.13 - 550}{550} \times 100\%$$

$\%VR = 25.47\%$

584 $R_{dc} = \dfrac{E_{dc}}{I_{dc}} = \dfrac{6}{87.6} = 0.0685\ \Omega$

$R_{ac} = 1.5\,R_{dc} = 1.5(0.0685) = 0.10275\ \Omega$

If wye - connected :

$$R_a = \frac{R_{ac}}{2} = \frac{0.10275}{2} = 0.051375\ \Omega$$

If delta - connected :

$$R_a = \frac{3}{2}R_{ac} = \frac{3(0.10275)}{2} = 0.154125\ \Omega$$

Note: By comparing the answers to the choices, the generator should be wye-connected.

585 $R_{dc} = \dfrac{E_{dc}}{I_{dc}} = \dfrac{6}{87.6} = 0.0685\ \Omega$

If wye - connected :

$$R_a = \frac{R_{dc}}{2} = \frac{0.0685}{2} = 0.03425\ \Omega$$

If delta - connected :

$$R_a = \frac{3}{2}R_{dc} = \frac{3(0.0685)}{2} = 0.10275\ \Omega$$

Note: By comparing the answers to the choices, the generator should be wye-connected.

586 $P_{out} = 2000(0.8) = 1600\ kW$

$$I = \frac{S}{\sqrt{3}V_L} = \frac{2 \times 10^6}{\sqrt{3}(2300)} = 502.04\ A$$

$$R_a = \frac{R_t}{2} = \frac{1.2(0.08)}{2} = 0.048\ \Omega$$

$P_a = 3I_p^2 R_a = 3(502.04)^2(0.048) = 36.294\ kW$

$P_f = V_f I_f = 125(70) = 8.75\ kW$

$P_{losses} = P_a + P_f + P_{iron} + P_{friction} + P_{stray\ load}$

$P_{losses} = 36.294 + 8.75 + 36 + 20 + 2 = 103.044\ kW$

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} = \frac{1600}{1600 + 103.044} \times 100\%$$

$\eta = 93.9\%$

587 $$I = \frac{S}{\sqrt{3}V_L} = \frac{1500 \times 10^3}{\sqrt{3}(2300)} = 376.53\ A$$

$$R_{dc} = \frac{R_t}{2} = \frac{0.08}{2} = 0.04\ \Omega \text{ per phase}$$

$R_a = 1.5R_{dc} = 1.5(0.04) = 0.06\ \Omega$ per phase

$P_a = 3I_p^2R_a = 3(376.53)^2(0.06) = 25.52\ kW$

$P_f = V_fI_f = 120(70) = 8.4\ kW$

$P_{losses} = P_a + P_f + P_{core} + P_{fw} + P_{stray\ load}$

$P_{losses} = 25.52 + 8.4 + 35 + 15 + 1.5 = 85.42\ kW$

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} = \frac{1500(0.8)}{1500(0.8) + 85.42} \times 100\%$$

$\eta = 93.35\%$

588 $P_{out} = (2{,}500)(0.85) = 2125\ kW$

$$I = \frac{S}{\sqrt{3}V_L} = \frac{2{,}500}{\sqrt{3}(6.6)} = 218.69\ A$$

$P_a = 3I_p^2R_a = 3(218.69)^2(0.05) = 7.174\ kW$

$P_f = I_f^2R_f = (200)^2(0.45) = 18\ kW$

$P_{losses} = P_a + P_f + P_{stray}$

$P_{losses} = 7.174 + 18 + 82 = 107.174\ kW$

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} = \frac{2125}{2125 + 107.174} \times 100\%$$

$\eta = 95.2\%$

589 $$P_d = \frac{E_LV_L}{X_s}\sin\delta$$

Note: At maximum power condition, $\sin\delta$ must be equal to 1.

$$P_{dmax} = \frac{E_LV_L}{X_s} = \frac{(6600)(\sqrt{3} \times 5800)}{6} = 11.05\ MW$$

590 $P_{d\,phase} = \dfrac{E_p V_p}{X_s} \sin\delta$

Note: δ is the angle between E_p and V_p. At maximum power condition, $\sin\delta$ is equal to 1, thus $\delta = 90°$.

$$E_p\angle\delta = V_p\angle 0° + IZ_p$$

$$\frac{12000}{\sqrt{3}}\angle 90° = \frac{11000}{\sqrt{3}}\angle 0° + (I)(6\angle 90°)$$

$$j6928.203 = 6350.85 + (I)(6\angle 90°)$$

$$I = \frac{-6350.85 + j6928.203}{6\angle 90°} = \frac{9398.579\angle 132.51°}{6\angle 90°}$$

$$I = 1566.43\angle 42.51°\ A$$

591 $E_p\angle\delta = V_p\angle 0° + IZ_p$

Note: At maximum power developed condition, $\delta = 90°$.

$$\frac{14000}{\sqrt{3}}\angle 90° = \frac{11000}{\sqrt{3}}\angle 0° + (I)(8\angle 90°)$$

$$j8082.9 = 6350.85 + (I)(8\angle 90°)$$

$$I = \frac{-6350.85 + j8082.9}{8\angle 90°} = \frac{10279.42\angle 128.157°}{8\angle 90°}$$

$$I = 1284.9\angle 38.157°\ A$$

$$pf = \cos\theta = \cos 38.157°$$

$$pf = 0.786$$

592 At 60% pf :

$$I_{p1} = \frac{P}{\sqrt{3}V_L pf} = \frac{3{,}200{,}000}{\sqrt{3}(2300)(0.6)} = 1338.78\ A$$

$$P_{a_1} = 3I_{p_1}{}^2 R_a = 3(1338.78)^2 R_a$$

$$P_{a1} = 0.1(3{,}200) = 320\ kW$$

$$R_a = \frac{320{,}000}{3(1338.78)^2} = 0.0595\ \Omega$$

At 80% pf :

$$I_{p2} = \frac{P}{\sqrt{3}V_L pf} = \frac{3{,}200{,}000}{\sqrt{3}(2300)(0.8)} = 1004.08\ A$$

$P_{a2} = 3I_{p2}{}^2R_a = 3(1004.08)^2(0.0595) \cong 180\text{ kW}$

Savings $= P_{a1} - P_{a2}$

Savings $= 320 - 180 = 140\text{ kW}$

593 $I = \dfrac{S}{\sqrt{V_L}} = \dfrac{50 \times 10^6}{\sqrt{3}(13{,}800)} = 2092\text{ A}$

$\theta = \cos^{-1}0.88 = 28.35°$

$E_{p1} = V_p + I_{p1}X_s$

$= \dfrac{13{,}800}{\sqrt{3}} + (2092\angle -28.35°)(3.8\angle 90°)$

$= 7967.43 + 7949.6\angle 61.65°$

$= 7967.43 + 3774.918 + j6996.15$

$E_{p1} = 13{,}668.53\angle 30.786°\text{ V}$

Note: Flux is directly proportional to the rotor current. Induced emf is directly proportional to flux, thus induced emf is directly proportional to the rotor current.

$E_p = kI_r$

$\dfrac{E_{p1}}{E_{p2}} = \dfrac{I_{r1}}{I_{r2}}$

$\dfrac{13668.53}{E_{p2}} = \dfrac{I_{r1}}{0.95I_{r1}}$

$E_{p2} = 12{,}985.104\text{ V}$

$E_{p2} = V_p + I_{p2}X_s$

$= 7967.43 + (I_2\angle -\theta_2)(j3.8)$

$= 7967.43 + (I_2\cos\theta_2 - jI_2\sin\theta_2)(j3.8)$

Note: Since load is the same, the real components of the vector currents are equal. Thus, $I_2 \cos\theta_2$ is equal to $I_1 \cos\theta_1$.

$E_{p2} = 7967.43 + 2092(0.88)(j3.8) + 3.8I_2\sin\theta_2$

$12{,}985.104\angle\delta = (7967.43 + 3.8I_2\sin\theta_2) + j6995.648$

By Pythagorean relations:

$$(12{,}985.104)^2 = (7967.43 + 3.8I_2 \sin\theta_2)^2 + (6995.648)^2$$
$$I_2 \sin\theta_2 = 782.138$$

$$\frac{I_2 \sin\theta_2}{I_2 \cos\theta_2} = \frac{I_2 \sin\theta_2}{I_1 \cos\theta_1} = \frac{782.138}{2092(0.88)} = 0.4248$$
$$\tan\theta_2 = 0.4248$$
$$\theta_2 = 23.015°$$
$$pf_2 = \cos 23.015° = 0.920$$

594 $$I_{p1} = \frac{S}{\sqrt{3}V_L} = \frac{50 \times 10^6}{\sqrt{3}(13.8 \times 10^3)} = 2092 \text{ A}$$
$$\theta = \cos^{-1} 0.88 = 28.35°$$

$$E_{p1} = V_p + I_{p1}Z_p$$
$$= \frac{13800}{\sqrt{3}}\angle 0° + (2092\angle -28.35°)(3.8\angle 90°)$$
$$= 7967.43 + 7946.6\angle 61.65°$$
$$= 7967.43 + 3774.918 + j6996.15$$
$$= 11742.348 + j6996.15$$
$$E_{p1} = 13668.53\angle 30.786° \text{ V}$$

Note: : Since the prime mover power is decreased by 25%, this means that the induced emf in the second condition is 75% of the induced emf in the first condition.

$$E_{p2} = 0.75\,E_{p1} = 0.75(13668) = 10251 \text{ V}$$

Note: Since input power drops by 25%, output power also drops by 25%.

$$I_{p2} = 0.75\,I_{p1} = 0.75(2092) = 1569 \text{ A}$$
$$E_{p2} = V_p + I_{p2}Z_p$$
$$= \frac{13800}{\sqrt{3}}\angle 0° + (1569\angle -\theta_2)(3.8\angle 90°)$$
$$= 7967.43 + (1569\cos\theta_2 - j1569\sin\theta_2)(j3.8)$$
$$= 7967.43 + j5962.2\cos\theta_2 - j^2 5962.2\sin\theta_2$$
$$10251\angle\delta = (7967.43 + 5962.2\sin\theta_2) + j5962.2\cos\theta_2$$

By Pythagorean relations:

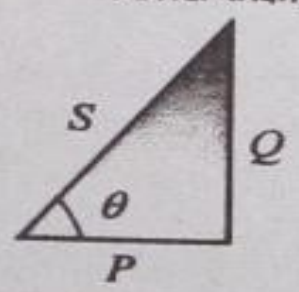
$$(10251)^2 = (7967.43 + 5962.2\sin\theta_2)^2 + (5962.2\cos\theta_2)^2$$
$$(10251)^2 = (7967.43)^2 + 2(7967.43)(5962.2\sin\theta_2) + (5962.2)^2(\sin^2\theta_2 + \cos^2\theta_2)$$

$$\sin\theta_2 = 0.0637347$$
$$\theta_2 = 3.654°$$
$$pf_2 = \cos\theta_2 = \cos 3.654°$$
$$pf_2 = 0.998$$

595 $$I = \frac{S}{\sqrt{3}V_L} = \frac{50 \times 10^6}{\sqrt{3}(13.8 \times 10^3)} = 2092\text{ A}$$
$$\theta = \cos^{-1} 0.88 = 28.35°$$

$$E_{P1} = V_p + I_p Z_p$$
$$= \frac{13800}{\sqrt{3}}\angle 0° + (2092\angle -28.35°)(3.8\angle 90°)$$
$$= 7967.43 + 7946.6\angle 61.65°$$
$$= 7967.43 + 3774.918 + j6996.15 = 11742.348 + j6996.15$$
$$E_{P1} = 13668.53\angle 30.786°\text{ V}$$

Power triangle



$$\sin\theta_2 = \frac{Q}{S} = \frac{40}{50} = 0.8$$
$$\theta_2 = \sin^{-1} 0.8 = 53.13°$$

$$E_{P2} = V_p + I_p Z_p$$
$$= \frac{13800}{\sqrt{3}}\angle 0° + (2092\angle -53.13°)(3.8\angle 90°)$$
$$= 7967.43 + 7946.6\angle 36.187°$$
$$= 7967.43 + 6359.67 + j4769.77 = 14327.1 + j4769.77$$
$$E_{P2} = 15100.21\angle 18.41°\text{ V}$$

Note: Flux is directly proportional to the rotor current. Induced emf is directly proportional to flux, thus induced emf is directly proportional to the rotor current.

$$\%\text{Increase} = \frac{E_{P2} - E_{P1}}{E_{P1}} = \frac{15100 - 13668}{13668} \times 100\%$$
$$\%\text{Increase} = 10.48\%$$

596 $\theta = \cos^{-1} 0.8 = 36.869°$

$E_{p1} = V_p + I_{p1}X_s$

$= \frac{12{,}000}{\sqrt{3}} \angle 0° + (250\angle -36.869°)(15\angle 90°)$

$= 6928.203 + 3750\angle 53.13°$

$= 6928.203 + 2250 + j3000$

$E_{p1} = 9656.055\angle 18.1°$ volts

Note: If steam admission is constant, thus the powers delivered in both conditions are the same

$P_1 = P_2$

$3V_pI_{p1}\cos\theta_1 = 3V_pI_{p2}\cos\theta_2$

$250(0.8) = I_{p2}\cos\theta_2$

$I_{p2}\cos\theta_2 = 200 \Rightarrow$ ❶

$E_{p2} = 1.2\,E_{p1} = (1.2)(9656.055) = 11{,}587.266$ V

$E_{p2} = V_p + I_{p2}X_s$

$= 6928.203 + (I_{p2}\angle -\theta_2)(15\angle 90°)$

$= 6928.203 + (I_{p2}\cos\theta_2 - jI_{p2}\sin\theta_2)\,j15$

$= 6928.203 + 15\,I_{p2}\sin\theta_2 + j15\,I_{p2}\cos\theta_2$

$11{,}587.266\angle\alpha = 6928.203 + 15\,I_{p2}\sin\theta_2 + j15(200)$

By Pythagorean relations:

$(11{,}587.266)^2 = (6928.203 + 15\,I_{p2}\sin\theta_2)^2 + [15(200)]^2$

$I_{p2}\sin\theta_2 = 284.265 \Rightarrow$ ❷

Divide Eq.2 by Eq.1:

$\frac{I_{p2}\sin\theta_2}{I_{p2}\cos\theta_2} = \frac{284.265}{200}$

$\tan\theta_2 = 1.421$

$\theta_2 = 54.86°$

$pf_2 = \cos\theta_2 = \cos 54.86°$

$pf_2 = 0.575$

597 Note: If steam admission is constant, thus the powers delivered in both conditions are the same.

$P_1 = P_2$

$3V_pI_{p1}\cos\theta_1 = 3V_pI_{p2}\cos\theta_2$

$I_{p2}\cos\theta_2 = 150(0.8) = 120 \Rightarrow$ ❶

$\theta = \cos^{-1}0.8 = 36.87°$

$E_{p1} = V_p + I_{p1}Z_p$

$= \frac{20000}{\sqrt{3}}\angle 0° + (150\angle -36.87°)(10\angle 90°)$

$= 11547 + 1500\angle 53.13° = 11547 + 900 + j1200$

$E_{p1} = 12447 + j1200 = 12504.7\angle 5.506°$ V

$E_{p2} = 1.25(12504.7) = 15630.87$ V

$E_{p2} = V_p + I_{p2}Z_p$

$= \frac{20000}{\sqrt{3}}\angle 0° + I_{p2}\angle -\theta_2°)(10\angle 90°)$

$= 11547 + (I_{p2}\cos\theta_2 - jI_{p2}\sin\theta_2)(j10)$

$= 11547 + j10I_{p2}\cos\theta_2 - j^2 10I_{p2}\sin\theta_2$

$15630.87\angle\delta = 11547 + 10I_{p2}\sin\theta_2 + j10(120)$

By Pythagorean relations:

$(15630.87)^2 = (11547 + 10I_{p2}\sin\theta_2)^2 + (1200)^2$

$I_{p2}\sin\theta_2 = 403.77 \Rightarrow$ ❷

Divide Eq.2 by Eq.1:

$\frac{I_{p2}\sin\theta_2}{I_{p2}\cos\theta_2} = \frac{403.77}{120}$

$\tan\theta_2 = 3.36475$

$\theta_2 = 73.448°$

$I_{p2} = \frac{120}{\cos 73.448°}$

$I_{p2} = 421.22$ A

598 $\theta_A = \cos^{-1} 0.8 = 36.87°$

$\theta_B = \cos^{-1} 1 = 0°$

$S_t = S_A + S_B = 50\angle -36.87° + 20\angle 0°$

$= 40 - j30 + 20 + j0 = 60 - j30$

$S_t = 67.08\angle -26.56°$

$pf_t = \cos 26.56°$

$pf_t = 0.894$ lagging

599 $\theta_A = \cos^{-1} 0.8 = 36.87°$

$\theta_B = \cos^{-1} 1 = 0°$

$$P_A = \frac{P_{out}}{\eta} = \frac{20(0.746)}{0.9} = 16.58 \text{ kW}$$

$$S_A = \frac{P_A}{pf_A} = \frac{16.58}{0.8} = 20.725 \text{ kVA}$$

$S_t = S_A + S_B = 20.725\angle -36.87° + 7\angle 0°$

$= 16.58 - j12.435 + 7 + j0$

$S_t = 23.58 - j12.435$

By inspection : $Q_t = 12.435$ kVAR

600 Note: Refer to the power triangle as shown for the trigonometric relations between real, apparent and reactive powers.

$\theta_t = \cos^{-1} 0.75 = 41.409°$

$\theta_A = \cos^{-1} 0.8 = 36.869°$

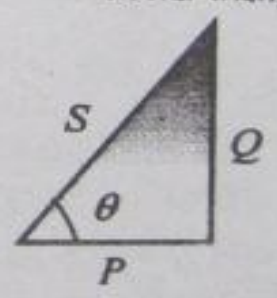
$Q_t = Q_A + Q_B$

$Q_B = Q_t - Q_A = P_t \tan\theta_t - P_A \tan\theta_A$

$= 1050 \tan 41.409° - (700 \tan 36.869°)$

$Q_B = 401$ kVAR

Power triangle



601 $\theta_t = \cos^{-1} 0.8 = 36.869°$

$\theta_A = \cos^{-1} 0.6 = 53.13°$

$S_T = S_A + S_B$

$S_B = S_T - S_A = 1000\angle -36.869 - 500\angle -53.13°$

$S_B = 800 - j600 - (300 - j400) = 500 - j200$

$S_B = 538.516\angle -21.8$ kVA

$pf_B = \cos\theta_B = \cos(-21.8)$

$pf_B = 0.928$ lagging

602 $I_t = \dfrac{P}{\sqrt{3}V_L pf} = \dfrac{10 \times 10^6}{\sqrt{3}(33000)(0.8)} = 218.7\ A$

$\theta_t = \cos^{-1} 0.8 = 36.87°$

Note: As given the alternators share the load equally.

$pf_a = \dfrac{P_a}{\sqrt{3}V_L I_a} = \dfrac{5 \times 10^6}{\sqrt{3}(33000)(125)} = 0.699$

$\theta_a = \cos^{-1} 0.699 = 45.58°$

$I_b = I_t - I_a = 218.7\angle -36.87° - (125\angle -45.58°)$

$= 174.96 - j131.22 - (87.489 - j89.278) = 87.471 - j41.942$

$I_b = 97\angle -25.6°\ A$

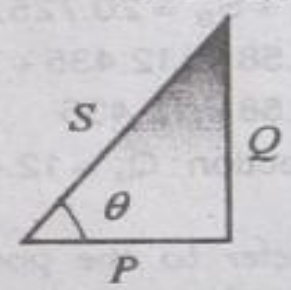
603 For alternator A :

$\theta_A = \cos^{-1} 0.9 = 25.84°$

$S_A = P_A - jQ_A = 60 - j60\tan 25.84°$

$S_A = 60 - j29.056$

Power triangle



For alternator B :

$\theta_B = \cos^{-1} 0.7 = 45.57°$

$S_B = P_B - jQ_B = 80 - j80\tan 45.57°$

$S_B = 80 - j81.608$

$S_t = S_A + S_B$

$= 60 - j29.056 + 80 - j81.608 = 140 - j110.664$

$S_t = 178.45\angle -38.32°$

$pf_t = \cos\theta_t = \cos 38.32°$

$pf_t = 0.784$ lagging

604 $I_t = \dfrac{P}{\sqrt{3}V_L pf} = \dfrac{3 \times 10^6}{\sqrt{3}(6600)(0.8)} = 328.04\ A$

$\theta_t = \cos^{-1} 0.8 = 36.87°$

Note: As given the alternators share the load equally.

$pf_1 = \dfrac{P_1}{\sqrt{3}V_L I_1} = \dfrac{1.5 \times 10^6}{\sqrt{3}(6600)(150)} = 0.0.875$

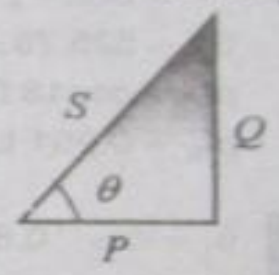
$\theta_1 = \cos^{-1} 0.875 = 28.955°$

$I_2 = I_t - I_1 = 328.04\angle -36.87° - (150\angle -28.955°)$
$= 262.43 - j196.824 - (131.25 - j72.618)$
$= 131.18 - j124.206$
$I_2 = 180.65\angle -43.435°$ A
$pf_2 = \cos\theta_2 = \cos 43.435°$
$pf_2 = 0.726$ lagging

605 Note: Refer to the power triangle as shown for the trigonometric relations between real, apparent and reactive powers.

$\theta_t = \cos^{-1} 0.85 = 31.788°$
$\theta_A = \cos^{-1} 0.92 = 23.07°$
$S_B = S_t - S_A$
$S_B = 4250 - j4250\tan 31.788° - \left(\dfrac{600}{\tan 23.07°} - j600\right)$
$S_B = 2841.276 - j2033.879$
$S_B = 3494.21\angle -35.596°$

Power triangle



$pf_B = \cos\theta_B = \cos 35.596$
$pf_B = 0.813$ lagging

606 $\theta_1 = \cos^{-1} 1.0 = 0°$
$\theta_2 = \cos^{-1} 0.8 = 36.869°$
$\theta_3 = \cos^{-1} 0.707 = 45°$
$P_t = P_1 + P_2 + P_3 = 800 + 600 + 400 = 1800$ kW
$Q_t = 800\tan 0° + 600\tan 36.869° + 400\tan 45° = 850$ kVAR
$S_t = P_t - jQ_t = 1800 - j850$
$I_t = \dfrac{S_t}{\sqrt{3}\,E_t} = \dfrac{1800 - j850}{\sqrt{3}(3.3)} = 314.918 - j148.711$ A

$\theta_A = \cos^{-1} 0.85 = 31.788°$
$I_A = 150\angle -31.788° = 127.5 - j79.016$ A

$I_B = I_t - I_A$
$= 314.918 - j148.711$ A $- (127.5 - j79.016)$
$= 187.418 - j69.695 = 199.95\angle -20.398°$
$I_B \cong 200$ A

607 $\theta_1 = \cos^{-1} 1.0 = 0°$

$\theta_2 = \cos^{-1} 0.9 = 25.84°$

$\theta_3 = \cos^{-1} 0.866 = 30°$

$S_t = S_1 + S_2 + S_3$

$= 400 + j0 + 500 - j500 \tan 25.84° + 600 - j600 \tan 30°$

$S_t = 1500 - j588.55$

$\theta_A = \cos^{-1} 0.92 = 23.07°$

$S_B = S_t - S_A$

$= 1500 - j588.55 - (1000 - j1000 \tan 23.07°)$

$= 500 - j162.63$

$S_B = 525.78\angle -18.02°$

$pf_B = \cos 18.02°$

$pf_B = 0.951$ lagging

608 $\theta_t = \cos^{-1} 0.85 = 31.788°$

$\theta_A = \cos^{-1} 0.707 = 45°$

$S_t = S_A + S_B$

$S_B = S_t - S_A$

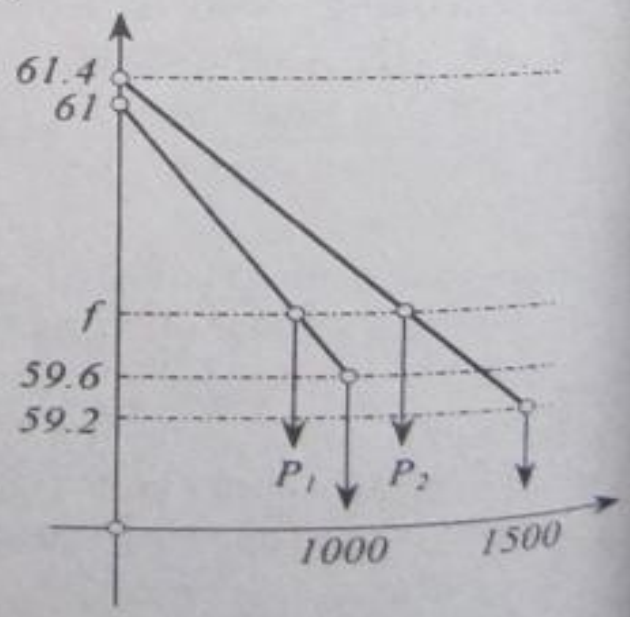
$S_B = (4250 - j4250 \tan 31.788°) - (2125 - j2125 \tan 45°)$

$= 2125 - j508.879$

$S_B = 2185.082\angle -13.467°$

By inspection : $S_B \cong 2185$ kVA

609 Note: Refer to the characteristic triangles:



$P_1 + P_2 = 2000 \Rightarrow$ ❶

By similar triangles :

$$\frac{61 - 59.6}{1000} = \frac{61 - f}{P_1}$$

$$P_1 = \frac{1000}{1.4}(61 - f) \Rightarrow ❷$$

$$\frac{61.4 - 59.2}{1500} = \frac{61.4 - f}{P_2}$$

$$P_2 = \frac{1500}{2.2}(61.4 - f) \Rightarrow ❸$$

Substitute Eq.2 & Eq.3 to Eq.1:

$$\frac{1000}{1.4}(61-f)+\frac{1500}{2.2}(61.4-f)=2000$$

$$43571.428-714.285f+41863.636-681.818f=2000$$

$$f=59.763\text{ Hz}$$

$$P_1=\frac{1000}{1.4}(61-f)=\frac{1000}{1.4}(61-59.763)\cong 884\text{ kW}$$

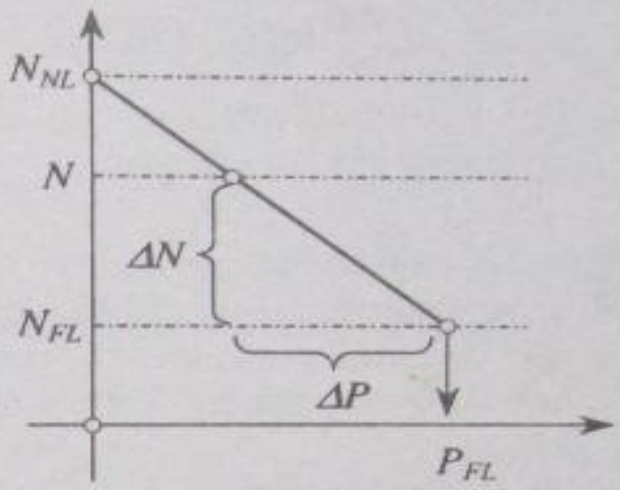
$$P_2=\frac{1500}{2.2}(61.4-f)=\frac{1500}{2.2}(61.4-59.763)\cong 1116\text{ kW}$$

610 $$\frac{\Delta N}{\Delta P}=\frac{N_{NL}-N_{FL}}{P_{FL}}=\frac{GSR\times N_{FL}}{P_{FL}}$$

$$\Delta P=\frac{(\Delta N)(P_{FL})}{GSR\times N_{FL}}$$

$$\Delta P_1=\frac{\Delta N(50)}{0.03\,N_{FL}}=\frac{1666.67\Delta N}{N_{FL}}\Rightarrow ❶$$

$$\Delta P_2=\frac{\Delta N(50)}{0.04\,N_{FL}}=\frac{1250\Delta N}{N_{FL}}\Rightarrow ❷$$



$$\Delta P=\Delta P_1+\Delta P_2\Rightarrow ❸$$

Substitute Eq. 1 and Eq.2 in Eq.3 :

$$50=\frac{1666.67\Delta N}{N_{FL}}+\frac{1250\Delta N}{N_{FL}}$$

$$\frac{\Delta N}{N_{FL}}=0.0171428$$

$$P_1=50+\Delta P_1=50+1666.67(0.0171428)=78.571\text{kW}$$

$$P_2=50+\Delta P_2=50+1250(0.0171428)=71.428\text{ kW}$$

611 Note: Refer to the characteristic triangles:

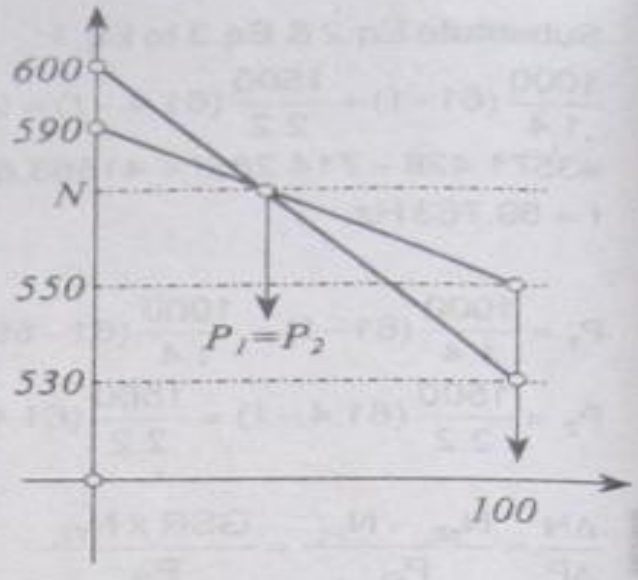
By similar triangles :

$$\frac{600-530}{100}=\frac{600-N}{P_1}$$

$$P_1=\frac{100}{70}(600-N)\Rightarrow ❶$$

$$\frac{590 - 550}{100} = \frac{590 - N}{P_2}$$

$$P_2 = \frac{100}{40}(590 - N) \Rightarrow ❷$$



$$P_1 = P_2$$

$$\frac{100}{70}(600 - N) = \frac{100}{40}(590 - N)$$

$$8.5714 - 0.01428N = 14.75 - 0.025N$$

$$N = 576.36 \text{ rpm}$$

$$P_1 = P_2 = \frac{100}{70}(600 - 576.36)$$

$$P_1 = P_2 = 33.77 \text{ kW}$$

$$P_t = 2(33.77) = 67.54 \text{ kW}$$

612 Note: Refer to the characteristic triangles. If operating speed is 530 rpm (full load condition of alternator #1), alternator #2 is overloaded. If operating speed is 550 rpm (full load condition of alternator #2), alternator #1 is not overloaded. Thus operate the machines at a speed of 550 rpm.

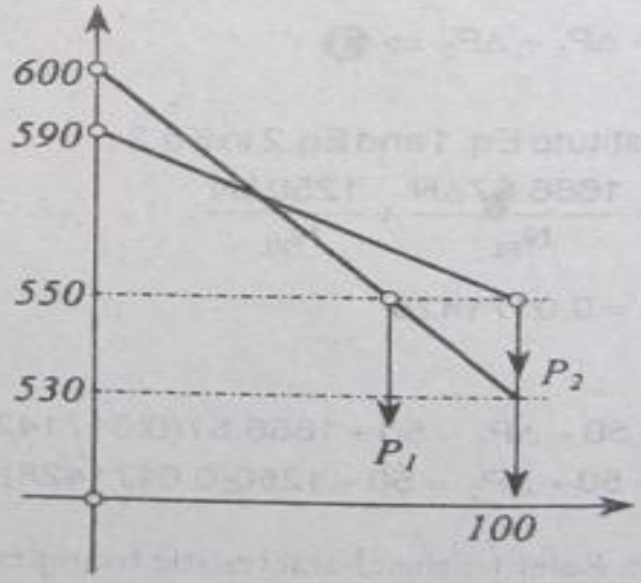
By similar triangles :

$$\frac{600 - 530}{100} = \frac{600 - N}{P_1}$$

$$P_1 = \frac{100}{70}(600 - N)$$

$$= \frac{100}{70}(600 - 550)$$

$$P_1 = 71.428 \text{ kW}$$



$$P_t = P_1 + P_2$$

$$= 71.428 + 100$$

$$P_t = 171.428 \text{ kW}$$

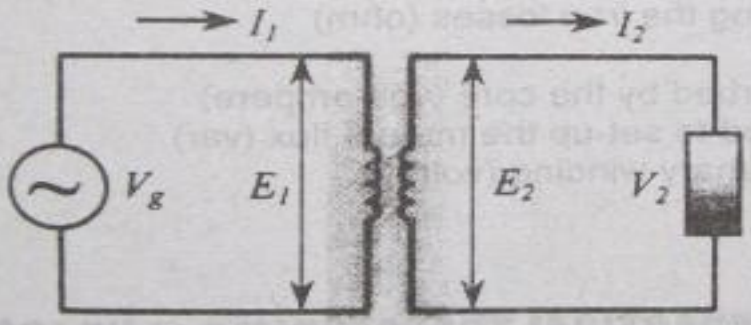
Transformers: Part 1

INDUCED EMF EQUATION

$$E = 4.44Nf\Phi_m$$

where: E = rms voltage induced (volt)
N = number of winding turns
f = frequency of the flux (Hertz)
Φ_m = peak value of the flux (weber)

EQUIVALENT CIRCUIT OF AN IDEAL TRANSFORMER



$$E_1 = 4.44N_1f_1\Phi_{m1}$$

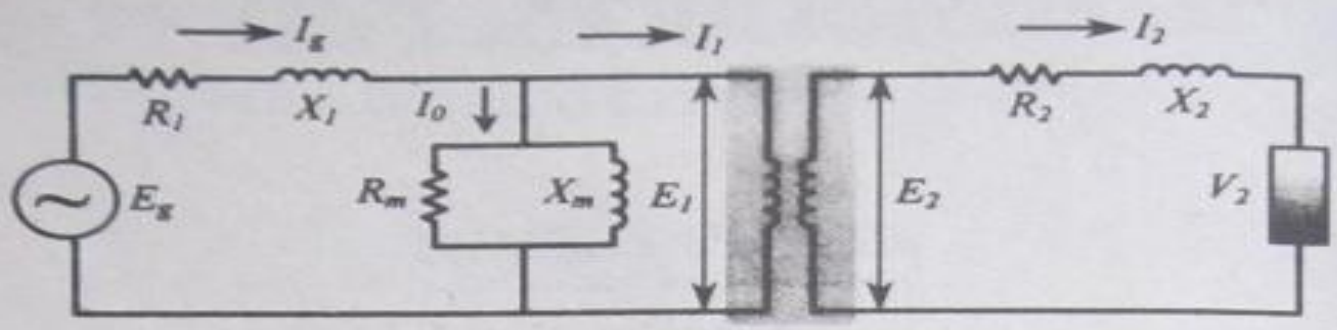
$$E_2 = 4.44N_2f_2\Phi_{m2}$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \qquad \frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2 = a^2$$

where: $N_1/N_2 = a$ = turns ratio
E_1/E_2 = voltage ratio
I_1/I_2 = current ratio
Z_1/Z_2 = impedance ratio
E_1 = voltage induced in the primary windings
E_2 = voltage induced in the secondary windings

Note: A transformer is ideal if its core is loss less, has no leakage flux and has no copper losses.

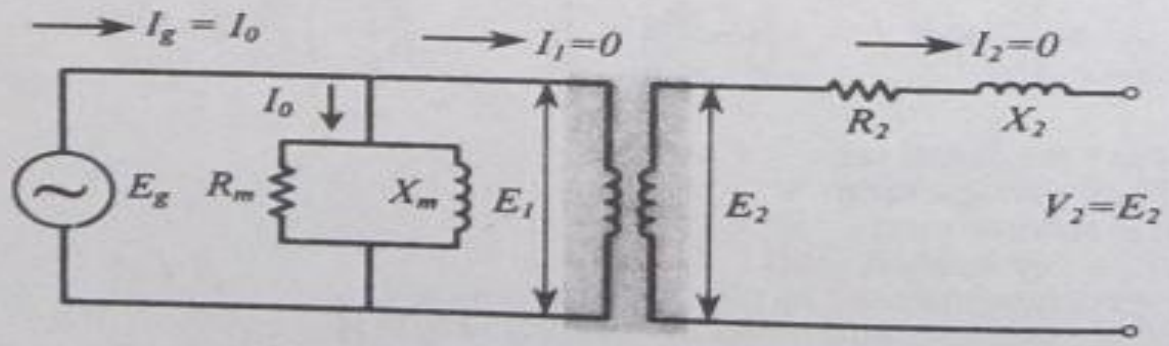
EQUIVALENT CIRCUIT OF A PRACTICAL TRANSFORMER



$$S_m = E_1 I_o \qquad Q_m = \sqrt{S_m^2 - P_m^2} \qquad R_m = \frac{E_1^2}{P_m} \qquad X_m = \frac{E_1^2}{Q_m}$$

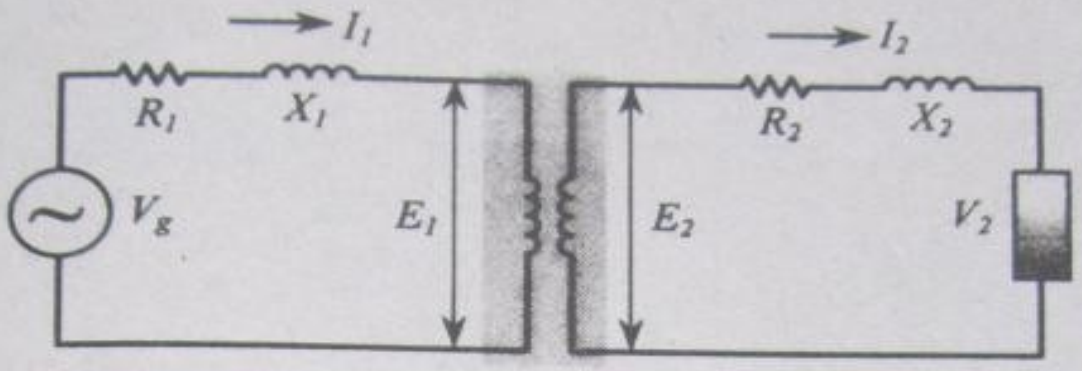
where: R_1 = resistance of primary winding (ohm)
X_1 = reactance of primary winding (ohm)
R_2 = resistance of secondary winding (ohm)
X_2 = reactance of secondary winding (ohm)
X_m = magnetizing reactance of primary winding (ohm)
R_m = resistance representing the iron losses (ohm)
P_m = iron losses (watt)
S_m = apparent power absorbed by the core (volt-ampere)
Q_m = reactive power needed to set-up the mutual flux (var)
E_1 = induced emf of the primary winding (volt)
E_g = supply voltage (volt)

EQUIVALENT CIRCUIT OF A PRACTICAL TRANSFORMER @ NO LOAD



Note: At no load $I_2 = 0$ and so is I_1. Thus only the exciting current I_0 flows in R_1 and X_1. And these impedances are so small that the voltage drop across them are negligible.

EQUIVALENT CIRCUIT OF A PRACTICAL TRANSFORMER @ FULL LOAD



Note: At full load I_g is at least 20 times bigger than I_0, consequently I_0 can be neglected and the corresponding magnetizing branch. This equivalent circuit can be used even if the loading is only 10% of the rated capacity of the transformer.

SHIFTING TRANSFORMER PARAMETERS FROM PRIMARY TO SECONDARY OR VICE-VERSA

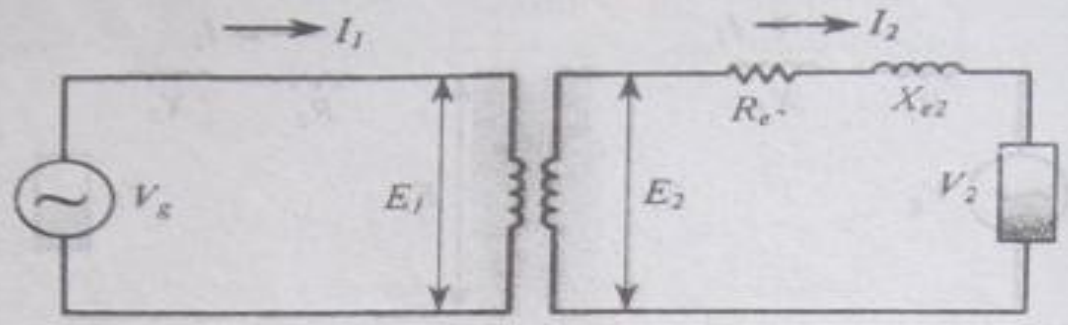
$$R_{e1} = R_1 + R_2\left(\frac{N_1}{N_2}\right)^2 \qquad R_{e2} = R_2 + R_1\left(\frac{N_2}{N_1}\right)^2$$

$$X_{e1} = X_1 + X_2\left(\frac{N_1}{N_2}\right)^2 \qquad X_{e2} = X_2 + X_1\left(\frac{N_2}{N_1}\right)^2$$

$$Z_{e1} = \sqrt{(R_{e1})^2 + (X_{e1})^2} \qquad Z_{e2} = \sqrt{(R_{e2})^2 + (X_{e2})^2}$$

where: R_{e1} = equivalent resistance referred to the primary side
R_{e2} = equivalent resistance referred to the secondary side
X_{e1} = equivalent reactance referred to the primary side
X_{e2} = equivalent reactance referred to the secondary side
Z_{e1} = equivalent impedance referred to the primary side
Z_{e2} = equivalent impedance referred to the secondary side

EQUIVALENT CIRCUIT OF A TRANSFORMER REFERRED TO THE SECONDARY SIDE



$$E_2 = \sqrt{(V_2 \cos\theta + I_2 R_{e2})^2 + (V_2 \sin\theta \pm I_2 X_{e2})^2}$$

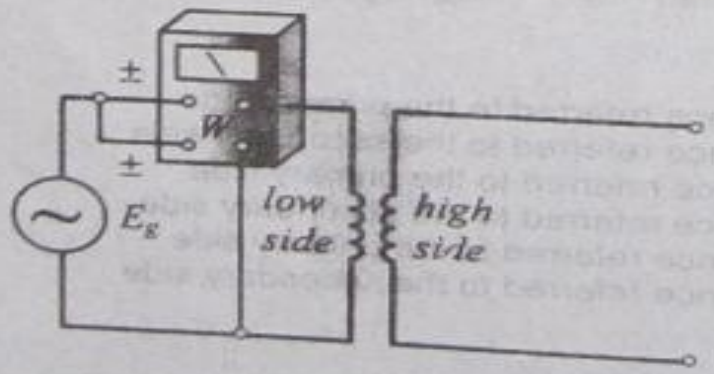
where: V_2 = load voltage (volt)
I_2 = load current (ampere)
θ = power factor angle
+ (sign) = if power factor is lagging
- (sign) = if power factor is leading

- Using complex number format

$$E_2\angle\pm\delta = V_2\angle 0^\circ + (I_2\angle\pm\theta)(R_{e2} + jX_{e2})$$

where: $+\theta$ = if power factor is leading
$-\theta$ = if power factor is lagging
δ = angle between E_2 and V_2

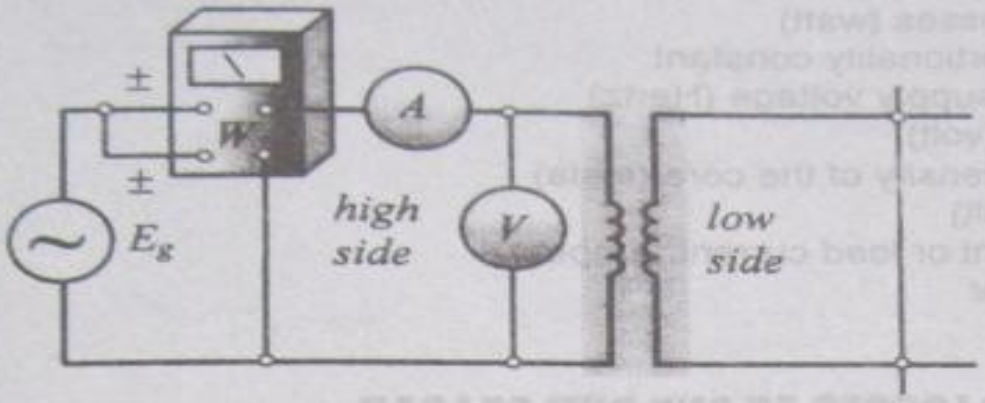
OPEN CIRCUIT TEST ON TRANSFORMERS



$$P_{core} = P_{oc}$$

where: P_{oc} = reading of the wattmeter during the test (watt)
E_g = rated low side voltage (volt)
P_{core} = core or iron losses (watt)

SHORT CIRCUIT TEST ON TRANSFORMERS



$$P_{cuFL} = P_{sc}$$

Note: The supply voltage is adjusted so that the reading of the ammeter is approximately equal to the rated high side current.

$$Z_e = \frac{E_{sc}}{I_{sc}} \qquad R_e = \frac{P_{sc}}{I_{sc}^2} \qquad X_e = \sqrt{Z_e^2 - R_e^2}$$

where: P_{sc} = wattmeter reading during the test
I_{sc} = ammeter reading during the test
E_{sc} = voltmeter reading during the test
R_e = equivalent resistance referred to the high side
X_e = equivalent reactance referred to the high side
Z_e = equivalent impedance referred to the high side
P_{cuFL} = copper losses at full load

POWER LOSSES AND EFFECIENCY

$$\eta = \frac{P_{out}}{P_{in}} \qquad P_{in} = P_{out} + P_{core} + P_{cu} \qquad P_{out} = V_2 I_2 pf$$

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2 \qquad P_{core} = P_{eddy} + P_{hysteresis}$$

$$P_{cu} = I_1^2 R_{e1} = I_2^2 R_{e2}$$

$$P_{eddy} = k_e (f\beta_m)^2 = k_e' E_g^2 \qquad P_{hysteresis} = k_h f \beta_m^{1.6} = k_h' \frac{E_g^{1.6}}{f^{0.6}}$$

here: P_{out} = output power (watt)
P_{in} = input power (watt)
P_{cu} = copper losses (watt)
P_{core} = core or iron losses (watt)
k_e, k_h, k_e', k_h' = proportionality constant
f = frequency of the supply voltage (Hertz)
E_g = supply voltage (volt)
β_m = maximum flux density of the core (tesla)
V_2 = load voltage (volt)
I_2 = secondary current or load current (ampere)
pf = load power factor

COPPER LOSSES AT ANY SIZE OF LOAD

$$P_{cu_x} = (x)^2 P_{cu_{FL}}$$

where: x = percentage loading on the transformer
P_{cu_x} = copper losses at x% load
$P_{cu_{FL}}$ = full load copper losses

MAXIMUM EFFICIENCY

Note: It will occur only when the core loss and the copper loss are equal.

$$\eta_{max} = \frac{P_{out}}{P_{out} + 2P_{core}}$$

ALL DAY EFFICIENCY

$$\eta_{all\ day} = \frac{W_{out}}{W_{out} + W_{core} + W_{cu}}$$

$$W_{out} = \sum (P_{out})(t) \qquad W_{out} = \sum (\text{size of load})(\text{VA rating})(pf)(t)$$

$$W_{core} = (P_{core})(24) \qquad W_{cu} = \sum (\text{size of load})^2 (P_{cu_{FL}})(t)$$

where: W_{out} = total output energy (watt-hour)
W_{core} = total energy losses due to core losses (watt-hour)
W_{cu} = total energy losses due to copper losses (watt-hour)
t = time of usage (hour)
Σ = symbol for a summation process

PER UNIT VALUES

$$\%IR = \frac{I_2 R_{e2}}{V_2} \qquad \%IR = \frac{P_{cuFL}}{VA_{rating}} = \frac{P_{sc}}{VA_{rating}}$$

$$\%IX = \frac{I_2 X_{e2}}{V_2} \qquad \%IZ = \frac{E_{sc}}{V_{rated}} \qquad \%IX = \sqrt{(\%IZ)^2 - (\%IR)^2}$$

where: P_{sc} = wattmeter reading during the short circuit test
E_{sc} = voltmeter reading during the short circuit test

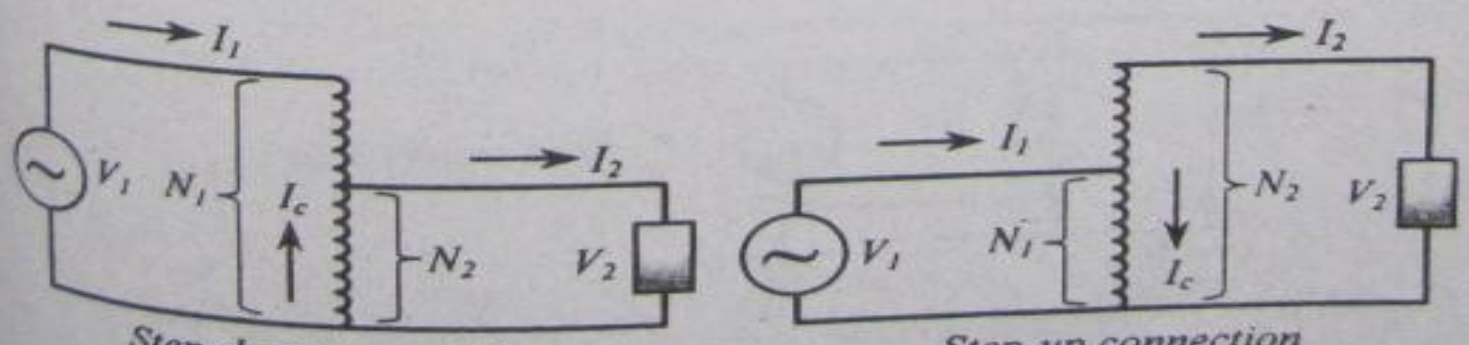
VOLTAGE REGULATION

- **Voltage regulation** - percentage rise in the terminal voltage of the transformer when the transformer load is removed.

$$\%VR = \frac{E_2 - V_2}{V_2} \qquad \%VR = \sqrt{(\cos\theta + \%IR)^2 + (\sin\theta \pm \%IX)^2} - 1$$

AUTOTRANSFORMER

An autotransformer is a transformer with only one winding common to both the primary and the secondary sides.



Step-down connection *Step-up connection*

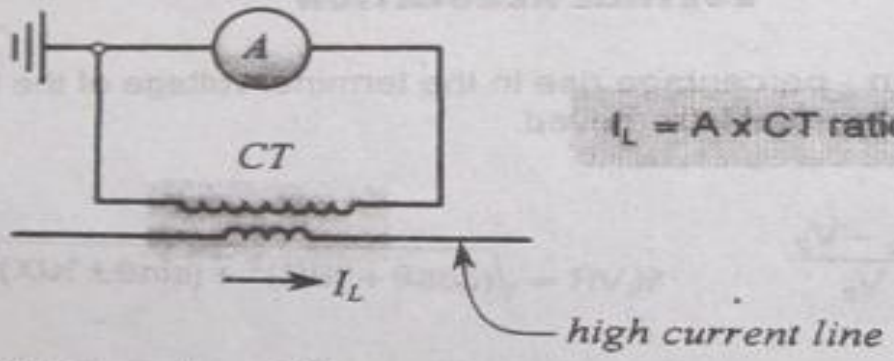
$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \qquad \frac{V_1}{V_2} = \frac{N_1}{N_2} = a \qquad P_{input} = V_1 I_1$$

$$P_{transformed} = P_{input}\left(1 - \frac{1}{a}\right) \qquad P_{conducted} = P_{input} - P_{transformed}$$

where: N_1 = number of turns of the primary winding
N_2 = number of turns of the secondary winding
I_c = current in the common winding
V_1 = supply voltage
V_2 = load voltage

INSTRUMENT TRANSFORMERS

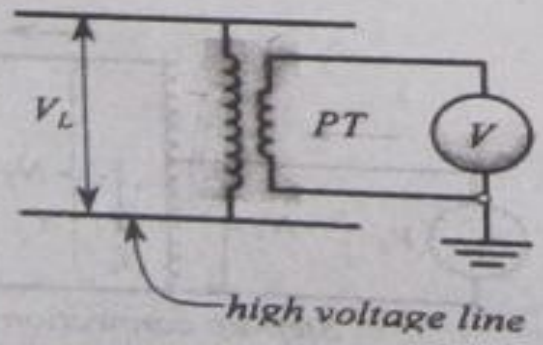
- **Current transformer (CT)** – to be used in conjunction with an ammeter to measure very high current.



I_L = A x CT ratio

where: A = current reading of the ammeter

- **Potential transformer (PT)** – to be used in conjunction with a voltmeter to measure very high voltage.



V_L = V x PT ratio

where: V = voltage reading of the voltmeter

Test 17

Problem 613:

The maximum flux density in the core of a 3000/240-V, 50 Hz single-phase distribution transformer is 1.25 Tesla. If the voltage induced per turn is 8 volts, determine the cross sectional of the core in cm^2.

A. 292
B. 375
C. 288
D. 362

Problem 614:

The net cross section of a transformer is 20 square inches. The maximum flux density is 11 kilogausses. The frequency is 60 Hz. If there are 1,000 turns in the primary, determine the primary induced emf.

A. 3780 V
B. 3945 V
C. 3590 V
D. 3850 V

Problem 615:

Calculate the total magnetic flux in a 60-cycle transformer in which the induced emf per turn of the winding is equal to 2 V.

A. 7.50 mWb
B. 7.02 mWb
C. 7.50 mWb
D. 7.42 mWb

Problem 616:

The low side of a certain welding transformer has 2 turns and delivers 300 A. If the high voltage side has 30 turns, how much current flows through it?

A. 10 A
B. 12 A
C. 20 A
D. 15 A

Problem 617: EE Board April 1997

A single-phase transformer is rated 4,160 V primary and 250 V secondary. It has 1,500 turns in the primary and a voltage regulation of 8%. What should be the number of turns in the secondary winding?

A. 88 turns
B. 92 turns
C. 90 turns
D. 86 turns

Problem 618:

The current densities in the primary and secondary windings of a step down transformer are 1250 and 1500 A per square inch, respectively. The ratio of transformation is 10:1 and the mean length per turn of the primary is 15% greater than that of the secondary. If the primary winding resistance is 20 ohms, calculate the resistance of the secondary windings.

A. 0.202 ohm
B. C.219 ohm
C. 0.209 ohm
D. 0.220 ohm

Problem 619: EE Board April 1991

A certain transformer has a ratio of 8 is to 1. The primary winding resistance is 0.00283 ohm per turn and has 1155 more turns than the secondary winding. If the copper losses in the primary is 20% less than in the secondary, find resistance of the secondary winding.

A. 0.073 ohm
B. 0.090 ohm
C. 0.068 ohm
D. 0.084 ohm

Problem 620: EE Board October 1991

A 10 kVA, 2400/240-V single-phase transformer has the following resistances and reactances. Find the primary voltage required to produce 240 V at the secondary terminals at full load, when the power factor is 0.8 lagging.

$r_1 = 3\ \Omega$, $x_1 = 15\ \Omega$, $r_2 = 0.03\ \Omega$, $x_2 = 0.150\ \Omega$

A. 2450 V
B. 2572 V
C. 2496 V
D. 2632 V

Problem 621:

A 10-kVA, 2,000/400 V, single-phase distribution transformer has a primary and secondary winding impedances of 5.0 + j 10 and 0.2 + j 0.45 ohms, respectively. Calculate the voltage across the secondary terminals at full load, 80% power factor lagging, when the primary is supplied with 2,000 volts?

A. 400 V
B. 394 V
C. 385 V
D. 379 V

Problem 622: EE Board April 1983

A single-phase transformer rated 75 kVA, 8000 volts primary 240 volts secondary, is given the short circuit test. With the secondary terminals short-circuited, 440 volts (60 Hz) is impressed on the primary, which then draws rated current and 1125 watts. Solve for the percentage impedance of the transformer.

A. 5.29%
B. 4.93%
C. 5.50%
D. 5.13%

Problem 623: EE Board April 1993

A short circuit test was performed upon a 10-kVA, 2300/230 volt transformer with the following results: E_{SC} = 137 volts; P_{SC} = 192 W; I_{SC} = 4.34 A. Calculate in primary terms the reactance of the transformer.

A. 31.56 ohms
B. 29.88 ohms
C. 10.19 ohms
D. 12.45 ohms

Problem 624: EE Board October 1984

A single-phase, 25 kVA, 4800/240 volt transformer was given a short circuit tests: With the secondary terminals short-circuited, 93.75 volts was applied on the primary so that the ammeter reads rated current and the wattmeter reads 162.74 watts, all on the primary side. Solve for the percentage resistance of the transformer.

A. 1.95%
B. 3.39%
C. 3.02%
D. 2.23%

Problem 625: EE Board April 1990

The following data were obtained when a short circuit test was performed upon a 100 kVA, 2400/240 volts distribution transformer: E_{SC} = 72 volts; I_{SC} = 41.6 A; P_{SC} = 1,180 W. All instruments are on the high side during the short circuit test. Calculate the equivalent resistance and reactance of the transformer.

A. 0.682 ohm, 1.59 ohms
B. 0.682 ohm, 1.73 ohms
C. 0.752 ohm, 1.73 ohms
D. 0.752 ohm, 1.59 ohms

Problem 626: EE Board March 1998

A 13.8 kV/480 V, 10 MVA three-phase transformer has 5% impedance. What is the impedance in ohms referred to the primary?

A. 0.952 ohm
B. 0.03 ohm
C. 5.125 ohms
D. 9.01 ohms

Problem 627:

A short circuit test was performed on a 20 kVA, 2000/200 V single-phase transformer. With the secondary winding short circuited, full load current flows in the primary winding with 100 V applied on it. The wattmeter connected in the primary read 300 W. Determine the terminal voltage on full load unity pf.

A. 198.6 V
B. 193.4 V
C. 196.7 V
D. 195.2 V

Problem 628: EE Board April 1990

The following data were obtained when a short circuit test was performed upon a 100 kVA, 2400/240 volts distribution transformer: E_{SC} = 72 volts; I_{SC} = 41.6 A; P_{SC} = 1,180 W. All instruments are on the high side during the short circuit test. Calculate the percent regulation at a power factor of 0.75 lagging.

A. 2.54%
B. 2.86%
C. 2.62%
D. 2.71%

Problem 629: EE Board October 1992

A 200-kVA transformer with impedance of 5% and the power factor of the load is 85%. The primary voltage is 6,000 volts while the copper loss is 5 kW. Find the percentage regulation.

A. 5.03%
B. 5.21%
C. 4.43%
D. 4.03%

Problem 630:

A 150 kVA, three-phase transformer supplies 100 kVA at 50% lagging power factor. The transformer's winding impedance is 2% resistance and 5% reactance, based on 150 kVA. Determine the voltage regulation of the transformer at this load. Assume 100% voltage.

A. 3.55%

B. 5.33%
C. 4.24%
D. 4.05%

Problem 631: EE Board April 1993

A short circuit test was performed upon a 10-kVA, 2300/230 volt transformer with the following results: E_{SC} = 137 volts; P_{SC} = 192 W; I_{SC} = 4.34 A. Calculate the percent regulation at a lagging power factor of 0.707.

A. 5.28%
B. 5.55%
C. 5.37%
D. 5.72%

Problem 632:

A 100 kVA, 6600/600 V, 60 Hz single-phase transformer operating at rated condition has the following losses: hysteresis loss = 520 W; eddy current loss = 370 W and copper loss = 2000 W. If this transformer is to be used on a 50 Hz system, determine its new kVA rating. Assume the same total losses and maximum core flux.

A. 83.8
B. 94.3
C. 87.4
D. 91.8

Problem 633:

When 120 V, 60 Hz is applied to the primary winding of a transformer, the core loss is 240 W. When 50 V, 25 Hz is applied the core loss is 50 W. Determine the core loss when 230 V, 50 Hz is applied to the primary winding.

A. 703.5 W
B. 942.6 W
C. 863.6 W
D. 874.7 W

Problem 634:

The eddy current and hysteresis losses of a 4,400-V, 60-Hz single-phase distribution transformer are 280 and 560 watts, respectively. Determine the core loss when the transformer is connected across a 4,600-volt, 50-Hz source.

A. 977 W
B. 983 W
C. 994 W
D. 967 W

Problem 635: EE Board October 1992

In a 400 V, 50 c/s transformer, the total iron loss is 2500 W. When the supplied p.d. is 220 V at 25 c/s, the corresponding loss is 850 W. Calculate the eddy current loss at normal frequency and p.d.

A. 2337 W
B. 2165 W
C. 2010 W
D. 2216 W

Problem 636:

When a 220 V, 60 Hz is impressed on a certain transformer at no load, the total core loss is 200 W. When the frequency of the impressed voltage is changed to 25 Hz and the magnitude of the voltage is made such as to maintain the same maximum flux density as before, the core loss falls to 75 W. Calculate the hysteresis losses at 60 Hz.

A. 163 W
B. 166 W
C. 171 W
D. 169 W

Problem 637:

A 100-kVA, 4160/230 V, 60 Hz single-phase transformer has percentage impedance of 1.75 + j 4.0. Its efficiency when operating at rated conditions at 75% pf lagging is 97%. Determine the efficiency, if the transformer is loaded at rated load and voltage but at 50 Hz. Assume the ratio of eddy current losses to hysteresis losses at 60 Hz to be 2.5.

A. 97.13%
B. 96.97%
C. 97.61%
D. 96.42%

Problem 638:

The full load copper loss of a 75 kVA single-phase transformer is thrice the iron losses. At half load unity pf, the efficiency is 97%. Determine the full load efficiency at 0.8 pf lagging.

A. 95.77%
B. 98.45%
C. 94.78%
D. 97.86%

Problem 639: EE Board April 1990

A 50 kVA, 2300/230 volt, 60-cycle transformer is tested in the laboratory so that its characteristics maybe determined. The standard test requires an open circuit test and short circuit test.

Open circuit test	I = 6.5 A	E = 230 V	P = 187 W
Short circuit test	I = 21.7 A	E = 115 V	P = 570 W

Determine the efficiency of the transformer at half load, unity pf.

A. 97.55%
B. 98.70%
C. 98.45%
D. 97.80%

Problem 640:

The iron loss of a 25-kVA, 60 Hz single-phase distribution transformer is 1% of the rated output at unity power factor and the full load copper loss is 360 W. Determine the efficiency when operating one half of the rated output at 70% power factor lagging.

A. 95.33%
B. 97.82%
C. 94.24%
D. 96.26%

Problem 641: EE Board April 1992

Given that full load copper losses are exactly twice the iron losses in a 50-kVA transformer, and that the quarter load efficiency is 96.5%. Calculate the full load efficiency at unity power factor.

A. 97.48%
B. 98.12%
C. 97.64%
D. 96.25%

Problem 642:

A 15 kVA, 2300/230-volt transformer is under test. With the low side supplied with rated voltage and the high side open circuited, input power through a wattmeter reads 245 W. With the high side supplied with 65 V while the low side short circuited, input power at rated current reads 350 W. Determine the maximum efficiency that can be expected from this transformer at unity pf.

A. 97.29%
B. 96.25%
C. 97.83%
D. 98.21%

Problem 643:

A 10-kVA, 2300/230 transformer has an equivalent resistance referred to the secondary winding of 0.05 ohm. The core loss of this transformer is 75 W. At what kVA load will this transformer operate at maximum efficiency?

A. 8.17 kVA
B. 9.08 kVA
C. 9.32 kVA
D. 8.90 kVA

Problem 644: EE Board April 1995

The core loss of a 5 kVA single-phase transformer with normal voltage applied to the primary is 75 watts. The maximum efficiency occurs at 60% of full load kVA. What is the full load efficiency of the transformer at 0.80 power factor?

A. 95.16%
B. 93.38%
C. 89.52%
D. 91.36%

Problem 645:

A 100-kVA transformer has 94% efficiency at full load and at 50% full load with unity pf in both cases. Determine the maximum efficiency of the transformer at unity pf load.

A. 96.27%
B. 95.68%
C. 93.44%
D. 94.32%

Problem 646:

A 100 kVA transformer has a maximum efficiency of 98% at 10% underload and at unity pf. Evaluate the efficiency at quarter load, 0.80 pf lagging.

A. 96.3%
B. 95.3%
C. 94.7%
D. 97.7%

Problem 647:

A 1 MVA, 66/11 kV, 60 Hz transformer has an equivalent series impedance of 1.0 + j 5 ohms referred to the low voltage side and a no-load loss of 5500 watts at rated terminal voltage. What is the maximum efficiency at 0.8 pf lagging?

A. 98.34%
B. 97.42%
C. 98.02%
D. 96.38%

Problem 648:

The all day efficiency of a 10 kVA single-phase transformer is 94.7% when loaded as follows: full load unity pf for 4 hours and no load the rest of the day. If the full load copper loss at unity pf is 400 watts, calculate the core loss.

A. 75 kW
B. 70 kW
C. 72 kW
D. 78 kW

Problem 649: EE Board October 1980, EE Board October 1982

A 25 kVA distribution transformer operates at 20% overload, 0.3 pf lagging for 1 hour; at full load, 0.9 lagging pf for 3 hours; 50% load at unity pf for 8 hours; and 10% load at 0.8 leading pf for the rest of the day. If core loss is 150 watts and full load copper loss is 500 W, what is the all day efficiency of the transformer?

A. 94.23%
B. 92.86%
C. 97.14%
D. 96.68%

Problem 650: EE Board October 1990

Find the all-day efficiency of a transformer having a maximum efficiency of 98 percent at 15 kVA at unity pf and loaded as follows:

12 hours ---------- 2 kW at 0.5 pf lagging
6 hours ---------- 12 kW at 0.8 pf lagging
6 hours ---------- at no-load

A. 95.3%
B. 91.7%
C. 94.6%
D. 93.3%

Problem 651: EE Board October 1991

A 30 kVA 2400/240 V, 60 Hz transformer has a full load power of unity over the period of 24 hours. The maximum efficiency is 95% and it occurs at full load. Calculate the all-day efficiency if loaded as follows:

6 hours at full load
6 hours at quarter load

A. 91.53%
B. 92.45%
C. 90.37%
D. 93.86%

Problem 652: EE Board October 1991, EE Board April 1994

A 1000 kVA, 6,600 /400 volts 60 cycle single phase core type transformer has the following average daily load:

Full load at 0.8 pf for 8 hours
Half load at 0.707 pf for 10 hours
No-load for 6 hours

Calculate the ratio of full load copper loss to the iron loss for the transformer to be most economical for the above loading.

A. 2.05
B. 3.09
C. 2.17
D. 2.28

Problem 653:

The full load copper loss and iron loss of a 75 kVA transformer are equal to 1 kW. During a given day, the transformer is loaded as follows: full load at unity pf for 8 hours, one-half load at unity for 8 hours and no-load the rest of the day. Calculate the all day efficiency.

A. 96.36%
B. 97.23%
C. 96.47%
D. 98.32%

Problem 654:

A 6.9 kV/600 V, multi-tap, single-phase distribution transformer is connected to the far end of a distribution line for which the rear end voltage is maintained at 6.9 kV. The equivalent impedance of the transformer referred to low voltage side is 0.05 + j0.25 Ω per phase. The impedance of each line is 1+ j2 Ω. It is required to maintain a terminal voltage of 600 V when the line current of 300 A at 80% pf lagging is delivered to the load. What approximate percentage tapping is required in the primary side? Neglect changes in impedance due to changes in turns ratio.

A. 94 %
B. 92 %
C. 90 %
D. 88 %

Problem 655:

A 60 Hz alternator generates a single-phase voltage of 120 V and has a reactance of 0.25 ohm. The alternator feeds a line, which is essentially resistive with an impedance of 0.1 ohm. The line is terminated with a 2:1 step up transformer. What should be the load impedance to be connected to the secondary side to have maximum power transfer to it?

A. 0.5 + j 1.2
B. 0.4 - j 1.0

C. 0.5 − j 1.2
D. 0.4 + j 1.0

Problem 656:

A transformer consists of a primary winding with 500 turns and two secondary windings of 125 turns and 36 turns. The 125-turn secondary winding has 60 ohms connected to its terminals and the 36-turn secondary winding has 3 ohms connected to its terminals. If the primary is connected to a 120-V, 60 Hz source, determine the current in the primary windings.

A. 0.332 A
B. 0.412 A
C. 0.235 A
D. 0.257 A

Problem 657:

A 3-phase, 60 Hz transformer has primary, secondary and tertiary windings. The primary winding is delta connected and rated 6,600 V. The secondary winding is delta connected and rated 1000 V. The tertiary winding is wye connected and rated 440 V. If the secondary winding has a 3-phase balanced load of 100 kVA at 80% pf lagging and the tertiary winding has a balanced 3-phase load of 50 kW at unity pf, what is the current in the primary winding and its operating pf?

A. 12.52 A, 0.908
B. 7.23 A, 0.902
C. 12.52 A, 0.902
D. 7.23 A, 0.908

Problem 658:

A 20-kVA, 500-V load is to be supplied by an ideal step-up autotransformer from a 400-V source. Find the current in the common winding.

A. 30 A
B. 40 A
C. 50 A
D. None of these

Problem 659:

The two windings of a 10-kVA, 2300/230 volt, distribution transformer is connected in series to form an autotransformer giving a small reduction to potential difference from a 2300-volt line. Determine its kVA rating as an ideal autotransformer, operating in the above conditions.

A. 10 kVA
B. 100 kVA
C. 110 kVA
D. None of these

Problem 660:

A 10 kVA, 440/110 volt, single-phase transformer has an efficiency of 96% at a rated load of unity power factor. The full load copper loss is 250 watts. If this transformer is to be converted as a 550/440-volt autotransformer, determine its efficiency when supplying full load at 85 percent power factor.

A. 99.25%
B. 99.03%
C. 97.45%
D. 98.61%

ANSWER KEY

613. C	624. B	635. B	646. B	657. D
614. A	625. A	636. B	647. A	658. D
615. A	626. A	637. B	648. B	659. B
616. C	627. C	638. A	649. D	660. B
617. C	628. D	639. B	650. A	
618. C	629. C	640. D	651. C	
619. A	630. A	641. C	652. D	
620. C	631. C	642. B	653. A	
621. D	632. C	643. D	654. C	
622. C	633. C	644. B	655. B	
623. B	634. A	645. D	656. A	

☑ RATING

43-48 - Topnotcher

33-42 - Passer

24-32 - Conditional

0-23 - Failed

Solutions to Test 17

622 Using primary quantities :

$$N_p = \frac{3000}{8} = 375 \text{ turns}$$

$$E_p = 4.44 N_p f_p \Phi_m$$

$$\Phi_m = \beta_m A$$

$$E_p = 4.44 N_p f_p \beta_m A$$

$$A = \frac{E_p}{4.44\, N_p f_p \beta_m} = \frac{3000}{4.44(375)(50)(1.25)}$$

$$A = 0.0288\ m^2 \times \frac{(100\ cm)^2}{1\ m^2}$$

$$A = 288\ cm^2$$

623 $E_p = 4.44 N_p f_p \beta_m A$

$$A = 20\ in^2 \times \left(\frac{2.54\ cm}{1\ in}\right)^2 = 129\ cm^2$$

Substitute :

$$E_p = 4.44(1000)(60)(11{,}000)(129) \times 10^{-8}$$

$$E_p = 3780\ V$$

Note: Since the flux as given is in Maxwells, the equation shall be multiplied by 10^{-8} (1 Weber = 1×10^8 Maxwells)

624 $E = 4.44 N f \Phi_m$

$$\Phi_m = \frac{E}{4.44Nf} = \left(\frac{1}{4.44f}\right)\left(\frac{E}{N}\right) = \left(\frac{1}{4.44(60)}\right)(2)$$

$$\Phi_m = 7.5\ mWb$$

625 $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

$$I_1 = \frac{N_2 I_2}{N_1} = \frac{2(300)}{30}$$

$$I_1 = 20\ A$$

626 $\frac{N_1}{N_2} = \frac{E_1}{E_2}$

$$N_2 = \frac{N_1E_2}{E_1} = \frac{(1500)(250)}{4160}$$

$N_2 = 90$ turns

627 Let : L_1 = total length of the primary winding

L_2 = total length of the secondary winding

x = mean length per turn of the secondary winding

I_{D_1} = current density of the primary winding

I_{D_2} = current density of the secondary winding

$$L_2 = xN_2$$

$$L_1 = (1.15\,x)N_1$$

$$R = \frac{\rho L}{A};\quad I_D = \frac{I}{A}$$

$$R = \frac{\rho L I_D}{I}$$

$$\frac{R_2}{R_1} = \frac{L_2 I_{D_2} I_1}{L_1 I_{D_1} I_2} = \left(\frac{xN_2}{1.15\,xN_1}\right)\left(\frac{I_{D_2}}{I_{D_1}}\right)\left(\frac{N_2}{N_1}\right) = \left(\frac{1}{1.15}\right)\left(\frac{N_2}{N_1}\right)^2\left(\frac{I_{D_2}}{I_{D_1}}\right)$$

$$R_2 = \left(\frac{R_1}{1.15}\right)\left(\frac{N_2}{N_1}\right)^2\left(\frac{I_{D_2}}{I_{D_1}}\right) = \left(\frac{20}{1.15}\right)\left(\frac{1}{10}\right)^2\left(\frac{1500}{1250}\right)$$

$R_2 = 0.2086\,\Omega$

628 $P_{cu_1} = 0.8P_{cu_2}$

$$I_1^2R_1 = 0.8\left(I_2^2R_2\right)$$

$$\left(\frac{I_1}{I_2}\right)^2 R_1 = 0.8\,R_2$$

$$\left(\frac{N_2}{N_1}\right)^2 (0.00283\,N_1) = 0.8\,R_2 \Rightarrow ❶$$

$$N_1 = N_2 + 1155 \Rightarrow ❷$$

$$\frac{N_1}{N_2} = 8$$

$$N_1 = 8N_2 \Rightarrow ❸$$

Substitute Eq. 3 in Eq.2 :

$$8N_2 = N_2 + 1155$$

$N_2 = 165$ turns

$N_1 = 165 + 1155 = 1320$ turns

Substitute in Eq.1:

$$\left(\frac{1}{8}\right)^2 (0.00283)(1320) = 0.8\,R_2$$

$R_2 = 0.073\,\Omega$

629 $a = \dfrac{N_1}{N_2} = \dfrac{2400}{240} = 10$

$I_2 = \dfrac{S}{V_2} = \dfrac{10{,}000}{240} = 41.667$ A

$\theta = \cos^{-1}0.8 = 36.87°$

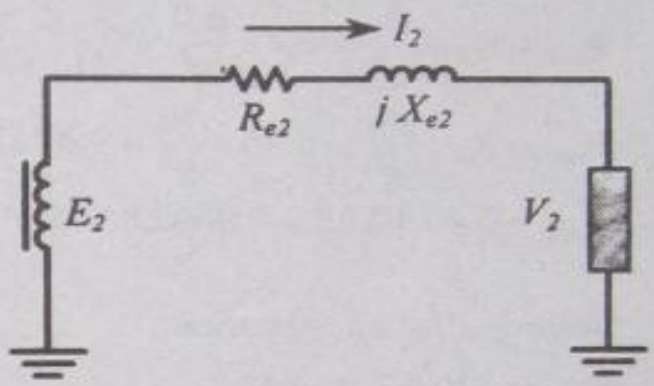
$R_{e2} = R_2 + \dfrac{R_1}{a^2}$

$= 0.03 + \dfrac{3.0}{10^2} = 0.06\,\Omega$

$X_{e2} = X_2 + \dfrac{X_1}{a^2}$

$= 0.15 + \dfrac{15}{10^2} = 0.30\,\Omega$

$Z_{e2} = 0.06 + j0.3 = 0.306\angle 78.69°$



Equivalent circuit of the transformer referred to the secondary side.

Assume V_2 as reference:

$E_2 = V_2 + I_2 Z_{e2}$

$E_2 = 240\angle 0° + (41.667\angle -36.869°)(0.306\angle 78.69°)$

$= 240 + 12.75\angle 41.82°) = 240 + 9.502 + j8.502$

$= 249.502 + j8.502$

$E_2 = 249.647\angle 1.95°$ V

$E_1 = aE_2$

$E_g = E_1 = 249.647(10)$

$E_g = 2496.47$ V

Note: E_g = voltage supplied at the primary side.
E_1 = voltage induced in the primary winding

630 Note: Refer to the circuit diagram of problem #629.

$$a = \frac{N_1}{N_2} = \frac{2000}{400} = 5$$

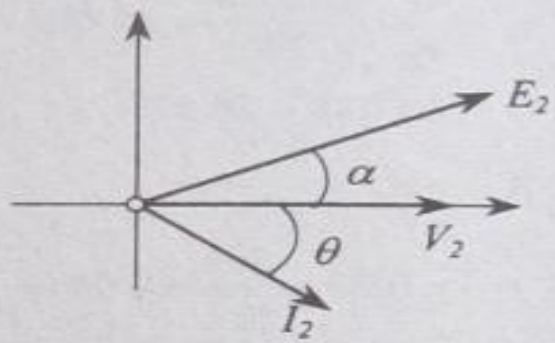
$$I_2 = \frac{S}{V_2} = \frac{10{,}000}{400} = 25\text{ A}$$

$$\theta = \cos^{-1}0.8 = 36.869°$$

$$R_{e2} = R_2 + \frac{R_1}{a^2} = 0.2 + \frac{5.0}{5^2} = 0.4\,\Omega$$

$$X_{e2} = X_2 + \frac{X_1}{a^2} = 0.45 + \frac{10}{5^2} = 0.85\,\Omega$$

$$Z_{e2} = 0.4 + j0.85 = 0.939\angle 64.798°$$



Assume V_2 as reference :

$$\theta = \cos^{-1}0.8 = 36.869°$$

$$V_2 = E_2 - I_2 Z_{e2}$$

$$V_2\angle 0° = 400\angle\alpha° - (25\angle -36.869°)(0.939\angle 64.798°)$$

$$= 400\cos\alpha + j400\sin\alpha - (23.475\angle 27.929°)$$

$$= 400\cos\alpha + j400\sin\alpha - (20.74 + j10.995)$$

$$= (400\cos\alpha - 20.74) + j(400\sin\alpha - 10.995)$$

Note: Since V_2 is the reference, "j-component must be zero.

$$400\sin\alpha = 10.995$$

$$\alpha = 1.575°$$

$$V_2 = 400\cos\alpha - 20.74 = 400\cos 1.575° - 20.74 = 379.108$$

$$V_2 \cong 379\text{ volts}$$

631 $$I_{sc} \cong I_p = \frac{\text{VA rating}}{V_p} = \frac{75{,}000}{8000} = 9.375\text{ A}$$

$$Z_{eq} = \frac{V_{sc}}{I_{sc}} = \frac{440}{9.375} = 46.933\,\Omega$$

$$\%IZ = \frac{I_p Z_{eq}}{V_p} = \frac{9.375(46.933)}{8000}$$

$$\%IZ = 5.5\%$$

632 $Z_p = \frac{E_{sc}}{I_{sc}} = \frac{137}{4.34} = 31.567\ \Omega$

$R_p = \frac{P_{sc}}{I_{sc}^2} = \frac{192}{4.34^2} = 10.193\ \Omega$

$X_p = \sqrt{Z_p^2 - R_p^2} = \sqrt{(31.567)^2 - (10.193)^2}$

$X_p = 29.88\ \Omega$

633 $I_{sc} \cong I_p = \frac{\text{VA rating}}{V_p} = \frac{25{,}000}{4800} = 5.208\ \text{A}$

$R_p = \frac{P_{sc}}{I_{sc}^2} = \frac{162.74}{5.208^2} = 31.248\ \Omega$

$\%IR = \frac{I_p R_p}{V_p} = \frac{5.208(31.248)}{4800} \times 100\%$

$\%IR = 3.39\%$

634 $R_{eq} = \frac{P_{sc}}{(I_{sc})^2} = \frac{1{,}180}{(41.6)^2} = 0.682\ \Omega$

$Z_{eq} = \frac{E_{sc}}{I_{sc}} = \frac{72}{41.6} = 1.73\ \Omega$

$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{(1.73)^2 - (0.682)^2}$

$X_{eq} = 1.59\ \Omega$

635 $\%IZ = \frac{IZ}{V} = \frac{Z_1(\text{VA rating})}{V_1^2}$

$Z_1 = \frac{\%IZ(V_1)^2}{\text{VA rating}} = \frac{0.05(13.8 \times 10^3)^2}{10 \times 10^6}$

$Z_1 = 0.952\ \Omega$

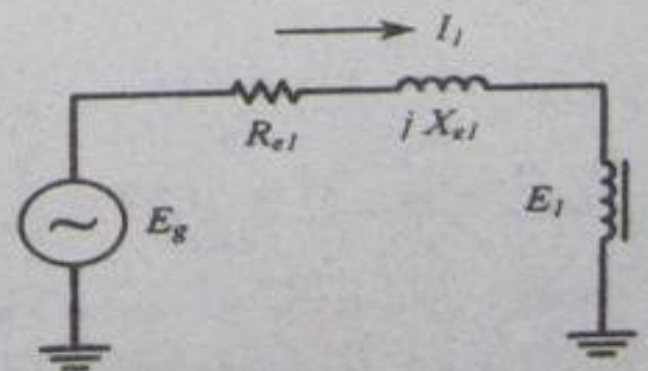
636 $I_{sc} \cong I_p = \frac{\text{VA rating}}{V_p} = \frac{20000}{2000} = 10\ \text{A}$

$R_{eq} = \frac{P_{sc}}{(I_{sc})^2} = \frac{300}{(10)^2} = 3\ \Omega$

$Z_{eq} = \frac{E_{sc}}{I_{sc}} = \frac{100}{10} = 10\ \Omega$

$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{(10)^2 - (3)^2}$

$X_{eq} = 9.54\ \Omega$



Equivalent circuit of the transformer referred to the primary side.

$Z_{eq} = 3 + j9.54 = 10\angle 72.54°$

Assume E_1 as reference :

$\theta = \cos^{-1}1 = 0°$

$E_1 = E_g - I_1 Z_{e1}$

$E_1\angle 0° = 2000\angle\alpha° - (10\angle 0°)(10\angle 72.54°)$

$E_1\angle 0° = 2000\cos\alpha + j2000\sin\alpha - (100\angle 72.54°)$

$= 2000\cos\alpha + j2000\sin\alpha - (30 + j95.39)$

$= (2000\cos\alpha - 30) + j(2000\sin\alpha - 95.39)$

Note: Since E_1 is the reference, "j-component must be zero.

$2000\sin\alpha - 95.39 = 0$

$\alpha = 2.734°$

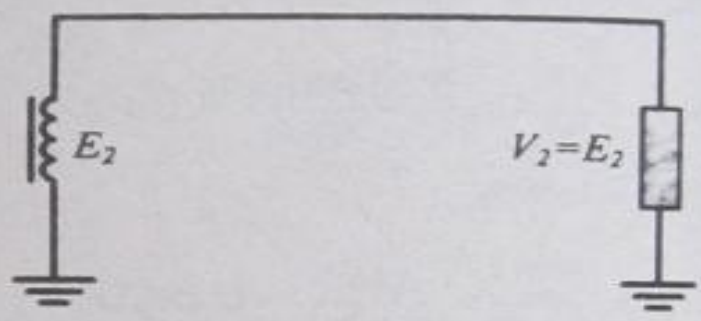
$E_1 = 2000\cos\alpha - 30$

$= 2000\cos 2.734° - 30$

$= 1967.72\text{ V}$

$E_2 = \dfrac{E_1}{a} = \dfrac{1967.72}{10} = 196.7\text{ V}$

$V_2 = E_2 = 196.7\text{ V}$



Equivalent circuit of the transformer referred to the secondary side.

637 $R_{eq} = \dfrac{P_{sc}}{(I_{sc})^2} = \dfrac{1{,}180}{(41.6)^2} = 0.682\,\Omega$

$Z_{eq} = \dfrac{E_{sc}}{I_{sc}} = \dfrac{72}{41.6} = 1.73\,\Omega$

$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$

$= \sqrt{(1.73)^2 - (0.682)^2} = 1.589\,\Omega$

$\theta = \cos^{-1}0.75 = 41.409°$

$\sin\theta = \sin 41.409° = 0.6614$

$\%IR = \dfrac{I_p R_{eq}}{V_p} = \dfrac{41.667(0.682)}{2400} = 0.01184$

$\%IX = \dfrac{I_p X_{eq}}{V_p} = \dfrac{41.667(1.589)}{2400} = 0.02758$

$\%VR = \sqrt{(\cos\theta + \%IR)^2 + (\sin\theta + \%IX)^2} - 1$

$= \sqrt{(0.75 + 0.01184)^2 + (0.6614 + 0.02758)} - 1$

$\%VR = 2.717\%$

638 $\%IR = \dfrac{P_{cuFL}}{\text{VA rating}} = \dfrac{5}{200} = 0.025$

$\%IX = \sqrt{\%IZ^2 - \%IR^2} = \sqrt{(0.05)^2 - (0.025)^2} = 0.0433$

$\theta = \cos^{-1} 0.85 = 31.788°$

$\sin\theta = \sin 31.788° = 0.5267$

$\%VR = \sqrt{(\cos\theta + \%IR)^2 + (\sin\theta + \%IX)^2} - 1$

$= \sqrt{(0.85 + 0.025)^2 + (0.5267 + 0.0433)^2} - 1$

$\%VR \cong 4.43\%$

639 $\%IR = \dfrac{IR}{V} \bullet \dfrac{V}{V} = \dfrac{\text{VA rating (R)}}{V^2}$

Note: Since V and R are constant, then %IR is directly proportional to its VA rating.

$\dfrac{\%IR_1}{\%IR_2} = \dfrac{VA_1}{VA_2}$

$\%IR_2 = \%IR_1\left(\dfrac{VA_2}{VA_1}\right)$

$\%IR_2 = 0.02\left(\dfrac{100}{150}\right) = 0.0133$

$\%IX_2 = 0.05\left(\dfrac{100}{150}\right) = 0.0333$

$\theta = \cos^{-1} 0.5 = 60°$

$\sin\theta = \sin 60° = 0.866$

$\%VR = \sqrt{(\cos\theta + \%IR_2)^2 + (\sin\theta + \%IX_2)^2} - 1$

$= \sqrt{(0.5 + 0.0133)^2 + (0.866 + 0.0333)^2} - 1$

$\%VR = 3.55\%$

640 $P_{cuFL} \cong P_{sc}$

$\%IR = \dfrac{P_{cuFL}}{\text{VA rating}} = \dfrac{195}{10000} = 0.0195$

$Z_{eq} = \dfrac{V_{sc}}{I_{sc}} = \dfrac{137}{4.34} = 31.566\ \Omega$

$\%IZ = \dfrac{IZ}{V} = \dfrac{4.34(31.566)}{2300} = 0.0595$

$\%IX = \sqrt{\%IZ^2 - \%IR^2} = \sqrt{(0.0595)^2 - (0.0195)^2} = 0.0562$

$\theta = \cos^{-1} 0.707 = 45°$

$\sin\theta = \sin 45° = 0.707$

$$\%VR = \sqrt{(\cos\theta + \%IR)^2 + (\sin\theta + \%IX)^2} - 1$$

$$= \sqrt{(0.707 + 0.0195)^2 + (0.707 + 0.0562)^2} - 1$$

$\%VR = 5.369\%$

641 $P_e = k_e f^2$

$$\frac{P_{e1}}{P_{e2}} = \left(\frac{f_1}{f_2}\right)^2$$

$$P_{e2} = P_{e1}\left(\frac{f_2}{f_1}\right)^2 = 370\left(\frac{50}{60}\right) = 256.94\text{W}$$

$P_h = k_h f$

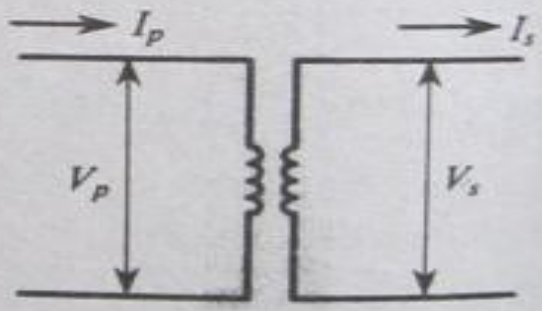
$$\frac{P_{h1}}{P_{h2}} = \frac{f_1}{f_2}$$

$$P_{h2} = P_{h1}\left(\frac{f_1}{f_2}\right) = 520\left(\frac{50}{60}\right) = 433.33\text{ W}$$

$P_{loss1} = P_{e1} + P_{h1} + P_{ou1}$

$P_{loss_1} = 370 + 520 + 2000 = 2890$

$P_{loss_2} = P_{loss_1}$



$$I_{s_1} = \frac{\text{VA rating}}{V_s} = \frac{100{,}000}{600} = 166.67\text{ A}$$

$$R_{e2} = \frac{P_{ou_1}}{I_{s_1}^2} = \frac{2000}{(166.67)^2} = 0.072\,\Omega$$

$$P_{ou2} = P_{loss2} - (P_{e2} + P_{h2}) = 2890 - (256.94 + 433.33)$$

$$= 2199.73\text{ W}$$

$$I_{s2} = \sqrt{\frac{P_{ou2}}{R_{e2}}} = \sqrt{\frac{2199.73}{0.072}} = 174.79\text{ A}$$

Note: With the maximum core flux constant, induced emf therefore is directly proportional to the frequency.

$E = kf$

$$\frac{E_1}{E_2} = \frac{f_1}{f_2}$$

$$E_2 = E_1\left(\frac{f_2}{f_1}\right) = 600\left(\frac{50}{60}\right) = 500\text{ V}$$

$$\text{New kVA rating} = \frac{E_2 I_{s2}}{1000} = \frac{500(174.79)}{1000}$$

New kVA rating = 87.4 kVA

642 $P_e = k_e E^2$

$$P_h = k_h\left(\frac{E^{1.6}}{f^{0.6}}\right)$$

$$P_{core} = P_e + P_h = k_e E^2 + k_h\left(\frac{E^{1.6}}{f^{0.6}}\right)$$

When supply is 120V, 60 Hz :

$$240 = k_e(120)^2 + k_h\left(\frac{120^{1.6}}{60^{0.6}}\right)$$

$$240 = 14400\,k_e + 181.886\,k_h$$

$$k_h = 1.319 - 79.17k_e \Rightarrow ❶$$

When supply is 50V, 25 Hz :

$$50 = k_e(50)^2 + k_h\left(\frac{50^{1.6}}{25^{0.6}}\right)$$

$$50 = 2500\,k_e + 75.785\,k_h \Rightarrow ❷$$

Substitute Eq.1 to Eq.2 :

$$50 = 2500k_e + 75.785(1.319 - 79.17k_e)$$

$$k_e = 0.01427$$

$$k_h = 1.319 - 79.17(0.01427) = 0.1892$$

When supply is 230V, 60 Hz :

$$P_{core} = k_e E^2 + k_h\left(\frac{E^{1.6}}{f^{0.6}}\right) = (0.01427)(230)^2 + (0.1892)\left(\frac{230^{1.6}}{50^{0.6}}\right)$$

$$P_{core} = 863.6\text{ W}$$

643 When supply is 4,400 V, 60 Hz :

$P_e = k_e E^2$

$$k_e = \frac{P_e}{E^2} = \frac{280}{4400^2} = 14.462 \times 10^{-6}$$

$$P_h = k_h\left(\frac{E^{1.6}}{f^{0.6}}\right)$$

$$k_h = \frac{P_h f^{0.6}}{E^{1.6}} = \frac{(560)(60)^{0.6}}{4400^{1.6}} = 9.673 \times 10^{-3}$$

When supply is 4,600 V, 50 Hz :

$$P_e = k_e E^2 = 14.462 \times 10^{-6}(4600)^2 = 306.016 \text{ watts}$$

$$P_h = k_h\left(\frac{E^{1.6}}{f^{0.6}}\right) = 9.673 \times 10^{-3}\left(\frac{4600^{1.6}}{50^{0.6}}\right) = 670.798 \text{ watts}$$

$$P_{core} = P_e + P_h = 306.016 + 670.798$$

$$P_{core} \cong 977 \text{ watts}$$

644 $P_e = k_e E^2$

$$P_h = k_h\left(\frac{E^{1.6}}{f^{0.6}}\right)$$

$$P_{core} = P_e + P_h = k_e E^2 + k_h\left(\frac{E^{1.6}}{f^{0.6}}\right)$$

When supply is 400V, 50 Hz :

$$2500 = k_e(400)^2 + k_h\left(\frac{400^{1.6}}{50^{0.6}}\right)$$

$$2500 = 160000\,k_e + 1392.88\,k_h$$

$$k_h = 1.795 - 114.87\,k_e \Rightarrow (1)$$

When supply is 220V, 25 Hz :

$$850 = k_e(220)^2 + k_h\left(\frac{220^{1.6}}{25^{0.6}}\right)$$

$$850 = 48400\,k_e + 811.17\,k_h \Rightarrow (2)$$

Substitute Eq.1 in Eq.2 :

$$850 = 48400\,k_e + 811.17(1.795 - 114.87k_e$$

$$k_e = 0.01353$$

$$P_e = 0.01353(400)^2$$

$$P_e = 2164.8 \text{ W}$$

645 $P_{co} = P_e + P_h = k_e f^2 + k_h f$

$200 = k_e(60)^2 + k_h(60)$

$k_e = 0.0555 - 0.01667k_h \Rightarrow$ ❶

$75 = k_e(25)^2 + k_h(25)$

$k_e = 0.12 - 0.04k_e \Rightarrow$ ❷

Equate Eq.1 to Eq.2 :

$0.0555 - 0.01667\,k_h = 0.12 - 0.04k_h$

$k_h = 2.765$

$P_h = (2.765)(60) = 165.9\text{ W}$

646 $\%IR = \dfrac{P_{cuFL}}{\text{VA rating}}$

$P_{cuFL} = (\%IR)(\text{VA rating}) = 0.0175(100{,}000)$

$P_{cuFL} = 1750\text{ W}$

$\eta = \dfrac{P_{out}}{P_{out} + P_{co} + P_{cuFL}}$

$P_{co} = \dfrac{P_{out}}{\eta} - (P_{out} + P_{cuFL})$

$= \dfrac{100{,}000(0.75)}{0.97} - [(100{,}000)(0.75) + 1750]$

$P_{co} = 569.587\text{ W}$

$P_{co} = P_e + P_h$

$P_{co} = 2.5\,P_h + P_h$

$3.5\,P_h = 569.587$

$P_h = 162.74\text{ W}$

$P_e = 2.5(162.74) = 406.85\text{ W}$

Note: Since supply voltage remains constant, eddy current loss is constant

At 50 Hz :

$P_h = k_h \dfrac{E^{1.6}}{f^{0.6}}$

$\dfrac{P_{h60}}{P_{h50}} = \dfrac{f_{50}^{\;0.6}}{f_{60}^{\;0.6}}$

$$P_{h50} = P_{h60}\left(\frac{f_{60}}{f_{50}}\right)^{0.6} = 162.74\left(\frac{60}{50}\right)^{0.6}$$

$$P_{h50} = 181.553 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cuFL}} = \frac{100{,}000(0.75) \times 100\%}{100{,}000(0.75) + (406.85 + 181.553) + 1750}$$

$$\eta = 96.97\%$$

647 $P_{cuFL} = 3P_{co}$

At quarter load :

$P_{out} = 0.5 \text{ Rating (pf)} = 0.5(75)(1) = 37.5 \text{ kW}$

$P_{cu} = (\text{size})^2 P_{cuFL} = (0.5)^2(3P_{co}) = 0.75P_{co}$

$$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cu}}$$

$$0.97 = \frac{37.5}{37.5 + P_{co} + 0.75P_{co}}$$

$P_{co} = 0.6627 \text{ kW}$

At full - load :

$P_{out} = 75(0.8) = 60 \text{ kW}$

$$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cu}} = \frac{60}{60 + 0.6627 + 3(0.6627)} \times 100\%$$

$\eta = 95.77\%$

648 $P_{co} \cong P_{oc} = 187 \text{ W}$

$P_{cuFL} \cong P_{sc} = 570 \text{ W}$

At half - load :

$P_{out} = (0.5)\text{Rating(pf)} = (0.5)(50)(1) = 25{,}000 \text{ W}$

$P_{cu} = (\text{size})^2 P_{cuFL} = (0.5)^2(570) = 142.5 \text{ W}$

$$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cu}} = \frac{25000}{25000 + 187 + 142.5} \times 100\%$$

$\eta = 98.69\%$

649 At half - load :

$P_{out} = (0.5)(\text{Rating})(\text{pf}) = (0.5)(25000)(0.7) = 8750 \text{ W}$

$P_{co} = 0.01(25)(1) = 250 \text{ W}$

$P_{cu} = (\text{size})^2 (P_{cuFL}) = (0.5)^2 (360) = 90 \text{ W}$

$$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cu}} = \frac{8750}{8750 + 250 + 90} \times 100\%$$

$\eta = 96.26\%$

650 $P_{cuFL} = 2P_{co}$

At quarter - load :

$P_{out} = (0.25)(50{,}000)(1) = 12500 \text{ W}$

$P_{cu} = (\text{size})^2 P_{cuFL} = (0.25)^2 (2P_{co}) = 0.125P_{co}$

$$\eta = \frac{P_{out}}{P_{out} + P_{cu} + P_{co}}$$

$$0.965 = \frac{12500}{12500 + 0.125P_{co} + P_{co}}$$

$P_{co} = 403 \text{ watts}$

At full - load :

$P_{out} = 50{,}000(1) = 50{,}000 \text{ W}$

$$\eta = \frac{P_{out}}{P_{out} + P_{cuFL} + P_{co}} = \frac{50{,}000}{50{,}000 + 2(403) + 403}$$

$\eta_{FL} = 97.64\%$

651 $P_{co} \cong P_{oc} = 245 \text{ W}$

$P_{cuFL} \cong P_{sc} = 350 \text{ W}$

Note: At maximum efficiency, copper loss is equal to core loss.

$P_{cu_m} = (\text{size})^2 P_{cuFL}$

$245 = (\text{size})^2 (350)$

$\text{size} = 84\%$

$$\eta_m = \frac{P_{out_m}}{P_{out_m} + 2P_{co}}$$

$$= \frac{(0.84)(15{,}000)(1)}{(0.84)(15{,}000)(1) + 2(245)} \times 100\%$$

$\eta_m = 96.25\%$

652 $I_2 = \dfrac{\text{VA rating}}{V_2} = \dfrac{10{,}000}{230} = 43.478\text{ A}$

$P_{cuFL} = I_2^2 R_{e2} = (43.478)^2(0.05) = 94.516\text{ W}$

$P_{cum} = (\text{size})^2 P_{cuFL}$

$75 = (\text{size})^2(94.516)$

size = 0.8908

Load = 0.8908(10)

Load = 8.908 kVA

653 $P_{cum} = P_{co} = (\text{size})^2 P_{cuFL}$

$75 = (0.6)^2 P_{cuFL}$

$P_{cuFL} = 208.33\text{ W}$

$P_{outFL} = \text{Rating(pf)} = 5{,}000(0.8) = 4000\text{ W}$

$$\eta = \frac{P_{out}}{P_{out} + P_{cu} + P_{co}} = \frac{4000}{4000 + 208.33 + 75} \times 100\%$$

$\eta = 93.38\%$

654 $$\eta_{FL} = \frac{P_{outFL}}{P_{outFL} + P_{co} + P_{cuFL}}$$

$$0.94 = \frac{100(1)}{100(1) + P_{co} + P_{cuFL}}$$

$P_{co} + P_{cuFL} = 6.38 \Rightarrow$ ❶

At half - load :

$P_{cu} = (\text{size})^2 P_{cuFL} = (0.5)^2 P_{cuFL} = 0.25 P_{cuFL}$

$$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cu}}$$

$$0.94 = \frac{50(1)}{50(1) + P_{co} + 0.25 P_{cuFL}}$$

$P_{co} = 3.19 - 0.25 P_{cuFL} \Rightarrow$ ❷

Substitute Eq.2 to Eq.1 :

$3.19 - 0.25 P_{cuFL} + P_{cuFL} = 6.38$

$P_{cuFL} = 4.25\text{ kW}$

$P_{co} = 3.19 - 0.25(4.25) = 2.13\text{ kW}$

At maximum efficiency:

$P_{cu_m} = P_{co} = (\text{size})^2 P_{cuFL}$

$\text{size} = \sqrt{\frac{2.13}{4.25}} = 0.708$

$\eta_m = \frac{P_{out_m}}{P_{out_m} + 2P_{co}} = \frac{0.708(100)(1)}{0.708(100)(1) + 2(2.13)}$

$\eta_m = 94.32\%$

655 $\eta_m = \frac{P_{out_m}}{P_{out_m} + 2P_{co}}$

$0.98 = \frac{0.9(100)(1)}{0.9(100)(1) + 2P_{co}}$

$P_{co} = 0.918\text{ kW}$

$P_{cu_m} = (\text{size})^2 P_{cuFL}$

$0.918 = (0.9)^2 P_{cuFL}$

$P_{cuFL} = 1.133\text{ kW}$

At quarter load:

$P_{cu} = (\text{size})^2 P_{cuFL} = (0.25)^2(1.133) = 0.0708\text{ kW}$

$P_{out} = 0.25(100)(0.8) = 20\text{ kW}$

$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{cu}} = \frac{20 \times 100\%}{20 + 0.918 + 0.0708}$

$\eta = 95.3\%$

656 $I_2 = \frac{\text{VA rating}}{V_2} = \frac{1 \times 10^6}{11000} = 90.9\text{ A}$

$P_{cuFL} = I_2^2 R_{e_2} = (90.9)^2(1) = 8262.81\text{ W}$

$P_{cu_m} = (\text{size})^2 P_{cuFL}$

$5500 = (\text{size})^2(8262.81)$

$\text{size} = 0.8158$

$P_{out_m} = 0.0.8158(1{,}000)(0.8) = 652.64\text{ kW}$

$\eta_m = \frac{P_{out_m}}{P_{out_m} + 2P_{co}} = \frac{652.64}{652.64 + 2(5.5)} \times 100\%$

$\eta_m = 98.34\%$

657

$$W_{out} = \sum(\text{VA rating})(\text{size})(\text{pf})(\text{time})$$
$$W_{out} = (10)(1)(1)(4) = 40\ \text{kW - hr}$$

$$W_{cu} = \sum(\text{size})^2(P_{cuFL})(\text{time})$$
$$W_{cu} = (1)^2(140)(4) = 0.56\ \text{kW - hr}$$
$$W_{co} = \frac{P_{co}(24)}{1000} = 0.024\,P_{co}$$
$$\eta = \frac{W_{out}}{W_{out} + W_{cu} + W_{co}}$$
$$0.947 = \frac{40}{40 + 0.56 + W_{co}}$$
$$W_{co} = 1.678648\ \text{kW - hr}$$
$$P_{co} = \frac{W_{co}}{0.024} = \frac{1.678648}{0.024} = 69.94\ \text{kW}$$
$$P_{co} \cong 70\ \text{kW}$$

658

$$W_{out} = \sum(\text{kVA rating})(\text{size})(\text{pf})(\text{time})$$
$$= (25)(1.2)(0.3)(1) + (25)(1)(0.9)(3) + (25)(0.5)(1)(8) + (25)(0.1)(0.8)(12)$$
$$W_{out} = 200.5\ \text{kW - hr}$$
$$W_{co} = \frac{P_{co}(24)}{1000} = \frac{150(24)}{1000} = 3.6\ \text{kW - hr}$$
$$W_{cu} = \sum(\text{size})^2(P_{cuFL})(\text{time})$$
$$= (1.2)^2(0.5)(1) + (1)^2(0.5)(3) + (0.5)^2(0.5)(8) + (0.1)^2(0.5)(12)$$
$$W_{cu} = 3.28\ \text{kW - hr}$$
$$\eta_{\text{all day}} = \frac{W_{out}}{W_{out} + W_{co} + W_{cu}} = \frac{200.5}{200.5 + 3.6 + 3.28} \times 100\%$$
$$\eta_{\text{all day}} = 96.68\%$$

659

$$\eta_m = \frac{P_{out_m}}{P_{out_m} + 2P_{co}}$$
$$0.98 = \frac{15}{15 + 2P_{co}}$$
$$P_{co} = 0.153\ \text{kW} = P_{cu}$$
$$W_{co} = P_{co} \times 24 = 0.153 \times 24 = 3.672\ \text{kW - hrs.}$$

$$W_{cu} = \sum(\text{size})^2(P_{cu})(\text{time})$$

$$W_{cu} = \left(\frac{2/0.5}{15}\right)^2 (0.153)(12) + \left(\frac{12/0.8}{15}\right)^2 (0.153)(6)$$

$$W_{cu} = 1.048 \text{ kW - hrs.}$$

$$W_{out} = \sum(\text{kW load})(\text{time}) = 2(12) + 12(6) = 96 \text{ kW - hrs.}$$

$$\eta_{all\,day} = \frac{W_{out}}{W_{out} + W_{cu} + W_{co}} = \frac{96 \times 100\%}{96 + 1.048 + 3.672}$$

$$\eta_{all\,day} = 95.3\%$$

CSI

660 $$\eta_m = \frac{P_{out}}{P_{out} + 2P_{co}}$$

$$0.95 = \frac{30(1)}{30(1) + 2P_{co}}$$

$$P_{co} = 0.789 \text{ kW}$$

$$P_{cuFL} = P_{co} = 0.789 \text{ kW}$$

$$W_{co} = P_{co} \times 24 = 0.789 \times 24 = 18.936 \text{ kW - hrs.}$$

$$W_{cu} = \sum(\text{size})^2 (P_{cuFL})(\text{time})$$

$$= (0.789)(6) + (0.25)^2 (0.789)(6) = 5.0298 \text{ kW - hrs.}$$

$$W_{out} = \sum(\text{rating})(\text{size})(\text{pf})(\text{time}) = (30)(1)(1)(6) + (30)(0.25)(1)(6)$$

$$= 225 \text{ kW - hrs.}$$

$$\eta_{all\,day} = \frac{W_{out}}{W_{out} + W_{cu} + W_{co}} = \frac{225 \times 100\%}{225 + 5.0298 + 18.936}$$

$$\eta_{all\,day} = 90.37\%$$

Note: Since not given, it should be understood that the transformer is only loaded for 12 hours and no-load the rest of the day.

661 Note: In order for the transformer to be most economical, its all day efficiency must be maximum. And it will occur only when the total energy lost in the copper windings is equal to the total energy lost in the core.

$$W_{co} = P_{co} \times 24 = 24P_{co}$$

$$W_{cu} = \sum(\text{size})^2 (P_{cuFL})(\text{time})$$

$$W_{cu} = P_{cuFL}(8) + (0.5)^2 P_{cuFL}(10) = 10.5P_{cuFL}$$

$$W_{co} = W_{cu}$$

$$24P_{co} = 10.5P_{cuFL}$$

$$\frac{P_{cuFL}}{P_{co}} = \frac{24}{10.5} = 2.28$$

662 $W_{out} = \sum(\text{rating})(\text{size of load})(pf)(t)$

$= 75(1)(1)(8) + 75(0.5)(1)(8) = 900\ \text{kW - hr}$

$W_{co} = P_{co} \times 24 = 1(24) = 24\ \text{kW - hr}$

$W_{cu} = \sum(\text{size of load})^2 (P_{cuFL})(t)$

$= (1)^2(1)(8) + (0.5)^2(1)(8) = 10\ \text{kW - hr}$

$$\eta_{\text{all day}} = \frac{W_{out}}{W_{out} + W_{co} + W_{cu}} = \frac{900 \times 100\%}{900 + 24 + 10}$$

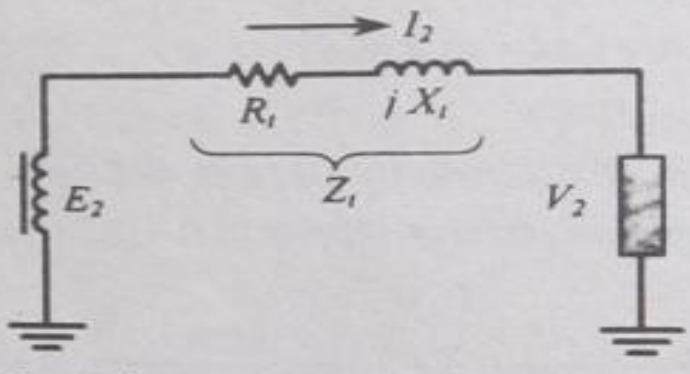
$\eta_{\text{all day}} = 96.36\%$

663 Let : Z_{L_2} = equivalent impedance of the TL referred to the low side

Z_t = total impedance referred to the low side

E_2 = induced emf of the secondary side

V_2 = terminal voltage



Equivalent circuit of the transformer referred to the secondary side.

$$Z_{L_2} = Z_{L_1}\left(\frac{N_2}{N_1}\right)^2 = (1 + j2)\left(\frac{660}{11000}\right)^2 = 0.0108 + j0.0216$$

$Z_t = Z_{L_2} + Z_{e_2} = 0.0108 + j0.0216 + 0.05 + j0.25$

$= 0.0608 + j0.2716 = 0.2783\angle 77.38°$

$\theta_L = \cos^{-1} 0.8 = 36.87°$

Assume V_2 as reference :

$E_2 = V_2 + I_2 Z_t$

$E_2 = V_2 + I_2 Z_t$

$= 600\angle 0° + (300\angle -36.87°)(0.2783\angle 77.38°)$

$= 600 + 83.49\angle 40.51° = 600 + 63.476 + j54.233$

$= 663.476 + j54.233$

$E_2 = 665.68\angle 4.67°\ \text{V}$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$N_1 = N_2\left(\frac{E_1}{E_2}\right) = 600\left(\frac{6900}{665.68}\right) = 6219.2$$

$$\%Tap = \frac{6219.2}{6900} \times 100\% = 90.13\%$$

$$\%Tap \cong 90\%$$

664 Let : Z_{L_1} = total impedance of the primary side

Z_{L_2} = equivalent impedance referred to the secondary side

$$Z_{L_1} = 0.1 + j0.25$$

$$Z_{L_2} = Z_{L_1}\left(\frac{N_2}{N_1}\right)^2 = (0.1 + j0.25)\left(\frac{2}{1}\right)^2 = 0.4 + j1\,\Omega$$

Note: For maximum power transfer, load impedance should be equal to the conjugate of the equivalent secondary line impedance.

$$Z_{load} = 0.4 - j1\,\Omega$$

665 $$E_2 = E_1\left(\frac{N_2}{N_1}\right) = \frac{120(125)}{500} = 30 \text{ volts}$$

$$I_2 = \frac{E_2}{R_2} = \frac{30}{60} = 0.5 \text{ A}$$

$$E_3 = E_1\left(\frac{N_3}{N_1}\right) = \frac{120(36)}{500} = 8.64 \text{ volts}$$

$$I_3 = \frac{E_3}{R_3} = \frac{8.64}{3} = 2.88 \text{ A}$$

$$I_1 = I_2\left(\frac{N_2}{N_1}\right) + I_3\left(\frac{N_3}{N_1}\right)$$

$$= \frac{0.5(125)}{500} + \frac{2.88(36)}{500}$$

$$I_1 = 0.332 \text{ A}$$

Note: Voltage ratio is directly proportional to the turns ratio while current ratio is inversely proportional to the turns ratio.

666 $\theta_2 = \cos^{-1} 0.8 = 36.87°$

$\theta_3 = \cos^{-1} 1.0 = 0°$

Assume V_1 (primary line voltage) as reference :

$I_1 = I_2 + I_3$

$$= \frac{S_2}{\sqrt{3}V_1}\angle -\theta_2 + \frac{P_3}{\sqrt{3}V_1 pf}\angle \theta_3$$

$$= \frac{100{,}000}{\sqrt{3}(6600)}\angle -36.87° + \frac{50{,}000}{\sqrt{3}(6600)(1)}\angle 0°$$

$$= 8.747\angle -36.87° + 4.37\angle 0° = 6.997 - j5.24 + 4.37$$

$I_1 = 12.52\angle -24.74°$

$$I_p = \frac{I_1}{\sqrt{3}} = \frac{12.52}{\sqrt{3}} = 7.23 \text{ A}$$

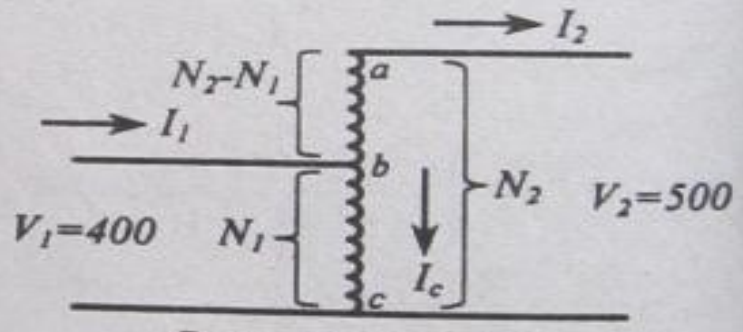
pf = cos24.74°

pf = 0.908 lagging

667 $I_2 = \dfrac{\text{VA rating}}{V_2} = \dfrac{20000}{500} = 40 \text{ A}$

$$I_c = I_2\left(\frac{N_2 - N_1}{N_1}\right) = I_2\left(\frac{V_{ab}}{V_{bc}}\right)$$

$$I_c = 40\left(\frac{100}{400}\right) = 10 \text{ A}$$



Step-up connection

Another solution:

$$I_1 = \frac{\text{VA rating}}{V_1} = \frac{20000}{400} = 50 \text{ A}$$

$I_c = I_1 - I_2 = 50 - 40 = 10$ A

668 Note: The phrase "small reduction to potential difference", means that the load of the autotransformer is connected across the 2300 V winding.

Solving for the rated curent in each winding :

$$I_1 = \frac{10000}{230} = 43.5 \text{ A}$$

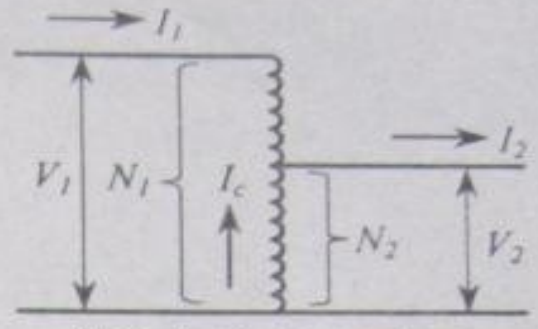
$$I_c = \frac{10000}{2300} = 4.35 \text{ A}$$

$I_2 = I_1 + I_c = 43.5 + 4.35 = 47.85\ A$

Solving for the turns ratio :

$$\frac{V_1}{V_2} = \frac{2300 + 230}{2300} = 1.1$$

$$V_2 = \frac{V_1}{1.1} = \frac{2300}{1.1} = 2091\ V$$



Step-down connection

$$\text{kVA rating} = \frac{V_2 I_2}{1000} = \frac{2091(47.85)}{1000}$$

kVA rating = 100.05 kVA

669 Solving for the rated curent of the 110 - V winding :

$$I_{rated} = \frac{10000}{110} = 90.9\ A$$

Note: This current is the high voltage (550V), full load current of the autotransformer.

Solving for the kVA rating of the autotransformer:

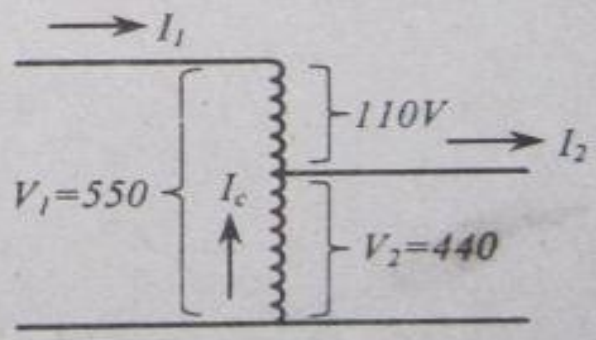
$$\text{kVA rating} = \frac{V_1 I_1}{1000} = \frac{550(90.9)}{1000} \cong 50\ \text{kVA}$$

As a two-winding transformer:

$$\eta = \frac{P_{out}}{P_{out} + P_{cu} + P_{co}}$$

$$0.96 = \frac{10(1)}{10(1) + 0.25 + P_{co}}$$

$$P_{co} = 0.166\ kW$$



Step-down connection

As an autotransformer:

P_{out} = Rating(pf) = 50(0.85)

$P_{out} = 42.5\ kW$

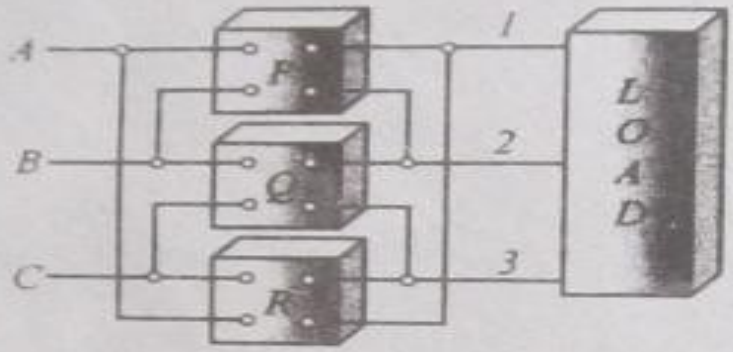
$$\eta = \frac{P_{out}}{P_{out} + P_{cu} + P_{co}}$$

$$= \frac{42.5}{42.5 + 0.25 + 0.166} \times 100\%$$

$\eta = 99.03\%$

Transformers: Part 2

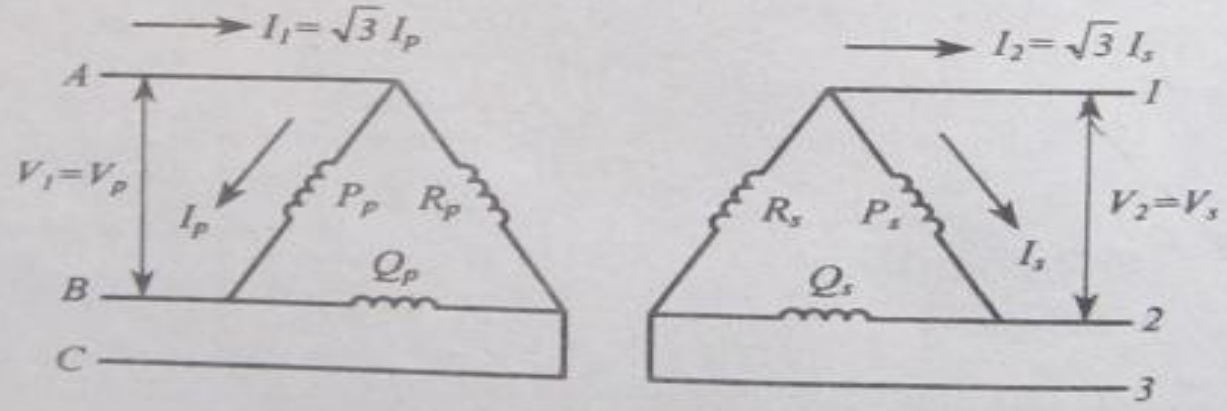
DELTA-DELTA TRANSFORMER BANK



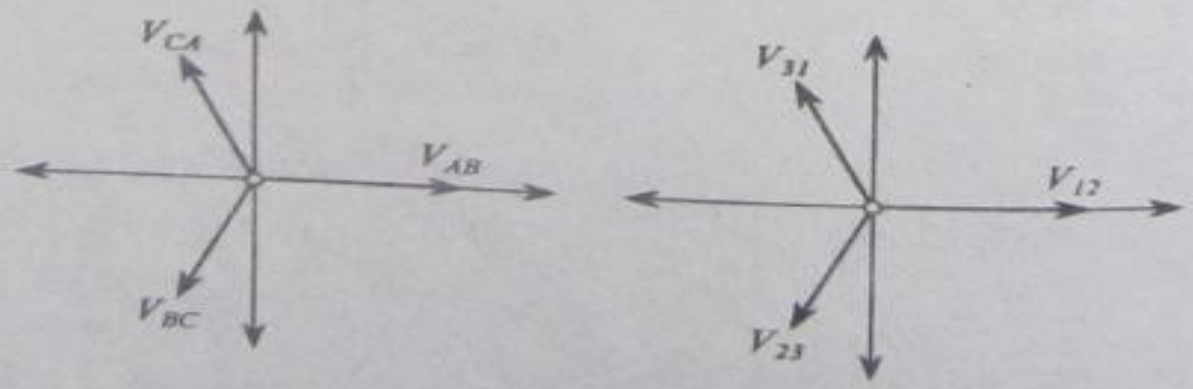
$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$\frac{I_p}{I_s} = \frac{N_2}{N_1}$$

Equivalent circuit diagram:

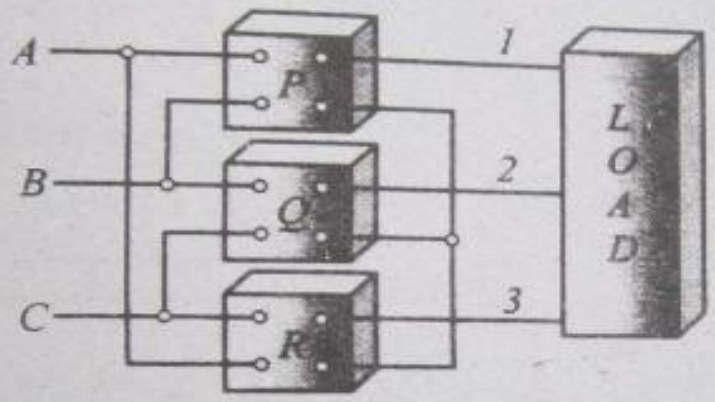


Phasor diagram:



where: V_p = voltage induced in the primary winding (volt)
V_s = voltage induced in the secondary winding (volt)
V_2 = line to line voltage in the secondary side of the bank (volt)
V_1 = line to line voltage in the primary side of the bank (volt)
I_p = primary winding current (ampere)
I_s = secondary winding current (ampere)
I_1 = line current in the primary side of the bank (ampere)
I_2 = line current in the secondary side of the bank (ampere)
N_1/N_2 = turns ratio of each transformer

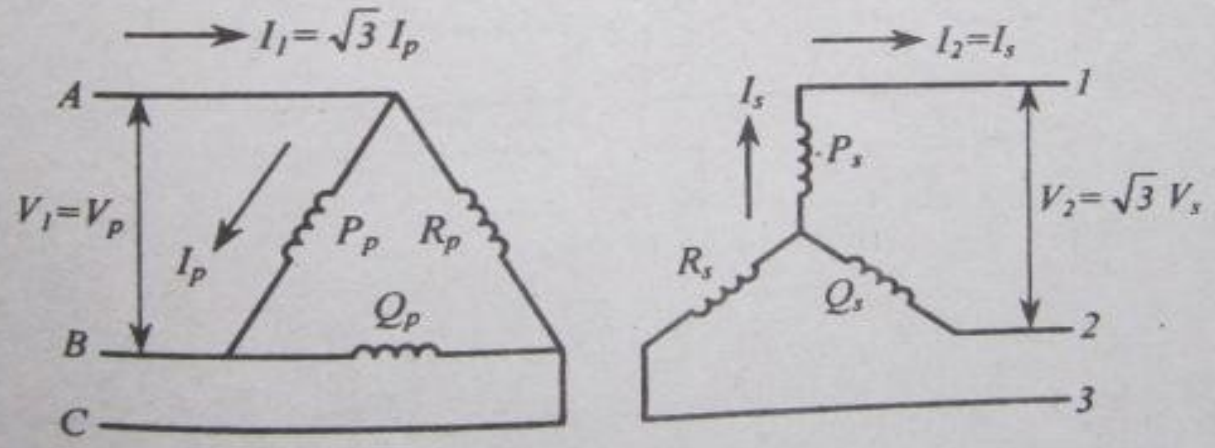
DELTA-WYE TRANSFORMER BANK



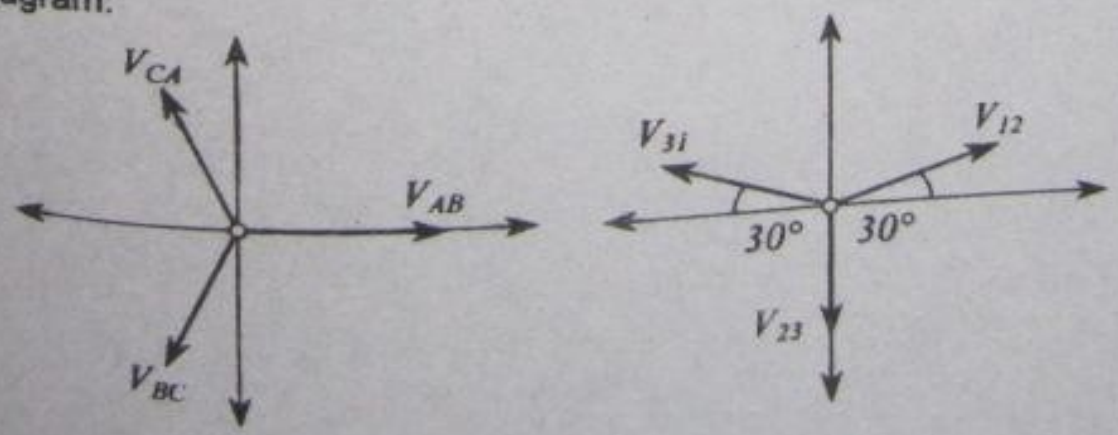
$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$\frac{I_p}{I_s} = \frac{N_2}{N_1}$$

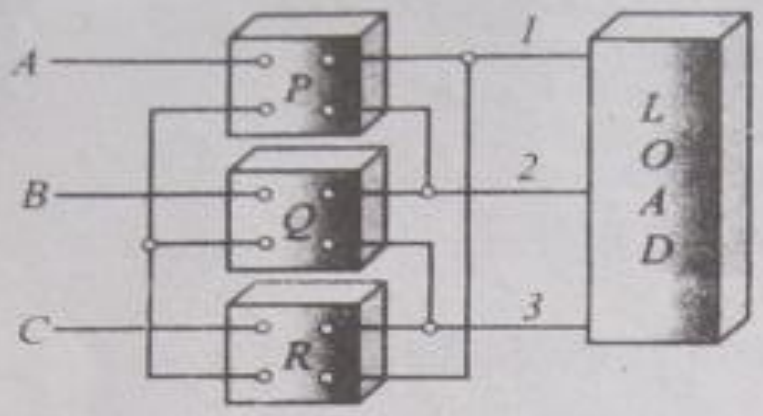
Equivalent circuit diagram:



Phasor diagram:



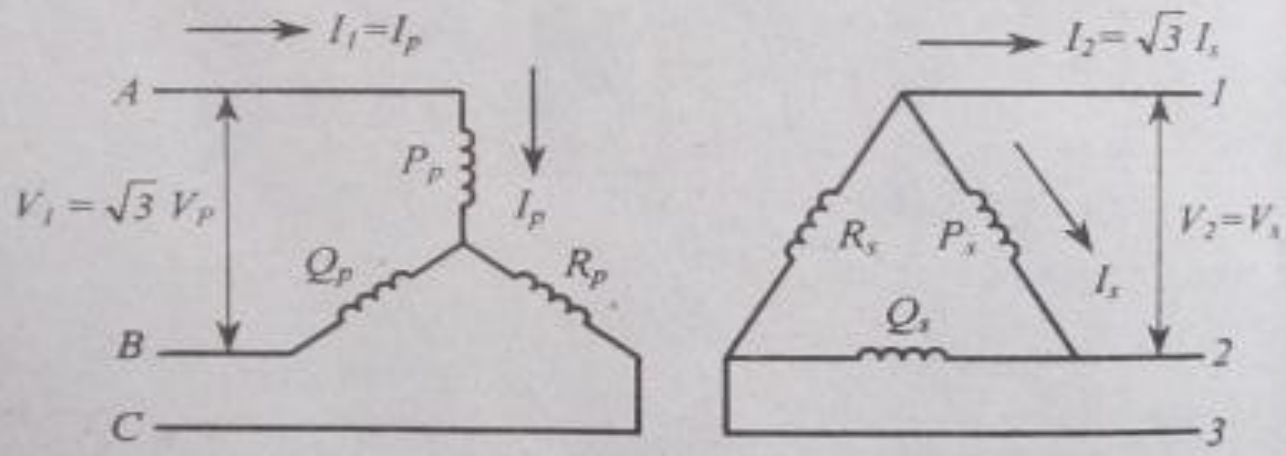
WYE-DELTA TRANSFORMER BANK



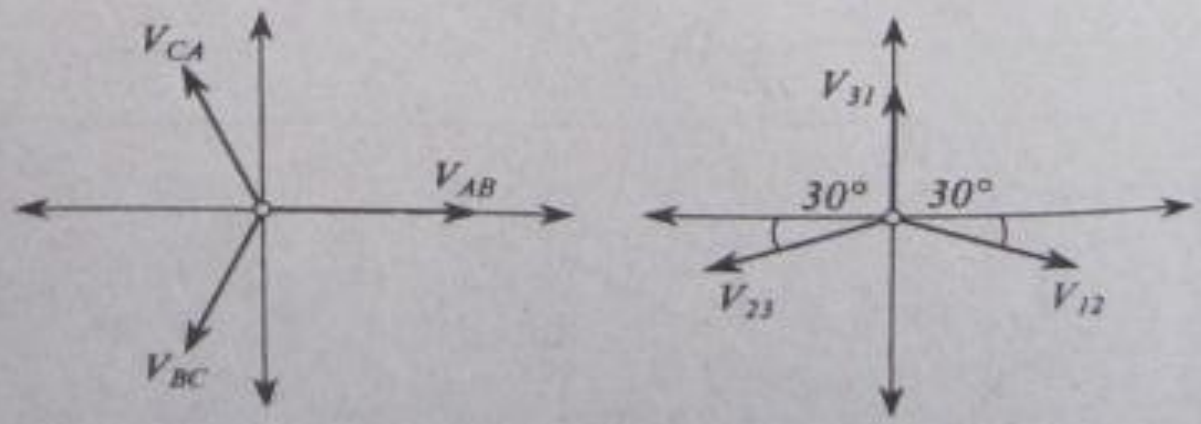
$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$\frac{I_p}{I_s} = \frac{N_2}{N_1}$$

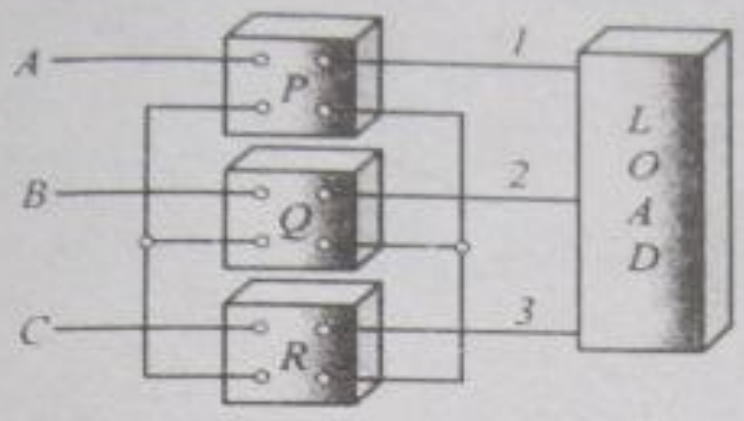
Equivalent circuit diagram:



Phasor diagram:



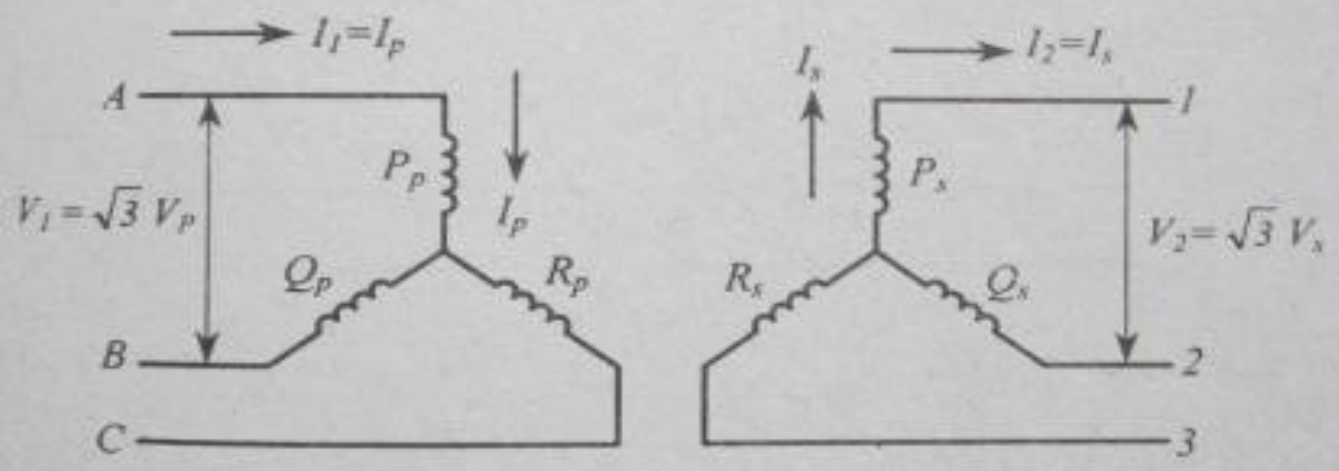
WYE-WYE TRANSFORMER BANK



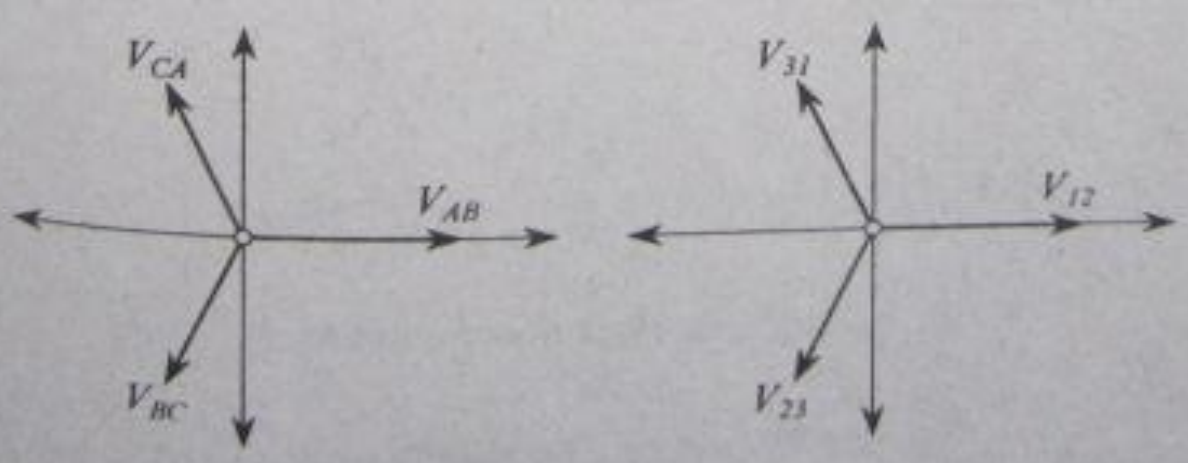
$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$\frac{I_p}{I_s} = \frac{N_2}{N_1}$$

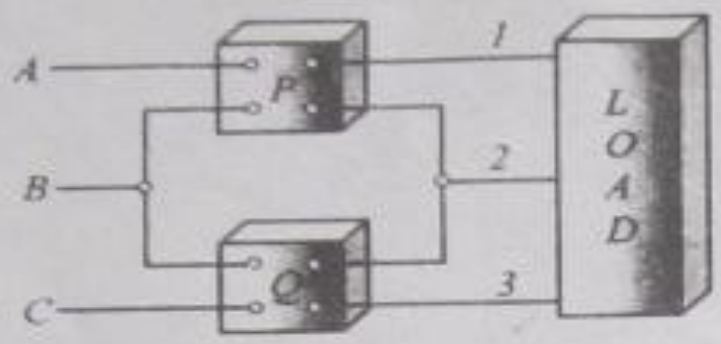
Equivalent circuit diagram:



Phasor diagram:



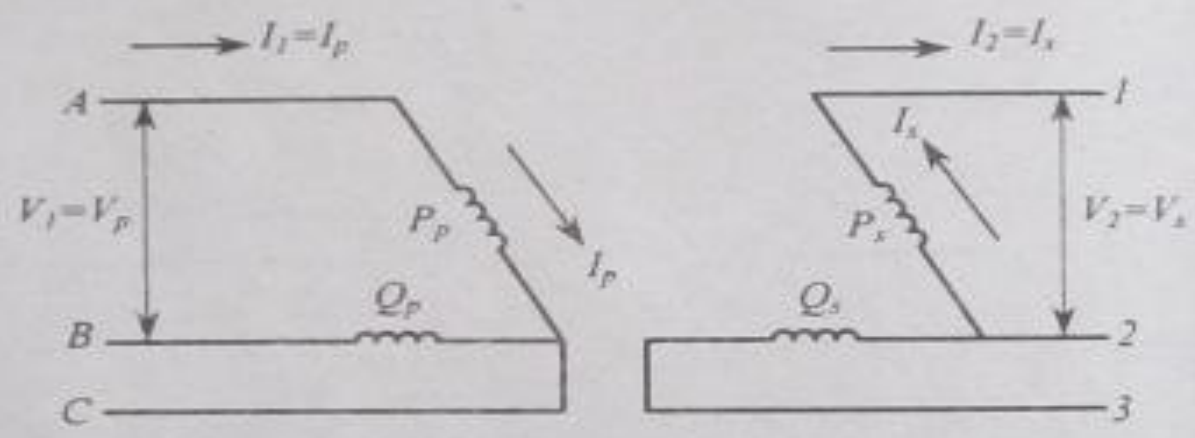
OPEN-DELTA TRANSFORMER BANK



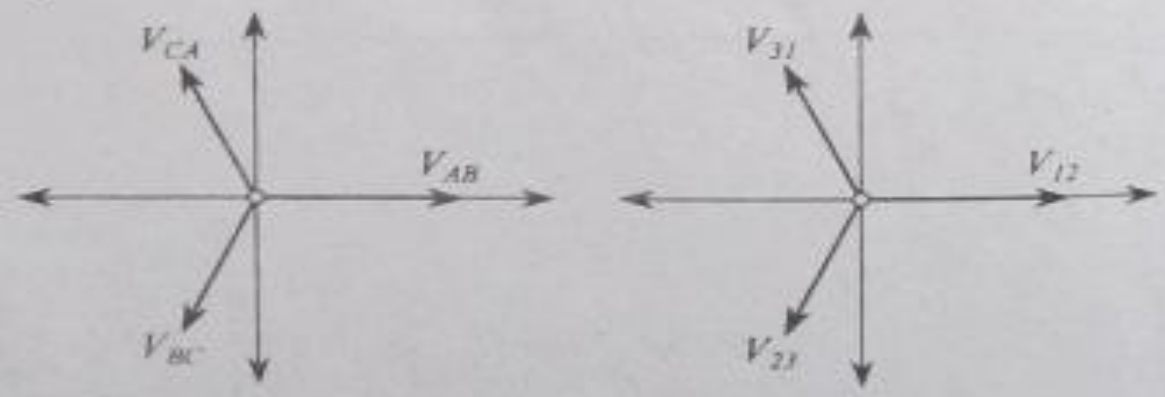
$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$\frac{I_p}{I_s} = \frac{N_2}{N_1}$$

Equivalent circuit diagram:



Phasor diagram:



$S_v = \sqrt{3}$ (VA rating of one transformer)

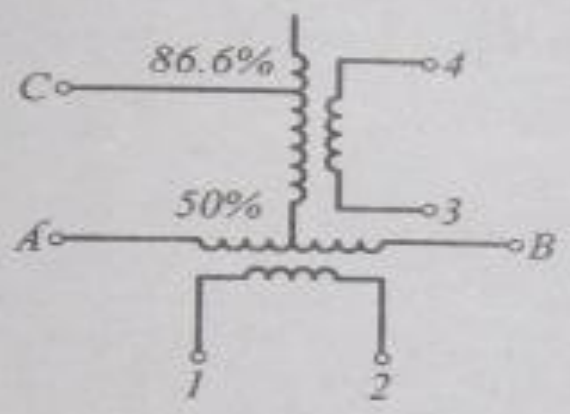
$S_\Delta = 3$ (VA rating of one transformer)

where: S_v = total apparent power that an open delta bank can deliver
S_Δ = total apparent power that a close delta bank can deliver

SCOTT OR T-CONNECTED TRANSFORMER BANK

Note: This connection is used to transform three-phase system to two phase system or vice-versa. It consist of two identical single phase transformers, one having a 50% tap and the other an 86.6% tap on their primary windings.

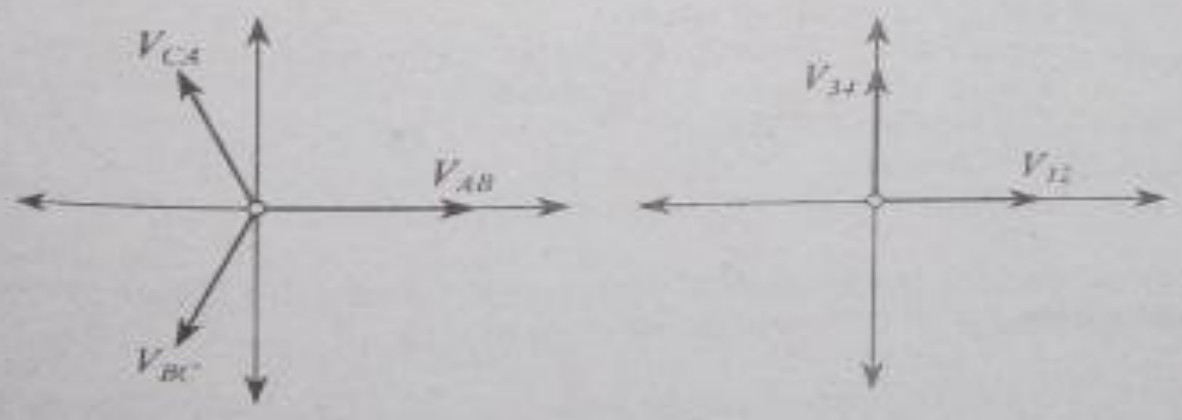
Equivalent circuit diagram:



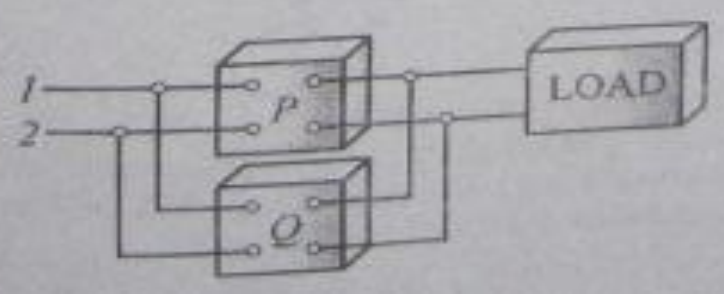
$$V_{12} = V_{34}$$

$$\frac{V_{AB}}{V_{12}} = \frac{N_1}{N_2} = a$$

Phasor diagram:



PARALLEL OPERATION OF TRANSFORMERS



Requirements:

- The voltage ratings of both primaries and secondaries must be identical
- The transformers must be properly connected with regards to polarity
- The ratio of the equivalent resistance to reactance (R_e:X_e) of all transformers should be the same

❖ **Load operations for transformers with unequal turns ratio.**

$$I_1 = \frac{(a_2 - a_1)V_2 + (a_2 Z_{e2} I_t)}{a_1 Z_{e1} + a_2 Z_{e2}} \qquad I_2 = \frac{(a_1 - a_2)V_2 + (a_1 Z_{e1} I_t)}{a_1 Z_{e1} + a_2 Z_{e2}}$$

❖ **Load operations for transformers with equal turns ratio (given their ohmic equivalent impedances)**

$$I_1 = \frac{I_t Z_{e2}}{Z_{e1} + Z_{e2}} \qquad I_2 = \frac{I_t Z_{e1}}{Z_{e1} + Z_{e2}}$$

$$S_1 = \frac{S_t Z_{e2}}{Z_{e1} + Z_{e2}} \qquad S_2 = \frac{S_t Z_{e1}}{Z_{e1} + Z_{e2}}$$

where: a_1, a_2 = voltage ratios of transformers 1 and 2
V_2 = load voltage or terminal voltage
Z_{e1}, Z_{e2} = equivalent impedances referred to the secondary side of transformers 1 and 2, respectively
I_t = total current or load current
I_1, I_2 = current delivered by transformers 1 and 2
S_1, S_2 = apparent powers delivered by transformers 1 and 2

❖ **Load operations for transformers with equal turns ratio (given their per unit impedances with different power base)**

Note: Assume a common power base and express all per unit impedances with respect to this new power base.

$$Z_e' = Z_e\left(\frac{kVA_{new}}{kVA_{old}}\right) \qquad S_1 = \frac{S_t Z_{e2}'}{Z_{e1}' + Z_{e2}'} \qquad S_2 = \frac{S_t Z_{e1}'}{Z_{e1}' + Z_{e2}'}$$

where: kVA_{new} = selected new power base
kVA_{old} = old power base (rating of the transformer considered)
Z_{e1}', Z_{e2}' = equivalent per unit impedances of transformers 1 and 2 based on the new selected power base.

Test 18

Problem 661: EE Board June 1990

What should be the kVA rating of each two transformers in an open-delta bank when the three-phase balanced load is 290.4 kVA?

A. 155.60 kVA
B. 173.40 kVA
C. 150.25 kVA
D. 167.66 kVA

Problem 662: EE Board April 1981

A V or open delta connected transformer bank, composed of two identical units serves a balanced 3-phase load of 16 kVA at 230 volts. Solve for the minimum size in kVA of the transformers needed.

A. 8 kVA
B. 9.24 kVA
C. 16 kVA
D. 8.25 kVA

Problem 663: EE Board April 1986

A V (open delta) connected transformer bank serves a balanced 3-phase load of 40 kVA at 230 V and 0.866 pf lagging. Solve for the minimum kVA of each transformer needed to accommodate the load without overloading.

A. 25
B. 24
C. 23
D. 20

Problem 664: EE Board April 1985

Two single-phase distribution transformers are banked in V (open delta). If each transformer is rated 37.5 kVA, solve for the maximum 3-phase load that the bank can carry without overloading, considering that the load has a power factor of 0.85 lagging.

A. 52.5 kW
B. 49.5 kW
C. 53.6 kW
D. 55.2 kW

Problem 665:

An open delta bank, consisting of two single-phase transformers is operating with a balanced three-phase load of 50 kVA, 440 V at 0.8 pf lagging and a single-phase load of 10 kW resistive connected across c-a leg. Determine the minimum ratings of the two transformers. Assume a phase sequence of a-b-c.

A. 38.80 kVA, 34.06 kVA
B. 34.32 kVA, 33.40 kVA
C. 35.48 kVA, 36.34 kVA
D. 30.83 kVA, 39.52 kVA

Problem 666: EE Board April 1980

Two (2) single-phase distribution transformers, connected in open delta will supply power to a 200 hp, 3-phase induction motor, operating at 0.70 power factor and 0.90 efficiency. Solve for the minimum size in kVA of each transformer needed to supply the power without being overloaded.

A. 136.73 kVA
B. 124.45 kVA
C. 130.38 kVA
D. 129.48 kVA

Problem 667: EE Board October 1986

The Utility Company supplies two single-phase transformers banked in open delta. If each transformer is rated 75-kVA capacity, what is the maximum 3-phase load in kilowatts that the bank can carry without suffering overload considering that the load has a lagging power factor of 0.80.

A. 104 kW
B. 112 kW
C. 130 kW
D. 114 kW

Problem 668: EE Board October 1980

Two identical 1 –phase transformers connected in open delta will serve a 3-phase motor load of 250 hp, 0.70 pf lagging, 0.90 efficiency. Solve for the minimum size in kVA of each transformer required to serve the load without overloading.

A. 150.33
B. 170.91
C. 160.43
D. 185.13

Problem 669: EE Board October 1985, EE Board April 1986

An open-delta connected transformer bank serves a balanced 3-phase load of 40 kVA at 230 V and 0.866 pf lagging. Solve for the real power and pf at which each transformer is operating.

A. 11.54 kW at 0.5 pf, 23.09 kW at unity pf
B. 12.22 kW at 0.5 pf, 22.42 at unity pf
C. 11.54 kW at 0.866 pf, 23.09 kW at 0.707 pf
D. 12.22 kW at 0.866 pf, 22.42 kW at 0.707 pf

Problem 670:

An open delta, three-phase transformer consisting of two single-phase transformers is operating with a balanced three-phase load of 50 kVA, 440-V at a lagging pf of 0.8. Determine the operating pf of each transformer.

A. 0.393 lagging, 0.993 lagging
B. 0.383 lagging, 0.887 lagging
C. 0.383 lagging, 0.993 lagging
D. 0.393 lagging, 0.887 lagging

Problem 671: EE Board April 1983

Two single-phase transformers are connected in V (open delta) and serving a delta connected impedance load, each impedance is equal to $16\angle 36.87°$ ohms. If the transformer voltages impressed on the impedances are:

$$V_{ab} = 240\angle 0°,\ V_{ab} = 240\angle 240°,\ V_{ab} = 240\angle 120°\ \text{volts}$$

Solve for the kVA delivered by each transformer.

A. 7.28, 5.20
B. 5.87, 6.61
C. 6.84, 5.64
D. 6.24, 6.24

Problem 672: EE Board June 1990

Two single-phase transformers in an open-delta bank served a three-phase balanced load of 290.4 kVA. If a third transformer is added for delta operation, what percent increase in load does this represent?

A. 73.2%
B. 84.6%
C. 80.5%
D. 79.4%

Problem 673: EE Board October 1983

Three-single phase transformers each rated 75 kVA are banked in delta and supplying a 3-phase load drawing 160 kVA at 0.8 lagging power factor. If one transformer is removed for repairs, solve for the amount of overloading of the remaining units.

A. 30 kVA
B. 10 kVA
C. 20 kVA
D. 25 kVA

Problem 674: EE Board April 1992

A 6600-volt, 60 Hz, 2-phase network is connected to a three-phase, 4-wire system by a Scott-connected transformer combination. If there are 1500 turns on the 2-phase side, calculate the number of turns on the 3-phase side required for the main and teaser transformers. Voltage per phase on the 3-phase side is 240 V.

A. 94 turns, 83 turns
B. 93 turns, 84 turns
C. 95 turns, 82 turns
D. 92 turns, 85 turns

Problem 675: EE Board April 1985

A balanced 3-phase transformer bank has delta-connected primary and wye-connected secondary windings. The primary to secondary winding voltage ratio is 2400/277. Solve for the primary line current if the low-side (secondary) load draws a line current of 1000 A.

A. 200 A
B. 115 A
C. 150 A
D. 175 A

Problem 676:

A three-phase 150 kVA, 4160 V, 0.8 lagging pf load is to be supplied by three single-phase transformers having a turns ratio of 2. Calculate the current that flows in the primary windings if the transformers are connected delta-delta.

A. 10.41 A
B. 6.01 A
C. 20.82 A
D. 12.02 A

Problem 677: EE Board October 1991

What should be the turns ratio of a three phase transformer to transform 10,000 kVA from 230 kV to 4160 V if the transformer is to be connected $\Delta - Y$?

A. 94
B. 98
C. 96
D. 97

Problem 678:

A bank of three 50:1 single-phase transformers connected Y-Δ is used to step down the HV transmission line rated 138 kV. Determine the magnitude of the step down voltage.

A. 126 kV
B. 6.90 kV

C. 138.55 kV
D. 159.35 kV

Problem 679: EE Board October 1998

The potential transformer of a line has a ratio 132,000/66. These are connected wye-delta. The voltmeter reads 64 volts. What is the voltage of the line?

A. 221.7 kV
B. 112.5 kV
C. 135.2 kV
D. 132 kV

Problem 680: EE Board October 1997

A power transformer rated 50,000 kVA, 34.5 kV/13.8 kV is connected wye-grounded primary and delta on the secondary. Determine the full load phase current on the secondary side.

A. 2092 A
B. 1725 A
C. 1449 A
D. 1208 A

Problem 681: EE Board October 1998

A 69/13.8 kV, 10 MVA three-phase transformer is connected wye-grounded delta. What is the kV rating of the secondary winding?

A. 7.97
B. 15
C. 23.9
D. 13.8

Problem 682: EE Board October 1997

The PTs of a 220 kV transmission line have ratio of 132.76 kV/66.375 kV and are connected wye-wye. A voltmeter connected line to ground reads 66 volts. What is the transmission line voltage?

A. 228.63 kV
B. 220.10 kV
C. 223.15 kV
D. 225.25 kV

Problem 683: EE Board October 1997

A power transformer rated 50,000 kVA, 34.5 kV/13.8 kV is connected wye-wye. What are the line currents at full load?

A. 525.2 A, 1725.4 A
B. 836.7 A, 2092 A
C. 483.1 A, 1207.7 A
D. 1449.3 A, 3623.2 A

Problem 684: EE Board October 1985

Two transformers are connected in parallel to supply a common load of 175 kVA. Transformer A is rated 100 kVA with equivalent impedance of 9.6 ohms while transformer B is rated 75 kVA with equivalent impedance of 12.5 ohms. Find the kVA load of each transformer.

A. 78, 97
B. 76, 99
C. 80, 95
D. 75,100

Problem 685: EE Board October 1981

Two distribution transformers are connected in parallel supplying a load of 125 kVA. If transformer A is rated 50 kVA, with 14 ohms equivalent reactance and transformer B is rated 75 kVA with 11 ohms equivalent reactance. Solve for the loading in kVA of each transformer.

A. 55 kVA, 70 kVA
B. 50 kVA, 75 kVA
C. 58 kVA, 67 kVA
D. 52 kVA, 73 kVA

Problem 686: EE Board March 1998

A 125-kVA distribution transformer with 4% impedance is connected in parallel with another transformer rated 75-kVA and with 3% impedance. Both have the same voltage ratio. Neglect resistance of each transformer. The total load is 140 kVA at 85 percent power factor. How much load does each carry

A. 80.5 kVA / 59.5 kVA
B. 77.78 kVA / 62.22 kVA
C. 80 kVA / 60 kVA
D. 87.50 kVA / 52.5 kVA

Problem 687:

Two parallel transformers have 8% reactance each. One is rated 25 MVA and the other at 15 MVA. For a given 30 MVA load with a lagging pf of 0.8, Determine the load carried by each transformer?

A. 18.50 MVA, 11.50 MVA
B. 18.75 MVA, 11.25 MVA
C. 15.25 MVA, 14.75 MVA
D. 16.45 MVA, 13.55 MVA

Problem 688:

A 500-kVA single-phase transformer "A" with percentage impedance of 0.010 + j 0.05 is connected in parallel with a 250-kVA transformer "B" with a percentage impedance of 0.015 + j 0.04. If the transformers are serving a single-phase load rated 800-kVA, 80% lagging pf, determine the operating pf of each transformer. Assume the transformers have the same turns ratio.

A. A = 0.762, B = 0.803
B. A = 0.772, B = 0.856
C. A = 0.762, B = 0.856
D. A = 0.772, B = 0.803

Problem 789: EE Board April 1993

The MERALCO has two single-phase transformers with equal turns ratio and ratings and are operated in parallel to supply a load of 280 kW at 0.80 lagging pf. Transformer A has a resistance of 2% and a reactance of 8%. Transformer B has a resistance of 1% and a reactance of 6%. Determine the power delivered by the transformers A and B to the load.

A. A = 123 kW, B = 157 kW
B. A = 125 kW, B = 155 kW
C. A = 122 kW, B = 158 kW
D. A = 127 kW, B = 153 kW

Problem 690: EE Board April 1994

Two single-phase transformers with equal ratings and turns ratio are operated in parallel to supply a load of 180 kW at a lagging pf of 0.90. Transformer A has a resistance of 1% and a reactance of 6%. Transformer B has a resistance of 2% and a reactance of 5%. Calculate the power delivered by each transformer.

A. 83 kW, 97 kW
B. 79 kW, 101kW
C. 88 kW, 92 kW
D. 80 kW, 100 kW

Problem 691:

Two single-phase transformers A and B are connected in parallel to serve a 600-A single-phase load at 0.80 pf lagging. The equivalent impedance of the transformers are 2 + j 3 and 2.3 + j 5 ohms, respectively. Determine the ratio of the kW output of transformer A to the kW output of transformer B. Assume the transformers have equal ratings and turns ratio.

A. 1.56
B. 1.88
C. 1.68
D. 1.74

ANSWER KEY

661. D	668. B	675. A	682. A	689. A
662. B	669. A	676. B	683. B	690. D
663. C	670. A	677. C	684. B	691. D
664. D	671. D	678. D	685. A	
665. A	672. A	679. A	686. B	
666. A	673. A	680. D	687. B	
667. A	674. C	681. D	688. C	

☑ RATING

27-31 - Topnotcher

21-26 - Passer

15-20 - Conditional

0-14 - Failed

Did you know that... the greatest inventive genius of all time, often called "the magician of electricity" who set 60 Hz as the standard electrical frequency in the US became a millionaire through his inventions but died nearly penniless and a lonely man?

Nikola Tesla was born in July 10, 1865 in Smiljan Austria-Hungary (now Yugoslavia). His father was a clergyman of the Greek Orthodox Church. As a young boy, Tesla had a talent for mathematics and an incredible memory. In his early days, he solved complex problems in his head without pencil and paper. His teachers suspected him of cheating, but young Tesla had actually memorized the whole logarithmic tables.

He spent two years at the Polytechnic Institute of Graz, Austria, where he conceived the idea of the rotating magnetic field that was the later the basis for his induction motor. At this point in life his father died and he decided to leave school and take a job in Paris with the Continental Edison Company.

In 1884 at the young age of 28 Nikola Tesla stepped off the ship in New York City. Edison although committed to direct current gave Tesla a job at the Edison Machine Works. By 1885 he had left Edison and worked on his own until he teamed up with George Westinghouse who later bought the rights of his AC dynamos, transformers and motors.

During his remarkable life he held over 700 patents, settled the ac versus dc dispute and was primarily responsible for the selection of 60 Hz as the standard ac frequency in the United States and throughout much of the world. He never acquired the habit of writing notes, always claiming and proving that he was able to retain all of his research in his mind.

In his last years, scientists & engineers who were engaged in developing his earlier ideas increasingly ignored Tesla. He was totally secluded. As poor as he was, in January 8, 1943 during his sleep in his hotel room, he died at the age of 87. He planned to die at the age of 150 and upon reaching the age of 100, would write his memoirs, which would include a detailed record of all the data he had compiled. At his death (during World War II), military authorities impounded the contents of his safe and nothing has been heard since as to what records, if any, were there.

Solutions to Test 18

661 $S_v = \sqrt{3}\text{Rating}$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{290.4}{\sqrt{3}}$$

Rating = 167.66 kVA

662 $S_v = \sqrt{3}\text{Rating}$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{16}{\sqrt{3}}$$

Rating = 9.24 kVA

663 $S_v = \sqrt{3}\text{Rating}$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{40}{\sqrt{3}}$$

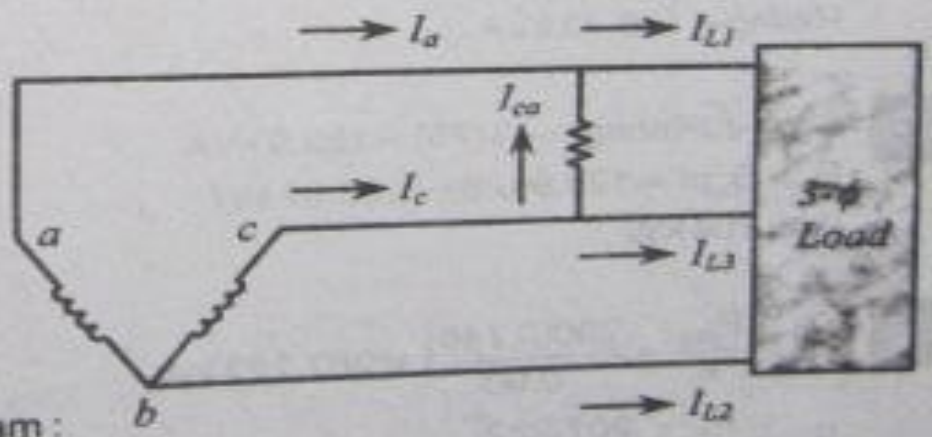
Rating = 23.09 kVA

664 $S_v = \sqrt{3}\text{Rating} = \sqrt{3}(37.5) = 64.95\text{ kVA}$

$P_L = (S_v)(pf) = 64.95(0.85)$

$P_L = 55.207\text{ kW}$

665 Note: Assume the 3-phase load is wye-connected.



$$I_L = \frac{S}{\sqrt{3}V_L} = \frac{50{,}000}{\sqrt{3}(440)}$$

$I_L = 65.61\text{ A}$

$\theta = \cos^{-1} 0.8 = 36.87°$

Refer to the vector diagram :

$I_{L1} = 65.61\angle -(36.87° + 30°)$

$= 25.772 - j60.336$

$I_{L3} = 65.61\angle(90° - 36.87°)$

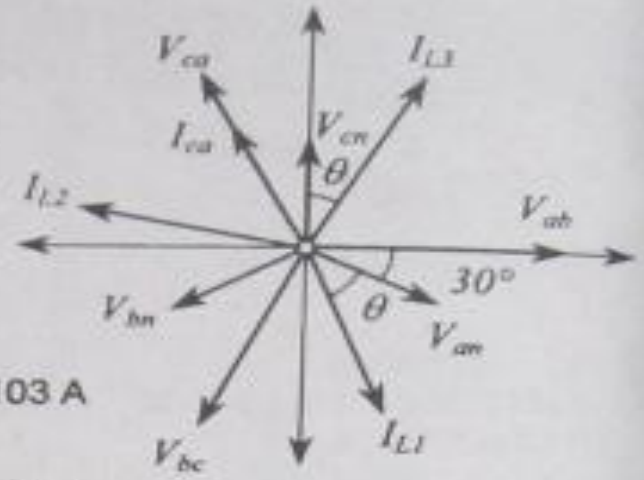
$= 39.366 + j52.488$

For the single - phase load :

$$I_{ca} = \frac{P}{V_{ca}} = \frac{10{,}000}{440} = 22.73\text{ A}$$

$$\theta_{ca} = \cos^{-1} 1.0 = 0°$$

$$I_{ca} = 22.73\angle 120° = -11.365 + j19.684$$



$$I_a = I_{L1} - I_{ca}$$
$$= 25.772 - j60.336 - 22.73$$
$$= 48.502 - (-11.365 + j19.684)$$
$$I_a = 37.134 - j80.011 = 88.208\angle -65.103\text{ A}$$
$$I_c = I_{L3} + I_{ca}$$
$$= 39.366 + j52.488 + (-11.365 + j19.684)$$
$$I_c = 28.001 + j72.172 = 77.413\angle 68.79°$$

$$kVA_1 = \frac{V_L I_a}{1000} = \frac{440(88.208)}{1000} = 38.8\text{ kVA}$$

$$kVA_2 = \frac{V_L I_c}{1000} = \frac{440(77.413)}{1000} = 34.06\text{ kVA}$$

666 $$S_v = \frac{P_{out}}{\eta\, pf} = \frac{200(0.746)}{0.9(0.7)} = 236.825\text{ kVA}$$

$$S_v = \sqrt{3}(\text{Rating})$$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{236.825}{\sqrt{3}}$$

$$\text{Rating} = 136.73\text{ kVA}$$

667 $$S_v = \sqrt{3}\text{Rating} = \sqrt{3}(75) = 129.9\text{ kVA}$$

$$P_L = S_v pf = 129.9(0.8) = 103.92\text{ kW}$$

$$P_L \cong 104\text{ kW}$$

668 $$P_{in} = \frac{P_{out}}{\eta} = \frac{250(0.746)}{0.90} = 207.222\text{ kW}$$

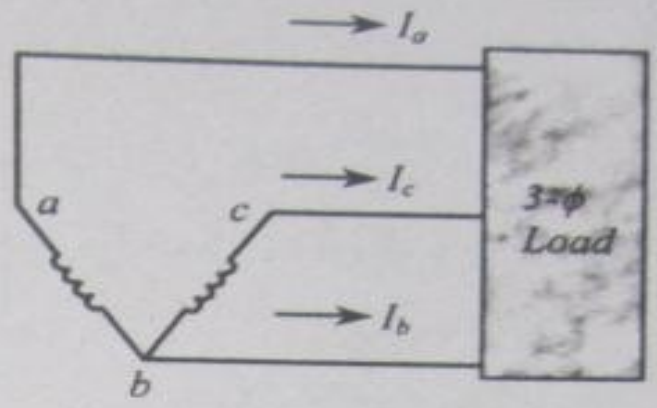
$$S_{in} = \frac{P_{in}}{pf} = \frac{207.222}{0.70} = 296.03\text{ kVA}$$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{296.03}{\sqrt{3}}$$

$$\text{Rating} = 170.913\text{ kVA}$$

669 $I_L = \dfrac{S}{\sqrt{3}V_L} = \dfrac{40{,}000}{\sqrt{3}(230)} = 100.41\text{ A}$

$\theta = \cos^{-1}0.866 = 30°$



Refer to the vector diagram :

$I_a = 100.41\angle -60°$

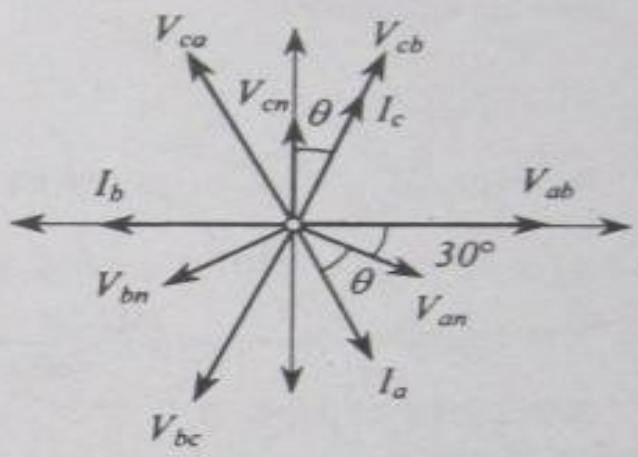
$I_c = 100.41\angle 60°$

$S_a = V_{ab}I_a\Big|_{I_a}^{V_{ab}} = (230)(100.41)\angle -60°$

$= 23.09\angle -60° = 11.54 - j20$

$P_a = 11.54\text{ kW}$

$pf_a = \cos 60° = 0.5$ lagging



$S_b = V_{cb}I_c\Big|_{I_c}^{V_{cb}} = (230)(100.41)\angle 0°$

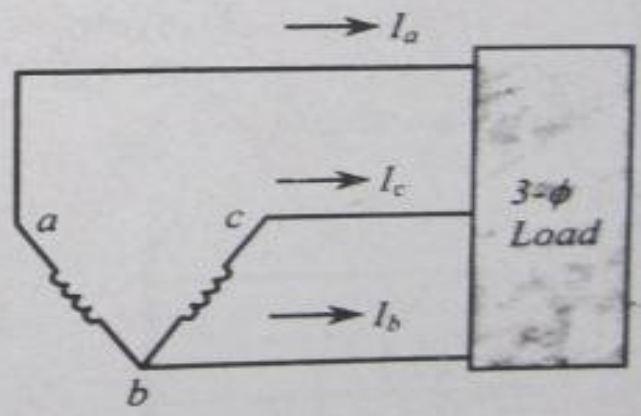
$= 23.09\angle 0° = 23.09 + j0$

$P_b = 23.09\text{ kW}$

$pf_b = \cos 0° = 1$ unity

670 $I_L = \dfrac{S}{\sqrt{3}V_L} = \dfrac{50{,}000}{\sqrt{3}(440)} = 65.608\text{ A}$

$\theta = \cos^{-1}0.8 = 36.87°$



Refer to the vector diagram :

$I_a = 65.608\angle -(30 + 36.87)°$

$= 65.608\angle -66.87$

$I_c = 65.608\angle (90 - 36.87)°$

$= 65.608\angle 53.13°$

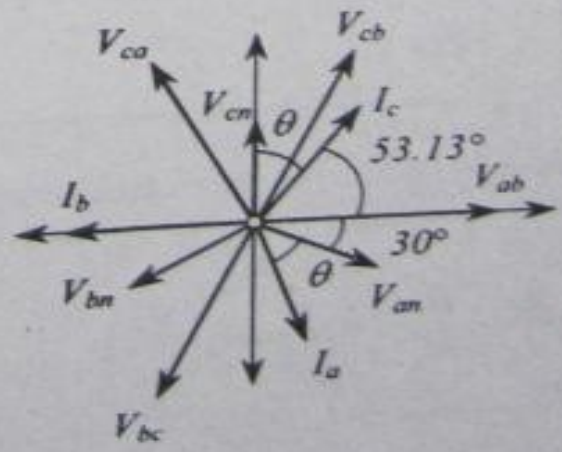
$S_a = V_{ab}I_a\Big|_{I_a}^{V_{ab}} = V_{ab}I_a\angle -66.87°$

$pf_a = \cos 66.87° = 0.393$ lagging

$S_b = V_{cb}I_c\Big|_{I_c}^{V_{cb}} = V_{cb}I_c\angle (60° - 53.13°)$

$pf_b = \cos 6.87°$

$pf_b = 0.993$ lagging



671 Note: Since load is balanced, the kVA delivered by transformer A is equal to the kVA delivered by transformer B.

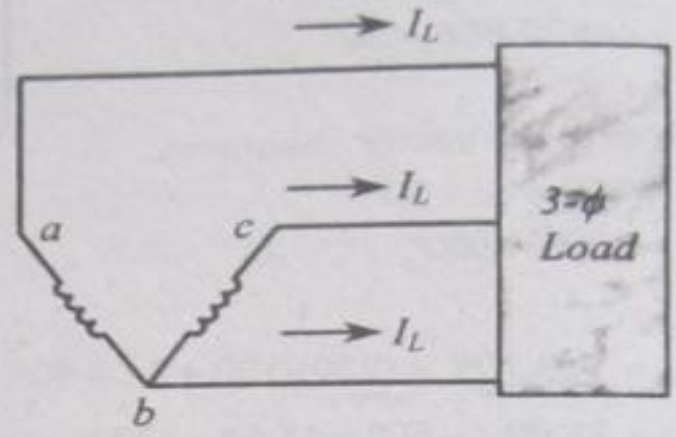
$$I_p = \frac{V_p}{Z_p} = \frac{240}{16} = 15\text{ A}$$

$$I_L = \sqrt{3}I_p = \sqrt{3}(15) = 25.98\text{ A}$$

$$S_a = S_b = \frac{V_L I_L}{1000}$$

$$= \frac{240(25.98)}{1000}$$

Rating = 6.235 kVA



672 $S_v = \sqrt{3}\text{Rating}$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{290.4}{\sqrt{3}} = 167.66\text{ kVA}$$

$$S_\Delta = 3\,\text{Rating} = 3(167.66) = 502.98\text{ kVA}$$

$$\%\text{Increase} = \frac{S_\Delta - S_v}{S_v} = \frac{502.98 - 290.4}{290.4} \times 100\%$$

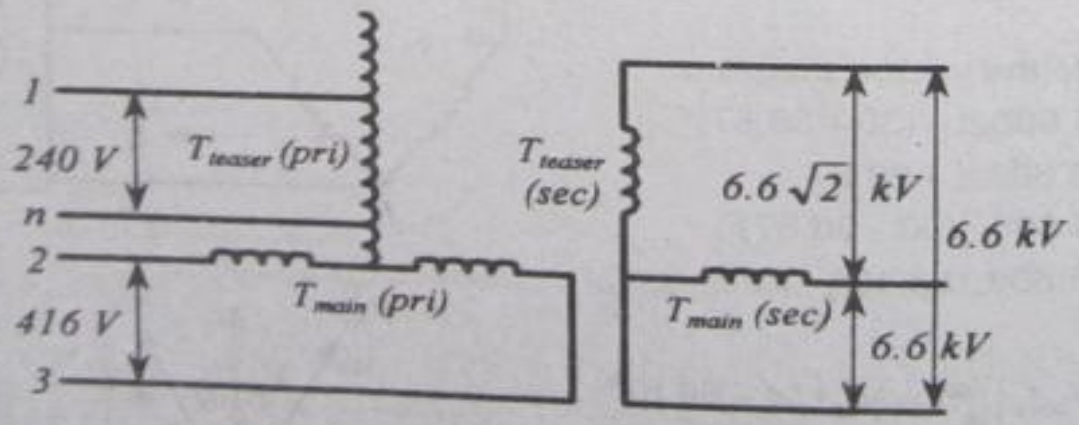
%Increase = 73.2%

673 $S_v = \sqrt{3}(\text{Rating}) = \sqrt{3}(75) = 129.9 \cong 130\text{ kVA}$

Overload = $S_L - S_v$ = 160 - 130

Overload = 30 kVA

674



Note: Refer to the connection diagram. Line to line voltage of the primary side is 416 V while line to neutral is 240 V as given.

$$\frac{V_p}{V_s} = \frac{N_1}{N_2}; \quad N_1 = N_2\left(\frac{V_p}{V_s}\right)$$

For the main transformer :

$V_p = \sqrt{3}(240) \cong 416\ V$

$N_1 = 1500\left(\frac{416)}{6600}\right) = 94.5 \cong 95$ turns

For the teaser transformer :

$V_p = 0.866(416) \cong 360\ V$

$N_1 = 1500\left(\frac{360}{6600}\right) = 81.8 \cong 82$ turns

675 $\frac{I_p}{I_s} = \frac{N_2}{N_1}$

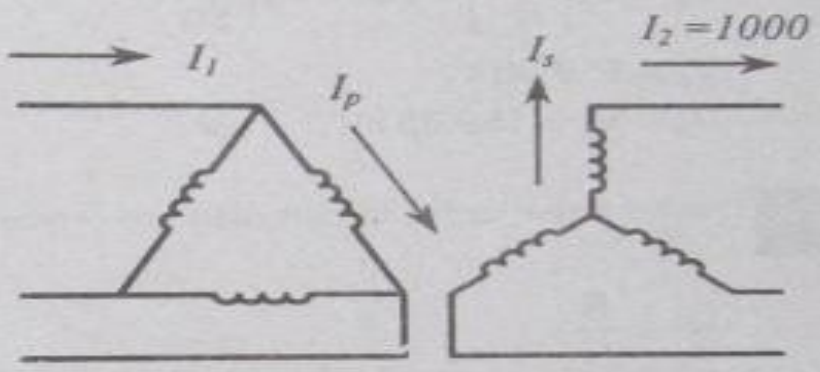
$I_p = \frac{I_s N_2}{N_1} = \frac{(1000)(277)}{2400}$

$I_p = 115.416\ A$

$I_1 = \sqrt{3} I_p = \sqrt{3}(115.416)$

$I_1 = 199.906$

$I_1 \cong 200\ A$

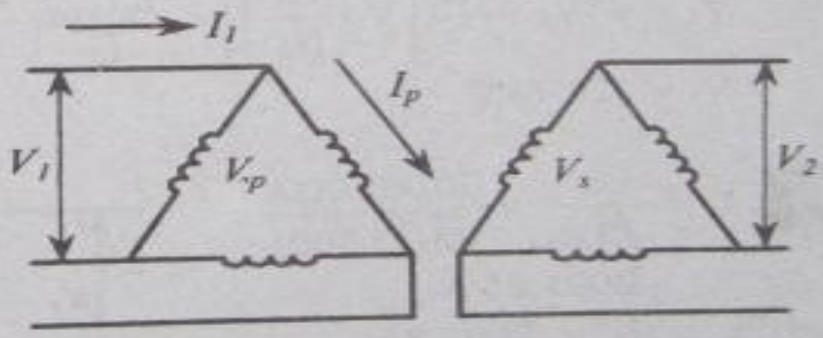


676 $V_s = V_2 = 4160\ V$

$\frac{V_p}{V_s} = \frac{N_1}{N_2}$

$V_p = V_s\left(\frac{N_1}{N_2}\right)$

$V_p = 4160(2) = 8320\ V$



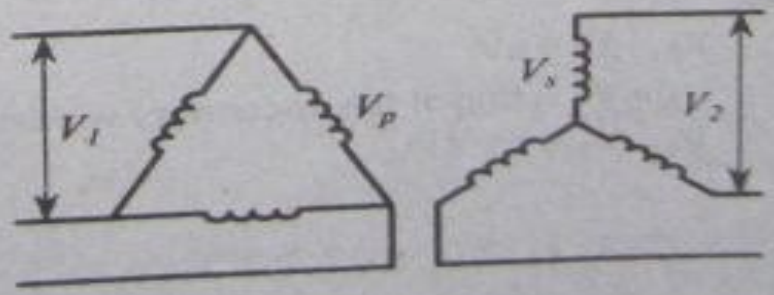
$I_1 = \frac{S}{\sqrt{3}V_1} = \frac{150{,}000}{\sqrt{3}(8320)} = 10.41A$

$I_p = \frac{I_1}{\sqrt{3}} = \frac{10.41}{\sqrt{3}} = 6.01A$

677 $\frac{V_p}{V_s} = \frac{N_1}{N_2}$

$V_p = V_1 = 230\ kV$

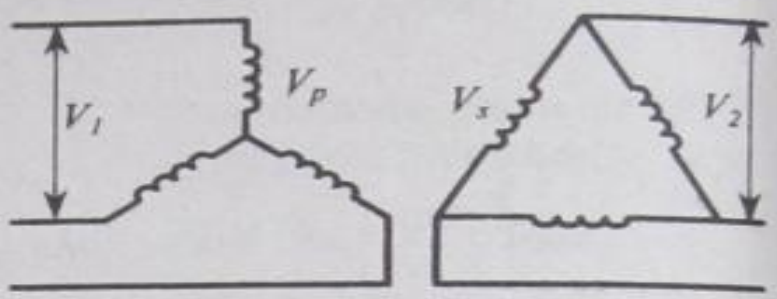
$V_s = \frac{V_2}{\sqrt{3}} = \frac{4160}{\sqrt{3}} = 2.4\ kV$



$$\frac{N_1}{N_2} = \frac{V_p}{V_s} = \frac{2.30}{2.4} = 95.83$$

Ratio ≅ 96

678 $V_L = 13800\ \text{kV}$



$$V_p = \frac{V_L}{\sqrt{3}} = \frac{13800}{\sqrt{3}}$$

$$= 7967.43\ \text{kV}$$

$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$V_s = V_p\left(\frac{N_2}{N_1}\right) = 7967.43\left(\frac{1}{50}\right)$$

$$V_s = 159.35\ \text{kV}$$

$$V_2 = V_s = 159.35\ \text{kV}$$

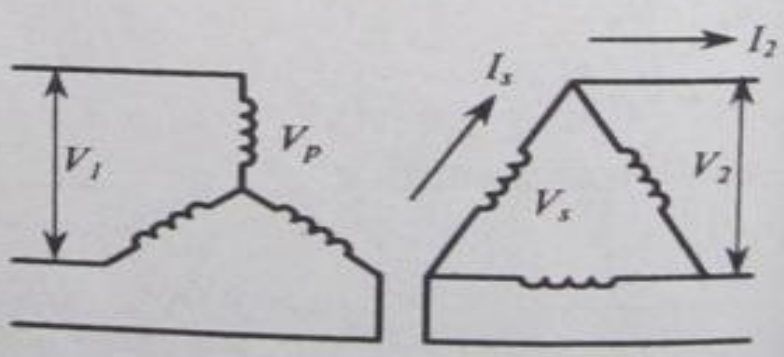
679 Note: Refer to the circuit diagram (wye-delta connection).

$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

$$V_1 = \sqrt{3}V_p = \sqrt{3}\left[V_s\left(\frac{N_1}{N_2}\right)\right] = \sqrt{3}\left[64\left(\frac{132{,}000}{66}\right)\right]$$

$$V_1 = 221.7\ \text{kV}$$

680 $I_2 = \dfrac{S}{\sqrt{3}V_2} = \dfrac{50{,}000{,}000}{\sqrt{3}(13{,}800)}$



$$= 2091.85\ \text{A}$$

$$I_s = \frac{I_2}{\sqrt{3}} = \frac{2091.85}{\sqrt{3}}$$

$$I_s = 1207.73$$

$$I_s \cong 1208\ \text{A}$$

681 Note: Refer to the circuit diagram (wye-delta connection).

$V_2 = 13.8\ \text{kV}$

V_s = kV rating of the secondary windings

$V_s = V_2 = 13.8\ \text{kV}$

682 $V_s = 66\ V$

$$\frac{V_p}{V_s} = \frac{N_1}{N_2}$$

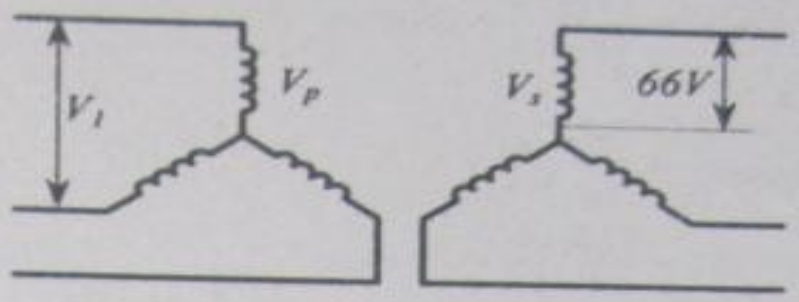
$$V_p = V_s\left(\frac{N_1}{N_2}\right)$$

$$= 66\left(\frac{132.760}{66.375}\right)$$

$$V_p = 132\ kV$$

$$V_1 = \sqrt{3}V_p = \sqrt{3}(132)$$

$$V_1 = 228.63\ kV$$

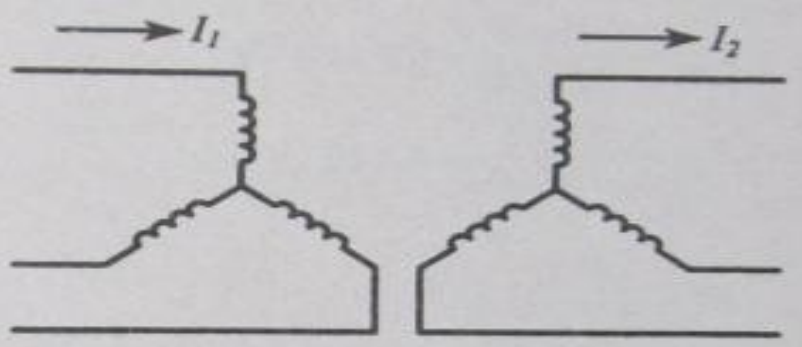


683 $$I_1 = \frac{\text{VA Rating}}{\sqrt{3}V_1} = \frac{50{,}000}{\sqrt{3}(34.5)}$$

$$= 836.7\ A$$

$$I_2 = \frac{\text{VA Rating}}{\sqrt{3}V_2} = \frac{50{,}000}{\sqrt{3}(13.8)}$$

$$I_2 \cong 2092\ A$$



684 Note: Assume the transformer have the same turns ratio and the given reactance is referred to the secondary side

$$S_1 = \frac{S_t X_{e2}}{X_{e1} + X_{e2}}$$

$$S_1 = \frac{175(12.5)}{9.6 + 12.5} = 98.98$$

$$S_1 \cong 99\ kVA$$

$$S_2 = \frac{S_t X_{e1}}{X_{e1} + X_{e2}}$$

$$S_2 = \frac{175(9.6)}{9.6 + 12.5} = 76.01$$

$$S_2 \cong 76\ kVA$$

685 Note: Assume the transformer have the same turns ratio and the given reactance is referred to the secondary side

$$S_1 = \frac{S_t X_{e2}}{X_{e1} \div X_{e2}}$$

$$S_1 = \frac{125(11)}{14+11} = 55\,\text{kVA}$$

$$S_1 = \frac{S_t X_{e2}}{X_{e1} + X_{e2}}$$

$$S_1 = \frac{125(14)}{14+11} = 70\,\text{kVA}$$

686 $$\%IZ = \frac{(\text{VA rating})Z_e}{V^2}$$

$$Z_e = \frac{\%IZ(V^2)}{\text{VA rating}}$$

$$Z_{e1} = \frac{0.04V^2}{125{,}000} = 3.2 \times 10^{-7}\,V^2$$

$$Z_{e2} = \frac{0.03V^2}{75{,}000} = 4 \times 10^{-7}\,V^2$$

$$S_1 = \frac{S_t Z_{e2}}{Z_{e1} + Z_{e2}}$$

$$S_1 = \frac{140(4 \times 10^{-7}\,V^2)}{(3.2 \times 10^{-7} + 4 \times 10^{-7})V^2} = 77.777\,\text{kVA}$$

$$S_1 \cong 77.78\,\text{kVA}$$

$$S_2 = \frac{S_t Z_{e1}}{Z_{e1} + Z_{e2}}$$

$$S_1 = \frac{140(3.2 \times 10^{-7}\,V^2)}{(3.2 \times 10^{-7} + 4 \times 10^{-7})V^2}$$

$$S_2 = 62.22\,\text{kVA}$$

Another solution:

Note: Since the equivalent impedances of the transformers are given in per unit values from different kVA base, select a common base kVA and transform all per unit impedance to the selected common kVA base.

Select the common kVA base equal to 125 kVA

$$X_{p1} = 0.04$$

$$X_{e2}' = X_{e2}\left(\frac{kVA_c}{kVA_2}\right)$$

$$X_{e2}' = 0.03\left(\frac{125}{75}\right) = 0.05$$

$$S_1 = \frac{S_t X_{e2}'}{X_{e1} + X_{e2}'}$$

$$S_1 = \frac{140(0.05)}{0.04 + 0.05} \cong 77.78\ kVA$$

$$S_2 = \frac{S_t X_{e1}}{X_{e1} + X_{e2}'}$$

$$S_2 = \frac{140(0.04)}{0.04 + 0.05} = 62.22\ kVA$$

687 Note: Select the common kVA base equal to 25 kVA

$$X_{e1} = 0.08$$

$$X_{e2}' = X_{e2}\left(\frac{kVA_c}{kVA_2}\right)$$

$$X_{e2}' = 0.08\left(\frac{25}{15}\right) = 0.133$$

$$S_1 = \frac{S_t X_{e2}'}{X_{e1} + X_{e2}'}$$

$$S_1 = \frac{30(0.1333)}{0.08 + 0.1333} = 18.75\ kVA$$

$$S_2 = \frac{S_t X_{e1}}{X_{e1} + X_{e2}'}$$

$$S_2 = \frac{30(0.08)}{0.08 + 0.1333} = 11.25\ kVA$$

688 Note: Select the common kVA base equal to 125 kVA

$$Z_{e1} = 0.01 + j0.05 = 0.051\angle 78.69^\circ$$

$$Z_{e2}' = Z_{e2}\left(\frac{kVA_c}{kVA_2}\right)$$

$$Z_{e2}' = 0.015 + j0.04\left(\frac{500}{250}\right) = 0.03 + j0.08$$

$$Z_{e2}' = 0.0854\angle 69.44°$$

$$Z_{e1} + Z_{e2}' = 0.01 + j0.05 + 0.03 + j0.08$$
$$Z_{e1} + Z_{e2}' = 0.04 + j0.13 = 0.136\angle 72.897°$$

$$\theta_t = \cos^{-1} 0.8 = 36.87°$$

$$S_1 = \frac{S_t Z_{e2}'}{Z_{e1} + Z_{e2}'}$$

$$S_1 = \frac{800\angle -36.87°(0.0854\angle 69.44°)}{0.136\angle 72.897°} = 502.35\angle -40.327° \text{ kVA}$$

$$pf_1 = \cos(40.327°)$$

$$pf_1 = 0.762 \text{ lagging}$$

$$S_2 = \frac{S_t X_{e1}}{X_{e1} + X_{e2}'}$$

$$S_2 = \frac{800\angle -36.87°(0.051\angle 78.69°)}{0.136\angle 72.897°} = 300\angle -31.077° \text{ kVA}$$

$$pf_2 = \cos(31.077°)$$

$$pf_2 = 0.856 \text{ lagging}$$

689 Note: Since the turns ratios and kVA ratings of the two transformers are equal, there is no need to transform the given per unit impedances based on the selected common kVA base.

$$Z_a = 0.02 + j0.08 = 0.08246\angle 75.96°$$
$$Z_b = 0.01 + j0.06 = 0.0608\angle 80.53°$$
$$Z_a + Z_b = 0.03 + j0.14 = 0.143\angle 77.9°$$

$$\theta_t = \cos^{-1} 0.8 = 36.87°$$

$$S_t = \frac{P_t}{pf} = \frac{280}{0.8} = 350 \text{ kVA}$$

$$S_a = \frac{(S_t)Z_b}{Z_a + Z_b} = \frac{(350\angle -36.87°)(0.0608\angle 80.53°)}{0.143\angle 77.9°}$$
$$= 148.81\angle -34.24° = 123.02 - j83.729$$
$$P_a \cong 123 \text{ kW}$$

$$S_b = \frac{(S_t)Z_a}{Z_a + Z_b} = \frac{(350\angle -36.87°)(0.08246\angle 75.96°)}{0.143\angle 77.9°}$$
$$= 201.825\angle -38.81° = 157.26 - j126.49$$
$$P_b \cong 157\text{ kW}$$

690

$$Z_a = 0.01 + j0.06 = 0.0608\angle 80.537°$$
$$Z_b = 0.02 + j0.05 = 0.0538\angle 68.198°$$
$$Z_a + Z_b = 0.03 + j0.11 = 0.114\angle 74.74°$$
$$\theta_t = \cos^{-1} 0.9 = 25.84°$$
$$S_t = \frac{P_t}{pf} = \frac{180}{0.9} = 200\text{ kVA}$$

$$S_a = \frac{(S_t)Z_b}{Z_a + Z_b} = \frac{(200\angle -25.84°)(0.0538\angle 68.198°)}{0.114\angle 74.74°}$$
$$S_a = 94.38\angle -32.382° = 79.703 - j50.546$$
$$P_a \cong 80\text{ kW}$$

$$S_b = \frac{(S_t)Z_a}{Z_a + Z_b} = \frac{(200\angle -25.84°)(0.0608\angle 80.537°)}{0.114\angle 74.74°}$$
$$= 106.67\angle -20.043° = 100.2 - j36.558$$
$$P_b \cong 100\text{ kW}$$

691

$$Z_a = 2 + j3 = 3.605\angle 56.31°$$
$$Z_b = 2.2 + j5 = 5.462\angle 66.25°$$
$$Z_a + Z_b = 4.2 + j8 = 9.035\angle 62.3°$$
$$\theta_t = \cos^{-1} 0.8 = 36.87°$$
$$I_a = \frac{I_t Z_b}{Z_a + Z_b} = \frac{(600\angle -36.87°)(5.462\angle 66.25°)}{9.035\angle 62.3°}$$
$$I_a = 362.72\angle -32.92°\text{ A}$$
$$P_a = V_L I_a pf_a = V(362.72)\cos 32.92° = 304.478V_L$$

$$I_b = \frac{(I_t)Z_a}{Z_a + Z_b} = \frac{(600\angle -36.87°)(3.605\angle 56.31°)}{9.035\angle 62.3°}$$
$$= 239.4\angle -42.86°\text{ A}$$
$$P_b = V_L I_b pf_b = V(239.4)\cos 42.86° = 175.484V_L$$
$$\text{Ratio} = \frac{P_a}{P_b} = \frac{304.487V_L}{175.484V_L} \cong 1.74$$

3-Phase Induction Motors

SLIP

- **Slip** is defined as the difference between the synchronous speed and the actual rotor speed expressed as a percent (or per unit) of the synchronous speed.

$$s = \frac{N_s - N}{N_s} \qquad N_s = \frac{120f}{P} \qquad N = \frac{120f}{P}(1 - s)$$

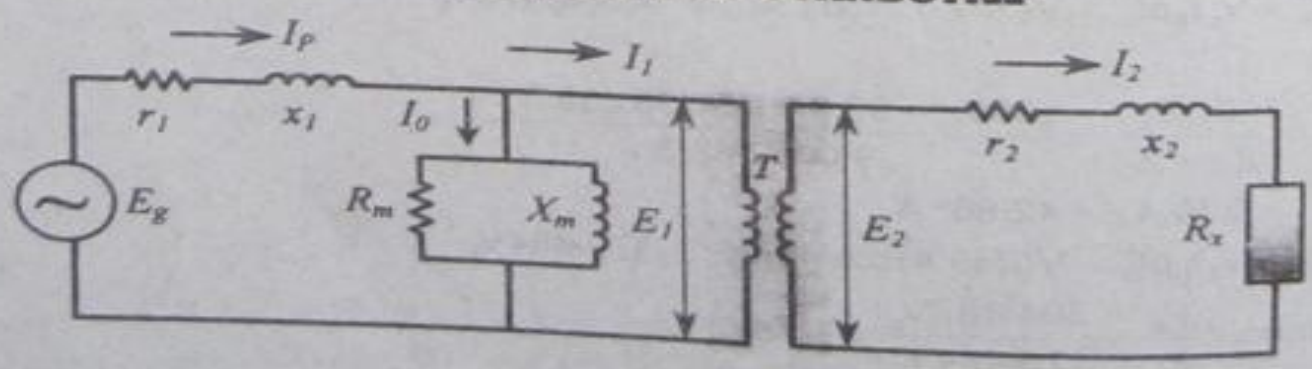
where: N_s = synchronous speed (rpm)
N = rotor speed (rpm)
f = frequency of the stator voltage (Hertz)
P = number of poles

ROTOR VOLTAGE AND FREQUENCY

$$f_r = sf \qquad E_2 = sE_{oc}$$

where: f_r = rotor frequency at slip s (Hertz)
E_2 = approximated voltage induced per phase in the rotor at slip s (volt)
E_{oc} = open circuit voltage induced in the rotor per phase also equal to the voltage per phase induced when the rotor is blocked (volt)

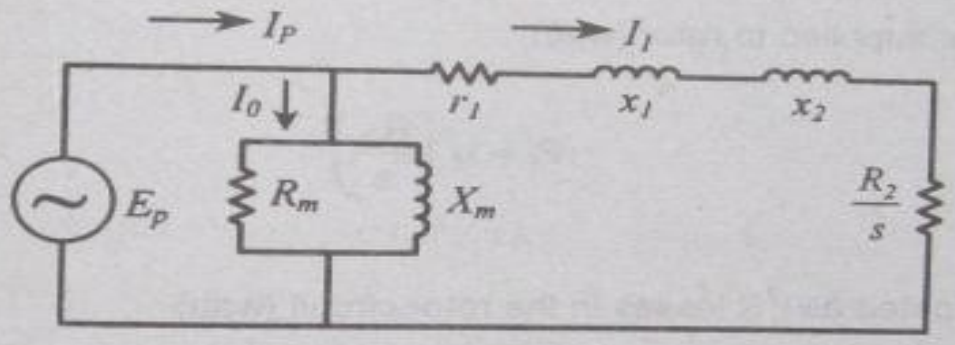
EQUIVALENT CIRCUIT PER PHASE OF A WOUND ROTOR INDUCTION MOTOR AT STANDSTILL



where : r_1 = stator resistance per phase (ohm)
r_2 = rotor resistance per phase (ohm)
R_x = external resistance, effectively connected between one slip ring and the neutral of the rotor (ohm)
R_m = resistance corresponding to the iron losses and friction and windage losses (ohm)
x_1 = stator leakage reactance (ohm)
x_2 = rotor leakage reactance (ohm)
X_m = magnetizing reactance (ohm)
T = ideal transformer having a turns ratio oi 1:1

Note: For squirrel-cage induction motor, $R_x = 0$.

EQUIVALENT CIRCUIT PER PHASE OF A WOUND ROTOR INDUCTION MOTOR REFERRED TO THE STATOR SIDE



where: $R_2 = r_2 + R_x$

$$I_1 = \frac{E_p}{\sqrt{\left(r_1 + \frac{R_2}{s}\right)^2 + (x_1 + x_2)^2}}$$

- Active power absorbed by the motor (watt)

$$P = \frac{E_p^2}{R_m} + I_1^2 r_1 + I_1^2\left(\frac{R_2}{s}\right)$$

- Reactive power absorbed by the motor (var)

$$Q = \frac{E_p^2}{X_m} + I_1^2(x_1 + x_2)$$

- Apparent power absorbed by the motor (volt-ampere)

$$S = \sqrt{P^2 + Q^2}$$

- Power factor of the motor (pf)

$$pf = \frac{P}{S}$$

- Line current drawn (ampere)

$$I_p = \frac{S}{E_p}$$

- Active power supplied to rotor (watt)

$$P_r = I_1^2\left(\frac{R_2}{s}\right)$$

- Power dissipated as I^2R losses in the rotor circuit (watt)

$$P_{cu2} = I_1^2 R_2 = sP_r$$

- Mechanical power developed by the motor (watt)

$$P_m = P_r - P_{cu2} \qquad P_m = P_r(1-s)$$

- Torque developed by the motor (newton-meter)

$$T = \frac{9.55\,P_m}{N} \qquad T = \frac{9.55\,P_r}{N_s}$$

- Efficiency of the motor

$$\eta = \frac{P_m}{P}$$

NO LOAD TEST OF AN INDUCTION MOTOR

When an induction motor runs at no load, the slip is exceedingly small, thus making R_2/s very large and I_1, becomes negligible compared to I_o. Thus at no load the circuit consist essentially of the magnetizing branch X_m and R_m only.

$$S_{NL} = \sqrt{3}E_{NL}I_{NL} \qquad Q_{NL} = \sqrt{(S_{NL})^2 - (P_{NL})^2}$$

$$X_m = \frac{(E_{NL})^2}{Q_{NL}} \qquad R_m = \frac{(E_{NL})^2}{P_{NL}}$$

where: P_{NL} = wattmeter reading during the test (power drawn by the motor)
E_{NL} = voltmeter reading during the test (line to line voltage of the supply)
I_{NL} = ammeter reading during the test (line current drawn by the motor)

- Assuming copper losses are negligible (ideal analysis).

$$P_{co} + P_{fw} \approx P_{NL}$$

- Assuming copper losses are not negligible (non-ideal analysis).

$$P_{co} + P_{fw} = P_{NL} - 3(I_{NL})^2 R_{e1}$$

Note: This formula shall be used only for a Y-connected motor or motor assumed Y-connected in the analysis.

where: R_{e1} = equivalent resistance per phase of the motor referred to the stator side
P_{co} = core losses
P_{fw} = friction and windage losses

LOCKED ROTOR TEST OF AN INDUCTION MOTOR

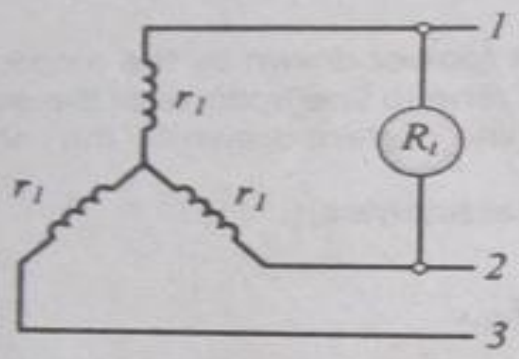
Under rated line voltage, when the rotor is locked, stator current I_P is almost 6 times its rated value. Furthermore the slip "s" equals one which means that R_2/s becomes R_2, where R_2 is the resistance of the rotor reflected into the stator and since I_P is much greater compared to I_o, thus the magnetizing branch can be neglected.

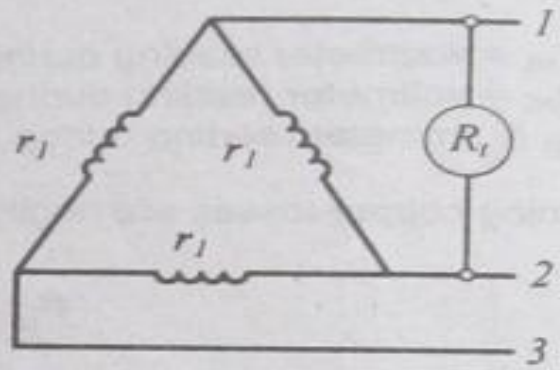
$$S_{LR} = \sqrt{3}E_{LR}I_{LR} \qquad Q_{LR} = \sqrt{(S_{LR})^2 - (P_{LR})^2}$$

$$R_{e1} = \frac{P_{LR}}{3(I_{LR})^2} \qquad X_{e1} = \frac{Q_{LR}}{3(I_{LR})^2}$$

where: P_{LR} = wattmeter reading during the test (power drawn by the motor)
E_{LR} = voltmeter reading during the test (line to line voltage of the supply)
I_{LR} = ammeter reading during the test (line current drawn by the motor)
X_{e1} = equivalent reactance per phase of the motor referred to the stator side

DC STATOR RESISTANCE TEST OF AN INDUCTION MOTOR

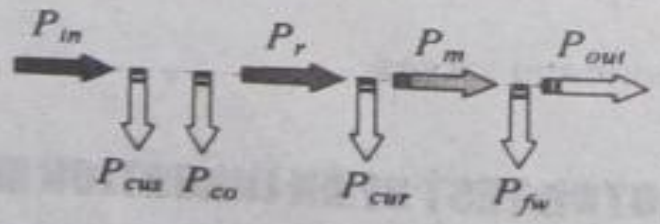




$$r_1 = \frac{1}{2}R_t \qquad r_1 = \frac{3}{2}R_t$$

where: R_t = DC resistance between any two terminals of the stator.
= ohmmeter reading during the test

POWER FLOW DIAGRAM



where: P_{in} = input power drawn by the motor
P_{out} = output power drawn by the motor
P_{cus} = stator copper losses
P_{co} = core losses
P_r = power input of the rotor
P_{cur} = rotor copper losses
P_m = mechanical power developed
P_{fw} = friction and windage losses

Test 19

Problem 692: EE Board October 1997

A single-phase induction motor is rated 5 hp, 75% power factor and 220 volts. What is its full load current?

A. 22.5 A
B. 20.5 A
C. 16.9 A
D. 18.5 A

Problem 693: EE Board April 1990

A 5 hp, 3-phase motor draws current when connected to 230 V. Determine the current drawn by the motor if the power factor is 0.90 and efficiency of 83%.

A. 12.53 A
B. 15.13 A
C. 13.25 A
D. 14.05 A

Problem 694: EE Board April 1984

A 3-phase motor is rated 750 HP at 460 volts line to line, 0.85 pf lagging and 0.95 efficiency. Solve for the rated current.

A. 869.64 A
B. 702.23 A
C. 405.43 A
D. 917.32 A

Problem 695: EE Board April 1985

A 40 HP, 3-phase, 4-pole, 230 volt, 60 Hz, induction motor operating at 0.90 efficiency, 0.85 power factor and 3 percent slip, drives a water pump. Determine the current at which the motor is operating.

A. 94.531 A
B. 96.234 A
C. 97.915 A
D. 93.351 A

Problem 696: EE Board April 1992

Calculate the speed of a 60-cycle, 14-pole motor if the slip is 5%.

A. 514.28 rpm
B. 488.57 rpm
C. 456.32 rpm
D. 429.63 rpm

Problem 697: EE Board April 1992

The rotor speed of a six-pole, 50-cycle induction motor is 960 rpm. Calculate the percent slip.

A. 5%
B. 2%
C. 4%
D. 3%

Problem 698: EE Board October 1997

What is the speed of an induction motor of six poles if the percent slip is 2.5%?

A. 1462 rpm
B. 1170 rpm
C. 877 rpm
D. 1755 rpm

Problem 699: EE Board October 1984

A squirrel cage induction motor with nameplate data of: 150 HP, 3-phase, 460-V, 60 Hz, 6-pole, 0.85 pf was subjected to certain performance tests. The test result readings were as follows:

Full load current = 202 A
Full load torque = 676.8 lb-ft.

Solve the percentage slip.

A. 3.28%
B. 4.15%
C. 3.05%
D. 2.25%

Problem 700:

A six-pole, three-phase squirrel-cage induction motor is connected to a 60-cps supply. At full load, the rotor's induced emf makes 72 complete cycles in 1 minute. Find the rotor speed.

A. 1176 rpm
B. 1200 rpm
C. 1054 rpm
D. 1124 rpm

Problem 701: EE Board March 1998

A 50-hp three-phase motor, rated 220 V, 75% power factor has 6 poles. The slip at full load is 3%. What is the frequency of the rotor current at full load?

A. 1.8 Hz
B. 60 Hz
C. 5.4 Hz
D. 58.2 Hz

Problem 702: EE Board April 1986

A 150 HP, 3-phase, 6-pole, 460 V, 60 Hz induction motor draws 195 A line current at full load speed of 1170 rpm and 0.85 pf. Solve for the percent slip at full load.

A. 2.5%
B. 3.0%
C. 2.8%
D. 2.0%

Problem 703: EE Board October 1986

The deep well pump motor is 50 hp, 3-phase, 4-poles, 230 volts, 60 Hz induction motor, operating at 0.90 efficiency, 0.85 power factor and 3% slip. Determine the current at which the motor is operating.

A. 112.7 A
B. 122.4 A
C. 120.5 A
D. 124.7 A

Problem 704: EE Board April 1993

The rotor of a 220-V, 60 Hz, 4-pole induction motor has a power input of 80 kW. Is observed that the rotor emf makes 90 cycles per minute. Calculate the rotor copper loss.

A. 2 kW
B. 5 kW
C. 3 kW
D. 4 kW

Problem 705:

An induction motor carries a rotor current per phase of 10 A when loaded until the slip is 5%. If the rotor effective resistance is 0.1 ohm per phase, determine the total power transferred across the air gap per phase.

A. 210 W
B. 240 W
C. 190 W
D. 200 W

Problem 706:

A 10 hp (nameplate), 6-pole, 60 Hz, three-phase induction motor delivers 9.9 hp with an input of 9200 watts. The core loss is 450 W. The stator copper loss is 650 watts and the rotational losses is 150 watts. What is the motor speed?

A. 1116 rpm
B. 1200 rpm
C. 1210 rpm
D. 1125 rpm

Problem 707:

A 3-phase squirrel-cage induction motor is drawing 45 kW from a 440-V, 60 Hz 3-phase line. If stator losses amount to 900 W, determine total mechanical power developed. Assume a slip of 3%.

A. 40.32 kW
B. 44.64 kW
C. 48.22 kW
D. 42.78 kW

Problem 708: EE Board October 1990

A 10-hp, 4-pole, 25 Hz, 3-phase induction motor is taking 9100 watts from the line. Core loss is 290 watts: stator copper loss is 568 watts; rotor copper loss is 445 watts and the friction and windage loss 121 watts. Determine the output torque in newton-meters.

A. 108.4
B. 112.5
C. 101.2
D. 103.3

Problem 709:

A 40 hp, 230-V, 8-pole, 25-Hz three-phase induction motor is running at 355 rpm at rated load. The torque lost by friction is 24 N-m. If total stator losses amount to 1000 W, evaluate the efficiency of the motor at this load.

A. 91.42%
B. 88.51%
C. 90.31%
D. 89.17%

Problem 710: EE Board April 1993

The rotor of a 220-V, 60 Hz, 4-pole induction motor has a power input of 80 kW. Is observed that the rotor emf makes 90 cycles per minute. Calculate the gross mechanical power developed.

A. 72 kW
B. 78 kW
C. 75 kW
D. 77 kW

Problem 711:

A 6-pole, 50-Hz, 3-phase induction motor runs at 960 rpm while delivering a shaft torque of 120 N-m. If friction and windage loss amount to 180 W, determine rotor copper loss.

A. 521 W
B. 518 W
C. 510 W
D. 526 W

Problem 712: EE Board April 1993

Determine the speed in rpm of the motor at full load with the following circuit elements per phase.

Rotor leakage reactance = 0.65 Rotor resistance = 0.10
Stator leakage reactance = 1.12 Stator resistance = 0.25
Connection = Y-Y

This motor is 500 HP, 3-phase, 2200 volt, 60-cycle, 6-pole induction motor with additional core loss of 2,000 watts and the friction and windage losses total is 10,000 watts.

A. 1200 rpm
B. 1196 rpm
C. 1190 rpm
D. 1188 rpm

Problem 713:

A 50 HP, 440 V, 3-phase, 60 Hz, 6-pole squirrel cage induction motor is operating at full load and 0.8 pf. The full load efficiency is 85% and the percentage slip is 5%. Determine full load torque.

A. 214.55 N-m
B. 206.72 N-m
C. 312.47 N-m
D. 323.24 N-m

Problem 714: EE Board April 1981, EE Board April 1989

A 30-hp, 4-poles, 3-phase, 230 volts, 60 Hz squirrel cage induction motor operating at 90% efficiency, 85% power factor and 2.5% slip drives a water pump for a reservoir. Find the current and speed at which the motor is operating.

A. 73.4 A, 1755 rpm
B. 62.39 A, 1755 rpm
C. 73.4 A, 1800 rpm
D. 62.39 A, 1800 rpm

Problem 715: EE Board October 1980

A 50 hp, 3-phase, 4-pole, 60 Hz, 230-volt squirrel cage induction motor has an efficiency of 90%, a power factor of 85% and a slip of 2% at full load. Find the torque in lb-ft at full load.

A. 152.45
B. 148.87
C. 121.34
D. 150.56

Problem 716: EE Board October 1992

The losses in a three-phase, 25-Hz, 4-pole induction motor at full load are as follows:

Core – 3%
Friction and windage – 2%
Stator copper - 4%
Rotor copper – 2%

At 70% of full load of the machine, determine its efficiency.

A. 90.52%
B. 88.15%
C. 87.24%
D. 89.81%

Problem 717: EE Board October 1980

A 100 hp, 440 volt, three-phase, 60 Hz induction motor operates with 92 percent efficiency, 0.85 power factor, at rated load. What is the current of the motor?

A. 125 A
B. 115 A
C. 98 A
D. 55 A

Problem 718:

A six-pole three-phase 440-V, 60 Hz induction motor develops 10-hp at 1150 rpm, the power factor being 80% lagging. Stator and core losses amount to 400 W and 350 respectively. Frictional losses amount to 0.5-hp. Calculate motor line current.

A. 12.24 A
B. 11.42 A
C. 13.41 A
D. 14.64 A

Problem 719:

A three-phase, 7.5 hp, 220-V, 6-pole, 60-Hz, Y-connected induction motors was tested and give the following results:

No-load test:	I = 6.5 A	P = 610 W	N_r = 1190 rpm
Load test:	I =18.4 A	P = 6700 W	N_r = 1164 rpm
Resistance test:	0.54 ohm between any two stator terminals		

Determine the efficiency of the motor during the load test.

A. 83.45%

B. 84.69%
C. 85.28%
D. 86.55%

Problem 720: EE Board October 1991

A three-phase wound rotor induction motor, 440 V, 60 cycle, 4-pole is connected to a pump that delivers 1000 cfm of water against an effective head of 8 ft. Under this load, the motor draws 15 kW at a power factor of 0.90. When operated without load, the motor draws 800 watts. The stator resistance per phase is 0.21 ohm and the rotor resistance per phase is 0.022 ohm. The turns ratio between stator and rotor is 4:1. Determine the efficiency of the pump.

A. 83.62%
B. 84.65%
C. 83.16%
D. 84.24%

Problem 721:

A three-phase, six-pole, 60 Hz, Y-connected squirrel-cage induction motor operates with a slip of 2% and draws 10 A and 3,500 W at full load. When running light, the motor draws 4.2 A and 300 W. When the rotor of this motor is blocked, the motor draws 450 W, 15 A at 50 V. Calculate the torque exerted when driving its full load.

A. 22.82 N-m
B. 24.65 N-m
C. 20.36 N-m
D. 26.74 N-m

Problem 722:

The following data were obtained from a 5-hp, 110-V, 60 Hz, 8-pole, 3-phase, Y-connected induction motor:

No-load test:	I = 10 A	W_1 = 725 W	W_2 = - 425 W	
Load test:	I = 28 A	W_1 = 3140 W	W_2 = 1580 W	Nr = 870 rpm
DC resistance test:	0.16 ohm between any two stator terminals			

Calculate the torque in N-m exerted by the motor during the load test. Assume the ac resistance is 1.25 times the dc resistance.

A. 44.74
B. 52.42
C. 46.10
D. 43.84

Problem 723: EE Board April 1997

A resistor assembly for a wound rotor consists of 20 elements of cast iron grid elements. The rating of each is 0.10 ohm. When joined in series and tested across 220 volts, the measured current was 100 A. How much power was lost in the joints?

A. 1.9 kW
B. 1.8 kW
C. 2.0 kW
D. 2.1 kW

Problem 724: EE Board October 1987

A wound rotor motor, 7.5 HP, 230 volts, 3-phase takes a line current of 18.4 amperes, when operating at rated output at an efficiency of 88%. Calculate the indications on wattmeters when these are inserted to measure power by the two-wattmeter method.

A. 4.035 kW, 2.323 kW
B. 4.232 kW, 2.126 kW
C. 3.902 kW, 2.456 kW
D. 3.855 kW, 2.503 kW

Problem 725: EE Board June 1990

Two-wattmeter method is used to test a 25 HP, 230 volt, 1800 rpm, 60 cycle, 3-phase induction motor. When the line voltages are 230 volts, one wattmeter reads +13,400 watts and the other +7,400 watts. Determine motor line current.

A. 55.6 A
B. 58.4 A
C. 52.3 A
D. 50.8 A

Problem 726: EE Board April 1992

An induction motor of 30 hp, 220 volts 3-phase draws 450 percent of the rated current with the rated voltage and delivers during the starting period 130 percent of the normal torque. The full load efficiency and full load pf of this motor is 80% and 70% respectively. If an autotransformer is used as a starting unit and starting torque of the load is only 50 percent of the rated torque of the motor, find the starting voltage.

A. 136.44 V
B. 130.65 V
C. 132.36 V
D. 134.20 V

Problem 727: EE Board October 1989

A 10 HP, 550 V, 60 Hz, 3-phase induction motor has a starting torque of 160% of full load torque and a starting current of 425% of full load current. If the motor is used on a 440 V, 60 Hz system, what will be the starting torque expressed in percent of full load value?

A. 102.4%
B. 101.8%
C. 104.2%
D. 103.5%

Problem 728: EE Board October 1989

A 10 HP, 550 V, 60 Hz, 3-phase induction motor has a starting torque of 160% of full load torque and a starting current of 425% of full load current. What will the voltage in order that the starting current be limited to the full load value?

A. 132.47 V
B. 129.41 V
C. 125.52 V
D. 136.75 V

Problem 729: EE Board October 1998

A 25-hp, 230 V three-phase motor with 85% power factor has a starting current of 5.5 times rated current. To reduce the starting current, a wye-delta starter is installed. What shall be the new starting current?

A. 195 A
B. 302 A
C. 135 A
D. 175 A

Problem 730: EE Board October 1997

A squirrel cage motor is started at 50% of its rated voltage. What is the starting torque relative to its rated voltage starting torque?

A. 50%
B. 100%
C. 25%
D. 75%

Problem 731: EE Board April 1989

A delta-connected load induction motor draws 100 A from each line of a three-phase source and develops 40 lb-ft torque at standstill. Find the torque when connected to the same service if the motor is re-connected in wye.

A. 13.3 lb-ft
B. 120 lb-ft
C. 40 lb-ft
D. None of these

Problem 732: EE Board April 1992

An induction motor of 30 HP, 220 V, 3-phase draws 450 percent of the rated current with the rated voltage and delivers during the starting period of 130% of the normal torque. The full load efficiency and full load pf of this motor is 80 percent and 70 percent respectively. If an autotransformer is used as a starting unit and the starting torque of the load is only 50 percent of the rated torque of the motor, find the starting current on the line.

A. 301.4 A
B. 292.7 A

C. 288.6 A
D. 227.3 A

Problem 733:

A 7.5 hp, 500 V three-phase motor induction motor is developing its full load output at 960 rpm. The starting torque is equal to the full load torque. What will be the starting torque if the supply voltage falls to 400 volts?

A. 32.34 N-m
B. 35.62 N-m
C. 28.27 N-m
D. 38.82 N-m

Problem 734: EE Board October 1986

The deep well pump motor is 50 hp, 3-phase, 4-poles, 230 volts, 60 Hz induction motor, operating at 0.90 efficiency, 0.85 power factor and 3% slip. Determine the discharge of the pump in cubic meters per minute, assuming that the total head against which the pump is working is 50 meters and that the density of water is 1000 kg per cubic meter.

A. 4.103
B. 4.224
C. 4.563
D. 4.205

Problem 735:

A three-phase, 230 V, 6-pole, 60 Hz induction motor is loaded by means of a pony brake. The length and dead weight of which are 2.5 ft and 2 lbs respectively. With 230-V impressed on this motor, the total force delivered by the pony brake is 30 lbs. The power input to the motor is measured by two wattmeters using the two-wattmeter method. These wattmeters register 4 and 8 kW respectively. If the motor has a slip of 5% under this condition, determine its efficiency.

A. 93.62%
B. 92.44%
C. 90.82%
D. 94.43%

Problem 736:

A 75 kVA, 3-phase transformer supplies the following initial loads: Capacitor bank drawing 25 kVAR and an inductive load drawing 36 kW, 0.715 pf. Determine the maximum hp of induction motor at 0.8 pf and 90% efficiency that can be added to the load without overloading the transformer.

A. 40.2 hp
B. 38.4 hp
C. 35.5 hp
D. 37.5 hp

Problem 737:

The feeder of an industrial plant is has a load of 400 kVA at a power factor of 75% lagging. An induction motor load drawing 100 kW is added to the feeder. Determine the operating power factor this motor such that the total feeder load will have a lagging pf of 90%.

A. 0.942
B. 0.894
C. 0.902
D. 0.818

ANSWER KEY

692. A	702. A	712. C	722. A	732. B
693. A	703. B	713. C	723. C	733. B
694. A	704. A	714. A	724. B	734. C
695. C	705. D	715. B	725. B	735. D
696. B	706. A	716. D	726. A	736. D
697. C	707. D	717. A	727. A	737. C
698. B	708. D	718. D	728. B	
699. A	709. D	719. B	729. D	
700 A	710. B	720. D	730. C	
701. A	711. C	721. B	731. A	

☑ RATING

41-46 - Topnotcher

32-40 - Passer

23-31 - Conditional

0-22 - Failed

Solutions to Test 19

692 Note: Since efficiency is not given, assume that the rating of the motor is equivalent to the same power drawn from the supply. (η = 100%)

$$I = \frac{P}{V_L pf} = \frac{5(746)}{(220)(0.75)}$$

$$I = 22.606 \text{ A}$$

693 $$P_{in} = \frac{P_{out}}{\eta} = \frac{5(746)}{0.83} = 4493.97 \text{ W}$$

$$I = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{4493.97}{\sqrt{3}(230)(0.9)}$$

$$I = 12.53 \text{ A}$$

694 $$P_{in} = \frac{P_{out}}{\eta} = \frac{750(746)}{0.95} = 588947.368 \text{ W}$$

$$I = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{588947.368}{\sqrt{3}(460)(0.85)}$$

$$I = 869.64 \text{ A}$$

695 $$P_{in} = \frac{P_{out}}{\eta} = \frac{40(746)}{0.9} = 33155.55 \text{ W}$$

$$I = \frac{P_{out}}{\sqrt{3}Epf} = \frac{33155.55}{\sqrt{3}(230)(0.85)}$$

$$I = 97.915 \text{ A}$$

696 $$N_r = \frac{120f}{P}(1-s)$$

$$N_r = \frac{120(60)}{14}(1-0.05)$$

$$N_r = 488.57 \text{ rpm}$$

697 $$N_r = \frac{120f}{P}(1-s)$$

$$s = 1 - \frac{N_r P}{120f} = 1 - \frac{960(6)}{120(50)} = 0.04$$

$$s = 4\%$$

698 Note: Since not given, assume a standard electrical frequency of 60 Hz

$$N_r = \frac{120f}{P}(1-s)$$

$$N_r = \frac{120(60)}{6}(1-0.025)$$

$$N_r = 1170 \text{ rpm}$$

699 $$HP = \frac{2\pi N_r T}{33000}$$

$$N_r = \frac{HP(33000)}{2\pi T} = \frac{150(33000)}{2\pi(678.8)} = 1160.6 \text{ rpm}$$

$$N_r = \frac{120f}{P}(1-s)$$

$$s = 1 - \frac{N_r P}{120f} = 1 - \frac{1160.6(6)}{120(60)} = 0.0328$$

$$s = 3.28\%$$

700 $$f_r = \frac{72 \text{ cycles}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 1.2 \text{ Hz}$$

$$f_r = sf$$

$$s = \frac{f_r}{f} = \frac{1.2}{60} = 0.02$$

$$N_r = \frac{120f}{P}(1-s) = \frac{120(60)}{6}(1-0.02)$$

$$N_r = 1176 \text{ rpm}$$

701 Note: Since stator frequency is not given and it can't be determined using the given data, assume a standard electrical frequency of 60 Hz.

$$f_{rotor} = (s)(f_{stator}) = (0.03)(60)$$

$$f_{rotor} = 1.8 \text{ Hz}$$

702 $$N_r = \frac{120f}{P}(1-s)$$

$$s = 1 - \frac{N_r P}{120f} = 1 - \frac{1170(6)}{120(60)}$$

$$s = 0.025$$

$$s = 2.5\%$$

703 $P_{in} = \dfrac{P_{out}}{\eta} = \dfrac{50(746)}{0.9} = 41444.44\ W$

$I = \dfrac{P_{in}}{\sqrt{3}V_L pf} = \dfrac{41444.44}{\sqrt{3}(230)(0.85)}$

$I = 122.4\ A$

704 $f_r = \dfrac{90\ cycles}{min} \times \dfrac{1\ min}{60\ sec} = 1.5\ Hz$

$fr = sf$

$s = \dfrac{f_r}{f} = \dfrac{1.5}{60} = 0.025$

$P_{cu2} = sP_r = 0.025(80)$

$P_{cu2} = 2\ kW$

705 $P_r = I_r^2\left(\dfrac{r_2}{s}\right) = (10)^2\left(\dfrac{0.1}{0.05}\right)$

$P_r = 200\ W$

706 Note: Refer to the power flow diagram.

$P_m = P_{out} + P_{fw}$

$= 9.9(746) + 150 = 7535.4\ W$

$P_r = P_{in} - (P_{cus} + P_{co})$

$= 9200 - (650 + 450) = 8100\ W$

$P_{cur} = P_r - P_m$

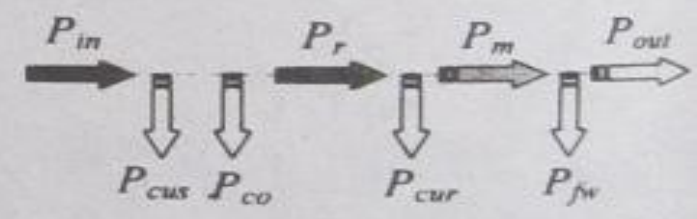
$= 8100 - 7535.4 = 564.6\ W$

$P_{cur} = sP_r$

$s = \dfrac{P_{cur}}{P_r} = \dfrac{564.6}{8100} = 0.0697$

$N_r = \dfrac{120f}{P}(1 - s) = \dfrac{120(60)}{6}(1 - 0.0697)$

$N_r = 1116.36\ rpm$



Power flow diagram

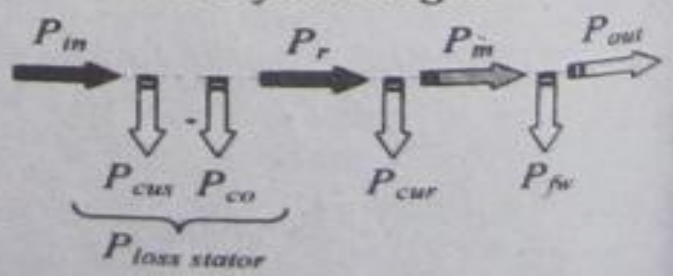
707 Note: Refer to the power flow diagram

$P_r = P_{in} - P_{loss\ stator}$

$P_r = 45{,}000 - 900 = 44{,}100\ W$

$P_m = P_r(1 - s)$

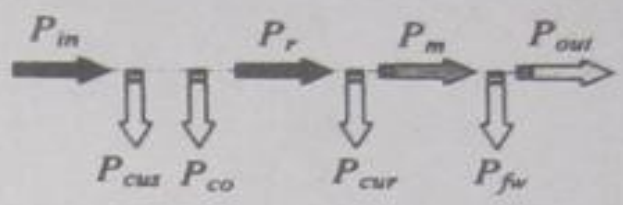
Power flow diagram



$P_m = 44{,}100\,(1 - 0.03) = 42{,}777\ W$

$P_m = 42.78\ kW$

708 Note: Refer to the power flow diagram



Power flow diagram

$P_{out} = P_{in} - (P_{cu_s} + P_{co} + P_{cu_r} + P_{fw})$

$= 9100 - (568 + 290 + 445 + 121)$

$P_{out} = 7676\ W$

$P_r = P_{in} - (P_{cu_s} + P_{co})$

$= 9100 - (568 + 290) = 8242\ W$

$P_{cu_r} = sP_r$

$$s = \frac{P_{cu_r}}{P_r} = \frac{445}{8242} = 0.0539$$

$$N_r = \frac{120f}{P}(1 - s) = \frac{120(25)}{4}(1 - 0.0539)$$

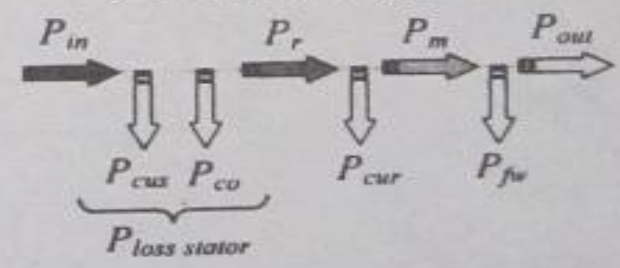
$= 709.575\ rpm$

$$T = \frac{9.55P_{out}}{N_r} = \frac{9.55(7676)}{709.575}$$

$T = 103.3\ N\text{-}m$

709 $$T_{out} = \frac{9.55P_{out}}{N_r}$$

Power flow diagram



$$T_{out} = \frac{9.55(40)(746)}{355} = 802.74\ N\text{-}m$$

$T_{dev} = T_{out} + T_{loss} = 802.74 + 24$

$T_{dev} = 826.74\ N\text{-}m$

$$N_s = \frac{120f}{P} = \frac{120(25)}{8} = 375\ rpm$$

$$T_{dev} = \frac{9.55P_r}{N_s}$$

$$P_r = \frac{T_{dev}N_s}{9.55} = \frac{(826.74)(375)}{9.55} = 32463.6\ W$$

Note: Refer to the power flow diagram.

$P_{input} = P_r + P_{loss_{stator}} = 32463.6 + 1000 = 33463.6\ W$

$$\eta = \frac{P_{out}}{P_{input}} = \frac{40(746)}{33463.6} \times 100\%$$

$\eta = 89.17\%$

710

$$f_r = \frac{90 \text{ cycles}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 1.5 \text{ Hz}$$

$$s = \frac{f_r}{f_s} = \frac{1.5}{60} = 0.025$$

$$P_m = P_r(1 - s) = 80(1 - 0.025)$$

$$P_m = 78 \text{ kW}$$

711

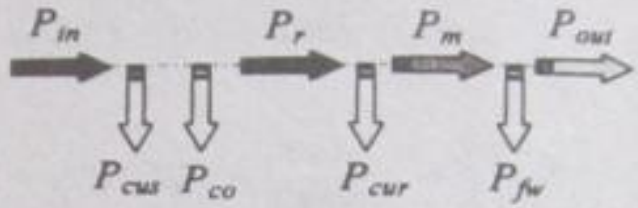
Note: Refer to the power flow diagram:

$$P_{out} = \frac{2\pi N_r T_{out}}{60} = \frac{2\pi(960)(120)}{60}$$

$$= 12063.7 \text{ W}$$

$$P_m = P_{out} + P_{fw} = 12063.7 + 180$$

$$= 12243.7 \text{ W}$$



Power flow diagram

$$s = 1 - \frac{N_r P}{120f} = 1 - \frac{960(6)}{120(50)} = 0.04$$

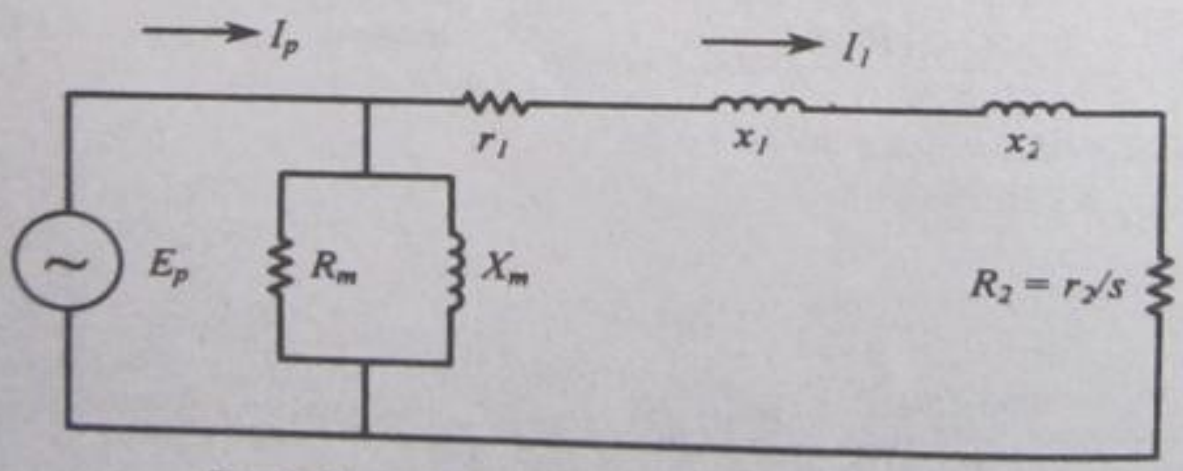
$$P_m = P_r(1 - s)$$

$$P_r = \frac{P_m}{1 - s} = \frac{12243.7}{1 - 0.04} = 12753.85 \text{ W}$$

$$P_{cu_r} = sP_r = 0.04(12753.85)$$

$$P_{cu_r} = 510.15 \text{ W}$$

712



Equivalent circuit of an induction motor (referred to the stator side)

Let: P_r = power developed in the rotor circuit

$$P_r = P_{out} + P_{fw} = 500(746) + 10{,}000$$

$$P_r = 383{,}000 \text{ W}$$

$P_r = 3I_1^2R_2$

$383{,}000 = 3I_1^2R_2$

$I_1^2 = \dfrac{1276666.667}{R_2} \Rightarrow ❶$

Refer to the equivalent circuit diagram:

$I_1 = \dfrac{E_p}{\sqrt{(r_1 + R_2)^2 + (x_1 + x_2)^2}}$

$I_1 = \dfrac{2200/\sqrt{3}}{\sqrt{(0.25 + R_2)^2 + (1.12 + 0.65)^2}}$

$I_1 = \dfrac{1270}{\sqrt{(0.25 + R_2)^2 + 3.133}}$

$I_1^2 = \dfrac{1612900}{(0.25 + R_2)^2 + 3.133} \Rightarrow ❷$

Equate Eq.1 to Eq.2:

$\dfrac{127666.667}{R_2} = \dfrac{1612900}{(0.25 + R_2)^2 + 3.133}$

$(0.25 + R_2)^2 + 3.133 = 12.633R_2$

$0.0625 + 0.5R_2 + R_2^2 + 3.133 = 12.633R_2$

$R_2^2 - 12.133R_2 + 3.1955 = 0$

Using the quadratic formula:

$R_2 = \dfrac{12.133 \pm \sqrt{(12.133)^2 - 4(1)(3.1955)}}{2(1)} = \dfrac{12.133 \pm 11.594}{2}$

$R_2^{(+)} = 11.86\Omega$

$R_2 = \dfrac{r_2}{s}$

$s = \dfrac{r_2}{R_2} = \dfrac{0.10}{11.86}$

$s = 0.00843$

$N_r = \dfrac{120f}{P}(1 - s) = \dfrac{(120)(60)}{6}(1 - 0.00843)$

$= 1189.884$

$N_r \cong 1190\text{rpm}$

713 $N_r = \frac{120f}{P}(1-s) = \frac{120(60)}{6}(1-0.05) = 1140 \text{ rpm}$

$T = \frac{9.55P_{out}}{N_r} = \frac{9.55(50)(746)}{1140}$

$T = 312.47 \text{ N-m}$

714 $N_r = \frac{120f}{P}(1-s) = \frac{120(60)}{4}(1-0.025) = 1755 \text{ rpm}$

$P_{in} = \frac{P_{out}}{\eta} = \frac{30(746)}{0.9} = 24866.67 \text{ W}$

$I = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{24866.67}{\sqrt{3}(230)(0.85)}$

$I = 73.436 \text{ A}$

715 $N_r = \frac{120f}{P}(1-s) = \frac{120(60)(1-0.02)}{4} = 1764 \text{ rpm}$

Note: At full load, the output power is 50 hp.

$HP = \frac{2\pi N_r T}{33{,}000}$

$T = \frac{HP(33{,}000)}{2\pi N_r} = \frac{(50)(33{,}000)}{(2\pi)(1764)}$

$T = 148.87 \text{ lb-ft}$

716 $P_{in} = P_{out} + P_{cu_s} + P_{co} + P_{cu_r} + P_{fw}$

$P_{co} = 0.03P_{outFL}$

$P_{fw} = 0.02P_{outFL}$

$P_{cuFL} = (0.04 + 0.02)P_{outFL} = 0.06P_{outFL}$

At 70% of rated load:

$P_{out} = 0.7P_{outFL}$

$P_{cu} = (\text{size})^2 P_{cuFL} = (0.7)^2(0.06P_{outFL}) = 0.0294P_{outFL}$

$\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{fw} + P_{cu}}$

$= \frac{0.7P_{outFL}}{0.7P_{outFL} + 0.03P_{outFL} + 0.02P_{outFL} + 0.0294P_{outFL}} \times 100\%$

$\eta = 89.81\%$

717 $P_{in} = \dfrac{P_{out}}{\eta} = \dfrac{100(746)}{0.92} = 81086.95\ W$

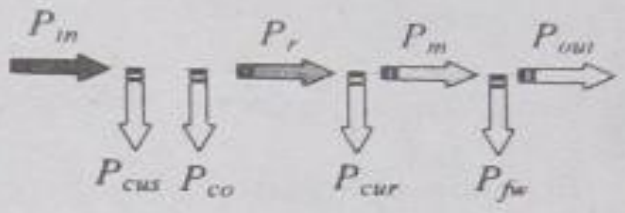
$I = \dfrac{P_{in}}{\sqrt{3}V_L pf} = \dfrac{81086.95}{\sqrt{3}(440)(0.85)}$

$I = 125.175\ A$

718 Note: Refer to the power flow diagram.

$s = 1 - \dfrac{PN_r}{120f} = 1 - \dfrac{6(1150)}{120(60)} = 0.0416$

$P_r = \dfrac{P_m}{1-s} = \dfrac{(10+0.5)(746)}{1-0.0416} = 8173\ W$



Power flow diagram

$P_{in} = P_r + P_{cu_s} + P_{co}$

$= 8173 + 400 + 350 = 8923\ W$

$I_L = \dfrac{P_{in}}{\sqrt{3}V_L pf} = \dfrac{8923}{\sqrt{3}(440)(0.8)}$

$I_L = 14.64\ A$

719 Note: Since not specified assume dc resistance of stator windings to be equal to the ac stator resistance.

$r_1 = \dfrac{R_t}{2} = \dfrac{0.54}{2} = 0.27\ \Omega$

At no - load :

$P_{co} + P_{fw} = P_{NL} - 3I_{NL}^2 r_1$

$P_{co} + P_{fw} = 610 - 3(6.5)^2(0.27) = 575.77\ W$

Note: R_t is the resistance of the stator windings measured between any two line terminals.

When loaded :

$P_{cu_s} = 3I_1^2 r_1 = 3(18.4)^2(0.27) = 274.23\ W$

$P_r = P_{in} - (P_{cu_s} + P_{co} + P_{fw})$

$= 6700 - (274.23 + 575.77) = 5850\ W$

Note: If there is no other way to solve, because of the given data, it is permissible to charge the friction and windage loss to the stator with no appreciable error, although this loss is actually a part of the rotor losses.

$$s = 1 - \frac{N_r P}{120f} = 1 - \frac{1164(6)}{120(60)} = 0.03$$

$$P_{cu_r} = sP_r = 0.03(5850) = 175.5\ W$$

$$P_{out} = P_{in} - (P_{cu_s} + P_{cu_r} + P_{co} + P_{fw})$$

$$= 6700 - (274.23 + 175.5 + 575.77) = 5674.5\ W$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{5674.5}{6700} \times 100\%$$

$$\eta = 84.69\%$$

720 Note: The output power of the motor is equal to the input power of the pump.

For the motor (at no - load) :

$$I_{NL} = \frac{P_{NL}}{\sqrt{3}V_{NL}pf} = \frac{800}{\sqrt{3}(440)(0.9)} = 1.166\ A$$

$$R_{e_1} = r_1 + r_2\left(\frac{N_1}{N_2}\right)^2 = 0.21 + 0.022\left(\frac{4}{1}\right)^2 = 0.562\ \Omega$$

$$P_{co} + P_{fw} = P_{NL} - 3I_{NL}^2 R_{e_1} = 800 - 3(1.166)^2(0.562) = 797.707\ W$$

For the motor (at rated - load) :

$$I_{FL} = \frac{P_{inFL}}{\sqrt{3}V_{FL}pf} = \frac{15000}{\sqrt{3}(440)(0.9)} = 21.869\ A$$

$$P_{cuFL} = 3I_{FL}^2 R_{e_1} = 3(21.869)^2(0.562) = 806.445\ W$$

$$P_{out} = P_{in} - (P_{cu} + P_{co} + P_{fw})$$

$$P_{out} = 15000 - (806.445 + 797.707) = 13395.848\ W$$

For the pump :

$$P_{out} = \frac{Qwh}{33000} = \frac{1000(62.4)(8)}{33000} = 15.127\ hp$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{15.127(746)}{13395.848} \times 100\%$$

$$\eta = 84.24\%$$

721 $$R_e = \frac{P_{LR}}{3I_{LR}^2} = \frac{450}{3(15)^2} = 0.667\ \Omega$$

$$P_{cuNL} = 3I_{NL}^2 R_e = 3(4.2)^2(0.667) = 35.29\ W$$

$$P_{cuFL} = 3I_{FL}^2 R_e = 3(10)^2(0.667) = 200.1\ W$$

$P_{stray} = P_{inNL} - P_{cuNL} = 300 - 35.29 = 264.71\,W$

$P_{outFL} = P_{inFL} - (P_{cuFL} + P_{stray})$

$P_{outFL} = 3500 - (200.1 + 264.71) = 3035.19\,W$

$$N_r = \frac{120f}{P}(1-s) = \frac{120(60)}{6}(1-0.02) = 1176\,rpm$$

$$T = \frac{9.55P_{out}}{N_r} = \frac{9.55(3035.19)}{1176}$$

$T \cong 24.65\,N\text{-}m$

722 $$r_{1dc} = \frac{R_L}{2} = \frac{0.16}{2} = 0.08\,\Omega$$

$r_{1ac} = 1.25(0.08) = 0.1\,\Omega$

At no - load :

$P_{co} + P_{fw} = P_{NL} - 3I_{NL}^2 r_1$

$P_{co} + P_{fw} = (725 - 425) - 3(10)^2(0.1) = 270\,W$

Note: R_L is the resistance of the stator windings measured between any two line terminals.

When loaded :

$P_{cu_s} = 3I_1^2 r_1 = 3(28)^2(0.1) = 235.2\,W$

$P_{in} = 3140 + 1580 = 4720\,W$

$P_r = P_{in} - (P_{cu_s} + P_{co} + P_{fw}) = 4720 - (235.2 + 270) = 4214.8\,W$

Note: If there is no other way to solve, because of the given data, it is permissible to charge the friction and windage loss to the stator with no appreciable error, although this loss is actually a part of the rotor losses.

$$s = 1 - \frac{N_r P}{120f} = 1 - \frac{870(8)}{120(60)} = 0.033$$

$P_{cu_r} = sP_r = 0.033(4214.8) = 139.088\,W$

$P_{out} = P_{in} - (P_{cu_s} + P_{cu_r} + P_{co} + P_{fw})$

$P_{out} = 4720 - (235.2 + 139.088 + 270) = 4075.712\,W$

$$T = \frac{9.55P_{out}}{N_r} = \frac{9.55(4075.712)}{870}$$

$T = 44.74\,N\text{-}m$

723 Since connected in series,

$R_{eq} = 20(0.1) = 2\,\Omega$

$R_{total} = \frac{V}{I} = \frac{220}{100} = 2.2\,\Omega$

$R_{joints} = R_{total} - R_{eq} = 2.2 - 2 = 0.2\,\Omega$

$P_{loss} = I^2 R_{joints} = (100)^2(0.2) = 2000\text{ W}$

$P_{loss} = 2\text{ kW}$

724 $P_{in} = \frac{P_{out}}{\eta} = \frac{7.5(746)}{0.88} = 6357.95\text{ W}$

$P_{in} = \sqrt{3}V_L I_L \cos\theta$

$\cos\theta = \frac{P_{in}}{\sqrt{3}V_L I_L} = \frac{6357.95}{\sqrt{3}(230)(18.4)}$

$\cos\theta = 0.8673$

$\theta = 29.85°$

Note: Since the load is balanced, the reading of the wattmeters shall be:

$W_1 = V_L I_L \cos(30° - \theta) = (230)(18.4)\cos(30° - 29.85°)$

$W_1 = 4.232\text{ kW}$

$W_2 = V_L I_L \cos(30° + \theta) = (230)(18.4)\cos(30° + 29.85°)$

$W_2 = 2.126\text{ kW}$

725 $P_{in} = W_1 + W_2 = 13400 + 7400$

$P_{in} = 20800$

$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{13400 - 7400}{20800}\right)$

$\theta = 26.54°$

$I_L = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{20800}{\sqrt{3}(230)(\cos 26.54°)}$

$I_L \cong 58.4\text{ A}$

726 $T_s = kV_s^2$

$\frac{T_{s1}}{T_{s2}} = \left(\frac{V_{s1}}{V_{s2}}\right)^2$

Substitute :

$$\frac{1.3T_{FL}}{0.5\,T_{FL}} = \left(\frac{220}{V_{s2}}\right)^2$$

$V_{s2} = 136.44$ volts

7 $T_s = kV_s^{\,2}$

$$\frac{T_{s1}}{T_{s2}} = \left(\frac{V_{s1}}{V_{s2}}\right)^2$$

Substitute:

$$\frac{1.6T_{FL}}{T_{s2}} = \left(\frac{550}{440}\right)^2$$

$T_{s2} = 1.024$ or 102.4% of full load value

8 $I_s = kV_s$

$$\frac{I_{s1}}{I_{s2}} = \frac{V_{s1}}{V_{s2}}$$

$$\frac{4.25\,I_{FL}}{I_{FL}} = \frac{550}{V_{s2}}$$

$V_{s2} = 129.41$ volts

9 Note: Since efficiency of the motor is not given and it's not possible to solve it using the data given, assume its value is 100%. This means that the input power drawn is equal to the output power delivered.

$$I_{FL} = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{25(746)}{\sqrt{3}(230)(0.85)} = 55\text{ A}$$

$I_s = 5.5(55) = 302.5$ A

Note: Starting current is directly proportional to the voltage (V_p) applied to the stator windings of the motor at the instant it was started.

Without starter connected, motor is delta - connected at start :

$V_{p1} = V_L$

With the starter connected, motor is wye - connected at start :

$$V_{p2} = \frac{V_L}{\sqrt{3}} = 0.577V_L$$

$I_s = kV_p$

$$\frac{I_{s1}}{I_{s2}} = \frac{V_{p1}}{V_{p2}}$$

$$\frac{302.5}{I_{s2}} = \frac{V_L}{0.577V_L}$$

$I_{s2} = 174.54$

$I_{s2} \cong 175\ A$

730 $T_s = kV_s^2$

$$\frac{T_{s1}}{T_{s2}} = \left(\frac{V_{s1}}{V_{s2}}\right)^2$$

Substitute :

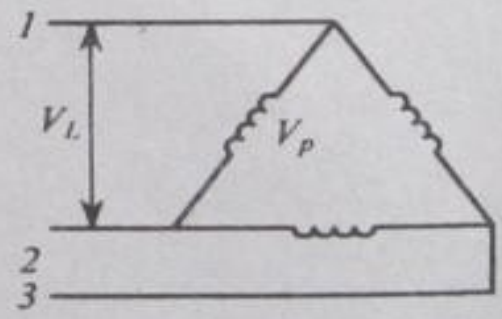
$$\frac{T_{FL}}{T_{s2}} = \left(\frac{550}{0.5(550)}\right)^2$$

$T_{s2} = 0.25\ T_{FL}$

T_{s2} is 25% of the starting torque

731 If delta - connected :

$V_P = V_L$

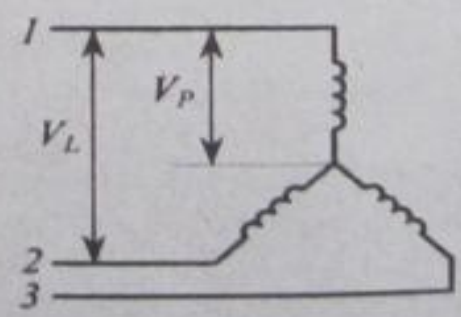


If wye - connected :

$$V_p = \frac{V_L}{\sqrt{3}} = 0.577\ V_L$$

$T = k\ V_P^2$

$$\frac{T_1}{T_2} = \left(\frac{V_{p1}}{V_{p2}}\right)^2$$



Substitute :

$$\frac{40}{T_2} = \left(\frac{V_L}{0.577V_L}\right)^2$$

$T_2 = 40(0.577)^2$

$T_2 = 13.3$ lb - ft

732 $P_{in} = \frac{P_{out}}{\eta} = \frac{30(746)}{0.8} = 27975\ W$

$I_{FL} = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{27975}{\sqrt{3}(220)(0.7)} = 104.88\ A$

$T_s = kV_s^2$

$\frac{T_{s1}}{T_{s2}} = \left(\frac{V_{s1}}{V_{s2}}\right)^2$

$\frac{1.3T_{FL}}{0.5\,T_{FL}} = \left(\frac{220}{V_{s2}}\right)^2$

$V_{s2} = 136.44$ volts

$I_s = kV_s$

$\frac{I_{s1}}{I_{s2}} = \frac{V_{s1}}{V_{s2}}$

$\frac{4.5(104.88)}{I_{s2}} = \frac{220}{136.44}$

$I_{s2} = 292.7\ A$

733 $T_{FL} = \frac{9.55P_{out}}{N_r} = \frac{9.55(7.5)(746)}{960} = 55.66\ N\text{-}m$

$T = kV^2$

$\frac{T_1}{T_2} = \left(\frac{V_1}{V_2}\right)^2$

Substitute :

$\frac{55.66}{T_2} = \left(\frac{500}{400}\right)^2$

$T_2 = 35.62\ N\text{-}m$

734 Note: Since not specified, assume the pump has an efficiency of 100%. The output of the motor is equivalent to the input of the pump.

$HP_{pump} = \frac{Qwh}{44760}$

$Q = \frac{HP_{pump}(44760)}{wh} = \frac{(50)(44760)}{(1000)(9.81)(50)}$

$Q = 4.563\ m^3$ per min.

735 T = (scale reading – dead weight) x length of arm

$T = (30 - 2)(2.5) = 70 \text{ lb-ft}$

$$N_r = \frac{120f}{P}(1-s) = \frac{120(60)}{6}(1-0.05) = 1140 \text{ rpm}$$

$$HP_{out} = \frac{2\pi N_r T}{33{,}000} = \frac{2\pi(1140)(70)}{33000} = 15.19 \text{ hp}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{15.19(0.746)}{4+8} \times 100\%$$

$$\eta = 94.43\%$$

736 Note: Refer to the power triangle, for the relations between real, apparent and reactive powers.

$\theta_{ind} = \cos^{-1} 0.715 = 44.356°$

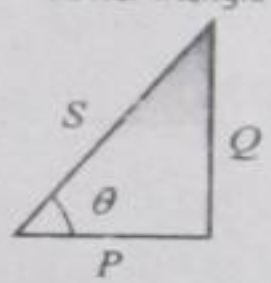
$S_{ind} = 36 - j36 \tan 44.356 = 36 - j35.2$

$S_{cap} = j25$

$\theta_m = \cos^{-1} 0.8 = 36.87°$

$S_m = P_m - jP_m \tan 36.87° = P_m - j0.75P_m$

Power triangle



$S_t = S_{cap} + S_{ind} + S_m$

$75\angle\alpha = j25 + (36 - j35.2) + (P_m - j0.75P_m)$

$75\angle\alpha = (P_m + 36) - j(10.2 + 0.75P_m)$

Using Pythagorean relations:

$(75)^2 = (P_m + 36)^2 + (10.2 + 0.75P_m)^2$

$5625 = P_m^2 + 72P_m + 1296 + 104.04 + 15.3P_m + 0.5625P_m^2$

$P_m^2 + 55.872P_m - 2703.974 = 0$

Using the quadratic formula;

$$P_m = \frac{-55.872 \pm \sqrt{(55.872)^2 - 4(1)(-2703.974)}}{2(1)} = \frac{-55.872 \pm 118.057}{2}$$

$$P_m^{(+)} = \frac{-55.872 + 118.057}{2} = 31.0925 \text{ kW}$$

$P_{out} = \eta P_m = (0.9)(31.0925)$

$$P_{out} = 27.98 \text{ kW} \times \frac{1 \text{ hp}}{0.746 \text{ kW}}$$

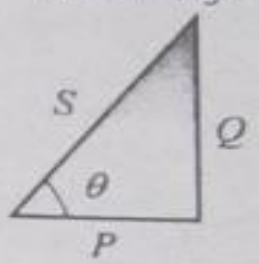
$P_{out} = 37.5 \text{ hp}$

737 Note: Refer to the power triangle, for the relations between real, apparent and reactive powers. If not specified, assume an induction motor has a lagging power factor.

$\theta_L = \cos^{-1} 0.75 = 41.409°$

$\theta_t = \cos^{-} 0.788 = 38°$

Power triangle



$S_t = S_L + S_{ind}$

$S_t\angle -38° = 400\angle -41.409° + (100 - j100\tan\theta_m)$

$P_t - jP_t\tan 38° = 300 - j264.57 + 100 - j100\tan\theta_m$

Equate reals :

$P_t = 300 + 100 = 400$ kW

Equate coefficients of j :

$P_t \tan 38° = 264.57 + 100\tan\theta_m$

$400\tan 38° = 264.57 + 100\tan\theta_m$

$\theta_m = 25.615°$

$pf_m = \cos 25.615°$

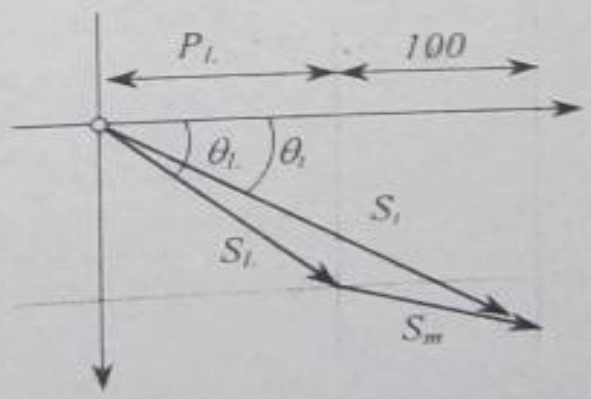
$pf_m \cong 0.902$ lagging

Another solution: (Refer to the vector diagram)

$P_L = S_L pf_L = 400(0.75) = 300$ kW

$P_t = 300 + 100 = 400$ kW

$S_t = \dfrac{P_t}{pf_t} = \dfrac{400}{0.788} = 507.614$ kVA



Using cosine law :

$S_m^2 = S_t^2 + S_L^2 - 2S_tS_m\cos(\theta_L - \theta_t)$

$= (507.614)^2 + (400)^2 - 2(507.614)(400)\cos(41.409° - 38°)$

$S_m = 110.902$ kVA

$pf_m = \dfrac{P_m}{S_m} = \dfrac{100}{110.902}$

$pf_m \cong 0.902$ lagging

Synchronous Motors

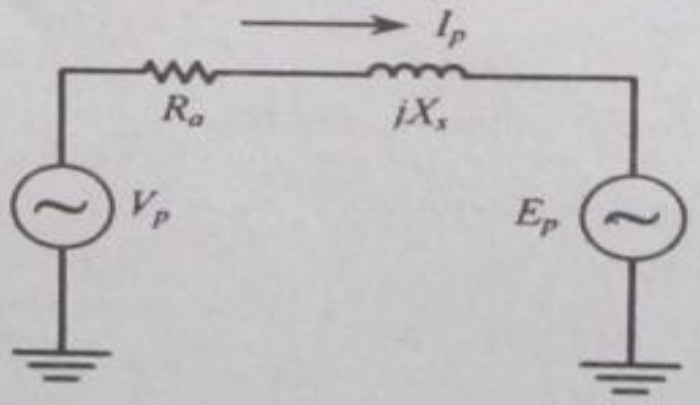
ROTOR SPEED

Note: Rotor speed of a synchronous motor is approximately equal to the synchronous speed.

$$N_r = N_s = \frac{120f}{P}$$

where: N_r = rotor speed (rpm)
f = frequency of the stator voltage (Hertz)
P = number of poles

EQUIVALENT CIRCUIT PER PHASE



$$E_p = \sqrt{(V_p \cos\theta + I_p R_a)^2 + (V_p \sin\theta \pm I_p X_s)^2}$$

where : E_p = back emf per phase (volt)
V_p = supply voltage per phase (volt)
I_p = motor current per phase (ampere)
R_a = armature resistance per phase (ohm)
X_s = synchronous reactance per phase (ohm)
+ (sign) = if motor is operating at a leading pf
- (sign) = if motor is operating at a lagging pf

- Using complex number format

$$E_p\angle \pm\delta = V_p\angle 0^\circ - (I_p\angle \pm\theta)(R_a + jX_s)$$

where : δ = angle between E_p and V_p (also called torque angle)
$+\theta$ = if motor is operating at a leading pf
$-\theta$ = if motor is operating at a lagging pf

POWER AND TORQUE DEVELOPED PER PHASE

$$P_d = \frac{E_p V_p}{X_s}\sin\delta \qquad T = \frac{9.55 P_d}{N_s}$$

where: T = torque developed per phase (newton-meter)
N_s = synchronous speed (rpm)
P_d = mechanical power developed per phase (watt)

Note: At maximum power developed condition, sin δ must be equal to 1 (δ = 90°)

$$P_{d_{max}} = \frac{E_p V_p}{X_s} \qquad T_{max} = \frac{9.55 P_{d_{max}}}{N_s}$$

POWER LOSSES AND EFFICIENCY

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} \qquad P_{cu(3\phi)} = 3I_p^2 R_a \qquad P_{rotor} = P_f$$

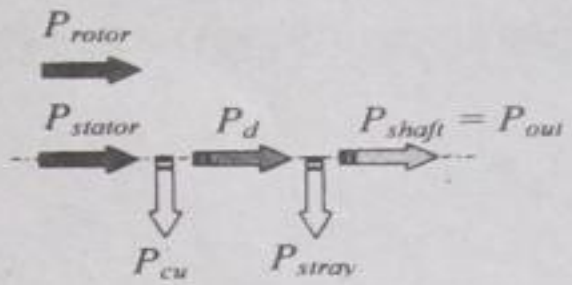
$$P_{losses} = P_{cu} + P_{stray} \qquad P_{stray} = P_{core} + P_{fw}$$

$$P_{in} = P_{stator} + P_{rotor} \qquad P_{stator} = 3V_p I_p \cos\theta$$

where : P_{cu} = stator copper losses
P_{core} = core losses
P_{fw} = friction and windage losses
P_f = field copper losses or exciter losses

P_{stray} = stray power or rotational losses
P_{shaft} = output power in the shaft
P_{stator} = input power of the stator
P_{rotor} = input power of the rotor (equal to field copper losses)
$\cos\theta$ = operating power factor
η = efficiency

POWER FLOW DIAGRAM



$$P_d = P_{stator} - P_{cu} \qquad P_{shaft} = P_{stator} - (P_{cu} + P_{stray})$$

where: P_d = power developed in the armature
P_{out} = power output

Note: The power input of the rotor (power losses in the exciter) will be taken into consideration only when the overall efficiency of the machine is to be determined, otherwise it is negligible.

Test 20

Problem 738:

A three-phase Y-connected synchronous motor with a line-to-line voltage of 440 V and a synchronous speed of 900 rpm operates with a power of 9 kW and a lagging power factor of 0.8. The synchronous reactance per phase is 10 ohms. Determine the torque angle in electrical degrees.

A. 36.33°
B. 33.51°
C. 35.51°
D. 38.46°

Problem 739:

A 10-hp, 230-V, three-phase Y-connected synchronous motor has a reactance of 3 ohms per phase and a negligible resistance. To what voltage must the motor is excited in order to deliver full load at unity power factor and 90% efficiency?

A. 262 V
B. 254 V
C. 248 V
D. 242 V

Problem 740:

A 1000 kVA, 11 kV, 3-phase Y-connected synchronous motor has a resistance of 3 ohms and a reactance of 25 ohms per phase. Determine the induced emf when fully loaded at 0.8 power factor lagging.

A. 5521 V
B. 5362 V
C. 5475 V
D. 5582 V

Problem 741:

A 500-hp, 2200 V, 3-phase star-connected synchronous motor has a synchronous impedance of 0.3 + j 3 Ω per phase. Determine the induced emf per phase if the motor works on full load with an efficiency of 94% and a power factor of 0.8 leading.

A. 1360 V
B. 1354 V
C. 1402 V
D. 1522 V

Problem 742:

A three-phase, star connected synchronous motor takes 50 kW at 660 V, the power factor being 0.8 lagging. This machine has a synchronous reactance of 1 ohm per phase with a negligible resistance. If the emf is increased by 30%, the power taken remaining the same, find the new leading pf current.

A. 94.1 A
B. 85.2 A
C. 90.3 A
D. 88.6 A

Problem 743:

A 6.6 kV, Y-connected synchronous motor has a synchronous reactance per phase of 10 ohms and a negligible resistance. For a certain load, the motor draws 990 kW at normal voltage. At this load, the induced emf per phase is 5160 V. Determine operating pf.

A. 0.478
B. 0.502
C. 0.462
D. 0.518

Problem 744:

A 75-hp, 600-volt, 1000-rpm, three-phase Y-connected synchronous motor has an armature resistance of 0.05 ohm per phase and a leakage reactance of 0.45 ohm per phase. The efficiency at rated load, 80% pf leading is 92%. Determine mechanical power developed within the armature.

A. 64 kW
B. 62 kW
C. 60 kW
D. 67 kW

Problem 745:

A 230 V, 60-Hz, 3-phase, Y-connected synchronous motor draws a current of 20 A and delivers 8 bhp. Armature resistance per phase is 0.5 Ω If the iron and friction losses amount to 300 W, determine the operating power factor of the motor.

A. 0.809
B. 0.807
C. 0.866
D. 0.862

Problem 746:

A 2200 V, 200 hp, 3-phase mesh connected synchronous motor is working on full load at an efficiency 0.88 and 0.8 pf leading. The armature reactance per phase is 5 ohms. Determine the induced emf per phase. Neglect resistance.

A. 2377 V
B. 2345 V
C. 2380 V
D. 2307 V

Problem 747:

A 15-hp, 230 V single-phase synchronous motor is taking a current of 50 A at a power factor of 0.85 leading. Effective armature resistance is 0.2 ohm. If this motor drives a mechanical load of 12 hp, determine its rotational losses.

A. 323 W
B. 340 W
C. 376 W
D. 314 W

Problem 748:

A 440-V, single-phase synchronous motor is drawing a line current of 50 A at 0.9 pf lagging. This machine has an armature resistance and reactance of 0.8 and 10 ohms respectively. If stray power and excitation losses amount to 500 W and 800 W respectively, determine bhp output of the motor.

A. 23.2
B. 20.4
C. 25.8
D. 18.5

Problem 749: EE Board April 1993

A single-phase synchronous motor gives an output of 9.46 kW at 0.9 pf lagging with 600 V. if stray power loss is 600 W and field losses is 900 W, while effective armature resistance is 0.6 ohm, find the efficiency of the synchronous motor.

A. 82.35%
B. 81.24%
C. 85.25%
D. 84.64%

Problem 750:

A 230-V, 60-Hz, single-phase synchronous motor is taking a current of 20 A at 0.866 lagging pf. Effective armature resistance is 0.3 ohm. Iron and friction losses amount to 450 W, Determine the efficiency of the motor at this load. Neglect losses in the excitation.

A. 86.15%
B. 85.30%
C. 88.52%
D. 83.20%

Problem 751:

A three-phase star-connected synchronous motor draws 70 A at 0.8 pf leading when connected from a 6.6 kV, 60-Hz, 3-phase source. At this load the machine operates with an efficiency of 90%. If the synchronous impedance of the machine is 2 + j15 ohms. Determine stray power losses.

A. 40.5 kW
B. 28.6 kW
C. 32.8 kW
D. 34.6 kW

Problem 752:

A 20 hp, 440 V, 60-Hz, 3-phase star-connected synchronous motor has an armature resistance per phase of 0.4 ohm. It is giving its full load output at a power factor of 0.9 leading. If the stray power losses amount to 500 watts, find the armature current.

A. 23.44 A
B. 25.28 A
C. 22.53 A
D. 20.67 A

Problem 753:

A 500-V, 60-Hz, single-phase synchronous motor develops 20 hp and is operating at a pf of 0.8 lagging. Effective resistance of armature is 1 ohm. Iron and friction losses amount to 500 W. Determine power drawn by the motor from the source.

A. 18,220 W
B. 21,546 W
C. 17,288 W
D. 20,065 W

Problem 754: EE Board April 1991

A three-phase synchronous motor is measured by two A and B wattmeters. It reads 28.6 kW and 46.5 kW, respectively. The input power of the motor is under excited and assumed constant. What are the readings of A and B wattmeters if the power factor is 85% and the motor is over-excited?

A. 48.6 kW, 26.5 kW
B. 42.5 kW, 32.6 kW
C. 52.6 kW, 22.5 kW
D. 51.0 kW, 24.1 kW

Problem 755:

The input power to a 600-V synchronous motor is measured by means of a two-wattmeter method. One wattmeter reads 64 kW and the other reads 36.5 kW. The motor is known to be operating at leading pf. Determine the line current drawn.

A. 104 A
B. 110 A
C. 107 A
D. 112 A

Problem 756:

A group of induction motors rated 2200-V, 3-phase, 60 Hz delivers a total of 600 hp and is operating at an average power factor of 0.8 lagging. If half of this total load is replaced by a group of synchronous motors operating at an average pf of 0.8 leading, determine the percentage reduction in the line current? Assume all motors operate at an average efficiency of 90%.

A. 23%
B. 25%
C. 20%
D. 21%

Problem 757:

A three-phase transmission line having a capacity of 10,000 kVA serves a balanced load that draws 5,000 kW at a lagging power factor such that the transmission line is loaded to its capacity. An additional load of induction motors operating at an average pf of 0.80 is connected to the line. In order to make use of the line to its full capacity in active power, a synchronous condenser having losses of 300 kW will be connected to the line. What is the kVA rating of the synchronous motor required?

A. 11,158
B. 12,189
C. 13,205
D. 10,202

Problem 758:

A three-phase Y-connected synchronous motor with a line to line voltage of 440 V and a synchronous speed of 900 rpm operates with a power of 9 kW and a lagging power factor of 0.8. The synchronous reactance per phase is 10 ohms. The machine is operating with a rotor current of 5 A. It is desired to continue carrying the same load but to provide 5 kVAR of power factor correction to the line. Determine the required rotor current to do this.

A. 7.88 A
B. 8.06 A
C. 8.62 A
D. 8.28 A

Problem 759: EE Board October 1991

A 3,600 volts three-phase star connected turbo synchronous generator of synchronous reactance of 0.5 ohm per phase and negligible resistance is supplying 30,000 kVA at 0.8 pf lagging to a large power system. If the steam supply is cut-off, calculate the amount of current that the generator will then carry assuming negligible losses.

A. 3901 A
B. 3823 A
C. 3094 A
D. 3870 A

Problem 760: EE Board October 1994

A wye-connected turbo alternator having a synchronous reactance of 0.5 ohm and negligible resistance is connected to a large power system having a busbar voltage of 13.8 kV supplying a load of 15,000 kVA at 0.80 lagging power factor. If the steam supply is cut-off, the armature current will the machine carry assuming negligible losses?

A. 256.55 A
B. 525.62 A
C. 384.26 A
D. 627.55 A

Problem 761: EE Board October 1994

A 1,000 kVA, 6,600 V wye-connected three-phase alternator having a reactance of 8.8 Ω and a negligible resistance is supplying power to a constant frequency bus bar. The open circuit emf at this instance is 4,311 V per phase. If steam supply is suddenly cut-off, the armature current in amperes is nearest to

A. 57
B. 60
C. 59
D. 56

ANSWER KEY

738. C	743. A	748. A	753. C	758. C
739. B	744. C	749. D	754. D	759. D
740. A	745. D	750. B	755. C	760. C
741. A	746. A	751. D	756. C	761. A
742. B	747. A	752. A	757. B	

☑ RATING

21-24 - Topnotcher
16-20 - Passer
12-15 - Conditional
0-11 - Failed

Solutions to Test 20

738 $I_L = \dfrac{P}{\sqrt{3}V_L pf} = \dfrac{9000}{\sqrt{3}(440)(0.8)} = 14.76\text{ A}$

$\theta = \cos^{-1}0.8 = 36.87°$

$E_p = V_p - I_p X_s$

$= \dfrac{440}{\sqrt{3}}\angle 0° - (14.76\angle -36.87°)(10\angle 90°)$

$= 254.03 - (147.6\angle 53.13°) = 254.03 - (88.56 + j118.08)$

$E_p = 165.47 - j118.08 = 203.281\angle -35.51°$

By inspection: $\delta = 35.51°$

739 $P_{in} = \dfrac{P_{out}}{\eta} = \dfrac{10(746)}{0.9} = 8288.88\text{ W}$

$I_L = \dfrac{P_{in}}{\sqrt{3}V_L pf} = \dfrac{8288.88}{\sqrt{3}(230)(1)} = 20.807\text{ A}$

$\theta = \cos^{-1}1 = 0°$

$E_p = V_p - I_p X_s$

$= \dfrac{230}{\sqrt{3}}\angle 0° - (20.807\angle -0°)(3\angle 90°)$

$= 132.79 - (62.421\angle 90°)$

$E_p = 132.79 - j62.421 = 146.73\angle 25.177°$

$E_L = \sqrt{3}E_p = \sqrt{3}(146.73)$

$E_L = 254.14\text{ V}$

740 $I_L = \dfrac{S}{\sqrt{3}V_L} = \dfrac{1\times10^6}{\sqrt{3}(11000)} = 52.486\text{ A}$

$\theta = \cos^{-1}0.8 = 36.87°$

$Z_s = 3 + j25 = 25.179\angle 83.157°$

$E_p = V_p - I_p Z_s$

$$E_p = V_p - I_p Z_s$$
$$= \frac{11000}{\sqrt{3}}\angle 0° - (52.486\angle -36.87°)(25.179\angle 83.157°)$$
$$= 6350.853 - (1321.54\angle 46.287°)$$
$$= 6350.853 - (913.245 + j955.22) = 5437.608 - j955.22$$
$$E_p = 5520.87\angle 9.963° \text{ V}$$

741 $$P_{in} = \frac{P_{out}}{\eta} = \frac{200(746)}{0.92} = 162173.9 \text{ W}$$
$$I_L = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{162173.9}{\sqrt{3}(2200)(0.8)} = 53.2 \text{ A}$$
$$\theta = \cos^{-1}0.8 = 36.87°$$
$$Z_s = 0.3 + j3 = 3.015\angle 84.289°$$

$$E_p = V_p - I_p Z_s$$
$$= \frac{2200}{\sqrt{3}}\angle 0° - (53.2\angle 36.87°)(3.015\angle 84.289°)$$
$$= 1270.17 - (160.398\angle 121.159°) = 1270.17 - (-83 + j137.258)$$
$$= 1353.17 - j137.258$$
$$E_p = 1360.113\angle -5.79° \text{ V}$$

742 $$I_L = \frac{P}{\sqrt{3}V_L pf} = \frac{50000}{\sqrt{3}(660)(0.8)} = 54.67 \text{ A}$$
$$\theta = \cos^{-1}0.8 = 36.87°$$

$$E_{p_1} = V_r - I_{p_1} X_s$$
$$= \frac{660}{\sqrt{3}}\angle 0° - (54.67\angle -36.87°)(1\angle 90°)$$
$$= 381.05 - (54.67\angle 53.13°) = 381.05 - (32.8 + j43.736)$$
$$E_{p_1} = 348.25 - j43.736 = 350.98\angle -7.158° \text{ V}$$

$$P_1 = P_2$$
$$3V_p I_{p2} \cos\theta_1 = 3V_p I_{p2} \cos\theta_2$$
$$I_{p2} \cos\theta_2 = 54.67 \cos 36.87° = 43.736 \Rightarrow ❶$$

$$E_{p2} = 1.3 E_{p1} = 1.3(350.98) = 456.27 \text{ V}$$

$E_{p2} = V_p - I_{p2}X_s$

$456.27\angle\delta = 381.05\angle 0° - (I_{p2}\angle\theta_2°)(1\angle 90°)$

$= 381.05 - (I_{p2}\cos\theta_2 + jI_{p2}\sin\theta_2)(j1)$

$= 381.05 - jI_{p2}\cos\theta_2 - j^2 I_{p2}\sin\theta_2$

$= 381.05 - j43.736 + I_{p2}\sin\theta_2$

$456.27\angle\delta = (381.05 + I_{p2}\sin\theta_2) - j43.736$

By Pythagorean relations :

$(456.27)^2 = (381.05 + I_{p2}\sin\theta_2)^2 + (43.736)^2$

$I_{p2}\sin\theta_2 = 73.119 \Rightarrow$ ❷

Divide Eq.2 by Eq.1:

$$\frac{I_{p2}\sin\theta_2}{I_{p2}\cos\theta_2} = \frac{73.119}{43.736} = 1.6718$$

$\tan\theta_2 = 1.6718$

$\theta_2 = 59.114°$

$$I_{p2} = \frac{73.119}{\sin 59.114°}$$

$I_{p2} = 85.2$ A

743 $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{6600}{\sqrt{3}} = 3810.5$ V

Note: Since the induced emf per phase is greater than the terminal voltage per phase, the motor is operating at a leading pf.

$E_p = V_p - I_pX_s$

$5160\angle\delta = 3810.5\angle 0° - (I_p\angle\theta°)(10\angle 90°)$

$= 3810.5 - (I_p\cos\theta + jI_p\sin\theta)(j10)$

$= 3810.5 - j10I_p\cos\theta - j^2 10I_p\sin\theta$

$5160\angle\delta = (3810.5 + 10I_p\sin\theta) - j(10I_p\cos\theta)$

By Pythagorean relations:

$(5160)^2 = (3810.5 + 10I_p\sin\theta)^2 + (10I_p\cos\theta)^2$

$(5160)^2 = (3810.5)^2 + 2(3810.5)(10I_p\sin\theta) + 100(\sin^2\theta + \cos^2\theta)$

$I_p\sin\theta = 158.845 \Rightarrow$ ❶

$$P_{in} = \sqrt{3}V_L I_p \cos\theta_2$$

$$I_p \cos\theta = \frac{P_{in}}{\sqrt{3}V_L} = \frac{990{,}000}{\sqrt{3}(6600)} = 86.602 \Rightarrow ❷$$

Divide Eq.1 by Eq.2:

$$\frac{I_p \sin\theta}{I_p \cos\theta} = \frac{158.845}{86.602} = 1.8342$$

$$\tan\theta = 1.8342$$

$$\theta = 61.4°$$

$$pf = \cos 61.4° = 0.478 \text{ leading}$$

744 $$P_{in} = \frac{P_{out}}{\eta} = \frac{75(746)}{0.92} = 60815.217 \text{ W}$$

$$I_L = \frac{P_{in}}{\sqrt{3}V_L pf} = \frac{60815.217}{\sqrt{3}(600)(0.8)} = 73.15 \text{ A}$$

$$\theta = \cos^{-1}0.8 = 36.87°$$

$$Z_s = 0.05 + j0.45 = 0.4527\angle 83.66°$$

$$E_p = V_p - I_p Z_s$$

$$= \frac{600}{\sqrt{3}}\angle 0° - (73.15\angle 36.87°)(0.4527\angle 83.66°)$$

$$= 346.41 - (33.115\angle 120.53°) = 346.41 - (-16..822 + j28.524)$$

$$E_p = 363.232 - j28.524 = 364.35\angle -4.49° \text{ V}$$

$$P_d = 3E_p I_p \cos\alpha = 3(364.35)(73.15)\cos(36.87° + 4.49°)$$

$$P_d = 60.01 \text{kW}$$

Another solution:

$$P_d = P_{stator} - 3I_p^2 R_a = 60815.217 - 3(73.15)^2(0.05)$$

$$P_d = 60.01 \text{kW}$$

745 Note: Since motor is wye-connected, phase and line currents are equal.

$$P_m = P_{stator} - (P_{cu} + P_{stray})$$

$$P_m = \sqrt{3}V_L I_L \cos\theta - (3I_p^2 R_a + P_{stray})$$

$$8(746) = \sqrt{3}(230)(20)\cos\theta - 3(20)^2(0.5) - 300$$

$\cos\theta = 0.862$

$pf = \cos\theta = 0.862$

746 $P_{in} = \dfrac{P_{out}}{\eta} = \dfrac{200(746)}{0.88} = 169545.45\ W$

$I_L = \dfrac{P_{in}}{\sqrt{3}V_L pf} = \dfrac{169545.45}{\sqrt{3}(2200)(0.8)} = 55.617\ A$

$\theta = \cos^{-1}0.8 = 36.87°$

Note: Since mesh connected, line voltage is equal to the phase voltage

$E_p = V_p - I_p Z_s$

$= 2200\angle 0° - (55.617\angle 36.87°)(5\angle 90°)$

$= 2200 - (278.085\angle 126.87°) = 2200 - (-166.85 + j222.467)$

$= 2366.85 - j222.467$

$E_p = 2377.28\angle -5.37°\ V$

747 $P_m = P_{stator} - (P_{cu} + P_{stray})$

$P_m = V_p I_p \cos\theta - (I_p^2 R_a + P_{stray})$

$12(746) = (230)(50)(0.85) - (50)^2(0.2) - P_{stray}$

$P_{stray} = 323\ W$

748 $P_m = P_{stator} - (P_{cu} + P_{stray})$

$P_m = V_p I_p \cos\theta - (I_p^2 R_a + P_{stray})$

$= (440)(50)(0.9) - (50)^2(0.8) - 500$

$P_m = 17300\ W \times \dfrac{1\ hp}{746\ W}$

$P_m = 23.19\ hp$

Note: The power lost in the excitation is not charge to the machine. It will be included in the computation, only if the overall efficiency of the machine is to be determined.

749 $P_m = P_{stator} - (P_{cu} + P_{stray})$

$P_m = V_p I_p \cos\theta - I_p^2 R_a - P_{stray}$

$9460 = (600)(I_p)(0.9) - I_p^2(0.6) - 600$

$0.6 I_p - 540 I_p + 10060 = 0$

$$I_p^2 - 900 I_p + 16766.667 = 0$$

Using the quadratic formula;

$$I_p = \frac{900 \pm \sqrt{(900)^2 - 4(1)(16766.667)}}{2(1)} = \frac{900 \pm 861.936}{2}$$

$$I_p^{(-)} = 19.032 \text{ A}$$

$$P_{stator} = V_p I_p \cos\theta = 600(19.032)(0.9) = 10.277 \text{ kW}$$

$$P_{in} = P_{stator} + P_f = 10.277 + 0.9 = 11.177 \text{ kW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{9.46}{11.177} \times 100\%$$

$$\eta = 84.64\%$$

Note: There are two input powers needed to run a synchronous motor, namely the AC power used to excite the stator windings and the DC power used to excite the field windings of the rotor circuit.

750 $P_{stator} = V_p I_p \cos\theta = (220)(25)(0.866) = 4763 \text{ W}$

$$P_{cu} = I_p^2 R_a = (25)^2(0.4) = 250 \text{ W}$$

$$\eta = \frac{P_m}{P_{stator}} = \frac{P_{stator} - (P_{cu} + P_{stray})}{P_{stator}}$$

$$\eta = \frac{4763 - (250 + 450)}{4763} \times 100\%$$

$$\eta = 85.3\%$$

751 $P_{stator} = \sqrt{3} V_L I_p \cos\theta = \sqrt{3}(6600)(70)(0.8)$

$$P_{stator} = 640.166 \text{ kW}$$

$$P_{cu} = 3 I_p^2 R_a = 3(70)^2(2) = 29.4 \text{ kW}$$

$$\eta = \frac{P_m}{P_{stator}}$$

$$P_m = \eta(P_{stator}) = 0.9(640.166) = 576.15 \text{ kW}$$

$$P_m = P_{stator} - (P_{cu} + P_{stray})$$

$$576.15 = 640.166 - 29.4 - P_{stray}$$

$$P_{stray} = 34.616 \text{ kW}$$

752 $P_{stator} = P_m + P_{cu} + P_{stray}$

$\sqrt{3}V_L I_p \cos\theta = P_m + 3I_p^2 R_a + P_{stray}$

$\sqrt{3}(440)(I_p)(0.9) = 20(746) + 3(I_p)^2(0.4) + 500$

$685.89\, I_p = 1.2\, I_p^2 + 15420$

$I_p^2 - 571.575\, I_p + 12850 = 0$

Using the quadratic formula :

$$I_p = \frac{571.575 \pm \sqrt{(571.575)^2 - 4(1)(12850)}}{2(1)} = \frac{571.575 \pm 524.688}{2}$$

$I_p^{(-)} = 23.44$ A

753 $P_{stator} = P_m + P_{cu} + P_{stray}$

$V_p I_p pf = P_m + I_p^2 R_a + P_{stray}$

$500(I_p)(0.8) = 20(746) + I_p^2(1) + 500$

$400\, I_p = 15420 + I_p^2$

$I_p^2 - 400 I_p + 15420 = 0$

Using the quadratic formula :

$$I_p = \frac{400 \pm \sqrt{400^2 - 4(1)(15420)}}{2(1)} = \frac{400 \pm 313.56}{2}$$

$I_p^{(-)} = 43.22$ A

$P_{stator} = V_p I_p pf = 500(43.22)(0.8)$

$P_{stator} = 17288$ W

754 $\theta = \cos^{-1} 0.85 = 31.788°$

$P_T = 28.6 + 46.5 = 75.1$ kW

$W_2 = 75.1 - W_1 \Rightarrow$ ❶

$$\tan\theta = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}(W_1 - W_2)}{75.1}$$

$$W_1 - W_2 = \frac{\tan 31.788°(75.1)}{\sqrt{3}}$$

$W_1 - W_2 = 26.87 \Rightarrow$ ❷

Substitute Eq.1 to Eq.2:

$W_1 - (75.1 - W_1) = 26.87$

$W_1 = 50.985 \cong 51\text{kW}$

$W_2 = 75.1 - 51 = 24.1\text{kW}$

755 $P_t = W_1 + W_2 = 64 + 36.5 = 100.5\text{ kW}$

$$\theta = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan^{-1}\sqrt{3}\left(\frac{64 - 36.5}{100.5}\right) = 25.358°$$

$$I_L = \frac{P_t}{\sqrt{3}V_L\cos\theta} = \frac{100500}{\sqrt{3}(600)(\cos 25.358°)}$$

$I_L = 107.01\text{A}$

756 Before the replacement:

$$P_t = \frac{P_{out}}{\eta} = \frac{600(746)}{0.92} = 486521.74\text{ W}$$

$$I_L = \frac{P_t}{\sqrt{3}(V_L)pf} = \frac{486521.74}{\sqrt{3}(2200)(0.8)} = 159.6\text{ A}$$

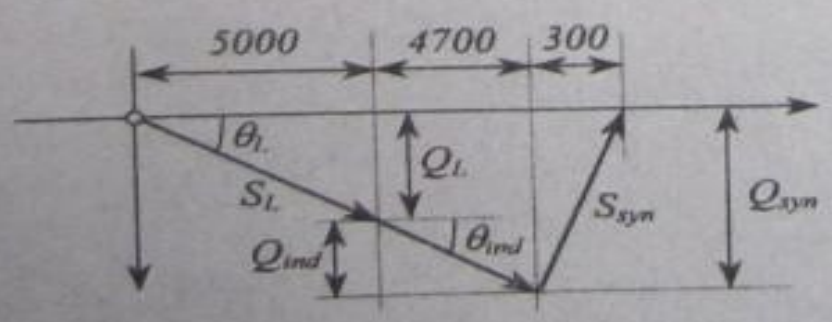
Note: After the replacement, the loads are: a group of induction motors delivering 300 hp operating at a pf of 0.8 lagging and a group of synchronous motors delivering also 300 hp while operating at a pf of 0.8 leading. Combining the two groups will result to a load of 600 hp operating at unity pf.

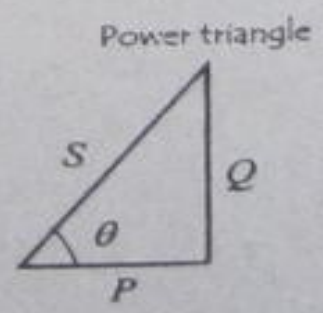
After the replacement:

$$I_L = \frac{P_t}{\sqrt{3}(V_L)pf} = \frac{486521.74}{\sqrt{3}(2200)(1.0)} = 127.68\text{ A}$$

$$\%\text{ Reduction} = \frac{159.6 - 127.68}{159.6} \times 100\% = 20\%$$

757 Note: Since the transmission line is loaded to its full capacity of 10 kW, its operating power factor therefore must be equal to unity (pf =1).





$$Q_L = \sqrt{S_L^2 - P_L^2} = \sqrt{(10{,}000)^2 - (5000)^2}$$
$$= 8660.25 \text{ kVAR}$$
$$\theta_{ind} = \cos^{-1} 0.8 = 36.869°$$
$$Q_{ind} = P_{ind} \tan\theta_{ind} = 4700 \tan 36.869°$$
$$= 3524.88 \text{ kVAR}$$
$$Q_{syn} = Q_L + Q_{ind} = 8660.25 + 3524.88$$
$$= 12{,}185.13 \text{ kVAR}$$
$$S_{syn} = \sqrt{P_{syn}^2 + Q_{syn}^2} = \sqrt{(300)^2 + (12{,}185.13)}$$
$$S_{syn} = 12{,}189 \text{ kVA}$$

758 $$I_L = \frac{P}{\sqrt{3}V_L pf} = \frac{9000}{\sqrt{3}(440)(0.8)} = 14.76 \text{ A}$$
$$\theta = \cos^{-1} 0.8 = 36.87°$$

$$E_{p_1} = V_p - I_{p1}X_s$$
$$= \frac{440}{\sqrt{3}}\angle 0° - (14.76\angle -36.87°)(10\angle 90°)$$
$$= 254.03 - (147.6\angle 53.13°) = 254.03 - (88.56 + j118.08)$$
$$E_{p_1} = 165.47 - j118.08 = 203.281\angle -35.51°$$

Note: When used as to correct the power factor, the synchronous motor should operate with a leading power factor.

$$\theta = \tan^{-1}\frac{5 \text{ kVAR}}{9 \text{ kW}} = 29.05°$$

$$E_{p_2} = V_p - I_{p2}X_s$$
$$= 254.03\angle 0° - (14.76\angle 29.05°)(10\angle 90°)$$
$$= 254.03 - (147.6\angle 119.05°) = 254.03 - (-71.67 + j129.03)$$
$$E_{p_2} = 325.7 - j129.03 = 350.33\angle -21.61°$$

Note: Flux is directly proportional to the rotor current. Induced emf is directly proportional to flux, thus induced emf is directly proportional to the rotor current.

$$E_p = kI_r$$
where : k = proprtionality constant

$$\frac{E_{p1}}{E_{p2}} = \frac{I_{r1}}{I_{r2}}$$

$$\frac{203.28}{350.33} = \frac{5}{I_{r2}}$$

$$I_{r2} = 8.62\ A$$

759 $$I = \frac{S}{\sqrt{3}V_L} = \frac{30 \times 10^6}{\sqrt{3}(3600)} = 4811.25\ A$$

$$\theta = \cos^{-1} 0.8 = 36.87°$$

As a synchronous generator:

$$E_{p1} = V_p + I_1 Z_p$$

$$= \frac{3600}{\sqrt{3}}\angle 0° + (4811.25\angle -36.87°)(0.5\angle 90°)$$

$$= 2078.46 + 2405.625\angle 53.13°$$

$$= 2078.46 + 1443.378 + j1924.497 = 3521.838 + j1924.497$$

$$E_{p1} = 4013.356\angle 28.654°\ V$$

Note: When steam supply is cut-off, the generator will continue to run as an unloaded synchronous motor operating at a leading pf with the magnitude of the emf unchanged at this instant.

As a synchronous motor:

$$E_{p2} = V_p - I_2 Z_p$$

$$4013.356\angle\delta = 2078.46\angle 0° - (I_2\angle 90°)(0.5\angle 90°)$$

$$= 2078.46 - 0.5 I_2\angle 180°$$

$$4013.356\angle\delta = 2078.46 + 0.5 I_2$$

Note: Since the right side of the equation does not contain imaginary component, then δ is equal to zero.

$$I_2 = \frac{4013.356 - 2078.46}{0.5}$$

$$I_2 = 3869.79\ A$$

760 Note: Refer to problem # 759 for a detailed explanation.

$$I = \frac{S}{\sqrt{3}V_L} = \frac{15 \times 10^6}{\sqrt{3}(13800)} = 627.55\ A$$

$$\theta = \cos^{-1} 0.8 = 36.87°$$

As a synchronous generator:

$$E_{p1} = V_p + I_1 Z_p$$

$$= \frac{13800}{\sqrt{3}}\angle 0° + (627.55\angle -36.87°)(0.5\angle 90°)$$

$$= 7967.43 + 313.775\angle 53.13°$$

$$= 7967.43 + 188.265 + j251.02$$

$$= 8155.695 + j251.02$$

$$E_{p1} = 8159.557\angle 1.763° \text{ V}$$

As a synchronous motor:

$$E_{p2} = V_p - I_1 Z_p$$

$$8159.557\angle\delta = 7967.43\angle 0° - (I_2\angle 90°)(0.5\angle 90°)$$

$$= 7967.43 - 0.5 I_2\angle 180°$$

$$8159.557\angle\delta = 7967.43 + 0.5 I_2$$

$$I_2 = \frac{8159.557 - 7967.43}{0.5}$$

$$I_2 = 384.254 \text{ A}$$

761 Note: Refer to problem # 759 for a detailed explanation.

As a synchronous generator:

$$E_p = 4311 \text{ V}$$

$$V_p = \frac{6600}{\sqrt{3}} = 3810.5 \text{ V}$$

As a synchronous motor:

$$E_p = V_p - I_1 Z_p$$

$$4311\angle\delta = 3810.5\angle 0° - (I_2\angle 90°)(8.8\angle 90°)$$

$$= 3810.5 - 8.8 I_2\angle 180°$$

$$4311\angle\delta = 3810.5 + 8.8 I_2$$

$$I_2 = \frac{4311 - 3810.5}{8.8} = 56.87$$

$$I_2 \cong 57 \text{ A}$$

Converters & Rectifiers

❖ **Synchronous converter** – a machine that receives alternating current through a set of slip rings at one end of the armature that rotates at synchronous speed and delivers direct current from the opposite end through a commutator and a set of brushes.

RELATIONSHIP BETWEEN AC AND DC VOLTAGES

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}} \sin\left(\frac{180°}{n}\right)$$

where: V_{dc} = dc voltage
V_{ac} = rms voltage between any two slip rings (line to line voltage)
n = number of slip rings
= 2, for a single-phase converter
= 3, for a three-phase converter
= 4, for a four-phase converter
....etc

EFFICIENCY OF CONVERTERS

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{dc}}{P_{ac}} \qquad P_{ac} = nV_pI_p\cos\theta \qquad P_{dc} = V_{dc}I_{dc}$$

❑ **If single-phase AC input**

$$V_{ac} = V_p$$

❑ **If polyphase AC input**

$$V_{ac} = 2V_p \sin\left(\frac{180°}{n}\right)$$

where: V_p = voltage per phase of the AC supply side.
V_{dc} = dc voltage
V_{ac} = rms voltage between any two slip rings
= line to line voltage of the AC supply side
P_{dc} = dc power delivered
P_{ac} = ac power drawn
n = number of phases
η = efficiency

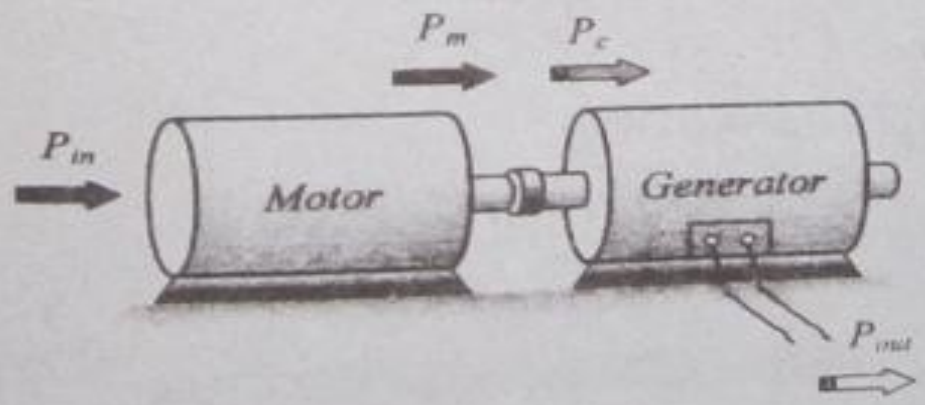
SYNCHRONOUS-SYNCHRONOUS FREQUENCY CONVERTERS

Two synchronous machines are coupled together so that both rotors, having the proper number of poles will operate at the same speed.

$$\frac{f_1}{f_2} = \frac{P_1}{P_2}$$

where: f_1, f_2 = frequency of the first and second synchronous machines
P_1, P_2 = number of poles of the first and second synchronous machines

MOTOR-GENERATOR SETS



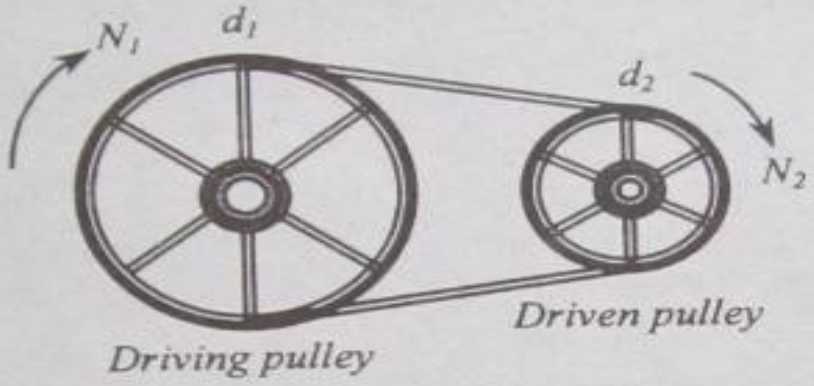
$$\eta_m = \frac{P_m}{P_{in}} \qquad \eta_c = \frac{P_c}{P_m} \qquad \eta_g = \frac{P_{out}}{P_c}$$

$$\eta_{overall} = \frac{P_{out}}{P_{in}} \qquad \eta_{overall} = \eta_m \eta_c \eta_g$$

here: η_m = motor efficiency
η_c = coupling efficiency
η_g = generator efficiency
P_{in} = electrical input of the motor (watt)
P_m = mechanical power output of the motor (watt)
P_c = output power of the coupling device (watt)
P_{out} = output power of the generator (watt)

RELATIONSHIP BETWEEN PULLEY DIAMETER AND SPEED

The speed of a pulley is inversely proportional to the diameter.

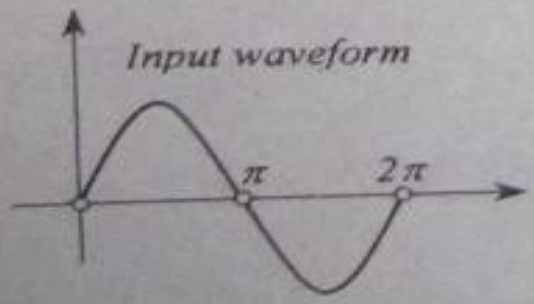


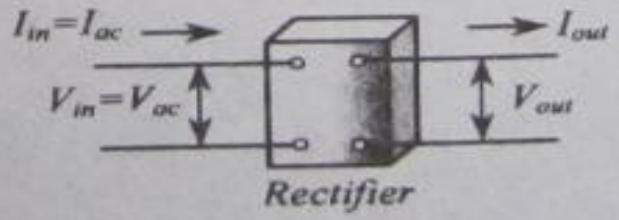
$$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

where: N_1 = speed of the driving pulley
N_2 = speed of the driven pulley
d_1 = diameter of the driving pulley
d_2 = diameter of the driven pulley

SINGLE-PHASE AC INPUT RECTIFIER

- **Rectifier** - a device which converts alternating current into unidirectional current by virtue of a characteristic permitting appreciable flow of current in only one direction.



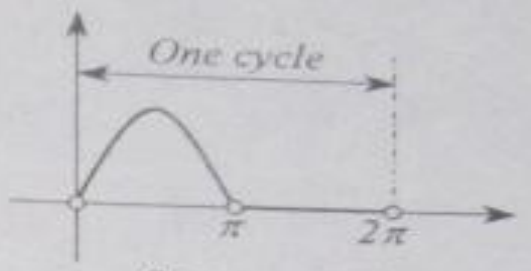


- **Half-wave output**

$V_m = \sqrt{2}V_{ac}$ $\quad$ $I_m = \sqrt{2}\,I_{ac}$

$V_{out_{dc}} = 0.318V_m$ $\quad$ $I_{out_{dc}} = 0.318I_m$

$V_{out_{rms}} = 0.5V_m$ $\quad$ $I_{out_{rms}} = 0.5I_m$



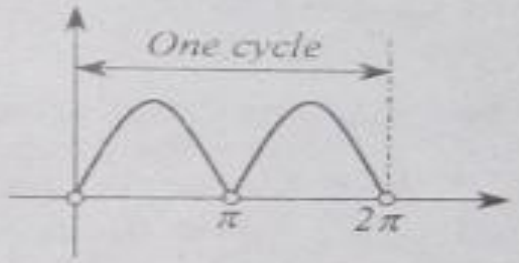
Output waveform

- **Full-wave output**

$V_m = \sqrt{2}V_{ac}$ $\quad$ $I_m = \sqrt{2}\,I_{ac}$

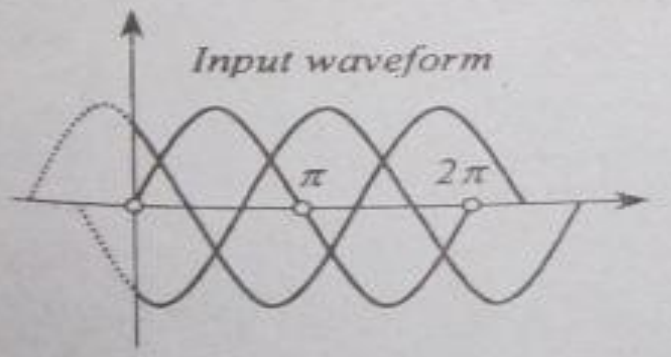
$V_{out_{dc}} = 0.636V_m$ $\quad$ $I_{out_{dc}} = 0.636I_m$

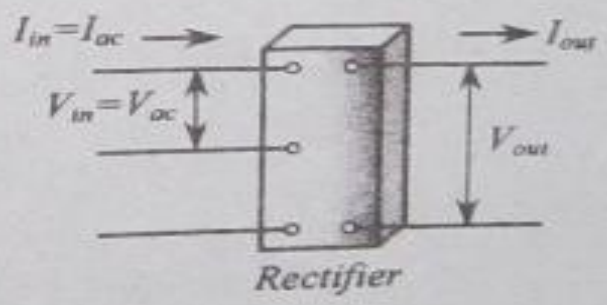
$V_{out_{rms}} = 0.707V_m$ $\quad$ $I_{out_{rms}} = 0.707I_m$



Output waveform

THREE-PHASE AC INPUT RECTIFIER



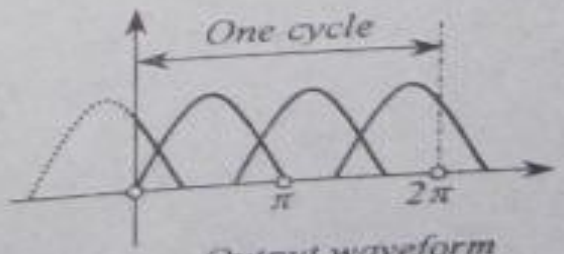


- **Half-wave output**

$V_m = \sqrt{2}V_{ac}$ $\quad$ $I_m = \sqrt{2}\,I_{ac}$

$V_{out_{dc}} = 0.827V_m$ $\quad$ $I_{out_{dc}} = 0.827I_m$

$V_{out_{rms}} = 0.841V_m$ $\quad$ $I_{out_{rms}} = 0.841I_m$



Output waveform

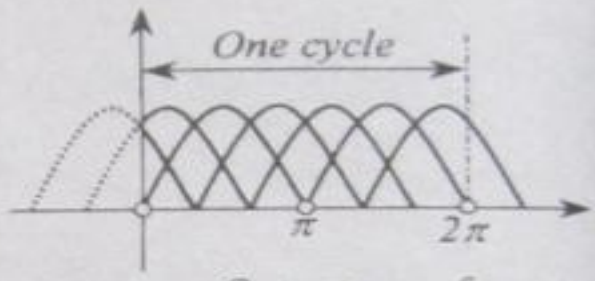
- **Full-wave output**

$V_m = \sqrt{2}V_{ac}$ $\quad$ $I_m = \sqrt{2}I_{ac}$

$V_{out_{dc}} = 0.955V_m$ $\quad$ $I_{out_{dc}} = 0.955I_m$

$V_{out_{rms}} = 0.95577V_m$

$I_{out_{rms}} = 0.95577I_m$



Output waveform

where: V_{ac} = rms voltage at the input (volt)
= line to line voltage at the input (volt)
V_m = maximum or peak value of the AC voltage at the input (volt)
$V_{out_{dc}}$ = dc or average voltage at the output (volt)
$V_{out_{rms}}$ = rms voltage at the output (volt)
I_{ac} = rms current at the input (ampere)
I_m = maximum or peak value of the AC current at the input (ampere)
$I_{out_{dc}}$ = dc or average current at the output (ampere)
$I_{out_{rms}}$ = rms current at the output (ampere)

Test 21

Problem 762:

A single-phase rotary converter has an output voltage of 110 volts. Determine the line current in the AC side when the machine delivers a DC output of 50 A. Assume the machine is operating with a pf of 0.85 and efficiency of 95%.

A. 70.71 A
B. 90.34 A
C. 87.57 A
D. 82.23 A

Problem 763: EE Board April 1994

In a three-phase synchronous converter, the ratio of the AC voltage to DC voltage is

A. 0.612
B. 1.0
C. 0.50
D. 0.707

Problem 764:

A three-phase rotary converter delivers 150 A at 500 VDC. Determine the current on the AC side if the operating power factor of the machine is 0.8 and the efficiency is 90%.

A. 179 A
B. 157 A
C. 189 A
D. 196 A

Problem 765:

Three single-phase transformers connected delta-delta fed a three-phase rotary converter at 2,300 V. This converter supplies power to a DC load that draws 100 kW at 250 V. If the rotary converter operates at an efficiency of 95% at unity pf, calculate the current in the secondary windings of the transformer bank.

A. 397 A
B. 355 A
C. 229 A
D. 240 A

Problem 766:

An open delta bank is serving a three-phase rotary converter. The converter supplies 175 A of current to a 1.4 ohm dc resistive load. If the machine draws an AC current of 100 A at 0.8 pf lagging, determine minimum size of transformer needed.

A. 15.0 kVA
B. 24.7 kVA
C. 25.9 kVA
D. 19.4 kVA

Problem 767:

A 4-phase star connected rotary converter operating at 0.90 efficiency and 0.85 power factor is connected across a 220 V, 60 Hz mains and draws 50 A. Determine the DC load current.

A. 208.46 A
B. 220.54 A
C. 216.37 A
D. 212.25 A

Problem 768:

The dc output of a six-phase star-connected converter is 500 kW at 400 V. If the pf and efficiency of the machine is 0.866 and 0.92 respectively, determine the ac current drawn from the supply?

A. 762 A
B. 740 A
C. 732 A
D. 729 A

Problem 769:

A 600 kW, 6-phase, Y-connected, 18-pole, synchronous converter has simplex lap wound armature with 1000 conductors. The flux per pole is 60 mWb. The machine runs at 600 rpm at rated load. Determine the ac current in the slip rings. Assume the operating power factor and efficiency of the machine equal to 1.0 and 0.9 respectively.

A. 496.21 A
B. 488.42 A
C. 462.25 A
D. 472.07 A

Problem 770: EE Board April 1992

A 500 kW, 600-V DC, 12-phase synchronous converter operates as a direct converter at a full load efficiency of 92% and a power factor of 0.93. Calculate the AC voltage between slip rings and the AC current drawn from a 12-phase transformer fed supply.

A. 112 V, 229 A

B. 112 V, 215 A
C. 110 V, 229 A
D. 110 V, 215 A

Problem 771:

A 4-pole 230-V, 60 Hz, three-phase induction motor directly drives a 6-pole alternator. Determine the output frequency. Assume a slip of 3% in the motor.

A. 90.0 Hz
B. 88.4 Hz
C. 86.6 Hz
D. 87.3 Hz

Problem 772:

A 4-pole, three-phase synchronous motor connected across a 60 Hz supply drives a 6-pole alternator through a belt. If the diameter of the driver and driven pulleys are 6 and 10 inches respectively, determine the operating frequency of the alternator.

A. 53 Hz
B. 54 Hz
C. 55 Hz
D. 52 Hz

Problem 773: EE Board April 1992

A generator is rated 600 kVA, 240 V, 60 cycles, 3-phase, 6-poles and wye-connected. What will be the speed of the driving pulley if the driven and driving pulleys are 1 ft and 2 ft in diameter respectively?

A. 2400 rpm
B. 1200 rpm
C. 600 rpm
D. 900 rpm

Problem 774: EE Board October 1998

The pulley of an old gen-set has a diameter of 20 inches. The belt exerts a pull of 353 pounds on the pulley. The gen-set runs at 900 rpm. What is the approximate kW rating of the gen-set?

A. 75
B. 250
C. 200
D. 37.5

Problem 775:

A 230-V, 3-phase motor belt drives a 50 kW, 250 V DC generator. The motor, belt and generator efficiencies are 0.91, 0.95 and 0.86 respectively. If the motor operates at a pf of 0.866 lagging, determine the current drawn.

A. 189 A
B. 195 A
C. 192 A
D. None of these

Problem 776:

A single-phase motor is connected across a 230-V, 60 Hz supply and draws 30 A at 0.85 pf lagging. This motor drives a 120-V DC generator through direct coupling. If the motor, coupling and generator efficiencies are 0.85, 0.96 and 0.88 respectively determine the current delivered by the generator using the above condition.

A. 35 A
B. 32 A
C. 38 A
D. 48 A

Problem 777:

A 230-V, 60 Hz, 3-phase motor drives a 25 kW, 250 V DC generator through belt and pulley. The motor and generator efficiencies are 0.90 and 0.88, respectively. If the motor draws a line current of 98 A at 0.85 pf lagging, how much is the combined efficiency of the belt and pulley?

A. 95.12%
B. 96.04%
C. 95.33%
D. 94.81%

Problem 778: EE Board October 1991

A moving coil ammeter, a resistance of 1500 ohms and a diode are connected in series across a 125 V sinusoidal AC supply. The diode has a linear current / voltage characteristics over the range I = 0, E = 0 V to I = 200 mA , E = 200 V. Calculate the power dissipated in the 1500-ohm resistor.

A. 0.76 W
B. 0.83 W
C. 0.71 W
D. 0.86 W

Problem 779:

A half-wave rectifier draws power from a power transformer at 24 volts and delivers 200 mA to a load. Determine power drawn by load.

A. 0.48 W
B. 1.25 W
C. 2.16 W
D. 2.05 W

Problem 780:

An AC ammeter is connected in series across the output of a full wave rectifier. This meter reads 8 A. If a moving coil ammeter is replaced instead of the AC ammeter, what will be its expected reading?

A. 8.0 A
B. 7.6 A
C. 7.2 A
D. 5.6 A

Problem 781: EE Board October 1987

In the electronic laboratory, a diode rectifier tube has a plate resistance of 600 ohms (assumed constant) and is connected as a half-wave rectifier to a load resistance of 1,400 ohms. The energizing supply voltage is 440 volts effective, 60 Hz alternating current. Find the average load voltage.

A. 138.5 V
B. 132.8 V
C. 133.6 V
D. 150.2 V

Problem 782:

Three identical transformers are used to supply power to an iodizing plant requiring 1000 A at 28 V DC through a 3-phase half wave rectifier. The transformers are connected delta-wye. What are the secondary current and voltage of each transformer?

A. 1042 A, 28.47 V
B. 1017 A, 16.44 V
C. 1042 A, 16.44 V
D. 1017 A, 28.47 V

Problem 783: EE Board October 1987

In the electronic laboratory, a diode rectifier tube has a plate resistance of 600 ohms (assumed constant) and is connected as a half-wave rectifier to a load resistance of 1,400 ohms. The energizing supply voltage is 440 volts effective, 60 Hz alternating current. Find the rms rectified current.

A. 0.162 A
B. 0.155 A
C. 0.208 A
D. 0.311 A

Problem 784: EE Board April 1997

A resistor is connected across an AC supply of 220 volts. The power drawn is 1,000 watts. If a diode were connected in series with the resistor, what would be the power absorbed by the resistor?

A. 850 W

B. 250 W
C. 750 W
D. 500 W

Problem 785: EE Board October 1996

A circuit consists of a resistor that is connected in series with a perfect diode. When connected across a 20-V battery the current measured is 10 A. However, when connected across a 30-V battery but with reversed polarity, what would be the new current?

A. 15 A
B. Zero current
C. 20 A
D. None of the answered listed

Problem 786: EE Board April 1997

A germanium diode, which has an offset voltage of 0.2 V and an incremental resistance of 20 ohms, is connected in series with a 10,000-ohm resistor across a 100 V DC source. What is the current?

A. 9.98 mA
B. 10 mA
C. 9.78 mA
D. 9.96 mA

ANSWER KEY

762. C	767. C	772. B	777. A	782. B
763. A	768. B	773. C	778. A	783. B
764. D	769. A	774. D	779. C	784. D
765. C	770. C	775. B	780. C	785. B
766. A	771. D	776. A	781. A	786. D

☑ RATING

22-25 - Topnotcher
17-21 - Passer
12-16 - Conditional
0-11 - Failed

Solutions to Test 21

762 $P_{dc} = V_{dc}I_{dc} = 110(50) = 5500\ W$

$$P_{ac} = \frac{P_{dc}}{\eta} = \frac{5500}{0.95} = 5789.47\ W$$

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{110}{\sqrt{2}}\sin\left(\frac{180}{2}\right) = 77.78\ V$$

$$P_{ac} = V_{ac}I_{ac}pf$$

$$I_{ac} = \frac{P_{ac}}{V_{ac}pf} = \frac{5789.47}{77.78(0.85)}$$

$$I_{ac} = 87.57\ A$$

763 $$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right)$$

$$\frac{V_{ac}}{V_{dc}} = \frac{\sin\left(\frac{180°}{3}\right)}{\sqrt{2}}$$

$$\frac{V_{ac}}{V_{dc}} = 0.612$$

764 $P_{dc} = V_{dc}I_{dc} = 500(150) = 75{,}000\ W$

$$P_{ac} = \frac{P_{dc}}{\eta} = \frac{75{,}000}{0.9} = 83333.33\ W$$

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{500}{\sqrt{2}}\sin\left(\frac{180}{3}\right) = 306.186\ V$$

$$P_{ac} = \sqrt{3}V_{ac}I_{ac}pf$$

$$I_{ac} = \frac{P_{ac}}{\sqrt{3}V_{ac}pf} = \frac{83333.33}{\sqrt{3}(306.186)(0.8)}$$

$$I_{ac} = 196.42\ A$$

765 $$P_{ac} = \frac{P_{dc}}{\eta} = \frac{100{,}000}{0.95} = 105263.15\ W$$

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{250}{\sqrt{2}}\sin\left(\frac{180}{3}\right)$$

$$V_{ac} = 153.1\ V$$

$$P_{ac} = \sqrt{3}V_{ac}I_{ac}pf$$

$$I_{ac} = \frac{P_{ac}}{\sqrt{3}V_{ac}pf} = \frac{105263.15}{\sqrt{3}(153.1)(1.0)} \cong 397\text{ A}$$

Note: As given the secondary side of the transformer bank is delta-connected.

Let: I_p = current that flows in the secondary windings.

$$I_p = \frac{I_{ac}}{\sqrt{3}} = \frac{397}{\sqrt{3}}$$

$$I_p = 229.2\text{ A}$$

766

$$V_{dc} = I_{dc}R_L = (175)(1.4) = 245\text{ V}$$

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{245}{\sqrt{2}}\sin\left(\frac{180}{3}\right) = 150.03\text{ V}$$

$$S_{ac} = \sqrt{3}V_{ac}I_{ac} = \sqrt{3}(150)(100) = 25.98\text{ kVA}$$

$$S_v = \sqrt{3}\text{Rating}$$

$$\text{Rating} = \frac{S_v}{\sqrt{3}} = \frac{25.98}{\sqrt{3}}$$

$$\text{Rating} \cong 15\text{ kVA}$$

767 Recall: Formula for the relationship between phase and line voltage for an n-phase system

$$P_{ac} = nV_pI_p\cos\theta$$

$$V_L = 2V_p\sin\left(\frac{180°}{n}\right)$$

Note: As given the input to the converter is a 4-phase star-connected system.

$$V_p = \frac{V_L}{2\sin\left(\frac{180°}{n}\right)} = \frac{220}{2\sin\left(\frac{180°}{4}\right)} = 155.56$$

$$P_{ac} = 4V_pI_p\cos\theta = 4(155.56)(50)(0.85) = 26445.2\text{ W}$$

$$P_{dc} = P_{ac}\eta = 26445.2(0.9) = 23800.68\text{ W}$$

$$V_{dc} = \frac{V_{ac}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{220}{\sqrt{2}}\sin\left(\frac{180°}{4}\right) = 110\text{ V}$$

$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{23800.68}{110}$$

$$I_{ac} = 216.37\text{ A}$$

768 $P_{ac} = \frac{P_{dc}}{\eta} = \frac{500}{0.92} = 543.478\text{ kW}$

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{400}{\sqrt{2}}\sin\left(\frac{180}{6}\right) = 141.42\text{ V}$$

$$P_{ac} = 6V_p I_{ac} pf$$

$$V_p = \frac{V_{ac}}{2\sin\left(\frac{180°}{n}\right)} = \frac{141.42}{2\sin\left(\frac{180°}{6}\right)} = 141.42\text{ V}$$

$$I_{ac} = \frac{P_{ac}}{6V_p pf} = \frac{543{,}478}{6(141.42)(0.866)}$$

$$I_{ac} = 739.6\text{ A}$$

769 $V_{dc} = \frac{PNZ\Phi}{60a}; \quad a = mP$

$$V_{dc} = \frac{NZ\Phi}{60m} = \frac{600(1000)(0.06)}{60(1)} = 600\text{ V}$$

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{600}{\sqrt{2}}\sin\left(\frac{180}{6}\right) = 212.132\text{ V}$$

$$P_{ac} = \frac{P_{dc}}{\eta} = \frac{600}{0.95} = 631.578\text{ kW}$$

$$P_{ac} = 6V_p I_{ac}\cos\theta$$

$$V_p = \frac{V_{ac}}{2\sin\left(\frac{180°}{n}\right)} = \frac{212.132}{2\sin\left(\frac{180°}{6}\right)} = 212.132\text{ V}$$

$$I_{ac} = \frac{P_{ac}}{6V_p pf} = \frac{631{,}578}{6(212.132)(1.0)}$$

$$I_{ac} = 496.21\text{ A}$$

770 Note: Assume the machine is Y-connected at the AC side.

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}}\sin\left(\frac{180°}{n}\right) = \frac{600}{\sqrt{2}}\sin\left(\frac{180°}{12}\right) \cong 110\text{ V}$$

$$P_{ac} = \frac{P_{dc}}{\eta} = \frac{500}{0.92} = 543.478\text{ kW}$$

$$P_{ac} = 12V_p I_{ac} pf$$

$$V_p = \frac{V_{ac}}{2\sin\left(\frac{180°}{n}\right)} = \frac{110}{2\sin\left(\frac{180°}{12}\right)} = 212.5\text{ V}$$

$$I_{ac} = \frac{P_{ac}}{12V_p pf} = \frac{543478}{12(212.5)(0.93)} = 229.17\ A$$

771 $$N_r = \frac{120f}{P}(1-s) = \frac{120(60)}{4}(1-0.03) = 1746\ rpm$$

Note: The rotor speed (shaft speed) of the motor is equivalent to the speed of the armature of the alternator.

$$f = \frac{PN_r}{120} = \frac{6(1746)}{120}$$

$$f = 87.3\ Hz$$

772 $$N_1 = \frac{120f}{P} = \frac{120(60)}{4} = 1800\ rpm$$

Note: Speed is inversely proportional to the diameter of the pulley.

$$N = k\frac{1}{d}$$

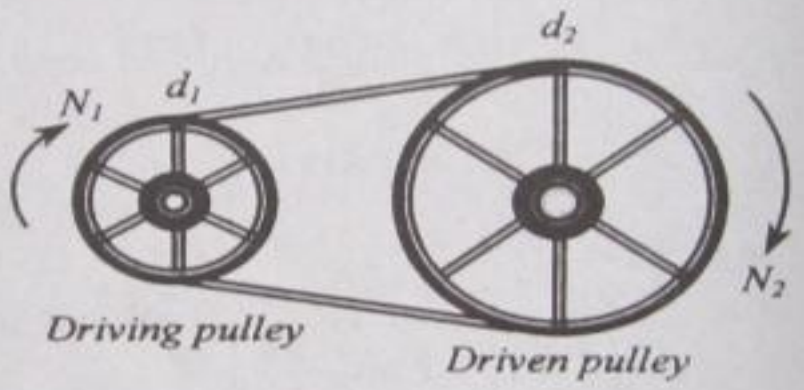
$$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

$$\frac{1800}{N_2} = \frac{10}{6}$$

$$N_2 = 1080\ rpm$$

$$f = \frac{PN}{120} = \frac{6(1080)}{120}$$

$$f = 54\ Hz$$



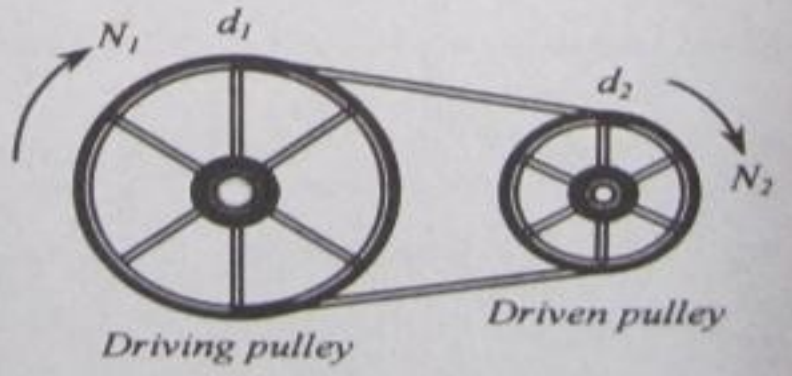
773 Note: Speed is inversely proportional to the diameter of the pulley.

$$N = k\frac{1}{d}$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

$$N_2 = \frac{120f}{P} = \frac{120(60)}{6}$$

$$N_2 = 1200\ rpm$$



Substitute :

$$\frac{N_1}{1200} = \frac{1}{2}$$

$$N_1 = 600\ rpm$$

774

$T = Fr$

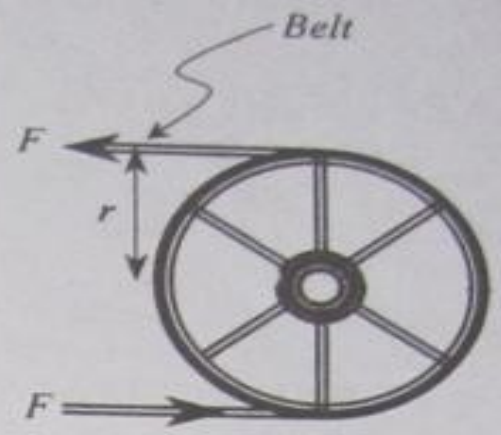
$$HP = \frac{2\pi NT}{33000}$$

$$HP = \frac{2\pi N(Fr)}{33000}$$

$$= \frac{2\pi(900)(353)(10/12)}{33000} = 50.41\,hp$$

$$P = 50.41\,hp \times \frac{0.746\,kw}{1\,hp}$$

$$P = 37.605\,kW$$



775 Note: For cascaded system, the overall efficiency of the set is equivalent to the product of their individual efficiencies respectively.

$$P_{ac} = \frac{P_{dc}}{\eta_m \eta_b \eta_g} = \frac{50}{(0.91)(0.95)(0.86)} = 67.25\,kW$$

$$P_{ac} = \sqrt{3} V_L I_L pf$$

$$I_L = \frac{P_{ac}}{\sqrt{3} V_L pf} = \frac{67250}{\sqrt{3}(230)(0.866)}$$

$$I_L \cong 195\,A$$

776 Note: For cascaded system, the overall efficiency of the set is equivalent to the product of their individual efficiencies respectively.

$$P_{ac} = V_L I_L pf = (230)(30)(0.85) = 5865\,W$$

$$P_{dc} = P_{ac}(\eta_m \eta_c \eta_g) = 5865(0.85)(0.96)(0.88) = 4211.54\,W$$

$$P_{dc} = V_{dc} I_{dc}$$

$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{4211.54}{120}$$

$$I_{dc} = 35.09\,A$$

777 Note: For cascaded system, the overall efficiency of the set is equivalent to the product of their individual efficiencies respectively.

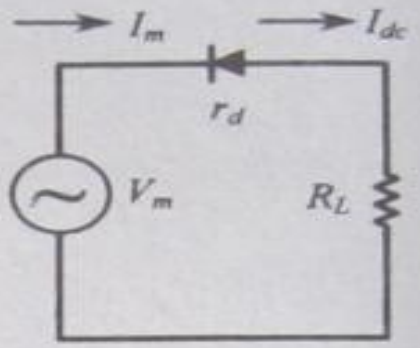
$$P_{ac} = \sqrt{3} V_L I_L pf = \sqrt{3}(230)(98)(0.85) = 33.184\,kW$$

$$\eta_m \eta_b \eta_g = \frac{P_{dc}}{P_{ac}}$$

$$\eta_b = \frac{P_{dc}}{\eta_m \eta_g P_{ac}} = \frac{25}{(0.9)(0.88)(33.184)} = 0.9512$$

$$\eta_b = 95.12\%$$

778 Let: rd = resistance of diode



$$V_m = \sqrt{2}V_{rms} = \sqrt{2}(125) = 176.776 \text{ volts}$$

$$r_d = \frac{V_d}{I_d} = \frac{200}{200 \times 10^{-3}} = 1{,}000\ \Omega$$

$$I_m = \frac{V_m}{r_d + R_L} = \frac{176.776}{1000 + 1500} = 0.0707 \text{ A}$$

$$I_{dc} = 0.318\, I_m = 0.318(0.0707) = 0.0225 \text{ A}$$

$$P_{dc} = I_{dc}^2 R_L = (0.0225)^2(1500)$$

$$P_{dc} = 0.76 \text{ watt}$$

779 $V_m = \sqrt{2}V_{rms} = \sqrt{2}(24) = 33.94$ volts

$$V_{dc} = 0.318V_m = 0.318(33.94) = 10.79 \text{ V}$$

$$P_{dc} = V_{dc}I_{dc} = 10.79(0.2)$$

$$P_{dc} \cong 2.16 \text{ W}$$

780 Note: The AC ammeter will register the rms value of the current while the moving- coil ammeter is used to measure dc current or average current.

$$I_{rms} = 0.707\, I_m$$

$$I_m = \frac{I_{rms}}{0.707} = \frac{8}{0.707} = 11.315 \text{ A}$$

$$I_{dc} = 0.636\, I_m = 0.636(11.315)$$

$$I_{dc} \cong 7.2 \text{ A}$$

781 $V_m = \sqrt{2}V_{rms} = \sqrt{2}(440) = 622.25$ volts

$$I_m = \frac{V_m}{r_p + R_L} = \frac{622.25}{600 + 1400} = 0.311 \text{ A}$$

$$I_{dc} = 0.318\, I_m = 0.318(0.311) = 0.0989 \text{ A}$$

$$V_{dc} = I_{dc}R_L = (0.0989)(1400)$$

$$V_{dc} = 138.46 \text{ V}$$

782 $I_{dc} = 0.827\, I_m$

$$I_m = \frac{I_{dc}}{0.827} = \frac{1000}{0.827} = 1209.19 \text{ A}$$

$$I_{rms} = 0.841 I_m = (0.841)(1209.19) = 1016.93 \text{ A}$$

$V_{dc} = 0.827\ V_m$

$$V_m = \frac{V_{dc}}{0.827} = \frac{28}{0.827} = 33.857\ V$$

$V_{rms} = 0.841 V_m = (0.841)(33.857) = 28.47\ V$

Note: As given the secondary winding of the transformer bank is wye-connected.

Let: V_p = voltage across the secondary winding of each transformer
I_p = current that flows in the secondary winding of each transformer

$I_p = I_{rms} = 1017\ A$

$$V_p = \frac{V_{rms}}{\sqrt{3}} = \frac{28.47}{\sqrt{3}}$$

$V_p = 16.437\ V$

783 $V_m = \sqrt{2}V_{rms} = \sqrt{2}(440) = 622.25$ volts

$$I_m = \frac{V_m}{r_p + R_L} = \frac{622.25}{600 + 1400} = 0.311\ A$$

$I_{rms} = 0.5 I_m = 0.5(0.311)$

$I_{rms} = 0.155\ A$

784 $$R_L = \frac{V_{rms}^2}{P} = \frac{(220)^2}{1000} = 48.4\ \Omega$$

Note: With the diode connected in series, the circuit will become a half- wave rectifier. Also, since not given, neglect resistance of the diode.

Let: V_m = maximum value of the supply voltage.

$V_m = \sqrt{2}V_{ac} = \sqrt{2}(220) = 311.127\ V$

Solving for the V_{rms} at the load side:

$V_{rms} = 0.5V_m = 0.5(311.127)$

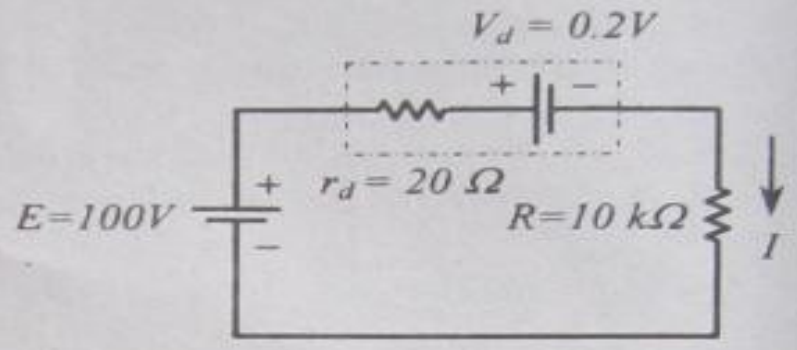
$V_{rms} = 155.56\ V$

$$P = \frac{V_{rms}^2}{R_L} = \frac{(155.56)^2}{48.4} = 499.977\ W$$

$P \cong 500\ W$

785 Note: Since the diode is reversed biased, then it will not conduct. Its equivalent diagram is an open circuit. Thus, zero or no current will flow.

786 Note: Refer to the circuit diagram



By KVL :

$$E - V_d - I(rd + R_L) = 0$$

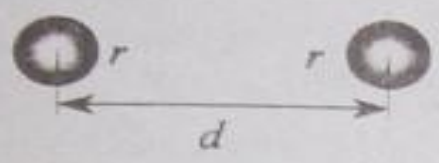
$$I = \frac{E - V_d}{rd + R_L}$$

$$= \frac{100 - 0.2}{20 + 10{,}000}$$

$$I = 9.96\ mA$$

Transmission Lines

INDUCTANCE OF A SINGLE-PHASE TWO-WIRE LINE



$$L = 4 \times 10^{-7} \ln \frac{d}{r'}$$

where: L = inductance (henry/meter)
d = distance between conductors (meter)
r = radius of conductor (meter)
r' = 0.7788 r

INDUCTANCE OF COMPOSITE CONDUCTOR LINES

conductor x *conductor y*

$$L_x = 2 \times 10^{-7} \ln \frac{D_{m\,xy}}{D_{s\,x}} \qquad L_y = 2 \times 10^{-7} \ln \frac{D_{m\,xy}}{D_{s\,y}}$$

$$L_{total} = L_x + L_y$$

where: Dm_{xy} = geometric mean distance (GMD) between conductors x and y
Ds_x = self GMD between strands of conductor x
Ds_y = self GMD between strands of conductor y
L_{total} = total inductance of the transmission line (henry/meter)

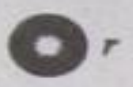
Note:

$$D_{s_x} = \sqrt[n]{\text{Product of all } n^2 \text{ distances between all strands of conductor x}}$$

$$D_{m_{xy}} = \sqrt[mn]{\text{Product of all m x n distances between n strands of conductor x to all m strands of conductor y}}$$

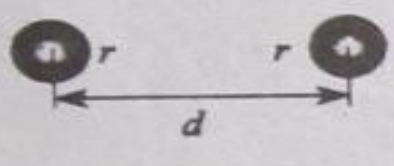
GEOMETRIC MEAN DISTANCE FOR VARIOUS CONDUCTOR SET-UP

- Solid round wire of radius r



$D_s = 0.7788\,r$

- Two round wires of distance d



$D_m = d$

- A point. line of area inside a circle of radius R

$D_m = R$

- Equally spaced points in a circle of radius R

$D_m = (R)\sqrt[n-1]{n}$

where: n = number of points

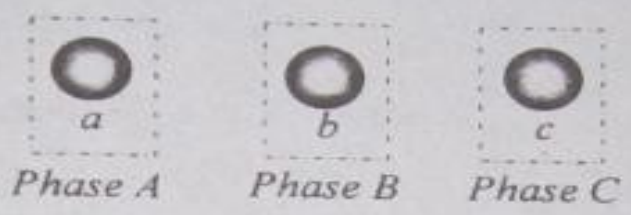
AVERAGE INDUCTANCE PER PHASE OF A 3-PHASE TRANSMISSION LINE

$$L = 2 \times 10^{-7} \ln \frac{D_m}{D_s} \qquad D_s = \sqrt[3]{D_{AA} D_{BB} D_{CC}} \qquad D_m = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$

where: D_m = mutual GMD
D_s = self GMD

where : D_{AB} = GMD of all conductors between lines A and B
D_{BC} = GMD of all conductors between lines B and C
D_{CA} = GMD of all conductors between lines C and A
D_{AA} = GMD of all conductors in line A
D_{BB} = GMD of all conductors in line B
D_{CC} = GMD of all conductors in line C

- **Single circuit transmission line**

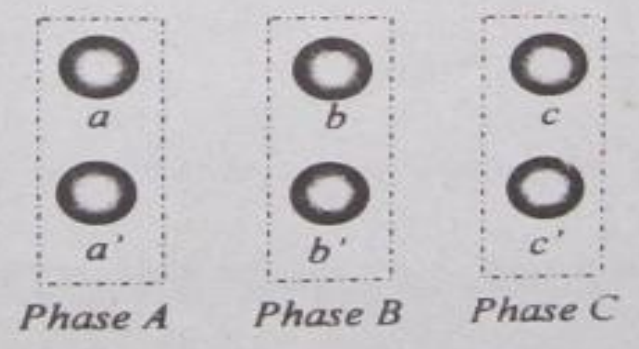


$$D_{AA} = D_{BB} = D_{CC} = 0.7788\,r$$

$$D_{AB} = D_{BC} = d_{ab} = d_{bc} \qquad D_{CA} = d_{ca}$$

where: d_{ab} = distance between conductors a and b
d_{bc} = distance between conductors b and c
d_{ca} = distance between conductors c and a
r = radius of each conductor

- **Double circuit transmission line**

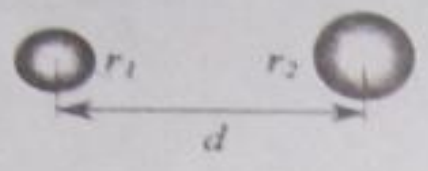


$$D_{AB} = \sqrt[4]{d_{ab}d_{ab}'d_{a'b}d_{a'b'}} \qquad D_{AA} = \sqrt[4]{d_{aa}d_{aa}'d_{a'a}d_{a'a'}}$$

$$D_{BC} = \sqrt[4]{d_{bc}d_{bc}'d_{b'c}d_{b'c'}} \qquad D_{BB} = \sqrt[4]{d_{bb}d_{bb}'d_{b'b}d_{b'b'}}$$

$$D_{CA} = \sqrt[4]{d_{ca}d_{ca}'d_{c'a}d_{c'a'}} \qquad D_{CC} = \sqrt[4]{d_{cc}d_{cc}'d_{c'c}d_{c'c'}}$$

LINE TO LINE CAPACITANCE OF A SINGLE-PHASE TWO WIRE LINE



$$C = \frac{2\pi\Sigma_o}{\ln \frac{d^2}{r_1 r_2}}$$

where: C = capacitance (farad per meter)
Σ_o = permittivity of medium (8.85×10^{-12} farad/meter)
r_1 = radius of the conductor 1
r_2 = radius of conductor 2

- If $r_1 = r_2 = r$

$$C = \frac{\pi\Sigma_o}{\ln \frac{d}{r}}$$

CAPACITANCE TO GROUND OF A SINGLE-PHASE TWO WIRE LINE

Note: Capacitance to ground is equal to twice the line-to-line capacitance.

$$C_n = \frac{2\pi\Sigma_o}{\ln \frac{d}{r}}$$

where: C_n = line to ground capacitance (farad per meter)

CAPACITANCE TO GROUND OF A THREE-PHASE TRANSMISSION LINE

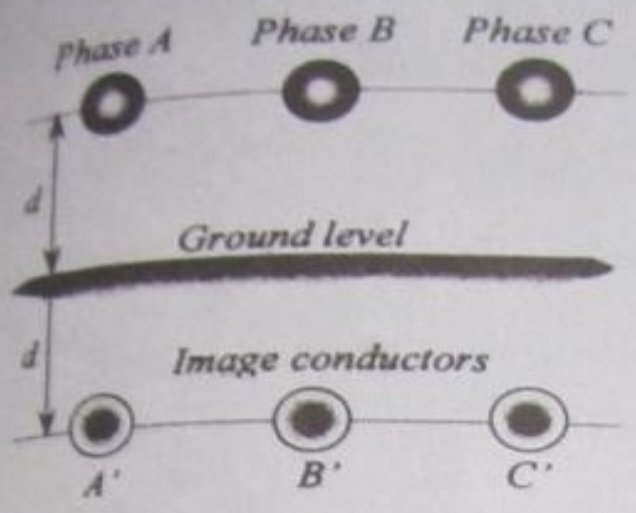
- **Neglecting ground effect**

$$C_n = \frac{2\pi\Sigma_o}{\ln \frac{D_m}{D_{s'}}}$$

where: D_m = mutual GMD
$D_{s'}$ = self GMD

Note: $D_{s'}$ is similar to D_s in the inductance formula, EXCEPT that each d_s is to be replaced by "r" instead of 0.7788 r

- With ground effects included



$$C_n = \frac{2\pi\Sigma_0}{\ln\frac{D_m}{D_{s'}} - \ln\frac{H_m}{H_s}}$$

$$H_m = \sqrt[3]{H_{AB'}H_{BC'}H_{CA'}}$$

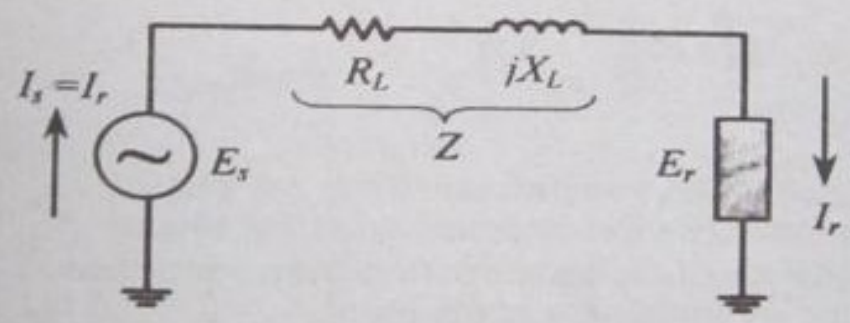
$$H_s = \sqrt[3]{H_{AA'}H_{BB'}H_{CC'}}$$

Note: Image conductors do not physically exist.

SHORT - TRANSMISSION LINES

A transmission line is classified a short line, if its length does not exceed 50 miles.

- **Equivalent circuit per phase**



$$E_s\angle\pm\theta_s = E_r\angle 0^\circ + (I_r\angle\pm\theta_r)(R_L + jX_L)$$

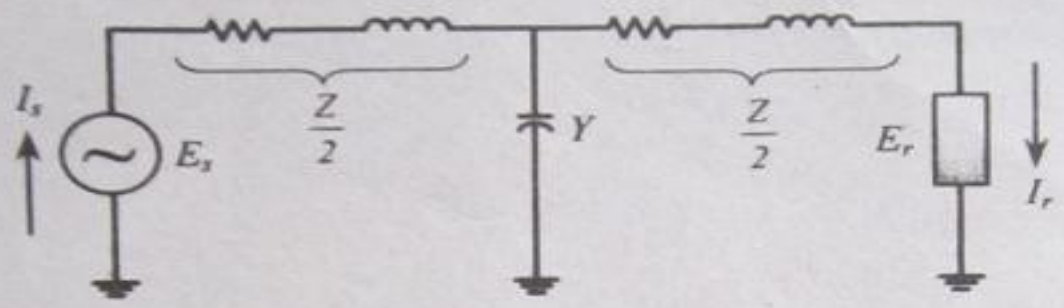
where: E_s = sending end voltage per phase (volt)
E_r = receiving end voltage per phase (volt)
I_r = receiving end current per phase (ampere)
I_s = sending end current per phase (ampere)
R_L = resistance of the transmission lines per phase (ohm)
X_L = reactance of the transmission lines per phase (ohm)
Z = impedance of the transmission lines per phase (ohm)
$+\theta_r$ = if pf of the receiving end is leading
$-\theta_r$ = if pf of the receiving end is lagging
θ_s = sending end power factor angle

Note: The sign of θ_s is dependent of the result after simplifying the right side of the equation.

MEDIUM - TRANSMISSION LINES

A transmission line is classified a medium line, if its length is more than 51 miles but less than 150 miles.

- **Nominal-T equivalent circuit per phase**



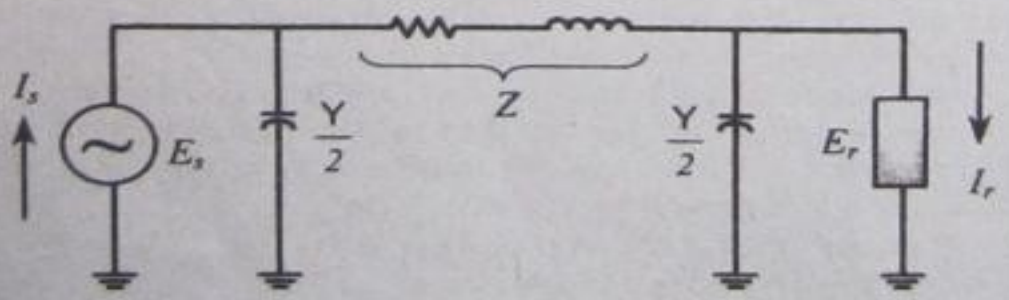
$$E_s = E_r\left[1+\frac{ZY}{2}\right]+I_rZ\left[1+\frac{ZY}{4}\right]$$

$$I_s = I_r\left[1+\frac{ZY}{2}\right]+E_rY \qquad Y = j\beta_C$$

where : Z = impedance of the transmission lines per phase
Y = admittance of the transmission lines per phase
β_C = susceptance of the transmission lines per phase
j = symbol for an imaginary component

Note: All operations (add, subtract, multiply and divide) for I_s and E_s equations above are to be done using rules in complex operations.

- **Nominal-π equivalent circuit per phase**

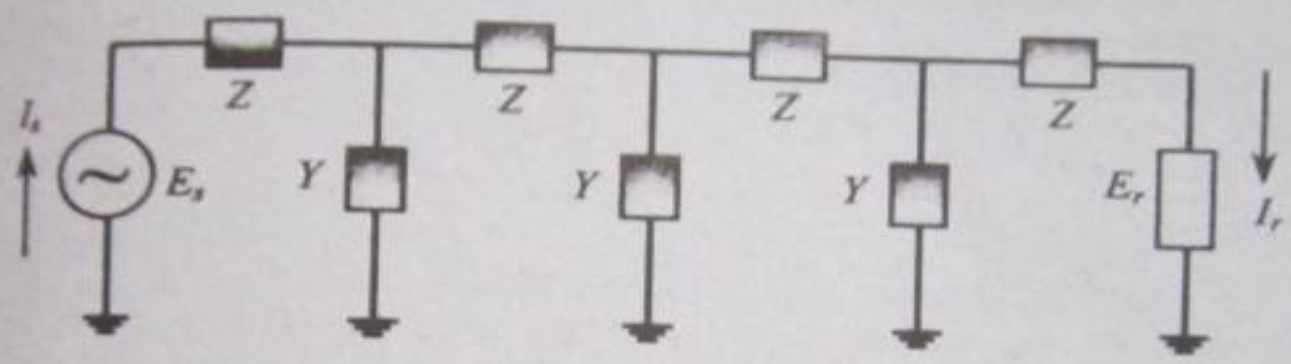


$$E_s = E_r\left[1+\frac{ZY}{2}\right]+I_rZ \qquad I_s = E_rY\left[1+\frac{ZY}{4}\right]+I_r\left[1+\frac{ZY}{2}\right]$$

LONG - TRANSMISSION LINES

A transmission line is classified a long line, if its length is more than 150 miles.

- **Equivalent circuit per phase**



$$E_s = \left[\frac{E_r + I_rZ_o}{2}\right]e^{\delta\ell} + \left[\frac{E_r - I_rZ_o}{2}\right]e^{-\delta\ell} \qquad Z_o = \sqrt{\frac{Z}{Y}}$$

$$I_s = \left[\frac{I_r + \frac{E_r}{Z_o}}{2}\right]e^{\delta\ell} + \left[\frac{I_r - \frac{Er}{Z_o}}{2}\right]e^{-\delta\ell} \qquad \frac{1}{Z_o} = \sqrt{\frac{Y}{Z}}$$

$$\delta\ell = \sqrt{ZY}$$

where: Z_o = characteristic impedance
$\delta\ell$ = propagation constant
ℓ = length of transmission lines

TRANSMISSION LINE REGULATION

$$\%VR = \frac{E_s - E_r}{E_r} \times 100\%$$

TRANSMISSION LINE LOSSES AND EFFICIENCY

$$P_{losses} = 3I_L^2 R_L \qquad P_r = 3E_r I_r \cos\theta_r$$

$$P_s = P_r + P_{losses} \qquad P_s = 3E_s I_s \cos\theta_s \qquad \eta = \frac{P_r}{P_s}$$

where: η= transmission efficiency
I_L = line current
P_s = sending end power
P_r = receiving end power
P_{losses} = power losses

Test 22

Problem 787:

A single solid round copper wire has a diameter of 0.50 inch. Determine its self GMD.

A. 0.3894 inch
B. 0.1947 inch
C. 0.1825 inch
D. 0.25 inch

Problem 788:

Determine the mutual GMD of a three-phase single-circuit transmission line whose conductors arranged in a triangular formation so that the two distances between conductors are 20 ft and the third is 38 ft, respectively?

A. 24.77 ft.
B. 25.35 ft
C. 23.67 ft
D. 26.43 ft

Problem 789:

A three-phase double circuit line has a horizontal spacing of 40 ft and a vertical spacing between adjacent levels of 25 ft. Determine the mutual GMD of the line.

A. 43.08 ft
B. 42.64 ft
C. 41.42 ft
D. 40.56 ft

Problem 790: EE Board March 1998

A 230 kV transmission line is 100 miles long. The conductor is ACSR of 1,113,000 CM. The conductors are horizontally arranged with 20 ft spacing. The resistance per mile is 0.0969 ohm and its GMR is 0.0435 ft. What is the impedance of the line?

A. 9.69 + j 95.25 ohms
B. 3.23 + j 95.25 ohms
C. 9.69 + j 77.19 ohms
D. 3.23 + j 25.93 ohms

Problem 791: EE Board October 1997

A 34.5 kV feeder line is 5 miles long. The conductors are space 4 feet horizontally. The conductor is 4/0 copper with GMR of 0.01668 ft. What is the reactance?

A. 2.36 ohms
B. 6.72 ohms
C. 3.46 ohms
D. 5.16 ohms

Problem 792:

A single-phase transmission line 15 km long is using copper conductors of diameter 0.8 cm. If the distance between conductors is 1.25 m, determine the self-inductance of the transmission lines.

A. 0.036 H
B. 0.024 H
C. 0.072 H
D. 0.055 H

Problem 793: EE Board October 1998

A 5 km long, three-phase line has a horizontal configuration of 4 ft spacing. The conductor is 336.4 MCM ACSR with GMR of 0.0244 ft and a resistance of 0.306 ohm per mile. What is the impedance?

A. 2.22 cis 65
B. 1.19 cis 46
C. 6.66 cis 65
D. 3.57 cis 46

Problem 794:

A 170-mile, 230-kV, 60-Hz, three-phase single-circuit transmission line uses a triangular arrangement with 20 ft, 20 ft, and 36 ft spacing respectively. If the line conductors have a GMR of 0.0217 ft, determine the capacitive susceptance to neutral per mile.

A. 4.81×10^{-6} mho per mi
B. 12.74×10^{-6} mho per mi
C. 7.92×10^{-6} mho per mi
D. 2.47×10^{-6} mho per mi

Problem 795:

A single-circuit, 3-phase, 60 Hz transmission is using a horizontal arrangement with a spacing distance of 4 m between adjacent conductors. If each conductor has an outside diameter of 0.8 cm, determine the capacitance to neutral per meter.

A. 8.352 pF/m
B. 7.793 pF/m
C. 6.742 pF/m.
D. None of these

Problem 796:

A single-phase transmission line has total impedance of 1.5 + j 0.3 ohms. It delivers 3 MW at 11 kV and at unity power factor. Determine the sending end voltage of the transmission lines.

A. 11.41 kV
B. 11.24 kV
C. 12.02 kV
D. 11.89 kV

Problem 797:

A single-phase transmission line has total impedance of $5\angle 60^\circ$ ohms and supplies a total load of 120 A, 3.3 kV and at 0.8 power factor lagging. Calculate the sending end voltage.

A. 3.93 kV
B. 3.62 kV
C. 3.54 kV
D. 3.86 kV

Problem 798: EE Board April 1981

A 3-phase, 3-wire, short transmission line has a resistance of 3 ohms and a reactance of 8 ohms per wire. At the receiving end, a balanced 3-phase load draws a line current of 60 A, at 13,500 volts line to line, 0.90 power factor lagging. Determine the voltage at the sending end.

A. 14,156 V
B. 14,143 V
C. 14,230 V
D. 15,055 V

Problem 799: EE Board October 1996, EE Board April 1997

A load of 12 MVA, 0.8 pf lagging, 22 kV is served by a transmission line which has a line resistance of 3 ohms and a line reactance of 10 ohms. Solve for the sending end voltage.

A. 24,345 V
B. 26,795 V
C. 23,335 V
D. 25,422 V

Problem 800: EE Board April 1982

A short 3-phase, 3-wire line has a resistance of 5 ohms and reactance of 12 ohms per wire and transmit power to a 3-phase load drawing 1000 kW at 13,120 volts line to line, 0.8 pf lagging, 60 Hz. Solve for the sending end voltage.

A. 14,200 V
B. 14,330 V
C. 14,130 V
D. 14,240 V

Problem 801: EE Board April 1991

A 3-phase line has a reactance of 4.32 ohms and a resistance of 4.15 ohms. The load at the receiving end is 3800 kW at 75%-power factor lagging and the voltage at the sending end is 36 kV. Determine the voltage at the receiving end.

A. 34.24 kV
B. 34.86 kV
C. 35.14 kV
D. 35.45 kV

Problem 802: EE Board March 1998

A short 230-kV transmission line has impedance of 5 cis 78 ohms. The sending end power is 100 MW at 230 kV and 85% power factor. What is the voltage at the other end?

A. 225.4 kV
B. 226.3 kV
C. 223.2 kV
D. 228.2 kV

Problem 803: EE Board October 1996

A three-phase transmission line, 15 km long serves a substation rated 15 MVA at 34.5 kV, 60 Hz. If the line resistance is 0.12 ohm per kilometer and the line reactance is 0.457 ohm per kilometer, what should the sending end voltage be so that the transformer can be fully loaded at its rated voltage and unity pf.

A. 37,200 V
B. 36,500 V
C. 35,408 V
D. 34,990 V

Problem 804:

A three-phase transmission line, 10 km long delivers a load of 5000 kW at 11 kV and 0.8 pf lagging at the receiving end. The resistance of the line per km per phase is 0.1 ohm and the reactance per phase per km is 0.2 ohm. Solve for the regulation of the line.

A. 12.44%

B. 11.05%
C. 10.45%
D. 10.12%

Problem 805: EE Board April 1980

A 3-phase, 3-wire short transmission line having an impedance of 3.6 + j16 ohms per wire is used to supply an inductive load of 100 A at 0.707 pf and a capacitive load of 50 A at 0.50 pf. The receiving end voltage is 4160 volts per phase. Find percent regulation.

A. 25.46%
B. 20.33%
C. 26.34%
D. 23.71%

Problem 806:

It is desired to deliver 4,000 kW, three-phase at a distance of 25 miles. The load voltage being 33 kV, 60 Hz and the power factor of the load being 85% lagging. The reactance of the line per phase is 0.692 ohm per mile. The line loss is 10% of the power delivered. Determine voltage regulation of the line.

A. 11.2%
B. 12.3%
C. 13.2%
D. 14.8%

Problem 807: EE Board October 1998

A short 230 kV transmission line has an impedance of 5 cis 78 ohms. The sending end power is 100 MW at 230 kV and 85% power factor. What is the percent regulation of the line?

A. 3.6%
B. 1.5%
C. 2.2%
D. 0.77%

Problem 808: EE Board October 1983

A short sub-transmission line serves at its end an induction motor rated 500 HP, 0.88 pf, 0.90 efficiency, at 2,400 volts phase to neutral. If the transmission line has a resistance of 1.5 ohms and a reactance of 2.4 ohms per phase, solve for the voltage regulation of the line.

A. 6.77%
B. 7.02%
C. 6.48%
D. 6.25%

Problem 809:

A short 3-phase, 3-wire transmission line has an impedance of 2 + j6 ohms per wire, at the receiving end are connected a 3-phase inductive load drawing 5,000 kW at 0.8 pf, line to line voltage of 13,800 V and a capacitor bank drawing 200 A line current. Solve for the power at the sending end.

A. 5,562 kW
B. 5,146 kW
C. 5,361 kW
D. 5,274 kW

Problem 810:

A short 3-wire transmission line has an impedance of 2 + j5 ohms per wire. At the receiving end, a balanced 3-phase load and capacitor bank draw 3000 kVA, 0.71 pf lagging and 600 kVAR respectively at 8000 volts per phase to neutral. Determine the power loss of the transmission line.

A. 81.32 kW
B. 69.57 kW
C. 90.74 kW
D. 71.09 kW

Problem 811:

A 10-km, three-phase transmission line delivers power to a load rated 200 kW, 6.9 kV and at 80% lagging power factor. The resistance and reactance of each line are 0.8 and 1.2 ohms, respectvely. What percentage of the power generated is lost in the transmission line?

A. 9.53%
B. 8.05%
C. 8.22%
D. 7.46%

Problem 812: EE Board October 1993

Each conductor of a three, phase, 3-wire transmission line has impedance of 15 + j 20 ohms at 60 Hz. The voltage between line conductors at the sending end is 13,200 volts. The load connected to this line is balanced and takes 1000 kW at a lagging power factor. The current per conductor is 70 A. What is the load power factor?

A. 0.803 lagging
B. 0.824 lagging
C. 0.850 lagging
D. 0.815 lagging

Problem 813:

A 15 MW, 132 kV, 80% pf three phase load is to be serve by a transmission line having conductors whose resistance is 0.5 ohm/km. If the losses on the line shall not exceed 4.5%, how long must this line be?

A. 68 km
B. 70 km
C. 66 km
D. 75 km

Problem 814:

A 3-phase, 3-wire transmission line supplies at the receiving end an inductive load of 5,000 kW at 0.8 pf and the line to line voltage of 13,250 V and in addition a capacitor bank that draws a line current of 90 A. The effective impedance of the transmission line is 2 + j3 ohms per wire. Determine the sending end power.

A. 5317 kW
B. 5530 kW
C. 5255 kW
D. 5410 kW

Problem 815: EE Board April 1982

A short 3-phase, 3-wire line has a resistance of 5 ohms and reactance of 12 ohms per wire and transmit power to a 3-phase load drawing 1000 kW at 13,120 volts line to line, 0.8 pf lagging, 60 Hz. Solve for the sending end power factor.

A. 0.755 lagging
B. 0.745 lagging
C. 0.754 lagging
D. 0.773 lagging

Problem 816: EE Board October 1982, EE Board October 1987

A short 3-phase transmission line has a resistance of 3 ohms and an inductive reactance of 4 ohms. At the receiving end, the voltage is 13,250 volts line to line, with a load which draws 200 A at 0.707 lagging power factor and a capacitor bank drawing 96 A. Solve for the power factor at the sending end.

A. 0.965 lagging
B. 0.902 lagging
C. 0.928 lagging
D. 0.935 lagging

Problem 817: EE Board April 1985

The sending end voltage of a balanced 3-phase transmission line is 8410 volts, phase to neutral. The line current is 200 amperes with a sending end lagging power factor of 0.803. The receiving end voltage is 7600 volts, phase to neutral, with the line current having a lagging power factor of 0.85. Solve for the impedance of the transmission line.

A. 1.47 + j 5.15 Ω
B. 1.42 + j 5.15 Ω
C. 1.47 + j 5.04 Ω
D. 1.42 + j 5.04 Ω

Problem 818: EE Board April 1986

A short, 3-phase, 3-wire transmission line has a receiving end voltage of 4160 V phase to neutral and serving a balanced 3-phase load of 998,400 volt-amperes at 0.82 pf lagging. At the sending end the voltage is 4600 V, phase to neutral and the pf is 0.77 lagging. Solve the resistance of the line.

A. 1.345 Ω
B. 1.462 Ω
C. 1.457 Ω
D. 1.635 Ω

Problem 819: EE Board April 1984, EE Board April 1987

At the sending end of a 3-phase transmission line, the voltage is measured to be 2,540 volts phase to neutral and the line current to be 60 A at a lagging power factor of 0.75. At the receiving end, the voltage measured is 2,200 volts phase to neutral and the power factor is 0.80, likewise lagging current. Solve for the resistance and reactance of the line.

A. 2.2 + j 4
B. 2.4 + j 6
C. 2.0 + j 5
D. 2.1 + j 3

Problem 820:

A three-phase transmission line 5 km long delivers 2 MVA at a power factor of 0.80 lagging. The resistance and reactance per km of each conductor are 0.3 ohm and 0.6 ohm, respectively. Calculate the voltage at the sending end. Assume a transmission line loss of 7.2% of the power delivered to the load.

A. 4405.45 V
B. 4166.67 V
C. 4810.50 V
D. 4652.85 V

Problem 821: EE Board April 1993

A certain station in Luzon has 36 kV, 3-phase, is to supply 12 MW load at 30 kV and 0.8 power factor lagging over a 3-phase transmission line. Find the resistance of the line if the length of the transmission line is 4,000 meters & the efficiency is 90%.

A. 5.33 ohms
B. 4.45 ohms
C. 6.31 ohms
D. 3.56 ohms

Problem 822:

A three-phase, 3-wire transmission line has an impedance per wire of 3 + j7 ohms, the receiving end load is 1950 kW, 0.65 pf lagging with the line voltage of 13,200 V. Determine the efficiency of the line.

A. 92.64%
B. 95.21%
C. 90.24%
D. 93.05%

Problem 823: EE Board October 1980

A short line has a resistance of 4 ohms and a reactance of 12 ohms per wire and transmits power to a concentrated load of 1000 kVA, 13,200 volts, 3-phase, 60 Hz, 0.8 pf inductive load. Find its efficiency.

A. 93.30%
B. 96.95%
C. 95.75%
D. 97.21%

Problem 824: EE Board October 1998

A 230 kV transmission line is sending 100 MW power at 230 kV and 90% power factor. The impedance is 5 + j 20 and its capacitive reactance is 2500 ohms. Determine the receiving end voltage.

A. 221.72 kV
B. 222.83 kV
C. 226.15 kV
D. 224.28 kV

Problem 825: EE Board October 1996

A 3-phase, 60 Hz transmission line delivers 20 MVA to a load at 66 kV at 80% pf lagging. The total series impedance of each line is 15 + j 75 ohms. If a nominal "pi" circuit is used, what would be the transmission efficiency if the admittance is j0.0006 mhos?

A. 90.8%
B. 91.7%
C. 93.5%
D. 92.6%

Problem 826: EE Board March 1998

A 230 kV transmission line has impedance of 50 cis 78 ohms and a capacitive reactance of 1200 ohms. It transmits the power of a base load plant. On a certain dry season the sending end power is 100 MW at 235 kV and 95% power factor continuously for a period of one month. If cost of generation is 1.30 pesos per kW-hr, what is the cost of the line losses for the one month period?

A. P 565,000
B. P 12.2 million
C. P 5.6 million
D. P 2.3 million

ANSWER KEY

787. B	795. B	803. C	811. B	819. B
788. A	796. A	804. C	812. A	820. D
789. D	797. D	805. D	813. B	821. A
790. C	798. A	806. A	814. A	822. A
791. C	799. B	807. D	815. D	823. D
792. A	800. A	808. A	816. D	824. B
793. A	801. C	809. D	817. C	825. A
794. A	802. D	810. D	818. D	826. D

☑ RATING

36-40 - Topnotcher

28-35 - Passer

20-27 - Conditional

0-19 - Failed

Did you know that... in 1888, a wealthy inventor named George Westinghouse offered Nikola Tesla a deal he can't refuse, a cash of one million dollar plus royalties for the right of his polyphase AC motor patent? Tesla suggested to make the royalty one dollar per horsepower. The deal was done !

George Westinghouse was born in Central Bridge, New York, the son of a prosperous machine factory owner.

He served the Union army and navy during the Civil war and then attended Union College before striking out of his own. He invented the air brake system to stop trains. By the time he was 40 years old, he had formed the Westinghouse Air Brake Company. The knowledge he gained from his work with compressed air guided him in his work with electricity. He invented a reduction valve that permitted high-pressure gas to be delivered at a lower pressure at the point of use. In 1886, he organized the Westinghouse Electric Company, which he used as a base to advocate successfully the ac system.

One day in 1888 George Westinghouse walked into Tesla's lab and introduced himself. Tesla was then 32 years old while Westinghouse was 42 years old. Westinghouse listened to Tesla explaining his theories, watched his demonstrations about the polyphase induction motor.

He quickly made up his mind and uttered these words to Tesla, " I will give you one million dollar cash for your AC patents, plus royalties" Tesla replied, "Make that royalty one dollar per horsepower and it's a deal". The two men then shook hands and it was a done deal.

Westinghouse was one of America's greatest inventors and one of the true giants in US industry. He used his money to buy up patents in the electrical field.

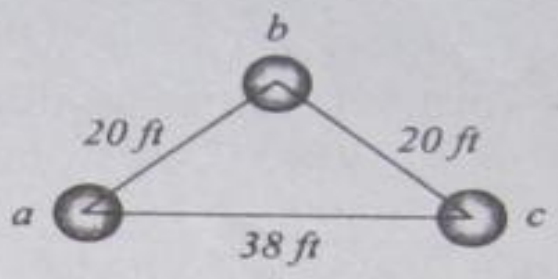
Solutions to Test 22

787

$D_s = 0.7788\,r = 0.7788(0.25)$

$D_s = 0.1947$ inch

788



$D_m = \sqrt[3]{(d_{ab})(d_{bc})(d_{ca})}$

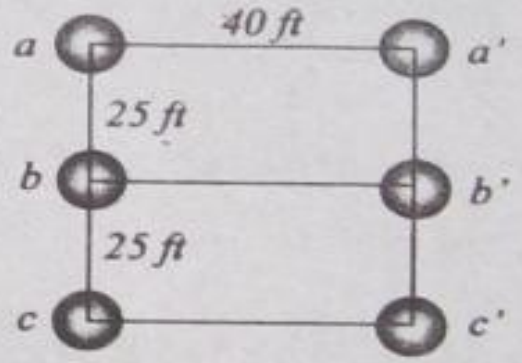
$= \sqrt[3]{(20)(20)(38)}$

$D_m = 24.77$ ft

789

$d_{ab'} = \sqrt{(40)^2 + (25)^2} = 47.17$

$d_{ac'} = \sqrt{(40)^2 + (50)^2} = 64.03$



$D_{AB} = \sqrt[4]{(d_{ab})(d_{ab'})(d_{a'b})(d_{a'b'})}$

$= \sqrt[4]{(25)(47.17)(47.17)(25)}$

$= 34.34$

$D_{BC} = \sqrt[4]{(d_{bc})(d_{bc'})(d_{b'c})(d_{b'c'})}$

$= \sqrt[4]{(25)(47.17)(47.17)(25)}$

$= 34.34$

$D_{CA} = \sqrt[4]{(d_{ca})(d_{ca'})(d_{c'a})(d_{c'a'})}$

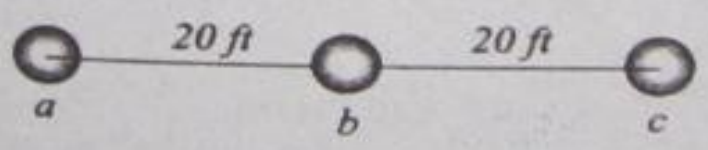
$= \sqrt[4]{(50)(64.03)(64.03)(50)}$

$= 56.58$

$D_m = \sqrt[3]{D_{AB}\,D_{BC}\,D_{CA}} = \sqrt[3]{(34.34)(34.34)(56.58)}$

$D_m = 40.56$ ft

790



$D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{(20)(20)(40)} = 25.1984$ ft

$L = 2 \times 10^{-7} \ln \dfrac{D_m}{D_s}$

$L = 2 \times 10^{-7} \ln \dfrac{25.1984}{0.0435} = 1.272355 \times 10^{-6}$ H/m

Note: Since frequency is not given, assume a standard electrical frequency of 60 Hz (w = 377).

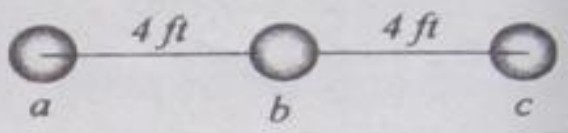
$$X_L = wL = 377(1.272355 \times 10^{-6}\ \text{H/m}) \times 100\ \text{mi} \times \frac{5280\ \text{ft}}{\text{mi}} \times \frac{1\text{m}}{3.281\ \text{ft}}$$

$X_L = 77.19\ \Omega$

$R = (0.0969\ \Omega/\text{mi})(100\ \text{mi}) = 9.69\ \Omega$

Thus, $Z_L = 9.69 + j77.19\ \Omega$

791 $D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{(4)(4)(8)} = 5.04\ \text{ft}$



$$L = 2 \times 10^{-7} \ln \frac{D_m}{D_s}$$

$$L = 2 \times 10^{-7} \ln \frac{5.04}{0.01668} = 1.1422 \times 10^{-6}\ \text{H/m}$$

Note: Since frequency is not given, assume a standard electrical frequency of 60 Hz (w = 377).

$$X_L = wL = 377(1.1422 \times 10^{-6}\ \text{H/m}) \times 5\ \text{mi} \times \frac{5280\ \text{ft}}{\text{mi}} \times \frac{1\text{m}}{3.281\ \text{ft}}$$

$X_L = 3.46\ \Omega$

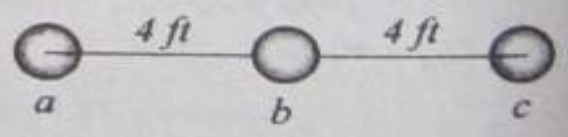
792 $L = 4 \times 10^{-7} \ln \frac{d}{r'} = 4 \times 10^{-7} \ln \frac{d}{0.7788\ r}$

$$= 4 \times 10^{-7} \ln \frac{1.25}{0.7788(0.004)} = 2.4\ \mu\text{H/m}$$

$L = 2.4\ \mu\text{H/m}\ (15{,}000\ \text{m})$

$L = 0.036\ \text{H}$

793 $D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{(4)(4)(8)} = 5.04\ \text{ft}$



$$L = 2 \times 10^{-7} \ln \frac{D_m}{D_s}$$

$$L = 2 \times 10^{-7} \ln \frac{5.04}{0.0244} = 1.066 \times 10^{-6}\ \text{H/m}$$

Note: Since frequency is not given, assume a standard electrical frequency of 60 Hz (w = 377).

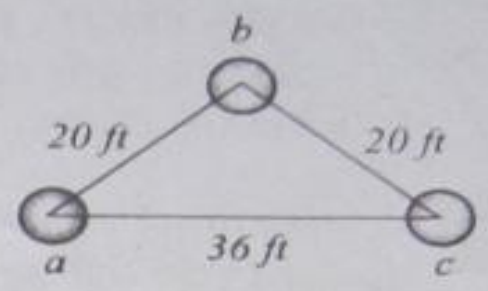
$$X_L = wL = 377(1.066 \times 10^{-6}\ \text{H/m}) \times 5\ \text{km} \times \frac{1000\ \text{ft}}{\text{km}} = 2.01\ \Omega$$

$$R = 0.306\ \Omega/\text{mi} \times 5\ \text{km} \times \frac{1\ \text{mi}}{1.609\ \text{km}} = 0.951\ \Omega$$

$$Z_L = 0.951 + j2.01 = 2.22\angle 64.68^\circ$$

$$Z_L = 2.22\ \text{cis}\ 65^\circ$$

794 $D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{(20)(20)(36)} = 24.328\ \text{ft}$



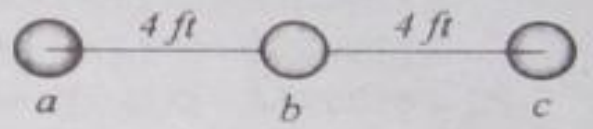
$$C_n = \frac{2\pi\Sigma_o}{\ln\frac{D_m}{D_s}} = \frac{2\pi(8.854 \times 10^{-12})}{\ln\frac{24.328}{0.0217}} = 7.922\ \text{pF/m}$$

$$C_n = 7.922 \times 10^{-12}\ \frac{F}{m} \times \frac{1\ \text{m}}{3.281\ \text{ft}} \times \frac{5280\ \text{ft}}{1\ \text{mi}} = 12.748\ \text{nF/mi}$$

$$\beta = wC = 377(12.748 \times 10^{-9})$$

$$\beta = 4.81 \times 10^{-6}\ \text{mho per mi}$$

795 $D_s = \sqrt[3]{D_{AA}D_{BB}D_{CC}}$



$$D_{AA} = D_{BB} = D_{CC} = r = 0.004\ \text{m}$$

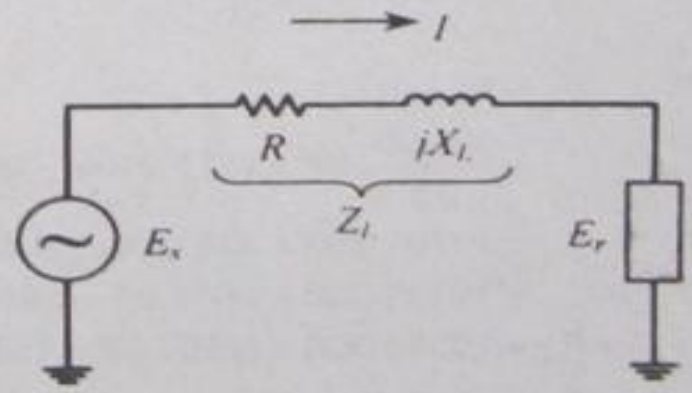
$$D_s = \sqrt[3]{(0.004)(0.004)(0.004)} = 0.004\ \text{m}$$

$$D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[3]{(4)(4)(8)} = 5.04\ \text{m}$$

$$C_n = \frac{2\pi\Sigma_o}{\ln\frac{D_m}{D_s}} = \frac{2\pi(8.854 \times 10^{-12})}{\ln\frac{5.04}{0.004}}$$

$$C_n = 7.793\ \text{pF/m}$$

796 $I = \frac{P}{Vpf} = \frac{3{,}000{,}000}{11000(1.0)} = 272.73\ \text{A}$



$$\theta = \cos^{-1}1.0 = 0^\circ$$

$$Z_L = 1.5 + j0.3 = 1.53\angle 11.3^\circ$$

$$E_s = E_r + IZ_L$$
$$= 11000 + 272.73\angle 0^\circ(1.53\angle 11.3^\circ)$$
$$= 11000 + 417.277\angle 11.3^\circ$$
$$= 11000 + 409.188 + j81.763$$
$$= 11409.188 + j81.763$$

$$E_s = 11409.48\angle 0.41^\circ\ \text{V}$$

797 $\theta = \cos^{-1} 0.8 = 36.87°$

$E_s = E_r + IZ_L$

$= 3300\angle 0° + (120\angle -36.87°)(5\angle 60°)$

$= 3300 + 600\angle 23.13°$

$= 3300 + 551.77 + j235.69$

$= 3851.77 + j235.69$

$E_s = 3858.97\angle 3.5°$ V

798 $Z_L = 3 + j8 = 8.544\angle 69.44°$

$\theta_R = \cos^{-1} 0.9 = 25.84°$

$E_s = E_r + I_r Z_L$

$= \dfrac{13{,}500}{\sqrt{3}}\angle 0° + (60\angle -25.84°)(8.544\angle 69.44°)$

$= 7794.228 + 512.64\angle 43.6°$

$= 7794.228 + 371.24 + j353.526$

$E_s = 8165.468 + j353.526 = 8173.117\angle 2.479°$

$E_{sLL} = \sqrt{3}(8173.117)$

$E_{sLL} = 14156$ V

799 $I_r = \dfrac{S_r}{\sqrt{3}E_{rLL}} = \dfrac{12{,}000{,}000}{\sqrt{3}(22000)} = 314.918$ A

$Z_L = 3 + j10 = 10.44\angle 73.3°$

$\theta_r = \cos^{-1} 0.8 = 36.87°$

$E_s = E_r + I_r Z_L$

$= \dfrac{22000}{\sqrt{3}}\angle 0° + (314.918\angle -36.87°)(10.44\angle 73.3°)$

$= 12701.706 + 3287.74\angle 36.43°$

$= 12701.706 + 2645.26 + j1952.39$

$E_s = 15346.966 + j1952.39 = 15470.655\angle 7.25°$

$E_{sLL} = \sqrt{3}(15470.655)$

$E_{sLL} = 26795.96$ V

800

$Z_L = 5 + j12 = 13\angle 67.38°$

$\theta_r = \cos^{-1} 0.8 = 36.869°$

$$I_r = \frac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \frac{1{,}000{,}000}{\sqrt{3}(13{,}120)(0.8)} = 55\text{ A}$$

$$E_s = E_r + I_r Z_L$$

$$= \frac{13{,}120}{\sqrt{3}}\angle 0° + (55\angle -36.869°)(13\angle 67.38°)$$

$$= 7574.835 + 715\angle 30.511°$$

$$E_s = 7574.835 + 616 + j363 = 8198.874\angle 2.537°\text{ V}$$

$$E_{sLL} = \sqrt{3}(8198.874)$$

$$E_{sLL} = 14{,}200.866\text{ V}$$

801

Let: E_r = receiving end voltage per phase in kV

$$I_r = \frac{P_r}{3E_r pf_r} = \frac{3{,}800}{3(0.75)E_r} = \frac{1688.89}{E_r}$$

$Z_L = 4.15 + j4.32$

$\theta = \cos^{-1} 0.75 = 41.409°$

$$E_s = E_r + I_r Z_L$$

$$\frac{36}{\sqrt{3}}\angle\delta = E_r\angle 0° + \left(\frac{1688.89}{E_r}\angle -41.409°\right)\left(\frac{4.15 + j4.32}{1000}\right)$$

$$20.784\angle\delta = E_r + \left(\frac{1.689\,(\cos 41.409°)}{E_r} - j\frac{1689\,(\sin 41.409°)}{E_r}\right)$$

$$(4.15 + j4.32)$$

$$20.784\angle\delta = E_r + \left(\frac{1.267}{E_r} - j\frac{1.117}{E_r}\right)(4.15 + j4.32)$$

$$20.784\angle\delta = E_r + \frac{5.258}{E_r} + j\frac{5.473}{E_r} - j\frac{4.635}{E_r} + \frac{4.825}{E_r}$$

$$20.784\angle\delta = \left(E_r + \frac{10.083}{E_r}\right) + j\frac{0.838}{E_r}$$

Using Pythagorean relations:

$$(20.784)^2 = \left(E_r + \frac{10.083}{E_r}\right)^2 + \left(\frac{0.838}{E_r}\right)^2$$

$$(20.784)^2 = E_r^{\,2} + 20.166 + \frac{102.369}{E_r^{\,2}}$$

$$411.808 = E_r^{\,2} + \frac{102.369}{E_r^{\,2}}$$

$$411.808\,E_r^{\,2} = E_r^{\,4} + 102.369$$

$$E_r^{\,4} - 411.808\,E_r^{\,2} + 102.369 = 0$$

Using the quadratic formula :

$$E_r^{\,2} = \frac{411.808 \pm \sqrt{(411.808)^2 - 4(1)(102.369)}}{2(1)} = \frac{411.808 \pm 411.31}{2}$$

$$E_r^{\,2} = 411.559$$

$$E_r = 20.286\text{ kV}$$

$$E_{rLL} = \sqrt{3}(20.286)$$

$$E_{rLL} = 35.136\text{ kV}$$

802 Note: Assume E_s as reference.

$$I_s = \frac{P_s}{\sqrt{3}E_{sLL}\,pf_s} = \frac{100 \times 10^6}{\sqrt{3}(230{,}000)(0.85)} = 295.32\text{ A}$$

$$\theta_s = \cos^{-1} 0.85 = 31.788°$$

$$E_s = E_r + I_s Z_L$$

$$E_r = E_s - I_s Z_L$$

$$= \frac{230{,}000}{\sqrt{3}}\angle 0° - (295.32\angle -31.788°)(5\angle 78°)$$

$$= 132{,}790.56 - (1476.6\angle 46.212°$$

$$= 132{,}790.56 - (1021.795 + j1065.965)$$

$$E_r = 131773.076\angle 0.463°\text{ V}$$

$$E_{rLL} = \sqrt{3}(131773.076)$$

$$E_{rLL} = 228.237\text{ kV}$$

803 $$I_r = \frac{S_r}{\sqrt{3}E_{rLL}} = \frac{15{,}000{,}000}{\sqrt{3}(34500)} = 251.02\text{ A}$$

$$Z_L = (0.12 + j0.457)15 = 1.8 + j6.855 = 7.087\angle 75.287°$$

$$\theta_r = \cos^{-1} 1.0 = 0°$$

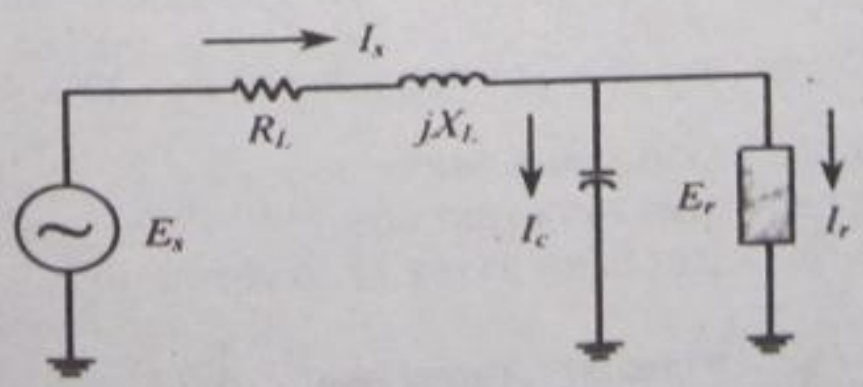
$$E_s = E_r + I_r Z_L$$
$$= \frac{34500}{\sqrt{3}}\angle 0^\circ + (251.02\angle 0^\circ)(7.087\angle 75.287^\circ)$$
$$= 19918.58 + 1778.978\angle 75.287^\circ$$
$$= 19918.58 + 451.82 + j1720.645$$
$$E_s = 20370.4 + j1720.645 = 20442.94\angle 4.83^\circ \text{ V}$$
$$E_{sLL} = \sqrt{3}(20442.94)$$
$$E_{sLL} = 35{,}408.21\text{V}$$

804 $$I_r = \frac{P_r}{\sqrt{3}E_{rLL}pf_r} = \frac{5{,}000{,}000}{\sqrt{3}(11000)(0.8)} = 328.04 \text{ A}$$
$$Z_L = (0.1 + j0.2)10 = 1 + j2 = 2.236\angle 63.435^\circ$$
$$\theta_r = \cos^{-1} 0.8 = 36.87^\circ$$

$$E_s = E_r + I_r Z_L$$
$$= \frac{11000}{\sqrt{3}}\angle 0^\circ + (328.04\angle -36.87^\circ)(2.236\angle 63.435^\circ)$$
$$= 6350.85 + 733.497\angle 26.565^\circ$$
$$= 6350.85 + 656.06 + j328.03$$
$$E_s = 7006.91 + j328.03 = 7014.58\angle 2.68^\circ \text{ V}$$

$$\%VR = \frac{E_s - E_r}{E_r} = \frac{7014.58 - 6350.85}{6350.85} \times 100\%$$
$$\%VR = 10.45\%$$

805



$$Z_L = 3.6 + j16 = 16.4\angle 77.32^\circ$$
$$\theta_r = \cos^{-1} 0.707 = 45^\circ$$
$$\theta_c = \cos^{-1} 0.5 = 60^\circ$$

$I_s = I_r + I_c = 100\angle -45° + 50\angle 60°$

$= 70.71 - j70.71 + 25 + j43.3$

$I_s = 95.71 - j27.41 = 99.557\angle -15.98°$ A

$E_s = E_r + I_r Z_L$

$= 4160\angle 0° + (99.557\angle -15.98°)(16.4\angle 77.32°)$

$= 4160 + 1632.73\angle 61.34°$

$= 4160 + 783.075 + j1432.689$

$E_s = 4943.075 + j1432.689 = 5146.51\angle 16.163°$ V

$$\%VR = \frac{E_s - E_r}{E_r} = \frac{5146.51 - 4160}{4160} \times 100\%$$

$\%VR = 23.714\%$

806 $$I_r = \frac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \frac{4{,}000{,}000}{\sqrt{3}(33{,}000)(0.85)} = 82.33 \text{ A}$$

$\theta_r = \cos^{-1} 0.85 = 31.788°$

$P_{Loss} = 3I_r^2 R_L$

$$R_L = \frac{P_{Loss}}{3I_r^2} = \frac{0.1(4{,}000{,}000)}{3(82.33)^2} = 19.67\ \Omega$$

$X_L = 0.692(25) = 17.3\ \Omega$

$Z_L = R_L + jX_L = 19.67 + j17.3 = 26.195\angle 41.33°$

$E_s = E_r + I_r Z_L$

$$= \frac{33{,}000}{\sqrt{3}}\angle 0° + (82.33\angle -31.788°)(26.195\angle 41.33°)$$

$= 19052.558 + 2156.634\angle 9.542°$

$= 19052.558 + 2126.795 + j357.506$

$E_s = 21179.353 + j357.506 = 21182.37\angle 0.967°$ V

$$\%VR = \frac{E_s - E_r}{E_r} = \frac{21{,}182.37 - 19{,}052.558}{19{,}052.558} \times 100\%$$

$\%VR \cong 11.2\%$

807 $I_r = \dfrac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \dfrac{100{,}000{,}000}{\sqrt{3}(230{,}000)(0.85)} = 295.32\text{ A}$

$\theta_R = \cos^{-1} 0.85 = 31.788°$

$E_s = E_r + I_r Z_L$

$= \dfrac{230{,}000}{\sqrt{3}}\angle 0° + (295.32\angle -31.788°)(5\angle 78°)$

$= 132790.56 + 1476.6\angle 46.212°$

$= 132790.56 + 1021.795 + j1065.956$

$E_s = 133812.355 + j1065.956 = 133816.6\angle 0.456°$

$\%VR = \dfrac{E_s - E_r}{E_r} = \dfrac{133816.6 - 132790.56}{132790.56} \times 100\%$

$\%VR = 0.7726\%$

808 $P_r = \dfrac{P_m}{\eta} = \dfrac{500(746)}{0.9} = 414{,}444.44\text{ W}$

$I_r = \dfrac{P_r}{3E_r pf_r} = \dfrac{414{,}444.44}{3(2400)(0.88)} = 65.41\text{ A}$

$\theta_r = \cos^{-1} 0.88 = 28.358°$

$Z_L = 1.5 + j2.4 = 2.83\angle 58°$

$E_s = E_r + I_r Z_L$

$= 2400\angle 0° + (65.41\angle -28.358°)(2.83\angle 58°)$

$= 2400 + 185.11\angle 29.642°$

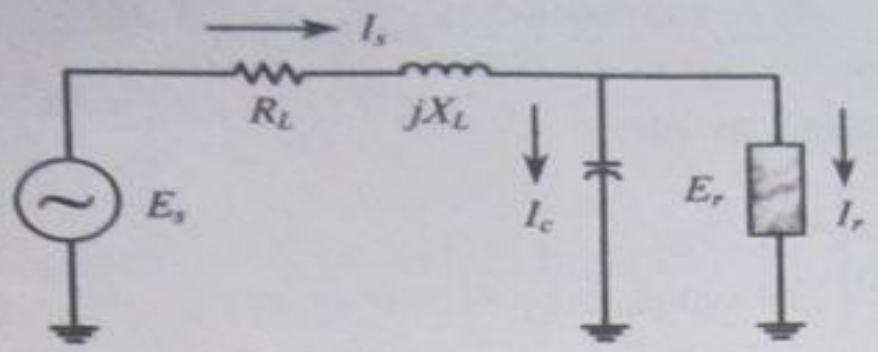
$= 2400 + 160.885 + j91.55$

$E_s = 2560.885 + j91.55 = 2562.52\angle 2.047°\text{ V}$

$\%VR = \dfrac{E_s - E_r}{E_r} = \dfrac{2562.52 - 2400}{2400} \times 100\%$

$\%VR = 6.77\%$

809



$$I_r = \frac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \frac{5{,}000{,}000}{\sqrt{3}(13{,}800)(0.8)} = 261.48\text{ A}$$

$$\theta_r = \cos^{-1} 0.8 = 36.87°$$

$$I_s = I_c + I_r$$
$$= 200\angle 90° + 261.48\angle -36.87°$$
$$= j200 + 209.183 - j156.888$$
$$I_s = 209.183 + j43.112 = 213.58\angle 11.645°\text{ A}$$

$$P_{loss} = 3I_s^2R_L = 3(213.58)^2(2) = 273.698\text{ kW}$$
$$P_s = P_r + P_{loss}$$
$$P_s = 5{,}000 + 273.698$$
$$P_s = 5{,}273.698\text{ kW}$$

810 Note: Refer to the circuit diagram of problem #823.

$$I_r = \frac{S_r}{3E_r} = \frac{3{,}000{,}000}{3(8000)} = 125\text{ A}$$

$$\theta_r = \cos^{-1} 0.71 = 44.765°$$

$$I_c = \frac{Q_c}{3E_r} = \frac{600{,}000}{3(8000)} = 25\text{ A}$$

$$I_s = I_c + I_r$$
$$= 25\angle 90° + 125\angle -44.765°$$
$$= j25 + 88.75 - j88.025$$
$$I_s = 88.75 - j63.025 = 108.85\angle -35.38°\text{ A}$$
$$P_{loss} = 3I_s^2R_L = 3(108.85)^2(2)$$
$$P_{loss} \cong 71.09\text{ kW}$$

811 $I_r = \dfrac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \dfrac{2{,}000{,}000}{\sqrt{3}(6900)(0.8)} = 209.185\text{ A}$

$Z_L = (0.2 + j1.2)10 = 2 + j12$

$P_{loss} = 3I_r^2R_L = 3(209.185)^2(2) = 262.55\text{ kW}$

$P_s = P_r + P_{loss} = 3000 + 262.55 = 3262.55\text{ kW}$

$\%P_{loss} = \dfrac{P_{loss}}{P_s} = \dfrac{262.55}{3262.55} \times 100\%$

$\%P_{loss} \cong 8.05\%$

812 Note: Assume E_s as the reference

$P_{loss} = 3I_s^2R_L = 3(70)^2(15) = 220.5\text{ kW}$

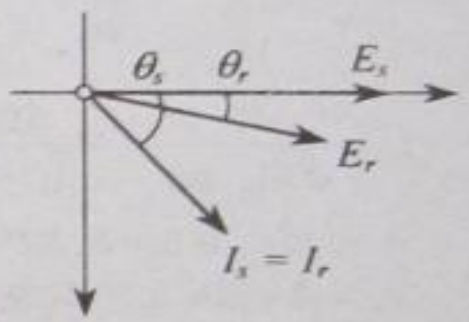
$P_s = P_r + P_{loss} = 1000 + 220.5 = 1220.5\text{ kW}$

$P_s = \sqrt{3}E_{sLL}I_s\cos\theta_s$

$\cos\theta_s = \dfrac{P_s}{\sqrt{3}E_{sLL}I_s} = \dfrac{1220500}{\sqrt{3}(13200)(70)} = 0.7626$

$\theta_s = \cos^{-1}0.7626 = 40.306°$

$Z_L = 15 + j20 = 25\angle 53.13°$



$E_s = E_r + I_sZ_L$

$E_r = E_s - I_sZ_L$

$= \dfrac{13200}{\sqrt{3}}\angle 0° - (70\angle -40.306°)(25\angle 53.13°)$

$= 7621.023 - 1750\angle 12.824°$

$= 7621.023 - (1706.348 + j388.424)$

$= 5914.675 - j388.424$

$E_r = 5927.415\angle -3.757°$

Note: Refer to the vector diagram.

$pf_r = \cos\theta_r = \cos(40.306° - 3.757°)$

$pf_r = 0.803$ lagging

813 $I_r = \dfrac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \dfrac{15{,}000{,}000}{\sqrt{3}(132{,}000)(0.8)} = 82\text{ A}$

$\theta_r = \cos^{-1}0.8 = 36.87°$

$P_s = P_r + P_{loss}$

$P_s = 15 + 0.045\,P_s$

$P_s = 15.706\text{ MW}$

$P_{loss} = 0.045(15.706) = 0.706\text{ MW} = 706\text{ kW}$

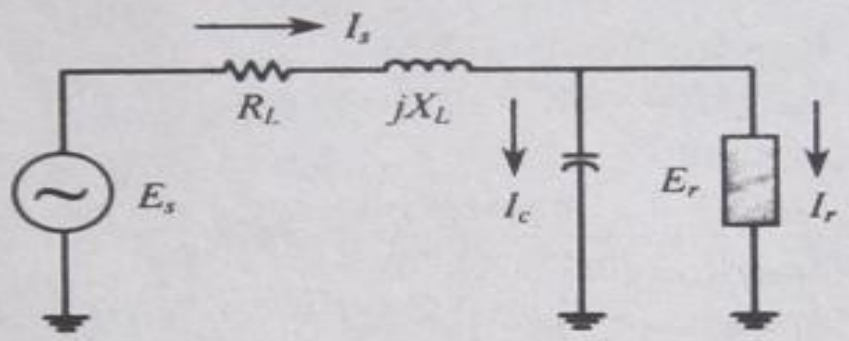
$P_{loss} = 3I_r^2 R_L$

$R_L = \dfrac{P_{loss}}{3I_r^2} = \dfrac{706{,}000}{3(82)^2} = 35\,\Omega$

Length of line $= \dfrac{35}{0.5}$

Length of the line = 70 km

814



$I_r = \dfrac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \dfrac{5{,}000{,}000}{\sqrt{3}(13{,}250)(0.8)} = 272.335\text{ A}$

$\theta_r = \cos^{-1} 0.8 = 36.87°$

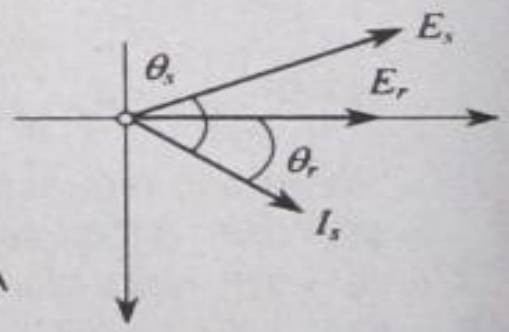
$Z_L = 2 + j3 = 3.605\angle 56.31°$

$I_s = I_c + I_r$

$= 90\angle 90° + 272.335\angle -36.87°$

$= j90 + 217.867 - j163.401$

$I_s = 217.867 - j73.401 = 229.89\angle -18.62°\text{A}$



$E_s = E_r + I_s Z_L$

$= \dfrac{13250}{\sqrt{3}}\angle 0° + (229.89\angle -18.62°)(3.605\angle 56.31°)$

$= 7649.89 + 828.753\angle 37.69°$

$= 7649.89 + 655.817 + j506.69$

$E_s = 8305.707 + j506.69$

$E_s = 8321.148\angle 3.491°\text{ V}$

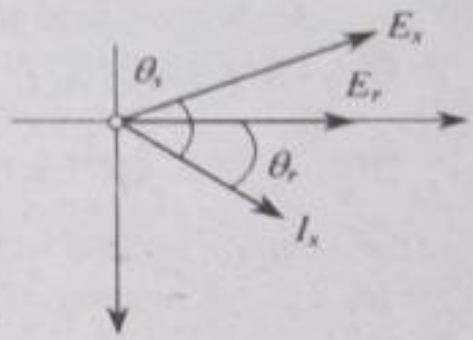
Note: Refer to the vector diagram.

$\theta_s = 18.62° + 3.491° = 22.11°$
$P_s = 3E_sI_s\cos\theta_s = 3(8321.148)(229.89)\cos 22.11°$
$P_s = 5316.828$
$P_s \cong 5317\text{ kW}$

Another solution:

$P_{loss} = 3I_s^2R_L = 3(229.89)^2(2) = 317.096\text{ kW}$
$P_s = P_r + P_{loss} = 5000 + 317.096$
$P_s = 5317.096\text{ kW}$

815 $I_r = \dfrac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \dfrac{1{,}000{,}000}{\sqrt{3}(13{,}120)(0.8)} = 55\text{ A}$



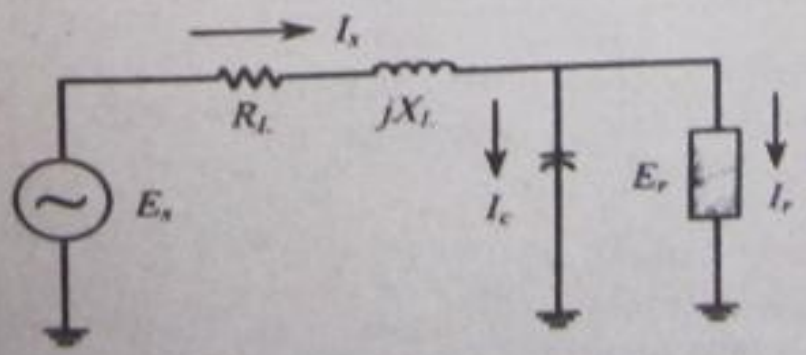
$\theta_r = \cos^{-1}0.8 = 36.87°$
$Z_L = 5 + j12 = 13\angle 67.38°$

$E_s = E_r + I_rZ_L$
$= \dfrac{13120}{\sqrt{3}}\angle 0° + (55\angle -36.87°)(13\angle 67.38°)$
$= 7574.835 + 715\angle 30.51°$
$= 7574.835 + 616 + j363$
$E_s = 8190.835 + j363 = 8198.874\angle 2.537°\text{ V}$

Note: Refer to the vector diagram.

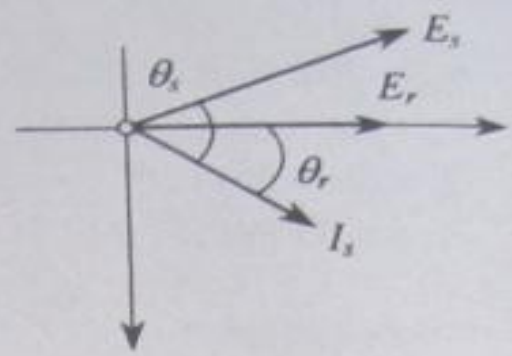
$pf_s = \cos\theta_s = \cos(36.87° + 2.537°)$
$pf_s = 0.7726$ lagging

816



$\theta_r = \cos^{-1} 0.707 = 45°$

$Z_L = 3 + j4 = 5\angle 53.13°$



$I_s = I_c + I_r$

$= 96\angle 90° + 200\angle -45°$

$= j96 + 141.42 - j141.42$

$I_s = 141.42 - j45.42 = 148.53\angle -17.805° A$

$E_s = E_r + I_s Z_L$

$= \dfrac{13250}{\sqrt{3}}\angle 0° + (148.53\angle -17.805°)(5\angle 53.13°)$

$= 7649.89 + 742.65\angle 35.325°$

$= 7649.89 + 605.917 + j429.41$

$= 8255.807 + j429.41$

$E_s = 8266.966\angle 2.977° V$

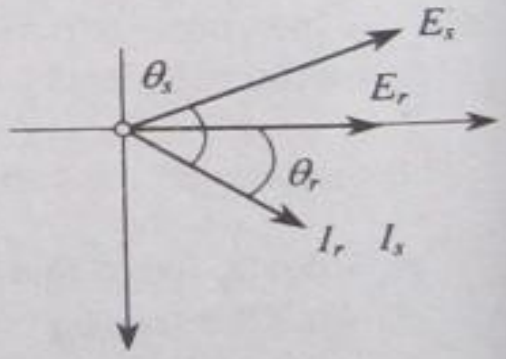
Note: Refer to the vector diagram.

$pf_s = \cos\theta_s = \cos(17.805° + 2.977°)$

$pf_s = 0.935$ lagging

817 $\theta_s = \cos^{-1} 0.803 = 36.582°$

$\theta_r = \cos^{-1} 0.850 = 31.788°$



Note: Refer to the vector diagram

$E_s = 8410\angle(36.582° - 31.788°)$

$= 8410\angle 4.794° = 8380.578 + j702.853$

$E_s = E_r + I_r Z_L$

$Z_L = \dfrac{E_s - E_r}{I_r} = \dfrac{8380.578 + j702.853 - 7600}{200\angle -31.788°}$

$= \dfrac{780.578 + j702.853}{200\angle -31.788°} = \dfrac{1050.383\angle 42°}{200\angle -31.788°} = 5.252\angle 73.788°$

$Z_L = 1.466 + j5.043\ \Omega$

818 $I_r = I_s = \dfrac{S_r}{3E_r} = \dfrac{998{,}400}{3(4160)} = 80$ A

$P_r = S_r pf_r = (998{,}400)(0.82) = 818{,}688$ W

$P_s = 3E_sI_s\cos\theta = 3(4600)(80)(0.77) = 850{,}080\ W$

$P_{loss} = P_s - P_r = 850{,}080 - 818{,}688 = 31{,}392\ W$

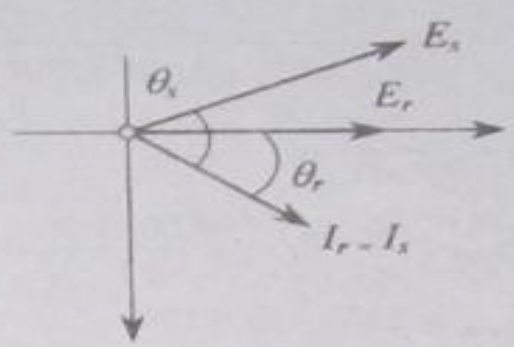
$P_{loss} = 3I_r^2R_L$

$R_L = \dfrac{P_{loss}}{3I_r^2} = \dfrac{31392}{3(80)^2}$

$R_L = 1.635\ \Omega$

819 $\theta_s = \cos^{-1}0.75 = 41.409°$

$\theta_r = \cos^{-1}0.80 = 36.869°$



Note: Refer to the vector diagram.

$E_s = 2540\angle(41.409° - 36.869°)$

$= 2540\angle 4.54°$

$E_s = 2532.03 + j201.054$

$E_s = E_r + I_rZ_L$

$Z_L = \dfrac{E_s - E_r}{I_r} = \dfrac{2532.03 + j201.054 - 2200}{60\angle -36.869°}$

$Z_L = \dfrac{332.03 + j201.054}{60\angle -36.869°} = \dfrac{388.158\angle 31.196°}{60\angle -36.869°} = 6.469\angle 68.065°$

$Z_L = 2.416 + j6\ \Omega$

820 $P_r = S_rpf_r = (2(0.8) = 1.6\ MW$

$P_{loss} = 0.072(1.6) = 0.1152\ MW = 115{,}200\ W$

$Z_L = (0.3 + j0.6)5 = 1.5 + j3$

$Z_L = 3.354\angle 63.435°$

$P_{loss} = 3I_r^2R_L$

$I_r = \sqrt{\dfrac{P_{loss}}{3R_L}} = \sqrt{\dfrac{115{,}200}{3(1.5)}} = 160\ A$

$S_r = 3E_rI_r$

$E_r = \dfrac{S_r}{3I_r} = \dfrac{2{,}000{,}000}{3(160)} = 4166.67\ V$

$\theta_r = \cos^{-1}0.8 = 36.87°$

$$E_s = E_r + I_r Z_L$$
$$= 4166.67\angle 0° + (160\angle -36.87°)(3.354\angle 63.435°)$$
$$= 4166.67 + 536.64\angle 26.565°$$
$$= 4166.67 + 479.986 + j240$$
$$= 4646.656 + j240$$
$$E_s = 4652.85\angle 2.956° \text{ V}$$

821 $$I_r = \frac{P_r}{\sqrt{3}E_{\eta LL}\,pf} = \frac{12{,}000{,}000}{\sqrt{3}(30{,}000)(0.8)} = 288.675 \text{ A}$$

$$\eta = \frac{P_r}{P_r + P_{loss}}$$

$$P_{loss} = \frac{P_r}{\eta} - P_r = \frac{12{,}000{,}000}{0.9} - 12{,}000{,}000$$

$$P_{loss} = 1{,}333{,}333.33 \text{ W}$$

$$P_{loss} = 3I_r^2 R_L$$

$$R_L = \frac{P_{loss}}{3I_r^2} = \frac{1{,}333{,}333.33}{3(288.675)^2}$$

$$R_L = 5.33\ \Omega$$

822 $$I_r = \frac{P_r}{\sqrt{3}E_{\eta LL}\,pf} = \frac{1{,}950{,}000}{\sqrt{3}(13{,}200)(0.65)} = 131.216 \text{ A}$$

$$\theta_r = \cos^{-1} 0.65 = 49.458°$$

$$P_{loss} = 3I_r^2 R_L = 3(131.216)^2(3) \cong 155 \text{ kW}$$

$$P_s = P_r + P_{loss} = 1950 + 155 = 2105 \text{ kW}$$

$$\eta = \frac{P_r}{P_s} = \frac{1950}{2105} \times 100\%$$

$$\eta \cong 92.64\%$$

823 $$I_r = \frac{S_r}{\sqrt{3}E_{\eta LL}} = \frac{1{,}000{,}000}{\sqrt{3}(13{,}200)} = 43.738 \text{ A}$$

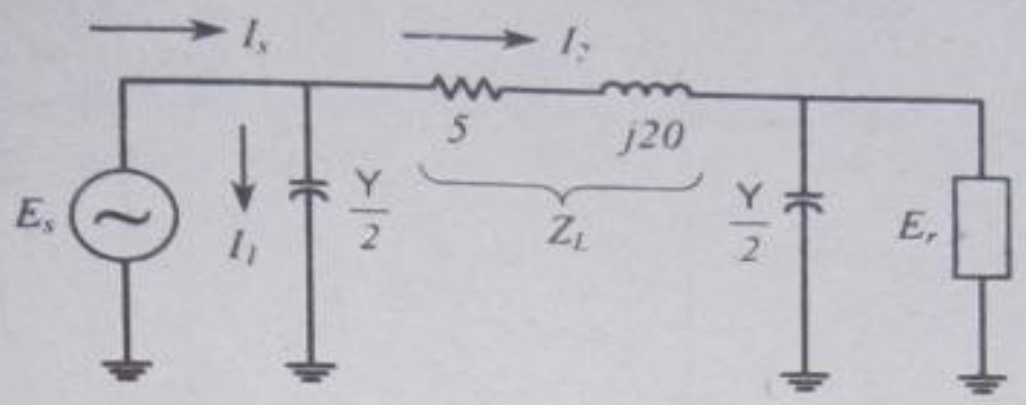
$$\theta_r = \cos^{-1} 0.8 = 36.87°$$

$$P_{loss} = 3I_r^2 R_L = 3(43.738)^2(4) = 22.956 \text{ kW}$$

$$P_r = S_r pf_r = 1000(0.8) = 800 \text{ kW}$$

$$P_s = P_r + P_{loss} = 800 + 22.956 = 822.956 \text{ kW}$$

$$\eta = \frac{P_r}{P_s} = \frac{800}{822.956} \times 100\% = 97.21\%$$



Nominal pi equivalent circuit

Note: Assume a nominal pi equivalent circuit. Also, assume E_s as the reference.

$$\theta_s = \cos^{-1} 0.9 = 25.84°$$

$$I_s = \frac{P_s}{\sqrt{3}E_{s_{LL}} pf_s} = \frac{100 \times 10^6}{\sqrt{3}(230{,}000)(0.9)} = 278.913 \text{ A}$$

$$I_s = 278.913\angle -25.84° = 251.026 - j121.567 \text{ A}$$

$$I_1 = E_s\left(\frac{Y}{2}\right) = \left(\frac{230{,}00}{\sqrt{3}}\angle 0°\right)\left(\frac{1}{2(2500)}\angle 90°\right) = 26.558\angle 90° \text{ A}$$

$$I_2 = I_s - I_1 = 251.026 - j121.567 - j26.558$$

$$= 251.026 - j148.125$$

$$I_2 = 291.47\angle -30.544° \text{ A}$$

$$Z_L = 5 + j20 = 20.615\angle 75.963°$$

$$E_r = E_s - I_2 Z_L$$

$$= \frac{230{,}00}{\sqrt{3}}\angle 0° - (291.47\angle -30.544°)(20.615\angle 75.963°)$$

$$= 132790.56 - 6008.65\angle 45.419°$$

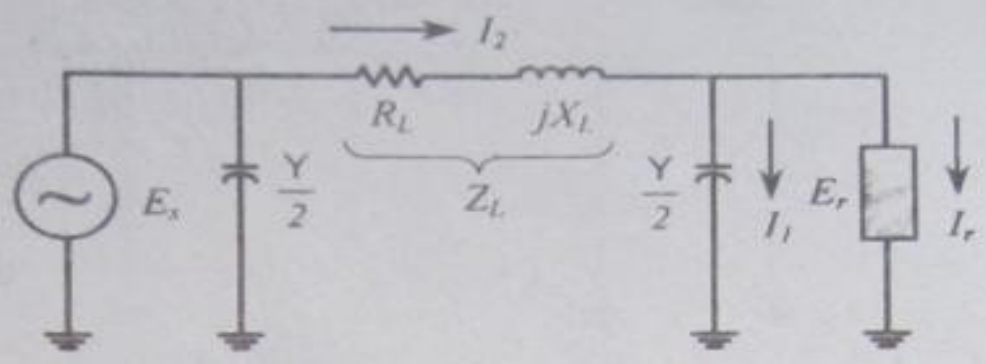
$$= 132790.56 - (4217.57 + j4279.714)$$

$$= 128572.99 - j4279.714 = 128644.198\angle 1.906°$$

$$E_{r_{LL}} = \sqrt{3}(128644.198)$$

$$E_{r_{LL}} \cong 222.82 \text{ kV}$$

825



Nominal pi equivalent circuit

Note: Assume E_r as the reference.

$$\theta_r = \cos^{-1} 0.8 = 36.87°$$

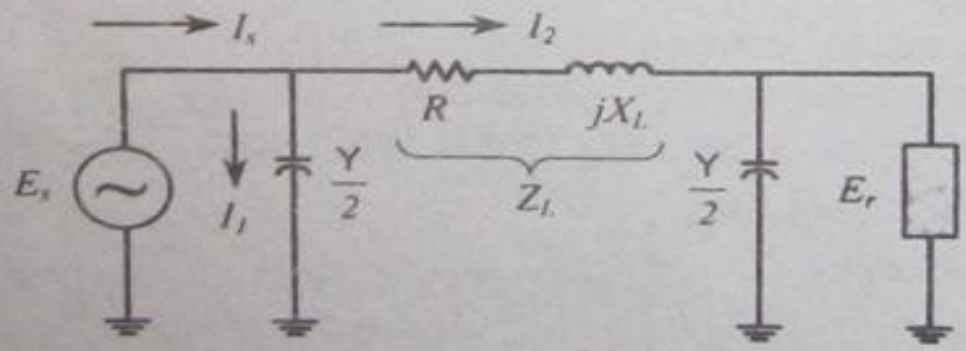
$$I_r = \frac{P_r}{\sqrt{3}E_{rLL}\,pf_r} = \frac{20 \times 10^6}{\sqrt{3}(66{,}000)(0.8)} = 218.693 \text{ A}$$

$$I_1 = E_r\left(\frac{Y}{2}\right) = \left(\frac{66{,}000}{\sqrt{3}}\angle 0°\right)\left(\frac{0.0006}{2}\angle 90°\right) = 11.43\angle 90° \text{ A}$$

$$I_2 = I_r + I_1 = 218.693\angle -36.87 + 11.43\angle 90°$$
$$= 174.95 - j131.216 + j11.43$$
$$I_2 = 174.95 - j119.786 = 212.028\angle -34.398° \text{ A}$$

$$P_{loss} = 3I_2^2 R_L = 3(212.028)^2(15) = 2.023 \text{ MW}$$
$$P_s = P_r + P_{loss} = 20 + 2.023 = 22.023 \text{ MW}$$
$$\eta = \frac{P_r}{P_s} = \frac{20}{22.023} \times 100\%$$
$$\eta = 90.814\%$$

826



Nominal pi equivalent circuit

Note: Assume a nominal pi equivalent circuit. Also, assume E_s as the reference and with a lagging pf.

$$I_s = \frac{P_s}{\sqrt{3}E_{sLL}\,pf_s} = \frac{100 \times 10^6}{\sqrt{3}(235 \times 10^3)(0.95)} = 258.6\text{ A}$$

$$\theta_s = \cos^{-1} 0.95 = 18.194°$$

$$Y = \frac{1}{X_c} = \frac{1}{-j1200} = \frac{1}{1200}\angle 90°$$

$$\frac{Y}{2} = \frac{1}{2(1200)} = \frac{1}{2400}\angle 90°$$

$$I_1 = E_s\left(\frac{Y}{2}\right) = \left(\frac{235{,}000}{\sqrt{3}}\angle 0°\right)\left(\frac{1}{2400}\right)\angle 90° = 56.532\angle 90°\text{ A}$$

$$I_2 = I_s - I_1$$

$$= 258.6\angle -18.194° - 56.532\angle 90°$$

$$= 245.67 - j80.744 - (j56.532)$$

$$I_2 = 245.67 - j137.276 = 281.422\angle -29.195°\text{ A}$$

$$Z_L = R + jX_L = 50\angle 78° = 10.395 + j48.9\,\Omega$$

$$P_{loss} = 3I_2^{\,2}R_L = 3(281.422)^2(10.395) = 2469.8\text{ kW}$$

$$W_{loss} = P_{loss} \times \frac{24\text{ hrs}}{\text{day}} \times \frac{30\text{ days}}{\text{month}}$$

$$= (2469.8)(24)(30)$$

$$W_{loss} = 1778256\text{ kW - hr}$$

$$\text{Total cost} = W_{loss} \times \text{ cost per kW - hr}$$

$$= 1778256(1.3)$$

$$\text{Total cost} \cong \text{P } 2.312\text{ million}$$

Power Factor Corrections

TYPICAL LOADS WHICH OPERATE ON LOW POWER FACTOR

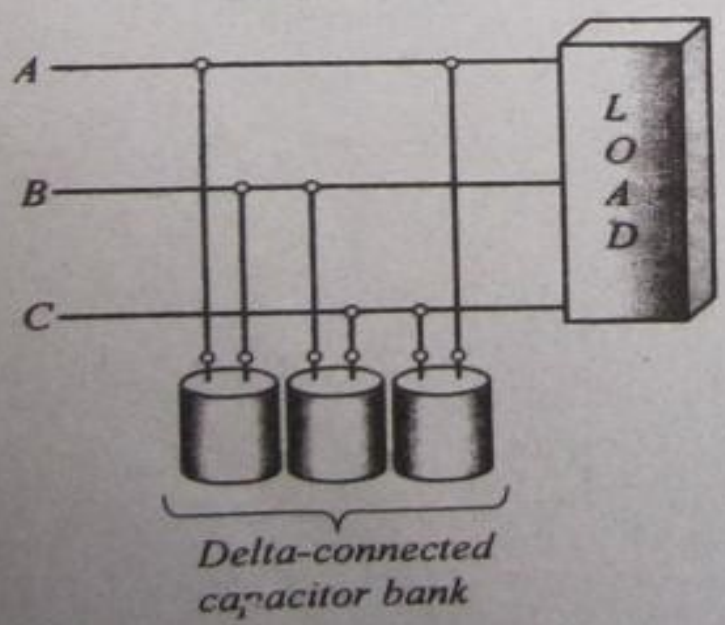
- Arc lamps
- Induction type AC motors
- Industrial heating furnaces

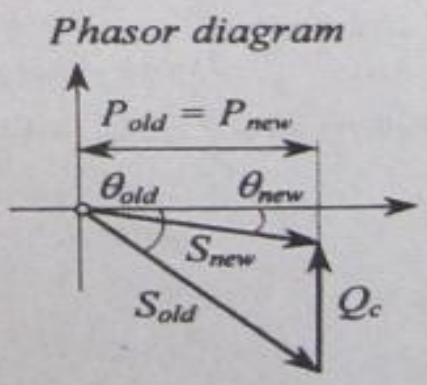
SOME UNWANTED EFFECTS WITH LOW POWER FACTOR

- For a given load, the current will increase which means larger size of wire is needed. Also it increases the losses and decreases the efficiency of the system.
- Generators, transformers, switches and transmission conductors become overloaded due to the high current.
- Voltage drop in transmission lines increases due the increase in current.

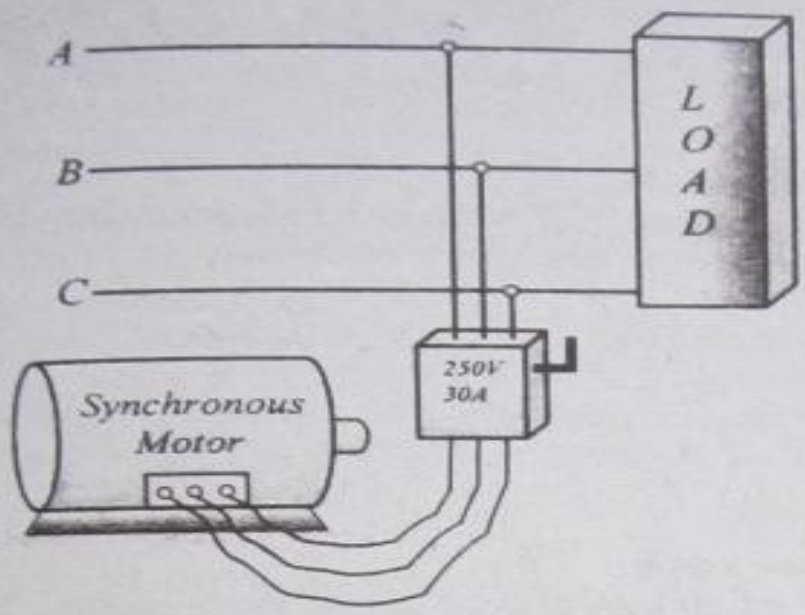
METHODS USED TO CORRECT LOW POWER FACTOR

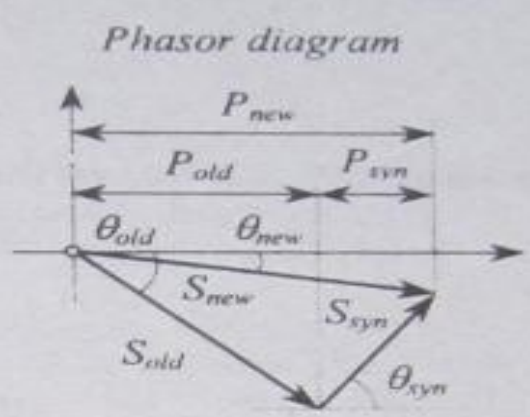
- Using a bank of **static condensers** (capacitors) connected across the load.





- Using a **synchronous condenser** (a synchronous motor operated at a leading power factor) connected across the load.





where: S_{old} = old apparent power, when power factor is not yet corrected.
S_{new} = new apparent power, after power factor is corrected.
S_{syn} = apparent power rating of the synchronous motor needed to correct the low power factor
Q_c = reactive power rating of the static condensers needed to correct the low power factor
P_{old} = old active power before correction
P_{new} = new active power after correction
P_{syn} = active power rating of the synchronous motor
$\cos\theta_{old}$ = power factor before correction
$\cos\theta_{new}$ = power factor after correction
$\cos\theta_{syn}$ = power factor of the synchronous motor

Test 23

Problem 827: EE Board October 1996

A 132kV line three-phase system delivers 70.7 MVA of a balanced delta load of power factor 70.7% lagging. Determine the reactance necessary to attain unity power factor.

A. X_C = 1,092 ohms
B. X_C = 965 ohms
C. X_C = 1,142 ohms
D. X_C = 1,045 ohms

Problem 828: EE Board October 1990

A single-phase inductive load takes 50 kVA at 0.60 power factor lagging. Solve for the kVAR of a capacitor required to improve the power factor to 1.0.

A. 30 kVAR
B. 20 kVAR
C. 22.5 kVAR
D. 40 kVAR

Problem 829:

A single-phase load on 220 V takes 5 kW at 0.6 lagging power factor. Find the kVAR size of capacitor, which maybe connected in parallel with this motor to bring the resultant power factor to 1.0.

A. 6.67
B. 7.32
C. 8.66
D. 6.26

Problem 830:

A 5 hp, 220 V, 60 Hz, single-phase induction motor operates at an efficiency and power factor of 0.88 and 0.8 respectively. What capacitance should be connected across the motor in order for the feeder supplying this motor to operate at unity power factor?

A. 201.55 μF
B. 183.52 μF
C. 174.22 μF
D. 212.56 μF

Problem 831: EE Board October 1987

Installed in one of the customer of CEPALCO are two single phase transformers each rated 75 kVA are connected V or open delta to serve a 3-phase load of 120 kW at 0.8 pf lagging. To prevent the overloading of the transformers, determine the size of the capacitor in kVAR.

A. 40
B. 41
C. 39
D. 42

Problem 832:

Three transformers each rated 100 kVA are connected delta supplying a load of 160 kW at 0.8 pf lagging. One of the transformers is taken for repair and the rest are connected open delta. What kVAR (minimum) of capacitor must be connected with the load so that the load of the remaining transformers will be 95% of their combined rated capacities.

A. 85.32
B. 81.62
C. 87.58
D. 80.92

Problem 833: EE Board October 1983

Three-single phase transformers each rated 75 kVA are banked in delta and supplying a 3-phase load drawing 160 kVA at 0.8 lagging power factor. If one transformer is removed for repairs, solve for the minimum amount in kVAR of a capacitor needed to prevent overloading of the remaining units.

A. 70.32
B. 73.64
C. 72.46
D. 73.28

Problem 834:

Two single-phase transformer each rated 150 kVA are connected open delta supplying a three-phase induction motor rated 250 HP, 0.7 pf and 80% efficiency. Determine the minimum size in kVAR of a capacitor needed to prevent overloading the transformers.

A. 135
B. 126
C. 120
D. 123

Problem 835: EE Board October 1982

Two single-phase transformers are each rated 75 kVA are connected in V or open delta to serve a 3-phase load of 120 kW at 0.80 power factor lagging. Determine the size in kVAR of the capacitor needed to prevent overloading of the transformers.

A. 40.25
B. 41.28
C. 45.24
D. 43.50

Problem 836: EE Board April 1984

A plant has a load of 290 kilowatt with an average power factor of 70%. The owner requests you to correct the power factor to reduce its power consumption. How much capacitor kVAR is required to increase the power factor to 90%?

A. 152.46
B. 155.39
C. 150.34
D. 154.58

Problem 837: EE Board October 1982

A 150-kVA-transformer bank will serve a load expected to draw 135 kW at 0.80 lagging power factor. Solve for the size of the capacitor bank needed to be added in order to prevent overloading of the transformer bank.

A. 32.506 kVAR
B. 35.866 kVAR
C. 40.391 kVAR
D. 28.266 kVAR

Problem 838: EE Board October 1981

A 3-phase generator has the following 3–phase loads: an inductive load drawing 400 kVA at 0.60 power factor and a resistive load drawing 80 kVA at 1.00 power factor. Solve for the size in kVAR of the capacitor bank needed to improve the power factor of the combined loads to 0.85 lagging.

A. 120.58 kVAR
B. 121.68 kVAR
C. 124.54 kVAR
D. 122.82 kVAR

Problem 839: EE Board April 1997

A load of 10,000 kVA, 80% pf lagging is connected to a 13,200 volts line. How much capacitive reactive power is needed to correct the power factor to 0.97 lagging?

A. 5,156 kVAR
B. 3,138 kVAR

C. 2,547 kVAR
D. 4,395 kVAR

Problem 840: EE Board April 1986

A short, 3-phase, 3-wire transmission line has a receiving end voltage of 4160 V phase to neutral and serving a balanced 3-phase load of 998,400 volt-amperes at 0.82 pf lagging. At the receiving end the voltage is 4600 V, phase to neutral and the pf is 0.77 lagging. Solve for the size in kVAR of a capacitor needed to improve the receiving end pf to 0.9 lagging maintaining 4160 V.

A. 181
B. 175
C. 172
D. 178

Problem 841:

A three-phase, 3-wire transmission line has an impedance per wire of 3 + j7 ohms, the receiving end load is 1950 kW, 0.65 pf lagging with the line voltage of 13,200 V. Determine the kVAR of the capacitor to be connected at the receiving end to make the pf at that end to 0.8 lagging.

A. 824.56
B. 871.45
C. 803.88
D. 817.32

Problem 842: EE Board October 1980

A balanced, 500 kVA, 3-phase, 440 volt, 60 Hz, inductive load operates at a pf of 75%. Determine the total capacitor kVAR required improving the pf to 95%.

A. 207.46
B. 176.42
C. 210.75
D. 192.21

Problem 843: EE Board March 1998

A single-phase induction motor is rated 5 hp, 75% power factor and 220 volts. What approximate size of capacitor size of capacitor is necessary to raise the power factor to about 95%?

A. 3 kVAR
B. 2 kVAR
C. 2.5 kVAR
D. 3.5 kVAR

Problem 844: EE Board October 1984

A balanced 3-phase load draws 150 A phase current at 7.5 kV phase to neutral, 0.891 power factor lagging. It is desired to raise the power factor to 0.96 leading. Solve for the amount of capacitor kVAR needed to achieve such pf.

A. 2273 kVAR
B. 2409 kVAR
C. 2509 kVAR
D. 2365 kVAR

Problem 845: EE Board April 1981

A 3-phase, 3-wire, short transmission line has a resistance of 3 ohms and a reactance of 8 ohms per wire. At the receiving end, a balanced 3-phase load draws a line current of 60 A, at 13,500 volts line to line, 0.90 power factor lagging. Assuming the receiving end voltage is maintained at 13,500 V, solve the size in kVAR of capacitors needed to raise the power factor at the receiving end to 0.95 leading.

A. 1043.5
B. 1154.2
C. 1026.5
D. 1232.2

Problem 846:

The input to an induction motor from a 13 kV, 60 Hz line is 1000 kVA, at 0.8 pf lagging. A capacitor is placed in parallel with the motor to improve the pf. Calculate the capacitance required raising the power factor to 0.9 leading.

A. 62 μF
B. 50 μF
C. 43 μF
D. 56 μF

Problem 847: EE Board April 1981

A three-phase balanced load draws a line current of 80 A at 0.90 lagging power factor. Solve the minimum size in kVAR of the capacitor bank needed to raise the power factor to 0.95 leading, if the line to line voltage is 13,200 volts.

A. 1310.15
B. 1338.25
C. 1247.54
D. 1430.12

Problem 848: EE Board October 1990

Two Y-connected, 50° rise induction motors are fed by a 4160 V, line to line, 3-phase 60 Hz motor-control center 20 feet away. Motor #1 drives a 600-hp compressor. The efficiency of this motor is 90% and its power factor is 0.5. Instruments of motor #2 indicate 1730 kW, 277 amperes. Determine the capacity in microfarads per phase of a wye-connected bank that is required to correct the power factor of the total load to 0.966 lagging.

A. 172.4 μF
B. 193.8 μF
C. 167.2 μF
D. 182.1 μF

Problem 849:

A 2.3 kV three-phase system supplies a 120-kW, 0.6 pf lagging balanced load. Determine the capacitance in each phase of a wye-connected capacitor bank to adjust the power to 0.90 lagging.

A. 51.1 μF
B. 52.2 μF
C. 62.5 μF
D. 78.2 μF

Problem 850: EE Board October 1996

A single-phase, 60 Hz, 5 hp squirrel cage induction motor draws a current of 53 A at 117 V. If it has a 78.5% electrical to mechanical conversion efficiency, what capacitance should be connected at the terminals of the motor in order to increase the power factor of the load combination to 92%?

A. 480 μF
B. 380 μF
C. 320 μF
D. 420 μF

Problem 851: EE Board April 1989

A three-phase, 60 Hz, 2200 volts induction motor develops 500 HP, 0.8 lagging pf and efficiency of 94%. The power factor is raised to 0.90 lagging by connecting a bank of condensers in delta across the lines. If each of the capacitance unit is built up of four similar 550 V condensers, calculate the required capacitance of each condenser.

A. 77.04 μF
B. 75.42 μF
C. 76.12 μF
D. 72.30 μF

Problem 852: EE Board April 1993

A star-connected 400 Hp (metric), 2000 V, 50 c/s motor works at a power factor of 0.7 lagging. A bank of mesh-connected condensers are used to raise the power factor to 0.93 lagging. Calculate the capacitance of each unit required if each is rated 500 V, 50 c/s. The motor efficiency is 85%.

A. 194 μF
B. 225 μF
C. 302 μF
D. 233 μF

Problem 853:

A factory draws a lagging load of 2000 kW at a power factor of 60% from a 6 kV bus line. A synchronous capacitor is installed to raise the overall power factor to unity. Assuming the synchronous capacitor losses are 275 kW, calculate the operating power factor of the synchronous capacitor.

A. 0.201 leading
B. 0.103 leading
C. 0.302 leading
D. 0.113 leading

Problem 854:

A 400-V three-phase balanced load takes 40 A at a lagging power factor of 0.8. An over-excited synchronous motor is connected to raise the power factor to unity. If the mechanical output of the motor is 15 hp and its efficiency is 85%, find the kVA input to the motor.

A. 24.2 kVA
B. 18.7 kVA
C. 20.3 kVA
D. 21.2 kVA

Problem 855:

A three-phase load of 100 kVA operates at 0.707-pf lagging. Solve for the operating power factor of a synchronous condenser necessary to raise the system power factor to 0.866 lagging, and so arrange that the system apparent power is the same as before the correction.

A. 0.632 leading
B. 0.882 leading
C. 0.705 leading
D. 0.609 leading

Problem 856:

An industrial plant has an electrical load of 5000 kW at 80% pf lagging. It has been decided to replace a 500-hp induction motor that drives a pump. This motor operates at an efficiency of 90% and a pf of 0.9 lagging. If a synchronous motor is purchased as a replacement, which is capable of operating at 0.80 pf leading. What will be the new plant pf. Assume the synchronous motor to have the same efficiency.

A. 0.863
B. 0.823
C. 0.837
D. 0.852

Problem 857:

A synchronous motor is drawing 60 kW is connected in parallel with a load drawing 250 kW at a lagging power factor of 0.8. If the combined load has a power factor of 0.9, at what power factor is the synchronous motor operating?

A. 0.862 leading
B. 0.849 leading
C. 0.868 leading
D. 0.874 leading

Problem 858: EE Board October 1985

An inductive load consumes 10 kW at 0.75 pf lagging. A synchronous motor with a pf of 0.9 leading is connected in parallel with the inductive load. What is the minimum required kW size of the synchronous motor so that the combined load will have a pf of 0.8 lagging.

A. 1.068 kW
B. 1.203 kW
C. 1.075 kW
D. 1.109 kW

Problem 859: EE Board April 1980, EE Board April 1992

An industrial plant draws 500 kW at 0.6 power factor from a 3-phase system. In order to raise the power factor to 0.866 lagging and to supply needed additional power, a synchronous motor is added. This motor draws 300 kW, bringing the new total plant load to 800 kW. Neglecting the losses of the synchronous motor, calculate its exact required kVA rating.

A. 363.24 kVA
B. 345.23 kVA
C. 393.34 kVA
D. 307.35 kVA

Problem 860:

An induction motor load of 1500 kW consists of several units in parallel, operating at an average power factor of 0.80 lagging. In order to improve the power factor, a portion of the induction motor loads is to be replaced by a synchronous motor, operating at the same efficiency as the induction motors and at a power factor of 0.70 leading. Find the kVA rating of the synchronous motor required bringing the power factor of the total load to 0.90 lagging. Assume the induction motor load, which has to be replaced, operates at the same pf as the induction motor group.

A. 332 kVA
B. 325 kVA
C. 300 kVA
D. 322 kVA

Problem 861:

An alternator rated 530-kVA, is operating at 0.60 pf lagging. A synchronous condenser is in parallel to improve the pf to 90% lagging. Determine the operating power factor of the synchronous condenser. Assume the alternator is not supposed to be overloaded.

A. 0.546 leading

B. 0.447 leading
C. 0.643 leading
D. 0.593 leading

Problem 862:

An over-excited synchronous motor is connected across a 150 kVA, 0.70 lagging power factor load. The motor takes 12 kW while running on no-load. Calculate the kVA rating of the motor required in order to bring the overall pf of the motor-inductive load combination to unity.

A. 106.22
B. 107.[illegible]8
C. 110.20
D. 105.5[illegible]

Problem 863:

An industrial load takes 250 kW at 60% pf from a 600 V, 60 Hz, 3-phase feeder. It is desired to raise the pf of the entire feeder to 90% by means of a synchronous motor, which at the same time is to drive a dc compound generator, requiring that the synchronous motor takes 70 kW from the line. Determine the required current rating of the synchronous motor.

A. 175.53 A
B. 184.37 A
C. 192.65 A
D. 188.44 A

Problem 864: EE Board April 1988

An existing industrial plant has an average load of 900 kW at 0.6 pf lagging. To raise the overall power factor to 0.92, a synchronous motor driving a DC generator is to be installed. If the input of the synchronous motor will be 250 kW, calculate its kVA input rating.

A. 753 kVA
B. 788 kVA
C. 723 kVA
D. 715 kVA

ANSWER KEY

827.	D	835.	A	843.	B	851.	A	859.	A
828.	D	836.	B	844.	B	852.	D	860.	D
829.	A	837.	B	845.	C	853.	B	861.	B
830.	C	838.	B	846.	B	854.	D	862.	B
831.	A	839.	D	847.	B	855.	D	863.	B
832.	B	840.	B	848.	B	856.	C	864.	A
833.	D	841.	D	849.	A	857.	B		
834.	D	842.	A	850.	B	858.	A		

☑ RATING

34-38 - Topnotcher

26-33 - Passer

19-25 - Conditional

0-18 - Failed

Did you know that.. a not well known scientist of his time, published the idea that heat is a form of energy in 1841? His works were slightly noticed. Not until a respected physicist named Lord Kelvin showed interest in his work and later accepted it.

James Prescott Joule is an English physicist whom we are indebted with the familiar formula I^2R for the power dissipated in a resistor. He published the result in 1841. He holds the reputation of being one of the history's most dedicated men of science. His vital discovery was the mechanical equivalent of heat that plays a major role in thermodynamics and many other branches of engineering.

Joule was born in Salford, England, and the second of five children of a wealthy brewer. As a boy, he taught himself electricity and magnetism and obtained his formal education in a nearby Manchester University.

Because of his father's wealth, Joule was able to devote himself to his interest, experimental science. His experiments on heat were conducted in his home laboratory. His idea was one that dominated all of his research, "THAT HEAT AND ENERGY ARE BASICALLY THE SAME". He devoted years to measure heat changes in every mechanical process he could think of.

Throughout most of his life, Joule was considered an isolated amateur scientist. Only few scientists took the slightest notice of his work. Not until towards the end of his of years, a respected physicist Lord Kelvin showed interest on his work and later accepted it.

In recognition to his valuable contributions, the SI unit of energy was named "joule" in his honor.

Solutions to Test 23

827 Note: The reactive (capacitive) power necessary to correct the pf to unity is equivalent to the reactive (inductive) power of the load.

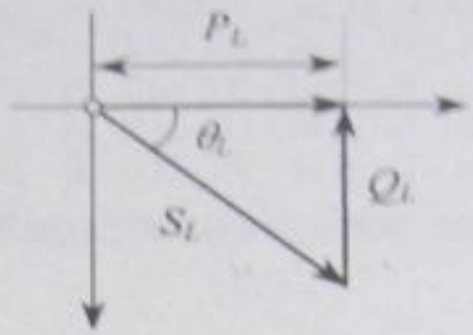
$\theta_L = \cos^{-1} 0.707 = 45°$

$Q_L = S_L \sin\theta_L = 70.7 \sin 45° = 50\ \text{MVAR}$

$Q_{phase} = \dfrac{Q_L}{3} = \dfrac{50}{3} = 16.67\ \text{MVAR}$

$X_C = \dfrac{E_L^2}{Q_{phase}} = \dfrac{(132{,}000)^2}{16.67 \times 10^6}$

$X_C = 1045.23\ \Omega$



828 Note: The reactive (capacitive) power necessary to correct the pf to unity is equivalent to the reactive (inductive) power of the load. Refer to the vector diagram of problem #841.

$\theta_L = \cos^{-1} 0.60 = 53.13°$

$Q_L = S_L \sin\theta_L = 50 \sin 45°$

$Q_L = 40\ \text{kVAR}$

829 Note: The reactive (capacitive) power necessary to correct the pf to unity is equivalent to the reactive power (inductive) of the load. Refer to the vector diagram of problem #841.

$\theta_L = \cos^{-1} 0.60 = 53.13°$

$Q_L = P_L \tan\theta_L = 5 \tan 53.13°$

$Q_L = 6.67\ \text{kVAR}$

830 Note: The reactive power of the capacitor necessary to correct the pf to unity must be equivalent to the reactive power of the motor load. Refer to the vector diagram of problem #841.

$\theta_L = \cos^{-1} 0.8 = 36.869°$

$P_L = \dfrac{P_{out}}{\eta} = \dfrac{5(746)}{0.88} = 4238.636\ \text{W}$

$Q_L = P_L \tan\theta_L = 4238.636 \tan 36.869° = 3178.87\ \text{VAR}$

$$X_C = \frac{E^2}{Q_L} = \frac{220^2}{3178.87} = 15.225\,\Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(15.225)}$$

$$C = 174.22\,\mu F$$

831 $S_V = \sqrt{3}\text{Rating} = \sqrt{3}(75) = 130\text{ kVA}$

$Q_V = \sqrt{S_V^2 - P_V^2} = \sqrt{130^2 - 120^2}$

$= 50\text{ kVAR}$

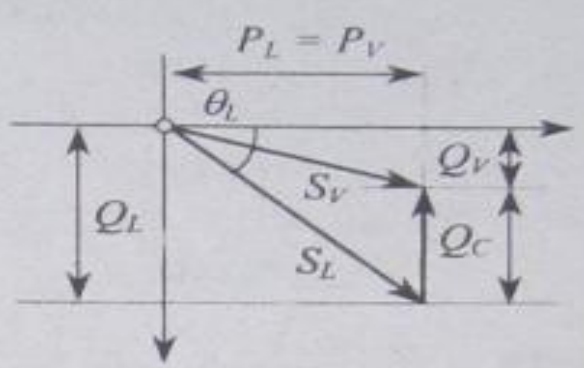
$\theta_L = \cos^{-1} 0.8 = 36.869°$

$Q_L = P_L \tan\theta_L$

$Q_L = 120 \tan 36.869° = 90\text{ kVAR}$

$Q_C = Q_L - Q_V = 90 - 50$

$Q_C = 40\text{ kVAR}$



832 Note: For minimum Q_C, the overall power factor must be lagging. Refer to the vector diagram of problem #831.

$S_V = (0.95)(\sqrt{3})(\text{Rating}) = (0.95)(\sqrt{3})(100) = 164.54\text{ kVA}$

$Q_V = \sqrt{S_V^2 - P_V^2} = \sqrt{164.54^2 - 160^2} = 38.385\text{ kVAR}$

$\theta_L = \cos^{-1} 0.8 = 36.869°$

$Q_L = P_L \tan\theta_L$

$Q_L = 160 \tan 36.869° = 120\text{ kVAR}$

$Q_C = Q_L - Q_V = 120 - 38.385$

$Q_C = 81.615\text{ kVAR}$

833 $S_V = \sqrt{3}(\text{Rating}) = \sqrt{3}(75) = 130\text{ kVA}$

$P_L = S_L pf_L = 160(0.8) = 128\text{ kW}$

$Q_V = \sqrt{S_V^2 - P_V^2} = \sqrt{130^2 - 128^2}$

$Q_V = 22.715\text{ kVAR}$

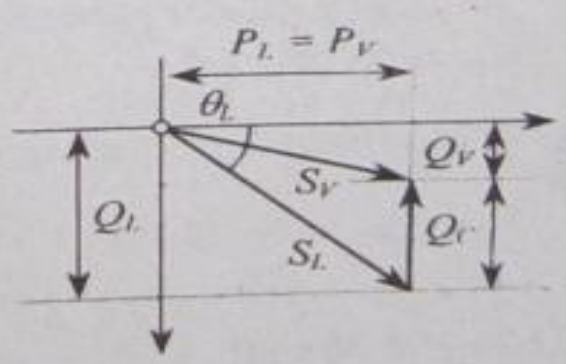
$\theta_L = \cos^{-1} 0.8 = 36.869°$

$Q_L = P_L \tan\theta_L$

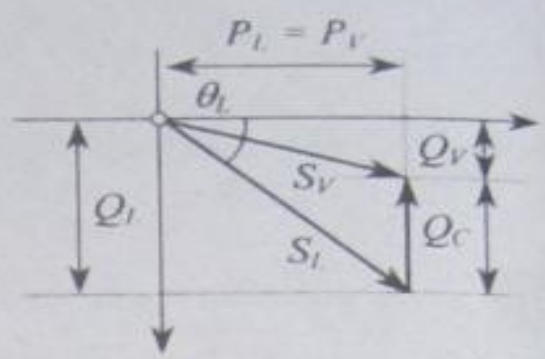
$Q_L = 128 \tan 36.869° = 96\text{ kVAR}$

$Q_C = Q_L - Q_V = 96 - 22.715$

$Q_C = 73.28\text{ kVAR}$



834 Note: Refer to the vector diagram



$\theta_L = \cos^{-1} 0.7 = 45.573°$

$S_V = \sqrt{3}\text{Rating} = \sqrt{3}(150) = 259.8\text{ kVA}$

$P_L = \dfrac{P_{out}}{\eta} = \dfrac{250(0.746)}{0.8} = 233.125\text{ kW}$

$Q_V = \sqrt{S_V^2 - P_V^2} = \sqrt{259.8^2 - 233.125^2}$

$Q_V = 114.668\text{ kVAR}$

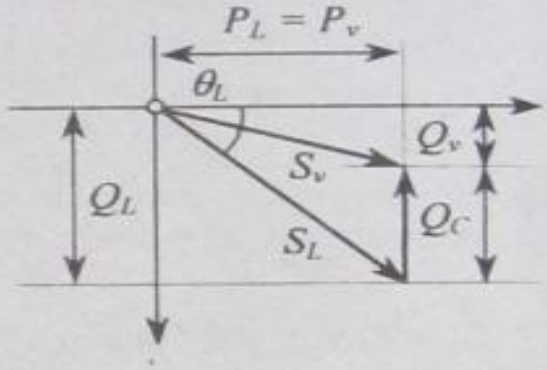
$Q_L = P_L \tan\theta_L = 233.125 \tan 45.573°$

$Q_L = 237.835\text{ kVAR}$

$Q_C = Q_L - Q_V = 237.835 - 114.668$

$Q_C = 123.167\text{ kVAR}$

835 $S_V = \sqrt{3}\text{ Rating} = \sqrt{3}(75) = 129.904\text{ kVA}$



$\theta_L = \cos^{-1} 0.8 = 36.869°$

$Q_L = P_L \tan\theta_L = 120 \tan 36.869°$

$= 90\text{ kVAR}$

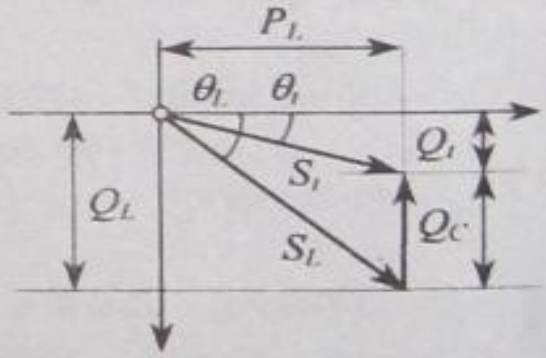
$Q_V = \sqrt{S_V^2 - P_V^2} = \sqrt{129.904^2 - 120^2}$

$= 49.75\text{ kVAR}$

$Q_C = Q_L - Q_V = 90 - 49.75$

$Q_C = 40.25\text{ kVAR}$

836 $\theta_L = \cos^{-1} 0.7 = 45.57°$



$\theta_t = \cos^{-1} 0.9 = 25.84°$

$Q_C = Q_L - Q_t$

$= P_L \tan\theta_L - P_L \tan\theta_t$

$= 290 \tan 45.57° - 290 \tan 25.84$

$Q_C = 155.39\text{ kVAR}$

837 Note: Refer to the vector diagram of problem #836.

$\theta_L = \cos^{-1} 0.8 = 36.869°$

$Q_t = \sqrt{S_t^2 - P_t^2} = \sqrt{150^2 - 135^2}$

$= 65.38\text{ kVAR}$

$Q_C = Q_L - Q_t$

$= P_L \tan\theta_L - Q_t = 135 \tan 36.869° - 65.38$

$Q_C = 35.866\text{ kVAR}$

838

$\theta_1 = \cos^{-1} 0.6 = 53.13°$

$\theta_2 = \cos^{-1} 1.0 = 0°$

$S_L = S_1 + S_2 = 400\angle -53.13° + 80\angle 0°$

$= 240 - j320 + 80$

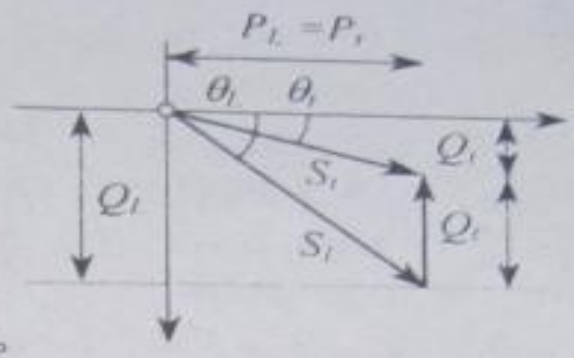
$S_L = 320 - j320$

$\theta_t = \cos^{-1} 0.85 = 31.788°$

$Q_C = Q_L - Q_t$

$= Q_L - P_t \tan\theta_t = 320 - 320 \tan 31.788°$

$Q_C = 121.68$ kVAR



839

Note: Refer to the vector diagram of problem #838.

$P_L = S_L pf_L = 10{,}000(0.8) = 8000$ kW

$\theta_L = \cos^{-1} 0.8 = 36.869°$

$\theta_t = \cos^{-1} 0.97 = 14.07°$

$Q_C = Q_L - Q_t$

$= P_t \tan\theta_t - P_t \tan\theta_t$

$= (8000)(\tan 36.869° - \tan 14.07°)$

$Q_C = 4395$ kVAR

840

Note: Refer to the vector diagram of problem #838.

$\theta_L = \cos^{-1} 0.82 = 34.915°$

$\theta_t = \cos^{-1} 0.9 = 25.84°$

$P_L = S_L pf_L = (998.4)(0.82) = 818.688$ kW

$Q_t = Q_L - Q_t$

$= P_L \tan\theta_L - P_t \tan\theta_t = (818.688)(\tan 34.915)° - \tan 25.84°)$

$Q_t = 175$ kVAR

841

Note: Refer to the vector diagram of problem #838.

$\theta_L = \cos^{-1} 0.65 = 49.458°$

$\theta_t = \cos^{-1} 0.8 = 36.869°$

$Q_L = P_L \tan\theta_L = 1950 \tan 49.458° = 2279.77$ kVAR

$Q_t = P_t \tan\theta_t = 1950 \tan 36.869° = 1462.45$ kVAR

$Q_C = Q_L - Q_t = 2279.77 - 1462.45$

$Q_C = 817.32$ kVAR

842 $\theta_L = \cos^{-1} 0.75 = 41.409°$

$\theta_t = \cos^{-1} 0.95 = 18.194°$

$Q_L = S_L \sin\theta_L = 500 \sin 41.409°$

$= 330.715 \text{ kVAR}$

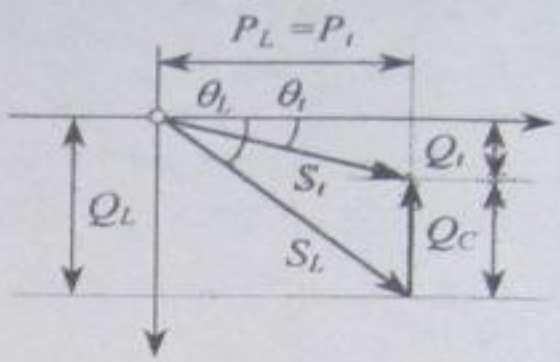
$P_L = S_L pf_L = 500(0.75) = 375 \text{ kW}$

$Q_t = P_t \tan\theta_t = 375 \tan 18.194°$

$= 123.25 \text{ kVAR}$

$Q_C = Q_L - Q_t = 330.715 - 123.25$

$Q_C = 207.465 \text{ kVAR}$



843 Note: Refer to the vector diagram of problem #842. Since efficiency of the motor is not given, assume the given power rating (output) equal to the input power drawn.

$\theta_L = \cos^{-1} 0.75 = 41.409°$

$\theta_t = \cos^{-1} 0.95 = 18.194°$

$P_L = 5(0.746) = 3.73 \text{ kW}$

$Q_L = P_L \tan\theta_L = 3.73 \tan 41.409° = 3.289 \text{ kVAR}$

$Q_t = P_t \tan\theta_t = 3.73 \tan 18.194° = 1.226 \text{ kVAR}$

$Q_C = Q_L - Q_t = 3.289 - 1.226 = 2.063$

$Q_C \cong 2 \text{ kVAR}$

844 $\theta_L = \cos^{-1} 0.891 = 27°$

$\theta_t = \cos^{-1} 0.96 = 16.26°$

$P_L = 3E_p I_p pf$

$= 3(7500)(150)(0.891)$

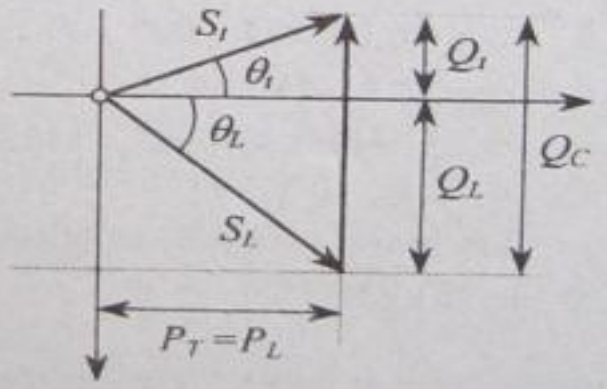
$= 3007 \text{ kW}$

$Q_C = Q_L + Q_t$

$= P_L \tan\theta_L + P_t \tan\theta_t$

$Q_C = 3007 \tan 27° + 3007 \tan 16.26°$

$Q_C = 2409 \text{ kVAR}$



845 $P_r = \sqrt{3} E_r I_r pf_r$

$P_r = \sqrt{3}(13{,}500)(60)(0.9)$

$= 1262.665 \text{ kW}$

$\theta_r = \cos^{-1} 0.9 = 25.84°$

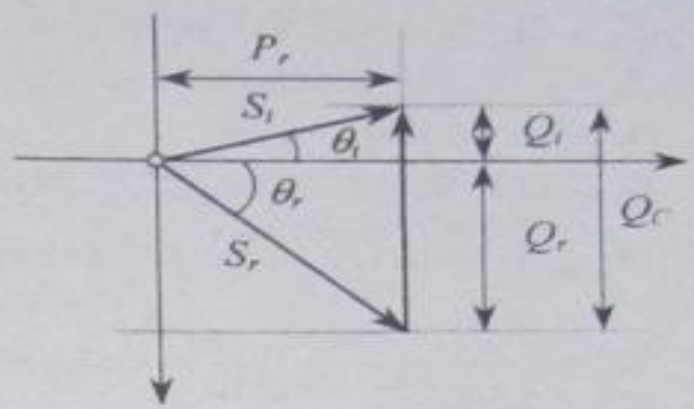
$\theta_t = \cos^{-1} 0.95 = 18.195°$

$Q_C = Q_r + Q_t$

$= 1262.665 \tan 25.84°$

$+ 1262.665 \tan 18.195°$

$Q_C = 1026.505 \text{ kVAR}$



846 $\theta_L = \cos^{-1} 0.8 = 36.869°$

$\theta_t = \cos^{-1} 0.9 = 25.84°$

$P_L = S_L pf = 1000(0.8) = 800 \text{ W}$

$Q_C = Q_L + Q_t$

$= P_L \tan\theta_L + P_t \tan\theta_t$

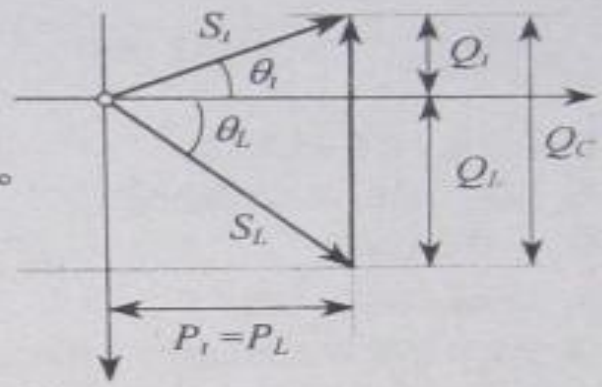
$= 800 \tan 36.869° + 800 \tan 25.84°$

$Q_C = 987.4 \text{ VAR}$

$X_C = \dfrac{E^2}{Q_C} = \dfrac{230^2}{987.4} = 53.575\ \Omega$

$C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(60)(53.575)} = 49.5\ \mu\text{F}$

$C \cong 50\ \mu\text{F}$



847 Note: Refer to the vector diagram of problem #846.

$\theta_L = \cos^{-1} 0.9 = 25.84°$

$\theta_t = \cos^{-1} 0.95 = 18.195°$

$P_L = \sqrt{3} EIpf = \sqrt{3}(13200)(80)(0.9) = 1646.14 \text{ kW}$

$Q_C = Q_L + Q_t$

$= P_L \tan\theta_L + P_t \tan\theta_t$

$= 1646.14 \tan 25.84° + 1646.14 \tan 18.195°$

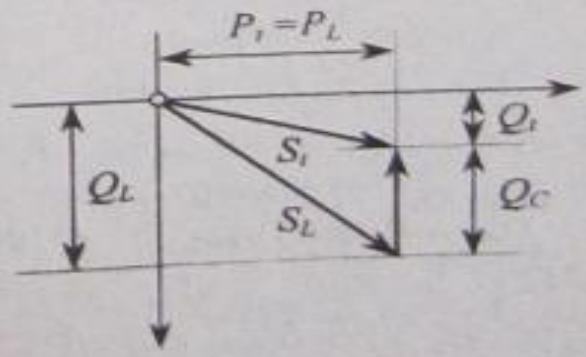
$Q_C = 1338.25 \text{ kVAR}$

848 $\theta_1 = \cos^{-1} 0.5 = 60°$

$P_1 = \dfrac{P_{out}}{\eta} = \dfrac{600(0.746)}{0.9} = 497.33 \text{ kW}$

$\cos\theta_2 = \dfrac{P_2}{\sqrt{3}EI} = \dfrac{1730 \times 10^3}{\sqrt{3}(4160)(277)} = 0.866$

$\theta_2 = \cos^{-1} 0.866 = 30°$



$S_L = S_1 + S_2 = 497.33 - j497.33\tan 60° + 1730 - j1730\tan 30°$

$S_L = 2227.33 - j1860.22$

$\theta_t = \cos^{-1} 0.966 = 14.98°$

$Q_C = Q_L - Q_t = 1860.22 - 2227.33\tan 14.98° = 1264.24\text{ kVAR}$

$$Q_{phase} = \frac{Q_C}{3} = \frac{1264.24}{3} = 421.413\text{ kVAR}$$

$$X_C = \frac{E_p^{\,2}}{Q_{phase}} = \frac{(4160/\sqrt{3})^2}{421{,}413} = 13.688\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(13.668)} = 193.8\ \mu F$$

849 $\theta_L = \cos^{-1} 0.6 = 53.13°$

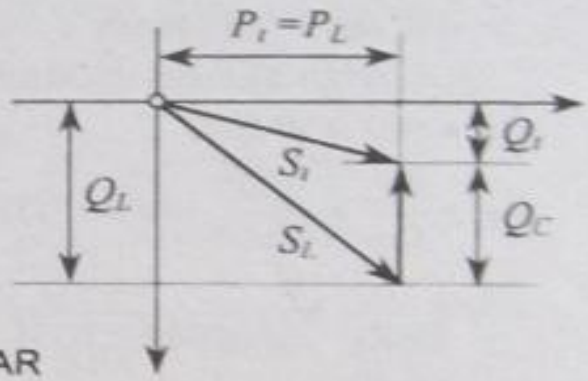
$Q_L = P_L \tan\theta_L = 120\tan 53.13°$

$Q_L = 160\text{ kVAR}$

$\theta_t = \cos^{-1} 0.90 = 25.84°$

$Q_t = P_L \tan\theta_t = 120\tan 25.84°$

$Q_t = 58.11\text{ kVAR}$



$Q_C = Q_L - Q_t = 160 - 58.11 = 101.89\text{ kVAR}$

$$Q_{phase} = \frac{Q_C}{3} = \frac{101.89}{3} = 33.963\text{ kVAR}$$

$$X_C = \frac{E_{phase}^{\,2}}{Q_{phase}} = \frac{(2300/\sqrt{3})^2}{33{,}963} = 51.92\ \Omega$$

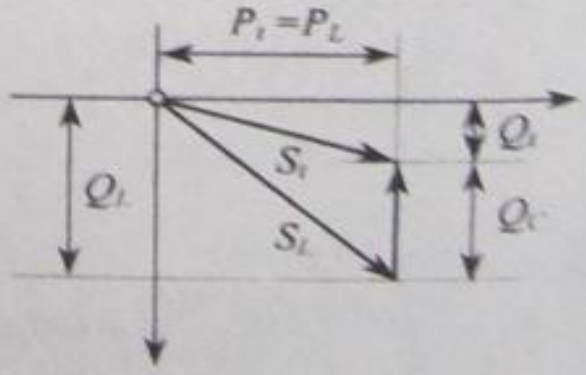
$$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60)(51.92)} = 51.089\ \mu F$$

850 $$P_L = \frac{P_{out}}{\eta} = \frac{5(746)}{0.785} = 4751.6\text{ W}$$

$$\cos\theta_L = \frac{P_L}{EI} = \frac{4751.6}{(117)(53)} = 0.766$$

$\theta_L = \cos^{-1} 0.766 = 40°$

$\theta_t = \cos^{-1} 0.92 = 23.073°$



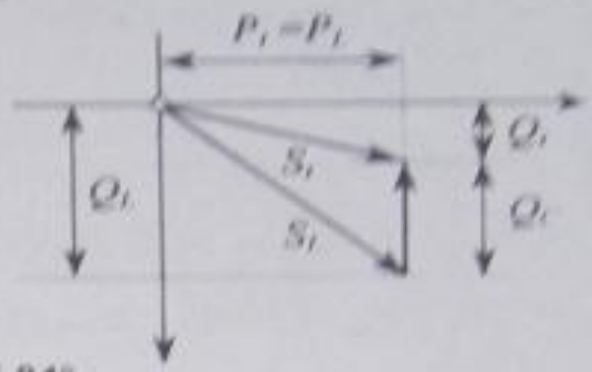
$Q_C = Q_L - Q_t = P_L \tan\theta_L - P_t \tan\theta_t$

$= 4751.6\tan 40° - 4751.6\tan 23.073°$

$Q_c = 1962.98\text{ VAR}$

$$X_C = \frac{E^2}{Q_C} = \frac{117^2}{1962.98} = 6.97\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(6.97)} = 380.57\ \mu F$$



851 $P_L = \frac{P_{out}}{\eta} = \frac{500(0.746)}{0.94} = 396.81\text{kW}$

$\theta_L = \cos^{-1} 0.8 = 36.869°$

$\theta_t = \cos^{-1} 0.9 = 25.84°$

$Q_C = Q_L - Q_t = P_L \tan\theta_L - P_t \tan\theta_t$

$= 396.81\tan 36.869° - 396.81\tan 25.84°$

$Q_C = 105.43\text{ kVAR}$

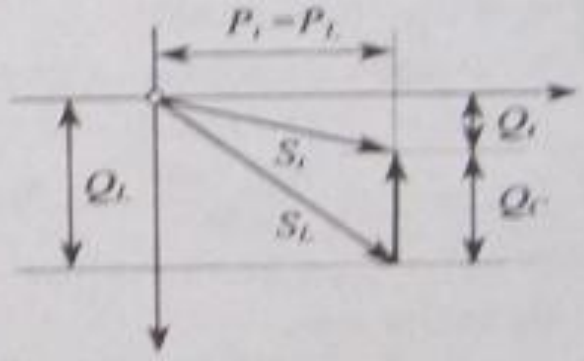
$$Q_{phase} = \frac{Q_C}{3} = \frac{105.43}{3} = 35.143\text{ kVAR}$$

$$Q_{cap} = \frac{Q_{phase}}{4} = \frac{35.143}{4} = 8.785\text{ kVAR}$$

$$X_C = \frac{E_{cap}^2}{Q_{cap}} = \frac{550^2}{8{,}785} = 34.43\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(34.43)}$$

$C = 77.04\ \mu F$



852 $P_L = \frac{P_{out}}{\eta} = \frac{400(0.746)}{0.85} = 351.06\text{ kW}$

$\theta_L = \cos^{-1} 0.7 = 45.57°$

$\theta_t = \cos^{-1} 0.93 = 21.565°$

$Q_C = Q_L - Q_t = P_L \tan\theta_L - P_t \tan\theta_t$

$= 351.06\tan 45.57° - 351.06\tan 21.565°$

$Q_C = 219.368\text{ kVAR}$

$$Q_{phase} = \frac{Q_C}{3} = \frac{219.368}{3} = 73.12\text{ kVAR}$$

$$n = \frac{E_{phase}}{E_{cap}} = \frac{2000}{500} = 4\text{ capacitors}$$

$$Q_{cap} = \frac{Q_{phase}}{4} = \frac{73.12}{4} = 18.28\text{ kVAR}$$

$$X_C = \frac{E_{cap}^2}{Q_{cap}} = \frac{500^2}{18280} = 13.676\ \Omega$$

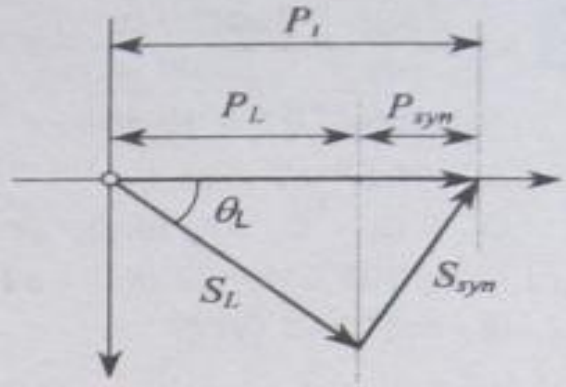
$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(13.676)}$$

$$C = 232.75\ \mu F$$

853 $\theta_L = \cos^{-1} 0.6 = 53.13°$

$$S_L = \frac{P_L}{pf_L} = \frac{2000}{0.6} = 3333.333\ kVA$$

$$P_t = P_L + P_{syn} = 2000 + 275 = 2275\ kW$$



By cosine law :

$$S_{syn} = \sqrt{S_L^2 + P_t^2 - 2S_L P_t \cos\theta_L}$$

$$= \sqrt{3333.333^2 + 2275^2 - 2(3333.333)(2275)\cos 53.13°}$$

$$S_{syn} = 2680.81\ kVA$$

$$pf_{syn} = \frac{P_{syn}}{S_{syn}} = \frac{275}{2680.81}$$

$$pf_{syn} = 0.103\ \text{leading}$$

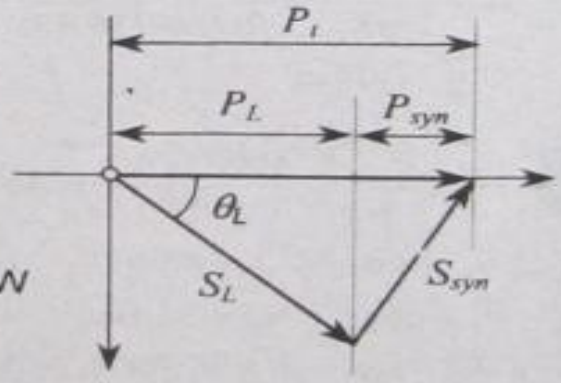
854 $\theta_L = \cos^{-1} 0.8 = 36.869°$

$$S_L = \sqrt{3}EI = \sqrt{3}(400)(40) = 27.713\ kVA$$

$$P_L = S_L pf_L = 27.713(0.8) = 22.17\ kW$$

$$P_{syn} = \frac{P_{out}}{\eta} = \frac{15(0.746)}{0.85} = 13.164\ kW$$

$$P_t = P_L + P_{syn} = 22.17 + 13.164 = 35.334\ kW$$



By cosine law :

$$S_{syn} = \sqrt{S_L^2 + P_t^2 - 2S_L P_t \cos\theta_L}$$

$$= \sqrt{27.713^2 + 35.334^2 - 2(27.713)(35.334)\cos 36.869°}$$

$$S_{syn} = 21.2\ kVA$$

855 $\theta_L = \cos^{-1} 0.707 = 45°$

$\theta_1 = \cos^{-1} 0.866 = 30°$

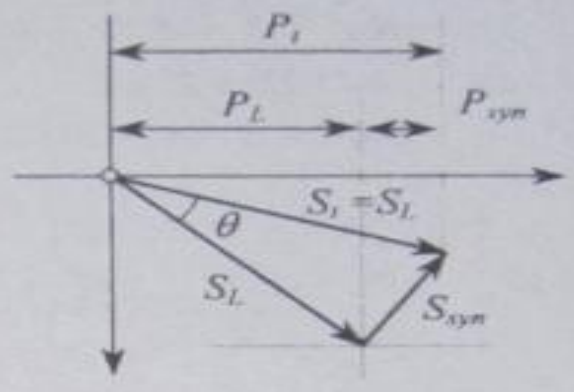
$\theta = \theta_L - \theta_1 = 45° - 30° = 15°$

By cosine law :

$$S_{syn} = \sqrt{S_L^2 + S_t^2 - 2S_L S_t \cos\theta}$$

$$S_{syn} = \sqrt{100^2 + 100^2 - 2(100)(100)\cos 15^\circ}$$

$$= 26.1\text{kVA}$$



$$P_L = S_L pf_L = 100(0.707) = 70.7\text{ kW}$$

$$P_t = S_t pf_t = 100(0.866) = 86.6\text{ kW}$$

$$P_{syn} = P_t - P_L = 86.6 - 70.7 = 15.9\text{ kW}$$

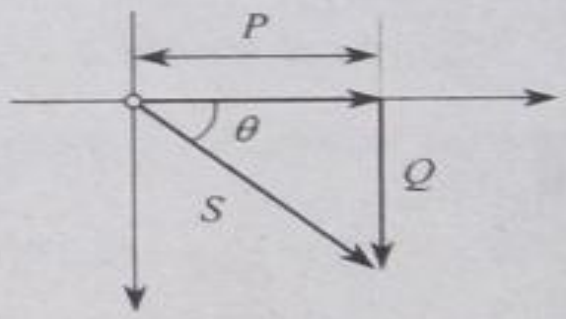
$$pf_{syn} = \frac{P_{syn}}{S_{syn}} = \frac{15.9}{26.1}$$

$$pf_{syn} = 0.609 \text{ leading}$$

856 $\theta_L = \cos^{-1} 0.8 = 36.869^\circ$

$$S_L = P_L - P_L \tan\theta = 5000 - j5000 \tan 36.869^\circ$$

$$= 5000 - j3750$$



For the induction motor :

$$P_{ind} = \frac{500(0.746)}{0.96} = 388.54$$

$$\theta_{ind} = \cos^{-1} 0.9 = 25.84^\circ$$

$$S_{ind} = 388.54 - j388.54 \tan 25.84^\circ$$

$$= 388.54 - j188.16$$

For the synchronous motor :

$$S_{syn} = P_{syn} + jQ_{syn}$$

$$\theta_{syn} = \cos^{-1} 0.8 = 36.869^\circ$$

$$P_{syn} = P_{ind}$$

$$S_{syn} = 388.54 + j388.54 \tan 36.869^\circ$$

$$= 388.54 + j291.39$$

$$S_{new} = S_L - S_{ind} + S_{syn}$$

$$= (5000 - j3750) - (388.54 - j188.54) + 388.54 + j291.39$$

$$= 5000 - j3270.45 = 5974.59\angle -33.188^\circ$$

$$pf_{new} = \cos\theta_{new} = \cos 33.188^\circ$$

$$pf_{new} = 0.837 \text{ lagging}$$

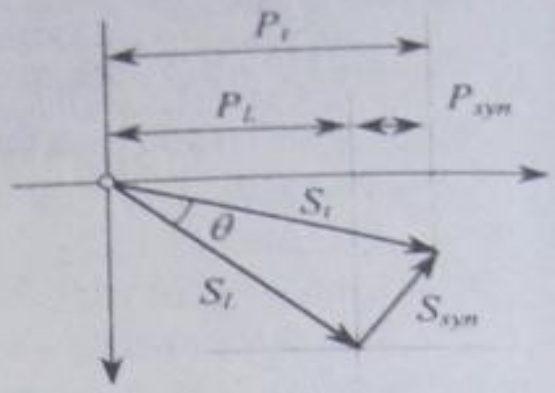
857

$$S_L = \frac{P_L}{pf_L} = \frac{250}{0.8} = 312.5\,kVA$$

$$\theta_L = \cos^{-1} 0.8 = 36.87°$$

$$\theta_t = \cos^{-1} 0.9 = 25.84°$$

$$\theta = 36.87° - 25.84° = 11.03°$$



$$P_t = P_L + P_{syn} = 250 + 60 = 310\,kW$$

$$S_t = \frac{P_t}{pf_t} = \frac{310}{0.9} = 344.44\,kVA$$

Using cosine law :

$$S_{syn}^2 = S_L^2 + S_t^2 - 2S_L S_t \cos\theta$$

$$= (312.5)^2 + (344.44)^2 - 2(312.5)(344.44)(\cos 11.03°)$$

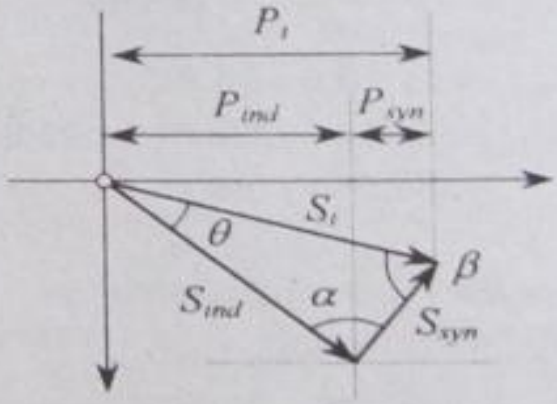
$$S_{syn} = 70.688\,kVA$$

$$pf_{syn} = \frac{P_{syn}}{S_{syn}} = \frac{60}{70.688}$$

$$pf_{syn} = 0.849 \text{ leading}$$

858

Note: Refer to the vector diagram.



$$S_{ind} = \frac{P_{ind}}{pf} = \frac{10}{0.75} = 13.333\,kVA$$

$$\theta_{ind} = \cos^{-1} 0.75 = 41.41°$$

$$\theta_{syn} = \cos^{-1} 0.9 = 25.84°$$

$$\theta_t = \cos^{-1} 0.8 = 36.87°$$

$$\theta = 41.41° - 36.87° = 4.54°$$

$$\alpha = 180° - (41.41° + 25.84°) = 112.75°$$

$$\beta = 180° - (\alpha + \theta) = 180° - (112.75° + 4.54°) = 62.71°$$

Using sine law :

$$\frac{\sin\theta}{S_{syn}} = \frac{\sin\beta}{S_{ind}}$$

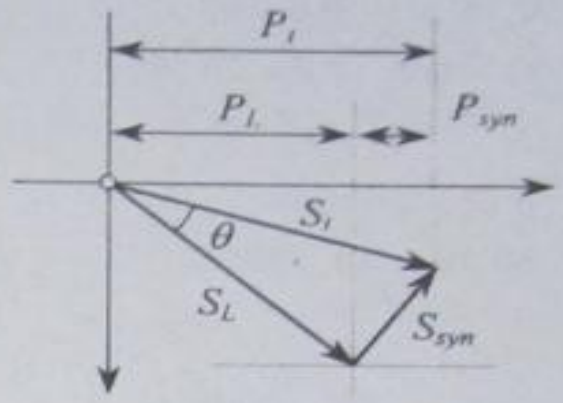
$$\frac{\sin 4.54°}{S_{syn}} = \frac{\sin 62.71°}{13.333}$$

$$S_{syn} = 1.1875\,kVA$$

$P_{syn} = S_{syn}(pf_{syn}) = 1.1875(0.9)$

$P_{syn} = 1.068\ kW$

859 Note: Refer to the vector diagram.



$$S_L = \frac{P_L}{pf_L} = \frac{500}{0.6} = 833.33\ kVA$$

$$S_t = \frac{P_t}{pf_t} = \frac{800}{0.866} = 923.787\ kVA$$

$\theta_L = \cos^{-1} 0.6 = 53.13°$

$\theta_t = \cos^{-1} 0.866 = 30°$

$\theta = 53.13° - 30° = 23.13°$

Using cosine law :

$S_{syn}^2 = S_L^2 + S_t^2 - 2S_L S_t \cos\theta$

$= (833.33)^2 + (923.787)^2 - 2(833.33)(923.787)(\cos 23.13°)$

$S_{syn} = 363.24\ kVA$

860 $\theta_L = \cos^{-1} 0.8 = 36.869°$

$S_L = 1500 - j1500 \tan 36.869° = 1500 - j1124.96$

$\theta_{syn} = \cos^{-1} 0.7 = 45.573°$

$S_{syn} = P + jP \tan 45.573° = P + j1.02P$

$S_{ind} = P - jP \tan 36.869 = P - j0.75P$

$\theta_{new} = \cos^{-1} 0.9 = 25.84°$

$S_{new} = 1500 - j1500 \tan 25.84° = 1500 - j726.42$

$S_{new} = S_L + S_{syn} - S_{ind}$

$1500 - j726.42 = 1500 - j1124.96 + P + j1.02P - (P - j0.75P)$

Equate coefficients of j :

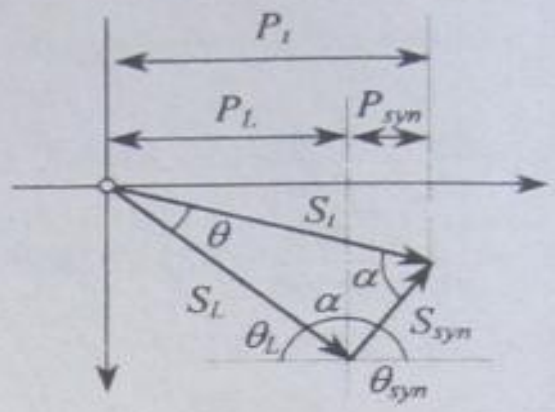
$-726.42 = -1124.96 + 1.02P + 0.75P$

$P = 225.164\ kW$

$$S_{syn} = \frac{P}{pf_{syn}} = \frac{225.164}{0.7} = 321.66$$

$S_{syn} \cong 322\ kVA$

861 Note: Refer to the vector diagram. Assume the total kVA load is equal to the kVA rating of the alternator. The triangle as shown is therefore an isosceles triangle since $S_t = S_L$



$\theta_L = \cos^{-1} 0.6 = 53.13°$

$\theta_t = \cos^{-1} 0.9 = 25.84°$

$\theta = 53.13° - 25.84° = 27.29°$

$\theta + 2\alpha = 180°$

$53.13° + 2\alpha = 180°$

$\alpha = 63.435°$

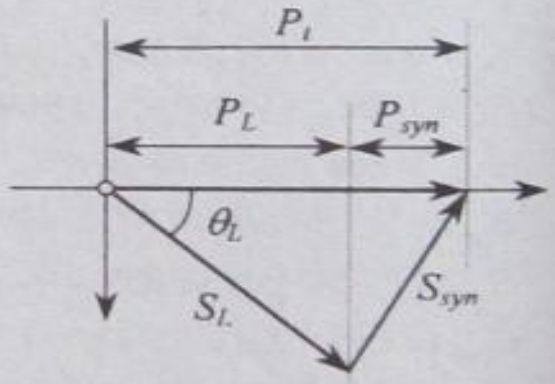
$\theta_L + \alpha + \theta_{syn} = 180°$

$53.13° + 63.435° + \theta_{syn} = 180°$

$\theta_{syn} = 63.435°$

$pf_{syn} = \cos 63.435° = 0.447$ leading

862 Note: An over-excited synchronous motor is operating at a leading pf. Refer to the vector diagram.



$P_L = S_L pf_L = 150(0.7) = 105$ kW

$P_t = P_L + P_{syn} = 105 + 12 = 117$ kW

$\theta_L = \cos^{-1} 0.7 = 45.57°$

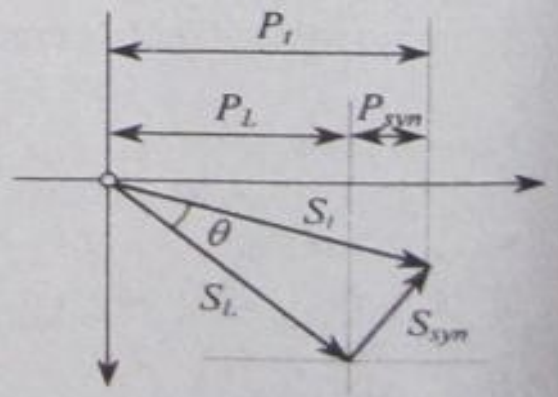
Using cosine law:

$S_{syn}^2 = S_L^2 + P_t^2 - 2S_L P_t \cos\theta_L$

$= (150)^2 + (117)^2 - 2(150)(117)(\cos 45.57°)$

$S_{syn} = 107.78$ kVA

863 $S_L = \dfrac{P_L}{pf_L} = \dfrac{250}{0.6} = 416.667$ kVA



$\theta_L = \cos^{-1} 0.6 = 53.13°$

$\theta_t = \cos^{-1} 0.9 = 25.84°$

$\theta = 53.13° - 25.84° = 27.29°$

$P_t = P_L + P_{syn} = 250 + 70$

$P_t = 320$ kW

$$S_t = \frac{P_t}{pf_t} = \frac{320}{0.9} = 355.55 \text{ kVA}$$

Using cosine law :

$$S_{syn}^2 = S_L^2 + S_t^2 - 2S_L S_t \cos\theta$$

$$= (416.667)^2 + (355.55)^2 - 2(416.667)(355.55)(\cos 27.29°)$$

$$S_{syn} = 191.607 \text{ kVA}$$

$$I_L = \frac{S_{syn.}}{\sqrt{3}V_L} = \frac{191607}{\sqrt{3}(600)}$$

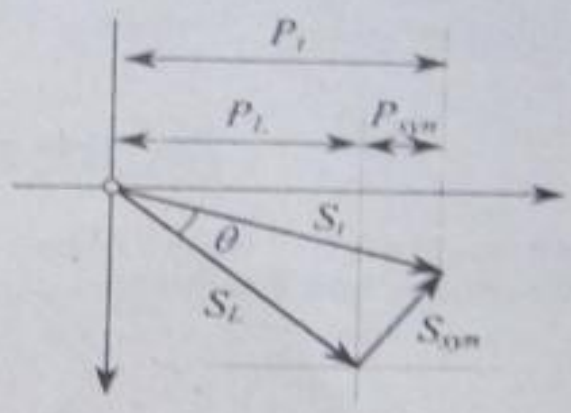
$$I_L = 184.37 \text{ A}$$

864 $$S_L = \frac{P_L}{pf_L} = \frac{900}{0.6} = 1500 \text{ kVA}$$

$$\theta_L = \cos^{-1} 0.6 = 53.13°$$

$$\theta_t = \cos^{-1} 0.92 = 23.07°$$

$$\theta = 53.13° - 23.07° = 30.06°$$



$$P_t = P_L + P_{syn} = 900 + 250 = 1150 \text{ kW}$$

$$S_t = \frac{P_t}{pf_t} = \frac{1150}{0.92} = 1250 \text{ kVA}$$

Using cosine law :

$$S_{syn}^2 = S_L^2 + S_t^2 - 2S_L S_t \cos\theta$$

$$S_{syn}^2 = (1500)^2 + (1250)^2 - 2(1500)(1250)(\cos 30.06°)$$

$$S_{syn} = 753 \text{ kVA}$$

Faults on Electrical System

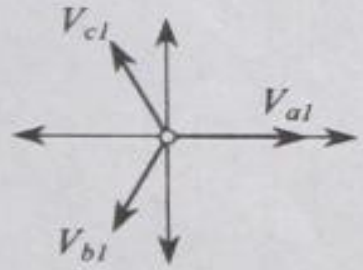
SYMMETRICAL COMPONENTS

An **unbalanced three-phase system vectors** can be broken down into three groups of balanced vectors, called the **symmetrical components** of the given unbalanced vectors.

Example: Given three unbalanced vectors V_a, V_b and V_c and a phase sequence of a-b-c.

POSITIVE SEQUENCE COMPONENTS

Three equal vectors whose phase sequence is **the same as the original sequence** of the unbalanced system of vectors. (It uses subscript 1)

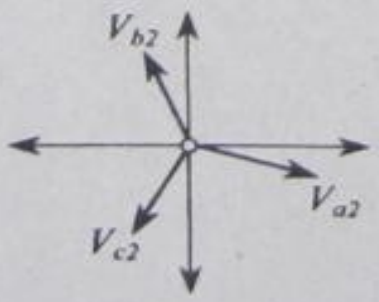


$$V_{b1} = a^2 V_{a1}$$

$$V_{c1} = aV_{a1}$$

NEGATIVE SEQUENCE COMPONENTS

Three equal vectors whose phase sequence is **opposite of the original sequence** of the unbalanced system of vectors. (It uses subscript 2)

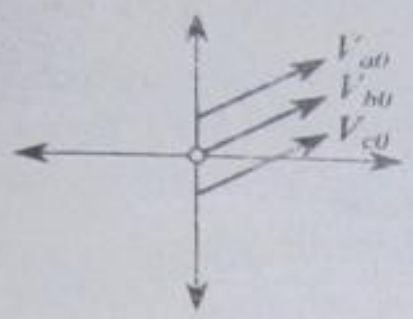


$$V_{b2} = aV_{a2}$$

$$V_{c2} = a^2 V_{a2}$$

ZERO SEQUENCE COMPONENTS

Three vectors which are equal and in phase. (It uses subscript 0)



$$V_{a0} = V_{b0} = V_{c0}$$

GENERAL FORMULAS FOR SYMMETRICAL COMPONENTS

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c] \qquad V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c]$$

$$V_{a2} = \frac{1}{3}[V_a + a^2V_b + aV_c]$$

$$V_a = V_{ao} + V_{a1} + V_{a2}$$
$$V_b = V_{bo} + V_{b1} + V_{b2}$$
$$V_c = V_{co} + V_{c1} + V_{c2}$$

$$a = 1\angle 120^\circ \qquad a^2 = 1\angle -120^\circ = 1\angle 240^\circ$$

PER UNIT IMPEDANCE USING A DIFFERENT POWER & VOLTAGE BASE

The numerical per unit value of any quantity is its ratio to the choosen base quantity of the same dimensions.

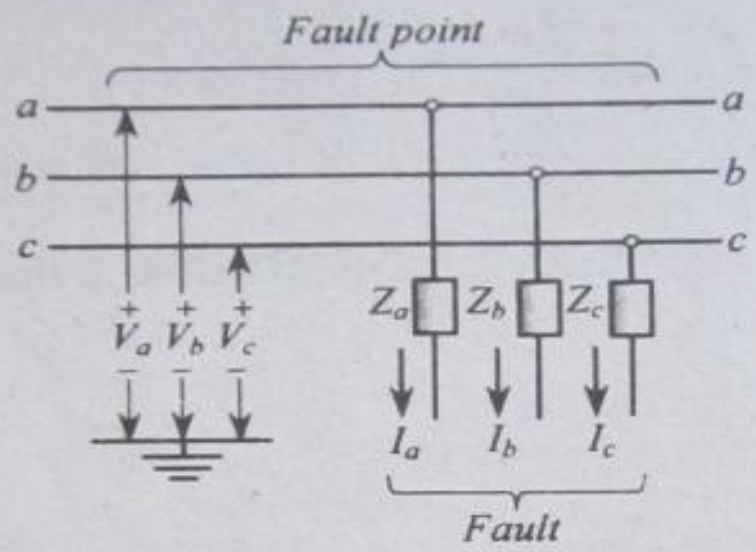
$$Z_{pu} = \frac{Z(S_{base})}{(V_{base})^2} \qquad Z_{pu2} = Z_{pu1}\left(\frac{S_{base2}}{S_{base1}}\right)\left(\frac{V_{base1}}{V_{base2}}\right)^2$$

where: Z_{pu} = per unit value of the impedance
Z = impedance per phase (ohms)
S_{base} = base apparent power (volt-ampere)
V_{base} = base line to line voltage (volt)
Z_{pu2} = per unit impedance using a new base
V_{base2} = new base line to line voltage (volt)
S_{base2} = new base apparent power (volt-ampere)

FAULT

Fault is defined as any connection or situation that causes an unbalance among the three phases.

- The **fault point** of a system or network is that point to which the unbalanced connection is attached in an otherwise balanced system.

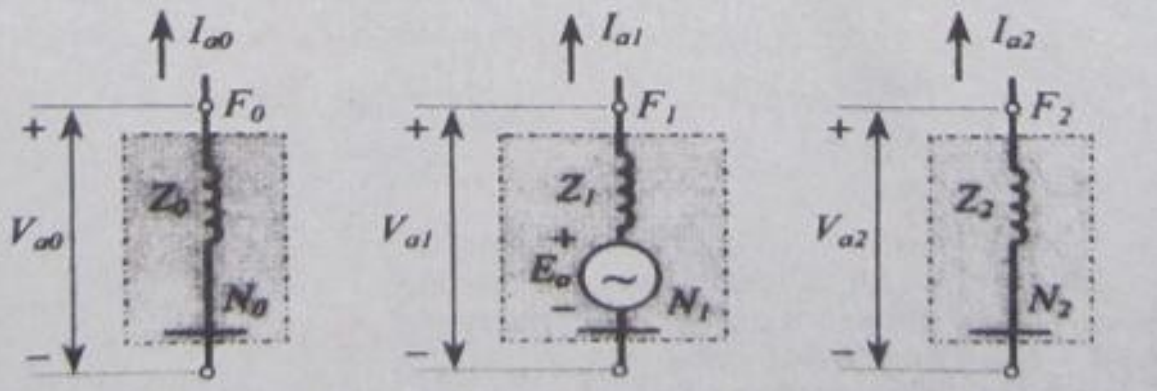


Circuit diagram of the fault point

where: Z_a, Z_b, Z_c = phase impedances at the fault point
V_a, V_b, V_c = line to ground voltages at the fault point
I_a, I_b, I_c = line currents at the fault point

THE SEQUENCE NETWORK

A **sequence network** is a copy of the original balanced system to which the **fault point** is connected and contains the same per phase impedances as the physical balanced system, arranged in the same way. The only difference being that the value of each impedance is a value unique to each sequence.

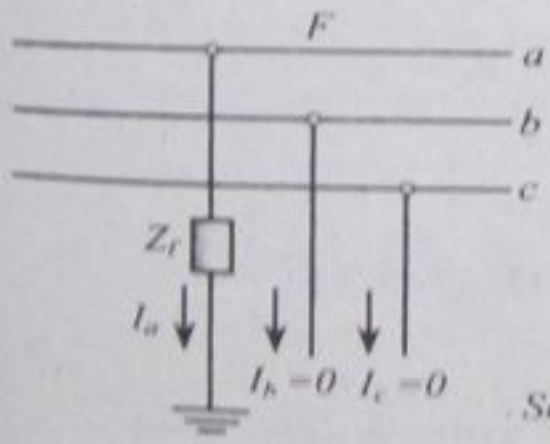


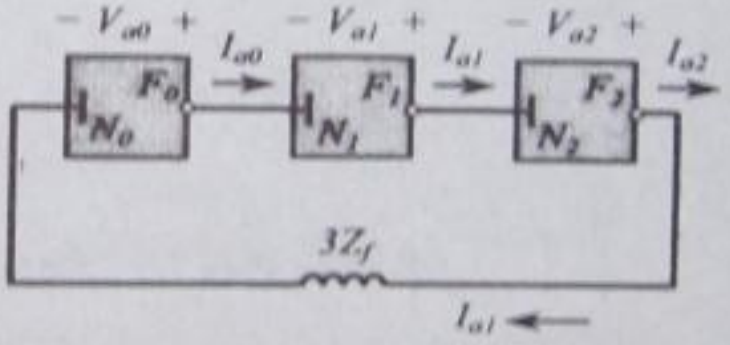
Zero Sequence *Positive Sequence* *Negative Sequence*

Note: The Thevenin's equivalent or open circuit voltages in the negative and zero sequence networks are zero because by definition the only voltages generated in the three-phase system are positive sequence (sequence a-b-c) voltages.

where: F = fault point
N = zero-potential bus
E_a = Thevenin's equivalent voltage (per unit) or the pre-fault (F open) voltage of phase a at point F.
Z_1 = positive sequence impedance (per unit)
Z_2 = negative sequence impedance (per unit)
Z_0 = zero sequence impedance (per unit)
I_{a1} = positive sequence current component of phase a (per unit)
I_{a2} = negative sequence current component of phase a (per unit)
I_{a0} = zero sequence current component of phase a (per unit)
V_{a1} = positive sequence voltage component of phase a (per unit)
V_{a2} = negative sequence voltage component of phase a (per unit)
V_{a0} = zero sequence voltage component phase a (per unit)

SINGLE-LINE TO GROUND (SLG) FAULT



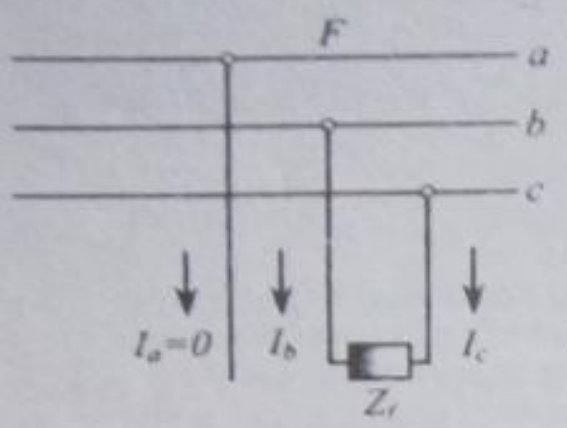


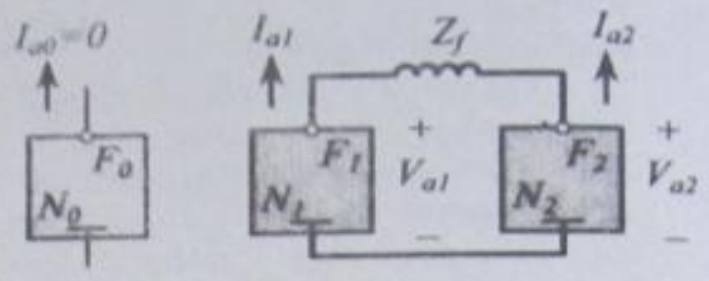
Sequence network connections for a SLG fault

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3Z_f} \qquad I_{a1} = I_{a2} = I_{a0} \qquad I_a = 3I_{a1}$$

Where: Z_f = phase impedance at the fault point (per unit)

LINE TO LINE (LL) FAULT





Sequence network connections for a LL fault

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_f}$$

$$I_{a2} = -I_{a1}$$

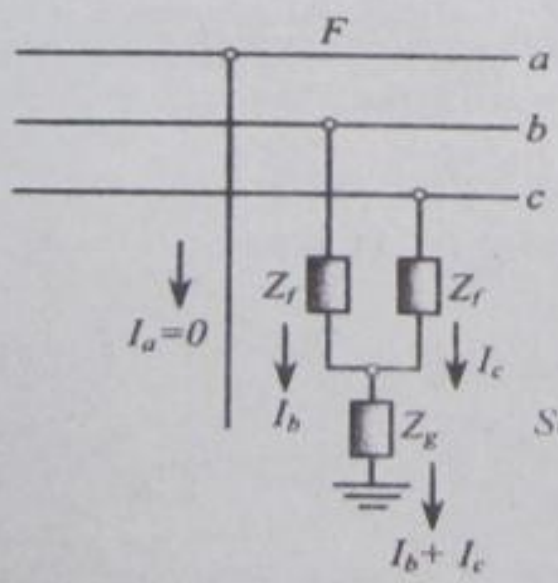
$$I_{a0} = 0$$

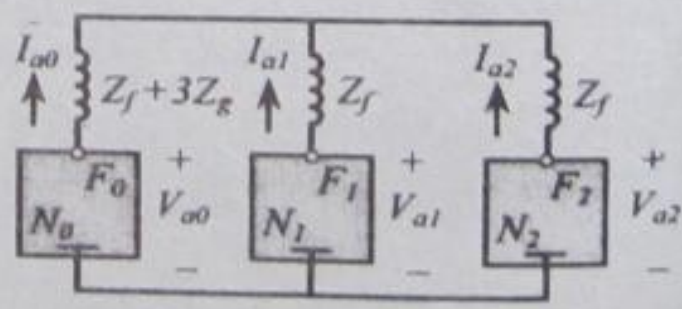
$$I_a = 0$$

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$

$$I_c = -I_b$$

DOUBLE LINE TO GROUND (2LG) FAULT





Sequence network connections for a 2LG fault

$$I_{a1} = \frac{E_a}{Z_1 + Z_f + \dfrac{(Z_2 + Z_f)(Z_0 + Z_f + 3Z_g)}{Z_0 + Z_2 + 2Z_f + 3Z_g}}$$

$$V_{a1} = E_a - I_{a1}Z_1$$

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$

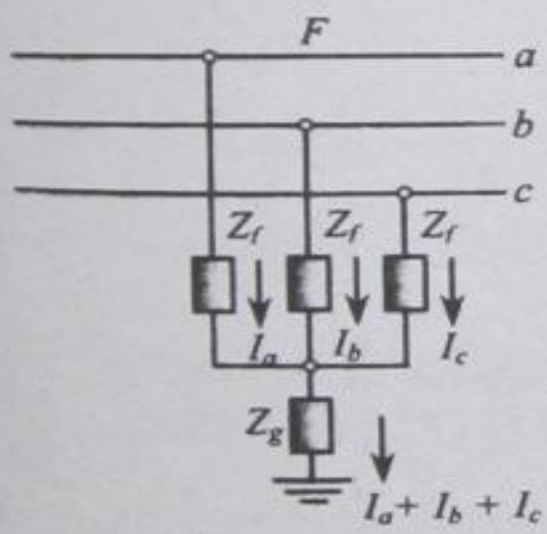
$$I_{a2} = -\frac{V_{a1} - I_{a1}Z_f}{Z_2 + Z_f}$$

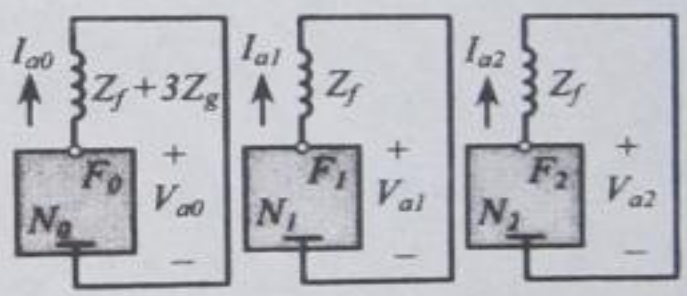
$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0}$$

$$I_{a0} = -\frac{V_{a1} - I_{a1}Z_f}{Z_0 + Z_f + 3Z_g}$$

where: Z_f = phase impedance at the fault point (per unit)
Z_g =neutral to ground impedance at the fault point (per unit)

THREE PHASE (3ϕ) FAULT OR THREE LINE TO GROUND FAULT





Sequence network connections for a 3ϕ fault

$$I_{a1} = \frac{E_a}{Z_1 + Z_f} \qquad I_{a2} = 0 \qquad I_{a0} = 0$$

$$I_a = I_{a1} \qquad I_a = I_b = I_c$$

Note: The three-phase fault is often the most severe type of fault.

THREE PHASE FAULT CALCULATIONS GIVEN A NETWORK

❖ Method I: kV-base method

Procedures:

- ❑ Draw the per phase diagram of the system
- ❑ Choose a common kV-base (V_c) and express all impedances on the selected kV-base.

$$Z_c = Z_a\left(\frac{V_c}{V_a}\right)^2$$

where: V_c = common kV-base
V_a = voltage base of Z_a
Z_c = new impedance based on V_c

- ❑ Calculate the total or equivalent impedance (Z_{total}) using circuit principles.
- ❑ Calculate the fault current (I_f) using Ohm's law.

$$I_f = \frac{V_f}{Z_{total}}$$

where: V_f = voltage per phase at the fault

- ❑ Evaluate the actual fault current using the relation

$$I_a = I_f\left(\frac{V_c}{V_a}\right)$$

where: I_a = actual fault current

❖ Method II: kVA-base method

Procedures:

- ❑ Draw the per phase diagram of the system.
- ❑ Express all impedances in percentage or per unit values.

- Choose a common kVA-base and express all Z_{pu} on the selected common kVA base.

$$Z_{c_{pu}} = Z_{a_{pu}}\left(\frac{kVA_c}{kVA_a}\right)$$

where: $Z_{c_{pu}}$ = new per unit impedance based on kVA_c
kVA_c = common kVA base

- Calculate the total per unit impedance of the system ($Z_{total_{pu}}$) using circuit principles.
- Calculate the fault power (kVA_f) using the relation,

$$kVA_f = \frac{kVA_c}{Z_{total_{pu}}}$$

- Evaluate the fault current (I_f) using the equation

$$I_f = \frac{kVA_f}{\sqrt{3}V_f}$$

where: V_f = line to line voltage at the fault point in kV

APPROXIMATE EQUIVALENTS BETWEEN A LINE TO LINE FAULT AND A THREE PHASE FAULT ON AN UNLOADED GENERATOR

$$I_{LL} \cong \frac{\sqrt{3}}{2} I_{3\phi}$$

where: I_{LL} = line current during a line to line fault
$I_{3\phi}$ = line current during a three-phase fault

Test 24

Problem 865:

In symmetrical components, what is the vector sum of $1 + a + a^2$?

A. 1
B. 0
C. -1
D. Infinity

Problem 866: EE Board October 1998

If the load of a wye-connected transformer are:

$I_a = 10 \text{ cis } (-30)$ $I_b = 12 \text{ cis } 215$ $I_c = 15 \text{ cis } 82$

What is the phase b positive sequence component?

A. 13.4 cis (-32.2)
B. 10.2 cis 240
C. 12.27 cis 208.4
D. 12.27 cis (-31.6)

Problem 867:

Given three unbalanced three-phase voltages:

$V_a = 150 + j0 \text{ V}$ $V_b = -90 - j120 \text{ V}$ $V_c = -120 + j90 \text{ V}$

Determine V_{a1}.

A. 142.43 + j 12.35
B. 135.32 - j 21.34
C. 145.62 + j 13.66
D. 140.23 – j 9.32

Problem 868: EE Board March 1998

The three-phase unbalanced currents are:

$I_a = 10 \text{ cis } (-30)$ $I_b = 0$ $I_c = 10 \text{ cis } 150$

Find the negative sequence current of phase a.

A. 8.66 cis 30
B. 5.77 cis (-60)
C. –5.77
D. 5.77

Problem 869: EE Board October 1984

Given the following currents:

$I_a = 60 + j0$ A $\quad I_b = -36 - j48$ A $\quad I_c = -48 + j36$ A

Solve for the negative sequence component of I_a.

A. 8.641 – j1.543
B. 9.436 + j1.346
C. 9.751 – j1.464
D. 8.354 + j1.034

Problem 870: EE Board October 1998

The three-phase unbalance currents are:

$I_a = 10 \text{ cis } (-30)$ $\quad I_b = 0$ $\quad I_c = 10 \text{ cis } 150$

Find the zero sequence current.

A. 3.33 cis 30
B. 0
C. 5.77
D. 3.33

Problem 871: EE Board October 1984

Given the following currents:

$I_a = 60 + j0$ A $\quad I_b = -36 - j48$ A $\quad I_c = -48 + j36$ A

Solve for zero sequence component of I_a.

A. 10 + j4
B. 8 - j6
C. –8 – j4
D. 12 – j6

Problem 872: EE Board October 1997

The sequence currents of phase a current are as follows:

Zero sequence current = $14.13\angle 17.34°$
Positive sequence current = $2.98\angle 10.06°$
Negative sequence current = $708.26\angle -31°$

Determine phase a current.

A. $720\angle -30°$
B. $730\angle -15.2°$
C. $710\angle 88°$
D. $695\angle 15.2°$

Problem 873: EE Board October 1998

The sequence components of phase a current are:

Zero sequence component = 0.47 + j 1.49
Positive sequence component = 18.4 cis (-31.6)
Negative sequence component = 3.23 cis 168.2

Determine the phase b current.

A. 18 cis 215
B. 15 cis 240
C. 19 cis 220
D. 20 cis 225

Problem 874: EE Board March 1998

The sequence components of phase a current are:

Zero sequence = 0.47 + j 1.49
Positive sequence = 18.4 cis (-31.6)
Negative sequence = 3.23 cis 168.2

Determine the phase c current.

A. 17.5 cis 91
B. 18 cis 216
C. 22.5 cis 82
D. 15 cis 100

Problem 875: EE Board April 1992

Determine the symmetrical components of the line current in line 'a' if one of the in phase impedance of its delta-connected load connected across lines 'ca' is removed. The delta load with impedance of $10\angle 0°$ ohms per phase is supplied from a 220 volts, 60 cycle, 3-phase. Assume a phase sequence of a-b-c.

A. $I_{a1} = 11\,A$, $I_{a2} = 11\,A$, $I_{a0} = 0\,A$
B. $I_{a1} = 7.33\,A$, $I_{a2} = 7.33\,A$, $I_{a0} = 7.33\,A$
C. $I_{a1} = 22\,A$, $I_{a2} = 22\,A$, $I_{a0} = 22\,A$
D. $I_{a1} = 25.4\,A$, $I_{a2} = 12.7\,A$, $I_{a0} = 0\,A$

Problem 876: EE Board April 1991

A star-connected balanced load takes 75 A from a balanced 3-phase, 4-wire supply. If the two supply lines of the fuses are removed determine the symmetrical components of the line currents after the fuses are removed.

A. $I_1 = 25\,A$, $I_2 = 25\,A$, $I_0 = 25\,A$
B. $I_1 = 25\,A$, $I_2 = 50\,A$, $I_0 = 0\,A$
C. $I_1 = 75\,A$, $I_2 = 75\,A$, $I_0 = 75\,A$
D. $I_1 = 75\,A$, $I_2 = 0\,A$, $I_0 = 0\,A$

Problem 877: EE Board March 1998

A 50 MVA, 33 kV/11 kV, three-phase, wye-delta connected transformer has a 3% impedance. What is the percent impedance at 100 MVA base and 34.5 kV base?

A. 1.639%
B. 5.49%
C. 5.74%
D. 6.56%

Problem 878:

At a certain location of an electrical system, the available short circuit MVA is 10 at 110 kV while its Thevenin's equivalent reactance is 0.05 pu. Determine the per unit reactance of this point using a base 20 MVA and 115kV?

A. 0.08
B. 0.07
C. 0.06
D. 0.09

Problem 879:

A 13.8 kV/440 V, 50 kVA single-phase transformer nas a leakage reactance of 300 ohms referred to the 13.8 kV side. Determine the per unit value of the leakage reactance for the voltage base.

A. 0.074
B. 0.082
C. 0.083
D. 0.079

Problem 880: EE Board April 1980

A 5 kVA, 2400-120/240 volt distribution transformer when given a short circuit test had 94.2 volts applied with rated current flowing in the short-circuited wiring. What is the per unit impedance of the transformer?

A. 0.0392 ohms
B. 0.0386 ohms
C. 0.0415 ohms
D. 0.0435 ohms

Problem 881:

A 3-phase, 375 kVA, 480 V, 50 Hz, wye connected alternator has an equal positive and negative sequence reactance of 10%. Find the symmetrical fault current, if a 3-phase fault occurs at the alternator terminals.

A. 4510.5 A
B. 4620.3 A
C. 4255.2 A
D. 4778.1 A

Problem 882: EE Board April 1992

A generator rated 600 kVA, 2400 V, 60 cycles, 3-phase, 6-poles and wye-connected has 10% synchronous reactance. If a three-phase fault occurs, what will be the short circuit current?

A. 1443 A
B. 1532 A
C. 1435 A
D. 1428 A

Problem 883:

A 20 MVA, 13.8 kV, 60 Hz, three-phase synchronous turbo alternator has a positive, negative and zero sequence reactances of 0.25, 0.35 and 0.15 respectively. If lines b and c are short-circuited, determine the subtransient fault current in line b. Assume the alternator is operating at no load and at rated voltage with the neutral of the alternator solidly grounded.

A. 2614 A
B. 2886 A
C. 2415 A
D. 2510 A

Problem 884: EE Board March 1998

At a certain location in an electric system, the available fault MVA is 400 MVA. A 15 MVA, 34.5 kV/6.24 kV, 2.5% impedance, wye-wye grounded transformer is installed at that location. Determine the short circuit MVA at the secondary side of the transformer.

A. 195 MVA
B. 150 MVA
C. 90 MVA
D. 240 MVA

Problem 885:

Two 3-phase transformers are connected in parallel at the primary as well as at the secondary sides. One is rated 10 MVA, 34.5/13.8 kV and 4% impedance while the other is rated 7.5 MVA, 34.5/13.8 kV and 5% impedance. The primary tapping point has a 3-phase short circuit MVA of 1000 MVA. Determine the fault current delivered to a 3-phase fault at the secondary side bus bars.

A. 13,501 A
B. 10,452 A
C. 12,532 A
D. 11,953 A

Problem 886:

A 100 MVA, 22 kV synchronous turbo generator has positive and negative reactances of 0.20 p.u. and a zero sequence reactance of 0.05 p.u. The neutral of the generator is grounded through a reactor of 0.242 ohm. Determine the ratio of the subtransient current for a single line to ground fault to the subtransient current for a three-phase fault. Assume the generator is operated without a load and at rated voltage

A. 1.50
B. 1.25
C. 1.00
D. 0.80

Problem 887: EE Board October 1992

A generator is rated 100 MVA, 15 kV, it is Y-connected, solid grounded and is operated at rated voltage at no-load and is disconnected from the rest of the system. Its positive and negative sequence reactances are each 0.10 p.u. and its zero sequence reactance is 0.05 p.u. Calculate in ohms of inductive reactance to be inserted in the neutral connection of the generator to limit the fault current for a single line to ground fault to the fault current of a symmetrical three-phase fault.

A. 0.0233 Ω
B. 0.0225 Ω
C. 0.0376 Ω
D. 0.0325 Ω

Problem 888: EE Board October 1982

The reactance of a three-phase alternator is 8 percent. If the alternator is rated 25 MVA at 13.25 kV output voltage, line to phase, solve for the magnitude of the fault current generated when a short circuit occurs between 2 phases at the terminals.

A. 6704 A
B. 6135 A
C. 6808 A
D. 6458 A

Problem 889:

A 3-phase, 375 kVA, 480 V, 50 Hz, wye connected alternator has an equal positive and negative sequence reactance of 10%. Find the unsymmetrical fault current, if a 2-phase fault (line to line) occurs at the alternator terminals.

A. 3890 A
B. 3906 A
C. 4510 A
D. 4180 A

Problem 890: EE Board March 1998

A 5 MVA, 13.8 kV / 480 V, 5% impedance transformer is tapped at 13.8 kV line where the Thevenin's equivalent impedance is 1/2 ohm. Determine the fault current at the primary for a three-phase fault at the secondary.

A. 10,500 A
B. 3,300 A
C. 4,200 A
D. 6,050 A

Problem 891: EE Board October 1980

A 10 kVA, 110 V, 3-phase, 4-wire, 60 Hz alternator generates 70 volts per phase when excited to give rated voltage at full load. The armature synchronous impedance per phase is 0.2 ohm. What is the current in each phase if the generator terminals are short-circuited?

A. 350 A
B. 875 A
C. 620 A
D. 534 A

Problem 892: EE Board October 1998

At a certain point of the system network the positive, negative and zero sequence impedances are 0.25 p.u., 0.25 p.u. and 0.3 p.u. respectively. The base MVA is 100. The voltage level at that point is 34.5 kV. Determine the zero sequence current for a one line to ground fault.

A. 6275 A
B. 8132 A
C. 7516 A
D. 2091 A

Problem 893:

In a short circuit analysis, the positive, negative and zero sequence impedances are 0.20 pu, 0.20 pu and 0.25 pu, respectively, using a base MVA of 50. Estimate the fault current on the faulted lines if a double line to ground fault at the 138 kV level occurs.

A. 1204 A
B. 1011 A
C. 1090 A
D. 1104 A

Problem 894:

A 15 MVA, 6,600 V, 60 Hz, Y-connected synchronous alternator has a positive, negative and zero sequence per unit reactance of 0.20, 0.20 and 0.10 respectively. The neutral of the generator is grounded through a reactor with a per unit reactance of 0.05 based on the generator rating. If a double line to ground fault occurs at the alternator terminals, estimate the current that flows in the reactor.

A. 5920 A
B. 5623 A
C. 6125 A
D. 6342 A

Problem 895: EE Board March 1998

In a short circuit study, the positive, negative and zero sequence impedance are 0.25 pu and 0.25 pu and 0.3 pu, respectively. The base MVA is 100. Determine the fault current for a three-phase fault at the115 kV level.

A. 3000 A
B. 2000 A
C. 2500 A
D. 3500 A

Problem 896:

A 3-phase 220-V, Y-connected alternator has a synchronous impedance of 0.15 + j2 ohms per phase. The alternator is connected through a short transmission line whose impedance per wire is 2 + j1 ohms to a Y-Y connected transformer bank whose total equivalent impedance referred to the secondary (high side) is 50 + j86.6 ohms. A three-phase load is connected from the secondary side of the bank through a transmission line whose impedance is 50 + j75 ohms per wire. If a symmetrical three-phase fault occurs at the load side, how much is the current will flow to the alternator windings. Assume the bank has a transformation ratio of 1:10

A. 22.73 A
B. 23.66 A
C. 21.54 A
D. 24.15 A

Problem 897: EE Board April 1984, EE Board April 1987

Three single-phase transformers each rated 50 kVA, 2400 volts primary, 277 volts secondary, four (4) percent impedance have their primaries connected in delta and their secondaries in wye. Calculate the fault current drawn if a short circuit occurs between two phases at the secondary terminals.

A. 2260 A
B. 2585 A
C. 2036 A
D. 2256 A

Problem 898: EE Board October 1986

La Tondeña Distillery Inc., located in Canlubang, Calamba, Laguna installed an emergency 3-phase generator, 3-wire GM generator is rated 350 kVA, 460 volts, 60 Hz, with a reactance of 8%. Solve for the symmetrical 3-phase short circuit.

A. 5,321 A
B. 5,703 A
C. 5,284 A

D. 5,491 A

Problem 899: EE Board October 1997

A 15 MVA, 34.5 kV/6.24 kV transformer is connected at an infinite bus. The percent impedance of the transformer is 2.5%. What is the current at the 34.5 kV side for a three phase short at the 6.24 kV side?

A. 55,500 A
B. 10,000 A
C. 5,000 A
D. 25,000 A

Problem 900: EE Board April 1985

A three-phase, 3-wire generator is rated 325 kVA, 480 volts, 60 Hz with a reactance of eight (8) percent. Solve for the fault current delivered by the generator during a 2-phase short circuit at the terminals.

A. 4232 A
B. 4462 A
C. 4012 A
D. 4156 A

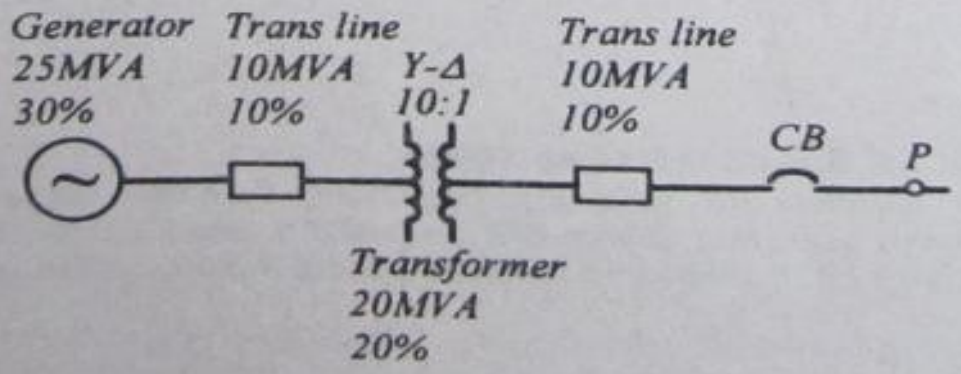
Problem 901:

The secondary side of 5 MVA, 34.5/13.8 kV, 3-phase transformer is connected to a bus bar served by a 20 MVA, 13.8 kV, 3-phase alternator. The primary side of the transformer is tapped from an infinite bus. Estimate the fault current delivered to a 3-phase fault near the common bus bar. The transformer and alternator have 4% and 7.5% reactance, respectively.

A. 12,760 A
B. 18,900 A
C. 15,340 A
D. 16,400 A

Problem 902:

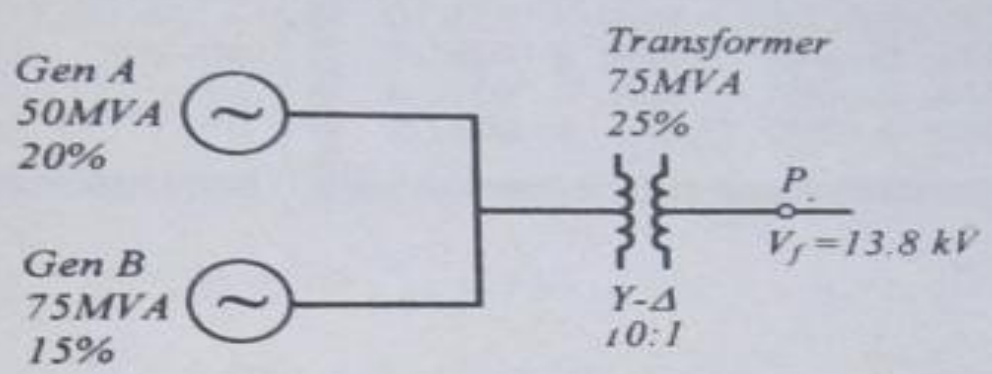
Determine the minimum rupturing capacity of the circuit breaker as shown, if a three-phase symmetrical fault occurs at point P.



A. 30 MVA
B. 20 MVA
C. 10 MVA
D. 15 MVA

Problem 903:

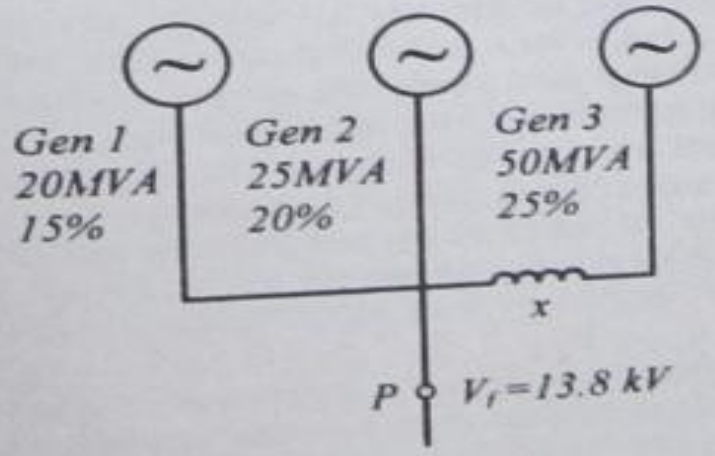
Given the one line diagram as shown, determine the current delivered by generator A when a three-phase fault occurs at point P.



A. 172.5 A
B. 167.1 A
C. 145.5 A
D. 198.4 A

Problem 904:

Given the one-line diagram as shown, determine the reactance of the limiting reactor x in order to limit the three-phase symmetrical fault power at point P to 400 MVA.



A. 0.65 Ω
B. 0.40 Ω
C. 0.52 Ω
D. 0.46 Ω

ANSWER KEY

865.	B	873.	A	881.	A	889.	B	897.	D
866.	C	874.	C	882.	A	890.	B	898.	D
867.	C	875.	D	883.	C	891.	A	899.	B
868.	B	876.	A	884.	D	892.	D	900.	A
869.	C	877.	B	885.	D	893.	B	901.	D
870.	B	878.	D	886.	C	894.	B	902.	B
871.	C	879.	D	887.	C	895.	B	903.	A
872.	A	880.	A	888.	C	896	A	904.	B

☑ RATING

36-40 - Topnotcher

28-35 - Passer

20-27 - Conditional

0-19 - Failed

Did you know that... even after the invention of the telegraph, people were still fearing electricity and the effects that it presented by passing their lands? They even demolished a whole line, blaming it for upsetting the weather and causing a sequence of bad harvests for the farmers.

The first practical applications of electricity as acclaimed by many, was the "TELEGRAPH". Developed by Samuel F. Morse, an American portrait painter and inventor. He built it on the ideas of the American physicist Joseph Henry, using the opening and closing of relays to produce the dots and dashes that represent letters and numbers.

Morse was born in Charles town, Massachusetts, the son of a minister and an author. He studied to be an artist at Yale and Royal Academy of Arts in London.

In 1826 he helped found and became the first president of the National Academy of Design. But tragedy struck the Morse family. The previous year his wife died. In 1826, his father died. And in 1828 his mother died. Morse went to Europe to recover and study further.

In 1832, while on a voyage, he worked out a design paper for an electromagnetic telegraph apparatus, the principle of which had already been considered by Henry.

In May 24, 1844 Morse sent the first message, "WHAT HATH GOD WROUGHT". The message was sent from Washington to Baltimore to an operator who repeated the message back to Morse.

And who would ever thought by 1851; there were over 50 telegraph companies who used the symbols designed by Morse, known as the "Morse Code".

Solutions to Test 24

865

$a = 1\angle 120^\circ = -0.5 + j0.866$

$a^2 = 1\angle 240^\circ = -0.5 - j0.866$

$$1 + a + a^2 = 1 + 1\angle 120^\circ + 1\angle 240^\circ$$
$$= 1 + (-0.5 + j0.866) + (-0.5 - j0.866)$$
$$= 0$$

866

$$I_{a1} = \frac{1}{3}\left(I_a + aI_b + a^2 I_c\right)$$
$$= \frac{1}{3}\left[10\angle -30^\circ + (1\angle 120^\circ)(12\angle 215^\circ) + (1\angle 240^\circ)(15\angle 82^\circ)\right]$$
$$= \frac{1}{3}\left[10\angle -30^\circ + 12\angle 335^\circ + 15\angle 322^\circ\right]$$
$$= \frac{1}{3}\left[8.66 - j5 + 10.875 - j5.071 + 11.820 - j9.235\right]$$

$I_{a1} = 10.45 - j6.435 = 12.27\angle -31.624^\circ$

$I_{b1} = a^2 I_{a1} = (1\angle 240^\circ)(12.27\angle -31.624^\circ)$

$I_{b1} = 12.27\angle 208.376^\circ$

867

$V_a = 150 + j0 = 150\angle 0^\circ$

$V_b = -90 - j120 = 150\angle -126.869^\circ$

$V_c = -120 + j90 = 150\angle 143.13^\circ$

$$V_{a1} = \frac{1}{3}\left(V_a + aV_b + a^2 V_c\right)$$
$$= \frac{1}{3}\left[150\angle 0^\circ + (1\angle 120^\circ)(150\angle -126.869^\circ) + (1\angle 240^\circ)(150\angle 143.13^\circ)\right]$$
$$= \frac{1}{3}\left[150\angle 0^\circ + 150\angle -6.869^\circ + 150\angle 383.13^\circ\right]$$
$$= \frac{1}{3}\left[150 + j0 + 148.923 - j17.94 + 137.942 + j58.923\right]$$

$V_{a1} = 145.62 + j13.66$

868

$$I_{a2} = \frac{1}{3}\left(I_a + a^2 I_b + aI_c\right)$$
$$= \frac{1}{3}\left[10\angle -30^\circ + (1\angle 240^\circ)(0) + (1\angle 120^\circ)(10\angle 150^\circ)\right]$$
$$= \frac{1}{3}\left[10\angle -30^\circ + 0 + 10\angle 270^\circ\right] = \frac{1}{3}\left[8.66 - j5 + 0 - j10\right]$$

$I_{a2} = 2.886 - j5 = 5.77\angle -60^\circ = 5.77 \text{ cis } (-60)$

$I_{a2} = 2.886 - j5 = 5.77\angle -60°$

$I_{a2} = 5.77 \text{ cis}(-60)$

869 $I_a = 60 + j0 = 60\angle 0°$

$I_b = -36 - j48 = 60\angle -126.869°$

$I_c = -48 + j36 = 60\angle 143.13°$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2 I_b + a I_c\right)$$

$$= \frac{1}{3}\left[60\angle 0° + (1\angle 240°)(60\angle -126.869°) + (1\angle 120°)(60\angle 143.13°)\right]$$

$$= \frac{1}{3}\left[60\angle 0° + 60\angle 113.13° + 60\angle 263.13°\right]$$

$$= \frac{1}{3}\left[60 + (-23.569 + j55.177) + (-7.177 - j59.569)\right]$$

$I_{a2} = 9.751 - j1.464$

870 $$I_{a0} = \frac{1}{3}\left(I_{a1} + I_{a2} + I_{a0}\right)$$

$$= \frac{1}{3}\left[10\angle -30° + (0) + (10\angle 150°)\right]$$

$$= \frac{1}{3}\left[8.66 - j5 + 0 + -8.66 + j5\right]$$

$I_{a0} = 0$

871 $$I_{a0} = \frac{1}{3}\left(I_{a1} + I_{a2} + I_{a0}\right)$$

$$= \frac{1}{3}\left[60 + j0 + (-36 - j48) + (-48 + j36)\right]$$

$I_{a0} = -8 - j4$

872 $I_a = I_{a1} + I_{a2} + I_{a0}$

$= \left[2.98\angle 10.06° + 708.26\angle -31° + 14.13\angle 17.34°\right]$

$= 2.934 + j0.52 + 607.097 - j364.78 + 13.487 + j4.211$

$I_a = 623.518 - j360.05$

$I_a = 720\angle 30°$

873 $I_{b1} = a^2 I_{a1} = (1\angle 240°)(18.4\angle -31.6°) = 18.4\angle 208.4° = -16.185 - j8.75$

$I_{b2} = a I_{a2} = (1\angle 120°)(3.23\angle 168.2°) = 3.23\angle 288.2° = 1.009 - j3.068$

$I_{b0} = I_{a0} = 0.47 + j1.49$

$I_b = I_{b1} + I_{b2} + I_{b0}$

$= -16.185 - j8.75 + 1.009 - j3.068 + 0.47 + j1.49$

$= -14.706 - j10.328 = 17.97\angle -145° \cong 18\angle(360° - 145°)$

$I_b = 18\angle 215°$

$I_b = 18 \text{ cis } (215)$

874 $I_{c1} = aI_{a1} = (1\angle 120°)(18.4\angle -31.6°) = 18.4\angle 88.4° = 0.513 + j18.39$

$I_{c2} = a^2 I_{a2} = (1\angle 240°)(3.23\angle 168.2°) = 3.23\angle 408.2° = 2.153 + j2.407$

$I_{c0} = I_{a0} = 0.47 + j1.49$

$I_c = I_{c1} + I_{c2} + I_{c0}$

$= 0.513 + j18.39 + 2.153 + j2.407 + 0.47 + j1.49$

$= 3.136 + j22.287 = 22.5\angle 82°$

$I_c = 22.5 \text{ cis } (82)$

875 $I_{ab} = \dfrac{V_{ab}}{Z} = \dfrac{220\angle 0°}{10\angle 0°} = 22\angle 0°$

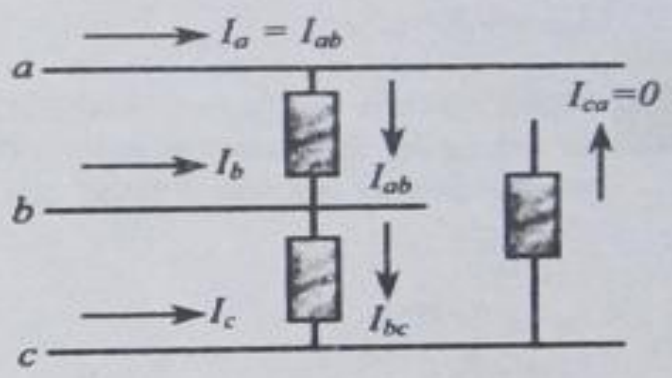
$= 22 \text{ A}$

$I_{bc} = \dfrac{V_{bc}}{Z} = \dfrac{220\angle -120°}{10\angle 0°}$

$= 22\angle -120° \text{ A}$

$= -11 - j19.052$

$I_a = I_{ab} = 22\angle 0° \text{ A}$



$I_b = I_{bc} - I_{ab} = -11 - j19.052 - 22$

$= -33 - j19.052$

$= 38.104\angle -150° \text{ A}$

$I_c = -I_{bc} = -(-11 - j19.052°) = 22\angle 60° \text{ A}$

$I_{a1} = \dfrac{1}{3}\left(I_a + aI_b + a^2 I_c\right)$

$= \dfrac{1}{3}[22\angle 0° + (1\angle 120°)(38.104\angle -150°) + (1\angle 240°)(22\angle 60°)]$

$= \dfrac{1}{3}[22\angle 0° + 38.104\angle -30° + 22\angle 300°]$

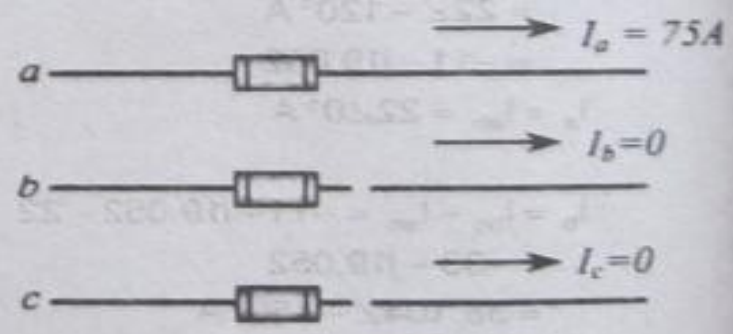
$= \dfrac{1}{3}[22 + 33 - j19.052 + 11 - j19.052]$

$I_{a1} = 22 - j12.7 = 25.4\angle -30° \text{ A}$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2 I_b + a I_c\right)$$
$$= \frac{1}{3}\left[22\angle 0^\circ + (1\angle 240^\circ)(38.104\angle -150^\circ) + (1\angle 120^\circ)(22\angle 60^\circ)\right]$$
$$= \frac{1}{3}\left[22\angle 0^\circ + 38.104\angle 90^\circ + 22\angle 180^\circ\right]$$
$$= \frac{1}{3}\left[22 + j38.104 - 22\right]$$
$$I_{a2} = j12.7 = 12.7\angle 90^\circ \text{ A}$$

$$I_{a0} = \frac{1}{3}\left(I_{a1} + I_{a2} + I_{a0}\right)$$
$$= \frac{1}{3}\left[22\angle 0^\circ + 38.104\angle -150^\circ + 22\angle 60^\circ\right]$$
$$= \frac{1}{3}\left[22 + (-33 - j19.052) + (11 + j19.052)\right]$$
$$I_{a0} = 0 \text{ A}$$

876 Note: Assume the fuses in lines "b and c" are removed and the current in line "a" as reference. By assuming different reference and lines to be open circuited, the magnitudes of the symmetrical component currents in the three lines will still be the same.



$$I_a = 75\angle 0^\circ \text{ A}$$
$$I_b = 0 \text{ A}$$
$$I_c = 0 \text{ A}$$

$$I_{a1} = \frac{1}{3}\left(I_a + a I_b + a^2 I_c\right)$$
$$= \frac{1}{3}\left[75\angle 0^\circ + (1\angle 120^\circ)(0) + (1\angle 240^\circ)(0)\right]$$
$$I_{a1} = 25\angle 0^\circ \text{ A}$$
$$I_{b1} = a^2 I_{a1} = (1\angle 240^\circ)(25\angle 0^\circ) = 25\angle 240^\circ \text{ A}$$
$$I_{c1} = a I_{a1} = (1\angle 120^\circ)(25\angle 0^\circ) = 25\angle 120^\circ \text{ A}$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2 I_b + a I_c\right)$$
$$= \frac{1}{3}\left[75\angle 0^\circ + (1\angle 240^\circ)(0) + (1\angle 120^\circ)(0)\right]$$
$$I_{a2} = 25\angle 0^\circ \text{ A}$$

$I_{b2} = aI_{a2} = (1\angle 120°)(25\angle 0°) = 25\angle 120°$ A

$I_{c2} = a^2 I_{a2} = (1\angle 240°)(25\angle 0°) = 25\angle 240°$ A

$$I_{a0} = \frac{1}{3}(I_{a1} + I_{a2} + I_{a0})$$

$$= \frac{1}{3}[75\angle 0° + 0 + 0]$$

$I_{a0} = 25\angle 0°$ A

$I_{b0} = I_{c0} = 25\angle 0°$ A

877 $$Z_{pu2} = Z_{pu1}\left(\frac{V_{base1}}{V_{base2}}\right)^2\left(\frac{S_{base2}}{S_{base1}}\right) = 0.03\left(\frac{33}{34.5}\right)^2\left(\frac{100}{50}\right) = 0.054896$$

$Z_{pu2} \cong 5.49\%$

878 $$Z_{pu2} = Z_{pu1}\left(\frac{V_{base1}}{V_{base2}}\right)^2\left(\frac{S_{base2}}{S_{base1}}\right) = 0.05\left(\frac{110}{115}\right)^2\left(\frac{20}{10}\right)$$

$Z_{pu2} \cong 0.09$

879 $$X_{pu} = \frac{XS_{base}}{V_{base}^2} = \frac{(300)(50{,}000)}{(13{,}800)^2}$$

$X_{pu} \cong 0.079$

880 $$I_{sc} \cong I_p = \frac{\text{VA rating}}{V_p} = \frac{5000}{2400} = 2.083 \text{ A}$$

$$Z = \frac{V_{sc}}{I_{sc}} = \frac{94.2}{2.083} = 45.22\ \Omega$$

$$Z_{pu} = \frac{I_p Z}{V_p} = \frac{2.083(45.22)}{2400}$$

$Z_{pu} = 0.0392$

881 $$I_{pu} = \frac{E_{pu}}{X_{pu}} = \frac{1}{j0.1} = -j10$$

$$I_{base} = \frac{S}{\sqrt{3}E} = \frac{375 \times 10^3}{\sqrt{3}(480)} = 451.05 \text{ A}$$

$I_f = I_{pu}(I_{base}) = 10(451.05)$

$I_f = 4510.5$ A

Another solution:

$$kVA_f = \frac{KVA_c}{X_t} = \frac{375}{0.1} = 3750\ kVA$$

$$I_f = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{3750}{\sqrt{3}(0.48)}$$

$$I_f = 4510.5\ A$$

882 $$I_{pu} = \frac{E_{pu}}{X_{pu}} = \frac{1}{j0.1} = -j10$$

$$I_{base} = \frac{S}{\sqrt{3}E} = \frac{600 \times 10^3}{\sqrt{3}(2400)} = 144.34\ A$$

$$I_f = I_{pu}(I_{base}) = 10(144.34)$$

$$I_f = 1443.4\ A$$

Another solution:

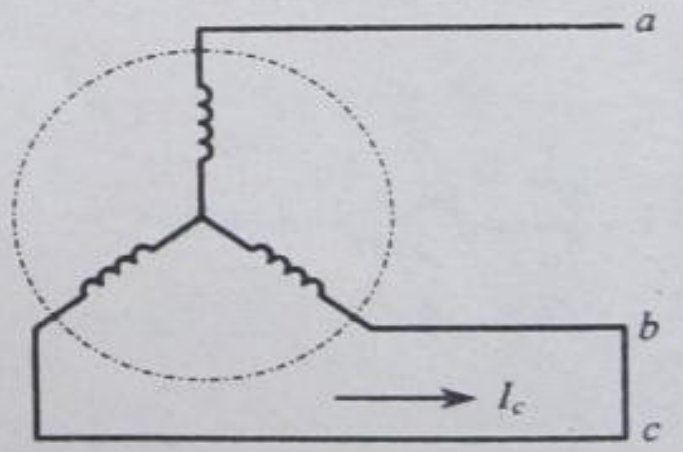
$$kVA_f = \frac{KVA_c}{X_{pu}} = \frac{600}{0.1} = 6000\ kVA$$

$$I_f = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{6000}{\sqrt{3}(2.4)}$$

$$I_f = 1443.4\ A$$

883 $$I_{base} = \frac{S}{\sqrt{3}E} = \frac{20{,}000{,}000}{\sqrt{3}(13800)}$$

$$I_{base} = 836.7\ A$$



For a line to line fault:

$$I_{a1} = \frac{E_{pu}}{X_{1pu} + X_{2pu}}$$

$$I_{a1} = \frac{1.0}{j0.25 + j0.35}$$

$$I_{a1} = -j1.667\ pu$$

$$I_{a2} = -I_{a1} = j1.667\ pu$$

$$I_{a0} = 0$$

Note: For a line to line fault: $I_{a0} = 0$, $I_{a1} = -I_{a2}$

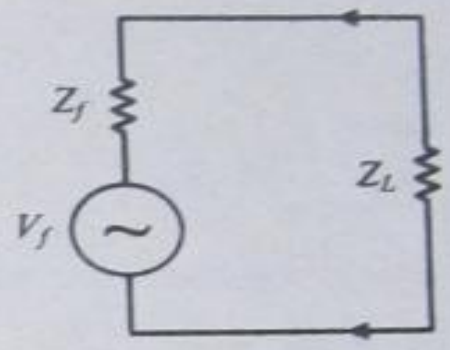
Using symmetrical components :

$$I_c = I_{c1} + I_{c2} + I_{c0}$$

$$I_c = aI_{a1} + a^2 I_{a2} + I_{a0}$$
$$= (1\angle 120°)(1.667\angle -90°) + (1\angle -120°)(1.667\angle 90°) + 0$$
$$= 1.667\angle 30° + 1.667\angle -30° + 0$$
$$= 1.443 + j0.8335 + (1.433 - j0.8335)$$
$$I_c = 2.886 \text{ pu}$$
$$I_{c\,fault} = I_{c\,pu}(I_{base}) = 2.886(836.7)$$
$$I_{c\,fault} = 2414.7 \text{ A}$$

884 $$S_f = \sqrt{3}V_f I_f = \sqrt{3}V_f\left(\frac{V_f/\sqrt{3}}{Z_f}\right) = \frac{V_f^2}{Z_f}$$

$$Z_f = \frac{V_f^2}{S_f} = \frac{(6.24 \times 10^3)^2}{400 \times 10^6} = 0.097344\ \Omega$$



Thevenin's equivalent circuit

Note: Z_f = equivalent impedance at the fault point per phase

$$Z_L = \frac{(\%IZ)V^2}{S} = \frac{(0.025)(6.24 \times 10^3)^2}{15 \times 10^6}$$
$$Z_L = 0.064896\ \Omega$$

Using the Thevenin's equivalent circuit:

$$Z_t = Z_f + Z_L = 0.097344 + 0.064896$$
$$Z_t = 0.16224\ \Omega$$
$$S_{f\,new} = \frac{V_f^2}{Z_t} = \frac{(6.24 \times 10^3)^2}{0.16224}$$
$$S_{f\,new} = 240 \text{ MVA}$$

Another solution:

Note: Since the transformer MVA is different from the fault MVA, select a common MVA and express all per unit reactance based on this MVA.

Use the 15 MVA as the common MVA base.

$$Z_{f_{pu}} = \frac{MVA_c}{MVA_f} = \frac{15}{400} = 0.0375$$
$$Z_{t_{pu}} = Z_{f_{pu}} + Z_{tran_{pu}} = 0.0375 + 0.025$$
$$Z_{t_{pu}} = 0.0625$$

$$MVA_{new} = \frac{MVA_c}{Z_{1_{pu}}} = \frac{15}{0.0625}$$

$$MVA_{new} = 240\,MVA$$

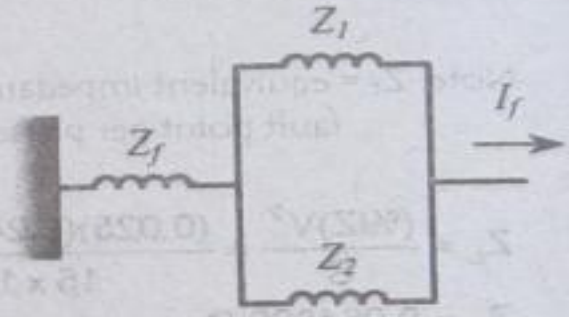
885 Note: Since the transformers MVA is different from the fault MVA, select a common MVA and express all per unit reactance based on this MVA.

Use the 10 MVA as the common MVA base.

$$Z_{1_{pu}} = \frac{MVA_c}{MVA_1} = \frac{10}{1000} = 0.01$$

$$Z_{pu2}' = Z_{pu2}\left(\frac{MVA_c}{MVA_2}\right) = 0.05\left(\frac{10}{7.5}\right) = 0.0667$$

Refer to the equivalent circuit diagram:



$$Z_{1_{pu}} = Z_{1_{pu}} + \frac{Z_{1_{pu}} Z_{2_{pu}}'}{Z_{1_{pu}} + Z_{2_{pu}}'}$$

$$= 0.01 + \frac{0.04(0.0667)}{0.04 + 0.0667}$$

$$Z_{1_{pu}} = 0.035$$

$$MVA_f = \frac{MVA_c}{Z_{1_{pu}}} = \frac{10}{0.035} = 285.714\,MVA$$

$$I_f = \frac{S_f}{\sqrt{3}V_f} = \frac{285.714 \times 10^6}{\sqrt{3}(13.8 \times 10^3)}$$

$$I_f = 11{,}953.41\,A$$

886

$$X_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(22{,}000)^2}{(100{,}000{,}000)}$$

$$X_{base} = 4.84\,\Omega$$

$$X_n = \frac{X_{neutral}}{X_{base}} = \frac{0.242}{4.84} = 0.05\,pu$$

For a single-line to ground fault:

$$I_{a1} = \frac{E_{pu}}{X_1 + X_2 + X_0 + 3X_n} = \frac{1.0}{j0.2 + j0.2 + j0.05 + j3(0.05)}$$

$$I_{a1} = -j1.667\,pu$$

$$I_a = 3I_{a1} = 3(-j1.667) = -j5.001\,pu$$

For a three-phase fault

$$I_a = \frac{E_{pu}}{X_1} = \frac{1.0}{j0.2} = -j5 \text{ pu}$$

$$\text{Ratio} = \frac{I_{a(LG)}}{I_{a(3\phi)}} = \frac{-j5.001}{-j5}$$

Ratio = 1.0

887 $$I_{3\phi} = \frac{E_{pu}}{Z_1} = \frac{1.0}{j0.1} = -j10 \text{ pu}$$

$$I_{LG} = 3I_{a1} = \frac{3E_{pu}}{Z_o + Z_1 + Z_2 + 3Z_n} = \frac{3(1.0)}{j(0.05 + 0.10 + 0.10 + 3Z_n)}$$

$$I_{LG} = \frac{-j3}{0.25 + 3Z_n}$$

$$I_{3\phi} = I_{LG}$$

$$-j10 = \frac{-j3}{0.25 + 3Z_n}$$

$$Z_n = j0.0167 \text{ pu}$$

$$X_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(15{,}000)^2}{(100{,}000{,}000)}$$

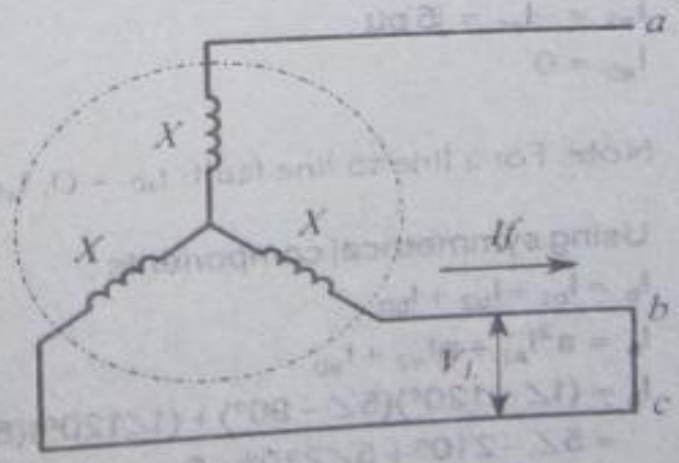
$$X_{base} = 2.25\ \Omega$$

$$X_{n\,pu} = \frac{X_{neutral}}{X_{base}}$$

$$X_{neutral} = X_{n\,pu}(X_{base}) = (0.0167)(2.25)$$

$$X_{neutral} = 0.0376\ \Omega$$

888 Note: Assume the alternator is wye-connected.



$$\%IX = \frac{SX}{V^2}$$

$$X = \frac{(\%IX)(V^2)}{S}$$

$$= \frac{(0.08)\left(\sqrt{3}(13.25 \times 10^3)\right)^2}{25 \times 10^6}$$

$$X = 1.6854\ \Omega$$

Refer to the circuit diagram:

$$I_f = \frac{V_L}{2X} = \frac{\sqrt{3}(13.25 \times 10^3)}{2(1.6854)}$$

$$I_f = 6808 \text{ A}$$

Another solution:

$$kVA_f = \frac{KVA_c}{X_{pu}} = \frac{25000}{0.08} = 312{,}500 \text{ kVA}$$

$$I_{f3\phi} = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{312{,}500}{\sqrt{3}\left(\sqrt{3}(13.25)\right)}$$

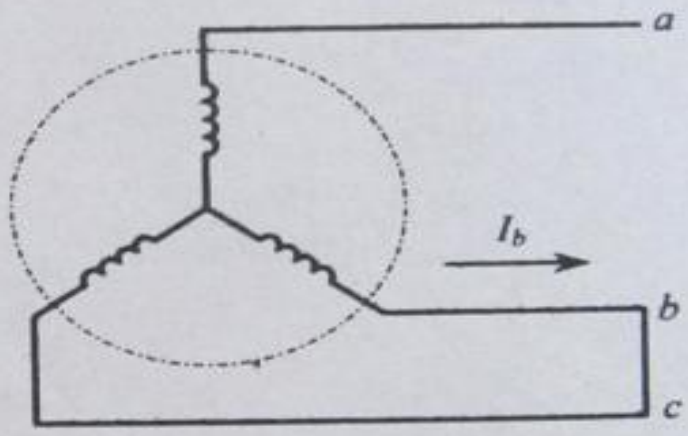
$$I_{f3\phi} = 7861.6 \text{ A}$$

$$I_{fLL} = \frac{\sqrt{3}}{2} I_{f3\phi} = \frac{\sqrt{3}}{2}(7861.6)$$

$$I_{fLL} = 6808 \text{ A}$$

889 $$I_{base} = \frac{S}{\sqrt{3}E} = \frac{375{,}000}{\sqrt{3}(480)}$$

$$I_{base} = 451.05 \text{ A}$$



Note: Assume line b and c are short circuited:

$$I_{a1} = \frac{E_{pu}}{X_{1pu} + X_{2pu}}$$

$$I_{a1} = \frac{1.0}{j0.1 + j0.1}$$

$$I_{a1} = -j5 \text{ pu}$$

$$I_{a2} = -I_{a1} = j5 \text{ pu}$$

$$I_{a0} = 0$$

Note: For a line to line fault: $I_{a0} = 0$, $I_{a1} = -I_{a2}$

Using symmetrical components :

$$I_b = I_{b1} + I_{b2} + I_{b0}$$

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$

$$I_b = (1\angle -120°)(5\angle -90°) + (1\angle 120°)(5\angle 90°) + 0$$

$$= 5\angle -210° + 5\angle 210° + 0$$

$$I_b = -4.33 + j2.5 + (-4.33 - j2.5)$$

$I_b = -8.66 = 8.66\angle -180°$ per unit

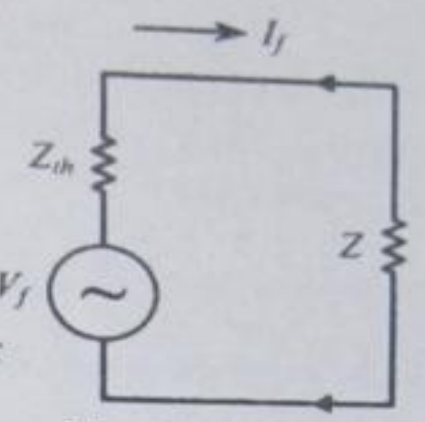
$I_{b\,fault} = I_{b\,pu}(I_{base}) = 8.66(451.05)$

$I_{b\,fault} = 3906.09$ A

890 $\%IZ = \dfrac{SZ}{V^2}$

$$Z = \frac{(\%IZ)(V^2)}{S} = \frac{(0.05)(13.8 \times 10^3)^2}{(5 \times 10^6)}$$

$Z = 1.9044\ \Omega$



Thevenin's equivalent circuit

Refer to the Thevenin's equivalent circuit referred to the primary side of the transformer:

$Z_t = Z_{th} + Z = 0.5 + 1.9044 = 2.4044\ \Omega$

$$I_f = \frac{V_f}{Z_t} = \frac{(13{,}800/\sqrt{3})}{2.4044}$$

$I_f = 3313.68$ A

Another solution:

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(13.8 \times 10^3)^2}{5 \times 10^6} = 38.088\ \Omega$$

$$Z_{th_{pu}} = \frac{Z_{th}}{Z_{base}} = \frac{0.5}{38.088} = 0.01313$$

$$Z_{t_{pu}} = Z_{th_{pu}} + Z_{tran_{pu}} = 0.01313 + 0.05$$
$$= 0.06313$$

$$S_f = \frac{S_{base}}{Z_{t_{pu}}} = \frac{5}{0.06313} = 79.202\ \text{MVA}$$

$$I_f = \frac{S_f}{\sqrt{3}V_f} = \frac{79.202 \times 10^6}{\sqrt{3}(13.8 \times 10^3)}$$

$I_f = 3313.57$ A

Note: From the choices, the closest answer is 3300 A.

891 Note: Assuming the generator is not loaded during the fault, a three-phase fault is approximately equivalent to a line to ground fault.

$$I_f = \frac{V_p}{Z_p} = \frac{70}{0.2} = 350\ \text{A}$$

892 $I_{a0\,pu} = I_{a1\,pu} = I_{a2\,pu}$

$$I_{a0\,pu} = \frac{E_{pu}}{Z_1 + Z_2 + Z_0} = \frac{1.0}{0.25 + 0.25 + 0.3} = 1.25\text{ pu}$$

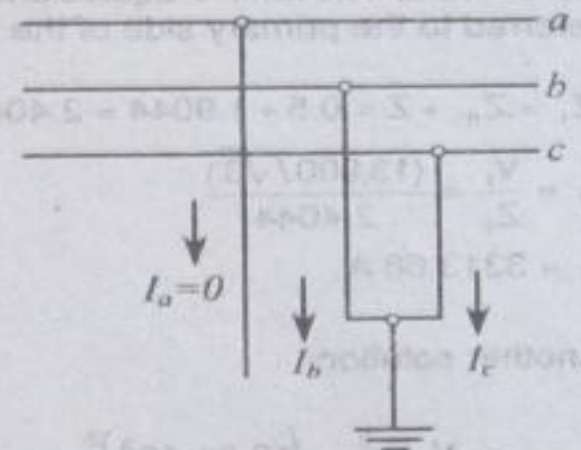
$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} = \frac{100 \times 10^6}{\sqrt{3}(34.5 \times 10^3)} = 1673.48\text{ A}$$

$$I_{a0} = I_{a0\,pu}(I_{base}) = 1.25(1673.48)$$

$$I_{a0} = 2091.81\text{ A}$$

893 Note: Assume lines b and c are the lines accidentally connected to the ground.

Refer to the equivalent circuit:

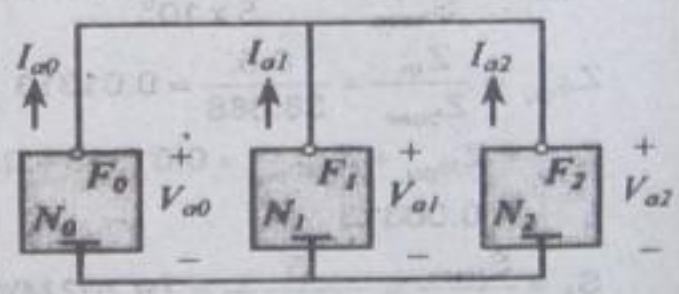


$$I_{a1} = \frac{E_{pu}}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$= \frac{1.0}{j0.2 + j\dfrac{(0.2)(0.25)}{0.2 + 0.25}}$$

$$I_{a1} = -j3.214$$

$$V_{a1} = V_{a2} = V_{a0} = E_{pu} - I_{a1}Z_1$$
$$= 1.0 - (-j3.214)(j0.2)$$
$$= 0.3572\text{ pu}$$



Equivalent network connections of a 2LG fault

$$I_{a0} = -\frac{V_{a0}}{Z_0} = -\frac{0.3572}{j0.25} = j1.4288$$

$$I_{a2} = -\frac{V_{a2}}{Z_2} = -\frac{0.3572}{j0.2} = j1.786$$

Using symmetrical components :

$$I_b = a^2 I_{a1} + aI_{a2} + I_{a0}$$
$$I_b = (1\angle -120°)(3.214\angle -90°) + (1\angle 120°)(1.786\angle 90°) + j1.4288$$
$$= 3.214\angle -210° + 1.786\angle 210° + j1.4288$$
$$= -2.783 + j1.607 + (-1.5467 - j0.893) + j1.4288$$
$$= -4.3297 + j2.1428$$
$$I_b = 4.831\angle 153.67°\text{ pu}$$

Note: Since there was no impedance connected to the faulted lines, fault currents in line b and c are equal.

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} = \frac{50 \times 10^6}{\sqrt{3}(138 \times 10^3)} = 209.184 \text{ A}$$

$I_f = I_{pu} I_{base} = 4.831(209.184) = 1010.568$ A

$I_f \cong 1011$ A

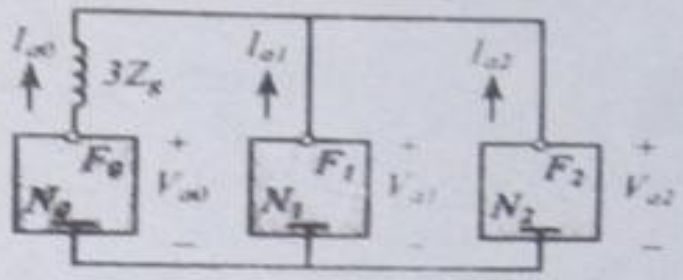
894 Note: Assume lines b and c are the lines accidentally connected to the ground

Refer to the equivalent circuit:

$$I_{a1} = \frac{E_{pu}}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_g)}{Z_2 + Z_0 + 3Z_g}}$$

$$= \frac{1.0}{j0.2 + j\dfrac{(0.2)(0.1 + 3(0.05))}{0.2 + 0.1 + 3(0.05)}}$$

$I_{a1} = -j3.214$



Equivalent network connections of a 2LG fault

$$V_{a1} = V_{a2} = V_{a0} = E_{pu} - I_{a1}Z_1$$
$$= 1.0 - (-j3.214)(j0.2)$$
$$= 0.3572 \text{ pu}$$

$$I_{a0} = -\frac{V_{a0}}{Z_0 + 3Z_n} = -\frac{0.3572}{j(0.1 + 3(0.05))} = j1.4288$$

$$I_{a2} = -\frac{V_{a2}}{Z_2} = -\frac{0.3572}{j0.2} = j1.786$$

Using symmetrical components :

$I_b = a^2 I_{a1} + aI_{a2} + I_{a0}$

$I_b = (1\angle -120°)(3.214\angle -90°) + (1\angle 120°)(1.786\angle 90°) + j1.4288$

$= 3.214\angle -210° + 1.786\angle 210° + j1.4288$

$= -2.783 + j1.607 + (-1.5467 - j0.893) + j1.4288$

$I_b = -4.3297 + j2.1428$

$I_c = aI_{a1} + a^2 I_{a2} + I_{a0}$

$I_c = (1\angle 120°)(3.214\angle -90°) + (1\angle -120°)(1.786\angle 90°) + j1.4288$

$= 3.214\angle 30° + 1.786\angle -30° + j1.4288$

$= 2.783 + j1.607 + (1.5467 - j0.893) + j1.4288$

$I_c = 4.3297 + j2.1428$

$I_n = I_b + I_c = -4.3297 + j2.1428 + 4.3297 + j2.1428$

$I_n = j4.2856$ pu

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} = \frac{15 \times 10^6}{\sqrt{3}(6600)} = 1312.16 \text{ A}$$

$I_f = I_{pu} I_{base} = 4.2856(1312.16) = 5623.39$ A

$I_f \cong 5623$ A

895 $$KVA_f = \frac{kVA_c}{X_{pu}} = \frac{100{,}000}{0.25} = 400{,}000 \text{ kVA}$$

$$I_f = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{400{,}000}{\sqrt{3}(115)}$$

$I_f = 2008$ A

Note: For a three-phase fault, use only the positive sequence components

Another solution:

$$I_{pu} = \frac{E_{pu}}{X_{pu}} = \frac{1}{j0.25} = -j4$$

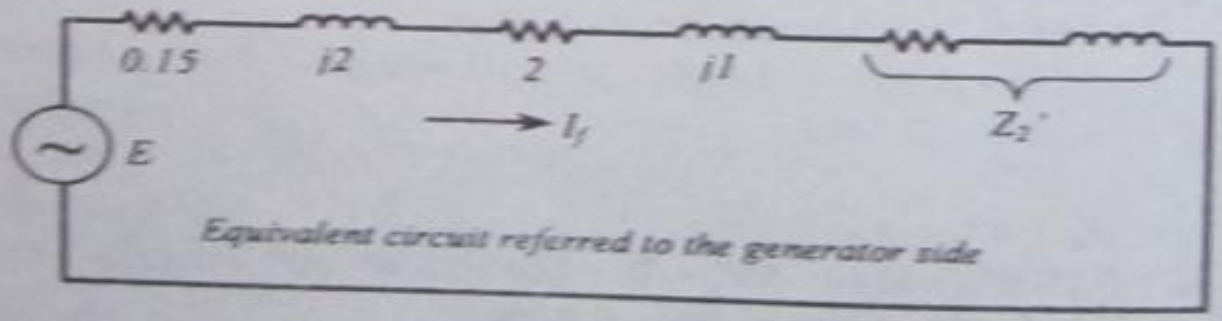
$$I_{base} = \frac{S}{\sqrt{3}E} = \frac{100 \times 10^6}{\sqrt{3}(115 \times 10^3)} = 502 \text{ A}$$

$I_f = I_{pu}(I_{base}) = 4(502)$

$I_f = 2008$ A

Note: From the choices, the closest answer is 2000 A.

896



Total impedance per phase at the secondary side of the transformer bank :

$Z_2 = (50 + j86.6) + (50 + j75)$

$Z_2 = 100 + j161.6\ \Omega$

Equivalent impedance of Z_2 referred to the primary side (generator side) :

$$Z_2' = a^2 Z_2 = \left(\frac{1}{10}\right)^2 (100 + j161.6)$$

$$Z_2' = 1 + j1.616\ \Omega$$

Refer to the equivalent circuit diagram :

$$Z_t = (0.15 + j2) + (2 + j1) + (1 + j1.616)$$

$$Z_t = 3.15 + j4.616 = 5.588\angle 55.69^\circ$$

$$I_f = \frac{E}{Z_t} = \frac{220/\sqrt{3}}{5.588}$$

$$I_f = 22.73\ A$$

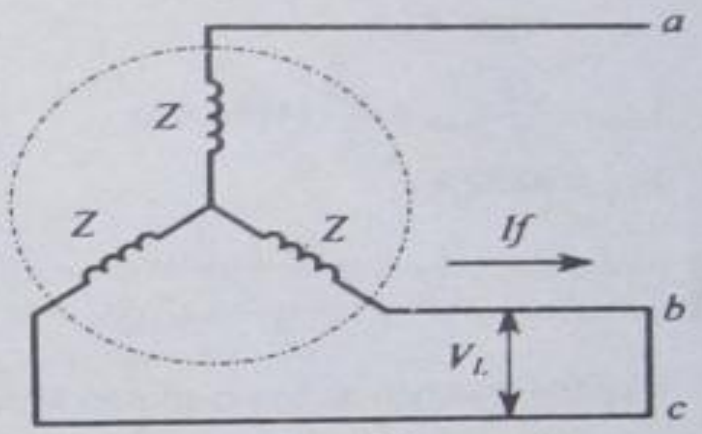
897 $$\%IZ = \frac{SZ}{V^2}$$

$$Z = \frac{(\%IZ)(V)^2}{S} = \frac{(0.04)(277)^2}{50{,}000}$$

$$Z = 0.06138\ \Omega$$

$$I_f = \frac{V_L}{2Z} = \frac{277}{2(0.06138)}$$

$$I_f = 2256.43\ A$$



Secondary side of the transformer

Another solution:

$$kVA_f = \frac{KVA_c}{X_{pu}} = \frac{50}{0.04} = 1250\ kVA$$

$$I_{f3\phi} = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{1250}{\sqrt{3}(0.277)}$$

$$I_{f3\phi} = 2605.37\ A$$

$$I_{fLL} \cong \frac{\sqrt{3}}{2} I_{f3\phi} = \frac{\sqrt{3}}{2}(2605.37)$$

$$I_{fLL} = 2256.3\ A$$

898 $$kVA_f = \frac{KVA_c}{X_{pu}} = \frac{350}{0.08} = 4375\ kVA$$

$$I_f = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{4375}{\sqrt{3}(0.46)}$$

$$I_f = 5491\ A$$

899 $kVA_f = \frac{KVA_c}{X_{pu}} = \frac{15000}{0.025} = 600{,}000\ kVA$

$I_f = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{600{,}000}{\sqrt{3}(34.5)}$

$I_f = 10{,}040.87\ A$

Note: From the choices, the closest answer is 10,000 A.

900 $kVA_f = \frac{KVA_c}{X_{pu}} = \frac{325}{0.08} = 4062.5\ kVA$

$I_{f3\phi} = \frac{kVA_f}{\sqrt{3}\,(kV_f)} = \frac{4062.5}{\sqrt{3}(0.48)}$

$I_{f3\phi} = 4886.43\ A$

$I_{fLL} = \frac{\sqrt{3}}{2} I_{f3\phi} = \frac{\sqrt{3}}{2}(4886.43)$

$I_{fLL} \cong 4232\ A$

901 Note: Since the transformer MVA is different from the alternator MVA, select a common MVA and express all per unit reactance based on this MVA.

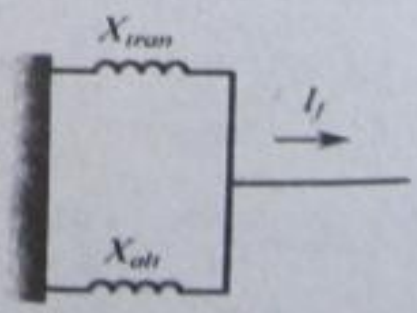
Use the 20 MVA as the common MVA base.

$$Z_{tranpu}' = Z_{tranpu}\left(\frac{MVA_c}{MVA_{tran}}\right) = 0.04\left(\frac{20}{5}\right) = 0.16$$

Refer to the equivalent circuit diagram:

$$Z_{tpu} = \frac{Z_{tranpu} Z_{altpu}}{Z_{tranpu} + Z_{altpu}} = \frac{0.16(0.075)}{0.16 + 0.075} = 0.051$$

$Z_{tpu} = 0.235$

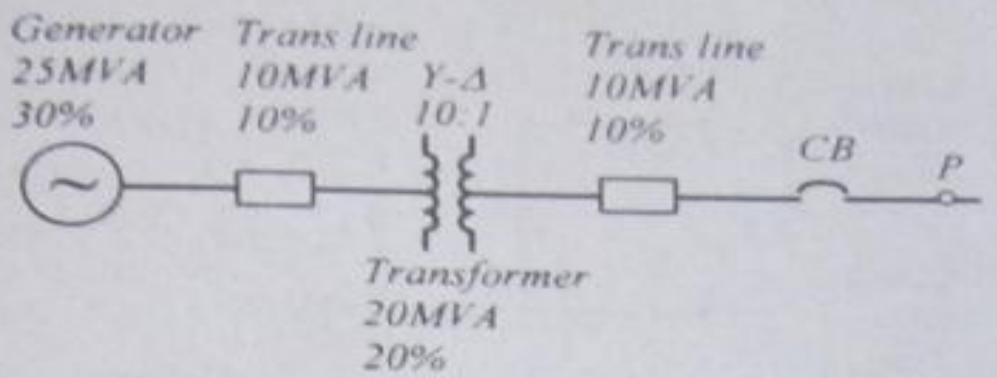


$S_f = \frac{MVA_c}{Z_{tpu}} = \frac{20}{0.051} = 392.157\ MVA$

$I_f = \frac{S_f}{\sqrt{3}\,(V_f)} = \frac{392.157 \times 10^6}{\sqrt{3}(13.8 \times 10^3)}$

$I_f = 16{,}406.66\ A$

902

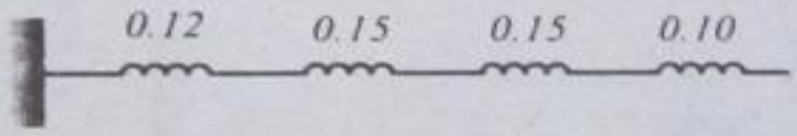


Use the 10 MVA as the common MVA base.

$$X_a = X_{pu}\left(\frac{MVA_c}{MVA_{base}}\right)$$

$$X_g = 0.30\left(\frac{10}{25}\right) = 0.12\,pu$$

$$X_{transformer} = 0.20\left(\frac{10}{20}\right) = 0.10$$



Refer to the equivalent circuit diagram:

$X_t = 0.12 + 0.15 + 0.15 + 0.10$

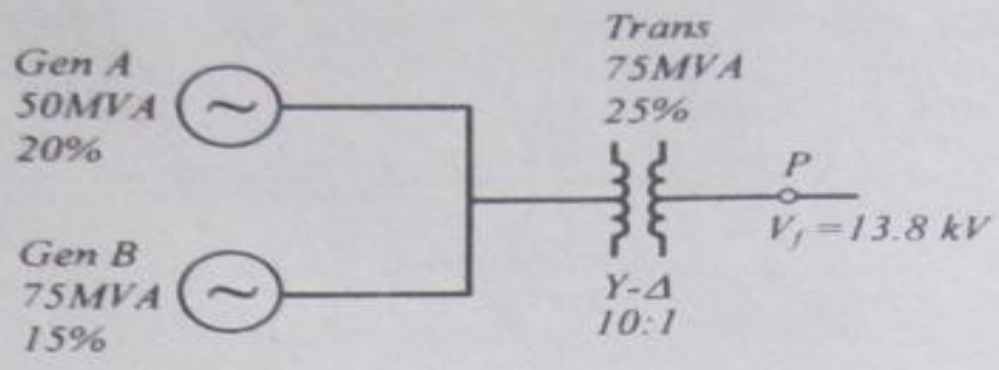
$X_t = 0.52\,pu$

$$MVA_f = \frac{MVA_c}{X_t} = \frac{10}{0.52}$$

$MVA_f = 19.23$

Rupturing capacity of CB = $MVA_f \geq 20$ MVA

903



Use the 75 MVA as the common MVA base.

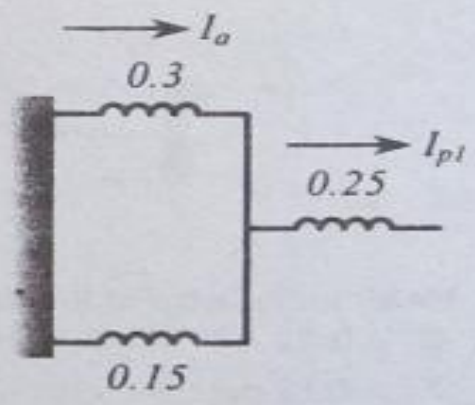
$$X_a = X_{pu}\left(\frac{MVA_c}{MVA_{base}}\right)$$

$$X_a = 0.20\left(\frac{75}{50}\right) = 0.30 \text{ pu}$$

Refer to the equivalent circuit diagram :

$$X_t = \frac{(0.3)(0.15)}{0.3 + 0.15} + 0.25$$

$$X_t = 0.35 \text{ pu}$$



$$S_f = \frac{kVA_c}{X_{tpu}} = \frac{75{,}000}{0.35}$$

$$S_f = 214285 \text{ kVA}$$

$$I_f = \frac{S_f}{\sqrt{3}(V_f)} = \frac{214285 \times 10^3}{\sqrt{3}(13.8 \times 10^3)}$$

$$I_f = 8965 \text{ A}$$

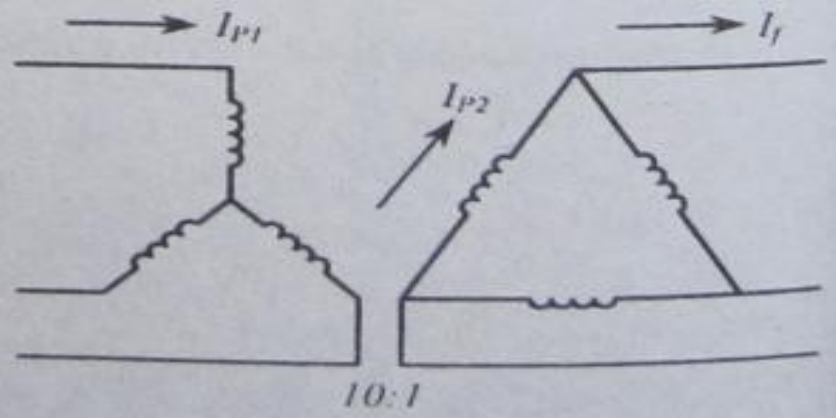
Refer to circuit diagram of the transformer bank:

$$I_{p2} = \frac{I_f}{\sqrt{3}} = \frac{8965}{\sqrt{3}} = 5176 \text{ A}$$

$$\frac{I_{p1}}{I_{p2}} = \frac{N_2}{N_1}$$

$$I_{p1} = I_{p2}\left(\frac{N_2}{N_1}\right) = 5176\left(\frac{1}{10}\right)$$

$$I_{p1} = 517.6 \text{ A}$$



Refer to the equivalent circuit diagram.

By CDT :

$$I_a = \frac{I_{p1}X_b}{X_b + X_a} = \frac{517.6(0.15)}{0.15 + 0.30}$$

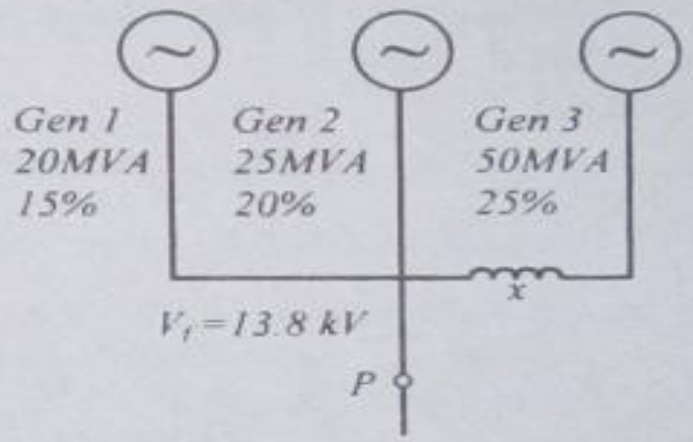
$$I_a = 172.5\ A$$

904 Use the 20 MVA as the common MVA base.

$$X_c = X_{pu}\left(\frac{MVA_c}{MVA_{base}}\right)$$

$$X_2 = 0.20\left(\frac{20}{25}\right) = 0.16$$

$$X_3 = 0.25\left(\frac{20}{50}\right) = 0.10$$



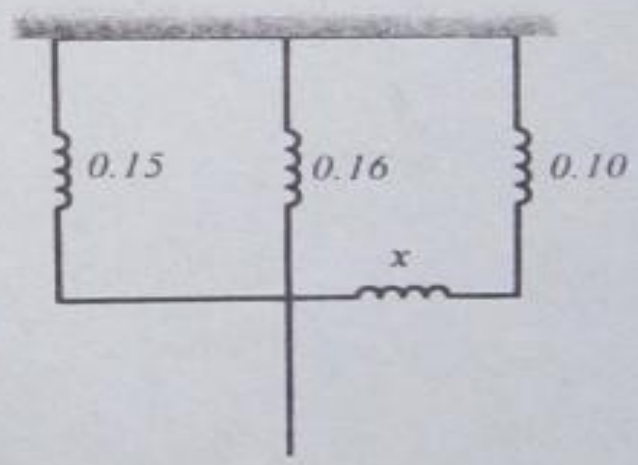
Refer to the equivalent circuit diagram:

$$X_t = \frac{1}{\frac{1}{0.15} + \frac{1}{0.16} + \frac{1}{0.1 + x}}$$

$$X_t = \frac{1}{12.916 + \frac{1}{0.1 + x}}$$

$$X_t = \frac{1}{\frac{1.2916 + 12.916x + 1}{0.1 + x}}$$

$$X_t = \frac{0.1 + x}{2.2916 + 12.916x} \Rightarrow ❶$$



$$MVA_t = \frac{MVA_c}{X_t}$$

$$X_t = \frac{MVA_c}{MVA_t} = \frac{20}{400}$$

$$X_t = 0.05$$

Substitute X_t in Eq.1:

$$\frac{0.1 + x}{2.2916 + 12.916x} = 0.05$$

$$0.11458 + 0.6458x = 0.1 + x$$

$$x = 0.04116\ pu$$

$$0.11458 + 0.6458x = 0.1 + x$$

$$x = 0.04116\,\text{pu}$$

$$X_{base} = \frac{V_{base}^{\ 2}}{S_{base}} = \frac{(13.8 \times 10^3)^2}{20 \times 10^6}$$

$$= 9.522\,\Omega$$

$$x = X_{pu}(X_{base}) = 0.04116(9.522)$$

$$x = 0.392\,\Omega$$

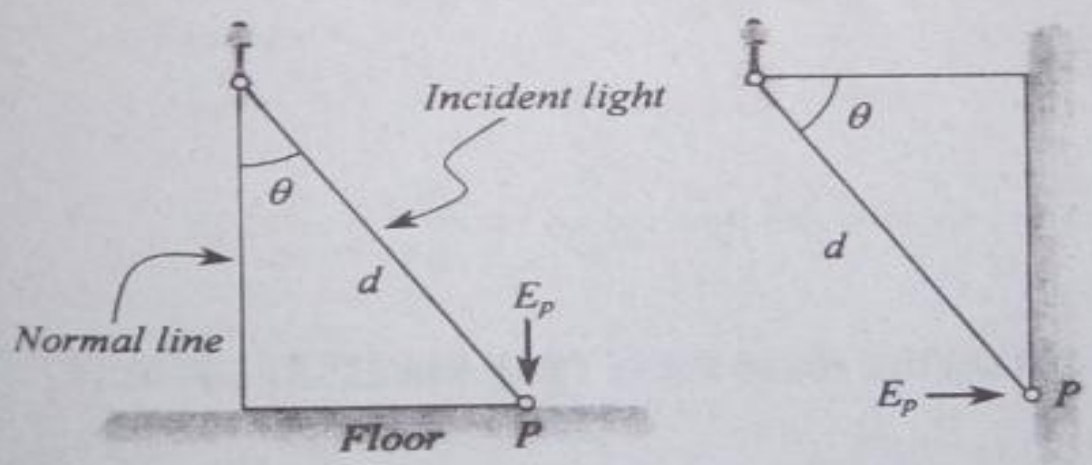
$$x \cong 0.4\,\Omega$$

Illumination

POINT BY POINT METHOD (two-dimensional plane)

Laws on illumination:

- The illumination is directly proportional to the luminous intensity of the light source.
- The illumination varies inversely as the square of the distance from the source of light.
- The illumination varies directly as the cosine of the angle between the normal to the surface and the direction of the incident light.



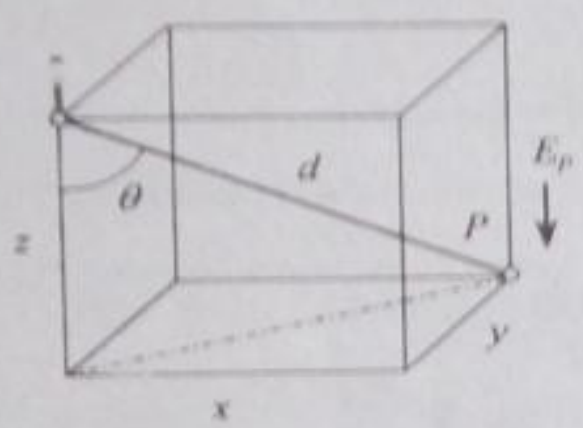
$$I = \frac{\Phi}{4\pi} \qquad E_p = \frac{I}{d^2}\cos\theta$$

Note: Normal line is a line perpendicular to the surface considered.
Incident light is the ray of light that strikes the point of interest.

where: E_p = intensity of illumination
d = distance of the lamp from the point of interest (point P)
I = candle power of the lamp (candela or candles)
θ = angle between the incident ray and the normal line also called angle of incidence
Φ = total lumen output of the lamp

Illumination (E)	Candle power (I)	Distance (d)
foot-candle	candela	foot
lux	candela	meter
phot	candela	centimeter

POINT BY POINT METHOD (three-dimensional plane)



$$E_P = \frac{I}{d^2}\cos\theta$$

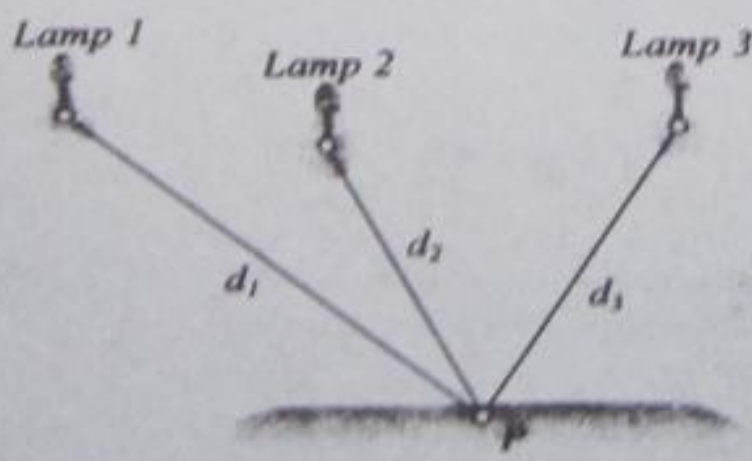
$$d = \sqrt{x^2 + y^2 + z^2}$$

$$\cos\theta = \frac{z}{d}$$

$$E_p = \frac{Iz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

where: x = length
y = width
z = elevation of the lamp from the x-y plane

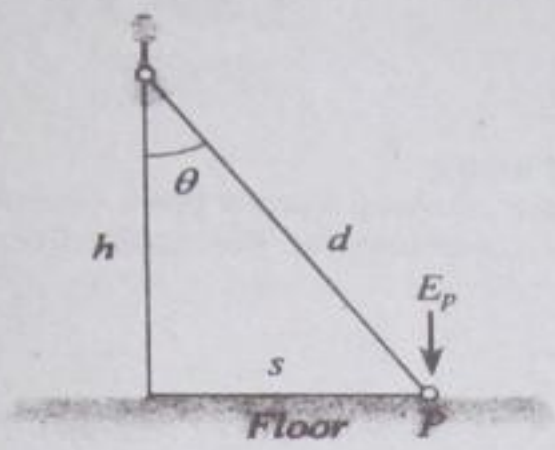
ILLUMINATION FROM MORE THAN ONE LIGHT SOURCES



$$E_p = E_{p1} + E_{p2} + E_{p3} + \ldots\ldots + E_{pn}$$

where E_{p1}, E_{p2}, E_{p3} = illuminations given by lamps 1, 2 and 3 respectively

VALUE OF MOUNTING HEIGHT "h" TO GIVE MAXIMUM ILLUMINATION ON A POINT (P) LOCATED "s" DISTANCE FROM A POINT DIRECTLY BELOW IT.



$$h = \frac{\sqrt{2}}{2} s$$

ZONAL CAVITY OR AVERAGE ILLLUMINATION METHOD

$$E_{initial} = \frac{\Phi \times N \times Cu}{A} \qquad E_{maintained} = \frac{\Phi \times N \times Cu \times Df}{A}$$

where: $E_{initial}$ = illumination when the installation is new
$E_{maintained}$ = average or maintained illumination
N = number of lamps
Cu = coefficient of utilization
Df = depreciation or maintenance factor
Φ = lumen output per lamp
A = surface area illuminated

Illumination (E)	Area (A)
foot-candle	ft^2
lux	m^2
phot	cm^2

Note: The values of Cu and Df are evaluated using standard graphs in the Electrical Eng'g Handbook. Usually in problem solving exercises, these values are always given (if not the question being asked).

Test 25

Problem 905: EE Board April 1992

A piece of paper lies on a table 2 m away from a point directly below a bulb of 100 cd and is 4 m. above the table. Calculate the illumination on the center of the paper in lux.

A. 5.2
B. 6.7
C. 4.5
D. 3.4

Problem 906:

An unshaded lamp is 6 m above a table. It is lowered down by 2 m. By how much is the illumination on the table increased?

A. 2.0 times
B. 2.25 times
C. 2.50 times
D. None of these

Problem 907: EE Board April 1989

A medium unshaded lamp hangs 8 m directly above the table. To what distance should it be lowered to increase the illumination to 4.45 times its former value?

A. 4.02 m
B. 3.86 m
C. 3.79 m
D. 4.21 m

Problem 908:

An unshaded lamp is placed 95 cm from the screen of a photometer. If a glass is placed between the screen and the lamp, the lamp must be moved 5 cm closer to the screen to produce the same illumination as before. The glass blocks what percent of the light produced by the lamp?

A. 11.5%
B. 10.2%
C. 12.4%
D. 9.08%

Problem 909: EE Board April 1994

A floodlight emitting 25,000 candelas in the center of its beam is aimed at 60° to a point on the ground 20 meters away. The illumination of the point in lux is

A. 54.12
B. 62.5
C. 31.25
D. 625

Problem 910:

An unknown lamp placed 6 m from a photometer screen provides the same illumination as a 90-cd lamp placed 4 m from the screen. What is the candlepower of the unknown lamp?

A. 202.5 cd
B. 215.2 cd
C. 211.6 cd
D. 208.4 cd

Problem 911:

Two lamps A and B having intensities of 300 cp and 500 cp respectively are situated 12 ft apart. A screen is placed between them in order that the illuminations on any sides are equal. How much is this illumination?

A. 11.45 fc
B. 10.92 fc
C. 12.45 fc
D. 13.15 fc

Problem 912: EE Board April 1985

A light is to be placed on a wall in order to obtain maximum brightness of illumination. To achieve this brightness, how high on the wall should this light bulb be placed to a point on the floor that is 3.6 m from the wall?

A. 2.54 m
B. 2.25 m
C. 2.08 m
D. 2.86 m

Problem 913:

Two lamps x and y are hanged at a distance of 20 feet between their centers and 10 feet above the working surface. Lamp x gives 140 cp while lamp y gives 230 cp. At what distance from a point directly below lamp A on the line joining their centers will the two lamps give equal amount of illumination?

A. 8.22 ft
B. 8.34 ft
C. 8.05 ft
D. 8.72 ft

Problem 914:

A 100-cd lamp, which emits light uniformly in all directions, is suspended 2.5 m above the center of the working table, which is 3 m square. Calculate the illumination at each corner of the table.

A. 29.56 lux
B. 26.63 lux
C. 24.94 lux
D. 28.38 lux

Problem 915: EE Board October 1991

A lamp of 500 cd is placed at the center of a room, 20 m x 10 m x 5 m. Calculate the illumination in each corner of the floor.

A. 0.83 lux
B. 0.81 lux
C. 0.78 lux
D. 0.98 lux

Problem 916:

Four lamps are suspended 6 m above the ground at the corners of a lawn 4 m on each side. If each lamp emits 250 cd, calculate the illumination at the center of the lawn.

A. 20.56 lux
B. 22.34 lux
C. 21.62 lux
D. 19.85 lux

Problem 917: EE Board October 1991

A lamp of 500 cd power is placed at the center of a room, 20 m x 10 m x 5 m. Calculate the illumination in a point in the middle of a 10 m wall at a height of 2 m from the floor.

A. 4.75 lux
B. 4.98 lux
C. 4.69 lux
D. 4.88 lux

Problem 918: EE Board October 1992

Luminaires of 2000 lumens output will be installed in a room. The desired illumination is 200 foot-candle. The coefficient of utilization is 0.45 and maintenance factor is 90%. Determine the number of luminaires needed if the size of the room is 120 m^2.

A. 30
B. 28
C. 32
D. 36

Problem 919: EE Board October 1993

A hall 35 m x 20 m is to be provided with a general illumination of 130 lux. If the depreciation factor is 1.4 and the coefficient of utilization is 0.50, determine the number of fluorescent lamp required. Assume luminous efficiency of the fluorescent tube as 50 lumens per watt for 80-watt lamp.

A. 62 lamps
B. 65 lamps
C. 63 lamps
D. 64 lamps

Problem 920: EE Board April 1990

A hall 30 m long and 12 m wide is to be illuminated and the illumination required is 50 meter-candles. Calculate the number of lamps required taking a depreciation factor of 1.3 and utilization coefficient of 0.5 given that the output of the lamp to be used is rated 100 W, 1615 lumens.

A. 30
B. 29
C. 28
D. 27

Problem 921: EE Board April 1993

A room 20 m x 10 m is to be illuminated by eight lamps and the average illumination is to be 50 lumens per square meter. If the utilization factor is 0.48 and depreciation factor 1.2, calculate the mean spherical power per lamp.

A. 249 cp
B. 255 cp
C. 235 cp
D. 224 cp

Problem 922: EE Board October 1991

It is required to provide an illumination of 150 meter-candle in a factory hall 45 m x 15 m. Assume that the depreciation factor is 0.80, coefficient of utilization is 0.40 and efficiency of the lamp is 20 lumens per watt. Calculate the number of 250 W lamps.

A. 60
B. 62
C. 63
D. 61

Problem 923: EE Board March 1998

The illumination of a room is being designed. The data as follows:

Room length = 30 ft
Room width = 20 ft
Coefficient of utilization = 0.6
Maintenance factor = 70%
Required foot-candle = 50
Lumens per lamp = 3300
Watts per lamp = 53
Voltage = 220
Power factor = 75%

What is the total current of the lamp required to satisfy the requirement?

A. 21 A
B. 15 A
C. 7 A
D. 25 A

Problem 924: EE Board October 1989

The floor area of a room is 10 meters by 10 meters. The desired illumination level is 150 lux. Assume the coefficient of utilization is 0.51, the maintenance factor is 0.95 and in the market a fluorescent luminaire provide 1200 lumens output of light. Determine the number of luminaires needed for the room.

A. 24 lamps
B. 28 lamps
C. 25 lamps
D. 26 lamps

ANSWER KEY

905. C	909. C	913. B	917. B	921. A
906. B	910. A	914. D	918. A	922. C
907. C	911. B	915. A	919. D	923. C
908. B	912. A	916. A	920. B	924. D

☑ RATING

18-20 - Topnotcher
14-17 - Passer
10-13 - Conditional
0-09 - Failed

Solutions to Test 25

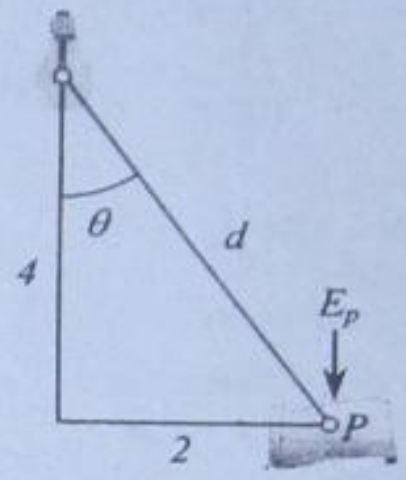
905 $d^2 = 2^2 + 4^2 = 20$

$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.565°$

$E_p = \frac{I}{d^2}\cos\theta$

$= \left(\frac{100}{20}\right)\cos 26.565°$

$E_p = 4.47 \cong 4.5\ \text{lux}$



906 Note: If the point concerned is directly below the lamp, Illumination varies inversely as the square of the distance from the light source to the point concerned.

$E_p = k\frac{I}{d^2}$

$\frac{E_{p2}}{E_{p1}} = \left(\frac{d_1}{d_2}\right)^2$

$\frac{E_{p2}}{E_{p1}} = \left(\frac{6}{4}\right)^2$

$E_{p2} = 2.25\,E_{p1}$

907 $E_p = k\frac{I}{d^2}$

$\frac{E_{p2}}{E_{p1}} = \left(\frac{d_1}{d_2}\right)^2$

$\frac{4.45\,E_{p1}}{E_{p1}} = \left(\frac{8}{d_2}\right)^2$

$d_2 = 3.79\ \text{m}$

Note: The lamp must be lowered to 3.79 m from the top of the table.

908 $E_{p1} = \dfrac{I_1}{d_1^2} = \dfrac{I_1}{(0.95)^2} = 1.108\,I_1$

$E_{p2} = \dfrac{I_2}{d_2^2} = \dfrac{I_2}{(0.9)^2} = 1.234\,I_2$

$E_{p1} = E_{p2}$

$1.108\,I_1 = 1.234\,I_2$

$I_2 = 0.898\,I_1$

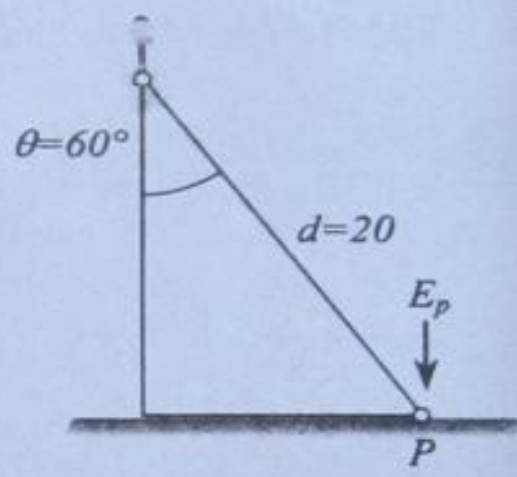
Let: x = percentage of light blocked by the glass

$x = (1 - 0.898) \times 100\%$

$x = 10.2\%$

909 $E_p = \dfrac{I}{d^2}\cos\theta = \left(\dfrac{25000}{(20)^2}\right)\cos 60°$

$E_p = 31.25$ lux



910 $E_{p1} = \dfrac{I_1}{d_1^2} = \dfrac{90}{(4)^2} = 5.625$ lux

$E_{p2} = \dfrac{I_2}{d_2^2}$

$E_{p1} = E_{p2}$

$I_2 = E_{p2}(d_2)^2 = 5.625(6)^2$

$I_2 = 202.5$ cd

911 $E_{p1} = E_{p2}$

$\dfrac{I_1}{x^2} = \dfrac{I_2}{(12-x)^2}$

$\dfrac{300}{x^2} = \dfrac{500}{(12-x)^2}$

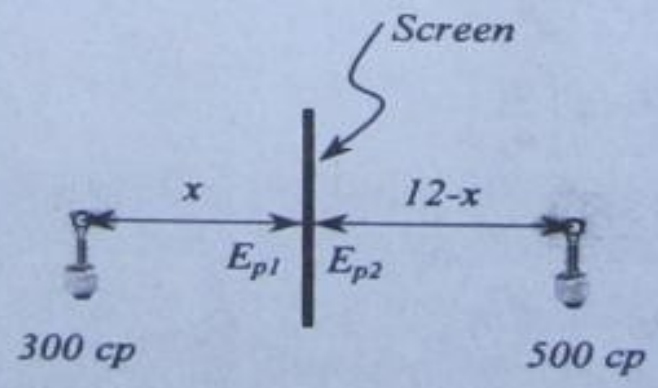
$\dfrac{12-x}{x} = \sqrt{\dfrac{500}{300}} = 1.29$

$12 - x = 1.29x$

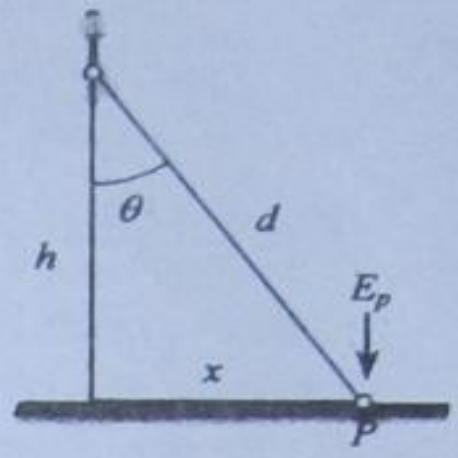
$x = 5.24$ ft

$E_p = \dfrac{I_1}{x^2} = \dfrac{300}{5.24^2}$

$E_p = 10.92$ fc



912 $E_p = \frac{I}{d^2}\cos\theta$



$\cos\theta = \frac{h}{d}$

$d = \sqrt{x^2 + h^2}$

$E_p = \frac{Ih}{d^3} = \frac{Ih}{(x^2 + h^2)^{3/2}}$

$$\frac{dE_p}{dh} = \frac{(x^2 + h^2)^{3/2}(I) - Ih(3/2)(x^2 + h^2)^{1/2}(2h)}{(x^2 + h^2)^3}$$

At maximum condition, $\frac{dE_p}{dh} = 0.$

$(x^2 + h^2)^{3/2}(I) = Ih(3/2)(x^2 + h^2)^{1/2}(2h)$

$x^2 + h^2 = 3h^2$

$2h^2 = x^2$

$h = \frac{x}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$

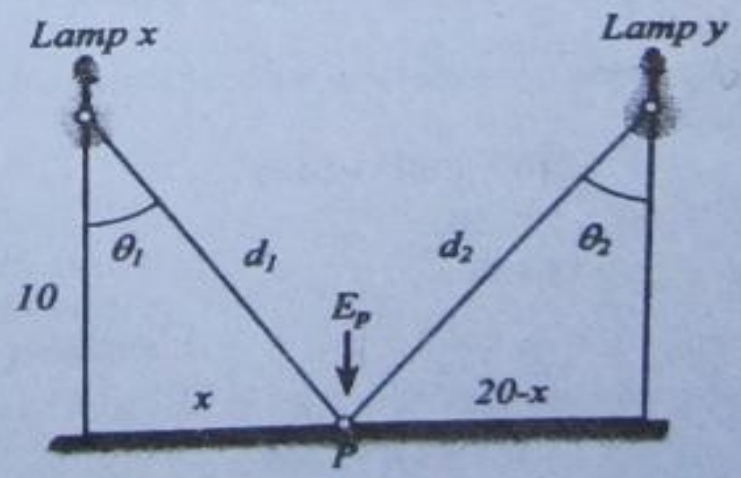
$h = \left(\frac{\sqrt{2}}{2}\right)x$

Substitute x = 3.6:

$h = \left(\frac{\sqrt{2}}{2}\right)3.6$

$h = 2.54\text{ m}$

913 $d_1 = \sqrt{x^2 + 100}$



$d_2 = \sqrt{(20 - x)^2 + 100}$

$E_{p_1} = E_{p_2}$

$\frac{I_1}{d_1^2}\cos\theta_1 = \frac{I_1}{d_1^2}\cos\theta_1$

Substitute :

$$\left(\frac{140}{x^2 + 100}\right)\left(\frac{10}{\sqrt{x^2 + 100}}\right) = \left(\frac{230}{(20 - x)^2 + 100}\right)\left(\frac{10}{\sqrt{(20 - x)^2 + 100}}\right)$$

$$\frac{230}{140} = \left(\frac{(20-x)^2+100}{x^2+100}\right)^{\frac{3}{2}}$$

$$\frac{(20-x)^2+100}{x^2+100} = \left(\sqrt[3]{\frac{230}{140}}\right)^2 = 1.3923$$

$$1.3923x^2 + 139.23 = 400 - 40x + x^2 + 100$$

$$0.3923x^2 + 40x - 360.77 = 0$$

$$x^2 + 101.96x - 919.627 = 0$$

Using the quadratic formula :

$$x = \frac{-101.96 \pm \sqrt{(101.96)^2 - 4(1)(-919.627)}}{2(1)} = \frac{-101.96 \pm 118.635}{2}$$

$$x \cong 8.34 \text{ ft}$$

914

$$d^2 = (1.5)^2 + (1.5)^2 + (2.5)^2$$

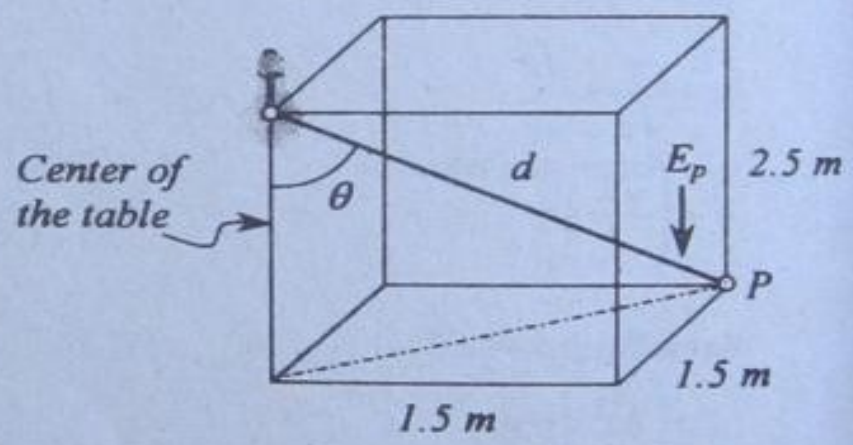
$$d^2 = 10.75$$

$$d = 3.278$$

$$E_p = \frac{I}{d^2}\cos\theta$$

$$E_p = \left(\frac{100}{10.75}\right)\left(\frac{10}{3.278}\right)$$

$$E_p = 28.38 \text{ lux}$$



Note: P is a point in one of the corners of the table.

915 Note: Since the lamp is placed at the middle, thus its elevation is 2.5 m.

$$d^2 = (10)^2 + (5)^2 + (2.5)^2$$

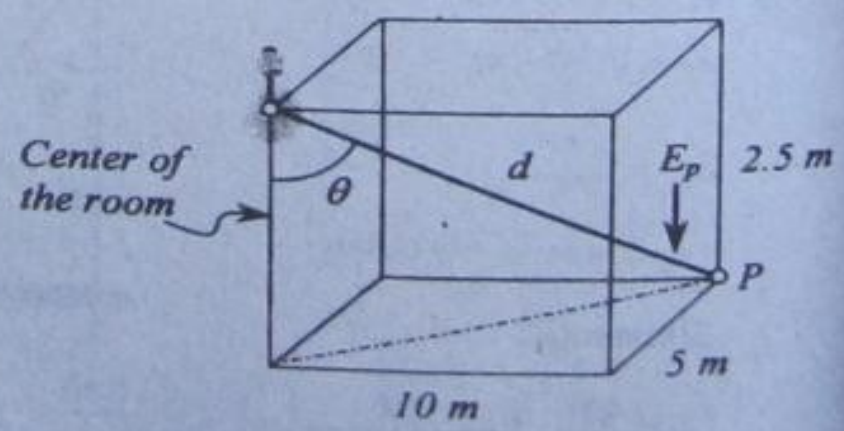
$$d^2 = 131.25$$

$$d = 11.456$$

$$E_p = \frac{I}{d^2}\cos\theta$$

$$E_p = \left(\frac{500}{131.25}\right)\left(\frac{2.5}{11.456}\right)$$

$$E_p = 0.83 \text{ lux}$$



916

$d^2 = (2)^2 + (2)^2 + (6)^2$

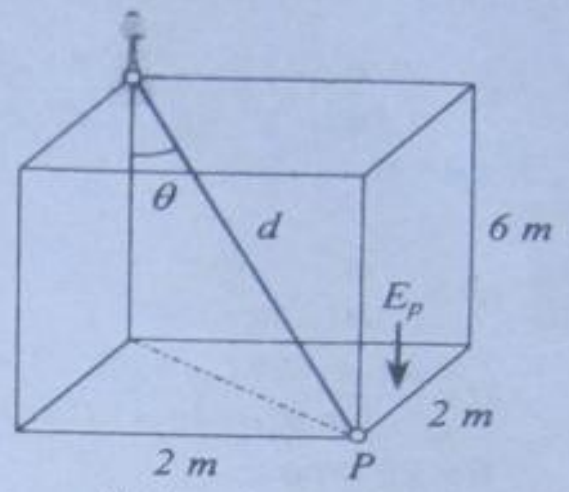
$d^2 = 44$

$d = 6.633$

$E_p = \frac{I}{d^2}\cos\theta$

$E_p = \left(\frac{250}{44}\right)\left(\frac{6}{6.633}\right)$

$E_p = 5.14$ lux



Note: P is a point in the center of the lawn. Since all the 4 lamps are identical, the total illumination is equal to 4 times the illumination offered by one lamp.

$E_{Ptotal} = 4(5.14)$

$E_{Ptotal} = 20.56$ lux

917 Note: Since the lamp is placed at the middle, thus its elevation is 2.5 m.

$d^2 = (0.5)^2 + (10)^2$

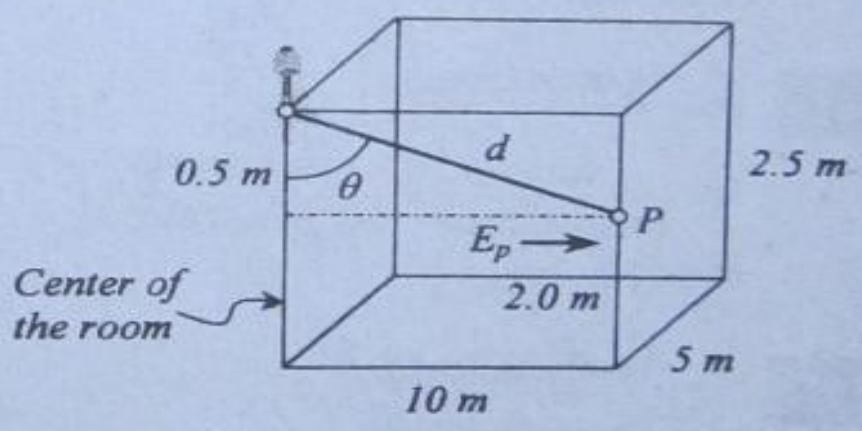
$d^2 = 100.25$

$d = 10.012$

$E_p = \frac{I}{d^2}\sin\theta$

$= \left(\frac{500}{100.25}\right)\left(\frac{10}{10.012}\right)$

$E_p = 4.98$ lux



Note: P is a point in the middle of the 10-m wall, 2 m above the floor.

918 $MI = \frac{N(\Phi)(m.f.)(c.u.)}{A}$

$N = \frac{MI(A)}{(\Phi)(m.f.)(c.u.)} = \frac{200(120)}{2000(0.9)(0.45)}$

N = 30 lamps

919 Note: If d.f. is given greater than 1, placed it at the denominator side of the formula.

$MI = \frac{N(\Phi)(c.u.)}{A(d.f.)}$

$$N = \frac{MI\,(A)\,(d.f.)}{\Phi(c.u.)} = \frac{130(35)(20)(1.4)}{50(80)(0.5)}$$

$N \cong 64$ lamps

920 $$MI = \frac{N\,(\Phi)\,(c.u.)}{A\,(d.f.)}$$

$$N = \frac{MI\,(A)\,(d.f.)}{\Phi(c.u.)} = \frac{50(30)(12)(1.3)}{1615(0.5)}$$

$N \cong 29$ lamps

921 $$MI = \frac{N\,(\Phi)\,(c.u.)}{A\,(d.f.)}$$

$$\Phi = \frac{MI\,(A)(d.f.)}{N\,(c.u.)} = \frac{50(20)(10)(1.2)}{8(0.48)} = 3125 \text{ lumens}$$

$$I = \frac{\Phi}{4\pi} = \frac{3125}{4\pi} = 248.679$$

$I \cong 249$ c.p.

922 $$MI = \frac{N(\Phi)(m.f.)(c.u.)}{A}$$

$$N = \frac{MI(A)}{(\Phi)(m.f.)(c.u.)} = \frac{150(45)(15)}{20(250)(0.8)(0.4)}$$

$N \cong 63$ lamps

923 $$MI = \frac{N(\Phi)(c.u.)(m.f.)}{A}$$

$$N = \frac{MI(A)}{\Phi(c.u.)(m.f.)} = \frac{50(30)(20)}{3300(0.6)(0.7)} \cong 22 \text{ lamps}$$

$$P = V_L I_L pf$$

$$I_L = \frac{P}{V_L pf} = \frac{22(53)}{220(0.75)}$$

$I_L = 7.06$ A

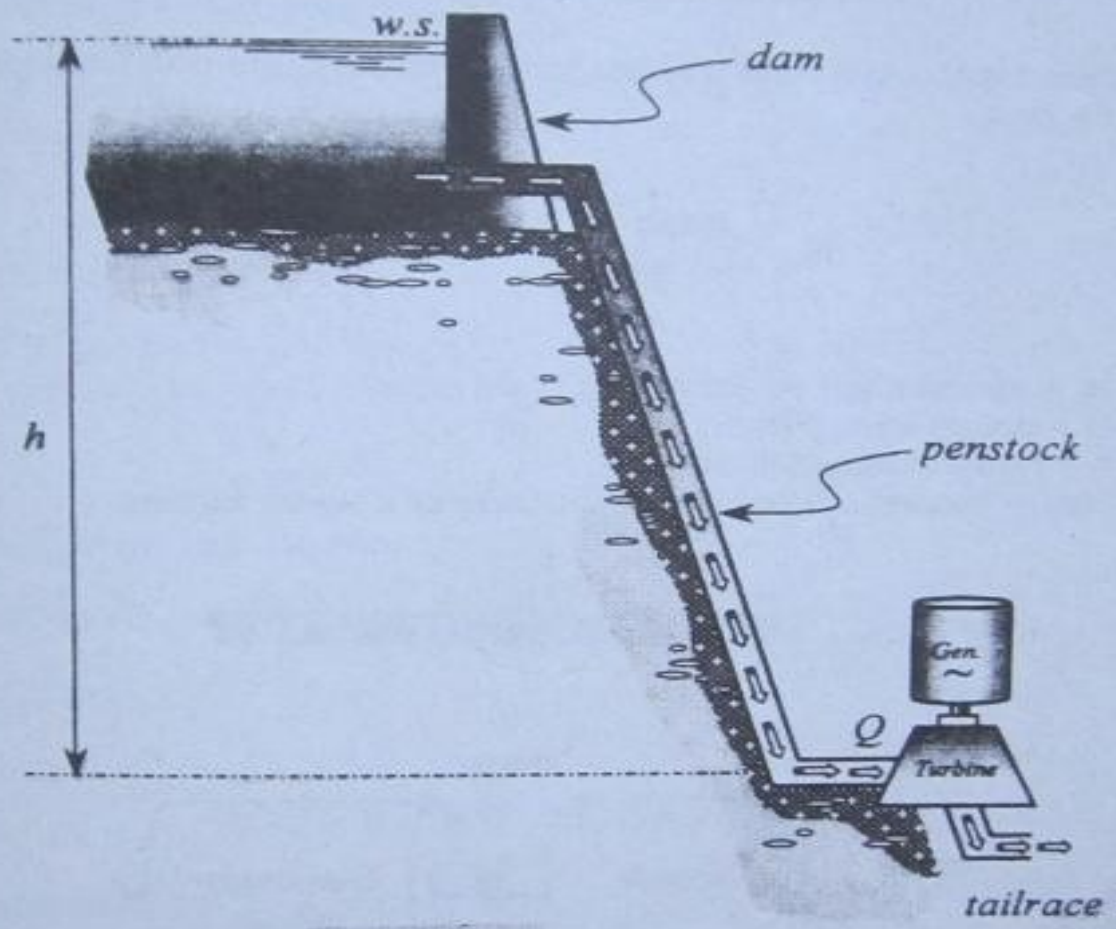
924 $$MI = \frac{N(\Phi)(c.u.)(m.f.)}{A}$$

$$N = \frac{MI(A)}{\Phi(c.u.)(m.f.)} = \frac{150(10)(10)}{1200(0.51)(0.95)}$$

$N \cong 26$ lamps

Electrical Power Plant

THEORETICAL POWER OF A WATER TURBINE



$$P = Qwh$$

where: P = theoretical power of the water turbine (watt)
Q = volume discharge of water (m^3/sec)
w = specific weight of water (9810 N/m^3)
h = head of water (m)

$$Hp_w = \frac{Qwh}{k}$$

where: Hp_w = theoretical horsepower of a water turbine

Note: k = constant factor for conversion

Q	w	h	k
ft^3/sec	62.4 lbs/ft^3	feet	550
ft^3/min	62.4 lbs/ft^3	feet	33000
m^3/sec	9810 N/m^3	meter	746
m^3/min	9810 N/m^3	meter	44760

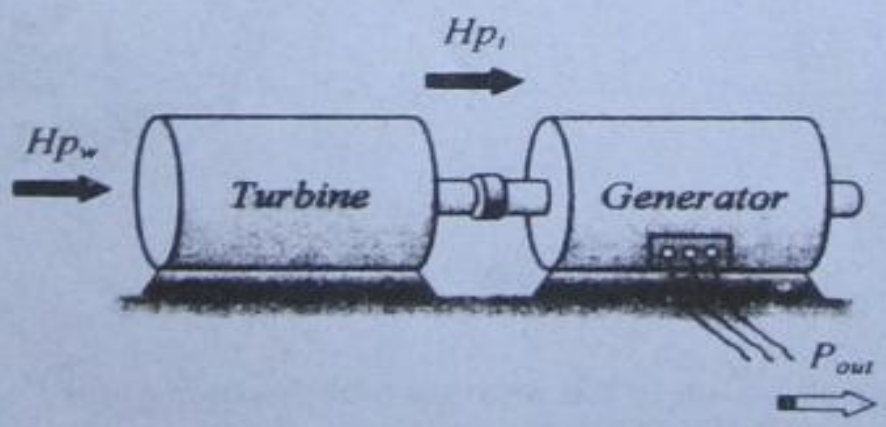
SPECIFIC SPEED OF A WATER TURBINE

Specific speed is the speed of a water turbine to generate one horsepower under a head of one foot.

$$N_s = \frac{5050}{h+32} + 19 \qquad N = \frac{N_s h^{1.25}}{\sqrt{Hp_w}}$$

where: N_s = specific speed (rpm)
N = actual speed (rpm)
h = head of water (foot)
Hp_w = theoretical output horsepower of a water turbine

TURBINE-GENERATOR SET UP



$$\eta_{overall} = \frac{P_{out}}{Hp_w(746)} \qquad \eta_{overall} = \eta_t \eta_g$$

where: P_{out} = output power of the generator (watt)
Hp_t = power output of the turbine (horsepower)
Hp_w = power input of the turbine (horsepower)
η_t = turbine efficiency
η_g = generator efficiency

HEAT ENERGY CONVERSION FACTORS

1calorie = 4.186 joules

1BTU = 252 calories

1kW - hr = 3.6×10^6 joules

where: calorie = SI or mks unit of heat energy
BTU (British Thermal Unit) = English or fps unit of heat energy
joule = unit of energy in SI unit

Note: In dealing with problems that involve heat rate, amount of coal needed, cost of production of energy, etc, use the technique in engineering called "CANCELLATION OF UNITS".

Sample problem:

The heat rate of a given coal-fired thermal plant is 12,500 BTU/lb. How many pounds of coal are needed to generate 1000 kW-hr?

Solution:

Let: W = weight of coal required

$$W = 1000\ \text{kW} - \text{hr} \times \frac{3.6 \times 10^6\ \text{J}}{1\ \text{kW - hr}} \times \frac{1\ \text{cal}}{4.186\ \text{J}} \times \frac{1\ \text{BTU}}{252\ \text{cal}} \times \frac{1\ \text{lb}}{12{,}500\ \text{BTU}}$$

$$W \cong 273\ \text{lbs}$$

TERMS USED IN PREDICTION OF LOAD

- **Average load (AL)** – the mean of all the demands that have occurred during the day, the month or the year.

$$AL_{day} = \frac{\sum \text{demand of the day}}{24} \qquad AL_{month} = \frac{\sum \text{demand of the month}}{(30)24}$$

$$AL_{year} = \frac{\sum \text{demand of the year}}{(365)24}$$

Note: One day has 24 hrs. One month has an average of 30 days while one year has an average of 365 days, unless otherwise specified.

- **Load factor (LF) – ratio of the average load (AL) to the peak load (PL)**

$$LF = \frac{AL}{PL}$$

Note: Peak load – highest or biggest load that occurred during the day, the month or the year

- **Demand factor (DF) – ratio of the peak load (PL) to the connected load (CL)**

$$DF = \frac{PL}{CL}$$

Note: Connected load – sum of all loads installed or connected to the system.

- **Diversity factor (dF) – ratio of the sum of individual peak load ($\sum PL$) to the peak load of the entire station ($PL_{station}$)**

$$dF = \frac{\sum PL}{PL_{station}}$$

Note: Station peak load – highest or biggest load served by the station that had occurred during the day, the month or the year

- **Utilization factor (UF) – ratio of the peak load of the entire station ($PL_{station}$) to the total or installed capacity of the station.**

$$UF = \frac{PL_{station}}{\text{Installed capacity}}$$

Note: Installed capacity of the station – sum of the ratings of the generators installed in the station.

- **Plant capacity factor (PCF)** – ratio of the annual energy delivered by the station to the rated energy that the station can generate or produced in a year.

$$PCF_{year} = \frac{\text{Annual energy in kW - hr}}{\text{Installed capacity in kW} \times 8760}$$

Note: For one year, unless otherwise specified a standard assumption of 365 days or 8760 hours shall be used.

- **Plant use factor (PUF)** – ratio of the annual energy delivered by the station to the actual energy that the station had generated in a year.

$$PUF_{year} = \frac{\text{Annual energy in kW - hr}}{\text{Installed capacity in kW} \times N}$$

where: N = number of hours the plant or station is in operation in one year

Test 26

Problem 925: EE Board March 1998

A deep well pumps water into an open reservoir at the rate of 100 gpm. The water table is 200 ft below the output pipe. The system has an overall efficiency of 70%. What shall be the minimum hp of the pump?

A. 5.3 hp
B. 8.7 hp
C. 12.4 hp
D. 7.2 hp

Problem 926: EE Board March 1998

A pump requires 100 hp to move water with a specific gravity of 1.0 at a certain flow rate to a given elevation. What horsepower does the pump require if the flow rate and elevation conditions are the same but the fluid pumped has a specific gravity of 0.8?

A. 80 hp
B. 100 hp
C. 60 hp
D. 120 hp

Problem 927: EE Board April 1993

An electric motor is used to pump water from a well 40 ft deep. The amount of water pumped per hour to the ground level is 237,000 gallons. What is the Hp of the motor assuming there is no power loss?

A. 43 hp
B. 42 hp
C. 41 hp
D. 40 hp

Problem 928:

A pump, driven by 440 V, 3-phase induction motor lifts 1,100 cubic feet of water per minute against a total head of 100 ft of water. The efficiency of the pump and that of its motor are 0.75 and 0.92 respectively. The power factor of the motor is 0.9. Calculate the cost of operating the pump for 24 hours a day duty when power costs P 0.30 per kW-hr.

A. P 1,723
B. P 2,035
C. P 1,619

D. P 1,905

Problem 929: EE Board October 1983, EE Board October 1990

A natural waterfalls, 50 meters high, consistently discharges 1.3 m^3 per second. A mini hydroelectric plant is to be constructed at the bottom of the waterfalls. Calculate the electric generator kW rating, assuming 90% mechanical to electrical conversion efficiency and water turbine design efficiency of 70%.

A. 401.72
B. 410.55
C. 420.42
D. 425.10

Problem 930: EE Board March 1998

A power plant gets water from a dam from a height of 122.45 m at the rate of 1000 cubic meters per minute. If the output of the plant is 15,000 kW, what is the plant efficiency?

A. 80%
B. 70%
C. 75%
D. 65%

Problem 931: EE Board April 1995

Calculate the current taken by a motor at 440 V having an efficiency of 90%. Moreover the motor is used to drive a pump, which lifts 240,000 kg of water per hour to a height of 30 meters. The pump is given to be 80% efficient.

A. 58 A
B. 75 A
C. 83 A
D. 62 A

Problem 932: EE Board April 1994

A three-phase motor driven pump at 1,800 rpm is discharging 500 gpm of water at a head of 25 meters. The pump efficiency is 75%. Determine the hp of the motor.

A. 15 hp
B. 12 hp
C. 14 hp
D. 10 hp

Problem 933: EE Board April 1993

A hydroelectric station has a turbine efficiency of 80% and a generator efficiency of 85%. Determine the volume of water required when delivering a load of 40 MW for 6 hours if its effective head is 120 m. Water weighs 1200 kg/m^3.

A. 899,424 m^3

B. 945,352 m^3
C. 845,041 m^3
D. 905,412 m^3

Problem 934:

A hydroelectric power plant is to be constructed. The place has an annual rainfall of 190 cm. The available catchment area is 250 sq. km with an available head of 30 meters. On the average, only 85% of the rainfall can be collected in and 70% of the impounded water is available for power. Assuming turbine and generator efficiencies of 80% and 92% respectively, what will be the average power generated.

A. 2,637.5 kW
B. 1,732.7 kW
C. 1,941.2 kW
D. 2,063.8 kW

Problem 935: EE Board June 1990, EE Board April 1991

A 100 MW hydroelectric station is supplying full load for 10 hrs. in a day. Calculate the volume of water, which has been used. Assume effect effective head of the water as 200 m and the overall efficiency of the station as 80%. Assume 1 m^3 of water = 1000 kg.

A. 2,232,435 m^3
B. 2,293,560 m^3
C. 2,345,456 m^3
D. 2,371,402 m^3

Problem 936:

A farmer has a small stream on his property, which he thinks might supply enough power to light his farm load. The stream was found to be flowing at 0.05 m^3/sec. The available fall is 5 m. A small hydraulic turbine and generator will have a combined efficiency of 75%. Assuming a transmission line efficiency of 82%, how many 50-W lamps can be served by the generator simultaneously?

A. 28 lamps
B. 32 lamps
C. 36 lamps
D. 30 lamps

Problem 937: EE Board April 1994

A waterfall is 60 meters high. It discharges at a constant rate of 1.0 cubic meter per second. A mini-hydro electric plant is to be constructed below the waterfalls. The turbine efficiency is 80% and the generator efficiency is 95%. Calculate the kW output of the generator.

A. 447.34 kW
B. 464.22 kW
C. 457.27 kW
D. 471.30 kW

Problem 938: EE Board April 1989

Maria Cristina falls located at south has a height of 100 meters, it discharges at constant rate 2.5 m^3/sec. National Power corporation has construct a hydroelectric plant below the water falls. If the turbine efficiency is 70% and generator efficiency is 90%, calculate the annual energy in kW-hr, if the discharge rate is constant throughout the year. Assume specific gravity constant.

A. 13,534,857
B. 13,467,546
C. 13,655,654
D. 13,504,231

Problem 939: EE Board October 1983, EE Board October 1990

A natural waterfall, 50 meters high, consistently discharges 1.3 m^3 per second. A mini hydroelectric plant is to be constructed at the bottom of the waterfalls. Calculate the annual energy in kW-hr that can be generated by the power plant. Assume the turbine and generator efficiencies equal to 70% and 90%, respectively.

A. 3,519,067
B. 3,462,356
C. 3,612,212
D. 3,345,815

Problem 940:

A water turbine operating at 85% efficiency is under a head of 100 m. If the constant discharge is 1000 gal/sec, evaluate the operating speed of the turbine.

A. 640 rpm
B. 654 rpm
C. 636 rpm
D. 608 rpm

Problem 941:

A mini-hydroelectric power plant is under a head of 275 feet. The turbine efficiency is 0.85 and the generator efficiency is 0.80. If the turbine speed is limited to 450 rpm, calculate the generator kW output.

A. 3402
B. 3948
C. 4024
D. 3902

Problem 942: EE Board October 1997

A power plant gets water from a dam from a height of 100 meters at the rate of 1000 cubic meters per minute. If the overall efficiency of the plant is 75%, what is the kW output of the plant?

A. 7,550 kW
B. 15,250 kW

C. 12,250 kW
D. 5,250 kW

Problem 943: EE Board October 1997

A power plant consumes 100,000 pounds of coal per hour. The heating value of the coal is 12,000 BTU per pound. The overall plant efficiency is 30%. What is the kW output of the plant?

A. 105,500 kW
B. 205,000 kW
C. 142,500 kW
D. 175,000 kW

Problem 944:

A power plant has a fuel consumption of 1 pound per kW-hr generated. Determine the heating value of the fuel used in BTU per pound. The overall efficiency of the plant is 36%.

A. 10,000
B. 9,500
C. 9,800
D. 10,050

Problem 945: EE Board October 1998

A certain coal-fired power plant has a heat rate of 2.88×10^6 calories per kW-hr. Coal costs P 2,500 per ton. How much is the fuel cost component of producing one kW-hr? Assume the heating value of coal used equal to 13,000 BTU/lb.

A. P 2.50
B. P 1.75
C. P 1.00
D. P 1.25

Problem 946:

A 100 MW thermal plant has an overall efficiency of 34%. If the coal used has a heating value of 10,800 BTU per pound, calculate the coal consumption of the plant per kW-hr output.

A. 0.84 lb
B. 0.96 lb
C. 0.89 lb
D. 0.93 lb

Problem 947: EE Board October 1997

A coal power plant has an overall plant efficiency of 28%. Coal with a heating value of 12,000 BTU per pound cost P 1.50 per pound. What is the fuel cost pf producing one kW-hr?

A. P 1.90

B. P 1.52
C. P 1.75
D. P 1.62

Problem 948:

A power plant has an overall efficiency of 30%. If this plant can consumed 4200 kilograms of coal per hour, estimate the total electric energy produced in one day? Assume the calorific value of the coal being used is 8000 kcal per kilogram.

A. 183 MW-hr
B. 168 MW-hr
C. 155 MW-hr
D. 176 MW-hr

Problem 949: EE Board April 1994

A plant has a total operating capacity of 800 kW. The coal consumption is 1900 lbs per hour. Heating value of coal is 9,500 BTU per lb. What approximate percent of the heat in the coal is converted into useful energy?

A. 12.3
B. 15.0
C. 8.7
D. 17.5

Problem 950: EE Board October 1997

A 100 MW coal fired power plant has an average heat rate of 9500 BTU per kW-hr. The plant load factor is 75%. The heating value of the coal is 12,000 BTU per pound. Calculate the amount of coal usage for one day.

A. 1.425×10^6 pounds
B. 2.235×10^6 pounds
C. 2.235×10^5 pounds
D. 1.826×10^6 pounds

Problem 951: EE Board October 1998

A diesel generator set burns diesel with a heating value of 18,000 BTU per pound. The diesel engine has an efficiency of 30% and the alternator has an efficiency of 95%. Determine the fuel cost component of producing one kW-hr if diesel costs P 2.80 per pound.

A. P 0.15
B. P 3.28
C. P 2.15
D. P 1.86

Problem 952:

A diesel electric generating unit supplies a load at 70 kW. The heating value of the oil used is 12,000 kcal per kg. If the overall efficiency of the unit is 40%, determine the mass of oil required per hour

A. 11.8 kg
B. 10.3 kg
C. 12.5 kg
D. 9.67 kg

Problem 953: EE Board April 1993

A 500 kW electric lighting cost P 1000 per kW installed. Fixed charges are 14%, operating cost is P 1.50 per kW-hr. The plant averages 150 kW for 5000 hrs of the year, 420 kW for 1000 hrs and 20 kW for the remainder. What is the unit cost production of electric energy?

A. P 1.96
B. P 1.84
C. P 2.15
D. P 2.05

Problem 954: EE Board October 1989

A coal fired Power Plant with 3000 kW rated capacity costs P 15,000 per kW installed. Annual operating cost P 12 million; annual maintenance cost P 8 million; annual depreciation P 15 million; interest on investment per year 8% and cost of coal P 800 per ton. If one pound of coal is needed to generate 1 kW-hr, find the total annual cost to operate the plant. Plant capacity factor is 50%

A. P 46.45 million
B. P 43.38 million
C. P 40.28 million
D. P 42.32 million

Problem 955: EE Board October 1998

To produce one kW –hr, a power plant burns 0.9 lb of coal with a heating value of 13,000 BTU. What is the heat rate of the plant?

A. 6,250 BTU/kW-hr
B. 9,550 BTU/kW-hr
C. 11,700 BTU/kW-hr
D. 8,700 BTU/kW-hr

Problem 956: EE Board March 1998

The current loads of four circuits are as follows:

Circuit No.1 = 25 amperes
Circuit No.2 = 38 amperes
Circuit No.3 = 18 amperes
Circuit No.4 = 45 amperes

If the diversity factor is 1.5, what is the minimum ampacity of the feeder conductor?

A. 126 A
B. 189 A
C. 84 A
D. 152 A

Problem 957: EE Board October 1998

The power customer has four feeder circuits of 220 V, three-phase. The circuits have maximum demand as follows:

Circuit no. 1 = 35 A
Circuit no. 2 = 46 A
Circuit no. 3 = 72 A
Circuit no. 4 = 57 A

The diversity factor is 1.5, determine the maximum kVA demand with the customer.

A. 46
B. 26.5
C. 80
D. 53

Problem 958: EE Board April 1980

A substation transformer is to serving the following loads:

Classification	Total load	Demand factor
Lighting	300 kW	60%
Power	1200 kW	80%
Heating	500 kW	90%

If the diversity factor among the load types is 1.5, determine the maximum demand on the transformer.

A. 2000 kW
B. 1060 kW
C. 1225 kW
D. 1180 kW

Problem 959: EE Board October 1998

The power customer has four circuits of 220 V, three-phase. The circuits having maximum demand as follows:

Circuit No.1 = 35 A
Circuit No.2 = 46 A
Circuit No.3 = 72 A
Circuit No.4 = 57 A

The diversity factor is 1.5 A, the load factor is 50%, and the power factor is 80%. What is the approximate kW demand of the customer?

A. 15 kW
B. 43 kW
C. 21 kW
D. 53 kW

Problem 960:

An industrial plant has an aggregate load of 100 kW. The demand factor is 60%. If the average total energy consumption of the plant in one year is 186 MW-hr, what is the yearly load factor of the plant?

A. 32.45%

B. 42.25%
C. 38.67%
D. 35.39%

Problem 961:

A power plant has two 75 MW generator sets each operating at 7200 hours per year and a 50 MW generator set operating at 3000 hours per year. If the plant output is 800 x 10^6 kW-hr per year, solve for the plant use factor.

A. 63.4%
B. 65.0%
C. 68.3%
D. 70.6%

Problem 962:

The average weekly energy generated by a thermal plant is 7,000,000 kW-hr. The peak load of the plant is 75,000 kW during the week. If the plant has an installed capacity of 100 MW solve for the load factor of the plant.

A. 48.92%
B. 56.34%
C. 50.44%
D. 55.55%

Problem 963: EE Board April 1990

A certain power plant has reserve capacity above the peak load of 10 MW. The annual factors are load factor = 59%, capacity factor = 41% and use factor = 46%. Determine installed capacity.

A. 33.45 MW
B. 31.85 MW
C. 32.77 MW
D. 30.47 MW

Problem 964: EE Board April 1990

A certain power plant has reserve capacity above the peak load of 10 MW. The annual factors are load factor = 59%, capacity factor = 41% and use factor = 46%. Determine hours per year not in service.

A. 952 hours
B. 965 hours
C. 924 hours
D. 937 hours

Problem 965: EE Board April 1993

A central generating station has an annual factor report as follows:

Plant capacity factor = 50%
Load factor = 60%
Use factor = 45%
Reserved = 10,500 kW

Determine the rated capacity of the station.

A. 63,000 kW
B. 60,000 kW
C. 65,000 kW
D. 70,000 kW

Problem 966: EE Board October 1990

A 55,000 kW thermal plant of National Power Corporation delivers an annual output of 238,000,000 kW-hr with a peak load of 44 MW. What is the capacity factor?

A. 49.4%
B. 42.4%
C. 48.2%
D. 44.6%

Problem 967:

A thermal power plant has two generating sets rated 600 kW and 800 kW respectively being operated at rated capacity. The total coal consumption is 680 kg per hour. If the coal has a heating value of 20,000 joules per gram, what fraction of heat produced from the coal is converted into useful electrical energy?

A. 37.06%
B. 40.20%
C. 33.67%
D. 38.28%

Problem 968: EE Board October 1998

The metering of a power customer was read and gave the following data.

kW-hr = 200,000
kVAR-hr = 180,000
Maximum demand = 380 kW
Billing days = 30

What is the load factor?

A. 68%
B. 73%
C. 80%
D. 62%

ANSWER KEY

925. D	934. C	943. A	952. C	961. B
926. A	935. B	944. B	953. A	962. D
927. D	936. D	945. C	954. B	963. C
928. C	937. A	946. D	955. C	964. A
929. A	938. A	947. B	956. C	965. A
930. C	939. A	948. D	957. D	966. A
931. D	940. B	949. B	958. B	967. A
932. C	941. B	950. A	959. C	968. B
933. A	942. C	951. D	960. D	

☑ RATING

39-44 - Topnotcher

30-38 - Passer

22-29 - Conditional

0-21 - Failed

Did you know that... the first intelligible telephone sentences, "Mr. Watson, come here. I want you." were spoken unintentional by the inventor himself in March 10, 1876? He was then only 29 years old.

A Scottish American scientist Alexander Graham Bell invented the most common and most widely used electrical instrument today, the "TELEPHONE".

Bell was born in Edinburgh, Scotland. His father, Alexander Melville Bell and his grandfather were both well-known speech teachers for the deaf. After attending at the University of Edinburgh and in the University of London, he became a speech teacher too.

In 1886 Bell became interested in trying to transmit speeches electrically after reading a book describing how vowel sounds could be made with tuning forks. Shortly after the death of his two brothers due to tuberculosis, the Bell family moved to Canada for health reasons.

In 1873, the young Bell became a professor at Boston University and began his electrical experiments in his spare time. It was there that he formed his partnership with his assistant Thomas Watson and went on to his great invention.

On March 10, 1876, the first intelligible telephone sentences, "Mr. Watson, come here. I want you" were spoken unintentional by Bell himself when he called Watson to come to an adjoining room to help him with some spilled acid. The patent on the telephone was granted. It was the most valuable patent ever issued. Bell was 29 years old.

And who would ever think in 1876 that in years after, over 100 million telephones would be in use in the world? Once said, " If Bell had known more about electricity he would never have patented the telephone".

Solutions to Test 26

925

$$Q = 100\frac{gal}{min} \times \frac{3.785\,li}{1\,gal} \times \frac{1\,m^3}{1000\,li} \times \left(\frac{3.281\,ft}{1\,m}\right)^3 = 13.368\frac{ft^3}{min}$$

$$HP_{out} = \frac{Qwh}{33{,}000} = \frac{(13.368)(62.4)(200)}{33{,}000} = 5.055\,hp$$

$$HP_{in} = \frac{HP_{out}}{\eta} = \frac{5.055}{0.7}$$

$$HP_{in} = 7.22\,hp$$

Note: Specific weight of water in English unit is 62.4 lbs/ft^3

926

$$Sp.gr. = \frac{w}{w_{water}}$$

$$HP = \frac{Qwh}{44760} = \frac{Q(Sp.gr.)w_{water}h}{44760}$$

Note: By inspection on the above equation, the horsepower required is directly proportional to the specific gravity of the liquid being pumped.

$$HP = k(Sp.gr.)$$

$$\frac{HP_2}{HP_1} = \frac{Sp.gr._2}{Sp.gr._1}$$

$$HP_2 = HP_1\left(\frac{Sp.gr._2}{Sp.gr._1}\right) = 100\left(\frac{0.8}{1.0}\right)$$

$$HP_2 = 80\,hp$$

927

$$Q = 237{,}000\frac{gal}{hr} \times \frac{1\,hr}{60\,min} \times \frac{3.785\,li}{1\,gal} \times \frac{1\,m^3}{1000\,li} \times \left(\frac{3.281\,ft}{1\,m}\right)^3 = 528.058\frac{ft^3}{min}$$

$$HP = \frac{Qwh}{33{,}000} = \frac{(528.058)(62.4)(40)}{33{,}000} = 39.94$$

$$HP \cong 40\,hp$$

928

$$HP_{out\,pump} = \frac{Qwh}{33{,}000} = \frac{(1100)(62.4)(100)}{33{,}000} = 208\,hp$$

$$HP_{in\,motor} = \frac{HP_{out\,pump}}{\eta_{motor}\,\eta_{pump}} = \frac{208}{(0.92)(0.75)} = 301.45\,hp$$

$P_{in} = 301.45(0.746) = 224.88\ kW$
$W = P_{in}t = 224.88(24) = 5397.12\ kW\text{-}hr$

$Cost = W \times (cost\ per\ kW\text{-}hr) = 5397.12(0.30)$
$Cost = P\ 1{,}619$

929 $P_{in} = Qwh = (1.3)(9810)(50) = 637650\ W = 637.65\ kW$
$P_{out} = P_{in}(\eta_t)(\eta_g) = 637.65(0.7)(0.9)$
$P_{out} = 401.72\ kW$

Note: Specific weight of water in SI units is 9810 N/m³.

930 $P_{in} = \dfrac{Qwh}{60} = \dfrac{(1000)(9810)(122.45)}{60} = 20{,}020{,}575\ W = 20{,}020.575\ kW$

$\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{15{,}000}{20{,}020.575} \times 100\%$

$\eta \cong 75\%$

931 Note: Since not clearly specified as given, assume the motor is a single-phase and is operating at unity pf.

$W = 240{,}000\ kg \times 9.81\ m/sec^2 = 2{,}354{,}400\ N$

$P_{out_{pump}} = \dfrac{work}{time} = \dfrac{Wh}{t} = \dfrac{(2{,}354{,}400)(30)}{3600} = 19620\ W$

$P_{in_{motor}} = \dfrac{P_{out_{pump}}}{\eta_{motor}\,\eta_{pump}} = \dfrac{19620}{(0.9)(0.8)} = 27{,}250\ W$

$I = \dfrac{P_{in}}{Epf} = \dfrac{27250}{(440)(1)}$

$I \cong 62\ A$

932 $Q = 500\dfrac{gal}{min} \times \dfrac{3.785\ li}{1\ gal} \times \dfrac{1\ m^3}{1000\ li} \times \dfrac{1\ min}{60\ sec} = 0.03154\dfrac{m^3}{sec}$

$HP_{out\,pump} = \dfrac{Qwh}{746} = \dfrac{(0.03154)(9810)(25)}{746} = 10.368\ hp$

$HP_{motor} = HP_{in_{pump}} = \dfrac{P_{out_{pump}}}{\eta} = \dfrac{10.368}{0.75} = 13.824$

$HP_{motor} \cong 14\ hp$

933 $P_{in} = \dfrac{P_{out}}{\eta_t \eta_g} = \dfrac{40}{(0.8)(0.85)} = 58.823\,MW$

$P_{in} = Qwh$

$Q = \dfrac{P_{in}}{wh} = \dfrac{58.823 \times 10^6}{(1200)(9.81)(120)} = 41.64\,\dfrac{m^3}{sec}$

$V = Qt = (41.64)\left(6\,hrs \times \dfrac{3600\,sec}{1\,hr}\right)$

$V = 899{,}424\,m^3$

Note: To convert kg to Newton, multiply kg by 9.81 m/sec^2

934 Let: V_t = total volume of water collected
V_p = volume of the impounded water which is available for power
Q = average discharge

$V_t = \left[\left(190\,cm \times \dfrac{1\,m}{100\,cm}\right) \times 250\,km^2 \times \left(\dfrac{1000\,m}{1\,km}\right)^2\right](0.85) = 403.75 \times 10^6\,m^3$

$V_p = 0.7(403.75 \times 10^6) = 282.625 \times 10^6\,m^3$

$Q = \dfrac{V_p}{t} = \dfrac{282.625 \times 10^6}{8760\,hrs \times \dfrac{3600\,sec}{1\,hr}} = 8.962\,\dfrac{m^3}{sec}$

$P_{in} = Qwh = (8.962)(9810)(30) = 2{,}637{,}516.6\,W = 2{,}637.516\,kW$

$P_{out} = P_{in}(\eta_t \eta_g) = 2637.516(0.8)(0.92)$

$P_{out} = 1941.2\,kW$

Note: One year is equivalent to 8760 hrs (365 days x 24 hrs/day)

935 $P_{in} = \dfrac{P_{out}}{\eta} = \dfrac{100}{0.8} = 125\,MW$

$Q = \dfrac{P_{in}}{wh} = \dfrac{125 \times 10^6}{1000(9.81)(200)} = 63.71\,\dfrac{m^3}{sec}$

$V = Qt = 63.71\left[10\,hrs \times \dfrac{3600\,sec}{1\,hr}\right]$

$V = 2{,}293{,}560\,m^3$

936 $P_{in} = Qwh = 0.05(9810)(5) = 2452.5$ watts

$P_{out} = P_{in}(\eta_t \eta_{trans}) = 2452.5(0.75)(0.82)$

$P_{out} = 1508.29$ watts

$$\text{No. of lamps} = \frac{P_{out}}{50} = \frac{1508.29}{50}$$

No. of lamps ≅ 30 lamps

937

$$P_{in} = Qwh = (1)(9810)(60) = 588{,}600\ W = 588.6\ kW$$

$$P_{out} = P_{in}(\eta_t \eta_g) = (588.6)(0.8)(0.95)$$

$$P_{out} = 447.34\ kW$$

938

$$P_{in} = Qwh = (2.5)(9810)(100) = 2{,}452{,}500\ W = 2452.5\ kW$$

$$P_{out} = P_{in}(\eta_t \eta_g) = 2452.5(0.7)(0.9) = 1545.075\ kW$$

$$W = P_{out}t = (1545.075)(8760)$$

$$W = 13{,}534{,}857\ kW\text{-}hr$$

Note: One year is equivalent to 8760 hrs.

939

$$P_{in} = Qwh = (1.3)(9810)(50) = 637650\ W = 637.65\ kW$$

$$P_{out} = P_{in}(\eta_t \eta_g) = 637.65(0.7)(0.9) = 401.72\ kW$$

$$W = P_{out}t = (401.72)(8760)$$

$$W = 3{,}519{,}067\ kW\text{-}hr$$

940

$$Q = 1000\frac{gal}{min} \times \frac{3.785\ li}{1\ gal} \times \frac{1\ m^3}{1000\ li} = 3.785\frac{m^3}{min}$$

$$Hp_w = \frac{Qwh}{746} = \frac{(3.785)(9810)(100)}{746} = 4977.32\ hp$$

$$N_s = \frac{5050}{h+32} + 19 = \frac{5050}{100\ m \times \frac{3.281\ ft}{1\ m} + 32} + 19 = 33.024\ rpm$$

$$N = \frac{N_s h^{1.25}}{\sqrt{Hp_w}} = \frac{(33.024)(100 \times 3.281)^{1.25}}{\sqrt{4977.32}} = 653.64$$

$$N \cong 654\ rpm$$

941

$$N_s = \frac{5050}{h+32} + 19 = \frac{5050}{275+32} + 19 = 35.45\ rpm$$

$$N = \frac{N_s h^{1.25}}{\sqrt{Hp_w}}$$

$$450 = \frac{(35.45)(275)^{1.25}}{\sqrt{Hp_w}}$$

$$Hp_w = 7782.86\ hp$$

$P_{out} = P_{in}(\eta_t \eta_g) = 7782.86(0.746)(0.85)(0.8)$

$P_{out} = 3948 \text{ kW}$

942 $P_{in} = \dfrac{Qwh}{60} = \dfrac{(1000)(9810)(100)}{60}$

$P_{in} = 16{,}350{,}000 \text{ W} = 16350 \text{ kW}$

$P_{out} = P_{in}\eta = 16350(0.75)$

$P_{out} = 12{,}262.5 \text{ kW}$

Note: From the choices, the closest answer is 12,250 kW. The slight difference in the magnitude is due to the constant used for specific weight.

943 $P_{in} = \dfrac{100{,}000 \text{ lbs}}{\text{hr}} \times \dfrac{12{,}000 \text{ BTU}}{\text{lb}} \times \dfrac{252 \text{ cal}}{\text{BTU}} \times \dfrac{4.186 \text{ J}}{\text{cal}} \times \dfrac{1 \text{ hr}}{3600 \text{ sec}}$

$= 351{,}624{,}000 \text{ watts}$

$P_{out} = \eta P_{in} = 0.3(351{,}624{,}000) = 105{,}487.2 \text{ kW}$

$P_{out} \cong 105{,}500 \text{ kW}$

944 Note: Assume an output of 1 kW-hr per 1 pound of fuel.

$W_{in} = \dfrac{W_{out}}{\eta} = \dfrac{1 \text{ kW - hr}}{0.36} = 2.778 \text{ kW - hr}$

$\text{Heating value} = \dfrac{2.778 \text{ kW - hr}}{\text{lb}} \times \dfrac{3.6 \times 10^6 \text{ J}}{\text{kW} - \text{hr}} \times \dfrac{1 \text{ cal}}{4.186 \text{ Jl}} \times \dfrac{1 \text{ BTU}}{252 \text{ cal}}$

$= 9480.58 \text{ BTU/lb}$

$\text{Heating value} \cong 9500 \text{ BTU/lb}$

945 $\text{Cost} = \dfrac{2.88 \times 10^6 \text{ cal}}{\text{kW} - \text{hr}} \times \dfrac{1 \text{ BTU}}{252 \text{ cal}} \times \dfrac{1 \text{ lb}}{13{,}000 \text{ BTU}} \times \dfrac{\text{P } 2{,}500}{1 \text{ ton}} \times \dfrac{1 \text{ ton}}{1000 \text{ kg}} \times \dfrac{1 \text{ kg}}{2.2 \text{ lb}}$

$\text{Cost} \cong \text{P } 1.00$

946 Note: Assume an output of 1 kW-hr.

$W_{in} = \dfrac{W_{out}}{\eta} = \dfrac{1 \text{ kW - hr}}{0.34} = 2.941 \text{ kW - hr}$

$\text{Amount of coal} = 2.941 \text{ kW - hr} \times \dfrac{3.6 \times 10^6 \text{ J}}{\text{kW} - \text{hr}} \times \dfrac{1 \text{ cal}}{4.186 \text{ Jl}} \times \dfrac{1 \text{ BTU}}{252 \text{ cal}} \times \dfrac{1 \text{ lb}}{10{,}800 \text{ BTU}}$

$= 0.9293 \text{ lb}$

$\text{Amount of coal} \cong 0.93 \text{ lb}$

947 $W_{in} = \frac{W_{out}}{\eta} = \frac{1}{0.28} = 3.57\ kW\text{-}hr$

$$Cost = 3.57\ kW\text{-}hr \times \frac{3.6 \times 10^6\ J}{kW-hr} \times \frac{1\ cal}{4.186\ J} \times \frac{1\ BTU}{252\ Cal} \times \frac{1\ lb}{12{,}000\ BTU} \times \frac{P\ 1.50}{1\ lb}$$

$Cost \cong P\ 1.523$

948 $W_{in} = 5000 \frac{kcal}{kg} \times \frac{4200\ kg}{1\ hr} \times \frac{24\ hrs}{1\ day} \times 1\ day$

$W_{in} = 504 \times 10^6\ kcal = 504 \times 10^9\ cal$

$W_{out} = \eta\ W_{in} = (0.30)(504 \times 10^9) = 1.512 \times 10^{11}\ cal$

$W_{out} = 1.512 \times 10^{11}\ cal \times \frac{4.186\ J}{1\ cal} \times \frac{1\ kW\text{-}hr}{3.6 \times 10^6\ J} = 175{,}812\ kW\text{-}hr$

$W_{out} \cong 176\ MW\text{-}hr$

949 $P_{in} = 1900 \frac{lbs}{hr} \times \frac{9500\ BTU}{lb} \times \frac{252\ cal}{1\ BTU} \times \frac{4.186\ J}{1\ cal} \times \frac{1\ hr}{3600\ sec}$

$P_{in} = 5{,}289{,}011 \frac{J}{sec}$ or W

$P_{in} \cong 5289\ kW$

$\eta = \frac{P_{out}}{P_{in}} = \frac{800}{5289} \times 100\%$

$\eta = 15.12\%$

950 $\text{Plant capacity factor} = \frac{\text{Average load}}{\text{Installed capacity}}$

Average load = 0.75(100) = 75 MW = 75,000 kW

$$\text{Amount of coal} = 75{,}000\ kW \times 24\ hrs \times \frac{9500\ BTU}{kW-hr} \times \frac{1\ lb}{12{,}000\ BTU}$$

Amount of coal = 1,425,000 lbs

951 $W_{in} = \frac{W_{out}}{\eta_{eng}\eta_{alt}} = \frac{1}{(0.3)(0.95)} = 3.51\ kW\text{-}hr$

$$\text{Lbs. of coal} = \frac{1\ lb}{18{,}000\ BTU} \times \frac{1\ BTU}{252\ cal} \times \frac{1\ cal}{4.186\ J} \times \frac{3.6 \times 10^6\ J}{kW\text{-}hr} \times 3.51\ kW\text{-}hr$$

Lbs. of coal = 0.66548 lbs

Cost = 0.66548 (P2.80) = 1.8633

$Cost \cong P\ 1.86$

952 $W_{in} = \frac{W_{out}}{\eta} = \frac{70}{0.4} = 175\ kW - hr$

$$\text{Mass of oil} = 175\ kW\text{-}hr \times \frac{3.6 \times 10^6\ J}{1\ kW\text{-}hr} \times \frac{1\ cal}{4.186\ J} \times \frac{1\ kg}{12{,}000{,}000\ cal}$$

Mass of oil = 12.54 kg

953 Note: Assume the plant operates continuously all throughout the year. One year is equal to 8760 hrs (365 days x 24 hrs/day).

Plant energy output per year = 150(5000) + 420(1000) + 20(8760 - 6000)
Plant energy output per year = 1,225,200 kW - hr

Cost of installed capacity = P 1,000(500) = P 500,000
Fixed cost = 0.14 (P 500,000) = P 70,000
Operating cost = P 1.5(1,225,200) = P 1,837,800
Total annual cost of production = P 500,000 + P 70,000 + P 1,837,800
Total annual cost of production = P 2,407,800

$$\text{Cost per kW - hr} = \frac{P\ 2{,}407{,}800}{1{,}225{,}200}$$

Cost per kW - hr = P 1.965

954 $\text{Plant capacity factor} = \frac{\text{Average load}}{\text{Installed capacity}}$

Average load = 0.50(3000 x 8760) = 13,140,000 kW - hr
First cost of installed units = P 15,000(3000) = P 45 million

$$\text{Cost of coal} = 13{,}140{,}000\ kW\text{-}hr \times \frac{1\ lb}{1\ kW\text{-}hr} \times \frac{1\ kg}{2.2\ lbs} \times \frac{1\ ton}{1000\ kg} \times \frac{P\ 800}{1\ ton}$$

Cost of coal = P 4,778,181.8 ≅ P 4.78 million
Interest on investments = First cost x rate = 45(0.08) = P 3.6 million

Annual cost = 12 + 8 + 15 + 3.6 + 4.778
Annual cost = P 43.378 million

Note: The interest of investments is only charge to the interest of the first cost. (Refer to theories/discussions in Eng'g Economics)

955 $\text{Heat rate} = \frac{13{,}000\ BTU}{lb} \times \frac{0.9\ lb}{1\ kW\text{-}hr}$

$$\text{Heat rate} = 11{,}700\ \frac{BTU}{kW\text{-}hr}$$

956

$$\text{Diversity factor} = \frac{\sum \text{Individual peak loads}}{\text{Peak load of the feeder}}$$

$$\text{Peak load} = \frac{25 + 38 + 18 + 45}{1.5} = 84\text{ A}$$

Minimum ampacity = Peak load of the feeder

Minimum ampacity = 84 A

957

$$\text{Diversity factor} = \frac{\sum \text{Individual peak loads}}{\text{Peak load of the feeder}}$$

$$\text{Peak load} = \frac{35 + 46 + 72 + 57}{1.5} = 140\text{ A}$$

$$\text{Max. kVA demand} = \frac{\sqrt{3}EI}{1000} = \frac{\sqrt{3}(220)(140)}{1000}$$

Max. kVA demand = 53.347 kVA

958

$$\text{Demand factor} = \frac{\text{Peak load}}{\text{Connected load}}$$

Peak load = Demand factor x Connected load

$$\text{Diversity factor} = \frac{\sum \text{Individual peak loads}}{\text{Peak load of the transformer}}$$

$$\text{Peak load} = \frac{(0.6)(300) + (0.8)(1200) + (0.9)(500)}{1.5}$$

Peak load = 1060 kW

959

$$\text{Diversity factor} = \frac{\sum \text{Individual peak loads}}{\text{Peak load of the transformer}}$$

$$\text{Peak load} = \frac{35 + 46 + 72 + 57}{1.5} = 140\text{ A}$$

$$\text{Load factor} = \frac{\text{Average load}}{\text{Peak load}}$$

Average load = 0.5(140) = 70 A

$P = \sqrt{3}EIpf = \sqrt{3}(220)(70)(0.8)$

P = 21.3 kW

960

$$\text{Demand factor} = \frac{\text{Peak load}}{\text{Connected load}}$$

Peak load = 0.60(100) = 60 kW

$$\text{Load factor} = \frac{\text{Average load}}{\text{Peak load}}$$

$$\text{Load factor} = \frac{186{,}000{,}000}{60{,}000 \times 8760} \times 100\%$$

$$\text{Load factor} \cong 35.39\%$$

961 $$\text{Plant use factor} = \frac{\text{Annual kW - hr produced}}{\sum \text{installed capacity} \times \text{Number of hours of operation}}$$

$$\text{Plant use factor} = \frac{800 \times 10^6}{2(75{,}000)(7200) + (50{,}000)(3000)} \times 100\%$$

$$\text{Plant use factor} = 65.04\%$$

962 $$\text{Load factor} = \frac{\text{Average energy per week}}{\text{Maximum demand} \times 166}$$

$$\text{Plant capacity factor} = \frac{7{,}000{,}000}{(75{,}000)(166)} \times 100\%$$

$$\text{Plant capacity factor} = 55.55\%$$

Note: 166 hrs per week (24 hrs/day x 7 days/week)

963 $$\text{Load factor} = \frac{\text{Average load}}{\text{Peak loaad}}$$

$$\text{Average load} = 0.59\ \text{Peak load} \Rightarrow (1)$$

$$\text{Installed capacity} = \text{Peak load} + \text{Reserved}$$

$$\text{Peak load} = \text{Installed capacity} - 10 \Rightarrow (2)$$

$$\text{Capacity factor} = \frac{\text{Average load}}{\text{Installed capacity}}$$

$$\text{Installed capacity} = \frac{\text{Average load}}{0.41} \Rightarrow (3)$$

Substitute Eq.1 and Eq.2 to Eq.3 :

$$\text{Installed capacity} = \frac{0.59\ \text{Peak load}}{0.41}$$

$$\text{Installed capacity} = \frac{0.59\ (\text{Installed capacity} - 10)}{0.41}$$

$$0.41(\text{Installed capacity}) = 0.59(\text{Installed capacity}) - 5.9$$

$$\text{Installed capacity} = \frac{5.9}{0.59 - 0.41}$$

$$\text{Installed capacity} = 32.77\ \text{MW}$$

964 $\text{Load factor} = \dfrac{\text{Average load}}{\text{Peak loaad}}$

Average load = 0.59 Peak load ⇒ ❶

Installed capacity = Peak load + Reserved

Peak load = Installed capacity − 10 ⇒ ❷

$\text{Capacity factor} = \dfrac{\text{Average load}}{\text{Installed capacity}}$

$\text{Installed capacity} = \dfrac{\text{Average load}}{0.41} \Rightarrow ❸$

Substitute Eq.1 and Eq.2 to Eq.3 :

$\text{Installed capacity} = \dfrac{0.59\ \text{Peak load}}{0.41}$

$\text{Installed capacity} = \dfrac{0.59\ (\text{Installed capacity} - 10)}{0.41}$

0.41(Installed capacity) = 0.59(Installed capacity) - 5.9

$\text{Installed capacity} = \dfrac{5.9}{0.59 - 0.41} = 32.77\ \text{MW}$

Let : n = number of hours of actual operation

N = number of hours not in operation

$\text{Use factor} = \dfrac{\text{Average load}}{\text{Installed capacity x n}}$

$n = \dfrac{\text{Average laod}}{\text{Installed capacity x Use factor}} = \dfrac{13.439(8760)}{32.778(0.46)} = 7807.8$

n ≅ 7808 hrs

N = 8760 - 7808

N = 952 hrs

965 Installed capacity = Peak load + Reserved

Installed capacity = Peak load + 10,500 ⇒ ❶

$\text{Load factor} = \dfrac{\text{Average load}}{\text{Peak load}}$

$\text{Peak load} = \dfrac{\text{Average load}}{0.6} \Rightarrow ❷$

$\text{Plant factor} = \dfrac{\text{Average load}}{\text{Installed capacity}}$

Average load = 0.5 Installed capacity ⇒ ❸

Substitute Eq.3 in Eq.2 :

$$\text{Peak load} = \frac{0.5\ \text{Capacity}}{0.6}$$

$$\text{Peak load} = 0.833333(\text{Installed capacity}) \Rightarrow ④$$

Substitute Eq.4 in Eq.1 :

$$\text{Installed capacity} = 0.833333(\text{Installed capacity}) + 10{,}500$$

$$\text{Installed capacity} = \frac{10{,}500}{1 - 0.833333}$$

$$\text{Installed capacity} = 63{,}000\ \text{kW}$$

966 $$\text{Capacity factor} = \frac{\text{Annual kW - hr produced}}{\text{Installed capacity} \times 8760}$$

$$\text{Capacity factor} = \frac{238{,}000{,}000}{(55{,}000)(8760)} \times 100\%$$

$$\text{Capacity factor} \cong 49.4\%$$

967 Energy generated for one (1) hour :

$$W = Pt$$

$$= (600 + 800)(1) = 1400\ \text{kw - hr}$$

$$= \frac{1400\ \text{kJ}}{\text{sec}} \times 1\text{hr} \times \frac{3600\ \text{sec}}{\text{hr}}$$

$$W = 5{,}040{,}000\ \text{kJ}$$

Energy produced from coal per hour :

$$W = \frac{20\ \text{kJ}}{\text{g}} \times 680\ \text{kg} \times \frac{1000\ \text{g}}{\text{kg}}$$

$$W = 13{,}600{,}000\ \text{kJ}$$

$$\%\ \text{Effective heat} = \frac{5{,}040{,}000}{13{,}600{,}000} \times 100\%$$

$$\%\ \text{Effective heat} = 37.06\%$$

968 $$\text{Load factor} = \frac{\text{Average monthly energy}}{\text{Peak load} \times 720} = \frac{200{,}000}{380(720)} \times 100\%$$

$$\text{Load factor} = 73\%$$

Note: In one month, there are 720 hours (30 days x 24 hrs/day)

Miscellaneous Topics - Test 27

Problem 969: EE Board October 1986, EE Board October 1993

An electric truck is propelled by a DC motor driven by a 110-V storage battery. The truck is required to exert a tractive effort of 220 lb at a speed of 5 mph. The overall efficiency of the motor and drive is 70 percent. What current is taken from the battery?

A. 25.24 A
B. 28.45 A
C. 30.22 A
D. 26.18 A

Problem 970: EE Board October 1990

An electric motor driven conveyor belt 80 feet long is required to take 2,200 lbs of dirt, rocks, etc per minute from an excavation site and raise it 15 ft vertically. If all friction in the machinery is neglected, what hp of motor is required?

A. 1.5
B. 1.0
C. 2.0
D. 2.5

Problem 971: EE Board June 1990

An electric motor is needed to drive an escalator in a department store. The escalator is 36 in wide and can accommodate two (2) riders side by side hence the loading can be considered as uniformly distributed with 250 lb/ft. Linear speed is 80 ft/min. The escalator angle is 38° and its lifting height is 6.5 meters. Mechanical efficiency of the system is 60%. Determine the motor size in hp.

A. 21.53
B. 22.01
C. 19.42
D. 20.84

Problem 972: EE Board June 1990

An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of the electrical energy supplied to the lift is lost in the motor and gearing. If the time required raising the load is 27 minutes find the current taken by the motor, the supply voltage being 230 V DC.

A. 4.83 A
B. 3.42 A
C. 5.26 A
D. 6.05 A

Problem 973: EE Board October 1992, EE Board April 1994

A conveyor belt of length 15-m is at 35° with respect to the ground or horizontal. It can hold 4,000 kg of ore over its total length and it travels the distance of its length in 30 seconds. Calculate the power rating of the motor in kW that drives the conveyor belt.

A. 10.50 kW
B. 11.25 kW
C. 13.58 kW
D. 12.75 kW

Problem 974: EE Board October 1998

What is the work in ergs needed to raise a 10-gram weight 100-m up?

A. 4.9×10^5
B. 98×10^7
C. 9.8×10^7
D. 1.96×10^7

Problem 975: EE Board October 1990

An electric lift makes 120 double journeys in a day and a load of 6 tonnes is raised to a height of 100 m in one and a half minutes. In the return journey, the cage of the lift is empty and it completes the journey in 70 seconds. The weight of the cage is 600 kg and the counter weight is 3 tonnes. Calculate the horsepower of the motor. Assume the efficiency of the lift to be 80%.

A. 64.24 hp
B. 65.75 hp
C. 58.70 hp
D. 70.43 hp

Problem 976: EE Board October 1989

The power required by a plane flying at 545 km/hr is 1800 kW. Determine the average resisting force in Newtons.

A. 11,890
B. 12,420
C. 11,520
D. 12,670

Problem 977: EE Board October 1990

A train weighing 1600 tons is pulled up a 2% grade by 600 hp. Train resistance is 19200 lbs. What is the speed in mph?

A. 22
B. 25
C. 32
D. 28

Problem 978: EE Board April 1993

A DC series motor operates a crane at normal voltage and lifts a load of 2-tonne at 50 m/min. Determine the speed at which it will lift a load of 5-tonne.

A. 30 m/min
B. 20 m/min
C. 25 m/min
D. 28 m/min

Problem 979: EE Board April 1994

How long could a 1000-hp motor be operated in the heat energy liberated by 1 cubic kilometer of ocean water when the temperature of the water is lowered by 1 °C if all its heat were converted to mechanical energy?

A. 170 yrs
B. 177 yrs
C. 182 yrs
D. 188 yrs

Problem 980: EE Board October 1991

Calculate the minimum rms HP required of an open type electric motor which operates at 15 hp load for 8 minutes, 10 hp load for 6 minutes, 20 hp load for 6 minutes and the motor is at rest for 12 minutes after which the cycle is repeated.

A. 15.00
B. 12.25
C. 10.32
D. 16.75

Problem 981: EE Board October 1993

A transformer is designed to carry 30 times its rated current for 1 second. The length of time that a current of 20 times the rated current can be allowed to flow is

A. 0.67 sec
B. 2 sec
C. 1.5 sec
D. 2.25 sec

Problem 982: EE Board October 1997

A three-phase motor draws 50 A of current at full load. What shall be th maximum inverse-time circuit breaker that maybe used for short circuit protection?

A. 50 A

B. 100 A
C. 125 A
D. 75 A

Problem 983: EE Board October 1990

A synchronous condenser costs P 1000 per kVA. If a consumer is charged at P 625 per annum per kVA of maximum demand and the interest and depreciation charges are 10% on the capital invested, determine his most economical operating power factor. The power factor of the consumer load is 0.80 lagging. Assume no losses in the synchronous condenser.

A. 0.998
B. 0.988
C. 0.989
D. 0.986

Problem 984: EE Board April 1982

In a group of 3 motors, one draws 15 A, the other 40 A and the third 52 A. what size of conductor in terms of ampacity must be used for the feeder circuit?

A. 120 A
B. 107 A
C. 134 A
D. 160 A

Problem 985: EE Board October 1992

Given the following three-phase induction motors operating at full load.

15-hp, 220 V, 0.95 eff., 0.80 pf
25-hp, 220 V, 0.90 eff., 0.707 pf
20-hp, 220 V, 0.92 eff., 0.866 pf

Determine the ampacity of the feeder circuit conductors supplying these motors.

A. 178.46 A
B. 183.93 A
C. 164.70 A
D. 205.07 A

Problem 986: EE Board April 1982

A single-phase motor has rated full load current of 28 amperes. How large must be the ampacity of the fuses to be used?

A. 35 A
B. 42 A
C. 70 A
D. 84 A

Problem 987: EE Board October 1998

What surge arrester's nominal rating shall you recommend to protect distribution transformer in a 34.5 kV grounded system?

A. 34.5 kV
B. 30 kV
C. 22 kV
D. 27 kV

Problem 988: EE Board April 1991

An induction motor is equipped with a flywheel of moment of inertia of 4 kg-m^2 for driving a rolling mill. If the speed of the induction motor is 1000 rpm, what extra energy the flywheel is capable of supplying when the load comes to the motor?

A. 22 kJ
B. 20 kJ
C. 28 kJ
D. 32 kJ

Problem 989: EE Board October 1992

A three-phase, 220 V, 60 Hz induction motor drives a fan, which provides 40 m^3/sec air requirements to a furnace. Assume fan efficiency of 60 % and 150 mm H_2O gauge pressure. Determine the line current drawn by the motor if efficiency and power factor is 80% and 70% respectively.

A. 460 A
B. 474 A
C. 482 A
D. 442 A

Problem 990: EE Board October 1998

A ten-year investigation of a river's potential gave an average water flow of 25 cm per second at the bottom and 90 cm per second at the surface. The average cross section at the same location is 80 sq. m. What is the average flow rate in cubic meter per hour?

A. 165,600
B. 46
C. 57,500
D. 2,760

Problem 991: EE Board April 1993, EE Board October 1994

A 500 kW electric lighting costs P 1000 per kW installed. Fixed charges are 14%, operating cost is P 1.50 per kW-hr. The plant averages 150 kW for 5000 hrs of the year, 420 kW for 1000 hrs and 20 kW for the remainder. What is the unit cost of producing electric energy?

A. P 2.10

B. P 1.96
C. P 2.01
D. P 1.82

Problem 992: EE Board October 1990

If the power gain in a circuit is 20 decibels, what is the actual ratio of output to input power?

A. 200
B. 100
C. 20
D. 10

Problem 993: EE Board October 1990

If 20 decibels were power losses, what is the power output to power input ratio?

A. 0.05
B. 0.01
C. 10
D. 5

Problem 994: EE Board October 1992

What is the output of a 27-dB gain amplifier whose input power is 25 μW?

A. 12.53 mW
B. 11.24 mW
C. 15.62 mW
D. 14.05 mW

Problem 995: EE Board April 1991

It is desired to deposit a thickness of 1 mm of nickel on the surface of a worn steel pin 10 inches long and 4 inches in diameter. The available current is 50 A. The electromechanical equivalent of nickel is 0.00038 gm/coulomb and its relative density is 8.9 Calculate the time required.

A. 11.32 hrs
B. 12.65 hrs
C. 10.08 hrs
D. 9.55 hrs

Problem 996: EE Board October 1987

The use of electricity for melting a snow in a driveway 10 ft wide by 30 ft long is being considered. At 0.20 cents a kilowatt-hour, what would be the cost of melting 6 inches of snow? Assume the following data: specific weight of snow, 10 lb per cu. ft. efficiency of operation, 50%.

A. P 20.25
B. P 25.35

C. P 28.40
D. P 30.63

Problem 997: EE Board April 1990

A car is being self started against a resisting torque of 60N-m and at each start the engine is cranked at 75 rpm for 8 seconds. For each start the energy is drawn from a lead acid accumulator. If the fully charged battery has a capacity of 100 wh per charge, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%

A. 26
B. 25
C. 20
D. 23

Problem 998: EE Board October 1990

In an electric heater, the inlet temperature is 15 °C. Water is flowing at the rate of 300 grams per minute. The voltmeter measuring voltage across the heating element reads 120 volts and an ammeter measuring current taken reads 10 A. When steady state is finally reached, what is the final reading of the outlet thermometer?

A. 72.6 °C
B. 81.2 °C
C. 75.3 °C
D. 78.8 °C

Problem 999: EE Board April 1992

A 230 volt is supplied to an electric soldering machine which draws a current of 0.8 A, fifty-five (55%) percent of the heat generated is lost in heating other metal parts of the machine and in radiation. The copper soldering tip weighs 150 grams.

Melting point of solder – 300°C
Ambient temperature –15 °C
Specific heat of copper – 0.094 cal/g-°C
Density of copper – 8.9 g/cc

Determine the time elapsed after switching on the machine before the soldering tip is heated to the melting point of solder and the machine ready for use.

A. 3.05 min
B. 2.26 min
C. 3.37 min
D. 4.24 min

Problem 1000: EE Board April 1990

A room measures 6 m x 6 m x 4 m. The air in it has to be renewed twice an hour and it is to be maintained at least 15 °C above the incoming air temperature. Find the capacity of the heater assuming a loss of 10% of the heat. Assume air density as 12.8×10^{-4} grams per cubic cm and specific heat of air as 0.24 cal/g-°C.

A. 1706 W
B. 1545 W
C. 1734 W
D. 1621 W

Problem 1001: EE Board April 1990, EE Board October 1993

A foundation 30 ft long, 12 ft wide and 9 ft deep is to be composed of a 1:2:4 concrete. How much gravel in cubic yards will be required?

A. 150
B. 106
C. 110
D. 100

ANSWER KEY

969. B	976. A	983. B	990. A	997. D
970. B	977. B	984. A	991. B	998. A
971. A	978. B	985. B	992. B	999. C
972. C	979. B	986. D	993. B	1000. A
973. B	980. B	987. D	994. A	1001. B
974. C	981. D	988. A	995. B	
975. B	982. C	989. A	996. B	

☑ RATING

29-33 - Topnotcher

23-28 - Passer

17-22 - Conditional

0-16 - Failed

Solutions to Test 27

968

$$V = 5\frac{mi}{hr} \times \frac{1\,hr}{3600\,s} \times \frac{5280\,ft}{1\,mi} \times \frac{1\,m}{3.281\,ft} = 2.235\,m/s$$

$$F = 220\,lb \times \frac{1\,kg}{2.202\,lb} \times 9.81 = 980.109\,N$$

$$P_{out} = \frac{Work}{time} = \frac{Fd}{t} = FV$$

$$P_{out} = (980.109)(2.235) = 2190.54\,W$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{V_L I_L}$$

$$I_L = \frac{P_{out}}{\eta V_L} = \frac{2190.54}{(0.7)(110)}$$

$$I_L \cong 28.45\,A$$

Note: To convert kilogram to newton, multiply kilogram by 9.81 m/s²

969

$$d = 15\,ft \times \frac{1\,m}{3.281\,ft} = 4.572\,m$$

$$F = 2200\,lb \times \frac{1\,kg}{2.202\,lb} \times 9.81 = 9801.09\,N$$

$$P_{out} = \frac{Work}{time} = \frac{Fd}{t} = \frac{(9801.09)(4.572)}{60}$$

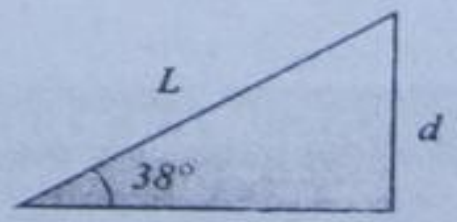
$$P_{out} = 746.84\,W$$

$$Hp \cong 1\,hp$$

970

$$d = 6.5\,m \times \frac{3.281\,ft}{1\,m} = 21.32\,ft$$

$$L = \frac{d}{\sin 38^\circ} = \frac{21.32}{\sin 38^\circ} = 34.63\,ft$$



$$t = \frac{L}{V} = \frac{34.63}{80} = 0.433\,min$$

$$F = 250\frac{lb}{ft} \times 34.63\,ft = 8657.5\,lb$$

$$Hp_{out} = \frac{Fd}{t\,(33{,}000)} = \frac{8657.5(21.32)}{0.433(33{,}000)} = 12.917\,hp$$

$$Hp_{in} = \frac{HP_{out}}{\eta} = \frac{12.917}{0.6}$$

$$Hp_{in} = 21.53\,hp$$

971 $F = 5000(9.81) = 49{,}050\ N$

$t = 27\ min \times \frac{60\ sec}{min} = 1620\ sec$

$P_{out} = \frac{Fd}{t} = \frac{(49050)(30)}{1620} = 908.33\ W$

$P_{in} = P_{out} + P_{losses} = P_{out} + 0.25\,P_{in}$

$0.75P_{in} = P_{out}$

$P_{in} = \frac{P_{out}}{0.75} = \frac{908.33}{0.75} = 1211.106\ W$

$I = \frac{P_{in}}{E} = \frac{1211.106}{230}$

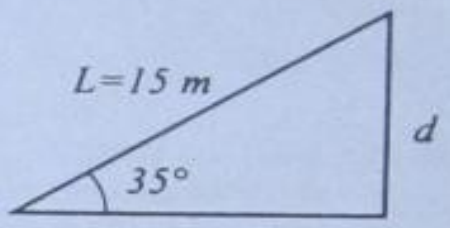
$I = 5.26\ A$

972 $F = 4000\,(9.81) = 39{,}240\ N$

$d = 15 \sin 35° = 8.6\ m$

$P = \frac{Fd}{t} = \frac{(39240)(8.6)}{30} = 11{,}248.8\ W$

$P = 11.25\ kW$



973 Work = Force x distance

$F = 10\ g \times 981\ cm/s^2 = 9810$ dynes

$d = 100\ m \times \frac{100cm}{1m} = 10{,}000\ cm$

$W = Fd = 9810(10{,}000)$

$W = 9.81 \times 10^7$ ergs

Note: To convert gram to dyne, multiply gram by 981 cm/s^2

974 Note: Since "lifting up the loaded cage" and "bringing the empty cage down" does not occur at the same time, to select the HP rating of the motor, the larger work between the two shall be considered, dropping the other. By inspection, "lifting up the loaded cage" requires the larger amount of work, thus the size of the motor is based on this aspect.

Let: W = weight raised during upward motion

$W = W_{load} + W_{cage} - W_{counter}$

$W = (6000 + 600 - 3000) \times 9.81$

$W = 35{,}316\ N$

$t = 1.5\ min \times \frac{60\ sec}{min} = 90\ sec$

$$Hp_{out} = \frac{Wd}{t(746)} = \frac{35{,}316(100)}{(90)(746)} = 52.6\ hp$$

$$Hp_{in} = \frac{HP_{out}}{\eta} = \frac{52.6}{0.8}$$

$$Hp_{in} = 65.75\ hp$$

975 Power = Force x Velocity

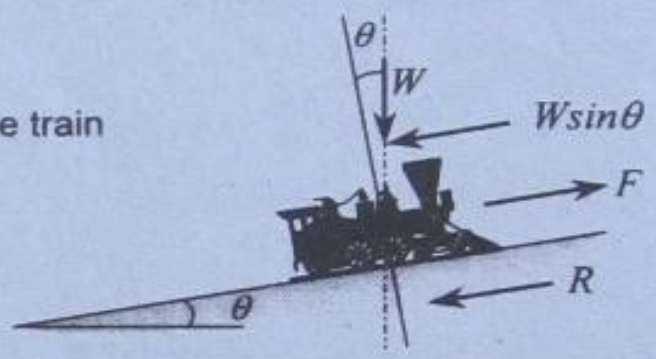
$P = FV$

$$V = 545\frac{km}{hr} x \frac{1000\ m}{km} x \frac{1\ hr}{3600\ sec} = 151.388\ m/s$$

$$F = \frac{P}{V} = \frac{1800{,}000}{151.388} = 11{,}889.97\ N$$

$$F \cong 11{,}890\ N$$

976 Let: R = train resistance
F = force exerted to pull up the train
W = weight of the train



$$\theta = \tan^{-1} 0.02 = 1.145°$$

$$W = 1600{,}000\ kg\ x \frac{2.202\ lbs}{1\ kg}$$

$$W = 3{,}523{,}200\ lbs$$

$$F = R + W\sin\theta = 19200 + 3523200(\sin 1.145°)$$

$$F = 89603\ lbs$$

Power = Force x Velocity

$$Hp = \frac{FV}{550}$$

$$V = \frac{Hp(550)}{F} = \frac{6000(550)}{89603} = 36.83\ ft/sec$$

$$V = 36.83\frac{ft}{sec} x \frac{1\ mi}{5280\ ft} x \frac{3600\ sec}{hr}$$

$$V = 25.11\ mph$$

977 Power = Force x Velocity

$P = FV$

Note: Assume the power delivered (full-load) by the motor in both conditions to be constant. And if the power (P) delivered is constant, the velocity (V) of the motor varies inversely as the force (F) exerted by the same motor.

$$V = k\left(\frac{1}{F}\right)$$

$$\frac{V_1}{V_2} = \frac{F_2}{F_1}$$

$$V_2 = V_1\left(\frac{F_1}{F_2}\right) = 50\left(\frac{2}{5}\right)$$

$$V_2 = 20 \text{ m/min}$$

978

$$m = 1\text{km}^3 \times \left(\frac{1000 \text{ m}}{1\text{km}}\right)^3 \times \frac{1000 \text{ kg}}{\text{m}^3} = 1 \times 10^{12} \text{ kg}$$

$$Q = mc\Delta_t$$

$$Q = (1 \times 10^{12}\text{kg})\left(\frac{1\text{kcal}}{\text{kg-°C}}\right)(1°\text{C}) = 1 \times 10^{12}\text{kcal}$$

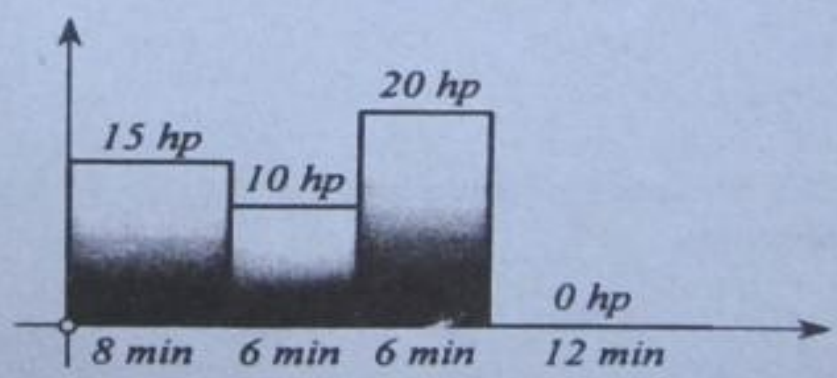
$$Q = 0.24Pt$$

$$t = \frac{Q}{0.24P} = \frac{1 \times 10^{12}}{0.24(1000)(0.746)}$$

$$t = 5.5853 \times 10^9 \text{ sec} \times \frac{1\text{hr}}{3600 \text{ sec}} \times \frac{1\text{yr}}{8760 \text{ hrs}}$$

$$t = 177.108 \text{ years}$$

979



$$Hp_{rms} = \sqrt{\frac{\sum(\text{hp}^2 \times \text{time})}{\text{running time} + k(\text{standstill time})}}$$

where: k = cooling factor

Note: Since not specified in the problem, as a standard assumption, assume that the heat dissipation of the motor at standstill is 100% or k = 1.

$$Hp_{rms} = \sqrt{\frac{(15)^2(8) + (10)^2(6) + (20)^2(6) + (0)^2(12)}{8 + 6 + 6 + (1)(12)}}$$

$$Hp_{rms} = 12.25 \text{ hp}$$

980 Notes:
The heat developed is directly proportional to the power drawn. Power drawn is directly proportional to the square of the current. Thus, the heat developed in a given wire is directly proportional to the square of the current that flows on it.

$Q = kI^2 \Rightarrow$ ❶

Also, the heat capacity of a given wire is inversely proportional to time

$Q = k'\left(\frac{1}{t}\right) \Rightarrow$ ❷

Equate Eq.1 to Eq.2:

$kI^2 = k'\left(\frac{1}{t}\right)$

$I^2 = k''\left(\frac{1}{t}\right)$

$\left(\frac{I_1}{I_2}\right)^2 = \frac{t_2}{t_1}$

$t_2 = t_1\left(\frac{I_1}{I_2}\right)^2 = (1)\left(\frac{30}{20}\right)^2$

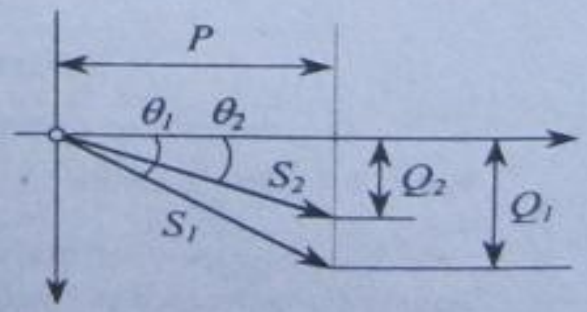
$t_2 = 2.25 \text{ sec}$

981 Note: The rating of an 'inverse time CB' shall not be greater than 250% of the full load current.

$\text{Size} = 2.5\, I_{FL} = 2.5(50)$

$\text{Size} = 125 \text{ A}$

982 Refer to the vector diagram:



Savings made due to reduction in kVA:

$\text{Savings} = 625(S_1 - S_2)$

$= 625\left(\frac{P}{\cos\theta_1} - \frac{P}{\cos\theta_2}\right)$

$= 625P\left(\frac{1}{\cos\theta_1} - \frac{1}{\cos\theta_2}\right)$

$\text{Savings} = 625P(\sec\theta_1 - \sec\theta_2)$

$$\text{Condenser rating} = Q_1 - Q_2$$
$$= P\tan\theta_1 - P\tan\theta_2$$
$$= P(\tan\theta_1 - \tan\theta_2)$$

$$\text{Cost to buy the condenser} = 1000\left[P(\tan\theta_1 - \tan\theta_2)\right]$$

$$\text{Depreciation \& interest} = 10\% \text{ of Capital}$$
$$= 0.1(1000P)(\tan\theta_1 - \tan\theta_2)$$
$$= 100P(\tan\theta_1 - \tan\theta_2)$$

Let: S = net savings

S = Savings due to change in kVA - depreciation & interest charges

$$S = 650P(\sec\theta_1 - \sec\theta_2) - 100P(\tan\theta_1 - \tan\theta_2)$$
$$S = 650P\sec\theta_1 - 650P\sec\theta_2 - 100P\tan\theta_1 + 100P\tan\theta_2$$
$$\frac{dS}{d\theta_2} = 0 - 650P\sec\theta_2\tan\theta_2 - 0 + 100P\sec^2\theta_2$$

At maximum condition: $\dfrac{dS}{d\theta_2} = 0$

$$100P\sec^2\theta_2 = 650P\sec\theta_2\tan\theta_2$$
$$\sec\theta_2 = 6.5\tan\theta_2$$
$$\frac{1}{\cos\theta_2} = 6.5\left(\frac{\sin\theta_2}{\cos\theta_2}\right)$$
$$\sin\theta_2 = \frac{1}{6.5}$$
$$\theta_2 = 8.85°$$
$$pf_2 = \cos\theta_2 = \cos 8.85°$$
$$pf_2 = 0.988$$

983 Note: The ampacity of the feeder conductors serving a group of motor loads is equivalent to the sum of the full load currents of each motor plus 25% of the largest motor load current in the group.

$$\text{Ampacity} = \sum I_{FL} + 25\% \text{ of the largest } I_{FL}$$
$$= 15 + 40 + 52 + 0.25(52)$$
$$\text{Ampacity} = 120 \text{ A}$$

984 $I_{FL} = \dfrac{P_{out}}{\sqrt{3}\,Epf\eta}$

$$I_{FL_1} = \frac{15(746)}{\sqrt{3}(220)(0.8)(0.95)} = 38.64\text{ A}$$

$$I_{FL_2} = \frac{25(746)}{\sqrt{3}(220)(0.707)(0.90)} = 76.92\text{ A}$$

$$I_{FL_3} = \frac{20(746)}{\sqrt{3}(220)(0.866)(0.92)} = 49.14\text{ A}$$

Ampacity = $\sum I_{FL}$ + 25% of the largest I_{FL}

$= 38.64 + 76.92 + 49.14 + 0.25(76.92)$

Ampacity = 183.93 A

985 Note: The rating or size of a 'non-time delay' fuse shall not be greater than 300% of the full load current.

Size = $3I_{FL} = 3(28)$

Size = 84 A

986 Note: For surge arrester the following formulas are to be used.

- For ungrounded system
  kV rating $\geq V_L$
- For grounded system
  kV rating $\geq 1.35\left(\dfrac{V_L}{\sqrt{3}}\right)$

Substitute:

$$\text{kV rating} = 1.35\left(\frac{V_L}{\sqrt{3}}\right) = 1.35\left(\frac{34.5}{\sqrt{3}}\right) = 26.89$$

From the choices, select 27 kV.

988 Note: The additional energy that the flywheel can provide is its kinetic energy due to rotational motion given by the following formula:

$$W = \frac{1}{2}I\omega^2$$

where: W = kinetic energy in joules
I = mass moment of inertia
ω = angular velocity in radians per second

$$\omega = 1000\,\frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{sec}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} = 104.72\,\text{rad/sec}$$

$$W = \frac{1}{2}(4)(104.72)^2 = 21{,}932.55\,\text{J}$$

$$W \cong 22\,\text{kJ}$$

989 $$P_{out\,fan} = Qwh = \left(40\,\frac{m^3}{sec} \times 9.81\,\frac{kN}{m^3} \times 0.15\,m\right)$$

$$P_{out\,fan} = 58.86\,\text{kW}$$

$$P_{in\,fan} = \frac{P_{out}}{\eta} = \frac{58.86}{0.6} = 98.1\,\text{kW}$$

$$P_{out_{motor}} = P_{in_{fan}}$$

$$P_{in_{motor}} = \frac{P_{out_{motor}}}{\eta} = \frac{98.1}{0.8} = 122.625\,\text{kW}$$

$$P_{in_{motor}} = \sqrt{3}E_L I_L pf$$

$$I_L = \frac{P_{in_{motor}}}{\sqrt{3}(E_L)pf} = \frac{122{,}625}{\sqrt{3}(220)(0.7)} = 459.7\,\text{A}$$

$$I_L \cong 460\,\text{A}$$

990 $$V_{ave} = \frac{25 + 90}{2} = 57.5\,\text{cm/s}$$

$$V_{ave} = 57.5\,\frac{cm}{sec} \times \frac{3600\,sec}{1\,hr} \times \frac{1\,m}{100\,cm} = 2{,}070\,m/hr$$

$$Q_{ave} = A_{ave}V_{ave} = 80(2070)$$

$$Q_{ave} = 165{,}600\,m^3/hr$$

991 Total number of hours in one year = 8760 hrs

$$W_{out} = 150(5000) + 420(1000) + 20[8760 - (5000 + 1000)]$$

$$W_{out} = 1{,}225{,}200\,\text{kW - hrs}$$

Installation cost = P 1000(500) = P 500,000
Fixed charges = 0.14(P 500,000) = P 70,000
Operating cost = P 1.5 (1,225,200) = P 1,837,800

Total annual cost = 500,000 + 70,000 + 1,837,800 = P 2,407,800

$$\text{Cost per kW - hr} = \frac{P\,2{,}407{,}800}{1{,}225{,}200} = P\,1.965$$

992

$$dB_{gain} = 10\log\frac{P_{out}}{P_{in}}$$

$$\frac{P_{out}}{P_{in}} = \text{antilog}\frac{dB}{10} = \text{antilog}\frac{20}{10}$$

$$\frac{P_{out}}{P_{in}} = 100$$

993

$$-dB_{loss} = 10\log\frac{P_{out}}{P_{in}}$$

$$\frac{P_{out}}{P_{in}} = \text{antilog}\frac{-dB}{10} = \text{antilog}\frac{-20}{10}$$

$$\frac{P_{out}}{P_{in}} = 0.01$$

994

$$dB_{gain} = 10\log\frac{P_{out}}{P_{in}}$$

$$\frac{P_{out}}{P_{in}} = \text{antilog}\frac{dB}{10}$$

$$P_{out} = P_{in}\text{antilog}\frac{dB}{10} = (25 \times 10^{-6})\left(\text{antilog}\frac{27}{10}\right)$$

$$P_{out} = 12.53\text{ mW}$$

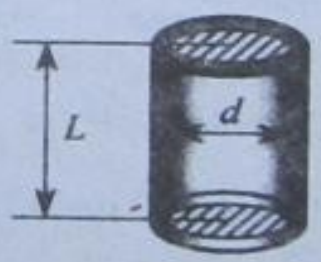
995

$$\text{Length} = 10\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} = 25.4\text{ cm}$$

$$\text{diameter} = 4\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} = 10.16\text{ cm}$$

$$\text{radius} = \frac{d}{2} = \frac{10.16}{2} = 5.08\text{ cm}$$

$$\text{thickness} = 1\text{ mm} \times \frac{1\text{ cm}}{10\text{ mm}} = 0.1\text{ cm}$$



Area of surface $= 2\pi rL = 2\pi(5.08)(25.4) = 810.73\text{ cm}^2$

Area of top & bottom $= 2(\pi r^2) = 2\pi(5.08)^2 = 162.15\text{ cm}^2$

Total area $= 810.73 + 162.15 = 972.88\text{ cm}^2$

Volume = Area x thickness

$= 972.88(0.1)$

Volume $= 97.288\text{ cm}^3$

Weight added = Volume x relative density

$= 97.288(8.9)$

Weight added = 865.86 grams

$$\text{Coulombs required} = \frac{865.86 \text{ grams}}{0.00038 \text{ grams/coul}} = 2.2785789 \times 10^6 \text{ coul}$$

$$I = \frac{Q}{t}$$

$$t = \frac{Q}{I} = \frac{2.2785789 \times 10^6}{50} = 45{,}571.578 \text{ sec}$$

$$t = 45{,}571.578 \text{ sec} \times \frac{1 \text{ hr}}{3600 \text{ sec}}$$

$t \cong 12.65$ hrs

996 Weight of snow = Volume of snow x Specific weight

$= (10 \times 30 \times 0.5)(10)$

$$= 1500 \text{ lbs} \times \frac{1 \text{ kg}}{2.202 \text{ lbs}} = 681.2 \text{ kg}$$

Heat required to melt = Heat of fusion (Weight of snow)

$$= \frac{80 \text{ kcal}}{\text{kg}} \times 681.2 \text{ kg}$$

$= 54{,}496$ kcal

$$\text{Heat input} = \frac{\text{Heat required}}{\eta} = \frac{54{,}496}{0.5}$$

$= 108{,}992$ kcal

$$\text{Energy input} = 108{,}992 \text{ kcal} \times \frac{4.186 \text{ kJ}}{\text{kcal}} = 456{,}240.5 \text{ kJ}$$

$$= 456{,}240.5 \text{ kW - sec} \times \frac{1 \text{ hr}}{3600 \text{ sec}}$$

$= 126.733$ kW - hr

Cost = (Energy input in kW - hr)(cost per kW - hr)

$= 126.733(0.2)$

Cost ≅ P 25.35

997 $$P = \frac{2\pi NT}{60} = \frac{2\pi(75)(60)}{60}$$

$P = 471.238$ W

Energy required per start = Pt = (471.238)(8)

$$= 3770 \text{ W - sec} \times \frac{1 \text{ hr}}{3600 \text{ sec}}$$

Energy required per start = Pt = (471.238)(8)

$$= 3770 \text{ W - sec} \times \frac{1 \text{hr}}{3600 \text{ sec}}$$

Energy required per start = 1.047 W - hr

Energy required per start with considerations to efficiency :

$$= \frac{1.047}{0.25} = 4.188 \text{ W - hr}$$

Number of possible starts $= \frac{100}{4.188} = 23.877$

Number of possible starts ≅ 23

998 Note: The time basis of the problem is 1 minute or 60 seconds (taken from the consideration that the mass flow rate is 300 grams/min)

$Q = mc\Delta_t = mc\,(t_{final} - t_{initial})$

$Q = 300\text{ g}\left(1\frac{\text{cal}}{\text{g - °C}}\right)(t_{final} - 15)\text{ °C}$

$Q = 300(t_{final} - 15) \Rightarrow$ ❶

$Q = 0.24Pt = 0.24EIt$

$Q = 0.24(120)(10)(60)$

$Q = 17280$ cal

Substitute Q in Eq.1 :

$17280 = 300(t_{final} - 15)$

$t_{final} = \frac{17280}{300} + 15$

$t_{final} = 72.6\text{ °C}$

Note: Specific heat of water is 1 calorie per gram-°C.

999 $Q = mc\Delta_t = mc(t_{final} - t_{initial})$

$Q = (150)(0.094)(300 - 15)$

$Q = 4018.5$ cal

$Q_{in} = Q_{out} + 0.55Q_{in}$

$0.45Q_{in} = Q_{out}$

$Q_{in} = \frac{Q_{out}}{0.45} = \frac{4018.5}{0.45} = 8930$ cal

$Q_{in} = 0.24Pt = 0.24EIt$

$$t = \frac{Q_{in}}{0.24EI} = \frac{8930}{0.24(230)(0.8)}$$

$$t = 202.22 \text{ sec} \times \frac{1 \text{ min}}{60 \text{ sec}}$$

$$t = 3.37 \text{ min}$$

1000 $\text{Density}(D) = \dfrac{\text{mass}(m)}{\text{volume}(V)}$

$V = 6(6)(4) = 144 \text{ m}^3$

$$D = 12.8 \times 10^{-4} \frac{g}{cm^3} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 1280 \text{ cm}^3$$

$$t = 30 \text{ min} \times \frac{60 \text{ sec}}{1 \text{ min}} = 1800 \text{ sec}$$

$m = DV = (1280)(144) = 184320 \text{ g}$

$Q = mc\Delta_t = (184320)(0.24)(15) = 663{,}552 \text{ cal}$

$Q = 0.24Pt$

$$P_{out} = \frac{Q}{0.24t} = \frac{663{,}552}{0.24(1800)} = 1536 \text{ W}$$

$$P_{in} = \frac{P_{out}}{\eta} = \frac{1536}{0.9}$$

$P_{in} = 1706.67 \text{ W}$

Note: As given air is to be replaced twice every hour, thus time required per work done is equal to 30 minutes or half an hour.

1001 $V_t = (\text{length})(\text{width})(\text{height}) = (30)(12)(9) = 3240 \text{ ft}^3$

$$V_t = 3240 \text{ ft}^3 \times \left(\frac{\text{yd}}{3 \text{ ft}}\right)^3 = 120 \text{ yd}^3$$

DATA ON CONCRETE MIXES TO YIELD "ONE CUBIC UNIT" OF CONCRETE

Mixture	Cement (sacks)	Sand (cu. unit)	Gravel (cu. unit)
1:2:3	9.2	0.51	0.77
1:2:4	7.8	0.44	0.88
1:2½ :4	7.3	0.52	0.83
1:3:5	6.2	0.52	0.86

Source: Power Plant Engineering by Frederick T. Morse

Refer to the table for the standards of a 1:2:4 concrete mixture:

$V_{gravel} = V_t(0.88) = 120(0.88) = 105.6 \text{ yd}^3$

$V_{gravel} \cong 106 \text{ yd}^3$

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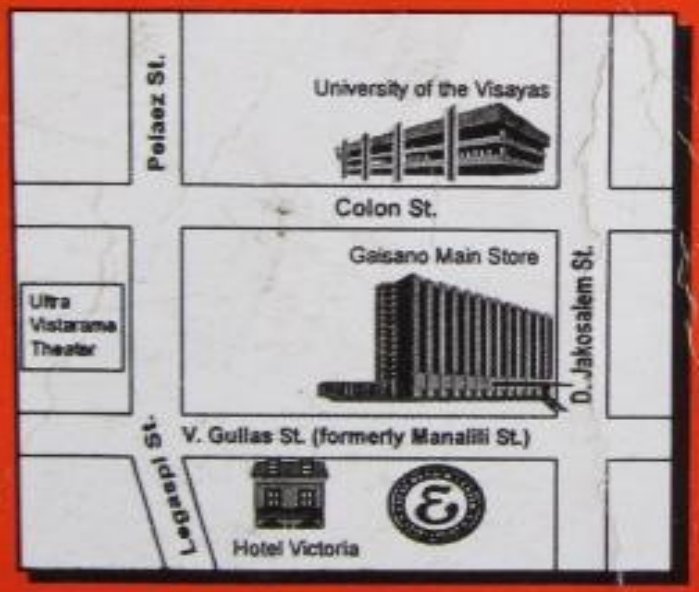

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9 789718 060100
ISBN 971-806-010-3