An Analysis of Uniswap

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1 Analyzing Uniswap

No-arbitrage conditions: The marginal price of coin α in Uniswap is defined as the price of an infinitesimally small trade. This price is described by the following formula:

$$\frac{d\Delta_\beta}{d\Delta_\alpha} = \frac{1}{\gamma}\frac{R_\beta}{R_\alpha} = \gamma^{-1} m_u,$$

where $m_u = R_\beta / R_\alpha$. From the above formula, we see that $d\Delta_\beta = \gamma^{-1} m_u d\Delta_\alpha$. Thus, we can always make a nonzeoro profit if the Uniswap marginal price of α, $\gamma^{-1} m_u$ is smaller than the market marginal price m_p. In other words, the following inequality holds: $m_p \Delta_\alpha \geq \gamma^{-1} m_u \Delta_\alpha$ after performing a small enough trade. Assume there is no arbitrage, this means we must have:

$$m_u \geq \gamma m_p.$$

Similarly, by swapping α for β we get the following bounds on the Uniswap market price:

$$\gamma m_p \leq m_u \leq \gamma^{-1} m_p.$$

In the case of no transaction fee, this means $\gamma = 1$, to obtain the no arbitrage conditions, we must have $m_p = m_u$.

1.1 Uniswap Properties

- **Increasing product constant**. For every trade, the product constant k is nondecreasing (and strictly increasing if $\gamma < 1$).
- **Splitting trade is more expensive**. performing many small transactions is more expensive than a large transaction. This is often referred to as the path-dependence property.
- **Cost of manipulation**. In the no-fee case with an infinitely liquid reference market, the cost of manipulating the price of the constant product market is to linearly increase with the reserve amounts.
- **Liquidity provider returns**. In the no-fee case, we see that the quotient of total pool value at the time t and $t-1$ which depends only on the relative ratios of R_α and R_β. Hence, if the pool size is large enough then the adding or removing tokens from the reserves that will barely affect the liquidity of the pool

1.2 Disadvantages of Uniswap

High slippage for large orders. An order size of half the liquidity pool causes the average price per token to double; a slippage rate of 100% would be outrageous on an order book exchange. To keep slippage under 1%, the liquidity pool would need to be 100x greater than the size of the order.
Uniswap exchanges are impractical for large orders as the respective liquidity pools would need to be massive

Risks for liquidity providers: we see that liquidity providers are most profitable when the token pair trades at around the same price ratio at which the liquidity provider supplied the token pair. Hence, whenever the price ratio changes, liquidity providers could loss money. This is particularly bad with the regular volatility of cryptocurrencies and is worst for low-volume markets that have huge price fluctuations. Liquidity providers could mitigate this risk by actively increasing or decreasing supplied liquidity in response to price changes.