

# Bounded Loss of Uniswap and Senswap Models

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## 1 Bounded Loss of Uniswap

Uniswap is an automated market maker based on *constant function market maker*. In this model, the cost function is described as follows:

$$C(x) = x_1 * x_2 = k, \text{ where } k \text{ is a constant.}$$

For this market, the price function for each token is defined as:

$$P_1(x) = \frac{\partial C(x)}{\partial x_1} = x_2.$$

$P_2(x)$  is defined in similar way. Hence, if  $x_1$  grow up as fast then the price of  $x_2$  as small.

**No-arbitrage conditions:** The marginal price of coin  $\alpha$  in Uniswap is defined as the price of an infinitesimally small trade. In other words, the marginal price is the exchange rate at the trading time. This price is described by the following formula:

$$\frac{d\Delta_2}{d\Delta_1} = \frac{1}{\gamma} \frac{x_2}{x_1} = \gamma^{-1} m_u,$$

where  $m_u = x_2/x_1$ . From the above formula, we see that  $d\Delta_2 = \gamma^{-1} m_u d\Delta_1$ . Thus, we can always make a nonzeoro profit if the Uniswap marginal price of  $\alpha$ ,  $\gamma^{-1} m_u$  is smaller than the market marginal price  $m_p$ . In other words, the following inequality holds:  $m_p \Delta_1 \geq \gamma^{-1} m_u \Delta_1$  after performing a small enough trade. Assume there is no arbitrage, this means we must have:

$$m_u \geq \gamma m_p.$$

Similarly, by swapping  $\alpha$  for  $\beta$  we get the following bounds on the Uniswap market price:

$$\gamma m_p \leq m_u \leq \gamma^{-1} m_p.$$

In the case of no transaction fee, this means  $\gamma = 1$ , to obtain the no arbitrage conditions, we must have  $m_p = m_u$ .

### 1.1 Bounded Loss Condition

Bounded loss is the condition that makes a market maker can control the loss in the worst-case. It is normally defined as:

$$\sup_i [\max(x) - C(x)] < \infty$$

The worse-case loss of a cost function  $C$  which begins from initial payout vector  $x^0$  is

$$\sup[\max_i(x_i) - C(x) + C(x^0)]$$

In the Uniswap model, the market maker can't be suffered loss from trading exclude the possible loss from the difference of exchange rate between two tradings at two different times. Therefore, if we want to define a bounded loss for Uniswap, we can regard the payout which pays for traders of the market maker as the number of tokens that are sent for traders after each trading.

With the above-defining payout along with the Uniswap formula, we noticed that the maximum payout that the market maker has to pay for traders in the worst case is to equal with either all reserve of  $x_1$  or  $x_2$ . Hence, using the definition of bounded loss condition, we have:

$$\sup[\max_i(x_i) - C(x)]$$

either equal to

$$x_1 - C(x) = x_1 - k < \infty, \text{ with } k \text{ is the constant}$$

or

$$x_2 - C(x) = x_2 - k < \infty, \text{ with } k \text{ is the constant}$$

and the worse-case loss is to equal to either  $x_1$  or  $x_2$ .