n=4

Encoding

$$v = \begin{pmatrix} 10 + 10i \\ 10i \end{pmatrix}$$

① $z := \pi^{-1}(v)$

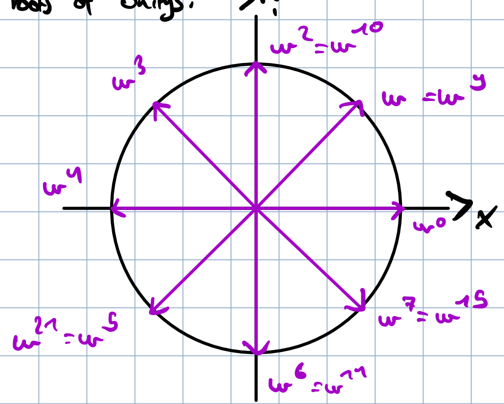
$$z = \begin{pmatrix} 10 + 10i \\ 10i \\ -10i \\ 10 - 10i \end{pmatrix}$$

Statt ξ_8^1 schreiben wir ω

② $$p_i = \frac{\langle z, V_n[i] \rangle}{\langle V_n[i], V_n[i] \rangle}$$

$$V_4 = V\begin{pmatrix} \omega \\ \omega^3 \\ \omega^5 \\ \omega^7 \end{pmatrix} = \begin{bmatrix} 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^3 & \omega^6 & \omega^9 \\ 1 & \omega^5 & \omega^{10} & \omega^{15} \\ 1 & \omega^7 & \omega^{14} & \omega^{21} \end{bmatrix}$$

8th roots of unitys:



$$p_1 = \frac{\langle (10+10i \;\; 10i \;\; -10i \;\; 10-10i)^\perp, (1 \;\; 1 \;\; 1 \;\; 1)^\perp \rangle}{\langle (1 \;\; 1 \;\; 1 \;\; 1)^\perp, (1 \;\; 1 \;\; 1 \;\; 1)^\perp \rangle}$$

$$= \frac{10 + 10i + 10i - 10i + 10 - 10i}{4} = \frac{20}{4} = 5$$

$$p_2 = \frac{\langle (10+10i \;\; 10i \;\; -10i \;\; 10-10i)^\perp, (\omega \;\; \omega^3 \;\; \omega^5 \;\; \omega^7)^\perp \rangle}{\langle (\omega \;\; \omega^3 \;\; \omega^5 \;\; \omega^7)^\perp, (\omega \;\; \omega^3 \;\; \omega^5 \;\; \omega^7)^\perp \rangle}$$

$$= \frac{10\overline{\omega} + 10i\overline{\omega} + 10i\overline{\omega^3} - 10i\,\overline{\omega^5} + 10\overline{\omega^7} - 10i\overline{\omega^7}}{\underbrace{\omega\overline{\omega}}_{=1,\text{ da } \omega\overline{\omega} = \|\omega\| = 1} + \omega^3\overline{\omega^3} + \omega^5\overline{\omega^5} + \omega^7\overline{\omega^7}}$$

$$= \frac{10 \cdot (\overline{\omega} + \overline{\omega^7}) + i \cdot (\overline{\omega} + \overline{\omega^3} - \overline{\omega^5} - \overline{\omega^7})}{4}$$

$$= \frac{10\left(\sqrt{2} + i\cdot(-\sqrt{2}\,i - \sqrt{2}\,i)\right)}{4}$$

$$= \frac{10\left(\sqrt{2} + 2\sqrt{2}\right)}{4} = \frac{30\sqrt{2}}{4}$$

$$p_3 = \frac{\langle (10+10i \quad 10i \quad -10i \quad 10i-10i)^T, (\omega^2 \quad \omega^6 \quad \omega^{10} \quad \omega^{14})^T \rangle}{4}$$

$$= \frac{10\left(\overline{\omega^2} + \overline{\omega^{14}} + i(\overline{\omega^2} + \overline{\omega^6} - \overline{\omega^{10}} - \overline{\omega^{14}})\right)}{4}$$

$$= \frac{10\left(0 + i(0+0)\right)}{4} = 0$$

$$p_4 = \frac{\langle (10+10i \quad 10i \quad -10i \quad 10i-10i)^T, (\omega^3 \quad \omega^9 \quad \omega^{15} \quad \omega^{21})^T \rangle}{4}$$

$$= \frac{10\left(\overline{\omega_3} + \overline{\omega^{21}} + i(\overline{\omega^3} + \overline{\omega^9} - \overline{\omega^{15}} - \overline{\omega^{21}})\right)}{4}$$

$$= \frac{10\left(-\sqrt{2} + i\cdot(-\sqrt{2}\,i - \sqrt{2}\,i)\right)}{4}$$

$$= \frac{10\left(-\sqrt{2} + 2\sqrt{2}\right)}{4} = \frac{10\sqrt{2}}{4}$$

③ $m_i = \text{round}(p_i)$

$m_1 = 5$

$$m_2 = \begin{cases} 10, & \text{with prob. } 1 - \left|\frac{30\sqrt{2}}{4} - 10\right| \approx 0{,}39 \\ 11, & \text{with prob. } 1 - \left|\frac{30\sqrt{2}}{4} - 11\right| \approx 0{,}61 \end{cases}$$

$m_3 = 0$

$$m_4 = \begin{cases} 3, & \text{with prob. } 1 - \left|\frac{10\sqrt{2}}{4} - 3\right| \approx 0{,}46 \\ 4, & \text{with prob. } 1 - \left|\frac{10\sqrt{2}}{4} - 4\right| \approx 0{,}54 \end{cases}$$

④ $m = \begin{pmatrix} 5 \\ 11 \\ 0 \\ 4 \end{pmatrix}$ „$\hat{=}$" $5 + 11x + 0x^2 + 4x^3$