Introduction to Homomorphic Cryptosystems
Exercise Sheet 2: Modular Arithmetic

Task 1 – Modular arithmetic

The lecture introduced the concept of congruence. For the following exercises you may need to research the basic properties of the congruence relationship.

a) Given that $4x \equiv 1 \mod 9$, find x.

b) Given that $12x \equiv 8 \mod 20$, find one solution for x.

c) Consider the same equation as in b)
Are there more solutions for x? If yes: Find all possible solutions.

d) Find the last digit of 7^{100}.

e) In year N, the 300th day of the year is a Tuesday. In year $N + 1$, the 200th day is also a Tuesday. On what day of the week did the 100th day of the year $N - 1$ occur?

f) If $n!$ denotes the product of integers 1 through n, what is the remainder when $(1! + 2! + 3! + 4! + 5! + 6! + ...)$ is divided by 9?