Geographic coordinates

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1 Introduction

The objective of this report is to compute real distance between two points on the globe. This question is important because people are used to use map, which makes them forget the spherical shape of the planet.

2 Conversion of degrees to distance

Because of the spherical shape of the planet, the length of a degree is different, according to the position of the observer on the globe. This means that if a traveler wants to move between two points situated at a distance of 1 degree from each other, for example 36° and 37°, he will have to walk a different distance, if he is near the Equator or on the North/South Pole [2].
at latitude phi, 1 degree latitude is equal in meters to :

$$Length_{lattitude}(\phi) = 111132.92 - 559.82 \cdot cos(2 \cdot \phi) + 1.175 * cos(4 \cdot \phi) - 0.0023 \cdot cos(6 \cdot \phi) = ...[m/degree] \quad (1)$$

1 degree longitude at latitude phi

$$Length_{longitude}(\phi) = 111412.84 - 93.5 \cdot cos(3 \cdot \phi) + 0.118 \cdot cos(5 \cdot \phi) = ...[m/degree] \quad (2)$$

2. https://en.wikipedia.org/wiki/Geographic_coordinate_system

3 Schéma cinématique

3.1 Vecteurs positions

origine : centre de rotation verticale se trouvant sous les pâles principales.

position des pâles principales (pp) : vecteur verticale

position de l'hélice arrière : vecteur allant de l'origine vers l'hélice (h) arrière.

4 Angular momentum

4.1 Formula

$$\vec{L} = \vec{OA} \otimes \vec{P} = \vec{r} \otimes \vec{P} = \vec{r} \otimes m \cdot \vec{v} = \vec{I} \otimes \vec{\omega} \tag{3}$$

$\vec{L}$: Angular Momentum $[kg \cdot \frac{m^2}{s}]$
$\vec{OA}$ and r : position of the mass $[m]$ according to a referance
$\vec{P}$: linear momentum $[kg \cdot \frac{m}{s}]$ [3]
$\vec{v}$: velocity $[\frac{m}{s}]$ I : moment of inertia $[m^2 \cdot kg \cdot]$
ω : angular speed $[\frac{rad}{s}]$
Torque :

$$M = \frac{d\vec{L}}{dt} = \frac{d(\vec{I} \otimes \vec{\omega})}{dt} \tag{4}$$

if we consider a particule of mass m, $\vec{r}$ is the position of the center of mass. If it is a solid object, L is first computed according to the axis of rotation of the object :

$$\vec{L}_{ar} = \vec{I}_{ar} \otimes \vec{\omega}_{ar} \tag{5}$$

To compute the angular moment according to an other axis of rotation (new referance), we use the Huygens-Steiner theorem (or the Parallel axis theorem) :

$$\vec{L}_0 = \vec{I}_0 \otimes \vec{\omega}_{cm} \tag{6}$$

$$\vec{I}_0 = \vec{I}_{ar} + m \cdot d^2 \tag{7}$$

with d the distance between the axis of rotation of the object and the new referance.

4.2 Condition of stability

Main rotor(s) :

$$\vec{L}_{mr} = \vec{r}_{mr} \otimes m_{mr} \cdot \vec{v_{mr}} = \vec{I_{mr}} \otimes \vec{\omega_{mr}} \tag{8}$$

Rear rotor :

$$\vec{L}_{rr} = \vec{r}_{rr} \otimes m_{rr} \cdot \vec{v_{rr}} = \vec{I_r r} \otimes \vec{\omega_{rr}} \tag{9}$$

assurer la stabilité lors du vol : les moments cinétiques doivent s'annuler. (poser la formule et résoudre)

$$\vec{L_{mr}} = \vec{L_{rr}} \tag{10}$$

or
The generated torque is compensated :

$$\sum \vec{M_{mr}} = \sum \vec{M_{rr}} \tag{11}$$

find a relation between ω_{mr} and ω_{rr} -> determine the transmission ratio

3. $\vec{L}$ is perpendicular to both $\vec{P}$ and $\vec{r}$

4.3 Pivots à droite et à gauche

pour tourner à gauche ou doite, on ne doit plus satisfaire la condition de stabilité. le pilote utiliser le pédalier pour accélérer/ralentir l'hélice arrière. ainsi les moments cinétiques ne sont plus égaux.

calculer l'effet de rotation sur l'hélicoptère si l'hélice est accélérée/ralentie de 10,20,30,.. %. mettre un tableau. calculer la vitesse de rotation dans ces cas-là.

$$\begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}_{R_1} \quad \vec{AB}_{R_2} = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}_{R_2} \quad \vec{BC}_{R_3} = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}_{R_3} \quad \vec{CD}_{R_4} = \begin{bmatrix} 0 \\ 0 \\ -l_4 \end{bmatrix}_{R_4} \tag{12}$$

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$$\begin{aligned} \vec{OE}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R \\ + \vec{CC}_{2R} + \vec{C_2D}_R + \vec{DD}_{1R} + \vec{D_1E}_R \end{aligned} \tag{13}$$

$$\begin{aligned} \vec{OF}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R \\ + \vec{CC}_{2R} + \vec{C_2D}_R + \vec{DD}_{1R} + \vec{D_1E}_R + \vec{EF}_R \end{aligned} \tag{14}$$

$$\begin{aligned} \vec{OG}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R + \vec{CC}_{2R} \\ + \vec{C_2D}_R + \vec{DD}_{1R} + \vec{D_1E}_R + \vec{EF}_R + \vec{FF}_{3R} + \vec{F_3G}_R \end{aligned} \tag{15}$$

$$\begin{aligned} \vec{OH}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R + \vec{CC}_{2R} + \vec{C_2D}_R \\ + \vec{DD}_{1R} + \vec{D_1E}_R + \vec{EF}_R + \vec{FF}_{3R} + \vec{F_3G}_R + \vec{GH}_R \end{aligned} \tag{16}$$

5 Conclusion