Engine output power

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Contents

1 Introduction

The main objective of this report is compute the thermodynamics cycle of a combustion engine and the corresponding output power generated. Secondary objective : torque profile $T(RPM)$, efficiency η, fuel consumption profile according to the engine RPM.

2 Engine power

The total engine power of the sum of the power produced from each piston. As each piston has the combustion stroke at different time (crankshaft angle), the piston power must be computed according to crankshaft angle and firing order of the engine. Step 1 : compute power produced by 1 piston Step 2 : shift the power profile for the piston according to its number and firing order. Example : I4 ->1-3-2-4 : $alpha_1 = alpha_crankshaft$
$alpha_2 = alpha_crankshaft + 360$
$alpha_3 = alpha_crankshaft + 180$
$alpha_4 = alpha_crankshaft + 540$

2.0.1 Combustion chamber pressure

Gasoline engine : .... cycle
Diesel engine : Diesel cycle
compute pressure and power with thermodynamic cycle Study the combustion to estimate its efficiency:

$$\eta_{combustion} = \frac{q_{out}}{q_{in}} \tag{1}$$

q : energy $[J/kg]$
The results is per $[\mathrm{kg}] \rightarrow \mathrm{J/kg}, W/\mathrm{kg}, \ldots$

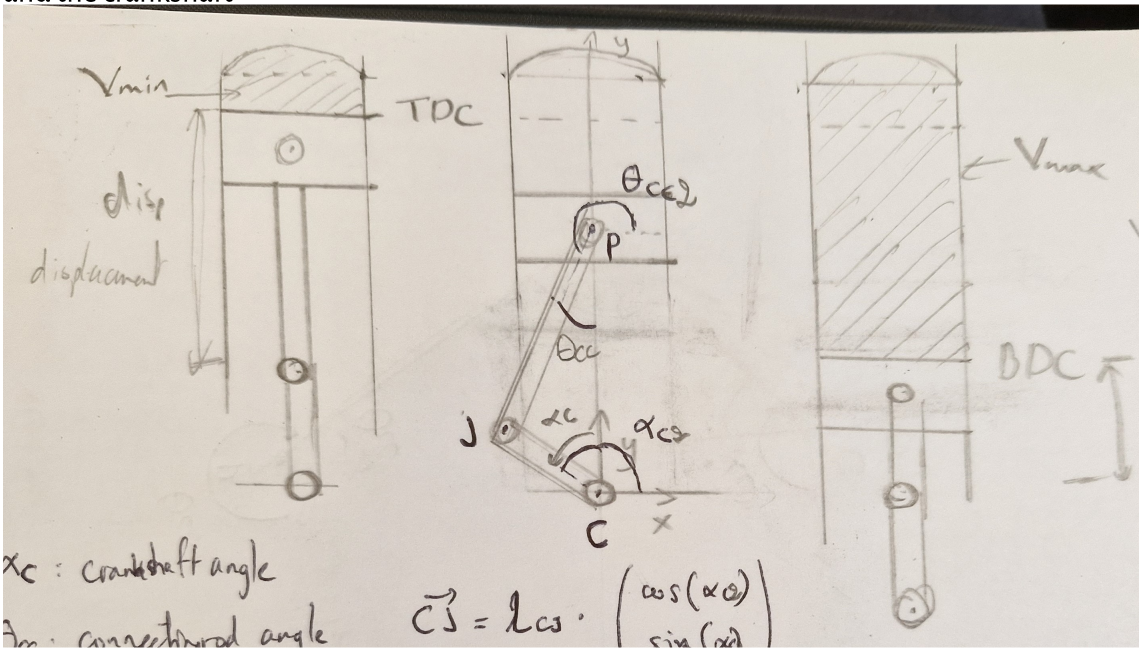
2.1 Engine dimensions

choose the size of the combustion chamber, the piston head diameter, the connecting rod length,
choose the mass flow $\dot{m}$, the displacement, and size of combustion chamber. -> we can compute power in W and pressure in Pa
$V_{\min}$ is the volume when the piston is at TDC
$V_{\max}$ is the volume when the piston is at BDC

$$V_{max} = V_min + disp \cdot S_{piston} = V_{min} + disp \cdot \pi(\frac{D}{2})^2 \tag{1}$$

2.2 Engine movement

The following figure shows the moving part in action : piston head, connecting rod and the crankshaft



main Referential, centered on the crankshaft :

- x to the right
- y to the top
- z perpendicular to the plan formed by x and y .
- The piston head is moving in the y direction.

we define angles by taking X-axis as reference
Let's define the angle and position vector of each component

1. Crankshaft

$$\overline{CJ} = L_{CJ} \tag{2}$$

$$\vec{CJ} = L_{CJ} \cdot \begin{bmatrix} cos(\alpha_c) \\ sin(\alpha_c) \\ 0 \end{bmatrix}_{R_1} \tag{3}$$

$$\alpha_c \in [0, 2\pi] \tag{4}$$

2. Connecting rod :

$$\overline{PJ} = L_{PJ} \tag{5}$$

$$\vec{PJ} = L_{PJ} \cdot \begin{bmatrix} cos(\theta_{cc}) \\ sin(\theta_{cc}) \\ 0 \end{bmatrix}_{R_1} \tag{6}$$

$$\theta_{cc} \in [\theta_{cc}^{min}, \theta_{cc}^{max}] \tag{7}$$

3. Piston head:

$$\vec{CP} = L_{CP} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{R_1} \tag{8}$$

$$L_{CP} \in [BDC, TDC] = [BDC, BDC + disp] \tag{9}$$

2.3 Newton-Raphson

The Newton-Raphson method is used to find the angle of the connecting rod. The following system of two equations must be solved :

$$\vec{CP} + \vec{PJ} + \vec{JC} = 0 \tag{10}$$

$$\vec{CP} + \vec{PJ} - \vec{CJ} = 0 \tag{11}$$

The two unknown variables are :

- angle of the connecting rod : θ_{cc}
- position of the piston head : L_{CP}

The driving variable is the crankshaft angle α_c. For each value of this angle (from 0 to 360), the iteration of the Newton-Raphson method are executed until the system is solved. The result will be the value of the unknown variables as function of the crankshaft angle.
with this we can define the range $[\theta_{cc}^{min}, \theta_{cc}^{max}]$

2.4 Thrust force

During the combustion stroke, the burned air-fuel mixture generated a pressure, which pushed the piston head in the negative direction of the Y-axis.

$$F_{thrust} = p \cdot S_p = p \cdot \pi \cdot \left(\frac{D}{2}\right)^2 \tag{2}$$

$$\vec{F}_{\text{piston}} = \begin{bmatrix} 0 \\ -F_{thrust} \\ 0 \end{bmatrix}_{R_1} \tag{3}$$

F_{thrust} : thrust force $[N]$
p : pressure inside the combustion chamber $[Pa]$
D : piston diameter, bore $[m]$
S_p : piston head section $[m^2]$

express the pressure as function of the crankshaft angle : $p(\alpha_c)$ is the profile of the pressure. As the combustion stroke is happening at different time, we need to establish this profile for each piston.
Thrust force applied on the crankshaft :
the thrust force in the connecting rod referential is computed by rotating the previous vector with a rotation matrix . The angle is the connecting rod angle θ_{cc}, found using the results of the previous section,

$$\vec{F}_{thrust}^{R_{cc}} = F_{thrust} \cdot \begin{bmatrix} \cos(\theta_{cc}) \\ \sin(\theta_{cc}) \\ 0 \end{bmatrix}_{R_{cc}} \tag{5}$$

θ_{cc} : angle between connecting rod and the y axis

Here, we can also add the energy consumed by the running gears : water pump, oil pump, alternator,

2.5 Produced torque

Torque is the vector product of this force with the position of the crankin journal of the crankshaft, from the center of the crankshaft. It should be oriented in the Z-direction.

$$\vec{T}_{\text{piston}} = \vec{F}_{\text{thrust}}^{\text{total}} \times \text{ position }_{cc} = \vec{F}_{\text{thrust}}^{\text{total}} \times \begin{bmatrix} \cos(\alpha_{cc}) \\ \sin(\alpha_{cc}) \\ 0 \end{bmatrix}_{R_1} \tag{7}$$

α_{cc} : connecting rod angle according to R_1

2.6 Output power from one piston

Power output is the scalar product of the torque with crankshaft angular speed

$$\text{Power} = \vec{T}_{\text{piston}} \cdot \vec{\omega}_{engine} \tag{8}$$

The result is the power profile depending on the crankshaft angle.
Engine mechanical efficiency: quotient of output power by the input heat power (fuel power)

$$\eta_m = \frac{\text{Power}}{\dot{Q}_{\text{in}}} \tag{9}$$

2.7 Total power

as each piston produced power at different time, the total power is the sum of each power produced by the pistons.

1. link cycle phases with 4 strokes with crankshaft angle then follow steps defined in Notion

2.8 Friction and consumption

The friction between The moving parts lead to power loss. There are several ways to describe these loses

- percentage of the total energy/power
- a friction coefficient to an existing force
- efficiency η lower than 100$\%$

Here a non-exhaustive list of possible energy loses in ICE:

- a part of energy is used for the compression stroke for another piston.
- mechanical loses : friction between moving part, between a part and a fluid (oil, water, ...)
- energy loses during combustion of the air- fuel mixture

- Piston friction : cylinder, piston pin, friction between piston head and cylinder:

$$F_{\text{thrust}_2} = F_{thrust} - F_{fpcy} \tag{4}$$

- Crankshaft friction : oil, bearing, connecting rod.

  power/energy lost because of between the crankshaft and oil

$$\vec{F}^{total}_{thrust} = \vec{F}^{R_{cc}}_{thrust} + \vec{F}_{\text{oil friction}} \tag{6}$$

- Water pump, oil pump, ... clutch, gearbox loses, transmission loses until reaching the wheels

these will reduce the overall efficiency of the engine.

$$\overrightarrow{BC}_{R_1} = \begin{bmatrix} \overline{BC}_{x_1} \\ \overline{BC}_{y_1} \\ 0 \end{bmatrix}_{R_1} \tag{10}$$

2. link cycle phases with 4 strokes with crankshaft angle.

$$\vec{BC}_{R_1} = \begin{bmatrix} \overline{BC}_{x_1} \\ \overline{BC}_{y_1} \\ 0 \end{bmatrix}_{R_1} \tag{12}$$

2.9 Engine thrust

Theorem of the preservation of linear momentum:

$$\int\int_{\Sigma} [P \cdot \vec{n} + \rho \cdot \vec{V}(\vec{V} \cdot \vec{n})] = 0 \tag{13}$$

The term $\vec{V} \cdot \vec{n}$ is equal to zero when the velocity is perpendicular to the normal vector, which is the case on the lateral surface. What remain are the terms corresponding to the surfaces A_0 and A_{10}, which are perpendicular to the axis of the jet engine. The thrust :

$$\begin{aligned} F &= (P_{10} + \rho_{10} \cdot V_{10}^2)A_{10} - (P_0 + \rho_0 \cdot V_0^2)A_0 \\ &= P_{10} \cdot A_{10} + \rho_{10} \cdot V_{10}^2 \cdot A_{10} - P_0 \cdot A_0 - \rho_0 \cdot V_0^2 \cdot A_0 \end{aligned} \tag{14}$$

mass flow :

$$\dot{m} = D = \rho \cdot V \cdot A \tag{15}$$

$$\begin{aligned} F &= P_{10} \cdot A_{10} + D_{10} \cdot V_{10} - P_0 \cdot A_0 - D_0 \cdot V_0 \\ &= D_{10} \cdot V_{10} - D_0 \cdot V_0 + P_{10} \cdot A_{10} - P_0 \cdot A_0 \end{aligned} \tag{16}$$

By using the simple trick of adding $P_0 \cdot A_{10} - P_0 \cdot A_{10} = 0$

$$\begin{aligned} F &= D_{10} \cdot V_{10} - D_0 \cdot V_0 + P_{10} \cdot A_{10} - P_0 \cdot A_0 + P_0 \cdot A_{10} - P_0 \cdot A_{10} \\ &= D_{10} \cdot V_{10} - D_0 \cdot V_0 + (P_{10} - P_0)A_{10} + P_0(A_{10} - A_0) \end{aligned} \tag{17}$$

It is useful to add the drag due to the engine housing:

$$X_c = -\int\int P_c \cdot dA\vec{n}\vec{x} - > X_c = P_c(A_{10} - A_0) \tag{18}$$

Where the P_c is the pressure at the surface of the housing. Historically, the first jet fighter, the Me-262, was equipped with two Junkers Jumo 004 B turbine engines with housings. So, these housings will produce each a drag force. Since then, engines were housed inside the body of the jet fighter. In this case, this term is equal to 0.
Consequently, the thrust generated by an engine is :

$$F = D_{10} \cdot V_{10} - D_0 \cdot V_0 + (P_{10} - P_0)A_{10} + P_0(A_{10} - A_0) - P_c(A_{10} - A_0) \tag{19}$$

The pure engine thrust is :

$$F_{engine} = D_{10} \cdot V_{10} - D_0 \cdot V_0 + (P_{10} - P_0)A_{10} \tag{20}$$

Thrust due to the external fluid:

$$F_{fluid} = P_0(A_{10} - A_0) \tag{21}$$

If the exhaust is adapted, then $P_{10} = P_0$, meaning that all the energy compressed in the engine will be converted to kinetic energy at the exhaust, in order to maximize the thrust. This is done thought modifiable nozzle to accommodate all working conditions. Depending on the measurements at the turbine (pressure, temperature, gas speed), the engine will modify the nozzle (section A_{10} and shape) accordingly.
To make the plane harder to detect from heat-seeking missiles, the nozzle must be designed in order to reduce the Temperature of the gases at the exit as much as possible. The plane manufacturer chooses the thrust needed to ensure that the plane has the specified performances, taking into account the drag due to the body shape of the plane. He also indicated the maximum volume that the engine can occupy in the plane. The engine manufacturer will design the engine to ensure to reach the specified thrust and air speed at the exit of the nozzle. The engine must fit into the volume specified by the plane manufacturer.
Naturally, other specifications must be fulfilled, like :

- low fuel consumption
-

3 Reminder

3.1 Gases

$$P \cdot v = r \cdot T \tag{22}$$

$$P \cdot V = m \cdot r \cdot T \tag{23}$$

$$P \cdot V = n \cdot R \cdot T \tag{24}$$

$$r = \frac{R}{M} = \frac{r}{m} \tag{25}$$

$$W = \int PdV \tag{26}$$

V : volume $[m^3]$
v : volume density $[\frac{m^3}{kg}]$

$$P \cdot V^\gamma = const \tag{27}$$

$$P_{in} \cdot (V_{in})^\gamma = P_{out} \cdot (V_{out})^\gamma \tag{28}$$

$$W = \int PdV \tag{29}$$

4 Engine computation

1. Combustion chamber
2. Turbine
3. Compressor
4. Inlet
5. Nozzle

$$\sum T_B = I_{BCE} \cdot \alpha \tag{30}$$

$\sum T_B$: total torques applied on point B in [Nm]
I_{BCE} : inertia of the bucket in $[kgm^2]$
α: angular acceleration in $[\frac{rad}{s^2}]$
The sum of torques in point B can be expressed as the vector/cross product of force verctor and position vertor.

$$\sum \vec{T_B} = \sum \vec{F} \times \vec{d} \tag{31}$$

The result is a vector that is perpendicular to both vectors : $\vec{T_B} \perp \sum \vec{F}$ and $\vec{T_B} \perp \vec{d}$.
Let's assume that F_3 is the sum of the torques applied on the triangle BCE. In this case, the application point of F_3 is C. The scalar value of $\vec{T_B}$ is the segment $\overline{BC}$ multiplied by the tangential force. This one is the projection of F_3 on x_2 axis, $F_3^{x_2}$:

$$T_B = ||\vec{T_B}|| = F_3^{x_2} \cdot \overline{BC} \tag{32}$$

The projection of F_3 on x_2 axis, $F_3^{y_2}$ is $\parallel$ to the segment BC. In this case, the vector/cross product is equal to 0. Or :

$$T_B = ||\vec{T_B}|| = F_3 \cdot \overline{BF} \tag{33}$$

$\overline{BF}$ is the shortest distance between the force F_3 and point B. $\overline{BF}$: projection of segment $\overline{BC}$ on axis y_1.

$$\vec{BC}_{R_1} = \begin{bmatrix} \overline{BC}_{x_1} \\ \overline{BC}_{y_1} \\ 0 \end{bmatrix}_{R_1} \tag{34}$$

or : F_3 multiplied by AD, the distance between F_3 and B. (add schema with this 2 examples)
Generally speaking, sum of Torque in B is 1. the sum of tangantial forces multiplied by 2. easier : vector/cross product of a vector and distance vector to the B point.
The Reynolds number is an non-dimensional number, and used to define if the fluid flow is laminar or turbulent.

$$Re = \frac{\rho \cdot u \cdot L}{\mu} \tag{35}$$

ρ : density of the fluid
u : flow speed
L : characteristic linear dimension
μ : dynamic viscosity of the fluid

The characteristic linear dimension L depends on the shape of the object of study. Here some example :

- Plane wing : length of the wing
- Hydraulic pipe : diameter of the pipe
- Complex shape : the biggest dimension

Depending of the result, the flow can have 3 regimes

- If $Re <$ 2040, the flow is still considered laminar
- If $Re >$ 2100, the flow is turbulent
- If 1800 < $Re <$ 2100, the flow is in the transition/intermediary range, which is a mix of laminar and turbulent [1]

For each regime, the drag force is different. For turbulent and laminar regimes, the formula is as follows:

$$F_d^{turbulent} = \frac{1}{2} \cdot \rho \cdot C_D \cdot S \cdot u^2 \tag{36}$$

$$F_d^{laminar} = C_F \cdot \rho \cdot D^2 \cdot u^2 \tag{37}$$

S: cross frontal section
C_D : turbulent drag coefficient
C_F : laminar drag coefficient

The intermediary regime is a mix from the 2 base regimes. In this situation, an approximation can be evaluated by computed the percent Re compared to 1800 and 2100.

$$F_d^{transition} = (1 - \frac{Re - 1800}{2100 - 1800}) \cdot F_d^{laminar} + \frac{Re - 1800}{2100 - 1800} \cdot F_d^{turbulent} \tag{38}$$

[1] https://en.wikipedia.org/wiki/Laminar_flow

5 Techniques

Prefabricated parts (PFP) /pre-manufactured parts (PMP)
Instead of creating the same parts again and again, design the components and make them easily modifiable. for example, combine 2 parts to create a third one, which is useful in an other application. Then add them to the system in the assembly Example : threads : try the smallest possible size, for example M3 to M20.
Test them with small parts and check the tolerances. The next step is to prepare small parts ready to be added to system parts. The final step is the merge the bodies.
table

Table 1: Holes and threaded holes

Screw	Threaded hole diameter	Bore Hole (clearance)
M3	2.8	3.2
4	35	144
5	45	300

Clearance To fix two part together, a clearance is needed. When two parts must be assembled together, a clearance of 0.1mm is enough. Then use the glue to fix the assembly.

Table 2: Nonlinear Model Results

Case	Method#1	Method#2	Method#3
1	50	837	970
2	47	877	230
3	31	25	415
4	35	144	2356
5	45	300	556

$$Length_{lattitude}(\phi) = 111132.92 - 559.82 \cdot cos(2 \cdot \phi) + 1.175 * cos(4 \cdot \phi) - 0.0023 \cdot cos(6 \cdot \phi) = ...[m/degree] \tag{39}$$

1 degree longitude at latitude phi

$$Length_{longitude}(\phi) = 111412.84 - 93.5 \cdot cos(3 \cdot \phi) + 0.118 \cdot cos(5 \cdot \phi) = ...[m/degree] \tag{40}$$

6 Schéma cinématique

6.1 Vecteurs positions

origine : centre de rotation verticale se trouvant sous les pâles principales.

position des pâles principales (pp) : vecteur verticale

position de l'hélice arrière : vecteur allant de l'origine vers l'hélice (h) arrière.

7 Angular momentum

7.1 Formula

$$\vec{L} = \vec{OA} \otimes \vec{P} = \vec{r} \otimes \vec{P} = \vec{r} \otimes m \cdot \vec{v} = \vec{I} \otimes \vec{\omega} \tag{41}$$

$\vec{L}$: Angular Momentum $[kg \cdot \frac{m^2}{s}]$
$\vec{OA}$ and r: position of the mass $[m]$ according to a referance
$\vec{P}$: linear momentum $[kg \cdot \frac{m}{s}]$[2]
$\vec{v}$: velocity $[\frac{m}{s}]$ I : moment of inertia $[m^2 \cdot kg \cdot]$
ω : angular speed $[\frac{rad}{s}]$
Torque :

$$M = \frac{d\vec{L}}{dt} = \frac{d(\vec{I} \otimes \vec{\omega})}{dt} \tag{42}$$

if we consider a particule of mass m, $\vec{r}$ is the position of the center of mass. If it is a solid object, L is first computed according to the axis of rotation of the object :

$$\vec{L}_{ar} = \vec{I}_{ar} \otimes \vec{\omega}_{ar} \tag{43}$$

To compute the angular moment according to an other axis of rotation (new referance), we use the Huygens-Steiner theorem (or the Parallel axis theorem) :

$$\vec{L}_0 = \vec{I}_0 \otimes \vec{\omega}_{cm} \tag{44}$$

$$\vec{I}_0 = \vec{I}_{ar} + m \cdot d^2 \tag{45}$$

with d the distance between the axis of rotation of the object and the new referance.

7.2 Condition of stability

Main rotor(s):

$$\vec{L}_{mr} = \vec{r}_{mr} \otimes m_{mr} \cdot \vec{v_{mr}} = \vec{I_{mr}} \otimes \vec{\omega_{mr}} \tag{46}$$

Rear rotor :

$$\vec{L}_{rr} = \vec{r}_{rr} \otimes m_{rr} \cdot \vec{v_{rr}} = \vec{I_r}r \otimes \vec{\omega_{rr}} \tag{47}$$

assurer la stabilité lors du vol: les moments cinétiques doivent s'annuler. (poser la formule et résoudre)

$$\vec{L_{mr}} = \vec{L_{rr}} \tag{48}$$

or
The generated torque is compensated :

$$\sum \vec{M_{mr}} = \sum \vec{M_{rr}} \tag{49}$$

find a relation between ω_{mr} and ω_{rr} -> determine the transmission ratio

[2] $\vec{L}$ is perpendicular to both $\vec{P}$ and $\vec{r}$

7.3 Pivots à droite et à gauche

pour tourner à gauche ou doite, on ne doit plus satisfaire la condition de stabilité. le pilote utiliser le pédalier pour accélérer/ralentir l'hélice arrière. ainsi les moments cinétiques ne sont plus égaux.

calculer l'effet de rotation sur l'hélicoptère si l'hélice est accélérée/ralentie de 10,20,30,.. %. mettre un tableau. calculer la vitesse de rotation dans ces cas-là.

$$\begin{bmatrix}0\\0\\l_1\end{bmatrix}_{R_1} \quad \vec{AB}_{R_2} = \begin{bmatrix}0\\l_2\\0\end{bmatrix}_{R_2} \quad \vec{BC}_{R_3} = \begin{bmatrix}l_3\\0\\0\end{bmatrix}_{R_3} \quad \vec{CD}_{R_4} = \begin{bmatrix}0\\0\\-l_4\end{bmatrix}_{R_4} \tag{50}$$

.

.

.

$$\begin{aligned}\vec{OE}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R \\ + \vec{CC}_{2R} + \vec{C_2D}_R + \vec{DD}_{1R} + \vec{D_1E}_R\end{aligned} \tag{51}$$

$$\begin{aligned}\vec{OF}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R \\ + \vec{CC}_{2R} + \vec{C_2D}_R + \vec{DD}_{1R} + \vec{D_1E}_R + \vec{EF}_R\end{aligned} \tag{52}$$

$$\begin{aligned}\vec{OG}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R + \vec{CC}_{2R} \\ + \vec{C_2D}_R + \vec{DD}_{1R} + \vec{D_1E}_R + \vec{EF}_R + \vec{FF}_{3R} + \vec{F_3G}_R\end{aligned} \tag{53}$$

$$\begin{aligned}\vec{OH}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB}_{1R} + \vec{B_1C}_{1R} + \vec{C_1C}_R + \vec{CC}_{2R} + \vec{C_2D}_R \\ + \vec{DD}_{1R} + \vec{D_1E}_R + \vec{EF}_R + \vec{FF}_{3R} + \vec{F_3G}_R + \vec{GH}_R\end{aligned} \tag{54}$$

8 Conclusion