Jet engine

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Contents

1 Introduction

The objective of this report is compute the thermodynamics cycle of a jet engine and the corresponding thrust generated.

2 Reminder

2.1 Gases

$$P \cdot v = r \cdot T \tag{1}$$
$$P \cdot V = m \cdot r \cdot T \tag{2}$$
$$P \cdot V = n \cdot R \cdot T \tag{3}$$
$$r = \frac{R}{M} = \frac{r}{m} \tag{4}$$
$$W = \int PdV \tag{5}$$

V : volume [m^3]
v : massic volume [$\frac{m^3}{kg}$]

2.2 Adiabatic process

$$P \cdot V^{\gamma} = const \tag{6}$$
$$P_{in} \cdot (V_{in})^{\gamma} = P_{out} \cdot (V_{out})^{\gamma} \tag{7}$$
$$W = \int PdV \tag{8}$$

2.3 Engine thrust

Theorem of the preservation of linear momentum:

$$\int \int_{\Sigma} [P \cdot \vec{n} + \rho \cdot \vec{V}(\vec{V} \cdot \vec{n})] = 0 \tag{9}$$

The term $\vec{V} \cdot \vec{n}$ is equal to zero when the velocity is perpendicular to the normal vector, which is the case on the lateral surface. What remain are the terms corresponding to the surfaces A_0 and A_{10}, which are perpendicular to the axis of the jet engine. The thrust :

$$\begin{aligned} F &= (P_{10} + \rho_{10} \cdot V_{10}^2)A_{10} - (P_0 + \rho_0 \cdot V_0^2)A_0 \\ &= P_{10} \cdot A_{10} + \rho_{10} \cdot V_{10}^2 \cdot A_{10} - P_0 \cdot A_0 - \rho_0 \cdot V_0^2 \cdot A_0 \end{aligned} \tag{10}$$

mass flow :

$$\dot{m} = D = \rho \cdot V \cdot A \tag{11}$$

$$\begin{aligned} F &= P_{10} \cdot A_{10} + D_{10} \cdot V_{10} - P_0 \cdot A_0 - D_0 \cdot V_0 \\ &= D_{10} \cdot V_{10} - D_0 \cdot V_0 + P_{10} \cdot A_{10} - P_0 \cdot A_0 \end{aligned} \tag{12}$$

By using the simple trick of adding $P_0 \cdot A_{10} - P_0 \cdot A_{10} = 0$

$$\begin{aligned} F &= D_{10} \cdot V_{10} - D_0 \cdot V_0 + P_{10} \cdot A_{10} - P_0 \cdot A_0 + P_0 \cdot A_{10} - P_0 \cdot A_{10} \\ &= D_{10} \cdot V_{10} - D_0 \cdot V_0 + (P_{10} - P_0) A_{10} + P_0 (A_{10} - A_0) \end{aligned} \tag{13}$$

It is useful to add the drag due to the engine housing:

$$X_c = -\int\int P_c \cdot dA\vec{n}\vec{x} - > X_c = P_c(A_{10} - A_0) \tag{14}$$

Where the P_c is the pressure at the surface of the housing. Historically, the first jet fighter, the Me-262, was equipped with two Junkers Jumo 004 B turbine engines with housings. So, these housings will produce each a drag force. Since then, engines were housed inside the body of the jet fighter. In this case, this term is equal to 0.
Consequently, the thrust generated by an engine is :

$$F = D_{10} \cdot V_{10} - D_0 \cdot V_0 + (P_{10} - P_0) A_{10} + P_0 (A_{10} - A_0) - P_c (A_{10} - A_0) \tag{15}$$

The pure engine thrust is :

$$F_{engine} = D_{10} \cdot V_{10} - D_0 \cdot V_0 + (P_{10} - P_0) A_{10} \tag{16}$$

Thrust due to the external fluid:

$$F_{fluid} = P_0 (A_{10} - A_0) \tag{17}$$

If the exhaust is adapted, then $P_{10} = P_0$, meaning that all the energy compressed in the engine will be converted to kinetic energy at the exhaust, in order to maximize the thrust. This is done thought modifiable nozzle to accommodate all working conditions. Depending on the measurements at the turbine (pressure, temperature, gas speed), the engine will modify the nozzle (section A_{10} and shape) accordingly.
To make the plane harder to detect from heat-seeking missiles, the nozzle must be designed in order to reduce the Temperature of the gases at the exit as much as possible.
The plane manufacturer chooses the thrust needed to ensure that the plane has the specified performances, taking into account the drag due to the body shape of the plane. He also indicated the maximum volume that the engine can occupy in the plane.
The engine manufacturer will design the engine to ensure to reach the specified thrust and air speed at the exit of the nozzle. The engine must fit into the volume specified by the plane manufacturer.
Naturally, other specifications must be fulfilled, like :

- low fuel consumption
-

3 Engine computation

1. Combustion chamber
2. Turbine
3. Compressor
4. Inlet
5. Nozzle

$$\sum T_B = I_{BCE} \cdot \alpha \tag{18}$$

$\sum T_B$: total torques applied on point B in [Nm]
I_{BCE} : inertia of the bucket in $[kgm^2]$
α: angular acceleration in $[\frac{rad}{s^2}]$
The sum of torques in point B can be expressed as the vector/cross product of force verctor and position vertor.

$$\sum \vec{T_B} = \sum \vec{F} \times \vec{d} \tag{19}$$

The result is a vector that is perpendicular to both vectors : $\vec{T_B} \perp \sum \vec{F}$ and $\vec{T_B} \perp \vec{d}$. Let's assume that F_3 is the sum of the torques applied on the triangle BCE. In this case, the application point of F_3 is C. The scalar value of $\vec{T_B}$ is the segment $\overline{BC}$ multiplied by the tangential force. This one is the projection of F_3 on x_2 axis, $F_3^{x_2}$:

$$T_B = ||\vec{T_B}|| = F_3^{x_2} \cdot \overline{BC} \tag{20}$$

The projection of F_3 on x_2 axis, $F_3^{y_2}$ is $\|$ to the segment BC. In this case, the vector/cross product is equal to 0. Or :

$$T_B = ||\vec{T_B}|| = F_3 \cdot \overline{BF} \tag{21}$$

$\overline{BF}$ is the shortest distance between the force F_3 and point B. $\overline{BF}$: projection of segment $\overline{BC}$ on axis y_1.

$$\vec{BC}_{R_1} = \begin{bmatrix} \overline{BC}_{x_1} \\ \overline{BC}_{y_1} \\ 0 \end{bmatrix}_{R_1} \tag{22}$$

or : F_3 multiplied by AD, the distance between F_3 and B. (add schema with this 2 examples)
Generally speaking, sum of Torque in B is 1. the sum of tangantial forces multiplied by 2. easier : vector/cross product of a vector and distance vector to the B point.
The Reynolds number is an non-dimensional number, and used to define if the fluid flow is laminar or turbulent.

$$Re = \frac{\rho \cdot u \cdot L}{\mu} \tag{23}$$

ρ : density of the fluid
u : flow speed
L : characteristic linear dimension
μ : dynamic viscosity of the fluid

The characteristic linear dimension L depends on the shape of the object of study. Here some example :

- Plane wing : length of the wing
- Hydraulic pipe : diameter of the pipe
- Complex shape : the biggest dimension

Depending of the result, the flow can have 3 regimes

- If Re < 2040, the flow is still considered laminar
- If Re > 2100, the flow is turbulent
- If 1800 < Re < 2100, the flow is in the transition/intermediary range, which is a mix of laminar and turbulent [1]

For each regime, the drag force is different. For turbulent and laminar regimes, the formula is as follows:

$$F_d^{turbulent} = \frac{1}{2} \cdot \rho \cdot C_D \cdot S \cdot u^2 \tag{24}$$

$$F_d^{laminar} = C_F \cdot \rho \cdot D^2 \cdot u^2 \tag{25}$$

S: cross frontal section
C_D : turbulent drag coefficient
C_F : laminar drag coefficient

The intermediary regime is a mix from the 2 base regimes. In this situation, an approximation can be evaluated by computed the percent Re compared to 1800 and 2100.

$$F_d^{transition} = (1 - \frac{Re - 1800}{2100 - 1800}) \cdot F_d^{laminar} + \frac{Re - 1800}{2100 - 1800} \cdot F_d^{turbulent} \tag{26}$$

[1] `https://en.wikipedia.org/wiki/Laminar_flow`

4 Techniques

Prefabricated parts (PFP) /pre-manufactured parts (PMP)
Instead of creating the same parts again and again, design the components and make them easily modifiable. for example, combine 2 parts to create a third one, which is useful in an other application. Then add them to the system in the assembly Example : threads : try the smallest possible size, for example M3 to M20.
Test them with small parts and check the tolerances. The next step is to prepare small parts ready to be added to system parts. The final step is the merge the bodies.
table

Table 1: Holes and threaded holes

Screw	Threaded hole diameter	Bore Hole (clearance)
M3	2.8	3.2
4	35	144
5	45	300

Clearance To fix two part together, a clearance is needed. When two parts must be assembled together, a clearance of 0.1mm is enough. Then use the glue to fix the assembly.

Table 2: Nonlinear Model Results

Case	Method#1	Method#2	Method#3
1	50	837	970
2	47	877	230
3	31	25	415
4	35	144	2356
5	45	300	556

$$Length_{lattitude}(\phi) = 111132.92 - 559.82 \cdot cos(2 \cdot \phi) + 1.175 * cos(4 \cdot \phi) - 0.0023 \cdot cos(6 \cdot \phi) = ...[m/degree] \tag{27}$$

1 degree longitude at latitude phi

$$Length_{longitude}(\phi) = 111412.84 - 93.5 \cdot cos(3 \cdot \phi) + 0.118 \cdot cos(5 \cdot \phi) = ...[m/degree] \tag{28}$$

5 Schéma cinématique

5.1 Vecteurs positions

origine : centre de rotation verticale se trouvant sous les pâles principales.

position des pâles principales (pp) : vecteur verticale

position de l'hélice arrière : vecteur allant de l'origine vers l'hélice (h) arrière.

6 Angular momentum

6.1 Formula

$$\vec{L} = \vec{OA} \otimes \vec{P} = \vec{r} \otimes \vec{P} = \vec{r} \otimes m \cdot \vec{v} = \vec{I} \otimes \vec{\omega} \tag{29}$$

$\vec{L}$: Angular Momentum $[kg \cdot \frac{m^2}{s}]$
$\vec{OA}$ and r: position of the mass $[m]$ according to a referance
$\vec{P}$: linear momentum $[kg \cdot \frac{m}{s}]$[2]
$\vec{v}$: velocity $[\frac{m}{s}]$ I : moment of inertia $[m^2 \cdot kg \cdot]$
ω : angular speed $[\frac{rad}{s}]$
Torque :

$$M = \frac{d\vec{L}}{dt} = \frac{d(\vec{I} \otimes \vec{\omega})}{dt} \tag{30}$$

if we consider a particule of mass m, $\vec{r}$ is the position of the center of mass. If it is a solid object, L is first computed according to the axis of rotation of the object :

$$\vec{L}_{ar} = \vec{I}_{ar} \otimes \vec{\omega}_{ar} \tag{31}$$

To compute the angular moment according to an other axis of rotation (new referance), we use the Huygens-Steiner theorem (or the Parallel axis theorem) :

$$\vec{L}_0 = \vec{I}_0 \otimes \vec{\omega}_{cm} \tag{32}$$

$$\vec{I}_0 = \vec{I}_{ar} + m \cdot d^2 \tag{33}$$

with d the distance between the axis of rotation of the object and the new referance.

6.2 Condition of stability

Main rotor(s):

$$\vec{L}_{mr} = \vec{r}_{mr} \otimes m_{mr} \cdot \vec{v_{mr}} = \vec{I_{mr}} \otimes \vec{\omega_{mr}} \tag{34}$$

Rear rotor :

$$\vec{L}_{rr} = \vec{r}_{rr} \otimes m_{rr} \cdot \vec{v_{rr}} = \vec{I_r} r \otimes \vec{\omega_{rr}} \tag{35}$$

assurer la stabilité lors du vol: les moments cinétiques doivent s'annuler. (poser la formule et résoudre)

$$\vec{L_{mr}} = \vec{L_{rr}} \tag{36}$$

or
The generated torque is compensated :

$$\sum \vec{M_{mr}} = \sum \vec{M_{rr}} \tag{37}$$

find a relation between ω_{mr} and ω_{rr} -> determine the transmission ratio

[2] $\vec{L}$ is perpendicular to both $\vec{P}$ and $\vec{r}$

6.3 Pivots à droite et à gauche

pour tourner à gauche ou doite, on ne doit plus satisfaire la condition de stabilité. le pilote utiliser le pédalier pour accélérer/ralentir l'hélice arrière. ainsi les moments cinétiques ne sont plus égaux.

calculer l'effet de rotation sur l'hélicoptère si l'hélice est accélérée/ralentie de 10,20,30,.. $\%$. mettre un tableau. calculer la vitesse de rotation dans ces cas-là.

$$\begin{bmatrix}0\\0\\l_1\end{bmatrix}_{R_1} \quad \vec{AB}_{R_2} = \begin{bmatrix}0\\l_2\\0\end{bmatrix}_{R_2} \quad \vec{BC}_{R_3} = \begin{bmatrix}l_3\\0\\0\end{bmatrix}_{R_3} \quad \vec{CD}_{R_4} = \begin{bmatrix}0\\0\\-l_4\end{bmatrix}_{R_4} \tag{38}$$

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$$\begin{aligned}\vec{OE}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB_1}_R + \vec{B_1C_1}_R + \vec{C_1C}_R \\ + \vec{CC_2}_R + \vec{C_2D}_R + \vec{DD_1}_R + \vec{D_1E}_R\end{aligned} \tag{39}$$

$$\begin{aligned}\vec{OF}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB_1}_R + \vec{B_1C_1}_R + \vec{C_1C}_R \\ + \vec{CC_2}_R + \vec{C_2D}_R + \vec{DD_1}_R + \vec{D_1E}_R + \vec{EF}_R\end{aligned} \tag{40}$$

$$\begin{aligned}\vec{OG}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB_1}_R + \vec{B_1C_1}_R + \vec{C_1C}_R + \vec{CC_2}_R \\ + \vec{C_2D}_R + \vec{DD_1}_R + \vec{D_1E}_R + \vec{EF}_R + \vec{FF_3}_R + \vec{F_3G}_R\end{aligned} \tag{41}$$

$$\begin{aligned}\vec{OH}_R = \vec{OA}_R + \vec{AB}_R + \vec{BB_1}_R + \vec{B_1C_1}_R + \vec{C_1C}_R + \vec{CC_2}_R + \vec{C_2D}_R \\ + \vec{DD_1}_R + \vec{D_1E}_R + \vec{EF}_R + \vec{FF_3}_R + \vec{F_3G}_R + \vec{GH}_R\end{aligned} \tag{42}$$

7 Conclusion