Integrales inmediatas

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \cos 2x\,dx = \frac{\sin 2x}{2} + C$$

$$\int \sin 2x\,dx = -\frac{\cos 2x}{2} + C$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C$$

$$\int \frac{1}{\sin^2 x}\,dx = -\cot x + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x\,\tan x\,dx = \sec x + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\begin{aligned}\int \tan x\,dx &= -\ln|\cos x| + C\\ &= \ln|\sec x| + C\end{aligned}$$

$$\int \tan^2 x\,dx = \tan x - x + C$$

$$\int x\sin x\,dx = \sin x - x\cos x + C$$

$$\int x\cos x\,dx = \cos x + x\sin x + C$$

Integral por partes

$$\int u\,dv = uv - \int v\,du$$

Derivadas

Regla de la cadena:

$$(f(g))' = f'g \cdot g'$$

Regla del producto

$$(f \cdot g)' = f'g + fg'$$

$$(k)' = 0$$

$$(x)' = 1$$

$$(nx)' = x$$

$$(x^n)' = nx^{n-1}$$

$$(e^x)' = e^x$$

$$(e^{kx})' = ke^{kx}$$

$$(\ln|x|)' = \frac{1}{x}$$

$$(a^x)' = a^x \ln a$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\cot x)' = -\csc^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$(\csc x)' = -\csc x \cot x$$

$$(\frac{1}{x})' = -\frac{1}{x^2}$$

$$\begin{aligned}(x^{-2})' &= -2x^{-3}\\ &= -\frac{2}{x^3}\end{aligned}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

Identidades trigonométricas

$$\sin^2 x + \cos^2 x = 1$$
$$\tan x = \frac{\sin x}{\cos x}$$
$$\cot x = \frac{\cos x}{\sin x}$$
$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = 2\cos^2 x - 1$$
$$\cos 2x = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\cot^2 x + 1 = \csc^2 x$$

Propiedades

$$\frac{u^{\frac{3}{5}}}{\frac{3}{5}} = \frac{5}{3}u^{\frac{3}{5}}$$

$$\frac{1}{u^{2/5}} = u^{-2/5}$$
$$\sqrt[n]{u^m} = u^{m/n}$$

$$\ln(e^x) = x$$
$$e^{\ln x} = x$$

Teoremas de ecuaciones diferenciales

Teorema 1:

Resolución de una E.D. de variables separables:
Dada la E.D. de variables separables M(x) dx + N(y) dy = 0, la solución es:

$$\int M(x)\,dx + \int N(y)\,dy = C$$

Teorema 2:

Resolución de una E.D. homogénea:
Dada la E.D. homogénea M(x,y) dx + N(x,y) dy = 0, tomamos el cambio de variable y=ux, transformandola en una E.D. de variable separable.

$$y = ux$$
$$dy = u\,dx + x\,du$$

Teorema 3:

Resolución de una E.D. exacta:
Dada la E.D. exacta M(x,y) dx + N(x,y) dy = 0, existe una función F(x,y) tal que:
$\frac{\partial F}{\partial x} = M(x,y)$ y $\frac{\partial F}{\partial y} = N(x,y)$

Teorema 4:

Factores integrantes:

Si la E.D. de la forma M(x,y) dx + N(x,y) dy = 0 no es exacta, entonces:

A: Si $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N(x,y)}$ sólo depende de ”x”, entonces el factor integrante es:

$\mu(x) = e^{\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N(x,y)}\, dx}$

B: Si $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M(x,y)}$ sólo depende de ”y”, entonces el factor integrante es:

$\mu(y) = e^{\int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M(x,y)}\, dy}$

Teorema 5:

Resolución de una E.D. lineal de primer orden:

Dada la E.D. lineal de primer orden: $\dfrac{dy}{dx} + p(x)y = q(x),$ La solución es:

$y = e^{-\int p(x)\,dx}\left(C + \displaystyle\int q(x)e^{\int p(x)\,dx}\,dx\right)$

Teorema 6:

Resolución de una E.D. Bernoulli

Dada la E.D. de Bernoulli: $\dfrac{dy}{dx} + p(x)y = q(x)y^n \quad (n \neq 1, \quad n \neq 0)$

Tomamos el cambio de variable: $u = y^{1-n}$

Esto transforma la E.D. de Bernoulli en una E.D. lineal de primer orden.

Teorema 7:

Resolución de una E.D. Riccati

Dada la E.D. de Riccati: $\dfrac{dy}{dx} + a_2(x)y^2 + a_1(x)y + a_0(x) = 0 \quad (a_0(x) \neq 0 \quad \wedge \quad a_2(x) \neq 0)$

siendo $y_{particular}(x)$ una solución, entonces:

A: Si tomamos el cambio de variable: $y = y_p(x) + u$

La E.D. de Riccati se transforma en una E.D. de Bernoulli.

B: Si tomamos el cambio de variable $y = y_p(x) + \dfrac{1}{u}$

La E.D. de Riccati se transforma en una E.D. lineal de primer orden.