

Q Time between accidents follow exponential distⁿ with a mean of 5 days between accidents. 1) Find the prob. that there will be an accident in next 48 hours.

Solⁿ T : Time for next accident

$$T \sim \exp(1/5)$$

$$f_T(t) = \frac{1}{5} e^{-t/5}, t > 0$$

$$P(T \leq 2) = 1 - e^{-2/5}$$

Find the prob. that in any given week at least 4 accidents will take place

Solⁿ $f_T(t) = \lambda e^{-\lambda t}, t > 0$ mean: $1/\lambda$

T_4 = Waiting time for 4th event.

$$P(T_4 \leq 7)$$

$$T_4 \sim G(4, \lambda)$$

$$T_4 \sim G(4, 1/5)$$

$$\rightarrow \int_0^7 \left(\frac{1}{5}\right)^4 t^3 e^{-0.2t} dt = \frac{(0.2)^4}{6} \int_0^7 t^3 e^{-0.2t} dt$$

$$\frac{(0.2)^4}{6} \int_0^{1.4} t^3 e^{-t} dt = \frac{1}{6} \int_0^{1.4} t^3 e^{-t} dt$$

$$\frac{1}{6} \left\{ -e^{-t} t^3 \Big|_0^{1.4} + 3 \int_0^{1.4} t^2 e^{-t} dt \right\}$$

$$\frac{1}{6} \left\{ -(1.4)^3 e^{-1.4} + 3 \left[-e^{-t} t^2 \Big|_0^{1.4} + 2 \int_0^{1.4} t e^{-t} dt \right] \right\}$$

$$\frac{1}{6} \left\{ -(1.4)^3 e^{-1.4} + 3 \left[-e^{-1.4} (1.4)^2 + 2 \left(-e^{-t} \Big|_0^{1.4} \right) \right] \right\}$$

$$\frac{1}{6} \left\{ -(1.4)^3 e^{-1.4} + 3 \left[-e^{-1.4} (1.4)^2 + 2 \left(-e^{-1.4} + e^{-0} \right) \right] \right\}$$

$$-6(1.4)e^{-1.4} - 6e^{-1.4} + 6$$

$$1 - e^{-1.4} - 1.4e^{-1.4} - 2(1.4)^2 e^{-1.4} - \frac{1}{6} e^{-1.4} (1.4)^3$$

Note that

$\{T_{0.1} \leq t\}$ equivalent to $\{X(t) \geq 1\}$

Alt

$$P(X(t) \geq 1)$$

$$X(t) \sim P(t/s)$$

$$P(X(t) \geq 1) = 1 - P(X(t) = 0) - P(X(t) = 1) - P(X(t) = 2) - P(X(t) = 3) - \dots$$

$$= 1 - e^{-7/5} - \frac{e^{-7/5} (7/5)^1}{1!} - \frac{e^{-7/5} (7/5)^2}{2!} - \frac{e^{-7/5} (7/5)^3}{3!} - \dots$$