Project I: Rocket Landing

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October 20th, 2021

1 Better Problem Formulation

Out of the options given, the following was chosen for the better problem formulation:
1) Rocket orientation (θ) and angular velocity ($\dot{\theta}$)

The rocket state is represented by the rocket's distance to the ground, d(t), its velocity, v(t), its orientation, θ(t), and its angular velocity, $\dot{\theta}$(t), i.e. $x(t) = [d(t), v(t), \theta(t), \dot{\theta}(t)]^T$ where t specifies time. The units for distance are kilometers (km), the units for velocity are kilometers per second (km/s), the units for orientation are in radians (rad), and the units for angular velocity are in radians per second (rad/s). The control input of the rocket is its acceleration a(t) and its angular acceleration $\ddot{\theta}$(t). The discrete-time dynamics are the following:

$$d(t+1) = d(t) + v(t)\Delta t \tag{1}$$

$$v(t+1) = v(t) + a(t)\Delta t * cos(\theta(t)) \tag{2}$$

$$\theta(t+1) = \theta(t) + \dot{\theta}(t)\Delta t \tag{3}$$

$$\dot{\theta}(t+1) = \dot{\theta}(t) + \ddot{\theta}(t)\Delta t \tag{4}$$

where Δt is a time interval. The $cos(\theta(t))$ term in equation 2 represents how the rocket's orientation affects its velocity and distance. Let the closed-loop controller be the following:

$$a_1(t) = a(t) = f_\alpha(x(t)) \tag{5}$$

$$a_2(t) = \ddot{\theta}(t) = f_\alpha(x(t)) \tag{6}$$

where $f_\alpha(*)$ is a neural network with parameters α, which are to be determined through optimization. a_1(t) controls the linear motion of the rocket while a_2(t) controls the rocket's rotation. a_1(t) is controlled by the booster at the bottom of the rocket while a_2(t) is controlled by a separate booster on the side of the rocket.

For each time step, we assign a loss as a function of the control inputs and the state: $l(x(t), a_1(t), a_2(t))$. Let $l(x(t), a_1(t), a_2(t)) = 0$ for all t = 1,

..., T - 1, where T is the final step, and $l(x(T), a_1(T), a_2(T)) = ||x(T)||^2 = d(T)^2 + v(T)^2 + \theta(T)^2 + \dot{\theta}(T)^2$. This loss encourages the rocket to reach $d(T) = 0$, $v(T) = 0$, $\theta(T) = 0$, and $\dot{\theta}(T) = 0$, which are proper landing conditions.

The optimization problem is now formulated as

$$min_\alpha \quad ||x(T)||^2 \tag{7}$$

$$d(t+1) = d(t) + v(t)\Delta t \tag{8}$$

$$v(t+1) = v(t) + a(t)\Delta t * cos(\theta(t)) \tag{9}$$

$$\theta(t+1) = \theta(t) + \dot{\theta}(t)\Delta t \tag{10}$$

$$\dot{\theta}(t+1) = \dot{\theta}(t) + \ddot{\theta}(t)\Delta t \tag{11}$$

$$a_1(t) = f_\alpha(x(t)), \ \forall \quad t = 1, ..., T-1 \tag{12}$$

$$a_2(t) = f_\alpha(x(t)), \ \forall \quad t = 1, ..., T-1 \tag{13}$$

While this problem is constrained, it is easy to see that the objective function can be expressed as a function of $x(T-1)$ and $a_1(T-1)$ and $a_2(T-1)$. Thus it is essentially an unconstrained problem with respect to α.

The following assumptions are made:

1. When $\theta = 0$, the rocket is oriented upright, which is the correct landing orientation.

2. The rocket is upright when the axis along the height of the rocket is perpendicular to the ground. θ equals zero when the rocket is upright. θ is positive when the rocket rotates counter-clockwise from zero and negative when the rocket rotates clockwise from zero.

3. The acceleration constants defined in the beginning of the code are all arbitrary.

4. It is assumed the gravity and rotation accelerations will always act on the rocket.

5. The system was designed so that the rotation acceleration always acts on the rocket, making it turn counter clockwise. The side thruster/booster was designed to act on the rocket, making it turn clockwise. It is assumed these acceleration directions will not change.

6. It is assumed the rocket only moves along the y-axis, which is the direction perpendicular to the ground. The x and z directions are not defined and are ignored.

7. When the rocket's orientation is anything other than zero, only the vertical component of the main booster's acceleration will contribute to the rocket's velocity.

8. There are no other forces acting on the rocket such as drag. It is also assumed that the environment (Ex: atmospheric conditions) are ignored and do not affect the rocket (Option number 2).
9. There are no other constraints in the state and action spaces (Option number 3).
10. The controller is only designed for the initial states specified in the results. It is assumed the system is not designed to handle initial states other than the values specified in the code (Option number 4).
11. The initial state of d(t) is always > 0.
12. There is no randomness in the dynamics or sensing of the rocket (Option number 5).
13. There is no discontinuity in modeling such as mechanical failures (Option number 6).
14. The rocket is treated as a point and not a rigid body. There are no dimensions for the rocket because they were not needed for any calculations.

2 Analysis of the Results

The initial states of the system are the following: $d(t) = 1$, $v(t) = 0$, $\theta(t) = -1$, and $\dot{\theta}(t) = 0$. These initial states mean the following:

1. $d(t) = 1$ means the rocket is initially at a distance of 1 km above the ground.
2. $v(t) = 0$ means the rocket is initially at the apex of its trajectory and is beginning its descent.
3. $\theta(t) = -1$ means the rocket is initially at angle of -1 radians clockwise with respect to its upright axis.
4. $\dot{\theta}(t) = 0$ means the rocket is initially not rotating.

The final results shown in Figure 1 show that the rocket landed upright ($\theta(T) = 0$) on the ground ($d(T) = 0$) with no linear velocity ($v(T) = 0$) and no angular velocity ($\dot{\theta}(T) = 0$). The results are validated because the loss function equals zero. $l(x(T), a_1(T), a_2(T)) = ||x(T)||^2 = d(T)^2 + v(T)^2 + \theta(T)^2 + \dot{\theta}(T)^2 = 0$. These values satisfy the conditions necessary for a correct landing.

The Velocity vs. Distance graph in Figure 1 shows the rocket falling towards the ground from a height of 1 km. Approximately half way to the ground, the booster slows the descent and brings the rocket to a safe landing.

The Angular Velocity vs. Angle graph in Figure 1 shows the rocket rotating counter-clockwise due to the rotation acceleration. This rotation is counter

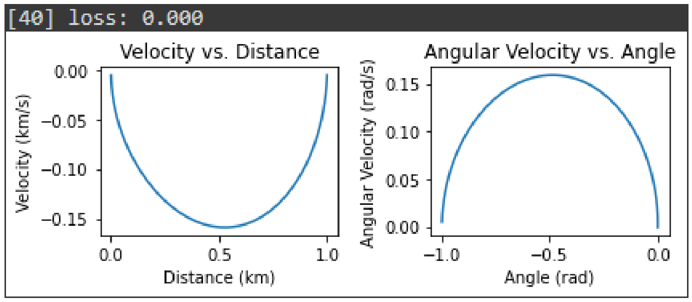

Figure 1: Final iteration of the system

acted by the side booster when the rocket is at approximately -0.5 radians, slowing the angular velocity until the rocket's angle reaches zero. This also shows the rocket landed correctly.

The optimizer reached these results in 40 iterations with 100 time steps per iteration. Based on the assumptions outlined for the defined system, this solution is acceptable and converged with a loss of 0. The appendices attached are drawings I made to help me visualize the problem.

Appendices

A Writing Out the Math

$\boxed{\text{Current Dynamics}}$

Gravity does not directly affect distance

$$\Delta^{State}_{Gravity} = \begin{bmatrix} 0 & -g\_a * \Delta t \end{bmatrix}$$

$$\Delta S = \underbrace{\begin{pmatrix} \text{thrust} \\ \text{const.} \end{pmatrix}}_{\text{Booster acceleration}} \Delta t \begin{bmatrix} 0, & \underbrace{1}_{\text{Upward Velocity}} \end{bmatrix} \underbrace{(\text{action})}_{\text{Value between 0 and 1}}$$

Change velocity based on booster firing on or off

$$\underbrace{State}_{\text{State is initialized in Simulation class}} = \underbrace{State}_{[1 \times 2]} + \underbrace{\Delta S}_{[1 \times 2]} + \underbrace{\Delta^{State}_{Gravity}}_{[1 \times 2]}$$

← Update velocity

$$State = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} d(t) + v(t)\Delta t \\ v(t) \end{bmatrix}$$

New Dynamics

Linear Velocity

$$\Delta \text{State}_{\text{Gravity}} = [0 \quad -g\_a * \Delta t \quad 0 \quad 0]$$

Assume angle of rocket and angular velocity of rocket affected by gravity is negligible.

$$\Delta S = \left(\text{thrust const.}\right) \Delta t \, [0, \ 1, \ 0, \ 0] \, (\text{action}) \cos(\theta)$$

Change velocity based on booster firing

Booster acceleration

Upward Velocity

No effect on others

Value between 0 and 1

Booster acceleration depends on orientation of rocket

Angular Velocity

state caused by rotation accel.

$$\Delta^{\text{State}}_{\text{Rotation}} = [0 \quad 0 \quad 0 \quad r\_a * \Delta t]$$

Angular velocity affected by rotation acceleration (Rotates rocket counter-clockwise)

$$\Delta \text{state}_{\text{angular}} = \left(\text{Rotation accel.}\right) \Delta t \, [0, \ 0, \ 0, \ -1] \, (\text{action\_side})$$

Value that represents how much side thruster affects angular velocity

The thruster is positioned so its force acts against force caused by rotation acceleration.

$$\underset{[1 \times 4]}{\text{State}} = \underset{[1 \times 4]}{\text{State}} + \underset{[1 \times 4]}{\Delta S} + \underset{[1 \times 4]}{\Delta^{\text{State}}_{\text{Gravity}}} + \underset{[1 \times 4]}{\Delta^{\text{State}}_{\text{Angular}}} + \Delta^{\text{State}}_{\text{rotation}}$$

Update linear and angular velocities

State is initialized in Simulation class

$$\text{State} = \underset{\substack{[4 \times 4] \\ \text{Step Matrix}}}{\begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{\substack{[4 \times 1] \\ \text{State vector}}}{\begin{bmatrix} d(t) \\ v(t) \\ \theta(t) \\ \dot{\theta}(t) \end{bmatrix}} = \underset{\substack{[4 \times 1] \\ \text{Updated state vector}}}{\begin{bmatrix} d(t) + v(t)\Delta t \\ v(t) \\ \theta(t) + \dot{\theta}(t)\Delta t \\ \dot{\theta}(t) \end{bmatrix}}$$

B Visualizing the Rocket Dynamics