Problem 1 (10 Points)

Sketch graphically the problem

$$\begin{array}{rl} \min_{x_1,\ x_2} & f(\mathbf{x}) = (x_1+1)^2 + (x_2-2)^2 \\ \text{subject to} & g_1 = x_1 - 2 \le 0, \quad g_3 = -x_1 \le 0, \\ & g_2 = x_2 - 1 \le 0, \quad g_4 = -x_2 \le 0. \end{array}$$

Find the optimum graphically. Determine directions of feasible descent at the corner points of the feasible domain. Show the gradient directions of f and g_is at these points. Verify graphical results analytically using the KKT conditions.

Problem 2 (10 Points)

Graph the problem

$$\begin{array}{rl} \min_{x_1,\ x_2} & f = -x_1 \\ \text{subject to} & g_1 = x_2 - (1-x_1)^3 \le 0 \quad \mathrm{a}nd \quad x_2 \ge 0. \end{array}$$

Find the solution graphically. Then apply the optimality conditions. Can you find a solution based on the optimality conditions? Why? (From Kuhn and Tucker, 1951.)

Problem 3 (30 Points)

Find a local solution to the problem

$$\begin{array}{rl} \max_{x_1,\ x_2,\ x_3} & f = x_1x_2 + x_2x_3 + x_1x_3 \\ \text{subject to} & h = x_1 + x_2 + x_3 - 3 = 0. \end{array}$$

Use two methods: reduced gradient and Lagrange multipliers.

Problem 4 (20 Points)

Use reduced gradient to find the value(s) of the parameter b for which the point $x_1 = 1$, $x_2 = 2$ is the solution to the problem

$$\begin{array}{rl} \max_{x_1,\ x_2} & f = 2x_1 + bx_2 \\ \text{subject to} & g_1 = x_1^2 + x_2^2 - 5 \le 0 \\ & g_2 = x_1 - x_2 - 2 \le 0. \end{array}$$

Problem 5 (30 Points)

Find the solution for

$$\begin{array}{rl} \min_{x_1,\ x_2,\ x_3} & f = x_1^2 + x_2^2 + x_3^2 \\ \text{subject to} & h_1 = x_1^2/4 + x_2^2/5 + x_3^2/25 - 1 = 0 \\ & h_2 = x_1 + x_2 - x_3 = 0, \end{array}$$