 *restart*

▼ Load Libraries

> with(*plots*)
[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*] **(1.1)**

> with(*LinearAlgebra*)
[*&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUdecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRdecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip*] **(1.2)**

> with(*Statistics*)
[*AbsoluteDeviation, AgglomeratedPlot, AreaChart, AutoCorrelation, AutoCorrelationPlot, BarChart, Biplot, Bootstrap, BoxPlot, BubblePlot, CDF, CGF, CentralMoment,* **(1.3)**

CharacteristicFunction, ChiSquareGoodnessOfFitTest, ChiSquareIndependenceTest, ChiSquareSuitableModelTest, ColumnGraph, Correlation, CorrelationMatrix, Correlogram, Count, CountMissing, Covariance, CovarianceMatrix, CrossCorrelation, Cumulant, CumulantGeneratingFunction, CumulativeDistributionFunction, CumulativeProduct, CumulativeSum, CumulativeSumChart, DataSummary, Decile, DensityPlot, Detrend, Difference, DiscreteValueMap, Distribution, ErrorPlot, EvaluateToFloat, Excise, ExpectedValue, ExponentialFit, ExponentialSmoothing, FailureRate, FisherInformation, Fit, FivePointSummary, FrequencyPlot, FrequencyTable, GeometricMean, GridPlot, HarmonicMean, HazardRate, HeatMap, Histogram, HodgesLehmann, Information, InteractiveDataAnalysis, InterquartileRange, InverseSurvivalFunction, Join, KernelDensity, KernelDensityPlot, KernelDensitySample, Kurtosis, LeastTrimmedSquares, Likelihood, LikelihoodRatioStatistic, LineChart, LinearFilter, LinearFit, LogLikelihood, LogarithmicFit, Lowess, MGF, MLE, MakeProcedure, MaximumLikelihoodEstimate, Mean, MeanDeviation, Median, MedianDeviation, MillsRatio, Mode, Moment, MomentGeneratingFunction, MovingAverage, MovingMedian, MovingStatistic, NonlinearFit, NormalPlot, OneSampleChiSquareTest, OneSampleTTest, OneSampleZTest, OneWayANOVA, OrderByRank, OrderStatistic, PCA, PDF, ParetoChart, Percentile, PieChart, PointPlot, PolynomialFit, PowerFit, PredictiveLeastSquares, PrincipalComponentAnalysis, Probability, ProbabilityDensityFunction, ProbabilityFunction, ProbabilityPlot, ProfileLikelihood, ProfileLogLikelihood, QuadraticMean, Quantile, QuantilePlot, Quartile, RandomVariable, Range, Rank, Remove, RemoveInRange, RemoveNonNumeric, RepeatedMedianEstimator, RousseeuwCrouxQn, RousseeuwCrouxSn, Sample, Scale, ScatterPlot, ScatterPlot3D, Score, ScreePlot, Select, SelectInRange, SelectNonNumeric, ShapiroWilkWTest, Shuffle, Skewness, Sort, Specialize, SplitByColumn, StandardDeviation, StandardError, StandardizedMoment, SunflowerPlot, Support, SurfacePlot, SurvivalFunction, SymmetryPlot, Tally, TallyInto, TreeMap, Trim, TrimmedMean, TwoSampleFTest, TwoSamplePairedTTest, TwoSampleTTest, TwoSampleZTest, Variance, Variation, VennDiagram, ViolinPlot, WeibullPlot, WeightedMovingAverage, Winsorize, WinsorizedMean]

```

> Type = "i5-2500 K CPU @ 3.30 GHz"
> FileL := FileTools[ListDirectory](".", 'all', 'returnonly' = ".txt")
      FileL := ["Speed.txt"] (1)
=
>
      Type = "i5-2500 K CPU @ 3.30 GHz" (2)
=
> FamSpeed := ImportMatrix(FileL[1], source = csv)

```

$$FamSpeed := \begin{bmatrix} 100. & 1.30000000000000 \cdot 10^{-6} & 2.66366000000000 \cdot 10^{-8} \\ 133. & 1.50000000000000 \cdot 10^{-6} & 1.50143000000000 \cdot 10^{-7} \\ 177. & 9.00000000000000 \cdot 10^{-7} & -3.37294000000000 \cdot 10^{-7} \\ 236. & 3.00000000000000 \cdot 10^{-6} & 2.98573000000000 \cdot 10^{-7} \\ 315. & 1.40000000000000 \cdot 10^{-6} & 2.29747000000000 \cdot 10^{-7} \\ 420. & 5.20000000000000 \cdot 10^{-6} & 1.55147000000000 \cdot 10^{-7} \\ 560. & 1.90000000000000 \cdot 10^{-6} & -1.94351000000000 \cdot 10^{-8} \\ 747. & 9.00000000000000 \cdot 10^{-6} & -3.93865000000000 \cdot 10^{-7} \\ 996. & 3.20000000000000 \cdot 10^{-6} & -8.63760000000000 \cdot 10^{-9} \\ 1328. & 4.10000000000000 \cdot 10^{-6} & -2.43901000000000 \cdot 10^{-7} \\ \vdots & \vdots & \vdots \end{bmatrix} \quad (3)$$

54 × 3 Matrix

$$\begin{aligned} &> N := Transpose(FamSpeed)[1]; \\ N &:= [100., 133., 177., 236., 315., 420., 560., 747., 996., 1328., 1771., 2362., 3150., \\ &4201., 5602., 7470., 9961., 13283., 17713., 23621., 31499., 42005., 56015., 74697., \\ &99610., 132832., 177134., 236212., 314994., 420051., 560147., 746968., 996098., \\ &1.328318 \cdot 10^6, 1.771341 \cdot 10^6, 2.362121 \cdot 10^6, 3.149939 \cdot 10^6, 4.200511 \cdot 10^6, 5.601471 \cdot 10^6, \\ &7.469682 \cdot 10^6, 9.960981 \cdot 10^6, 1.3283182 \cdot 10^7, 1.7713408 \cdot 10^7, 2.3621209 \cdot 10^7, \\ &3.1499388 \cdot 10^7, 4.2005109 \cdot 10^7, 5.6014713 \cdot 10^7, 7.4696820 \cdot 10^7, 9.9609810 \cdot 10^7, \\ &1.32831816 \cdot 10^8, 1.77134074 \cdot 10^8, 2.36212084 \cdot 10^8, 3.14993877 \cdot 10^8, 4.20051086 \cdot 10^8] \end{aligned} \quad (4)$$

$$\begin{aligned} &> DeltaT := Transpose(FamSpeed)[2]; \\ DeltaT &:= [1.30000000000000 \cdot 10^{-6}, 1.50000000000000 \cdot 10^{-6}, 9.00000000000000 \cdot 10^{-7}, \\ &3.00000000000000 \cdot 10^{-6}, 1.40000000000000 \cdot 10^{-6}, 5.20000000000000 \cdot 10^{-6}, \\ &1.90000000000000 \cdot 10^{-6}, 9.00000000000000 \cdot 10^{-6}, 3.20000000000000 \cdot 10^{-6}, \\ &4.10000000000000 \cdot 10^{-6}, 0.000020600000000000, 0.000026600000000000, \\ &9.10000000000000 \cdot 10^{-6}, 0.000012100000000000, 0.000063100000000000, \\ &0.000021000000000000, 0.000027900000000000, 0.000037200000000000, \\ &0.000057200000000000, 0.000066400000000000, 0.000348900000000000, \\ &0.000116800000000000, 0.000643800000000000, 0.000201500000000000, \\ &0.000279700000000000, 0.001527800000000000, 0.000497700000000000, \\ &0.001850100000000000, 0.002607600000000000, 0.001158300000000000, \\ &0.002286200000000000, 0.004366000000000000, 0.002877800000000000, \\ &0.010292400000000000, 0.009195700000000000, 0.009264500000000000, \\ &0.011986400000000000, 0.026059500000000000, 0.017151000000000000, \\ &0.030229800000000000, 0.027073600000000000, 0.060565200000000000, \\ &0.065932300000000000, 0.065965400000000000, 0.096574700000000000, \end{aligned} \quad (5)$$

```
0.1323700000000000, 0.1855770000000000, 0.2244900000000000, 0.3175850000000000,
0.4738950000000000, 0.6048140000000000, 0.6945540000000000, 0.9374840000000000,
1.2319300000000000 ]
```

```
> Err := Transpose(FamSpeed)[3];
```

```
Err := [ 2.66366000000000 10-8, 1.50143000000000 10-7, -3.37294000000000 10-7,
2.98573000000000 10-7, 2.29747000000000 10-7, 1.55147000000000 10-7,
-1.94351000000000 10-8, -3.93865000000000 10-7, -8.63760000000000 10-9,
-2.43901000000000 10-7, 6.02531000000000 10-7, -1.49169000000000 10-6,
-8.96918000000000 10-7, 1.46654000000000 10-7, 9.37750000000000 10-7,
2.23334000000000 10-6, -2.43484000000000 10-6, -2.53902000000000 10-6,
-2.25544000000000 10-6, 1.36195000000000 10-6, -9.30775000000000 10-7,
6.20667000000000 10-6, 8.43616000000000 10-7, 3.99471000000000 10-6,
-2.92095000000000 10-6, 3.42400000000000 10-6, 4.74537000000000 10-7,
-0.000014445000000000, -3.94483000000000 10-6, -3.72046000000000 10-6,
5.74396000000000 10-6, 5.11234000000000 10-7, -3.26537000000000 10-6,
-2.45651000000000 10-6, -8.69195000000000 10-6, 9.95978000000000 10-6,
6.64839000000000 10-6, -9.54467000000000 10-6, -2.82228000000000 10-6,
0.000044327400000000, 0.000057989600000000, 0.000035561300000000,
0.000022001200000000, -0.000089516700000000, 0.000075560600000000,
-0.2011850000000000, -0.4009530000000000, -0.5507780000000000,
-0.6631440000000000, -0.7473640000000000, -0.8105620000000000,
-0.8579480000000000, -0.8934760000000000, -0.9201170000000000 ]
```

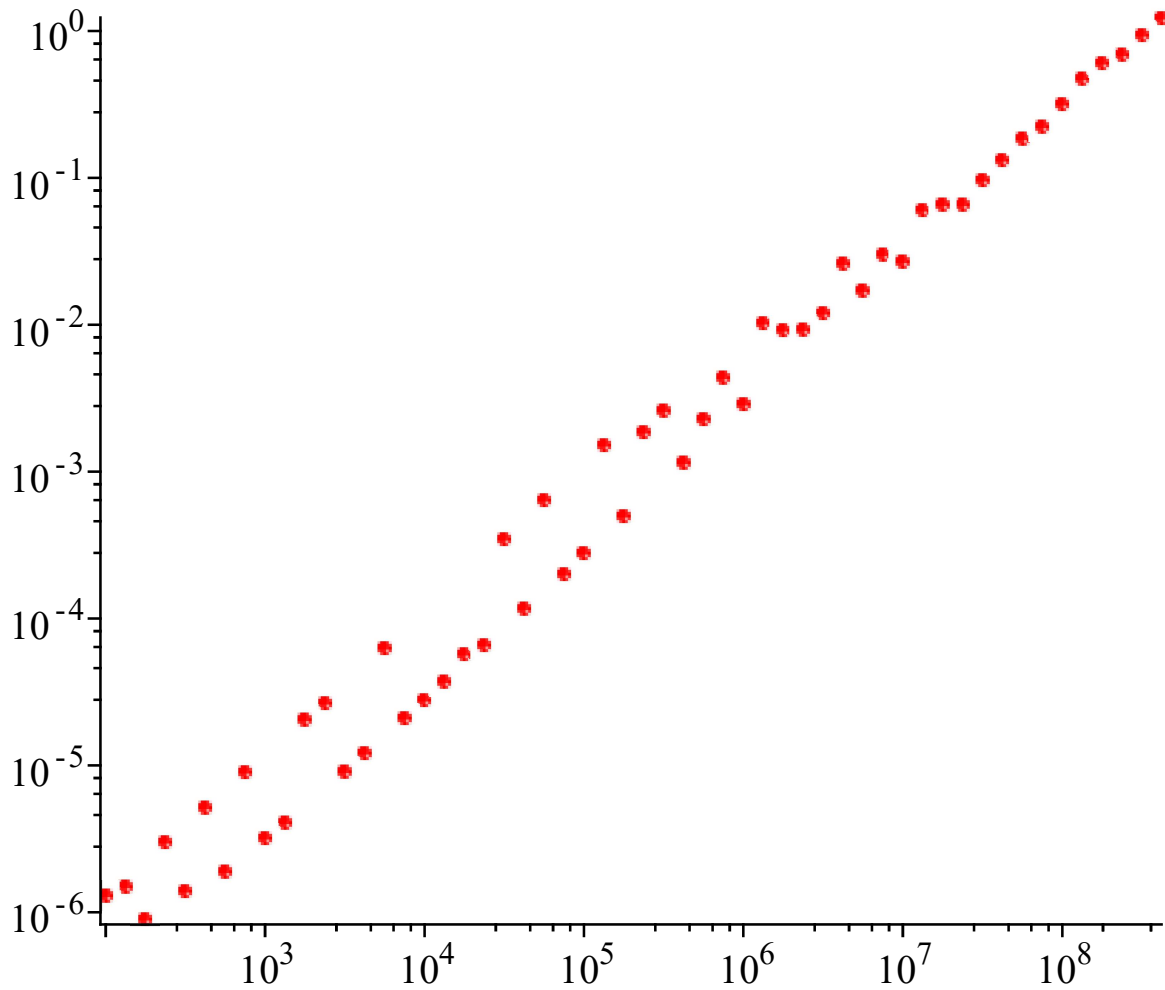
(6)

```
> PlotOpts0 := style=point, symbol=solidcircle, color=red, symbolsize=8
```

```
PlotOpts0 := style=point, symbol=solidcircle, color=red, symbolsize=8
```

(7)

```
> PData := loglogplot(N, DeltaT, PlotOpts0)
```



```
> LogN := map(log, N);
LogN := [4.60517018598809, 4.89034912822175, 5.17614973257383, 5.46383180502561,
5.75257263882563, 6.04025471127741, 6.32793678372919, 6.61606518513282,
6.90374725758460, 7.19142933003638, 7.47929963778283, 7.76726399675731,
8.05515773181968, 8.34307787116938, 8.63087895582005, 8.91865027812686,
9.20643274714516, 9.49424030113250, 9.78205411225260, 10.0698914258577,
10.3577110782781, 10.6455439377989, 10.9333747909928, 11.2211952096719,
11.5090178401392, 11.7968404508528, 12.0846617872759, 12.3724849858579,
12.6603088700073, 12.9481314114591, 13.2359545282642, 13.5237776251786,
13.8116009253007, 14.0994240381844, 14.3872474448141, 14.6750705022004,
14.9628936455349, 15.2507167425195, 15.5385397997828, 15.8263629859217,
16.1141861186865, 16.4020092817448, 16.6898324248993, 16.9776555528596,
17.2654786750357, 17.5533018117069, 17.8411249463342, 18.1289480789158,
18.4167712116805, 18.7045943446256, 18.9924174840207, 19.2802406204528,
19.5680637585057, 19.8558868951783 ]
```

```
> LogDeltaT := map(log, DeltaT);
LogDeltaT := [ -13.5531462934968, -13.4100454498561, -13.9208710736221,
-12.7168982692962, -13.4790383213431, -12.1668519323769,
```

```

-13.1736566717919, -11.6182859806281, -12.6523597481586,
-12.4045235842540, -10.7902194821687, -10.5345993421766,
-11.6072361444415, -11.3223051053616, -9.67078978841711,
-10.7709881202409, -10.4868838691370, -10.1992017966852,
-9.76895665957852, -9.61981350148189, -7.96072520977155,
-9.05504748757015, -7.34812243909957, -8.50972117657754,
-8.18179295775636, -6.48392648685701, -7.60551307209977,
-6.29251558729872, -5.94932502091389, -6.76080186602597,
-6.08086422837260, -5.43390802085438, -5.85072916563496,
-4.57634952017990, -4.68901929549285, -4.68156538724180,
-4.42398260523534, -3.64737289396221, -4.06569879803219,
-3.49892708615708, -3.60919619564991, -2.80403480828156,
-2.71912682098081, -2.71862491684306, -2.33743847682861,
-2.02215424725646, -1.68428538872285, -1.49392411622176,
-1.14700977994363, -0.746769500813357, -0.502834306236580,
-0.364485366030709, -0.0645555879498751, 0.208582045315296 ]

```

```

> LogLogFit := Fit(a·x + b, LogN, LogDeltaT, x)
      LogLogFit := 0.943613956315727 x - 18.5199994827706 (10)

```

```

> LinFit := simplify(exp(subs(x = log(n), LogLogFit)))
      LinFit := 9.05454058667457 10-9 n0.9436139563 (11)

```

```

> PlotOpts1 := color = blue, thickness = 3
      PlotOpts1 := color = blue, thickness = 3 (12)

```

```

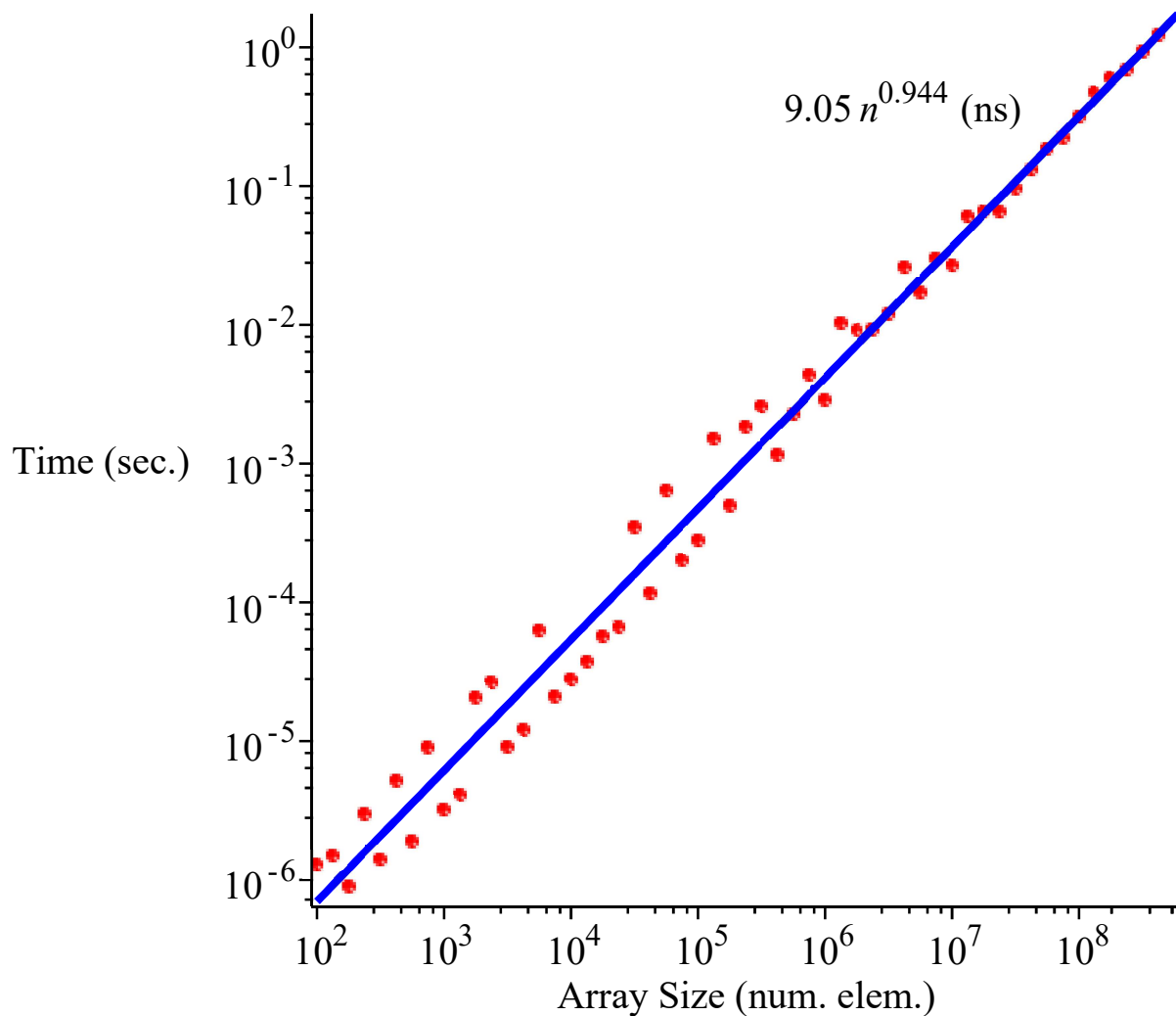
> PFit := loglogplot(LinFit, n = 1e2 .. 6e8, PlotOpts1) :
> PLegend := textplot([4e6, 0.4, typeset(9.05 · n0.944, "(ns)")] ) :
> PlotOpts2 := labels = ["Array Size (num. elem.)", "Time (sec.)"];
      PlotOpts2 := labels = ["Array Size (num. elem.)", "Time (sec.)"] (13)

```

```

> display(PData, PFit, PLegend, PlotOpts2)

```



Not a good fit

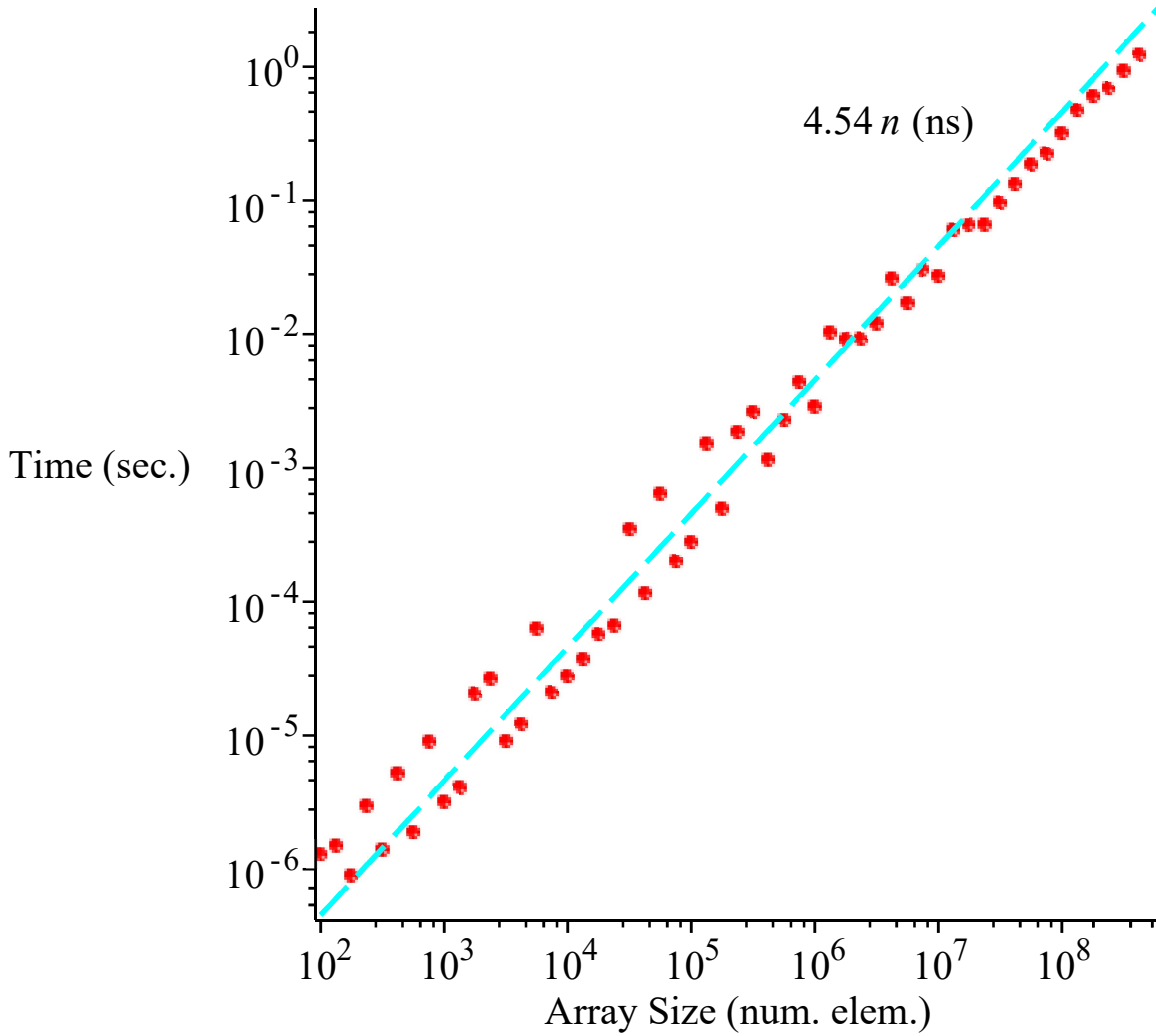
$$\begin{aligned} &> \text{LogLogFitA} := \text{Fit}(x + b, \text{LogN}, \text{LogDeltaT}, x) \\ &\quad \text{LogLogFitA} := -19.2095233430932 + x \end{aligned} \quad (2.1)$$

$$\begin{aligned} &> \text{LinFitA} := \text{simplify}(\exp(\text{subs}(x = \ln(n), \text{LogLogFitA}))) \\ &\quad \text{LinFitA} := 4.54370379735632 \cdot 10^{-9} n \end{aligned} \quad (2.2)$$

$> \text{PFitA} := \text{loglogplot}(\text{LinFitA}, n = 1\text{e}2 \dots 6\text{e}8, \text{color} = \text{cyan}, \text{linestyle} = \text{dash}, \text{thickness} = 2) :$

$> \text{PLegendA} := \text{textplot}([4\text{e}6, 0.4, \text{typeset}(4.54 \cdot n, " \text{(ns)}")]) :$

$> \text{display}(\text{PData}, \text{PFitA}, \text{PLegendA}, \text{PlotOpts2})$



```
> NumPoint := numelems(N)
```

```
NumPoint := 54
```

(14)

```
> TimePer := [seq( DeltaT[k] / N[k], k = 1 .. NumPoint )];
```

```
TimePer := [1.30000000000000 10^-8, 1.12781954887218 10^-8, 5.08474576271186 10^-9,
1.27118644067797 10^-8, 4.44444444444444 10^-9, 1.23809523809524 10^-8,
3.39285714285714 10^-9, 1.20481927710843 10^-8, 3.21285140562249 10^-9,
3.08734939759036 10^-9, 1.16318464144551 10^-8, 1.12616426756986 10^-8,
2.88888888888889 10^-9, 2.88026660318972 10^-9, 1.12638343448768 10^-8,
2.81124497991968 10^-9, 2.80092360204799 10^-9, 2.80057215990364 10^-9,
3.22926664032067 10^-9, 2.81105795690276 10^-9, 1.10765421124480 10^-8,
2.78062135460064 10^-9, 1.14933499955369 10^-8, 2.69756482857411 10^-9,
2.80795100893485 10^-9, 1.15017465670923 10^-8, 2.80973726105660 10^-9,
7.83237092103703 10^-9, 8.27825291910322 10^-9, 2.75752230086347 10^-9,
4.08142862498594 10^-9, 5.84496256867764 10^-9, 2.88907316348391 10^-9,
7.74844577879694 10^-9, 5.19137760600585 10^-9, 3.92211067934285 10^-9,
3.80528003875631 10^-9, 6.20388805076335 10^-9, 3.06187428266611 10^-9,
```

(15)

4.04699959114725 10^{-9} , 2.71796522852518 10^{-9} , 4.55954002587633 10^{-9} ,
 3.72216910489500 10^{-9} , 2.79263436515887 10^{-9} , 3.06592305856863 10^{-9} ,
 3.15128333555806 10^{-9} , 3.31300456721076 10^{-9} , 3.00534882207837 10^{-9} ,
 3.18829039027381 10^{-9} , 3.56763171859368 10^{-9} , 3.41444187638342 10^{-9} ,
 2.94038301613731 10^{-9} , 2.97619753415080 10^{-9} , 2.93280993921773 10^{-9}]

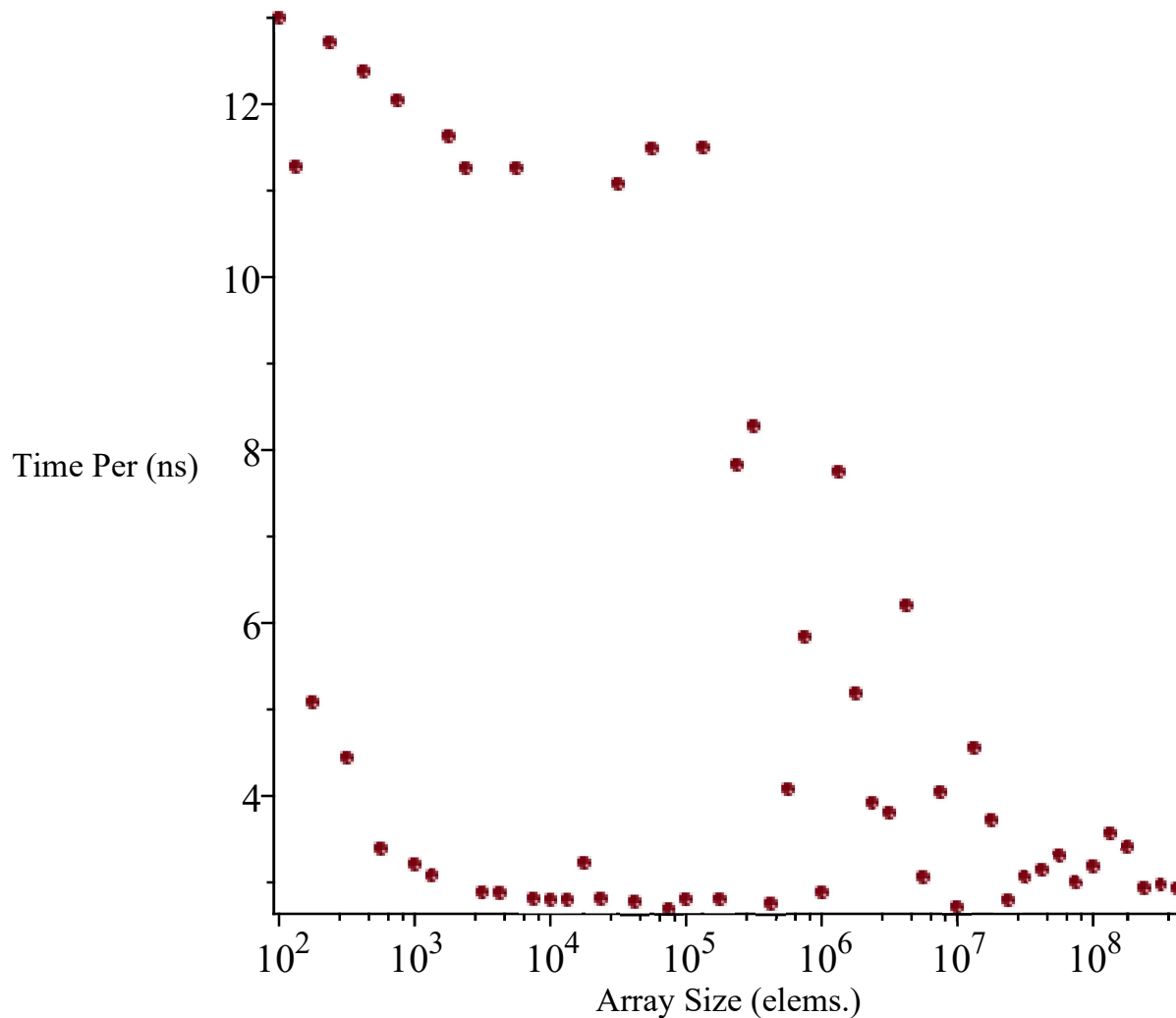
```
> PlotOptsB0 := style=point, symbol=solidcircle, symbolsize=8, labels=["Array Size (elems.)",  

  "Time Per (ns)"]  

PlotOptsB := style=point, symbol=solidcircle, symbolsize=8, labels=["Array Size (elems.)",  

  "Time Per (ns)"]  

> PTimePerData := semilogplot(N, TimePer·1e9, PlotOptsB0)
```



```
> PlotOptsB1 := style=point, symbol=solidcircle, symbolsize=8, labels=["Array Size (elems.)",  

  "Time Per\n(clock cycles)"]  

PlotOptsB1 := style=point, symbol=solidcircle, symbolsize=8, labels  

  = ["Array Size (elems.)", "Time Per  

  (clock cycles)"]
```

```
> ClockRate := 3.3e9  

> PTimePerData := semilogplot(N, TimePer·ClockRate, PlotOptsB1) :
```

Add Lower bound graph

```
> min(TimePer)·ClockRate, max(TimePer)·ClockRate
      8.90196393429455
      42.90000000000000
```

(3.1)

```
> P9 := semilogplot(9, 1e2..6e8, color = green, thickness = 2) :
```

```
> CacheSize := 6e6
```

$CacheSize := 6 \cdot 10^6$

(18)

```
> L2 :=  $\frac{CacheSize}{4}$ 
```

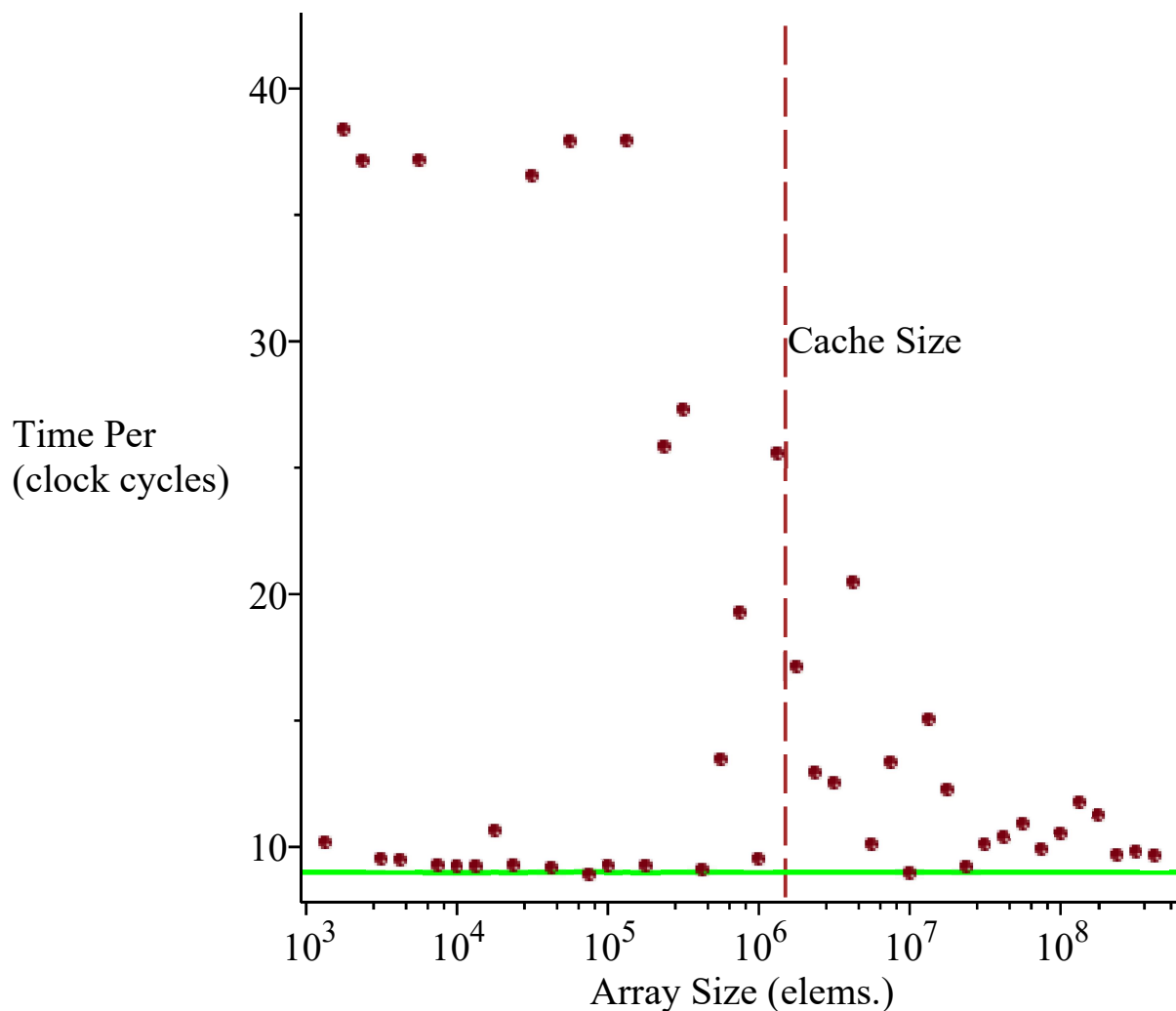
```
> PCache := semilogplot([L2, t, t = 8..43], color = brown, linestyle = dash) :
```

```
> PLegendA := textplot([6e6, 30, "Cache Size"]) :
```

```
> PlotOpts := labels = ["Array Size (elems.)", "Time Per\n(clock cycles)"]
      PlotOpts := labels = ["Array Size (elems.)", "Time Per
      (clock cycles)"]
```

(19)

```
> display(PCache, PLegendA, P9, PTimePerData, view = [1e3..6e8, 8..43], PlotOpts)
```



Not data rate limited

$$> \text{DataRate} := 21\text{e}9$$

$$\text{DataRate} := 2.1 \cdot 10^{10} \quad (4.1)$$

$$> \text{ElemLoadTime} = \frac{4}{\text{DataRate}}$$

$$\text{ElemLoadTime} = 1.904761905 \cdot 10^{-10} \quad (4.2)$$

$$> \text{BytesPerClock} = \frac{\text{DataRate}}{\text{ClockRate}}$$

$$\text{BytesPerClock} = 6.363636364 \quad (4.3)$$

$$> \text{LoadPerComp} = \frac{9}{4} \cdot \frac{\text{DataRate}}{\text{ClockRate}}$$

$$\text{LoadPerComp} = 14.31818182 \quad (4.4)$$