Rotor routing on 2-manifolds

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Outline

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- Rotor routing on $\mathbb{Z}^2$

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- Rotor routing on other 2-manifolds
- Visualizations and experimental results

What is rotor routing?

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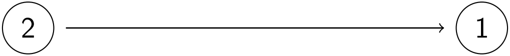
Rotor routing is a deterministic version of the random walk.

What is rotor routing?

Definition (Rotor)

An arrow pointing from one vertex to another.



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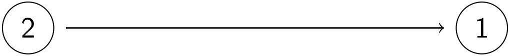

Definition (Rotor Configuration)

An assignment of a single rotor to every vertex on a graph.

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Given a set of possible rotors $\mathcal{E}$ starting at a given point, a rotor mechanism is a cyclic permutation of $\mathcal{E}$.

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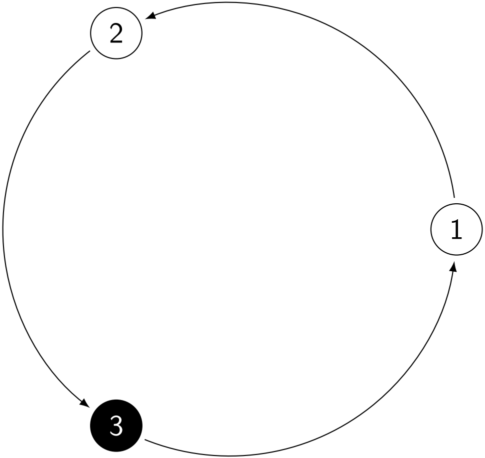
Definition (Rotor Walk)

A traversal of a particle along a initial rotor configuration. The particle follows the direction of rotors, which then change direction according to the rotor mechanism.

Rotor walk example



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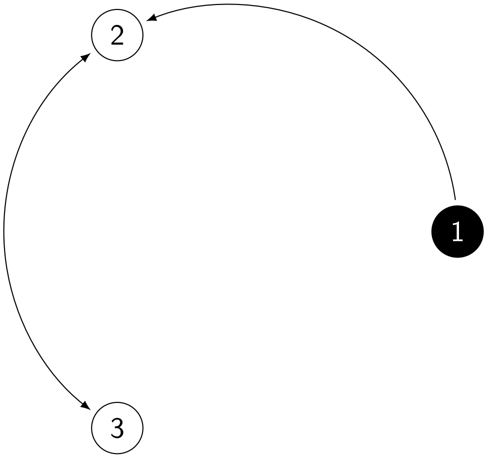

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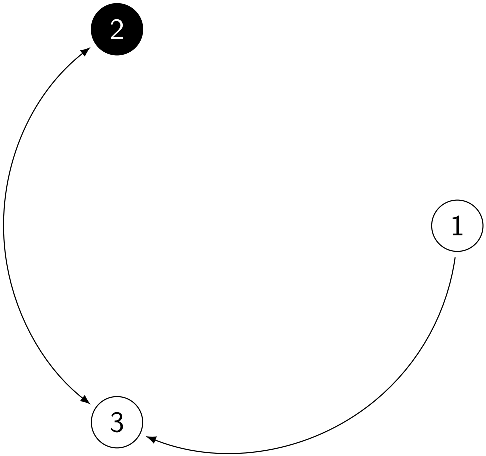

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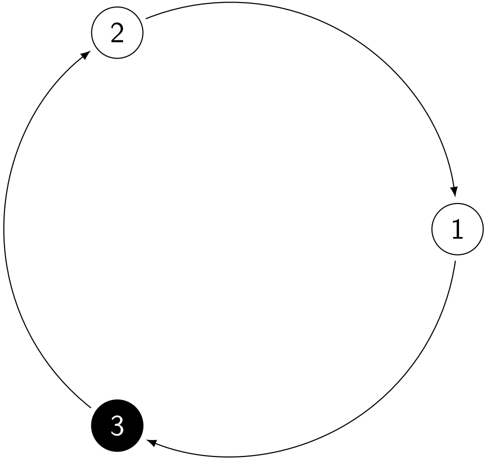

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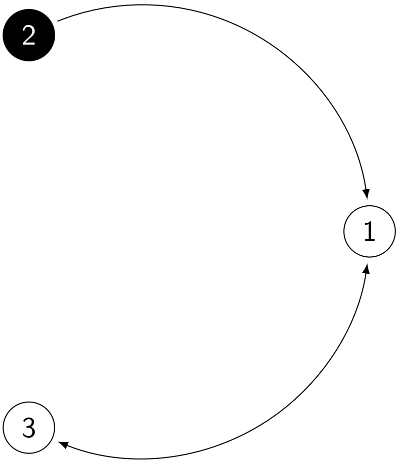

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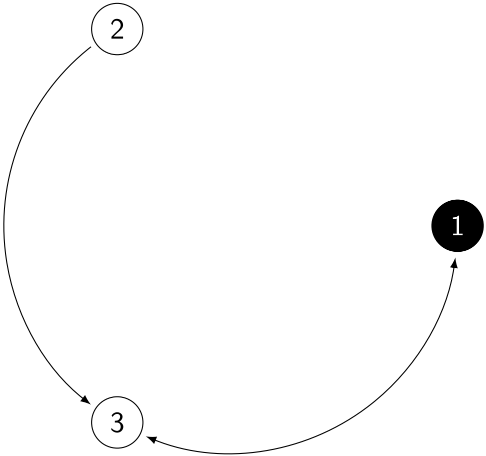

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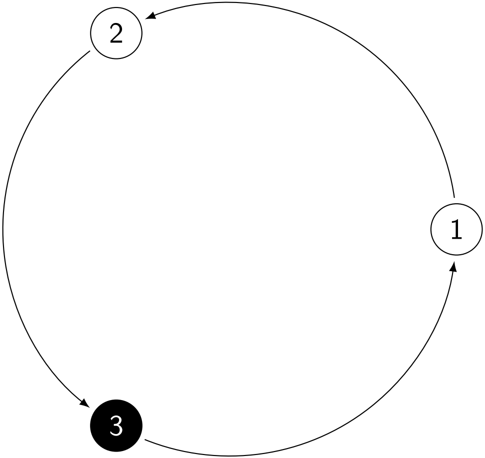

Rotor routing on $\mathbb{Z}^2$

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Definition (Rotor mechanism on $\mathbb{Z}^2$)

There are four possible rotors starting at a given point:

$$\mathcal{E} = \{North, East, South, West\}.$$

Rotors turn counter-clockwise, repeatedly cycling through $\mathcal{E}$.

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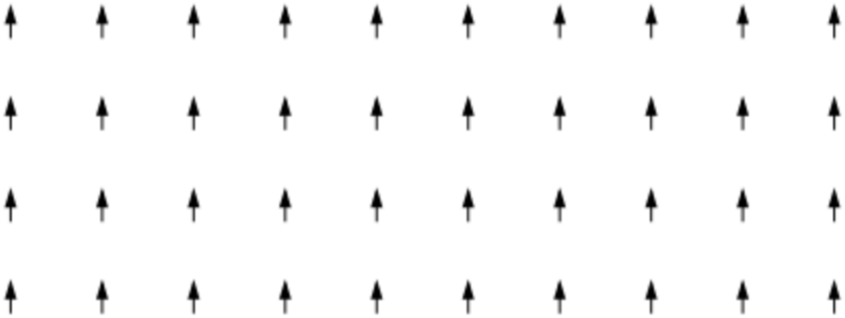

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- Random walk on $\mathbb{Z}$, $\mathbb{Z}^2$
- Repeated rotor walks on $\mathbb{Z}$, $\mathbb{Z}^2$ with single-direction configuration

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A walk is transient if it visits each point finitely many times.

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Definition (Transience)

A walk is transient if it visits each point finitely many times.

- Random walk on $\mathbb{Z}^d$ where $d \geq 3$
- Repeated rotor walks on $\mathbb{Z}^3$ with single-direction configuration

Theorems about recurrence and transience

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Theorem (Angel-Holroyd 2011)

Given a rotor mechanism and a configuration, a single rotor walk is either recurrent for every starting vertex, or transient for every starting vertex.

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Corollary

Given a rotor mechanism, two rotor configurations differing at only finitely many vertices are either both recurrent or both transient.

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Given a rotor mechanism, two rotor configurations differing at only finitely many vertices are either both recurrent or both transient.

Proof

We need only consider two rotor configurations r, r' differing at only one vertex v, such that a single rotation of the rotor at v produces r' from r. If r is recurrent, a rotor walk started at v produces r' with the particle moved to some other vertex. By the aforementioned theorem, r' must also be recurrent. $\square$

Random configuration on $\mathbb{Z}^2$

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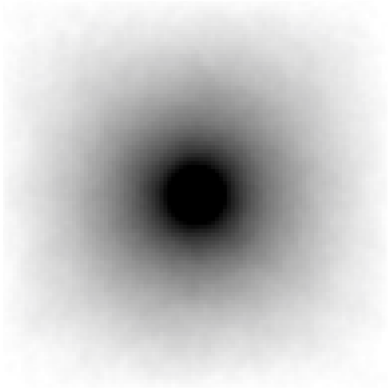
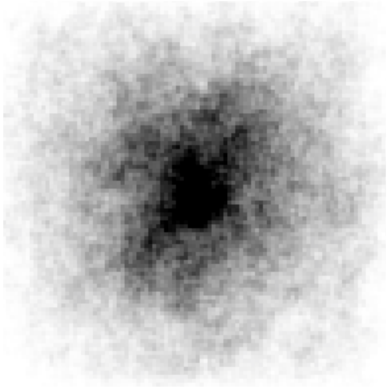
Conjecture (Levine-Peres 2015)

Given a random configuration of rotors on $\mathbb{Z}^2$, a rotor walk on that configuration is recurrent with probability 1.

Random configuration on $\mathbb{Z}^2$



Random Rotor Configuration



Random Walk

Rotor routing on other 2-manifolds

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A topological space whose points all have open disks as neighborhoods.

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Some 2-manifolds:

- Cylinder
- Möbius strip
- Torus
- Sphere
- Klein bottle
- Real projective plane

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Definition (Compactness)

If a space is compact, every open cover has a finite sub-cover.

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Non-compact 2-manifolds:

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- Cylinder
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Compact 2-manifolds:

- Torus
- Sphere
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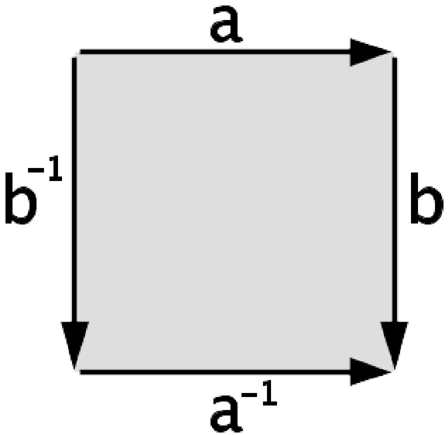
Rotor routing on other 2-manifolds

A gluing diagram shows how edges of a polygon are connected to create a surface. When a particle hits an edge that is associated to another, the particle "reappears" on the associated edge.



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Repeated rotor walks on the Möbius strip, cylinder, and plane will eventually create congruent octagonal patterns for the single-direction configuration.

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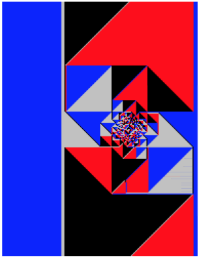
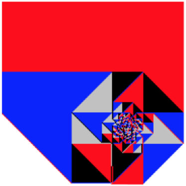
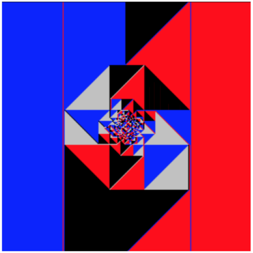


Cylinder



Plane



Mobius Strip

Experimental results

Observation

For any rotor configuration and rotor mechanism, a rotor walk on a compact surface eventually results in a finite loop of configurations.

Future work

- Testing alternate configurations, including different lattices and transient configurations.
- Experimenting with rotor routing in higher dimensions.
- Developing a predictive model for surface type based on loop length on compact surfaces.