Sección 1.5.7

Punto 2

- $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}} = x^i \hat{\mathbf{i}}_i$,
- $\mathbf{a} = \mathbf{a}(\mathbf{r}) = \mathbf{a}(x,y,z) = a^i(x,y,z)\hat{\mathbf{i}}_i$ y $\mathbf{b} = \mathbf{b}(\mathbf{r}) = \mathbf{b}(x,y,z) = b^i(x,y,z)\mathbf{i}_i$,
- $\phi = \phi(\mathbf{r}) = \phi(x,y,z)$ y $\psi = \psi(\mathbf{r}) = \psi(x,y,z)$.

a) $\nabla(\phi\psi) = \phi\nabla\psi + \nabla\phi\psi$

$$\nabla\phi = \frac{\partial\phi(x,y,z)}{\partial x}\hat{\imath} + \frac{\partial\phi(x,y,z)}{\partial y}\hat{\jmath} + \frac{\partial\phi(x,y,z)}{\partial z}\hat{k}$$

$$= \partial^1\phi(x,y,z)\hat{\imath} + \partial^2\phi(x,y,z)\hat{\jmath} + \partial^3\phi(x,y,z)\hat{k}$$

$$\nabla\phi = \partial^i\phi(x^j)\hat{\imath}_i$$

$$\nabla(\phi\psi) = \partial^i\phi(x^j)\psi^j\hat{\imath}_i + \phi(x^j)\partial^i(\psi^j)\hat{\imath}_i$$

$$= \underbrace{\partial^i\phi(x^j)\psi^j\hat{\imath}_i} + \underbrace{\phi(x^j)\partial^i(\psi^j)\hat{\imath}_i}$$

$$\nabla(\phi\psi) = \nabla\phi\psi + \phi\nabla\psi$$

e) $\nabla \cdot (a \times b) \overset{?}{=} (\nabla \times a) \cdot b - a \cdot (\nabla \times b)$

$$\nabla \cdot (a \times b) = \partial_i ((a \times b)^i)$$

$$\partial_i (\varepsilon^{ijk} a_j b_k) = \underbrace{\varepsilon^{ijk} \partial_i a_j b_k}_{1} + \underbrace{\varepsilon^{ijk} a_j \partial_i b_k}_{2}$$

Lycbe

1) $\varepsilon^{ijk} \partial_i a_j b_k \quad , \quad (\nabla \times A)^m = \varepsilon^{mnp} \partial_n A_p$

$$\hookrightarrow \underbrace{\varepsilon^{kij} \partial_i a_j}_{(\nabla \times a)^k} b_k = (\nabla \times a)^k b_k \Big\} \; (\nabla \times a) \cdot b$$

2) $\varepsilon^{ijk} a_j \partial_i b_k = a_j \, \varepsilon^{ijk} \partial_i b_k \quad , \quad \varepsilon^{ijk} = - \varepsilon^{jik}$

$$\hookrightarrow -a_j \underbrace{\varepsilon^{jik} \partial_i b_k}_{(\nabla \times b)^j} = -a_j (\nabla \times b)^j \Big\} - a \cdot (\nabla \times b)$$

$$\nabla \cdot (a \times b) = (\nabla \times a) \cdot b - a (\nabla \times b) \quad \checkmark$$

F) $\nabla \times (\nabla \times a) \overset{?}{=} \nabla(\nabla \cdot a) - \nabla^2 a$

$$\varepsilon^{ijk}\partial_j(\nabla \times a)_k = \varepsilon^{ijk}\partial_j(\underbrace{\varepsilon_{knp}}_{\hookrightarrow \text{cte}}\partial^n a^p)$$

$$\underbrace{\varepsilon^{ijk}\varepsilon_{knp}}\partial_j(\partial^n a^p)$$

$$\varepsilon^{ijk}\varepsilon_{knp} = \delta^i_n\delta^j_p - \delta^i_p\delta^j_n$$

$$\downarrow$$

$$(\underbrace{\delta^i_n\delta^j_p}_{1} - \underbrace{\delta^i_p\delta^j_n}_{2})\partial_j(\partial^n a^p)$$

$i=n \quad j=p$

1) $(\delta^i_n\delta^j_p)\partial_j(\partial^n a^p)$

$\partial_j(\partial^i a^j) = \partial^i(\partial_j a^j) \longrightarrow$ Teorema de Schawrz

$i=p \quad , \quad j=n$

2) $-(\delta^i_p\delta^j_n)\partial_j(\partial^n a^p)$

$-(\partial_j\partial^j)a^i$

$\hookrightarrow \underbrace{\partial^i(\partial_j a^j)} - (\partial_j\partial^j)a^i$

$$[\nabla(\nabla \cdot a)]^i - [\nabla^2 a]^i \quad \checkmark$$