Sección 1.4.5

Punto 3.

$$A^{i'}_{k}\,\tilde{A}^{j}_{i'} = \delta^{j}_{k} \quad , \quad \delta^{j}_{k} \begin{cases} 1, & si \quad j = k \\ 0, & si \quad j \neq k \end{cases}$$

$$A^{i'}_{k} = \frac{\partial i'}{\partial k} \quad , \quad \tilde{A}^{j}_{i'} = \frac{\partial j}{\partial i'}$$

$$\hookrightarrow \frac{\partial i'}{\partial k}\,\frac{\partial j}{\partial i'} = \frac{\partial j}{\partial k} = \delta^{j}_{k}$$

Porque la derivada parcial es cero, a menos que sea con respecto a la misma coordenada

$$\frac{\partial j}{\partial k} = \begin{cases} 1, & si \quad j = k \\ 0, & si \quad j \neq k \end{cases}$$

que es la definición del delta de Kronecker.

Ejemplo

$$x_1' = x_1 \cos\phi + x_2 \operatorname{sen}\phi$$

$$x_2' = -x_1 \operatorname{sen}\phi + x_2 \cos\phi$$

$$x_3' = x_3$$

$$A^{i'}_{j} = \frac{\partial x_i'}{\partial j}$$

$$\frac{\partial x_1'}{\partial x_1} = \cos\phi \qquad \frac{\partial x_1'}{\partial x_2} = \operatorname{sen}\phi \qquad \frac{\partial x_1'}{\partial x_3} = 0$$

$$\frac{\partial x_2'}{\partial x_1} = -\operatorname{sen}\phi \qquad \frac{\partial x_2'}{\partial x_2} = \cos\phi \qquad \frac{\partial x_2'}{\partial x_3} = 0$$

$$\frac{\partial x_3'}{\partial x_1} = 0 \qquad \frac{\partial x_3'}{\partial x_2} = 0 \qquad \frac{\partial x_3'}{x_3} = 1$$

$$A^{i'}_{j} = \begin{pmatrix} \cos\phi & \operatorname{sen}\phi & 0 \\ -\operatorname{sen}\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{i'}_{j} x_j = x_i'$$

Punto A

$r = x^i \hat{\imath}_i = x_1 \hat{\imath}_1 + x_2 \hat{\imath}_2$, en 2 dimensiones

1) $(x, y) \rightarrow (-y, x)$

$$x_1' = -x_2 \qquad x_2' = x_1$$

$$A^{i'}_{j} : \frac{\partial x_1'}{\partial x_1} = 0 \qquad \frac{\partial x_1'}{\partial x^2} = -1$$

$$\frac{\partial x_2'}{\partial x_1} = 1 \qquad \frac{\partial x_2'}{\partial x_2} = 0$$

$$A^{i'}_{j} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$x_i' = A^{i'}_{j} x_j = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$A^{i'}_{j} \cdot (A^{i'}_{j})^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$A^{i'}_{j}$ = ortogonal

$\hookrightarrow$ transforman como verdaderas componentes

2) $(x, y) \to (x, -y)$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$x_1' = x_1 \quad , \quad x_2' = -x_2$$

$$A^{i'}_{j} = \quad \frac{\partial x_1'}{\partial x_1} = 1 \qquad \frac{\partial x_1'}{\partial x_2} = 0$$

$$\frac{\partial x_2'}{\partial x_1} = 0 \qquad \frac{\partial x_2'}{\partial x_2} = -1$$

$$A^{i'}_{j} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left(A^{i'}_{j}\right)^T \left(A^{i'}_{j}\right) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$\Downarrow$

3) $(x, y) \to (x-y, x+y)$

$$x_1' = x_1 - x_2 \quad , \quad x_2' = x_1 + x_2$$

$$\left.\begin{aligned} A^{i'}_{j} = \frac{\partial x_1'}{\partial x_1} = 1 \qquad & \frac{\partial x_1'}{\partial x_2} = -1 \\ \frac{\partial x_2'}{\partial x_1} = 1 \qquad & \frac{\partial x_2'}{\partial x_2} = 1 \end{aligned}\right\} A^{i'}_{j} = \frac{\partial x_i'}{\partial x_j}$$

i, j de 1 a 2

$$A^{i'}_{j} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$(A^{i'}_{j})^T (A^{i'}_{j}) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$$

$\hookrightarrow$ No es ortogonal, no transforma como verdaderas componentes

A) $(x, y) \rightarrow (x+y, x-y)$

$x_1' = x_1 + x_2 \,, \quad x_2' = x_1 - x_2$

$A^{i'}_{\ j} = \frac{\partial x_i'}{\partial_j} = \frac{\partial x_i'}{\partial x_1} = 1 \qquad \frac{\partial x_i'}{\partial x_2} = 1$

$\frac{\partial x_2'}{\partial x_i} = 1 \qquad \frac{\partial x_2'}{\partial x_2} = -1$

$A^{i'}_{\ j} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$(A^{i'}_{\ j})^T (A^{i'}_{\ j}) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I \neq I$

$\hookrightarrow$ NO ORTOGONAL