

## Sección 1.6.6

### Punto 2

$$a) \cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\sin^2(\alpha)$$

$$b) \sin(3\alpha) = 3\cos^2(\alpha)\sin(\alpha) - \sin^3(\alpha)$$

$$e^{i\alpha} = \cos \alpha + i \sin \alpha$$

$$e^{i3\alpha} = \underbrace{\cos(3\alpha)}_{1} + i \underbrace{\sin(3\alpha)}_{2}$$

$$\hookrightarrow (e^{i\alpha})^3 = (\cos(\alpha) + i \sin(\alpha))^3$$

$$= (\cos^3(\alpha) + 3\cos^2(\alpha)i\sin(\alpha) + 3\cos(\alpha)(i\sin(\alpha))^2 + (i\sin(\alpha))^3)$$

$$\rightarrow (i\sin(\alpha))^2 = -\sin^2(\alpha), (i\sin(\alpha))^3 = (-i\sin^3(\alpha))$$

$$\hookrightarrow \cos^3(\alpha) + 3\cos^2(\alpha)i\sin(\alpha) - 3\cos(\alpha)\sin^2(\alpha) - i\sin^3(\alpha)$$

$$\underbrace{\cos^3(\alpha) - 3\cos(\alpha)\sin^2(\alpha)}_{1} + i \underbrace{(3\cos^2(\alpha)\sin(\alpha) - \sin^3(\alpha))}_{2}$$

$$1 = 2$$

$$\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\sin^2(\alpha)$$

$$\sin(3\alpha) = 3\cos^2(\alpha)\sin(\alpha) - \sin^3(\alpha)$$



## Punto 5

a)  $\sqrt{2i}$ ,  $r = |2i| = 2$

$$\theta = \arg(2i) = \pi/2$$

$$L, 2i = 2e^{i\pi/2}$$

$$\sqrt{2i} = 2^{1/2} e^{i(\frac{\pi}{2} + 2\pi k)/2} = \sqrt{2} e^{i(\frac{\pi}{4} + k\pi)}, k=0,1$$

$$\bullet k=0, \sqrt{2} e^{i\frac{\pi}{4}} = \sqrt{2} \left( \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2} \right) = 1 + i$$

$$\bullet k=1, \sqrt{2} e^{i(\frac{5\pi}{4})} = -1 - i$$

b)  $\sqrt{1 - \sqrt{3}i}$ ,  $z = 1 - \sqrt{3}i$ ,  $r = \sqrt{1 + (\sqrt{3})^2} = 2$

$$\theta = \arctan(-\sqrt{3}, 1) = -\pi/3$$

$$\sqrt{2} = \sqrt{2} e^{i(-\frac{\pi}{6} + k\pi)}, k=0,1$$

$$\bullet k=0$$

$$\begin{aligned} \sqrt{2} e^{i(-\frac{\pi}{6})} &= \sqrt{2} (\cos(-\frac{\pi}{6}) + \sin(-\frac{\pi}{6})) \\ &= \frac{\sqrt{6}}{2} - i \frac{\sqrt{2}}{2} \end{aligned}$$

$$\bullet k=1$$

$$= \frac{\sqrt{6}}{2} + i \frac{\sqrt{2}}{2}$$

c)  $\sqrt[3]{-1}$

$$-1 = e^{i(\pi + 2\pi n)}$$

$$(-1)^{1/3} = e^{i(\pi + 2\pi n)/3}, \quad n = 0, 1, 2$$

$$\theta_n: \frac{\pi + 2\pi n}{3} = \left\{ \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$$

$$\hookrightarrow e^{i\pi/3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}, -1, e^{i5\pi/3} = \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

d)  $\sqrt[6]{8}$

$$r = 8, \quad \theta = 0.8^{1/6} = (8)^{1/6} e^{i2\pi k/6}$$

$$\hookrightarrow 8 = 2^3 = \sqrt{2}^{\uparrow}, \quad \sqrt{2} e^{i\pi k/3}, \quad k = 0, 1, \dots, 5$$

$$e) \sqrt[4]{-8 - 8\sqrt{3}i}$$

$$r = \sqrt{256} = 16$$

$$\theta = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$$

$$z = 16 e^{i(-2\pi/3)}$$

$$\begin{aligned} \sqrt[4]{z} &= (16)^{1/4} e^{i(-2\pi/3 + 2\pi k)/4} \\ &= 2 e^{i(-\frac{\pi}{6} + \frac{\pi k}{2})}, \quad k = 0, 1, 2, 3 \end{aligned}$$

$$\cdot k=0 = \sqrt{3} - i$$

$$\cdot k=1 = 1 + i\sqrt{3}$$

$$\cdot k=2 = -\sqrt{3} + i$$

$$\cdot k=3 = -1 - i\sqrt{3}$$

## Punto 6

a)  $\text{Log}(-ie)$

$$r = |-ie| = e, \quad \arg(-ie) = -\frac{\pi}{2}$$

$$\text{Log}(-ie) = \text{Lne} + i\left(-\frac{\pi}{2} + 2\pi n\right) = 1 - i\frac{\pi}{2} + 2\pi in$$

$$n=0, \quad \boxed{1 - i\frac{\pi}{2}}$$

b)  $\text{Log}(1-i)$

$$r = |1-i| = \frac{1}{2}\text{Ln}2, \quad \arg(1-i) = -\frac{\pi}{4}$$

$$\text{Log}(1-i) = \frac{1}{2}\text{Ln}2 + i\left(-\frac{\pi}{4} + 2\pi n\right), \quad n=0$$

$$\boxed{\frac{1}{2}\text{Ln}2 - i\frac{\pi}{4}}$$

c)  $\text{Log}(e)$

$$r = e, \quad \arg(e) = 0$$

$$\text{Log}(e) = \text{Lne} + i(0 + 2\pi n) = 1 + 2\pi in$$

$$\boxed{1 + 2\pi in, \quad n \in \mathbb{Z}}$$

$$n=0, \quad 1 \quad \checkmark$$

d  $\text{Log}(i)$

$$r = |i| = 1, \quad \arg(i) = \frac{\pi}{2}$$

$$\log(i) = \ln|i| + i \left( \frac{\pi}{2} + 2\pi n \right)$$

$$\log(i) = \boxed{\left( 2n + \frac{1}{2} \right) \pi i, \quad n \in \mathbb{Z}}$$