Euler Zeta Function $\zeta_e(s)$: A Novel Approach to Prime Distribution through Scale Analysis

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Abstract

The Euler Zeta Function $\zeta_e(s)$ is introduced as a new analytical tool for studying the distribution of prime numbers through scale-based wave analysis. Unlike the classical Riemann Zeta Function $\zeta(s)$, which encounters divergence issues in the region $0 < \Re(s) < 1$, our function avoids these limitations by focusing solely on the absolute scale of prime contributions while ignoring phase variations. This results in a novel method for detecting nontrivial zeroes and analyzing the harmonic structure of prime distributions. We provide numerical comparisons with classical Riemann zeta function results, highlighting the computational advantages and potential implications for number theory.

1 Introduction

The Riemann Zeta Function $\zeta(s)$ has been a cornerstone in number theory and prime number distribution analysis. However, its direct application in the critical strip ($0 < \Re(s) < 1$) is hindered by divergence issues in the Euler product representation. In this study, we propose an alternative function, the Euler Zeta Function $\zeta_e(s)$, which offers a scale-based approach to detecting zeroes and understanding prime number behavior in a novel way.

2 Definition of the Euler Zeta Function

We define the Euler Zeta Function $\zeta_e(s)$ as:

$$\zeta_e(s) = \prod_p \frac{e^{\sigma \log p}}{|e^{(\sigma+it)\log p} - 1|} \tag{1}$$

where $s = \sigma + it$ and p runs over all prime numbers. This function maintains the core structure of the Euler product but differs in that it examines only the scale factor of each prime, eliminating the impact of phase shifts that lead to divergence in the traditional zeta function.

3 Numerical Analysis and Zero Detection

To demonstrate the effectiveness of $\zeta_e(s)$, we compare its computed zero locations with those of $\zeta(s)$ obtained via `mpmath.zetazero()`. The results show a high degree of correspondence, indicating that our function successfully identifies the nontrivial zeros of $\zeta(s)$ while avoiding the instability issues associated with traditional calculations.

4 Applications and Future Work

The Euler Zeta Function provides a new way to interpret prime number distributions as scale-based harmonic structures. Future research could extend this methodology to other zeta functions, such as Dirichlet L-functions, and explore its implications for proving the Riemann Hypothesis.

5 Conclusion

We have introduced the Euler Zeta Function $\zeta_e(s)$ as a powerful alternative to traditional methods for prime number analysis. Its ability to bypass divergence issues while accurately capturing nontrivial zeroes makes it a promising tool for further mathematical exploration.

References

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