Research Proposal

Dazhi Huang

Abstract

This research proposal outlines a plan to enhance the topological accuracy of deep learning-based image segmentation. To address topological errors in existing methods, I propose a two-stage research approach. First, I will develop a framework incorporating topological constraints for myocardial pathology segmentation, aiming to improve precision for features like scar and edema. Second, I will design a computationally efficient topological loss function that provides informative gradients for robust optimization. The proposed method will be evaluated across diverse datasets to ensure generalizability. The expected outcome is a practical approach that significantly improves segmentation performance and topological accuracy.

Keywords: Image Segmentation, Topological Data Analysis, Deep Learning, Loss Function

1 Research Background

In many image segmentation and structural analysis applications, preserving the topological integrity of segmented objects is arguably more critical than solely optimizing pixel-wise accuracy metrics. For instance, in medical image analysis, the correct topology of structures such as vascular networks [1], cardiac chambers [2], or neural tracts [3] is fundamental for reliable diagnosis and subsequent functional assessment. Despite achieving high pixel-level accuracy, segmentation algorithms frequently introduce topological errors, such as spurious connections or disconnected components. Standard pixel-based loss functions, including the widely used Dice coefficient, are inherently limited in addressing these topological concerns; they minimize local discrepancies but remain insensitive to global structural changes that can arise from minor, even single-pixel, errors. Consequently, seemingly small inaccuracies at the pixel level can cascade into significant topological failures.

While topological loss functions have been proposed to enforce topological correctness, existing methods often suffer from several drawbacks: **(a)** they can be computationally expensive, hindering practical application; **(b)** they typically focus on global topological invariants, potentially overlooking finer local structural details; and **(c)** their gradient computation, often relying only on critical pixels (e.g., birth and death pixels of topological features), can lead to slow or sparse parameter updates during training. This highlights a pressing need for novel, efficient methods that can overcome these limitations and provide robust topological guarantees for image segmentation.

1.1 Cubical Homology and Image Segmentation

In image segmentation, where data typically originates from 2D images or 3D volumes, a cubical complex provides a natural framework for representing the underlying structure. A cubical complex comprises elements such as points, unit line segments, unit squares, cubes, and higher-dimensional hypercubes. Formally, we define an *elementary interval* as a closed subset of the real line, denoted $I = [z, z+1]$ for $z \in \mathbb{Z}$. *Elementary cubes*, which correspond to pixels in 2D or voxels in 3D, are constructed as the Cartesian product of these intervals. For a k-dimensional space, an elementary cube is expressed as $Q = I_1 \times I_2 \times \cdots \times I_k$. In this study, we focus on the two-dimensional case, where the region associated with a pixel at row i and column j in an image is represented by $Q_{ij} = [i, i+1] \times [j, j+1]$.

Consider an image of size $N_x \times N_y$, represented as a 2D array X with pixel intensities X_{ij}. Alongside this, a predicted binary segmentation S, also a 2D array, assigns values $S_{ij} \in [0, 1]$, where S_{ij} denotes the predicted probability that the pixel at position (i, j) belongs to the segmented object. This segmentation is derived from a function $S = f(X; \omega)$, where f is typically a convolutional neural network (CNN) parameterized by weights ω.

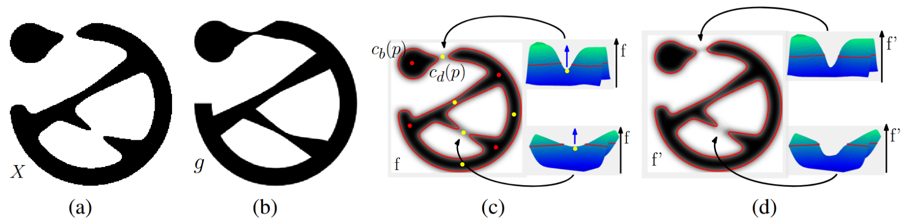


Figure 1: Illustration of topology in segmentation likelihoods. Higher function values are depicted as darker areas for visualization. (a) An example segmentation X with two components and one handle. (b) The ground truth with one connected component and two handles, also interpretable as a binary function g. (c) A likelihood map f yielding segmentation X (bounded by the red curve), with landscape views near the broken bridge and handle; critical points are highlighted. (d) Another likelihood map f' with the same segmentation as f, yet landscape views reveal deeper gaps, indicating poorer topological fidelity.

To explore the topological properties of the segmentation, we analyze the *super-level sets* of S, defined as the collection of pixels whose predicted probabilities exceed a threshold p. Mathematically, the super-level set $B(p)$ is given by:

$$B(p) = \bigcup_{\{i,j|S_{ij} \geq p\}} Q_{ij}. \tag{1}$$

As the threshold p decreases, a nested sequence of sets emerges:

$$\emptyset = B(1) \subseteq B(p_1) \subseteq B(p_2) \subseteq \cdots \subseteq B(0) \subseteq [0, N_x] \times [0, N_y], \tag{2}$$

where $p_1 > p_2 > \cdots > 0$. With each decrease in p, the topology of $B(p)$ evolves—new features, such as connected components or holes, may appear, while existing ones may merge or vanish.

This topological evolution is critical for understanding segmentation quality. For example, Figure 1 from [4] illustrates the limitations of relying on a single threshold, such as $p = 0.5$. The figure compares two likelihood maps that produce identical segmentations at a fixed threshold but exhibit distinct topological properties across varying thresholds. This underscores the need to examine the entire filtration of super-level sets to ensure topological correctness in image segmentation.

2 Literature Review

Significant progress has been made in developing image segmentation methods that preserve topological accuracy. Notably, the use of Persistent Homology (PH) based loss functions for training segmentation networks has shown promise [4, 5]. These approaches aim to ensure global topological correctness, often by encouraging the Betti numbers of the predicted segmentation to match known target values when the loss is minimized.

A specific topological loss function was developed by Clough et al. [5] which possesses the distinct advantage of not requiring a ground truth segmentation mask. Instead, it only requires prior knowledge of the target Betti numbers (β_k^*) for the object being segmented. The loss function is defined as follows:

$$\mathcal{L}_k(\beta_k^*) = \sum_{\ell=1}^{\beta_k^*} (1 - |b_{k,\ell} - d_{k,\ell}|^2) + \sum_{\ell=\beta_k^*+1}^{\infty} |b_{k,\ell} - d_{k,\ell}|^2 \tag{3}$$

$$\mathcal{L}_{\text{topo}} = \sum_k \mathcal{L}_k(\beta_k^*) \tag{4}$$

Here, $\mathcal{L}_k(\beta_k^*)$ represents the loss component for the k-th dimension, and β_k^* encodes the prior knowledge about the desired number of k-dimensional topological features. For instance, β_0^* specifies the target number of connected components, β_1^* the number of loops (or holes), and β_2^* the number of enclosed voids. The terms $b_{k,\ell}$ and $d_{k,\ell}$ denote the birth and death filtration values (often corresponding to probability thresholds in the segmentation map) of the ℓ-th most persistent

k-dimensional feature identified by PH analysis of the cubical complex derived from the segmentation output. This loss function $\mathcal{L}_{\text{topo}}$ is minimized when the resulting persistence barcode diagram contains precisely β_k^* features (bars) of maximum persistence (length 1) for each dimension k, and no other persistent features.

A key characteristic of this loss function is its independence from the ground truth segmentation mask. This property facilitates the development of semi-supervised learning frameworks, which can be particularly valuable in scenarios where acquiring extensive pixel-level annotations is costly or impractical, but prior topological information is readily available.

However, a limitation of topological loss functions like the one proposed by Clough et al. [5] is that they primarily enforce global topological properties (i.e., correct Betti numbers overall) but may not guarantee the spatial localization of these features. Preserving topological features accurately in space can be crucial for meaningful segmentation. Addressing this gap, Stucki et al. [6] introduced the concept of induced matchings derived from persistent homology theory. This allows for a spatially coherent matching between the persistence barcodes of a predicted segmentation and a ground truth or reference. Building upon this, they proposed the Betti matching error, a metric designed to be interpretable and sensitive to both topological and feature-wise spatial accuracy in image segmentations, thereby overcoming some limitations associated with relying solely on Betti number comparison.

However, there are still two issues with the methods of both Clough et al. [5] and Stucki et al. [6]. First, the computational cost is too high. Another limitation is the gradient's dependence on only two critical values (corresponding to the birth and death times, b and d, of the topological structure). More computationally efficient methods were introduced by Hu et al. [7, 8]. They used discrete Morse theory and persistent homology to learn the structural representation of images for fine-scale structure segmentation, while offering limited guarantees of topological correctness.

Shifting the focus from direct loss function design to addressing practical constraints in medical imaging, Santhirasekaram et al. [2] capitalized on the observation that medical images often exhibit limited structural variability across patients. Their approach involves two main stages. First, they constrain the latent space of a deep learning-based segmentation model using a learned dictionary [9] comprising fundamental structural components. Second, they incorporate a topological prior, leveraging persistent homology, during the sampling process from this dictionary. This strategy aims to ensure the topological correctness of the final segmentation formed by the composition of these dictionary components, offering an alternative route to topologically accurate results by constraining the model's generative process.

Non-TDA-based methods also exist to ensure topological correctness. For example, TopoGraph [10] is a graph-based loss function designed for topologically accurate image segmentation that is presented as both computationally efficient and generally applicable. This method constructs a component graph that fully encodes the topological information of both the prediction and the ground truth. This representation allows for the efficient identification of topologically critical regions and the aggregation of a loss based on local neighborhood information. However, this method is currently designed primarily for 2D images and reportedly lacks experiments demonstrating its applicability to 3D images.

3 Research Aim

The purpose of this research is to develop and evaluate a novel topological regularization method for deep learning-based image segmentation. This method will be specifically designed to simultaneously address the aforementioned limitations by: (1) incorporating mechanisms to explicitly encourage the accurate spatial placement of topological features, moving beyond global invariants, and (2) formulating a loss component that provides denser and more informative gradients with respect to the segmentation output, thereby facilitating more stable and efficient optimization. Ultimately, this research seeks to provide a more robust and practical approach for ensuring both global and spatially-precise topological fidelity in image segmentation tasks, particularly in domains like medical imaging where structural integrity is paramount.

4 Research Methods

The research methodology is structured into two sequential stages, each designed to address the limitations of existing topological approaches in image segmentation while prioritizing computational efficiency and practical applicability.

4.1 Stage One: Enhancing Topological Accuracy in Myocardial Pathology Segmentation

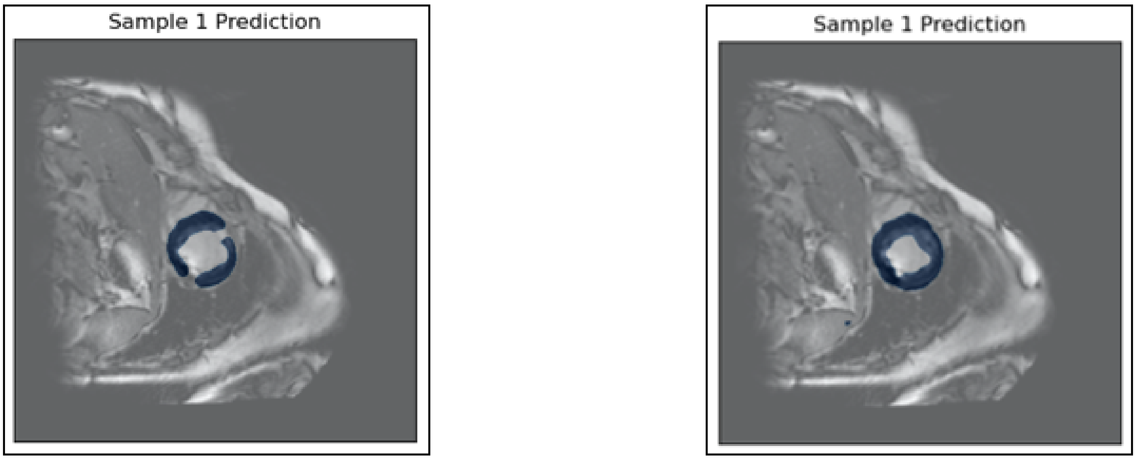
In the first stage, I will focus on the myocardial pathology segmentation task [11], aiming to improve the topological accuracy in segmenting scar and edema regions. Initially, I will acquire and preprocess relevant datasets, ensuring that the images are standardized in terms of resolution and intensity, and that ground truth annotations for scar and edema are properly prepared. Following this, I will replicate established benchmark methods to establish a performance baseline, which is crucial for quantifying the improvements of my proposed framework. The core of this stage involves developing an innovative framework that integrates topological constraints into the segmentation process. This will be achieved by capturing and preserving critical topological features, such as connected components and holes, in the segmented regions, enhancing the precision of segmentation predictions for scar and edema by ensuring that the topological properties of the predicted segments align with the ground truth.

4.2 Stage Two: Development of a Precise and Efficient Topological Loss Function

The second stage is dedicated to designing a more precise and computationally efficient topological loss function, combining both empirical and theoretical efforts. I will begin by conducting experiments to evaluate the performance of the proposed loss function against existing topological loss functions, focusing on its ability to enforce topological correctness while maintaining or enhancing segmentation accuracy. Concurrently, I will derive the theoretical foundations necessary to ensure that the loss function provides informative gradients for optimization, analyzing the gradient flow relative to the segmentation output to guide the model toward solutions that are both topologically accurate and spatially precise. To validate the generalizability of the proposed methodology, I will collect and apply datasets from diverse domains where topological accuracy is critical, including vessel segmentation, road network segmentation, and neural segmentation. By evaluating the loss function's performance across these varied tasks, I aim to demonstrate its broad applicability. A key aspect of this stage is optimizing the loss function's computational efficiency, leveraging GPU-accelerated persistent homology tools, such as those proposed by Kim et al. [12] and Gabrielsson et al. [13]. These tools will streamline the computation of topological features, enabling the loss function to support stable and efficient optimization within deep learning-based segmentation architectures. Ultimately, the goal is to create a loss function that not only improves topological accuracy but also enhances the training process by delivering denser, more informative gradients, addressing the limitations of existing methods that rely on sparse critical points.

5 Expected Results

The anticipated outcomes of this research are aligned with the two methodological stages outlined previously. During the first stage, I am currently engaged in a research internship at the **Hong Kong University of Science and Technology (Guangzhou)**, where preliminary work on the myocardial pathology segmentation task has already commenced. Leveraging this ongoing internship, preliminary experiments on myocardial datasets have demonstrated promising results, with the incorporation of a topological loss function leading to a 2–3% improvement in the Dice score on the validation set compared to models that do not account for topological accuracy. It is projected that by December 2025, the devised framework will achieve superior performance in the myocardial pathology segmentation task, surpassing existing benchmarks that neglect topological considerations across key metrics such as Cross-Entropy (CE) and Dice Similarity Coefficient (DSC). The findings from this stage will be synthesized into a scholarly manuscript, intended for submission to a prestigious journal or conference.



(a) CE loss (b) CE + topological loss

Figure 2: Topological loss function can optimize the topological inaccuracies effectively

Building upon the success of the first stage, the second stage will focus on developing a novel topological loss function that not only enforces topological correctness but also enhances gradient computation and accelerates parameter optimization. This loss function will be meticulously designed to capture both global topological invariants and fine-grained structural details, ensuring its broad applicability across diverse segmentation tasks. To validate its effectiveness and generalizability, the proposed loss function will be rigorously evaluated on multiple datasets from various domains, including vessel segmentation, road network segmentation, and neural segmentation. The completion of this stage is anticipated during the latter phase of the research internship and the initial months of the MPhil program at the **Hong Kong University of Science and Technology (Guangzhou)**. The insights and methodologies developed in this stage will be disseminated through high-impact academic publications, contributing to the advancement of topological data analysis in image segmentation.

Ultimately, this research endeavors to make significant strides in the field of intelligent medical health by providing robust tools for topologically accurate image segmentation. Furthermore, to maximize the impact and accessibility of this work, all developed methodologies and code will be made open-source on GitHub, fostering collaboration and further innovation within the scientific community.

References

[1] Mihail Ivilinov Todorov, Johannes Christian Paetzold, Oliver Schoppe, Giles Tetteh, Suprosanna Shit, Velizar Efremov, Katalin Todorov-Völgyi, Marco Düring, Martin Dichgans, Marie Piraud, et al. Machine learning analysis of whole mouse brain vasculature. *Nature methods*, 17(4):442–449, 2020.

[2] Ainkaran Santhirasekaram, Mathias Winkler, Andrea Rockall, and Ben Glocker. Topology preserving compositionality for robust medical image segmentation. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 543–552, 2023.

[3] Rajitha Kv, Keerthana Prasad, and Prakash Peralam Yegneswaran. Segmentation and classification approaches of clinically relevant curvilinear structures: A review. *Journal of Medical Systems*, 47(1):40, 2023.

[4] Xiaoling Hu, Fuxin Li, Dimitris Samaras, and Chao Chen. Topology-preserving deep image segmentation. *Advances in neural information processing systems*, 32, 2019.

[5] James R Clough, Nicholas Byrne, Ilkay Oksuz, Veronika A Zimmer, Julia A Schnabel, and Andrew P King. A topological loss function for deep-learning based image segmentation using persistent homology. *IEEE transactions on pattern analysis and machine intelligence*, 44(12):8766–8778, 2020.

[6] Nico Stucki, Johannes C Paetzold, Suprosanna Shit, Bjoern Menze, and Ulrich Bauer. Topologically faithful image segmentation via induced matching of persistence barcodes. In *International Conference on Machine Learning*, pages 32698–32727. PMLR, 2023.

[7] Xiaoling Hu, Yusu Wang, Li Fuxin, Dimitris Samaras, and Chao Chen. Topology-aware segmentation using discrete morse theory. In *International Conference on Learning Representations*, 2021.

[8] Xiaoling Hu, Dimitris Samaras, and Chao Chen. Learning probabilistic topological representations using discrete morse theory. In *International Conference on Learning Representations*, 2023.

[9] Aaron Van Den Oord, Oriol Vinyals, et al. Neural discrete representation learning. *Advances in neural information processing systems*, 30, 2017.

[10] Laurin Lux, Alexander H Berger, Alexander Weers, Nico Stucki, Daniel Rueckert, Ulrich Bauer, and Johannes C. Paetzold. Topograph: An efficient graph-based framework for strictly topology preserving image segmentation. In *The Thirteenth International Conference on Learning Representations*, 2025.

[11] Lei Li, Fuping Wu, Sihan Wang, Xinzhe Luo, Carlos Martín-Isla, Shuwei Zhai, Jianpeng Zhang, Yanfei Liu, Zhen Zhang, Markus J Ankenbrand, et al. Myops: A benchmark of myocardial pathology segmentation combining three-sequence cardiac magnetic resonance images. *Medical Image Analysis*, 87:102808, 2023.

[12] Kwangho Kim, Jisu Kim, Manzil Zaheer, Joon Kim, Frédéric Chazal, and Larry Wasserman. Pllay: Efficient topological layer based on persistent landscapes. *Advances in Neural Information Processing Systems*, 33:15965–15977, 2020.

[13] Rickard Brüel Gabrielsson, Bradley J Nelson, Anjan Dwaraknath, and Primoz Skraba. A topology layer for machine learning. In *International Conference on Artificial Intelligence and Statistics*, pages 1553–1563. PMLR, 2020.