Research Proposal

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Abstract

This research proposal outlines a plan to enhance the topological accuracy of deep learning-based image segmentation. To address topological errors in existing methods, I propose a two-stage research approach. First, I will develop a framework incorporating topological constraints for myocardial pathology segmentation, aiming to improve precision for features like scar and edema. Second, I will design a computationally efficient topological loss function that provides informative gradients for robust optimization. The proposed method will be evaluated across diverse datasets to ensure generalizability. The expected outcome is a practical approach that significantly improves segmentation performance and topological
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1 Research Background

In many image segmentation and structural analysis applications, preserving the topological integrity of segmented objects is arguably more critical than solely optimizing pixel-wise accuracy metrics. For instance, in medical image analysis, the correct topology of structures such as vascular networks [1], cardiac chambers [2], or neural tracts [3] is fundamental for reliable diagnosis and subsequent functional assessment. Despite achieving high pixel-level accuracy, segmentation algorithms frequently introduce topological errors, such as spurious connections or disconnected components. Standard pixel-based loss functions, including the widely used Dice coefficient, are inherently limited in addressing these topological concerns; they minimize local discrepancies but remain insensitive to global structural changes that can arise from minor, even single-pixel, errors. Consequently, seemingly small inaccuracies at the pixel level can cascade into significant topological failures.

While topological loss functions have been proposed to enforce topological correctness, existing methods often suffer from several drawbacks: **(a)** they can be computationally expensive, hindering practical application; **(b)** they typically focus on global topological invariants, potentially overlooking finer local structural details; and **(c)** their gradient computation, often relying only on critical pixels (e.g., birth and death pixels of topological features), can lead to slow or sparse parameter updates during training. This highlights a pressing need for novel, efficient methods that can overcome these limitations and provide robust topological guarantees for image segmentation.

1.1 Cubical Homology and Image Segmentation

In image segmentation, where data typically originates from 2D images or 3D volumes, a cubical complex provides a natural framework for representing the underlying structure. A cubical complex comprises elements such as points, unit line segments, unit squares, cubes, and higher-dimensional hypercubes. Formally, we define an *elementary interval* as a closed subset of the real line, denoted $I = [z, z+1]$ for $z \in \mathbb{Z}$. *Elementary cubes*, which correspond to pixels in 2D or voxels in 3D, are constructed as the Cartesian product of these intervals. For a k-dimensional space, an elementary cube is expressed as $Q = I_1 \times I_2 \times \cdots \times I_k$. In this study, we focus on the two-dimensional case, where the region associated with a pixel at row i and column j in an image is represented by $Q_{ij} = [i, i+1] \times [j, j+1]$.

Consider an image of size $N_x \times N_y$, represented as a 2D array X with pixel intensities X_{ij}. Alongside this, a predicted binary segmentation S, also a 2D array, assigns values $S_{ij} \in [0, 1]$, where S_{ij} denotes the predicted probability that the pixel at position (i, j) belongs to the segmented object. This segmentation is derived from a function $S = f(X; \omega)$, where f is typically a convolutional neural network (CNN) parameterized by weights ω.

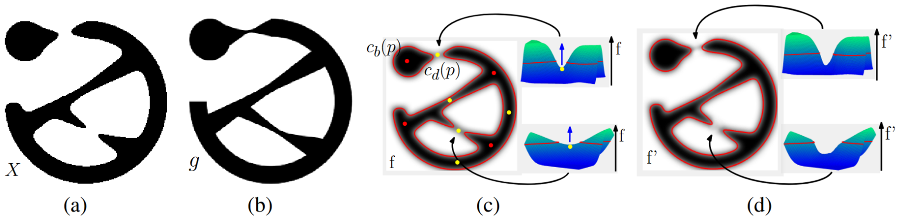

Figure 1: Illustration of topology in segmentation likelihoods. Higher function values are depicted as darker areas for visualization. (a) An example segmentation X with two components and one handle. (b) The ground truth with one connected component and two handles, also interpretable as a binary function g. (c) A likelihood map f yielding segmentation X (bounded by the red curve), with landscape views near the broken bridge and handle; critical points are highlighted. (d) Another likelihood map f' with the same segmentation as f, yet landscape views reveal deeper gaps, indicating poorer topological fidelity.

To explore the topological properties of the segmentation, we analyze the *super-level sets* of S, defined as the collection of pixels whose predicted probabilities exceed a threshold p. Mathematically, the super-level set $B(p)$ is given by:

$$B(p) = \bigcup_{\{i,j|S_{ij}\geq p\}} Q_{ij}. \tag{1}$$

As the threshold p decreases, a nested sequence of sets emerges:

$$\emptyset = B(1) \subseteq B(p_1) \subseteq B(p_2) \subseteq \cdots \subseteq B(0) \subseteq [0, N_x] \times [0, N_y], \tag{2}$$

where $p_1 > p_2 > \cdots > 0$. With each decrease in p, the topology of $B(p)$ evolves—new features, such as connected components or holes, may appear, while existing ones may merge or vanish.

This topological evolution is critical for understanding segmentation quality. For example, Figure 1 from [4] illustrates the limitations of relying on a single threshold, such as $p = 0.5$. The figure compares two likelihood maps that produce identical segmentations at a fixed threshold but exhibit distinct topological properties across varying thresholds. This underscores the need to examine the entire filtration of super-level sets to ensure topological correctness in image segmentation.

2 Literature Review

Significant progress has been made in developing image segmentation methods that preserve topological accuracy. Notably, the use of Persistent Homology (PH) based loss functions for training segmentation networks has shown promise [4, 5]. These approaches aim to ensure global topological correctness, often by encouraging the Betti numbers of the predicted segmentation to match known target values when the loss is minimized.

A specific topological loss function was developed by Clough et al. [5] which possesses the distinct advantage of not requiring a ground truth segmentation mask. Instead, it only requires prior knowledge of the target Betti numbers (β_k^*) for the object being segmented. The loss function is defined as follows:

$$\mathcal{L}_k(\beta_k^*) = \sum_{\ell=1}^{\beta_k^*}(1 - |b_{k,\ell} - d_{k,\ell}|^2) + \sum_{\ell=\beta_k^*+1}^{\infty} |b_{k,\ell} - d_{k,\ell}|^2 \tag{3}$$

$$\mathcal{L}_{\text{topo}} = \sum_k \mathcal{L}_k(\beta_k^*) \tag{4}$$

Here, $\mathcal{L}_k(\beta_k^*)$ represents the loss component for the k-th dimension, and β_k^* encodes the prior knowledge about the desired number of k-dimensional topological features. For instance, β_0^* specifies the target number of connected components, β_1^* the number of loops (or holes), and β_2^* the number of enclosed voids. The terms $b_{k,\ell}$ and $d_{k,\ell}$ denote the birth and death filtration values (often corresponding to probability thresholds in the segmentation map) of the ℓ-th most persistent

k-dimensional feature identified by PH analysis of the cubical complex derived from the segmentation output. This loss function $\mathcal{L}_{\text{topo}}$ is minimized when the resulting persistence barcode diagram contains precisely β_k^* features (bars) of maximum persistence (length 1) for each dimension k, and no other persistent features.

A key characteristic of this loss function is its independence from the ground truth segmentation mask. This property facilitates the development of semi-supervised learning frameworks, which can be particularly valuable in scenarios where acquiring extensive pixel-level annotations is costly or impractical, but prior topological information is readily available.

However, a limitation of topological loss functions like the one proposed by Clough et al. [5] is that they primarily enforce global topological properties (i.e., correct Betti numbers overall) but may not guarantee the spatial localization of these features. Preserving topological features accurately in space can be crucial for meaningful segmentation. Addressing this gap, Stucki et al. [6] introduced the concept of induced matchings derived from persistent homology theory. This allows for a spatially coherent matching between the persistence barcodes of a predicted segmentation and a ground truth or reference. Building upon this, they proposed the Betti matching error, a metric designed to be interpretable and sensitive to both topological and feature-wise spatial accuracy in image segmentations, thereby overcoming some limitations associated with relying solely on Betti number comparison.

However, there are still two issues with the methods of both Clough et al. [5] and Stucki et al. [6]. First, the computational cost is too high. Another limitation is the gradient's dependence on only two critical values (corresponding to the birth and death times, b and d, of the topological structure). More computationally efficient methods were introduced by Hu et al. [7, 8]. They used discrete Morse theory and persistent homology to learn the structural representation of images for fine-scale structure segmentation, while offering limited guarantees of topological correctness.

Shifting the focus from direct loss function design to addressing practical constraints in medical imaging, Santhirasekaram et al. [2] capitalized on the observation that medical images often exhibit limited structural variability across patients. Their approach involves two main stages. First, they constrain the latent space of a deep learning-based segmentation model using a learned dictionary [9] comprising fundamental structural components. Second, they incorporate a topological prior, leveraging persistent homology, during the sampling process from this dictionary. This strategy aims to ensure the topological correctness of the final segmentation formed by the composition of these dictionary components, offering an alternative route to topologically accurate results by constraining the model's generative process.

Non-TDA-based methods also exist to ensure topological correctness. For example, TopoGraph [10] is a graph-based loss function designed for topologically accurate image segmentation that is presented as both computationally efficient and generally applicable. This method constructs a component graph that fully encodes the topological information of both the prediction and the ground truth. This representation allows for the efficient identification of topologically critical regions and the aggregation of a loss based on local neighborhood information. However, this method is currently designed primarily for 2D images and reportedly lacks experiments demonstrating its applicability to 3D images.

3 Research Aim

The purpose of this research is to develop and evaluate a novel topological regularization method for deep learning-based image segmentation. This method will be specifically designed to simultaneously address the aforementioned limitations by: (1) incorporating mechanisms to explicitly encourage the accurate spatial placement of topological features, moving beyond global invariants, and (2) formulating a loss component that provides denser and more informative gradients with respect to the segmentation output, thereby facilitating more stable and efficient optimization. Ultimately, this research seeks to provide a more robust and practical approach for ensuring both global and spatially-precise topological fidelity in image segmentation tasks, particularly in domains like medical imaging where structural integrity is paramount.

4 Research Methods

The research methodology is structured into two sequential stages. The initial stage concentrates on the myocardial pathology segmentation task [11], with the objective of enhancing the topological accuracy in segmenting scar and edema regions. This stage encompasses the following steps: (1) acquisition and preprocessing of pertinent datasets; (2) replication of established benchmark methods to ascertain baseline performance; and (3) formulation of an innovative framework that integrates topological constraints to augment the precision of segmentation predictions. This framework is anticipated to deliver heightened topological fidelity, thereby elevating the overall efficacy of the segmentation process.

The subsequent stage is devoted to the creation of a more precise and computationally efficient topological loss function. This phase will entail both empirical and theoretical investigations, including: (1) execution of experiments to assess the loss function's performance; (2) derivation of theoretical underpinnings to guarantee the provision of informative gradients for optimization; and (3) aggregation and application of datasets from diverse domains—such as vessel segmentation, road network segmentation, and neural segmentation—to substantiate the generalizability of the proposed methodology. The overarching aim of this stage is to establish a loss function that not only ameliorates topological accuracy but also promotes stable and efficient optimization within deep learning-based segmentation architectures.

5 Expected Results

The anticipated outcomes of this research are aligned with the two methodological stages outlined previously. During the first stage, I am currently engaged in a research internship at the **Hong Kong University of Science and Technology (Guangzhou)**, where preliminary work on the myocardial pathology segmentation task has already commenced. Leveraging this ongoing internship, it is projected that by December 2025, the devised framework will demonstrate superior performance in the myocardial pathology segmentation task, surpassing existing benchmarks across key metrics such as Cross-Entropy (CE) and Dice Similarity Coefficient (DSC). The findings from this stage will be synthesized into a scholarly manuscript, intended for submission to a prestigious journal or conference.

In the second stage, the research aims to develop a novel topological loss function that enhances gradient computation and accelerates parameter optimization. This loss function will be designed to capture both global and fine-grained topological features, ensuring its applicability across diverse segmentation tasks. Its effectiveness and generalizability will be rigorously evaluated on multiple datasets from various domains, including vessel segmentation, road network segmentation, and neural segmentation. The completion of this stage is anticipated during the latter phase of the research internship and the initial months of the MPhil program at the **Hong Kong University of Science and Technology (Guangzhou)**. The insights gained from this stage will be disseminated through high-impact academic publications.

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