THE STATIC MAGNETIC FIELD

Magnetic field is a force field associated with a region in which charges in motion experience forces. Fundamentaly, when a conductor is carrying current, there may be a mechanical force exerted upon it. This is quite distinct from electrostatic force and from all non-electrical forces, for it disappears when current ceases to flow. This force is observed when the conductor that is also carrying current or when it is in the neighborhood of a magnet. It is therefore called magnetic force. The concept of magnetic field is introduced here by considering charges that are moving in a region of space with constant velocity and experience forces. These forces experienced by moving charges are in addition to any electric forces experienced by them by virtue of an electric field in the region. When charges are moving with constant velocity, the resulting force field is magnetic field that is called a magnetostatic field. A magnetic field is therefore said to exist at a point if a force (over and above any electrostatic force) is exerted on a moving charge at the point. Just as the electric field is an effect of electrical charge distributed throughout space, the magnetic field is an effect of electrical current distributed throughout space. Although there are many similarities between magnetic and electric fields, there are also some significant differences.

The resulting formula for the magnetic field is commonly referred to as the Biot-Savart law, named after Jean-Baptiste (1774-1862) and Felix Savart (1791-1841). Ampere's force law, developed in the early nineteenth century by Andre Marie Ampere (1775-1836), was based on the Analysis of experimental measurements and resulted in a quantitative formula describing the forces between pairs of current elements.

The concept of a current element is then extended to include current elements for electrical currents flowing on surface (e.g; electrical current flowing on a metal sheet) and throughout volumes (e.g, electrical current flowing in solid conductor).

In this chapter, the Biot-Savart law will be treated as a fundamental postulate. We will also obtain Ampere's circuital law, which describes the relation between the circulation integral of the magnetic field and the amount of current encircled by the circulation path.

3.1 TYPES OF MAGNETIC FIELD SOURCES

There are three types of magnetic field sources:

(i) Steady current

(ii) Permanent magnet

(iii) An electric field changing linearly with time.

3.2 MAGNETOSTATIC FIELD INTENSITY.

Our study of magnetostatic field shall begin with types of current configuration and the experimental law of Biot and Savart. The three basic current configuration or distributions: (a) filamentary, (b) Surface, (c) volume.

BIOT - SAVART LAW.

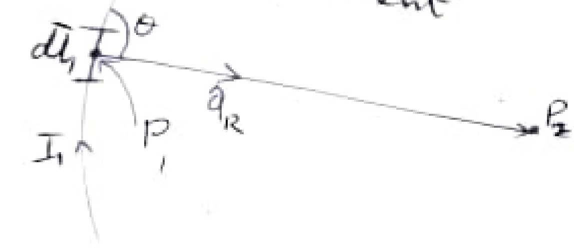


Fig 1 Field of a current carrying element.

This law states that at any point P_2 the magnitude of the magnetic field intensity produced by a differential vector length $d\bar{l}_1$, is proportional to the product of the current I flowing through the differential element, the magnitude of the differential element and the sine of the angle b/w the element and a line connecting the element to a point where the field is desired. The magnitude is also inversely proportional to the square of the distance between the source (differential element) and the point where the field is desired. Refering to fig 1, this law can best be stated in vector form as:

$$d\bar{H}_2 = \frac{I_1 d\bar{l}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2} \quad Am^{-1} \qquad (1)$$

The constant of proportionality is $\frac{1}{4\pi}$ where the subscripts indicate the point to which the quantities refer, and

I_1 = filamentary current at P_1 (A)

$d\bar{l}_1$ = Vector length of current path (vector direction same as conventional current) at P_1 (m)

$a_{R_{12}}$ = Unit vector directed from the current element $I_1 d\bar{l}_1$ to the location of dH_2, the distance between P_1 and P_2 (m)

$d\bar{H}_2$ = Vector magnetostatic field intensity at P_2 (Am^{-1})

Biot-Savart law is similar to coulombs law written for a differential element of charge

$$d\bar{E} = \frac{dQ\,\hat{a}_{R_{12}}}{4\pi\varepsilon_0 R_{12}^2} \quad (Vm^{-1}) \qquad (2)$$

in that they share the same inverse square law dependence on distance and show the same linear relationship between field and source.

The differences appear only in the direction of their fields, while the electric field is directed along the line joining the source and the point where it is to be measured the magnetic field intensity is normal to the plane containing the differential element and the line drawn from the element to the point where it is desired.

Since d.c sources are independent of time then applying the point form of the continuity equation

$$\nabla \cdot \bar{J} = -\frac{d\rho_v}{dt}, \qquad (3)$$

since ρ_v does not depend on time

$$\frac{d\rho_v}{dt} = 0 \qquad (4)$$

$\therefore$ $\nabla \cdot \bar{J} = 0$ or applying the divergence theorem.

$$\oint \bar{J} \cdot d\bar{s} = 0 \qquad (5)$$

This means that the total current crossing any closed surface is equal to zero.

This is satisfied by assuming that the current flows through a closed path. The current flowing through a closed path must be our source and not the differential element which can not be isolated. It then follows that only the integral form of Biot - Savart law that can be verified experimentally

$$\bar{H} = \frac{\oint i_1 d\bar{l}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2} \qquad (6)$$

Generally. for a current density $\bar{J}$

$$\bar{H} = \frac{\oint \bar{J} \times \hat{a}_{R_{12}}\, dv}{4\pi R_{12}^2} \qquad (7)$$

where $\bar{J} \times dv = d\bar{I}$

and $d\bar{I} = i_1 d\bar{l}_1$

3.3 APPLICATION OF BIOT SAVART LAW TO AN INFINITELY LONG STRAIGHT FILAMENT

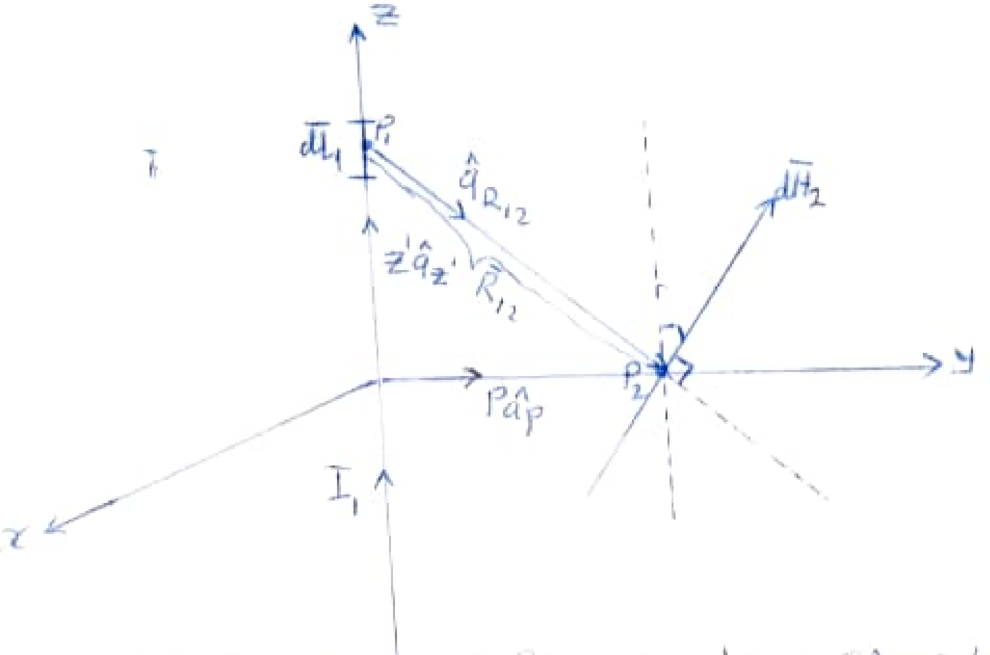


Fig Field of an infinitely long straight filament.

Fig shows graphically the relationship between the quantities found in the Biot-Savart law. The direction of $d\bar{H}_2$ comes from $\bar{dL}_1 \times a_{R_{12}}$ and thus is perpendicular to $\bar{dL}_1$ and $\hat{a}_{R_{12}}$. Point 2 at which we are to determine the field is chosen in the $z=0$ plane and we have

$$\bar{r} = \rho\hat{a}_\rho \quad \text{and} \quad r' = z'\hat{a}_{z'} \tag{1}$$

adding vectorially,

$$\bar{R}_{12} = \bar{r} - r' = \rho\hat{a}_\rho - z'\hat{a}_{z'} \tag{2}$$

so that the unit vector

$$\hat{a}_{R_{12}} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\rho\hat{a}_\rho - z'\hat{a}_{z'}}{\sqrt{\rho^2 + z'^2}} \tag{3}$$

If we take $\bar{dL}_1 = dz'\hat{a}_{z'}$ then equation (3.2.1) becomes

$$d\bar{H}_2 = \frac{I_1 dz'\hat{a}_{z'} \times (\rho\hat{a}_\rho - z'\hat{a}_{z'})}{4\pi \times (\sqrt{\rho^2 + z'^2})^2 \times (\sqrt{\rho^2 + z'^2})}$$

$$= \frac{I_1 dz'\hat{a}_{z'} \times (\rho\hat{a}_\rho - z'\hat{a}_{z'})}{4\pi(\rho^2 + z'^2)^{3/2}}$$

since the I_1 is directed towards increasing value of z', the limits are $-\infty$ and $+\infty$ on the integral and we have

$$\bar{H}_2 = \int_{-\infty}^{\infty} \frac{I_1 dz' \hat{a}_{z'} \times (\rho \hat{a}_\rho - z' \hat{a}_{z'})}{4\pi(\rho^2 + z'^2)^{3/2}}$$

$$= \frac{I_1}{4\pi} \int_{-\infty}^{\infty} \frac{\rho dz' \hat{a}_\phi}{(\rho^2 + z'^2)^{3/2}}$$

A vector is constant if its magnitude and direction are both constant, the unit vector $\hat{a}_\phi$ has a constant magnitude and varies only with ϕ. Since the integration here is with respect only to z', $\hat{a}_\phi$ is here a constant and can be removed from under the integral sign.

Thus

$$\bar{H}_2 = \frac{I_1 \rho \hat{a}_\phi}{4\pi} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2 + z'^2)^{3/2}} \qquad (a)$$

The integration is of the form

$$\int \frac{dx}{(c^2 + x^2)^{3/2}} = \frac{x}{c^2(c^2 + x^2)^{1/2}} \quad \ldots (b)$$

applying (b) to (a) we have

$$\bar{H}_2 = \frac{I_1 \rho \hat{a}_\phi}{4\pi} \frac{z'}{\rho^2 \sqrt{\rho^2 + z'^2}} \Bigg|_{-\infty}^{\infty}$$

$$= \frac{I_1}{2\pi\rho} \hat{a}_\phi \ \mathrm{Am^{-1}} \qquad \text{——} (4)$$

CONCLUSION

(a) The magnitude of the field is not a function of ϕ and z and it varies inversely with distance from the filament.

(b) The direction of the magnetic field intensity vector is circumferential

(c) Using the Biot-Savart law to find $\bar{H}$ is similar to the use of Coulombs to find E. A comparism of the electric field about an infinitely long charged filament shows that the streamlines of the magnetic field corresponds exactly to the equipotentials of the electric field and perpendicular family of the magnetic field corresponds to the streamlines of the electric field.

8.4 AMPERE'S CIRCUITAL LAW

Ampere's circuital law states that the line integral of the tangential component of the magnetic field strength around a closed path is equal to the current enclosed by the path:

$$\oint_l \bar{H} \cdot d\bar{l} = I \quad (1)$$

Through Ampere's circuital law, we will be able to solve quite formidable magnetostatic problems in cases of symmetrical current distribution.

Let us evaluate the integral $\oint \bar{H} \cdot d\bar{l}$ about a concentric closed loop that encloses the filamentary current I of infinite length, as suggested in fig. 1

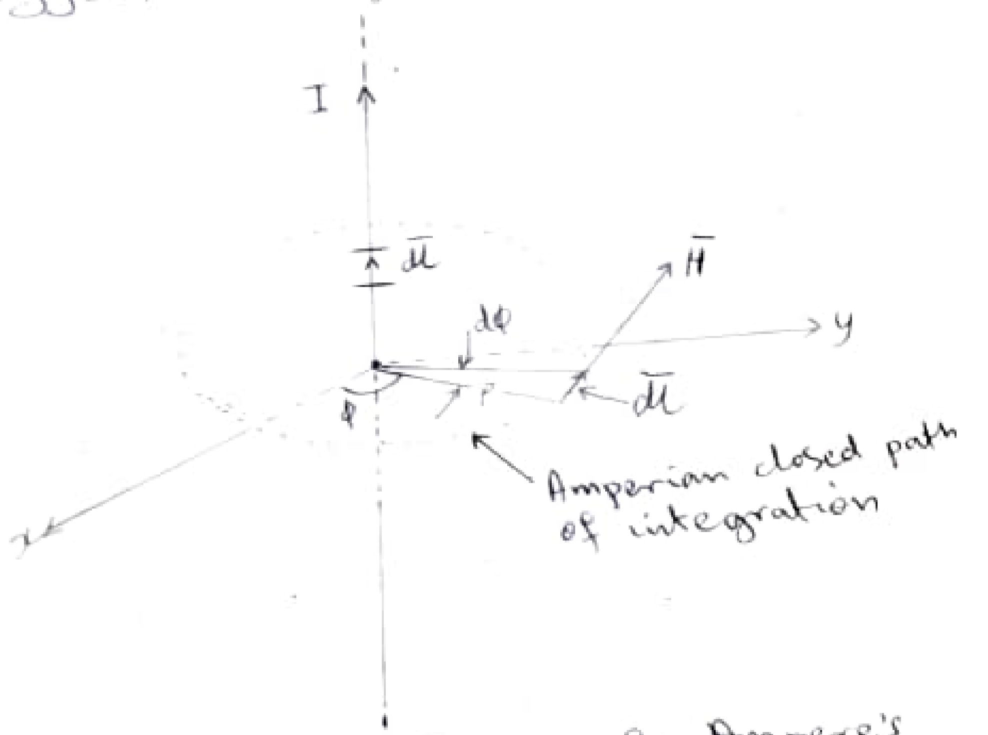


Fig 1 Graphical display for Ampere's circuital law.

Through the use of the equation $\bar{H} = \dfrac{I \hat{a}_\phi}{2\pi\rho}\ Am^{-1}$ we obtain

$$\oint_l \bar{H} \cdot d\bar{l} = \oint_l \left(\frac{I \hat{a}_\phi}{2\pi\rho}\right) \cdot (\rho\, d\phi\, \hat{a}_\phi) = \frac{I}{2\pi}\int_0^{2\pi} d\phi = I = I_{en}\ (A) \quad (2)$$

where I_{en} is the current enclosed by the closed loop. The formulation obtained by equating the first and last terms of (2), is called Ampere's circuital law.

$$\oint_l \bar{H} \cdot d\bar{l} = I_{en}\ (A) \quad (3)$$

3.5 APPLICATION OF AMPERE'S CIRCUITAL LAW TO FIELD OF AN INFINITELY LONG FILAMENT CARRYING CURRENT I

In the application, the direction of current is in the direction of increasing z as shown in fig- $I = I\hat{a}_z$

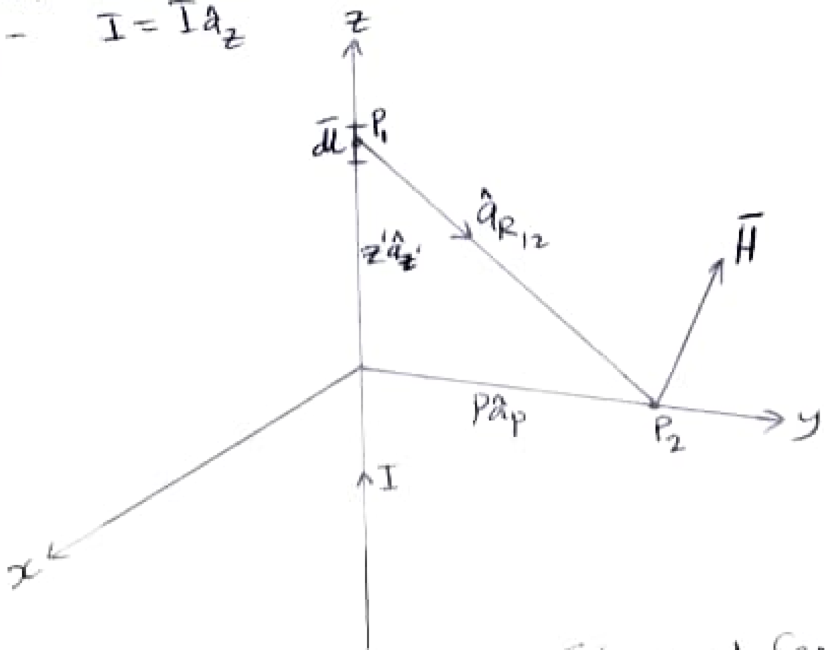


Fig 2 An Infinitely Long filament Carrying Current I.

The field dose not vary with z and does not also vary with ϕ, employing Biot Savart law gives

$$\bar{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

shows that the only component present is the ϕ component and it is the function of the radius ρ. Ampere's circuital law is applied to a part to which vector $\bar{H}$ is either perpendicular or tangential and along which it is constant. The pependicularity or tangenticy allows us to replace the dot product with a product of scalar magnitude, the constancy permits us to remove the magnetic field intensity from the integral sign, ~~therefore~~;

$$\oint \bar{H}\cdot\bar{dl} = \int_0^{2\pi} H_\phi \rho\, d\phi$$

$$= H_\phi \rho \int_0^{2\pi} d\phi = H_\phi 2\pi\rho$$

We know that

$$\oint \bar{H}\cdot\bar{dl} = I_{en}$$

therefore $I = H_\phi 2\pi\rho$

$$H_\phi = I/2\pi\rho \quad (Am^{-1})$$

Therefore let us construct a closed concentric loop perpendicular to the infinite length current as suggested in fig. 1. Through the use of the Biot-Savart law, we can argue that $d\bar{H}$ from any current element $Id\bar{l}$ will be in the a_ϕ direction at any point on the closed amperian path. Thus, $\bar{H}$ can be expressed as

$$\bar{H} = H_\phi a_\phi$$

Along the amperian closed path, $d\bar{l} = \rho d\phi a_\phi$. Substituting these expressions and $I_{en} = I$ into Ampere's circuital law, we obtain

$$\oint_L \bar{H} \cdot d\bar{l} = \oint_L (H_\phi \hat{a}_\phi) \cdot (\rho d\phi a_\phi) = H_\phi \rho \int_0^{2\pi} d\phi = H_\phi 2\pi\rho = I \ (A)$$

where H_ϕ and ρ were taken out from under the integral since they are constants over the amperian path of integration selected. Solving for H_ϕ, we obtain

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\therefore \bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi \quad (A m^{-1})$$

2.6 THE OPERATION OF CURL TO AMPERE'S CIRCUITAL LAW.

In our study of static electric fields, we applied Gauss's law to a point in space to obtain the divergence concept. Here we shall apply Ampere's circuital law to a point in space to obtain the curl concept.

To obtain an expression for the curl, we shall choose cartesian co-ordinate and incremental closed path of sides Δx and Δy as shown in fig — over which we will apply Amperes' circuital law.

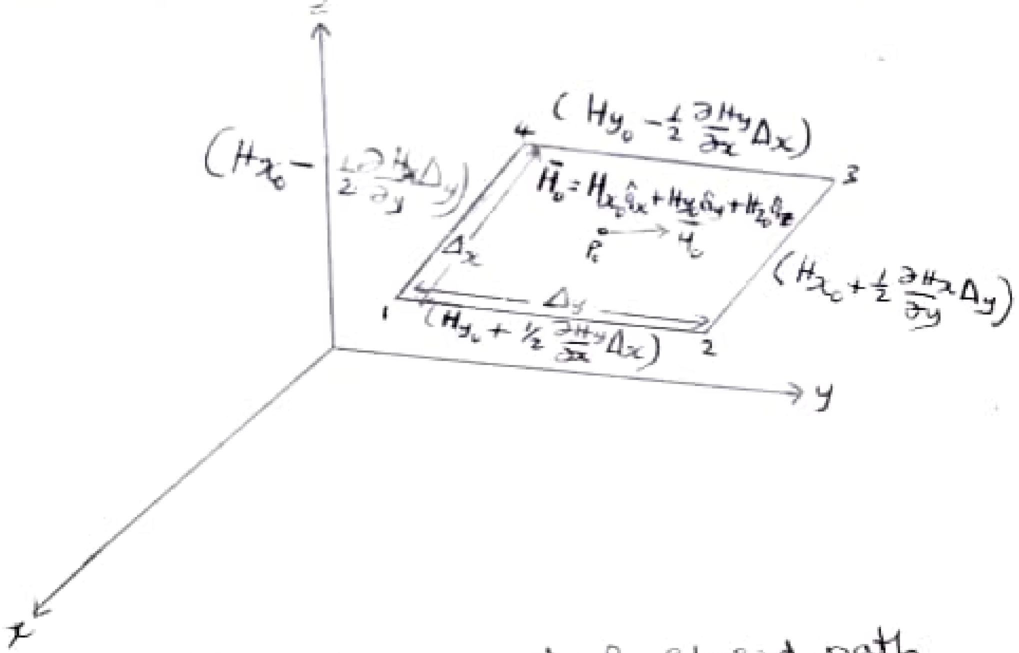


Fig — An incremental closed path in cartesian co-ordinate selected for the application of Ampere's circuital law in order to determine the spatial rate of change of $\bar{H}$

The result will be divided by the area Δs of the loop, and the limit as $\Delta s \to 0$ will be taken to obtain a scalar component of the curl of $\bar{H}$ at P_0. Let us assume that the amperian closed loop is embedded in an $\bar{H}$ field that we shall designate as $\bar{H}_0 = H_{x_0}\hat{a}_x + H_{y_0}\hat{a}_y + H_{z_0}\hat{a}_z$ at P_0. All $\bar{H}$ components perpendicular to the direction of travel about the loop will not contribute to the closed path integral of $\bar{H}\cdot d\bar{l}$. If we use the first two terms of Taylor's expansion for $\bar{H}$ and only the field components that are directed along the path of integration, we will obtain approximate $\bar{H}$ field values along each path segment, in fig -- Using these results, we obtain

$$\oint_L \bar{H}\cdot d\bar{l} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 = I_{en} = J_z \Delta_y \Delta_x \quad (A) \quad \ldots (1)$$

The value of scalar H_y on this section of the path may be given in terms of the reference value H_{y_0} at the centre of the rectangle plus the product of the rate of change of H_y with x, and distance $\Delta x/2$ from the centre to the mid point of 1-2

$$H_{y_{1-2}} = \int_1^2 = \left[H_{y_0} + \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta_x\right]\Delta_y \qquad (2)$$

Along the next sections of the paths we have

$$H_{x,2-3} = \int_2^3 = -\left[H_{x_0} + \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta_y\right)\Delta_x \qquad (3)$$

$$H_{y_{3-4}} = \int_3^4 = -\left[H_{y_0} - \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta_x\right]\Delta_y \qquad (4)$$

$$H_{x_{4-1}} = \int_4^1 = \left[H_{x_0} - \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta_y\right)\Delta_x \qquad (5)$$

Adding equations (2) to (5) give

$$\oint \bar{H}\cdot d\bar{l} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1$$

$$= H_{y_0}\Delta_y + \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta_x\Delta_y - H_{x_0}\Delta_x - \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta_y\Delta_x$$

$$- H_{y_0}\Delta_y + \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta_x\Delta_y + H_{x_0}\Delta_x - \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta_y\Delta_x$$

$$= \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\Delta_y\Delta_x \qquad (6)$$

Through the use of (1) and (6), let us form

$$(\text{Curl}H)_{a_z} = \lim_{\Delta_y\Delta_x\to 0}\frac{\oint_{L(z)} \bar{H}\cdot d\bar{l}}{\Delta_y\Delta_z} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial x}\right) = \lim_{\Delta_y\Delta_z\to 0}\left[\frac{I_{en}}{\Delta_x\Delta_y}\right] = J_z \quad (\ldots$$

The y and x components can be determined in a similar fashion as follows:

$$\oint_{L(x)} \bar{H}\cdot d\bar{l} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 = I_{en} = J_x \Delta_y \Delta_z (A) \quad (8)$$

where

$$\int_1^2 = \left[H_{y_0} + \frac{\partial H_y}{\partial z}\left(-\frac{\Delta_z}{2}\right)\right]\Delta_y \quad (9)$$

$$\int_2^3 = \left[H_{z_0} + \frac{\partial H_z}{\partial y}\left(\frac{\Delta_y}{2}\right)\right]\Delta_z \quad (10)$$

$$\int_3^4 = -\left[H_{z_0} + \frac{\partial H_y}{\partial z}\left(\frac{\Delta_z}{2}\right)\right]\Delta_y \quad (11)$$

$$\int_4^1 = -\left[H_{z_0} + \frac{\partial H_z}{\partial y}\left(-\frac{\Delta_y}{2}\right)\right]\Delta_z \quad (12)$$

Thus,

$$\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\Delta_y \Delta_z \quad (13)$$

Through the use of (8) and (13) let us form

$$\lim_{\Delta_y \Delta_z \to 0}\left[\frac{\oint_{L(x)} \bar{H}\cdot d\bar{l}}{\Delta_y \Delta_z}\right] = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = \lim_{\Delta_y \Delta_z \to 0}\left[\frac{I_{en}}{\Delta_y \Delta_z}\right] = J_x \quad (14)$$

where the first term is the definition for the x component of the curl of $\bar{H}$, and J_x is the x component of the current density vector at P_0.

Similarly for y component

$$\lim_{\Delta_x \Delta_z \to 0}\left[\frac{\oint_{L(y)} \bar{H}\cdot d\bar{l}}{\Delta_x \Delta_z}\right] = \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) = \lim_{\Delta_x \Delta_z \to 0}\left[\frac{I_{en}}{\Delta_x \Delta_z}\right] = J_y \quad (15)$$

from (7), (14), and (15) it should be noted that the scalar curl component of $\bar{H}$ are equal to scalar components of a current density $\bar{J}$ at P_0. Thus, we define the curl of $\bar{H}$ as

$$\text{Curl}\ \bar{H} \triangleq \sum_{k=x,y,z} \hat{k} \lim_{\Delta s(k) \to 0} \left[\frac{\oint_{L(k)} \bar{H} \cdot \bar{dl}}{\Delta s(k)} \right] \quad (\text{A m}^{-2}) \quad (\text{aap})$$

where $\Delta s(k)$ is the area bounded by the k^{th} amperian closed loop. If we combine in vector form the second and fourth terms of (7), 14, and (15), we obtain

$$\text{Curl}\ \bar{H} = \left[\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{a}_z \right] = \bar{J} \quad (17)$$

where $\bar{J} = J_x \hat{a}_x + J_y \hat{a}_y + J_z \hat{a}_z$

The left hand side of (17) can be written in shorthand vector operator form as

$$\nabla \times \bar{H} = \bar{J} \quad (\text{A m}^{-2}) \quad (\text{aap}) \quad (18)$$

where (aap) at a point.

Equation (18) is commonly referred to as the point form of Ampere's circuital law as well as Maxwell's second of four equations for static fields. Equation (17) can be expressed as third-order determinant, the expansion of which gives the cartesian curl of $\bar{H}$

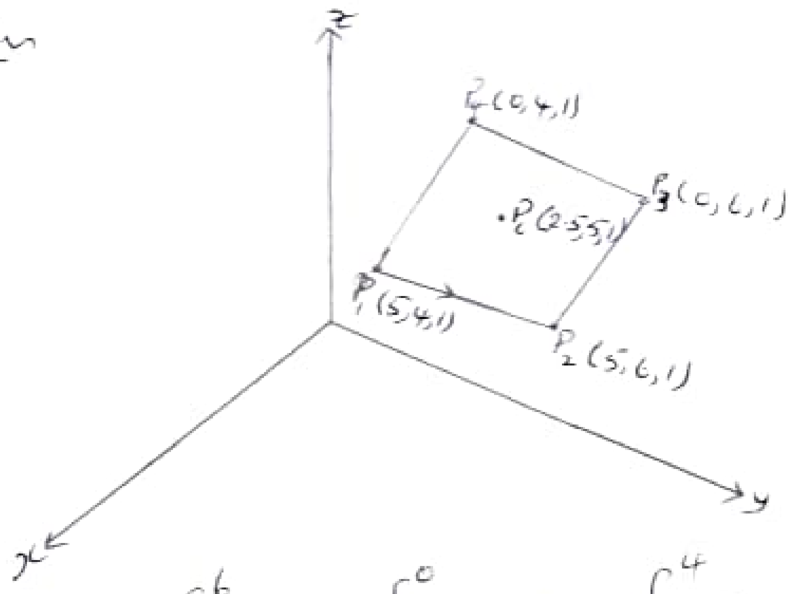
$$\text{Curl}\ \bar{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} \quad (19)$$

Expressions for curl $\bar{H}$ in cylindrical and spherical coordinates can be derived in the same manner as above, respectively,

Example 2

Evaluate the close line integral of
(a) Vector $\bar{H}$ from $P_1(5,4,1)$ to $P_2(5,6,1)$ to $P_3(0,6,1)$ to $P_4(0,4,1)$ to P_1 using straight line segments of $\bar{H} = 0.1y^3\bar{a}_x + 0.4x\bar{a}_y$ A/m
(b) Determine the quotient of the close line integral and the area enclosed by the path as an approximation to $(\nabla \times \bar{H})_z$
(c) Determine $(\nabla \times \bar{H})_z$ at the center of the area.

Solution



(a) $$\oint \bar{H}\cdot d\bar{l} = \int_{y=4}^{6}(0)\,dy + \int_{x=5}^{0} 0.1(6)^3 dx + \int_{x=6}^{4}(0)\,dy + \int_{x=0}^{5} 0.1(4)^3 dx$$

$$= 0 - 108 + 0 + 32 = -76\text{A}$$

(b) $$\frac{\oint \bar{H}\cdot d\bar{l}}{\Delta_s} = \frac{-76}{\Delta_s}$$

$$\Delta_s = 5\times 2 = 10\text{m}^2$$

$$\frac{\oint \bar{H}\cdot d\bar{l}}{\Delta_s} = -\frac{76}{10} = -7.6\,\text{A/m}^2$$

(c)

$$\text{Curl } \bar{H}_z = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{a}_z$$

Since $H_y = 0$, $\frac{\partial H_y}{\partial x} = 0$

Thus $\text{Curl } \bar{H}_z = -\frac{\partial H_x}{\partial y} \hat{a}_z$

Given $\bar{H}_x = 0.1 y^3 \hat{a}_x$;

At the center of the loop $P_c(2.5, 5, 1)$

$$\text{Curl } H_z = -0.3 \times (5)^2 \hat{a}_z$$

$$= -7.5 \hat{a}_z \ A/m^2$$

Example 1

Find the incremental field $\Delta\bar{H}_2$ at P_2 caused by a source at P_1 of $I_1\Delta L_1$ given as:

(i) $2\pi\hat{a}_z$ μA.m given $P_1(4,0,0)$ and $P_2(0,3,0)$

(ii) $2\pi(0.6\hat{a}_x - 0.8\hat{a}_y)$ μA.m given $P_1(4,0,0)$ and $P_2(0,3,0)$

$P_1(4,-2,3)$ $P_2(1,3,2)$

Solution

(i) $$\Delta\bar{H}_2 = \frac{I_1\Delta L_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$\bar{R}_{12} = (0-4)\hat{a}_x + (3-0)\hat{a}_y = -4\hat{a}_x + 3\hat{a}_y$$

$$\hat{a}_{R_{12}} = \frac{-4\hat{a}_x + 3\hat{a}_y}{\sqrt{4^2+3^2}} = -0.8\hat{a}_x + 0.6\hat{a}_y$$

$$\Delta\bar{H}_2 = \frac{2\pi\times10^{-6}\hat{a}_z \times (-0.8\hat{a}_x + 0.6\hat{a}_y)}{4\pi\, 5^2}$$

$$= (-1.2\hat{a}_x - 1.6\hat{a}_y)\times10^{-6}\ A/M^2$$

(ii)

$$\Delta\bar{H}_2 = \frac{I_1\Delta L_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$\bar{R}_{12} = (1-4)\hat{a}_x + (3-(-2))\hat{a}_y + (2-3)a_z = -3\hat{a}_x + 5\hat{a}_y - \hat{a}_z$$

$$\hat{a}_{R_{12}} = \frac{-3\hat{a}_x + 5\hat{a}_y - \hat{a}_z}{\sqrt{35}} = -0.507\hat{a}_x + 0.845\hat{a}_y - 0.169\hat{a}_z$$

$$\Delta\bar{H}_2 = \frac{2\pi(0.6\hat{a}_x - 0.8\hat{a}_y)\times10^{-6}\times(-0.507\hat{a}_x + 0.845\hat{a}_y - 0.169\hat{a}_z)}{4\pi\times35}$$

$$= \frac{(2\pi\times0.6\times0.846\hat{a}_z + 2\pi\times0.6\times0.169\hat{a}_y - 2\pi\times0.8\times0.507\hat{a}_z + 2\pi\times0.8\times0.169\hat{a}_x)\times10^{-6}}{4\pi\times35}$$

$$= (1.931\hat{a}_x + 1.449\hat{a}_y + 1.449\hat{a}_z)\times10^{-9}\ A/M^2$$