~~THEVENIN'S TH~~

1.0 INTRODUCTION

In electrical circuits, various ~~#~~ methods have been developed for use in solving for unknown currents and voltage drops in dc networks.
The laws which determine the current and voltage drops in dc networks are:
- Ohm's law
- Laws of resistors in series and in parallel
- Kirchhoff's laws (current and voltage laws)

Beside these three laws, there are other circuit theorems created for solving problems in electrical networks, namely
- Superposition theorem
- Thevenin's theorem
- Norton's theorem
- Maximum power transfer theorem.

Recall that Kirchhoff's current law states that at any junction in a network, the total current flowing towards the junction is equal to the total current flowing away from the junction.

Also recall that Kirchhoff's voltage law states that in any closed loop in a network the algebraic sum of the voltage drops taken round the loop is equal to the resultant emf acting in that loop.
Now, let's look at Thevenin's theorem.

1.1 THEVENIN'S THEOREM

Any two terminal network may be represented by a series combination of a voltage source and a resistor. The value of this equivalent source is the voltage appearing at the open circuited terminals. The value of the equivalent resistance is the value of that resistance seen looking back into the network with all energy sources replaced by their respective internal resistances.

OR

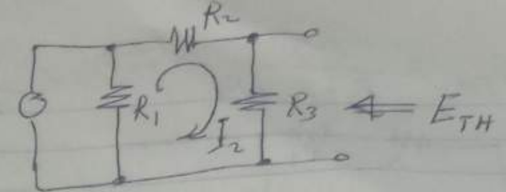
The current in any passive circuit element (say a resistor R_c) in a network is the same as would be obtained if R_c were supplied with a source voltage E_s in series with an equivalent resistance R_s. E_s being the open circuit voltage at the terminals from which R_c has been removed, and R_s being the resistance that would be measured at these terminals after all sources have been replaced by their internal resistances.

OR

Thevenin's theorem states that any linear two-terminal network may be replaced by a voltage source whose value is equal to the open circuit terminal voltage (say E_{oc}) in series with an equivalent impedance (Z_{eq}) whose value is the impedance seen at the pair of network terminals.

The following examples will be used to demonstrate the theorem.

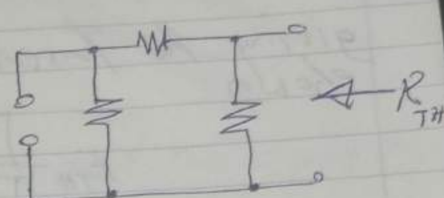
Solution



$$I_{TH} = \frac{R_1}{R_1+R_2+R_3} \times I_s = \frac{5}{5+15+5} \times 20mA = 4mA$$

$$E_{TH} = V_{R3} = I_2 R_3 = I_{TH} R_3 = 4mA \times 5k\Omega = 20V$$

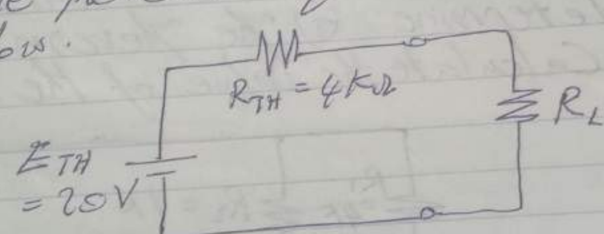
Resulting cct.



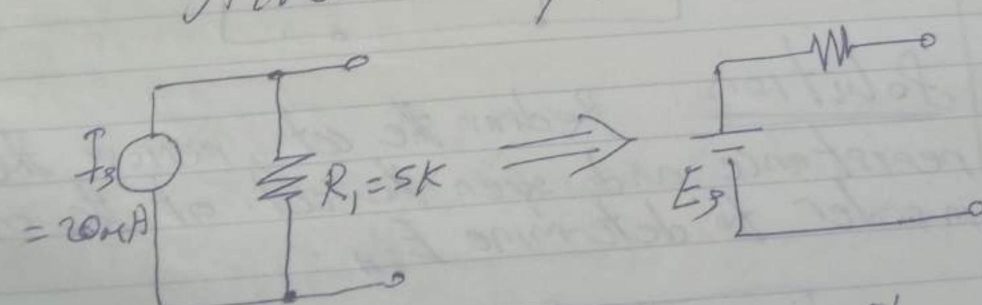
$$R_{TH} = R_3 // (R_1 + R_2)$$

$$= \frac{5 \times 20}{25} = \frac{100k}{25} = 4k\Omega$$

Therefore, the Thevenin's equivalent cct. is shown below.

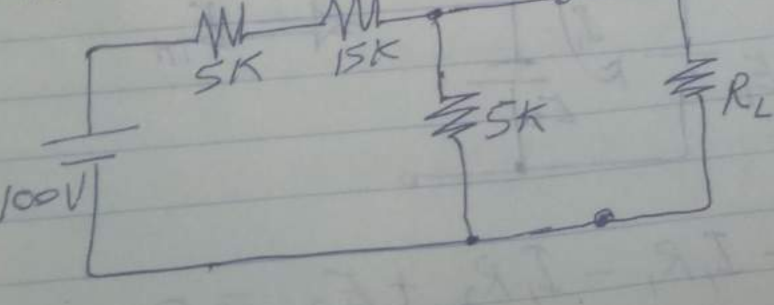


Alternatively,

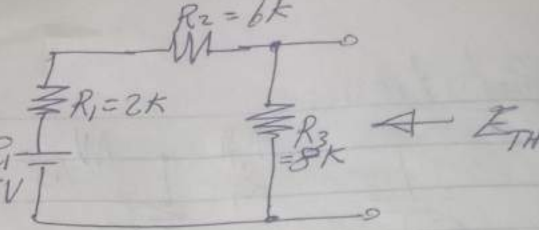


$$E_s = I_s R_1 = 20mA \times 5k\Omega = 100V$$

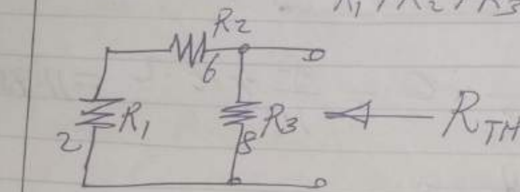
$\Longrightarrow$



Solution:

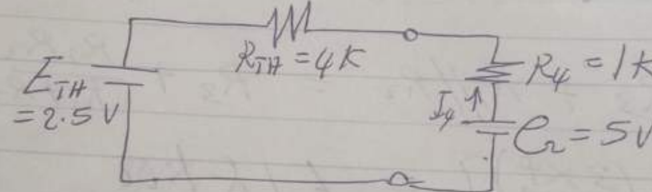


$$E_{TH} = e_1 \times \frac{R_3}{R_1 + R_2 + R_3} = 8/16 \times 5 = 2.5\,V$$



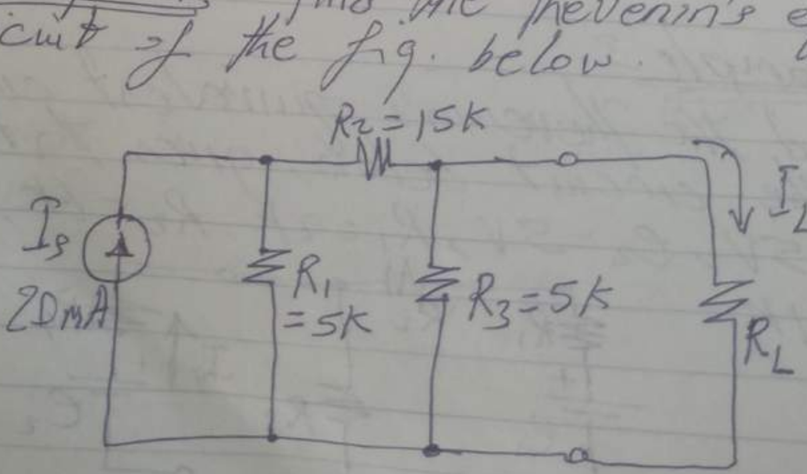
$$R_{TH} = (R_1 + R_2) // R_3$$
$$= 8//8 = \frac{8 \times 8}{16} = 4\,k\Omega$$

The Thevenin's equivalent circuit is thus



$$e_2 - E_{TH} = I_4 R_4 + I_4 R_{TH}$$
$$\Rightarrow (5 - 2.5)V = I_4 (R_4 + R_{TH})$$
$$= I_4 (1 + 4) = I_4 5$$
$$\Rightarrow I_4 = \frac{2.5}{5} = 0.5\,mA$$

Example 4 Find the Thevenin's equivalent circuit of the fig. below.



$85I_1 - 25I_2 = 60$

$\Rightarrow \quad 17I_1 - 5I_2 = 12 \quad \cdots (1)$

Loop 2

$E_2 - R_3(I_2 - I_1) - R_4 I_2 - R_5 I_2 + E_3 = 0$

$\Rightarrow 100 = 25I_2 - 25I_1 + 60I_2 + 50I_2$

$-25I_1 + 135I_2 = 100$

$-5I_1 + 27I_2 = 20 \quad \cdots (2)$

Solving eq^{n}s (1) and (2)

$17I_1 - 5I_2 = 12$

$-5I_1 + 27I_2 = 20$

Using Determinant,

$$\begin{bmatrix} 17 & -5 \\ -5 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$$

$$D = \begin{vmatrix} 17 & -5 \\ -5 & 27 \end{vmatrix} = 17 \times 27 - 25 = 459 - 25 = 434$$

$$I_1 = \frac{1}{D}\begin{vmatrix} 12 & -5 \\ 20 & 27 \end{vmatrix} = \frac{1}{434}(324 + 100) = \frac{424}{434}$$

$$= 0.977 \text{ Amps}$$

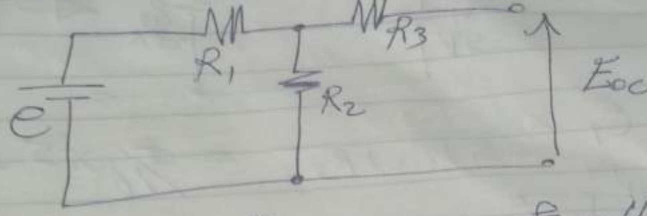
$$\text{and } I_2 = \frac{1}{D}\begin{vmatrix} 17 & 12 \\ -5 & 20 \end{vmatrix} = \frac{1}{434}(340 + 60)$$

$$= \frac{400}{434} = \underline{0.922 \text{ Amps}}$$

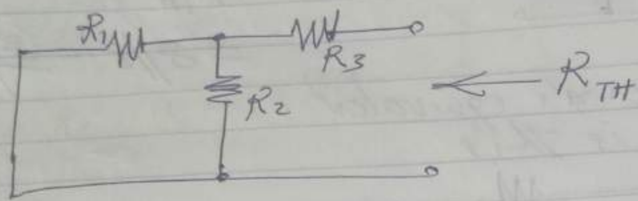
1.5 NODE VOLTAGE ANALYSIS

Procedure: Unlike in Mesh current analysis where variables are used in conjunction with KVL, nodal analysis depends on a choice of voltage used as variable, in conjunction with KCL. A general

Solution:



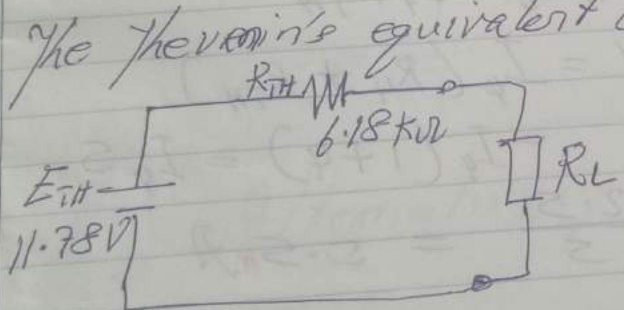
$$E_{oc} = E_{TH} = \frac{R_2}{R_1 + R_2} \times e = \frac{8}{11} \times 16.2 = 11.78V$$



$$R_{TH} = R_3 + R_1 // R_2 = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

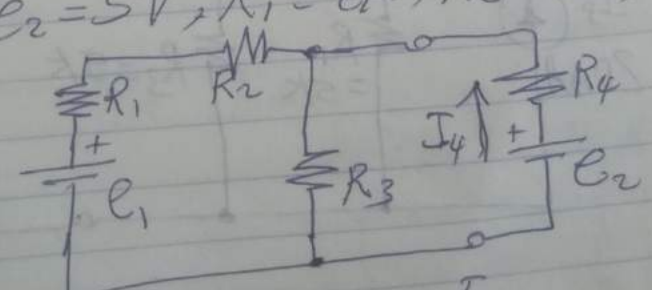
$$= \left[4 + \left(\frac{3 \times 8}{11}\right)\right] = 6.18\, k\Omega$$

The Thevenin's equivalent circuit is thus

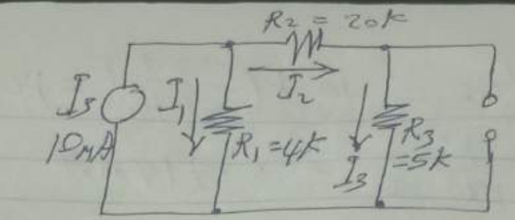


Example 3

Find the Thevenin's equivalent circuit of the circuit below, given that $e_1 = 5V$, $e_2 = 5V$, $R_1 = 2k$, $R_2 = 6k$, $R_3 = 8k$, $R_4 = 1k$.



Find also the current I_4.

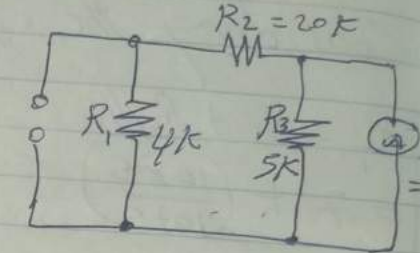


$$I_2 = \frac{R_1}{R_1 + R_2 + R_3} \times I_{s1} = \frac{4}{29} \times 10mA$$

$$= 1.379mA$$

Considering I_{s2} alone, we have

$$I_2 = \frac{R_3}{R_1 + R_2 + R_3} \times I_{s2}$$



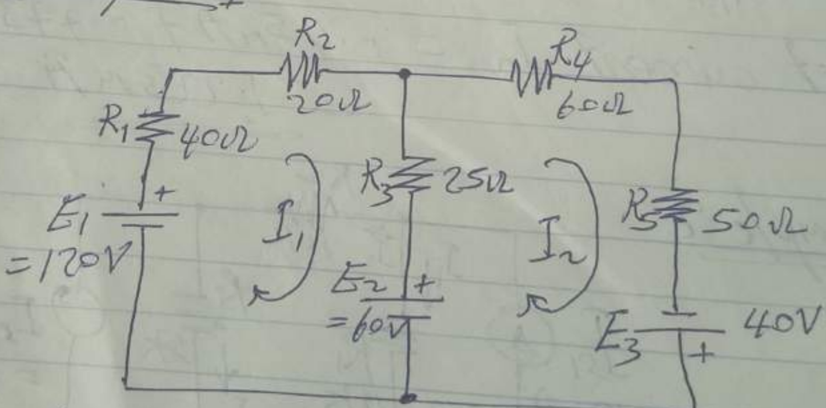
$$= \frac{5}{29} \times 4mA$$

$$= 0.69mA$$

Net $I_2 = (1.379 - 0.69) = 0.69mA$

1.4 MESH CURRENT ANALYSIS

Example 1.



Determine the branch currents in the network.

Sol^n: Loop 1:

$$E_1 - E_2 - R_1 I_1 - R_2 I_1 - R_3 (I_1 - I_2) = 0$$

$$120 - 60 - 40I_1 - 20I_1 - 25(I_1 - I_2) = 0$$

$$60 = 20I_1 + 25I_1 - 25I_2 + 40I_1$$

Fig 2. completely replaces fig (1) but with its characteristics maintained (E_{oc} is the same, I_{sc} the same).

To determine short circuit current,

$$I_{sc} = \frac{e}{R_1}$$

and this will be the current if we short-circuit terminals A and B.

From Thevenin's equivalent values,

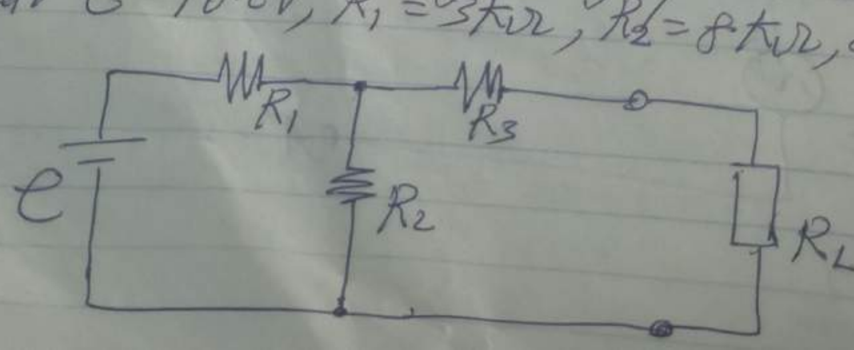
$$I_{sc} = \frac{E_{oc}}{R_{eq}} = \frac{e \times R_2/(R_1+R_2)}{R_1R_2/(R_1+R_2)} = \frac{e}{R_1} \quad \text{--(iii)}$$

which confirms that I_{sc} is the same for either circuits (fig 1 and fig 2).

It is good to note that E_{oc}, Z_{eq} and I_{sc} are always related by the expression $E_{oc} = I_{sc} \times Z_{eq}$ --- (iv)

Z_{eq} is used to replace R_{eq}, since it is a more general expression and it applies to networks containing capacitors and inductors as well.

Example 2: Determine the Thevenin's equivalent circuit of the fig. below, given that $e = 16.2V$, $R_1 = 3k\Omega$, $R_2 = 8k\Omega$, and $R_3 = 4k\Omega$



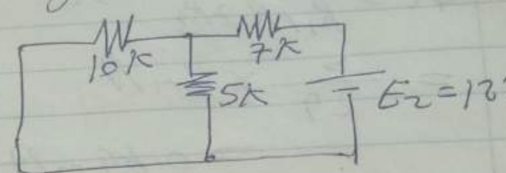
$R_T = R_1 + R_2 // R_3 = 10 + \left(\frac{5 \times 7}{5+7}\right) = 12.92k$

$I_1 = \frac{E_1}{R_T} = \frac{10}{12.92} = 0.774mA$

Using current divider rule,

$I_2 = \frac{R3}{R_2 + R_3} \times I_1 = \frac{7}{12} \times 0.774mA = 0.45mA$

Considering the effect of E_2 alone



$R_T = R_3 + R_2 // R_1$

$= 7 + \left(\frac{10 \times 5}{10+5}\right)$

$= 10.33k$

$I_3 = \frac{E_2}{R_T} = \frac{12}{10.33} = 1.16mA$

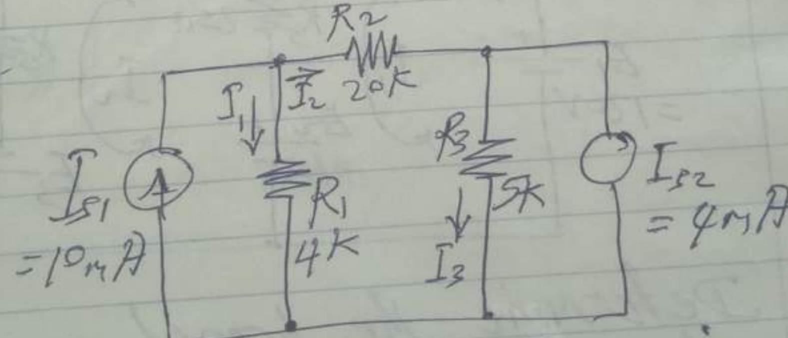
Again, using current divider rule,

$I_2 = \frac{R_1}{R_1 + R_2} \times I_3 = \frac{10}{15} \times 1.16mA = 0.773mA$

$\therefore$ Net current $I_2 = 0.45mA + 0.773mA$

$= \underline{1.223mA}$

Example 2



Find the value of current I_2 using superposition theorem.

Solution Considering I_{S1} acting alone, we have the circuit below.

Using current divider rule,

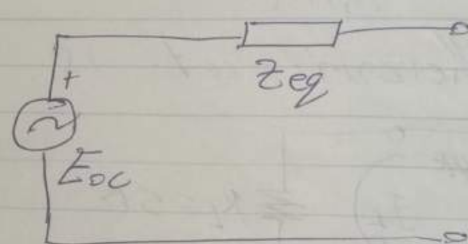
$$= \frac{e}{1+jR\omega c} = 10 \times \frac{1}{\sqrt{2}} \angle -45^\circ$$

Z_{eq} is found by substituting zero ohms as the impedance of the source (e), and finding R and C in parallel.

$$Z_{eq} = R // C = \frac{R \cdot (1/j\omega c)}{R + (1/j\omega c)} = \frac{R}{1 + jR\omega c}$$

$$= \frac{5(-j5)}{5 - j5} = \frac{5}{\sqrt{2}} \angle -45^\circ \ \Omega$$

Therefore the equivalent Thevenin's cct. is



To verify the results, we determine I_{sc}.

$$I_{sc} = e/R = \frac{10}{5} = 2A$$

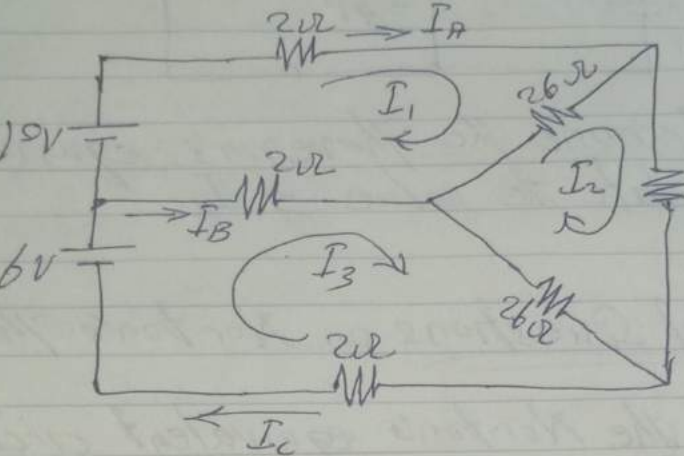
A short cct. across A-B terminals will exclude consideration of the capacitor since the capacitor is shorted.
From the equivalent network,

$$I_{sc} = \frac{E_{oc}}{Z_{eq}} = \frac{e\left(1/(1+j\omega Rc)\right)}{\left(R/(1+j\omega Rc)\right)} = e/R$$

which is the same.

(b) Add a 30V battery in series with the 300Ω resistor so that its positive terminal is connected to node A, and its negative terminal to the top of the resistor. Solve for the current in the 400Ω resistor.

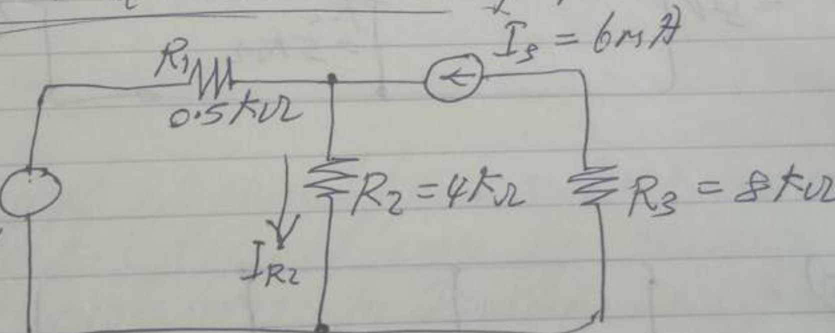
(3)



Determine the value of I_B using any method.

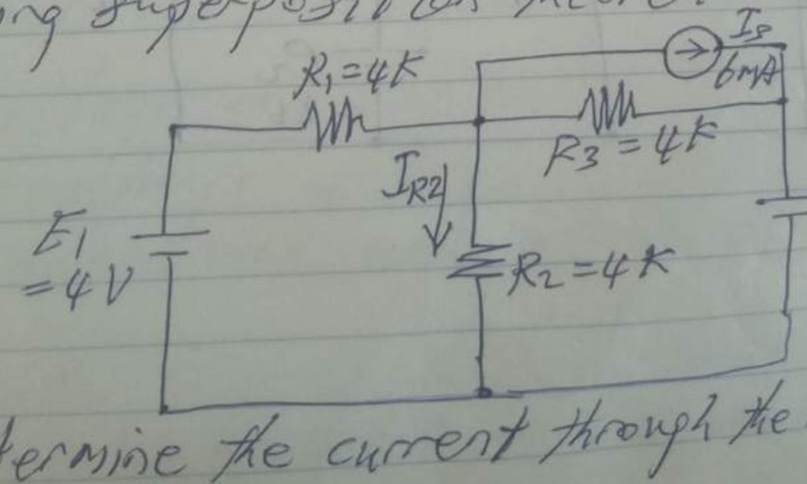
Tutorial Questions (Superposition Theorem).

(1)



Determine the branch current I_{R2} by using superposition theorem.

(2)



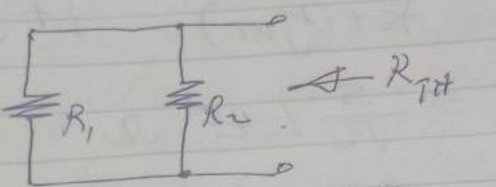
Determine the current through the resistor R_2.

$$E_1 + E_2 = I_1 R_1 + I_1 R_2 = I_1 (R_1 + R_2)$$

$$I_1 = \frac{E_1 + E_2}{R_1 + R_2} = \frac{14\,V}{10\,k\Omega} = 1.4\,mA$$

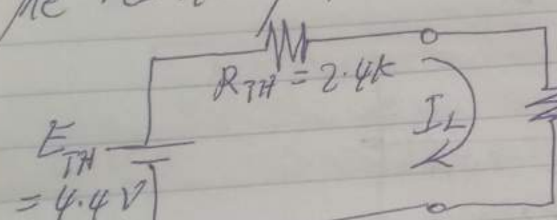
$$I_1 R_2 = E_{TH} + E_2 \Rightarrow E_{TH} = I_1 R_2 - E_2$$

$$= (1.4\,mA)(6\,k\Omega) - 4\,V = 8.4 - 4 = 4.4\,V$$



$$R_{TH} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 6}{10}\,k\Omega = 2.4\,k\Omega$$

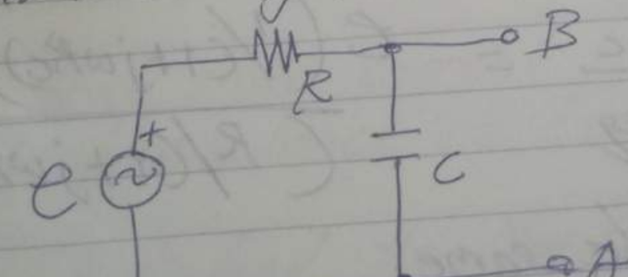
The resulting Thevenin's cct. is



$$I_L = \frac{E_{TH}}{R_{TH} + R_L} = \frac{4.4}{(2.4 + 5) \times 10^3} = \frac{4.4}{7.4} \times 10^{-3}\,A$$

$$= 0.595\,mA$$

Example 6. Determine the Thevenin's equivalent cct. of the cct. below.



$e = 10 \sin 10t$
$R = 5$
$C = 0.02$
$X_C = -j5$

Sol^n: $$E_{OC} = e \times \frac{X_C}{R + X_C} = \frac{e \times 1/j\omega C}{R + 1/j\omega C}$$

$$V_A = \frac{1}{D}\begin{vmatrix} 10 & -0.05 \\ 4 & 0.25 \end{vmatrix} = \frac{1}{0.0725}(2.5 + 0.2)$$

$$= \frac{2.7}{0.0725} = \underline{\underline{37.24\ V}}$$

$$V_B = \frac{1}{D}\begin{vmatrix} 0.3 & 10 \\ -0.05 & 4 \end{vmatrix} = \frac{1}{0.0725}(1.2 - 0.5)$$

$$= \frac{1.7}{0.0725} = \underline{\underline{23.45\ V}}$$

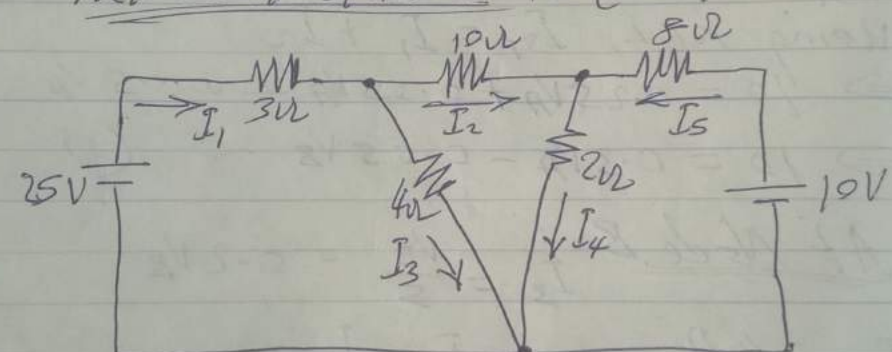
$$I_1 = \frac{V_A}{4} = \frac{37.24}{4} = 9.31\ \text{Amps}$$

$$I_2 = \frac{V_A - V_B}{20} = \frac{37.24 - 23.45}{20} = 0.69\ \text{Amps}$$

$$I_3 = \frac{V_B}{5} = \frac{23.45}{5} = 4.69\ \text{Amps}$$

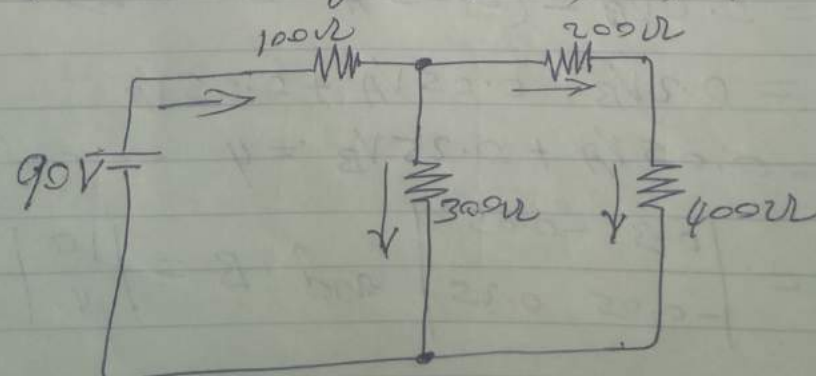
Tutorial Questions (Mesh/Node Analysis)

(1)



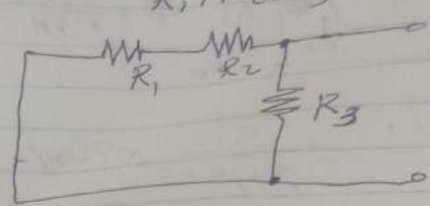
Find the values of I_1, I_2, I_3, I_4 and I_5

(2)



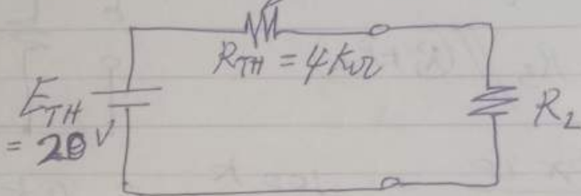
(a) Using nodal analysis, solve for the current in the 300Ω resistor.

$$E_{TH} = \frac{R_3}{R_1+R_2+R_3} \times E_S = \frac{5}{25} \times 100 = 20V$$

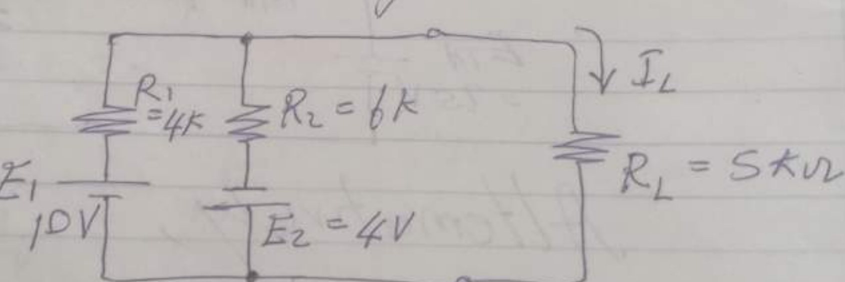


$$R_{TH} = R_3 // (R_1+R_2) = \frac{5 \times 20}{25} = \underline{4 k\Omega}$$

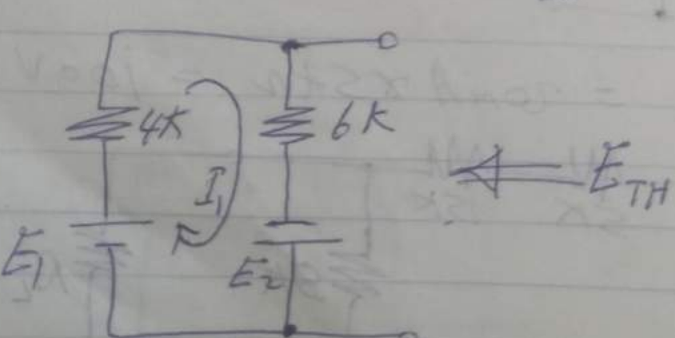
giving the Thevenin's equivalent cct. as shown



Example 5 In the circuit below, determine (a) the Thevenin's equivalent cct. (b) Calculate the value of the load current I_L



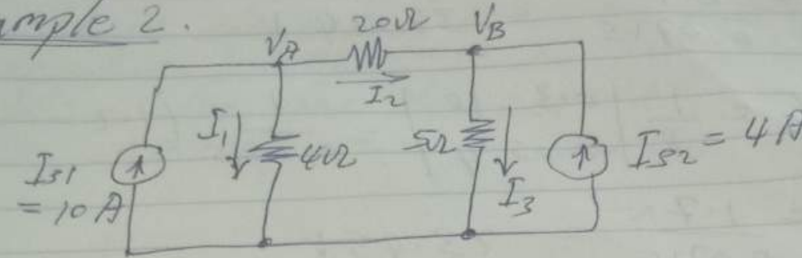
Solution: Redraw the cct., remove the load resistance and open the cct. at the end in order to determine E_{TH}.



$$E_1 - I_1R_1 - I_1R_2 + E_2 = 0$$

$\therefore I_3 = \frac{V_A}{5} = \frac{8.42}{5} = 1.684$ Amps

Example 2.



Determine the current I_1, I_2 and I_3 at node 'A'.

Solution

$$I_1 = \frac{V_A}{4} = 0.25 V_A$$

$$I_2 = \frac{V_A - V_B}{20} = 0.05 V_A - 0.05 V_B$$

$I_{S1} = 10$ A

Using KCL, $I_{S1} = I_1 + I_2$

$\Rightarrow 10 = 0.25 V_A + 0.05 V_A - 0.05 V_B$

$\Rightarrow 10 = 0.3 V_A - 0.05 V_B \quad \ldots (1)$

At Node B

$$I_3 = \frac{V_B}{5} = 0.2 V_B$$

$I_{S2} = 4A \qquad I_{S2} = I_3 - I_2$

$4 = 0.2 V_B - (0.05 V_A - 0.05 V_B)$

$4 = 0.2 V_B - 0.05 V_A + 0.05 V_B$

$\Rightarrow -0.05 V_A + 0.25 V_B = 4 \quad \ldots (11)$

$$D = \begin{vmatrix} 0.3 & -0.05 \\ -0.05 & 0.25 \end{vmatrix} \text{ and } B = \begin{vmatrix} 10 \\ 4 \end{vmatrix}$$

$D = 0.075 - 0.0025 = 0.0725$

procedure for analysis of a network by node voltage analysis is as follows:

(1) Convert all voltage sources to current sources, if necessary.

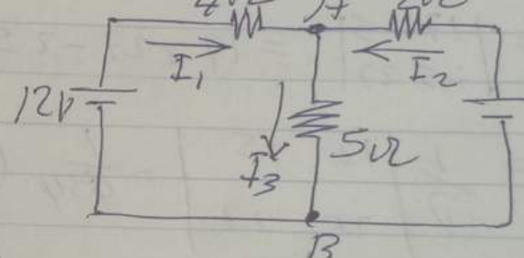
(2) Indicate one node as a reference node from which the node voltage are measured.

(3) The number of node voltages is generally given by $(N-1)$ where N is the total number of nodes in the network. The number of linear equations = the number of node voltages.

(4) Apply KCL at each node except the reference node.

(5) Solve the set of simultaneous linear equations for the node voltages.

__Example 1.__



Determine the current in the 5Ω resistor.

__Solution__: At node A, $I_1 + I_2 = I_3$

$$I_1 = \frac{12 - V_A}{4} = 3 - 0.25V_A$$

$$I_2 = \frac{10 - V_A}{2} = 5 - 0.5V_A$$

$$I_3 = \frac{V_A}{5} = 0.2V_A$$. But $I_3 = I_1 + I_2$

So, $I_1 + I_2 - I_3 = 0$

$$3 - 0.25V_A + 5 - 0.5V_A - 0.2V_A = 0$$

$$\Rightarrow 8 - 0.95V_A = 0$$

$$\Rightarrow V_A = \frac{8}{0.95} = 8.42V$$

Example 1

Looking at fig. 1, the open circuit voltage may be determined from voltage divider theory to be

$$V_{OC} = V_{AB} = e \times \frac{R_2}{R_1 + R_2} \quad \ldots (1)$$

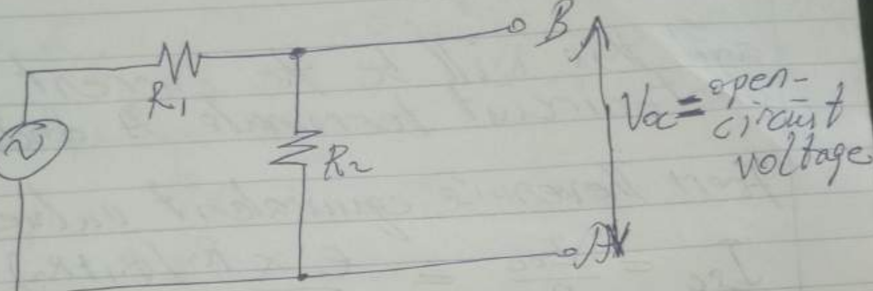


fig. 1

The equivalent resistance seen at the AB terminals is generally determined by replacing any sources with their internal resistances. For this circuit, R_1 is in parallel with R_2, so the equivalent resistance R_{eq} is given by

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \ldots (ii)$$

The thevenin's equivalent circuit is thus as shown in fig 2.

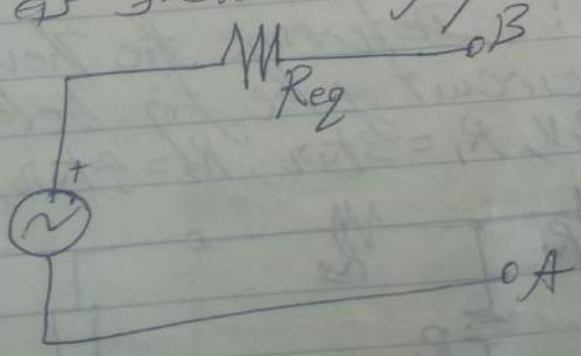
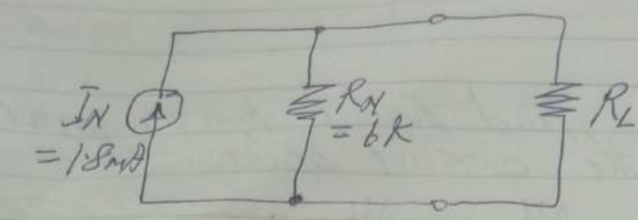


fig. 2

The Norton's equivalent circuit is thus

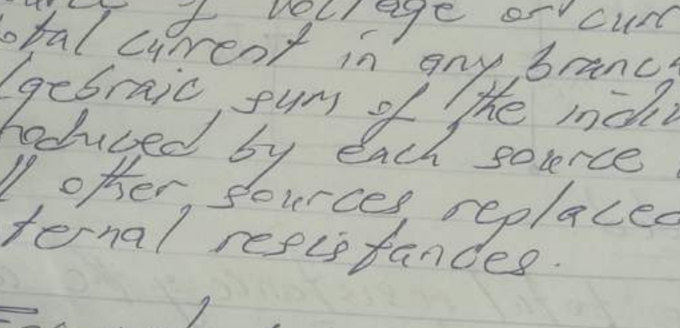


1.3 SUPERPOSITION THEOREM

In a network containing more than a source of voltage or current, the total current in any branch is the algebraic sum of the individual currents produced by each source acting alone, all other sources replaced by their internal resistances.

Example 1.

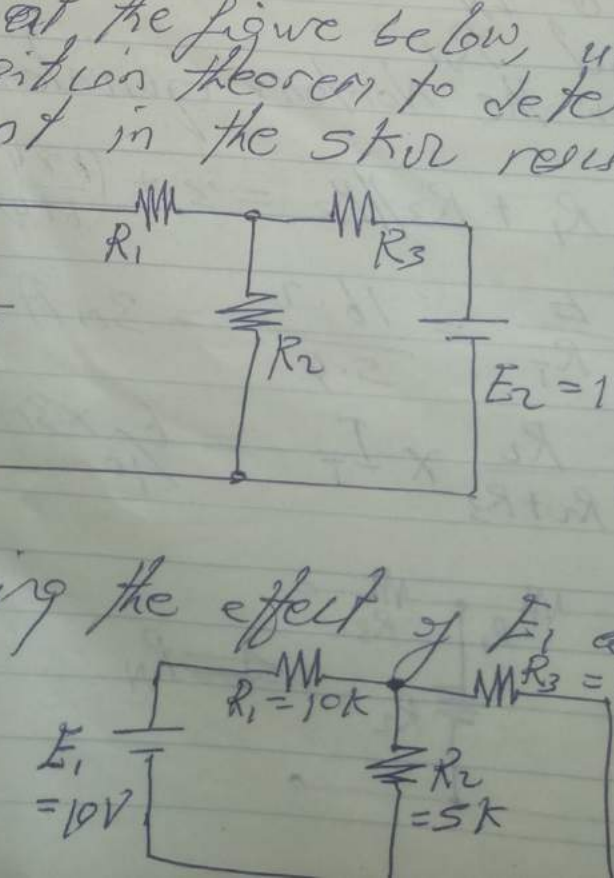
Looking at the figure below, use superposition theorem to determine the current in the 5kΩ resistor.

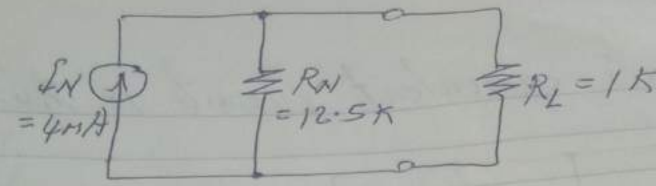


$R_1 = 10k$
$R_2 = 5k$
$R_3 = 7k$

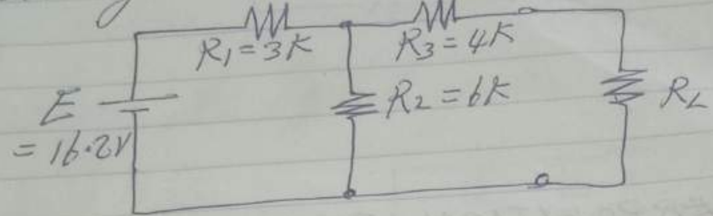
Solution.

Considering the effect of E_1 alone, the cct. is as follows.

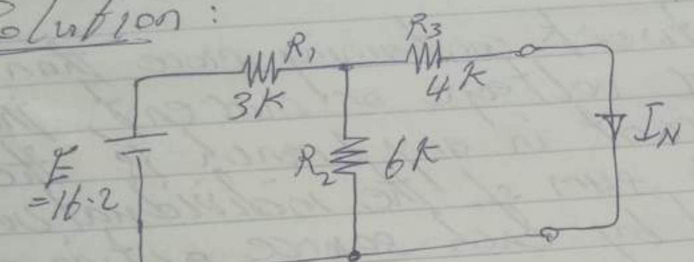




Example 3: Find the Norton's equivalent circuit of the circuit below.



Solution:



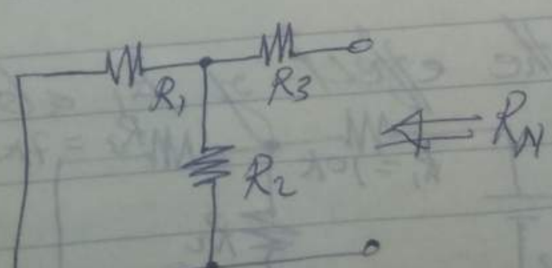
Steps needed:

(1) Find the total resistance of the cct.
(2) Use this to obtain I_T
(3) Find I_N
(4) Find R_N
(5) Draw the Norton's equivalent circuit.

$$R_T = R_1 + R_2 // R_3 = 3 + \left(\frac{6 \times 4}{6+4}\right) = 5.4\ k\Omega$$

$$I_T = \frac{E}{R_T} = \frac{16.2}{5.4} = 3mA$$

$$I_N = \frac{R_2}{R_2 + R_3} \times I_T = \frac{6}{10} \times 3mA = 1.8\ mA$$



$$R_N = R_3 + R_2 // R_1$$
$$= 4 + \left(\frac{3 \times 6}{9}\right)$$
$$= 6k\Omega$$

equations and solutions, only that the functions of some complimentary element are interchanged. It is this switchability or interchangeability that is known as the principle of Duality.

Dual Pairs

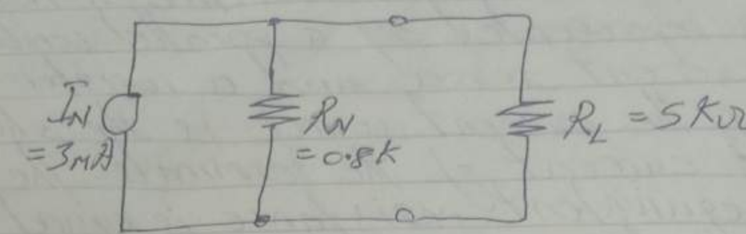
Resistance R	Conductance G
Voltage V	Current I
Voltage source	Current source
KVL	KCL
Thevenin	Norton
Inductance L	Capacitance C
Open circuit	Short circuit
Node	Mesh
Serial path	Parallel path

It is note worthy that power does not appear in the table, because power has no dual. This is because of the principle of linearity; since power is not linear, power cannot be dual.
Also note that the principle extends to circuit elements and theorems.

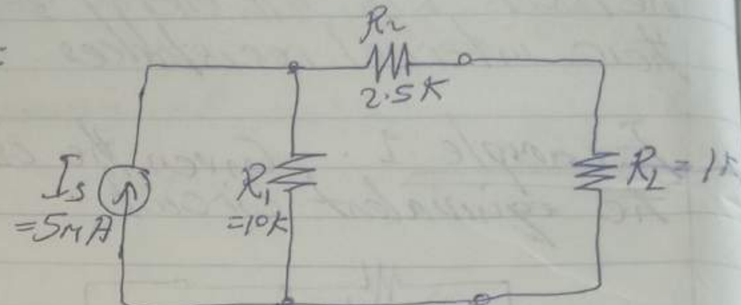
Two circuits that are described by equations of the same form, but in which the variables are switched are said to be dual of each other.

$R_N = R_1 // R_2 = 0.8 K\Omega$

Hence the equivalent Norton's cct. is

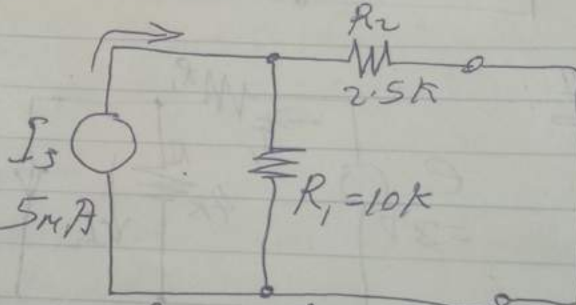


<u>Example 2</u>:



Find Norton's equivalent circuit and determine the load current

<u>Solution</u>:

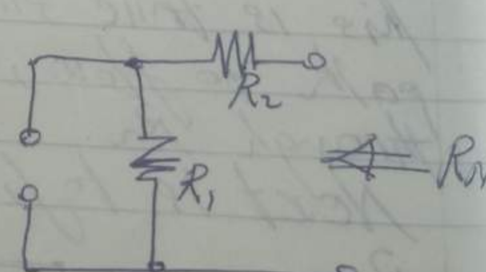


Using current divider rule,

$$I_N = \frac{R_1}{R_1 + R_2} \times I_S = \frac{10}{12.5} \times 5mA = \underline{4mA}$$

$R_N = R_1 + R_2 = 12.5K$

Hence Norton's equivalent circuit is shown below.



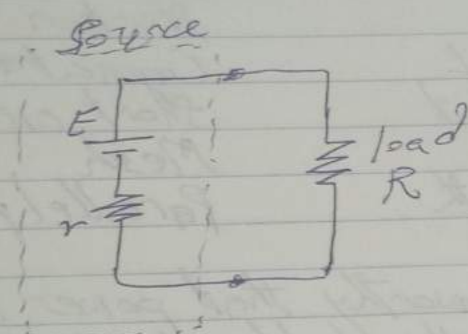
The load current is evaluated as

$$I_L = \frac{R_N}{R_N + R_L} \times I_N = 3.7mA$$

1.6 MAXIMUM POWER TRANSFER THEOREM

It states that the power transferred from a supply source to a load is at its maximum when the resistance of the load is equal to the internal resistance of the source.

Hence, in the figure below, when R = r, the power transferred from the source to the load is a maximum



1.7 DUALITY

The concept of Duality is a time saving and effort-efficient way of solving circuit problems. In Duality, a circuit is changed in that the components and or quantities are interchanged. The quantities that often changed include

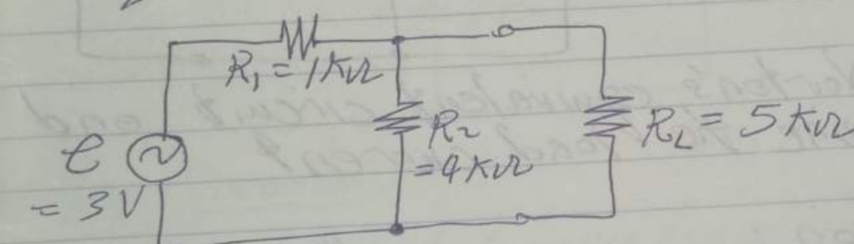
(1) voltage and current
(2) resistance and conductance
(3) capacitance and inductance

It so happens that in circuit analysis, the two different circuits have the same

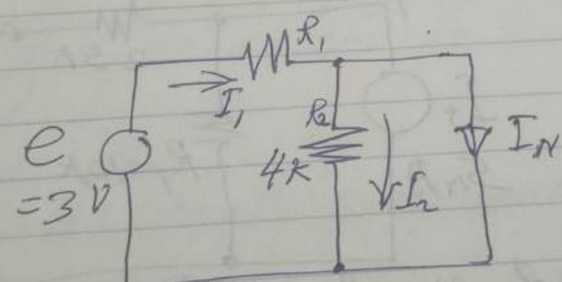
1.2 NORTON'S THEOREM

Norton's theorem states that any two terminal of an active resistive network may be represented by a parallel combination of a current source and a resistor. The value of the current source is the short circuit current of the terminals. The value of the equivalent resistance is equal to that resistance seen looking back into the network with all energy sources replaced by their internal resistances.

Example 1. Given the circuit below, find the equivalent circuit.



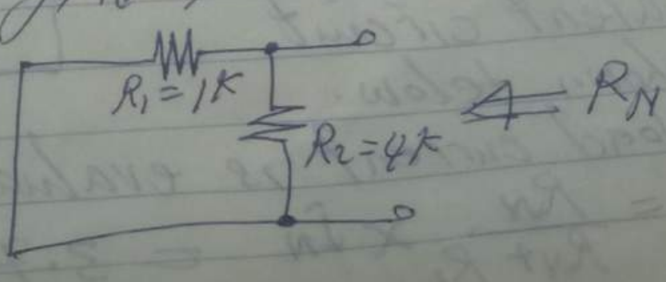
Sol^n:



$I_N = e/R_1 = 3mA$

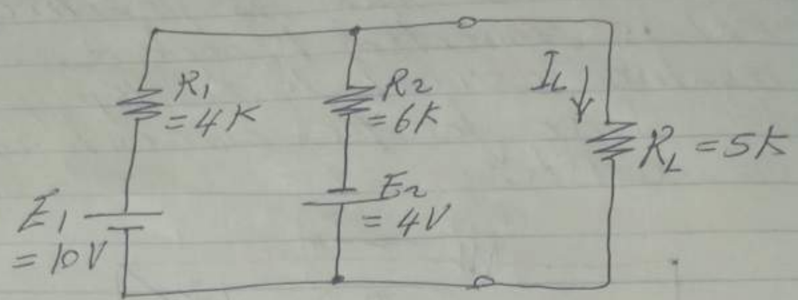
This is true since current finds easiest path to flow, hence no current flow through R_2.

Next is to find the Norton's resistance R_N.



Tutorial Questions (Thevenin's theorem)

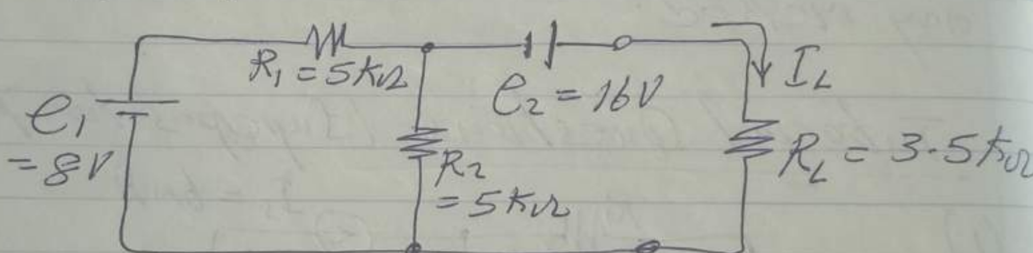
(3)



(a) Determine the Thevenin's equivalent cct.
(b) Calculate the value of I_L

Tutorial Questions on Norton's Theorem

(4) Find the Norton's equivalent circuit and determine the load current I_L for questions 4 and 5.



(5)