Exercises:

$$\psi = |+z\rangle \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad L_{xy}(\theta) \rightarrow \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{+i\theta/2} \end{bmatrix}$$

$$\phi = |+x\rangle \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad L_{tz}(\phi) \rightarrow \begin{bmatrix} e^{-\phi/2} & 0 \\ 0 & e^{+\phi/2} \end{bmatrix}$$

1. Calculate the following:

$$L_{xy}|+z\rangle \qquad L_{tz}|+z\rangle$$

$$L_{xy}|+x\rangle \qquad L_{tz}|+x\rangle$$

(Bonus: Draw on Bloch Sphere from Video #5 for various θ and ϕ angles.)

2. Take the above are left-chiral spinors and $SL(2,\mathbb{C})$ matrices...
Write out the left dual, right dual, and right spinors and $SL(2,\mathbb{C})$ matrices.

$$L_{xy}|+z\rangle$$

$$\begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{-i\theta/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$L_{tz}|+z\rangle$$

$$\begin{bmatrix} e^{-\phi/2} & 0 \\ 0 & e^{+\phi/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{-\phi/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$L_{xy}|+x\rangle$$

$$\begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = e^{-i\theta/2} \begin{bmatrix} 1 \\ e^{i\theta} \end{bmatrix}$$

$$L_{tz}|+x\rangle$$

$$\begin{bmatrix} e^{-\phi/2} & 0 \\ 0 & e^{+\phi/2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = e^{-\phi/2} \begin{bmatrix} 1 \\ e^{\phi/2} \end{bmatrix}$$

Weyl Spinor Type	Spinor Formula	$\vert +z\rangle$	$\vert +x\rangle$
Left	ψ	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}1\\1\end{bmatrix}$
Left Dual	$\psi^T \epsilon$	$\begin{bmatrix}0 & 1\end{bmatrix}$	$\begin{bmatrix}-1 & 1\end{bmatrix}$
Right Dual	ψ^*	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}1\\1\end{bmatrix}$
Right	$\psi^\dagger \epsilon$	$\begin{bmatrix}0 & 1\end{bmatrix}$	$\begin{bmatrix}-1 & 1\end{bmatrix}$

Weyl Spinor Type	Spinor Formula	$\lvert +z\rangle$	$\lvert +x\rangle$
Left	ψ	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}1\\1\end{bmatrix}$
Left Dual	$-\epsilon\psi$	$\begin{bmatrix}0\\1\end{bmatrix}$	$\begin{bmatrix}-1\\1\end{bmatrix}$
Right Dual	ψ^*	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}1\\1\end{bmatrix}$
Right	$-\epsilon\psi^*$	$\begin{bmatrix}0\\1\end{bmatrix}$	$\begin{bmatrix}-1\\1\end{bmatrix}$

Weyl Spinor Type	Lorentz Transformation	$L_{xy}(\theta)$	$L_{tz}(\phi)$
Left	L	$\begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{+i\theta/2} \end{bmatrix}$	$\begin{bmatrix} e^{-\phi/2} & 0 \\ 0 & e^{+\phi/2} \end{bmatrix}$
Left Dual	L^{-1} (row spinor)	$\begin{bmatrix} e^{+i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{bmatrix}$	$\begin{bmatrix} e^{+\phi/2} & 0 \\ 0 & e^{-\phi/2} \end{bmatrix}$
Right Dual	L^*	Same as Left Dual	Same as Left
Right	$(L^{-1})^*$ (row spinor)	Same as Left	Same as Left Dual

Weyl Spinor Type	Lorentz Transformation	$L_{xy}(\theta)$	$L_{tz}(\phi)$
Left	L	$\begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{+i\theta/2} \end{bmatrix}$	$\begin{bmatrix} e^{-\phi/2} & 0 \\ 0 & e^{+\phi/2} \end{bmatrix}$
Left Dual	$(L^{-1})^T$ (column spinor)	$\begin{bmatrix} e^{+i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{bmatrix}$	$\begin{bmatrix} e^{+\phi/2} & 0 \\ 0 & e^{-\phi/2} \end{bmatrix}$
Right Dual	L^*	Same as Left Dual	Same as Left
Right	$(L^{-1})^\dagger$ (column spinor)	Same as Left	Same as Left Dual