Exercise: Calculate the Pauli vectors for these Pauli spinors (and check that they make sense)

$\begin{bmatrix}1\\ i\end{bmatrix}$	$\begin{bmatrix}-1\\ -i\end{bmatrix}$	$\begin{bmatrix}1\\ -i\end{bmatrix}$	$\begin{bmatrix}-1\\ i\end{bmatrix}$

$$\begin{bmatrix}-\xi^2 & \xi^1\end{bmatrix}$$

$$\begin{bmatrix}\xi^1\\ \xi^2\end{bmatrix}\begin{bmatrix}z & x-yi\\ x+yi & -z\end{bmatrix}$$

$$\boxed{\begin{aligned}\xi^1 &= \sqrt{x-yi}\\ \xi^2 &= -i\sqrt{x+yi}\end{aligned}}$$

$$\boxed{\begin{aligned}x &= \tfrac{1}{2}(\xi^1\xi^1-\xi^2\xi^2)\\ y &= \tfrac{i}{2}(\xi^1\xi^1+\xi^2\xi^2)\\ z &= -\xi^1\xi^2\end{aligned}}$$

$$\begin{bmatrix} \xi^1 = 1 \\ \xi^2 = i \end{bmatrix} \quad \boxed{\begin{aligned} x &= \frac{1}{2}(\xi^1\xi^1 - \xi^2\xi^2) \\ y &= \frac{i}{2}(\xi^1\xi^1 + \xi^2\xi^2) \\ z &= -\xi^1\xi^2 \end{aligned}} \quad \Rightarrow \begin{bmatrix} x = 1 \\ y = 0 \\ z = -i \end{bmatrix}$$

$$\begin{aligned} x &= \frac{1}{2}\left((\xi^1)^2 - (\xi^2)^2\right) \\ &= \frac{1}{2}\left((1)^2 - (i)^2\right) \\ &= \frac{1}{2}\left(1 - (-1)\right) \\ &= \frac{1}{2}(2) = 1 \end{aligned}$$

$$\begin{aligned} y &= \frac{i}{2}\left((\xi^1)^2 + (\xi^2)^2\right) \\ &= \frac{i}{2}\left((1)^2 + (i)^2\right) \\ &= \frac{i}{2}\left(1 + (-1)\right) = 0 \end{aligned}$$

$$z = -\xi^1\xi^2 = -(1)(i) = -i$$

$$\begin{bmatrix} \xi^1 = -1 \\ \xi^2 = -i \end{bmatrix} \quad \boxed{\begin{aligned} x &= \frac{1}{2}(\xi^1\xi^1 - \xi^2\xi^2) \\ y &= \frac{i}{2}(\xi^1\xi^1 + \xi^2\xi^2) \\ z &= -\xi^1\xi^2 \end{aligned}} \quad \Rightarrow \begin{bmatrix} x = 1 \\ y = 0 \\ z = -i \end{bmatrix}$$

$$\begin{aligned} x &= \frac{1}{2}\left((\xi^1)^2 - (\xi^2)^2\right) \\ &= \frac{1}{2}\left((-1)^2 - (-i)^2\right) \\ &= \frac{1}{2}\left(1 - (-1)\right) \\ &= \frac{1}{2}(2) = 1 \end{aligned}$$

$$\begin{aligned} y &= \frac{i}{2}\left((\xi^1)^2 + (\xi^2)^2\right) \\ &= \frac{i}{2}\left((-1)^2 + (-i)^2\right) \\ &= \frac{i}{2}\left(1 + (-1)\right) = 0 \end{aligned}$$

$$z = -\xi^1\xi^2 = -(-1)(-i) = -i$$

$$\begin{bmatrix} \xi^1 = 1 \\ \xi^2 = -i \end{bmatrix} \quad \boxed{\begin{aligned} x &= \tfrac{1}{2}(\xi^1\xi^1 - \xi^2\xi^2) \\ y &= \tfrac{i}{2}(\xi^1\xi^1 + \xi^2\xi^2) \\ z &= -\xi^1\xi^2 \end{aligned}} \quad \Rightarrow \begin{bmatrix} x = 1 \\ y = 0 \\ z = i \end{bmatrix}$$

$$\begin{aligned} x &= \frac{1}{2}\left((\xi^1)^2 - (\xi^2)^2\right) \\ &= \frac{1}{2}\left((1)^2 - (-i)^2\right) \\ &= \frac{1}{2}\left(1 - (-1)\right) \\ &= \frac{1}{2}(2) = 1 \end{aligned}$$

$$\begin{aligned} y &= \frac{i}{2}\left((\xi^1)^2 + (\xi^2)^2\right) \\ &= \frac{i}{2}\left((1)^2 + (-i)^2\right) \\ &= \frac{i}{2}\left(1 + (-1)\right) = 0 \end{aligned}$$

$$z = -\xi^1\xi^2 = -(1)(-i) = i$$

$$\begin{bmatrix} \xi^1 = -1 \\ \xi^2 = i \end{bmatrix} \quad \boxed{\begin{aligned} x &= \tfrac{1}{2}(\xi^1\xi^1 - \xi^2\xi^2) \\ y &= \tfrac{i}{2}(\xi^1\xi^1 + \xi^2\xi^2) \\ z &= -\xi^1\xi^2 \end{aligned}} \quad \Rightarrow \begin{bmatrix} x = 1 \\ y = 0 \\ z = i \end{bmatrix}$$

$$\begin{aligned} x &= \frac{1}{2}\left((\xi^1)^2 - (\xi^2)^2\right) \\ &= \frac{1}{2}\left((-1)^2 - (i)^2\right) \\ &= \frac{1}{2}\left(1 - (-1)\right) \\ &= \frac{1}{2}(2) = 1 \end{aligned}$$

$$\begin{aligned} y &= \frac{i}{2}\left((\xi^1)^2 + (\xi^2)^2\right) \\ &= \frac{i}{2}\left((-1)^2 + (i)^2\right) \\ &= \frac{i}{2}\left(1 + (-1)\right) = 0 \end{aligned}$$

$$z = -\xi^1\xi^2 = -(-1)(i) = i$$

$$\begin{bmatrix} \xi^1 = 1 \\ \xi^2 = i \end{bmatrix} \text{ and } \begin{bmatrix} \xi^1 = -1 \\ \xi^2 = -i \end{bmatrix}$$

have the same component ratio, and so are associated with the same vector:

$$\begin{bmatrix} x = 1 \\ y = 0 \\ z = -i \end{bmatrix}$$

$$\begin{bmatrix} \xi^1 = -1 \\ \xi^2 = i \end{bmatrix} \text{ and } \begin{bmatrix} \xi^1 = 1 \\ \xi^2 = -i \end{bmatrix}$$

have the same component ratio, and so are associated with the same vector:

$$\begin{bmatrix} x = 1 \\ y = 0 \\ z = i \end{bmatrix}$$

These spinors have the opposite sign compared to the ones on the left, and so the z-component has a reversed sign.