Exercise:

Hint: $\sigma_z|+z\rangle = |+z\rangle$

$$\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\sigma_z\sigma_x$$

$$\xi^1|+z\rangle + \xi^2\sigma_x|+z\rangle$$

$$\left(\cos\frac{\theta}{2}\xi^1 + \sin\frac{\theta}{2}\xi^2\right)|+z\rangle + \left(-\sin\frac{\theta}{2}(\xi^1) + \cos\frac{\theta}{2}\xi^2\right)\sigma_x|+z\rangle$$

$$\begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} \xi^1 & 0 \\ \xi^2 & 0 \end{bmatrix} = \begin{bmatrix} \tilde{\xi}^1 & 0 \\ \tilde{\xi}^2 & 0 \end{bmatrix}$$

$$\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\sigma_z\sigma_x\right)\ (\xi^1|+z\rangle+\xi^2|-z\rangle)$$

$$=\cos\frac{\theta}{2}\ \xi^1|+z\rangle+\sin\frac{\theta}{2}\sigma_z\sigma_x\ \xi^1|+z\rangle$$

$$+\cos\frac{\theta}{2}\ \xi^2|-z\rangle+\sin\frac{\theta}{2}\sigma_z\sigma_x\xi^2|-z\rangle$$

$$=\cos\frac{\theta}{2}\xi^1|+z\rangle-\sin\frac{\theta}{2}\xi^1\sigma_x|+z\rangle$$

$$+\cos\frac{\theta}{2}\xi^2\sigma_x|+z\rangle+\sin\frac{\theta}{2}\xi^2\sigma_z|+z\rangle$$

$$=\cos\frac{\theta}{2}\xi^1|+z\rangle-\sin\frac{\theta}{2}\xi^1|-z\rangle+\cos\frac{\theta}{2}\xi^2|+z\rangle+\sin\frac{\theta}{2}\xi^2|+z\rangle$$

$$=\left(\cos\frac{\theta}{2}\xi^1+\sin\frac{\theta}{2}\xi^2\right)|+z\rangle+\left(-\sin\frac{\theta}{2}\xi^1+\cos\frac{\theta}{2}\xi^2\right)|-z\rangle$$

Hint:

$$|-z\rangle=\sigma_x|+z\rangle$$

$$|+z\rangle=\sigma_z|+z\rangle$$