__Exercise__ - check that $J^\dagger J = 1$ for the following Jones Vectors:

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \begin{bmatrix} i\cos\theta \\ i\sin\theta \end{bmatrix}$$

$$\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}\right)^{\dagger}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\big((1)(1)+(-1)(-1)\big)$$

$$=\frac{1}{2}(1+1)=\frac{2}{2}=1$$

$$\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}\right)^{\dagger}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -i\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -i\end{bmatrix}\begin{bmatrix}1\\ i\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\Big((1)(1)+(-i)(i)\Big)$$

$$=\frac{1}{2}(1+1)=\frac{2}{2}=1$$

$$
\left(\begin{bmatrix} i\cos\theta \\ i\sin\theta \end{bmatrix}\right)^{\dagger} \begin{bmatrix} i\cos\theta \\ i\sin\theta \end{bmatrix}
$$

$$
= \begin{bmatrix} -i\cos\theta & -i\sin\theta \end{bmatrix} \begin{bmatrix} i\cos\theta \\ i\sin\theta \end{bmatrix}
$$

$$
= \Big((-i\cos\theta)(i\cos\theta) + (-i\sin\theta)(i\sin\theta)\Big)
$$

$$
= (-i^2(\cos\theta)^2 + -i^2(\sin\theta)^2)
$$

$$
= -i^2\Big((\cos\theta)^2 + (\sin\theta)^2\Big)
$$

$$
= (1)(1) = 1
$$

Exercise - check that $U^{-1} = U^{\dagger}$ for the following matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$
\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}^{\dagger} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & (-i)(i) \end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
&\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\right)^{\dagger} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\
&= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{\dagger}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} (1)(1)+(-1)(-1) & (1)(1)+(-1)(1) \\ (1)(1)+(1)(-1) & (1)(1)+(1)(1) \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} 1+1 & 1-1 \\ 1-1 & 1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\right)^{\dagger}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}$$

$$=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}^{\dagger}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}$$

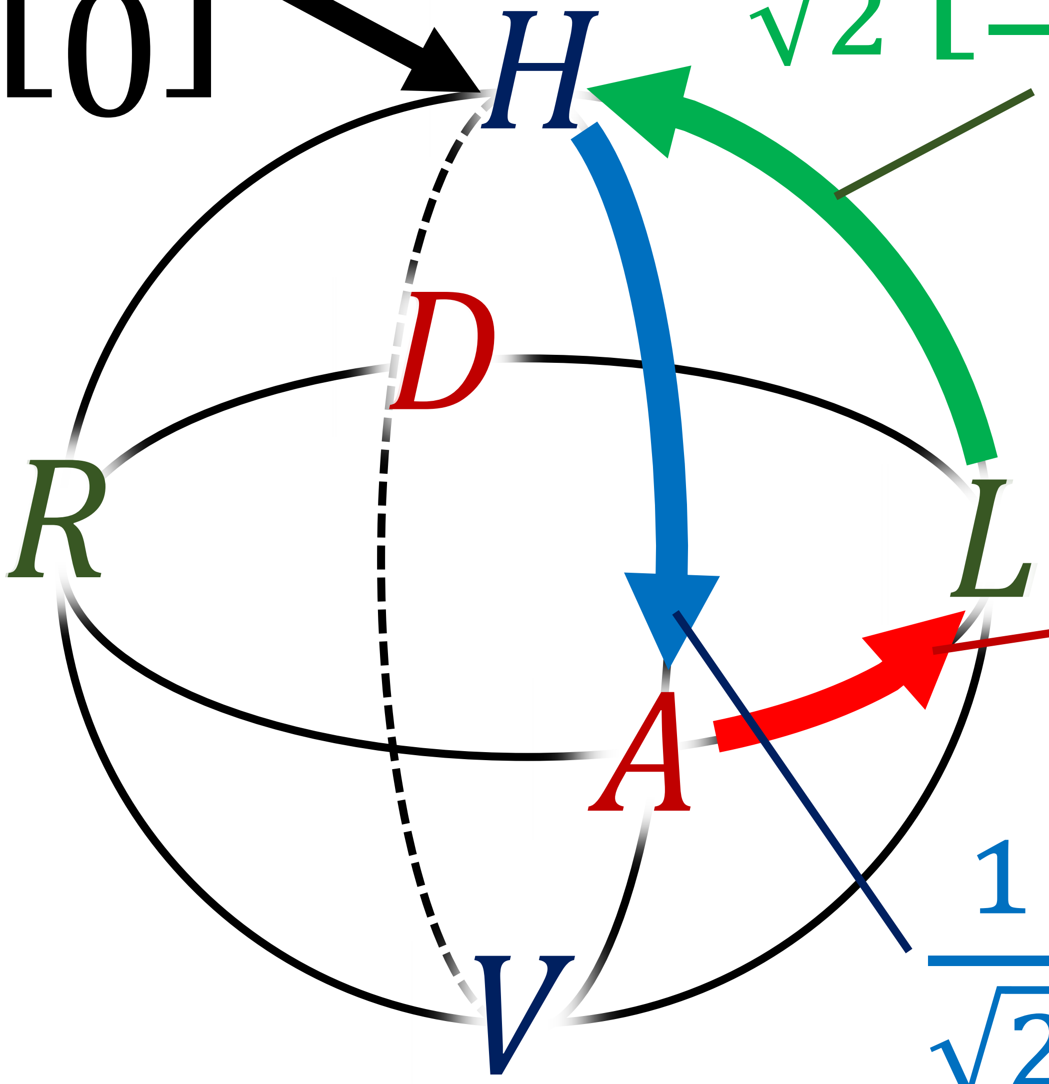
$$=\frac{1}{2}\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}(1)(1)+(-i)(i) & (-i)(1)+(1)(i)\\ (-i)(1)+(1)(i) & (-i)(i)+(1)(1)\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1+1 & -i+i\\ -i+i & 1+1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Exercise

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1+i & 0 \\ 0 & 1-i \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1+i & 0\\ 0 & 1-i\end{bmatrix}$$

$$=\begin{bmatrix}e^{i\pi/4} & 0\\ 0 & e^{-i\pi/4}\end{bmatrix}$$

$$=e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & e^{-i\pi/2}\end{bmatrix}$$

$$=e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$e^{i\frac{\pi}{4}}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = e^{i\frac{\pi}{4}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}e^{i\frac{\pi}{4}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} = e^{i\frac{\pi}{4}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} (1)(1) + (-i)(i) \\ (-i)(1) + (1)(i) \end{bmatrix}$$

$$= e^{i\frac{\pi}{4}}\frac{1}{2}\begin{bmatrix} 1 + 1 \\ -i + i \end{bmatrix} = e^{i\frac{\pi}{4}}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$