Born Rule Results:

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

$$|-x\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle$$

$$|\langle +z| +x\rangle|^2 = ½$$

$$|\langle -z| +x\rangle|^2 = ½$$

$$|\langle +z| -x\rangle|^2 = ½$$

$$|\langle -z| -x\rangle|^2 = ½$$

$$|\langle +x| +x\rangle|^2 = 1$$

$$|\langle -x| -x\rangle|^2 = 1$$

$$|\langle +x| -x\rangle|^2 = 0$$

$$\langle +x| -x\rangle$$

$$= \left(\langle +z|\frac{1}{\sqrt{2}} + \langle -z|\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle +z| +z\rangle + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)\langle +z| -z\rangle$$

$$+ \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle -z| +z\rangle + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)\langle -z| -z\rangle$$

$$= \frac{1}{2} + 0 + 0 - \frac{1}{2}$$

$$= 0$$

Left as exercise:

$$|\langle +x| +x\rangle|^2 = 1$$

$$|\langle -x| -x\rangle|^2 = 1$$

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

$$|-x\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle$$

Left as exercise:

$$|\langle +x|+x\rangle|^2 = 1$$

$$|\langle -x|-x\rangle|^2 = 1$$

$$\langle +x|+x\rangle$$

$$= \left(\langle +z|\frac{1}{\sqrt{2}} + \langle -z|\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle +z|+z\rangle + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle +z|-z\rangle$$

$$+ \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle -z|+z\rangle + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle -z|-z\rangle$$

$$= \frac{1}{2} + 0 + 0 + \frac{1}{2}$$

$$= 1$$

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

$$|-x\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle$$

Left as exercise:

$$|\langle +x|+x\rangle|^2 = 1$$

$$|\langle -x|-x\rangle|^2 = 1$$

$$\langle -x|-x\rangle$$

$$= \left(\langle +z|\frac{1}{\sqrt{2}} - \langle -z|\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\langle +z|+z\rangle + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)\langle +z|-z\rangle$$

$$+ \left(-\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}\langle -z|+z\rangle + \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)\langle -z|-z\rangle$$

$$= \frac{1}{2} + 0 + 0 + \frac{1}{2}$$

$$= 1$$

Exercise: Show that…

$$|\langle +x | +y \rangle|^2 = ½$$
$$|\langle -x | +y \rangle|^2 = ½$$
$$|\langle +x | -y \rangle|^2 = ½$$
$$|\langle -x | -y \rangle|^2 = ½$$

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$
$$|-x\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle$$
$$|+y\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{i}{\sqrt{2}}|-z\rangle$$
$$|-y\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{i}{\sqrt{2}}|-z\rangle$$

What does this mean in terms of the results of the corresponding Stern Gerlach experiments?

$$\langle +x | +y \rangle$$

$$= \left(\langle +z | \frac{1}{\sqrt{2}} + \langle -z | \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} | +z \rangle + \frac{i}{\sqrt{2}} | -z \rangle \right)$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle +z | +z \rangle + \frac{1}{\sqrt{2}} \frac{i}{\sqrt{2}} \langle +z | -z \rangle + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle -z | +z \rangle + \frac{1}{\sqrt{2}} \frac{i}{\sqrt{2}} \langle -z | -z \rangle$$

$$= \frac{1}{2} + 0 + 0 + \frac{i}{2}$$

$$= \frac{1}{2}(1 + i)$$

$$|\langle +x | +y \rangle|^2 = \left| \frac{1}{2}(1 + i) \right|^2$$

$$= \frac{1}{2}(1 + i)^* \frac{1}{2}(1 + i) = \frac{1}{2} \frac{1}{2}(1 - i)(1 + i)$$

$$= \frac{1}{4}(1 - i + i + 1) = \frac{2}{4} = \frac{1}{2}$$

$$\langle -x | +y \rangle$$

$$= \left(\langle +z | \frac{1}{\sqrt{2}} - \langle -z | \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} | +z \rangle + \frac{i}{\sqrt{2}} | -z \rangle \right)$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle +z | +z \rangle + \frac{1}{\sqrt{2}} \frac{i}{\sqrt{2}} \langle +z | -z \rangle + \left(-\frac{1}{\sqrt{2}} \right) \frac{1}{\sqrt{2}} \langle -z | +z \rangle + \left(-\frac{1}{\sqrt{2}} \right) \frac{i}{\sqrt{2}} \langle -z | -z \rangle$$

$$= \frac{1}{2} + 0 + 0 - \frac{i}{2}$$

$$= \frac{1}{2}(1 - i)$$

$$|\langle -x | +y \rangle|^2 = \left| \frac{1}{2}(1 - i) \right|^2$$

$$= \frac{1}{2}(1 - i)^* \frac{1}{2}(1 - i) = \frac{1}{2}\frac{1}{2}(1 + i)(1 - i)$$

$$= \frac{1}{4}(1 + i - i + 1) = \frac{2}{4} = \frac{1}{2}$$

$$\langle +x | -y \rangle$$

$$= \left(\langle +z | \frac{1}{\sqrt{2}} + \langle -z | \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} |+z\rangle - \frac{i}{\sqrt{2}} |-z\rangle \right)$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle +z | +z \rangle + \frac{1}{\sqrt{2}} \left(-\frac{i}{\sqrt{2}} \right) \langle +z | -z \rangle + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle -z | +z \rangle + \frac{1}{\sqrt{2}} \left(-\frac{i}{\sqrt{2}} \right) \langle -z | -z \rangle$$

$$= \frac{1}{2} + 0 + 0 - \frac{i}{2}$$

$$= \frac{1}{2}(1 - i)$$

$$|\langle +x | -y \rangle|^2 = \left| \frac{1}{2}(1 - i) \right|^2$$

$$= \frac{1}{2}(1 - i)^* \frac{1}{2}(1 - i) = \frac{1}{2} \frac{1}{2}(1 + i)(1 - i)$$

$$= \frac{1}{4}(1 + i - i + 1) = \frac{2}{4} = \frac{1}{2}$$

$$\langle -x | -y \rangle$$

$$= \left(\langle +z | \frac{1}{\sqrt{2}} - \langle -z | \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} | +z \rangle - \frac{i}{\sqrt{2}} | -z \rangle \right)$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle +z | +z \rangle + \frac{1}{\sqrt{2}} \left(-\frac{i}{\sqrt{2}} \right) \langle +z | -z \rangle + \left(-\frac{1}{\sqrt{2}} \right) \frac{1}{\sqrt{2}} \langle -z | +z \rangle + \left(-\frac{1}{\sqrt{2}} \right) \left(-\frac{i}{\sqrt{2}} \right) \langle -z | -z \rangle$$

$$= \frac{1}{2} + 0 + 0 + \frac{i}{2}$$

$$= \frac{1}{2}(1 + i)$$

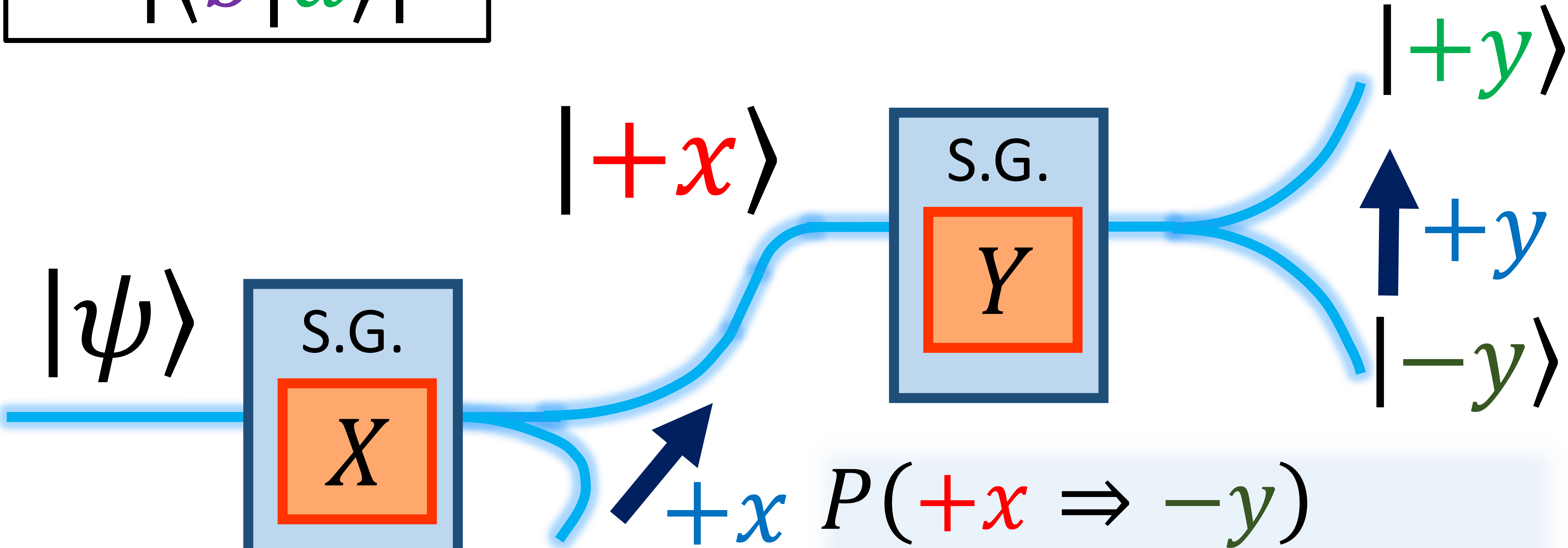
$$|\langle -x | -y \rangle|^2 = \left| \frac{1}{2}(1 + i) \right|^2$$

$$= \frac{1}{2}(1 + i)^* \frac{1}{2}(1 + i) = \frac{1}{2} \frac{1}{2}(1 - i)(1 + i)$$

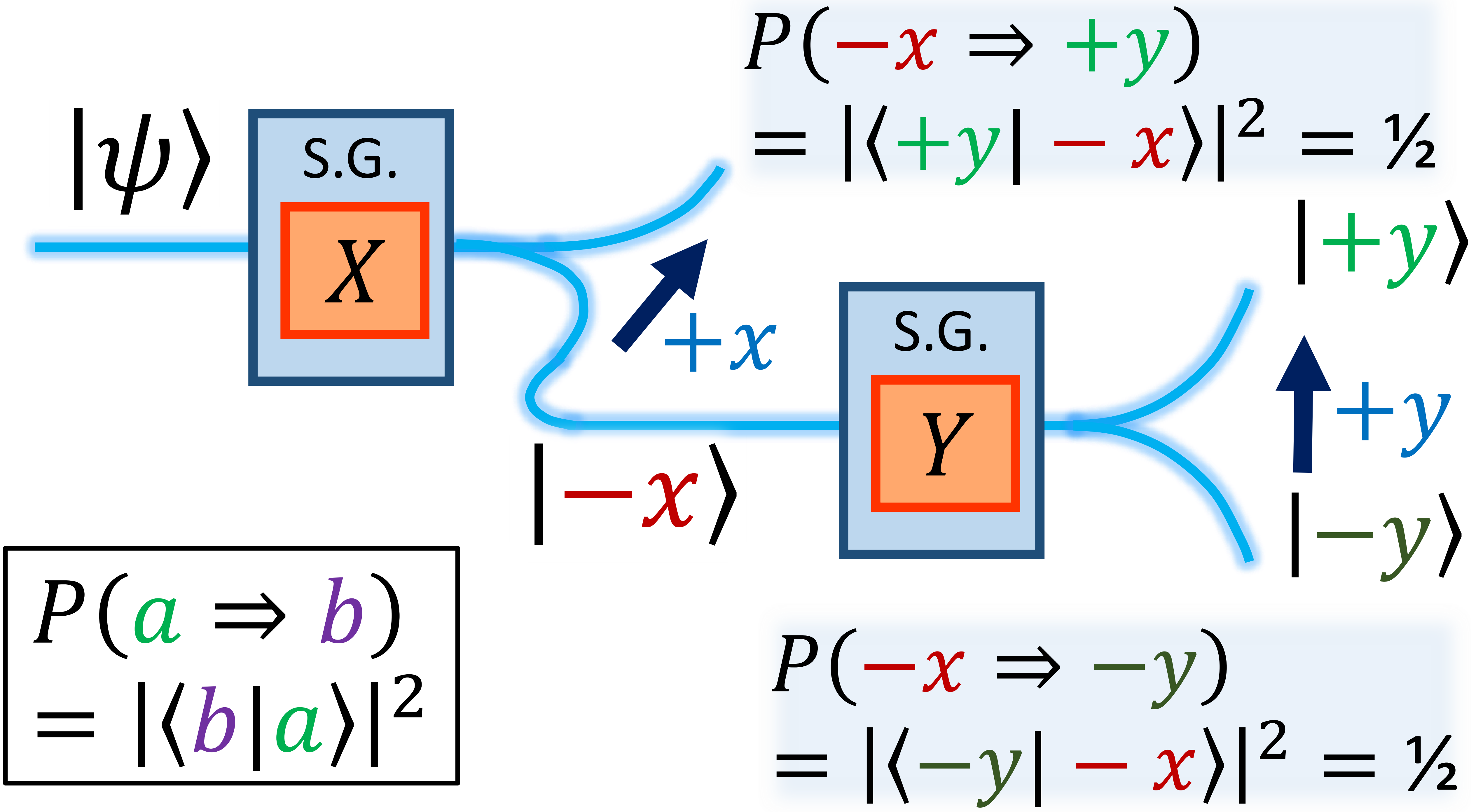
$$= \frac{1}{4}(1 - i + i + 1) = \frac{2}{4} = \frac{1}{2}$$

$$\boxed{\begin{aligned} &P(a \Rightarrow b) \\ &= |\langle b|a\rangle|^2 \end{aligned}}$$

$$P(+x \Rightarrow +y) = |\langle +y| + x\rangle|^2 = \tfrac{1}{2}$$



$$P(+x \Rightarrow -y) = |\langle -y| + x\rangle|^2 = \tfrac{1}{2}$$


$|\psi\rangle$
S.G.
X
$+x$
$|-x\rangle$
S.G.
Y
$P(-x \Rightarrow +y) = |\langle +y| - x\rangle|^2 = ½$
$|+y\rangle$
$+y$
$|-y\rangle$
$P(a \Rightarrow b) = |\langle b|a\rangle|^2$
$P(-x \Rightarrow -y) = |\langle -y| - x\rangle|^2 = ½$