

第三章

第六题

- 算法:

核心函数work():

```
def work(dim, eps):
    H = create_H(dim)                                #创建希尔伯特
    s_1 = np.max(abs(np.sum(H, 1)))                  #无穷范数
    H_I = np.mat(H).I                                #H的逆矩阵
    s_2 = np.max(abs(np.sum(H_I, 1)))                 #逆矩阵的无穷范数

    x_f = np.ones([dim], dtype=np.float64)           #创建x向量
    b = H.dot(x_f)                                    #b = H*x
    b_eps = b + eps                                   #b加上扰动
    x_L = figure_x(H, b_eps, dim)                    #求解方程Hx=b
    b_L = H.dot(x_L)                                  #b^ = H*x^

    r_b = b - b_L                                     #计算残差r = b - b^
    r_x = x_f - x_L                                   #计算误差x - x^
    print ("Dim = %d\teps_b = %.10f" % (dim, eps))
    print ("cond(H) = %d\t|H| = %d\t|H(-1)| = %d" % (s_1*s_2, s_1, s_2))
    print ("|r_b| = %.20f" % (np.max(abs(r_b))))
    print ("|r_x| = %.20f" % (np.max(abs(r_x))))
    print ()
    return (np.max(abs(r_b)), np.max(abs(r_x)), s_1*s_2)
```

Cholesky分解以及求解方程的函数将书上的伪代码翻译成Python代码即可，细节见代码

- $n = 10$ ，计算残差和误差的无穷范数

work(10, 0.0)，效果:

```
PS C:\Users\Adil\Desktop\大三第二学期\数值分析\上机实验\3第三章\6第六题> python .\1.py
Dim = 10      eps_b = 0.0000000000
cond(H) = 20503568      |H| = 2 |H(-1)| = 7000269
|r_b| = 0.000000000000000022204
|r_x| = 0.00044458507134448322
```

- 对右端项施加 10^{-7} ，观察残差和误差的无穷范数

work(10, 1e-7)，效果:

```

PS C:\Users\Adil\Desktop\大三 第二学期\数值分析\上机实验\3第三章\6第六题> python .\1.py
Dim = 10          eps_b = 0.0000000000
cond(H) = 20503568    |H| = 2 |H(-1)| = 7000269
|r_b| = 0.000000000000000022204
|r_x| = 0.00044458507134448322

Dim = 10          eps_b = 0.0000001000
cond(H) = 20503568    |H| = 2 |H(-1)| = 7000269
|r_b| = 0.000000100000000005839
|r_x| = 0.70070827001717650440

```

- 改变n的值为8、12，观察残差和误差的无穷范数

```

work(8, 0.0)
work(12, 0.0)

```

效果:

```

Dim = 8 eps_b = 0.0000000000
cond(H) = 587644    |H| = 2 |H(-1)| = 216215
|r_b| = 0.000000000000000022204
|r_x| = 0.00000041154382102171

Dim = 12          eps_b = 0.0000000000
cond(H) = 738986424    |H| = 3 |H(-1)| = 238136079
|r_b| = 0.000000000000000044409
|r_x| = 0.33580581043297352828

```

- 通过这个实验说明了什么问题
 - 巨大的条件数说明Hilbert矩阵的病态矩阵
 - 右端项施加扰动之后，误差变化不大，说明Cholesky分解算法求解方程比较稳定