第四章

第二题

- 规定
  - 取初始解 **x(0) = [0] * n-1**
  - 将 **|| x(k+1) - x(k) || < 10^{-4}** 作为终止迭代的判断依据
- 算法:
  核心函数work():

```
def work(eps):
    A = create_A(eps)                                            #创建A矩阵
    b = [a*(h**2) for x in range(n-1)]                           #b = [ah**2] * n-1
    e = 1e-4
    x_0 = [np.float64(0.0) for x in range(n-1)]                  #x(0) = [0] * n-1

    A_I = np.mat(A).I                                            #求A的逆矩阵
    x_r = A_I.dot(b)                                             #求解b的精确值
    print ("Exact:")
    print (x_r)                                                  #输出精确值
    print ()

    e_r, x_p = Jacobi(A, b, x_0, e, n-1)                         #Jacobi迭代法计算
    r = x_r - x_p                                                #计算误差
    print ("Jacobi:")
    print ("|r| = %.20f" % (np.max(abs(r))))                     #误差的无穷范数
    print ("x = ", np.mat([np.float64("%.3E" % x) for x in x_p]))       #输出结果，保留4位有效

    print ()

    e_r, x_p = GS(A, b, x_0, e, n-1)                             #GS迭代法计算
    r = x_r - x_p
    print ("GS:")
    print ("|r| = %.20f" % (np.max(abs(r))))
    print ("x = ", np.mat([np.float64("%.3E" % x) for x in x_p]))
    print ()

    e_r, x_p, k = SOR(A, b, 1.25, x_0, e, n-1)                   #SOR迭代法计算
    r = x_r - x_p
    print ("SOR:")
    print ("|r| = %.20f" % (np.max(abs(r))))
    print ("x = ", np.mat([np.float64("%.3E" % x) for x in x_p]))
```

各种迭代法的函数将书上给出的伪代码翻译为Python代码即可，细节见代码

- eps = 1.0 时
  - Jacobi迭代法：

```
Jacobi:
|r| = 0.06132757104799099745
x =  [[-2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05

  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05 -2.488e-05
  -2.488e-05 -2.488e-05 -2.488e-05]]
```

  - GS迭代法：

```
GS:
|r| = 0.06127806609749594791
x =  [[-3.738e-05 -5.597e-05 -6.522e-05 -6.982e-05 -7.211e-05 -7.325e-05
  -7.382e-05 -7.410e-05 -7.424e-05 -7.431e-05 -7.435e-05 -7.436e-05
  -7.437e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05 -7.438e-05
  -7.438e-05 -7.438e-05 -6.188e-05]]
```

  - SOR迭代法：

```
SOR:
|r| = 0.06119582925544748975
x =  [[-5.691e-05 -9.346e-05 -1.168e-04 -1.315e-04 -1.409e-04 -1.468e-04
  -1.505e-04 -1.528e-04 -1.542e-04 -1.551e-04 -1.557e-04 -1.560e-04
  -1.563e-04 -1.564e-04 -1.565e-04 -1.565e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04

  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04 -1.566e-04
  -1.566e-04 -1.488e-04 -1.081e-04]]
```

- eps = 0.1 时
  - Jacobi迭代法：

```
Jacobi:
|r| = 0.12019242244416511456
x =  [[-0.02536  -0.048    -0.06819  -0.08619  -0.1022   -0.1165   -0.1292
  -0.1404   -0.1504   -0.1593   -0.1671   -0.174    -0.1801   -0.1854
  -0.1901   -0.1941   -0.1977   -0.2007   -0.2033   -0.2055   -0.2074
  -0.2089   -0.2101   -0.2111   -0.2118   -0.2124   -0.2127   -0.2128
  -0.2127   -0.2125   -0.2122   -0.2117   -0.2111   -0.2103   -0.2095
  -0.2085   -0.2074   -0.2063   -0.205    -0.2037   -0.2022   -0.2007
  -0.1991   -0.1974   -0.1956   -0.1938   -0.1918   -0.1898   -0.1878
  -0.1856   -0.1834   -0.1811   -0.1787   -0.1762   -0.1737   -0.1711
  -0.1685   -0.1658   -0.163    -0.1601   -0.1571   -0.1541   -0.1511

  -0.1479   -0.1447   -0.1415   -0.1381   -0.1348   -0.1313   -0.1278
  -0.1242   -0.1206   -0.1169   -0.1131   -0.1093   -0.1055   -0.1016
  -0.0976   -0.09359  -0.08953  -0.08541  -0.08126  -0.07705  -0.07281
  -0.06852  -0.06419  -0.05982  -0.05541  -0.05096  -0.04648  -0.04196
  -0.03742  -0.03284  -0.02822  -0.02359  -0.01892  -0.01422  -0.009507
  -0.004765]]
```

  - GS迭代法：

```
GS:
|r| = 0.03264207621270182047
x =  [[-0.0361   -0.06849  -0.09752  -0.1235   -0.1467   -0.1674   -0.1859
  -0.2023   -0.2168   -0.2296   -0.2409   -0.2508   -0.2595   -0.2669
  -0.2734   -0.2789   -0.2836   -0.2875   -0.2906   -0.2932   -0.2951
  -0.2965   -0.2974   -0.2979   -0.298    -0.2977   -0.2971   -0.2962
  -0.295    -0.2935   -0.2918   -0.2899   -0.2877   -0.2854   -0.283
  -0.2803   -0.2776   -0.2747   -0.2716   -0.2685   -0.2652   -0.2619
  -0.2585   -0.2549   -0.2513   -0.2477   -0.2439   -0.2401   -0.2362
  -0.2323   -0.2283   -0.2243   -0.2202   -0.216    -0.2119   -0.2076

  -0.2034   -0.1991   -0.1947   -0.1904   -0.186    -0.1815   -0.1771
  -0.1726   -0.1681   -0.1635   -0.159    -0.1544   -0.1498   -0.1451
  -0.1405   -0.1358   -0.1311   -0.1264   -0.1217   -0.117    -0.1122
  -0.1075   -0.1027   -0.09788  -0.09307  -0.08825  -0.08342  -0.07857
  -0.07372  -0.06886  -0.06399  -0.05911  -0.05422  -0.04932  -0.04442
  -0.0395   -0.03459  -0.02966  -0.02473  -0.0198   -0.01485  -0.009907
  -0.004956]]
```

- SOR迭代法：

```
SOR:
|r| = 0.01946460713744957438
x =  [[-0.03781  -0.07175  -0.1022   -0.1294   -0.1538   -0.1755   -0.1948
  -0.212    -0.2272   -0.2406   -0.2524   -0.2628   -0.2718   -0.2795
  -0.2862   -0.2919   -0.2967   -0.3006   -0.3038   -0.3063   -0.3082
  -0.3096   -0.3104   -0.3107   -0.3106   -0.3102   -0.3093   -0.3082
  -0.3067   -0.305    -0.303    -0.3008   -0.2984   -0.2958   -0.2931
  -0.2901   -0.2871   -0.2839   -0.2805   -0.2771   -0.2736   -0.2699
  -0.2662   -0.2624   -0.2585   -0.2545   -0.2505   -0.2464   -0.2423

  -0.2381   -0.2338   -0.2295   -0.2252   -0.2208   -0.2164   -0.212
  -0.2075   -0.203    -0.1984   -0.1939   -0.1893   -0.1847   -0.18
  -0.1754   -0.1707   -0.166    -0.1613   -0.1565   -0.1518   -0.147
  -0.1423   -0.1375   -0.1327   -0.1278   -0.123    -0.1182   -0.1133
  -0.1085   -0.1036   -0.09874  -0.09385  -0.08896  -0.08406  -0.07916
  -0.07425  -0.06933  -0.06441  -0.05948  -0.05454  -0.0496   -0.04466
  -0.03971  -0.03476  -0.0298   -0.02484  -0.01988  -0.01492  -0.009946
  -0.004974]]
```

- eps = 0.01 时
  - Jacobi迭代法：

```
Jacobi:
|r| = 0.00266374232870697503
x =  [[-0.2433 -0.3626 -0.4198 -0.4461 -0.4569 -0.4599 -0.459  -0.4562 -0.4524
  -0.4481 -0.4435 -0.4388 -0.434  -0.4292 -0.4243 -0.4194 -0.4145 -0.4095
  -0.4046 -0.3997 -0.3947 -0.3898 -0.3848 -0.3798 -0.3749 -0.3699 -0.3649
  -0.3599 -0.3549 -0.3499 -0.345  -0.34   -0.335  -0.33   -0.325  -0.32
  -0.315  -0.31   -0.305  -0.3    -0.295  -0.29   -0.285  -0.28   -0.275
  -0.27   -0.265  -0.26   -0.255  -0.25   -0.245  -0.24   -0.235  -0.23
  -0.225  -0.22   -0.215  -0.21   -0.205  -0.2    -0.195  -0.19   -0.185
  -0.18   -0.175  -0.17   -0.165  -0.16   -0.155  -0.15   -0.145  -0.14

  -0.135  -0.13   -0.125  -0.12   -0.115  -0.11   -0.105  -0.1    -0.095
  -0.09   -0.085  -0.08   -0.075  -0.07   -0.065  -0.06   -0.055  -0.05
  -0.045  -0.04   -0.035  -0.03   -0.025  -0.02   -0.015  -0.01   -0.005 ]]
```

- GS迭代法：

```
GS:
|r| = 0.00163245247583437836
x =  [[-0.2439 -0.3635 -0.4209 -0.4472 -0.4579 -0.4608 -0.4599 -0.457  -0.4531
  -0.4487 -0.4441 -0.4393 -0.4344 -0.4295 -0.4246 -0.4197 -0.4147 -0.4098
  -0.4048 -0.3998 -0.3949 -0.3899 -0.3849 -0.3799 -0.3749 -0.3699 -0.365
  -0.36   -0.355  -0.35   -0.345  -0.34   -0.335  -0.33   -0.325  -0.32
  -0.315  -0.31   -0.305  -0.3    -0.295  -0.29   -0.285  -0.28   -0.275

  -0.27   -0.265  -0.26   -0.255  -0.25   -0.245  -0.24   -0.235  -0.23
  -0.225  -0.22   -0.215  -0.21   -0.205  -0.2    -0.195  -0.19   -0.185
  -0.18   -0.175  -0.17   -0.165  -0.16   -0.155  -0.15   -0.145  -0.14
  -0.135  -0.13   -0.125  -0.12   -0.115  -0.11   -0.105  -0.1    -0.095
  -0.09   -0.085  -0.08   -0.075  -0.07   -0.065  -0.06   -0.055  -0.05
  -0.045  -0.04   -0.035  -0.03   -0.025  -0.02   -0.015  -0.01   -0.005 ]]
```

- SOR迭代法：

```
SOR:
|r| = 0.00106139253479908824
x =  [[-0.2443 -0.364  -0.4214 -0.4477 -0.4584 -0.4613 -0.4603 -0.4574 -0.4535
  -0.449  -0.4443 -0.4395 -0.4346 -0.4297 -0.4248 -0.4198 -0.4148 -0.4099
  -0.4049 -0.3999 -0.3949 -0.3899 -0.3849 -0.38   -0.375  -0.37   -0.365

  -0.36   -0.355  -0.35   -0.345  -0.34   -0.335  -0.33   -0.325  -0.32
  -0.315  -0.31   -0.305  -0.3    -0.295  -0.29   -0.285  -0.28   -0.275
  -0.27   -0.265  -0.26   -0.255  -0.25   -0.245  -0.24   -0.235  -0.23
  -0.225  -0.22   -0.215  -0.21   -0.205  -0.2    -0.195  -0.19   -0.185
  -0.18   -0.175  -0.17   -0.165  -0.16   -0.155  -0.15   -0.145  -0.14
  -0.135  -0.13   -0.125  -0.12   -0.115  -0.11   -0.105  -0.1    -0.095
  -0.09   -0.085  -0.08   -0.075  -0.07   -0.065  -0.06   -0.055  -0.05
  -0.045  -0.04   -0.035  -0.03   -0.025  -0.02   -0.015  -0.01   -0.005 ]]
```

- eps = 0.0001 时
    - Jacobi迭代法：

```
Jacobi:
|r| = 0.00014234056032735865
x =  [[-0.4899 -0.4899 -0.485  -0.48   -0.475  -0.47   -0.465  -0.46   -0.455
  -0.45   -0.445  -0.44   -0.435  -0.43   -0.425  -0.42   -0.415  -0.41
  -0.405  -0.4    -0.395  -0.39   -0.385  -0.38   -0.375  -0.37   -0.365
  -0.36   -0.355  -0.35   -0.345  -0.34   -0.335  -0.33   -0.325  -0.32
  -0.315  -0.31   -0.305  -0.3    -0.295  -0.29   -0.285  -0.28   -0.275
  -0.27   -0.265  -0.26   -0.255  -0.25   -0.245  -0.24   -0.235  -0.23
  -0.225  -0.22   -0.215  -0.21   -0.205  -0.2    -0.195  -0.19   -0.185
  -0.18   -0.175  -0.17   -0.165  -0.16   -0.155  -0.15   -0.145  -0.14

  -0.135  -0.13   -0.125  -0.12   -0.115  -0.11   -0.105  -0.1    -0.095
  -0.09   -0.085  -0.08   -0.075  -0.07   -0.065  -0.06   -0.055  -0.05
  -0.045  -0.04   -0.035  -0.03   -0.025  -0.02   -0.015  -0.01   -0.005 ]]
```

    - GS迭代法：

```
GS:
|r| = 0.00004745115022036783
x =  [[-0.49   -0.4899 -0.485  -0.48   -0.475  -0.47   -0.465  -0.46   -0.455
  -0.45   -0.445  -0.44   -0.435  -0.43   -0.425  -0.42   -0.415  -0.41
  -0.405  -0.4    -0.395  -0.39   -0.385  -0.38   -0.375  -0.37   -0.365
  -0.36   -0.355  -0.35   -0.345  -0.34   -0.335  -0.33   -0.325  -0.32
  -0.315  -0.31   -0.305  -0.3    -0.295  -0.29   -0.285  -0.28   -0.275
  -0.27   -0.265  -0.26   -0.255  -0.25   -0.245  -0.24   -0.235  -0.23
  -0.225  -0.22   -0.215  -0.21   -0.205  -0.2    -0.195  -0.19   -0.185
  -0.18   -0.175  -0.17   -0.165  -0.16   -0.155  -0.15   -0.145  -0.14
  -0.135  -0.13   -0.125  -0.12   -0.115  -0.11   -0.105  -0.1    -0.095
  -0.09   -0.085  -0.08   -0.075  -0.07   -0.065  -0.06   -0.055  -0.05
  -0.045  -0.04   -0.035  -0.03   -0.025  -0.02   -0.015  -0.01   -0.005 ]]
```

    - SOR迭代法：

```
SOR:
|r| = 0.00002181856461624943
x =  [[-0.49   -0.4899 -0.485  -0.48   -0.475  -0.47   -0.465  -0.46   -0.455
  -0.45   -0.445  -0.44   -0.435  -0.43   -0.425  -0.42   -0.415  -0.41
  -0.405  -0.4    -0.395  -0.39   -0.385  -0.38   -0.375  -0.37   -0.365

  -0.36   -0.355  -0.35   -0.345  -0.34   -0.335  -0.33   -0.325  -0.32
  -0.315  -0.31   -0.305  -0.3    -0.295  -0.29   -0.285  -0.28   -0.275
  -0.27   -0.265  -0.26   -0.255  -0.25   -0.245  -0.24   -0.235  -0.23
  -0.225  -0.22   -0.215  -0.21   -0.205  -0.2    -0.195  -0.19   -0.185
  -0.18   -0.175  -0.17   -0.165  -0.16   -0.155  -0.15   -0.145  -0.14
  -0.135  -0.13   -0.125  -0.12   -0.115  -0.11   -0.105  -0.1    -0.095
  -0.09   -0.085  -0.08   -0.075  -0.07   -0.065  -0.06   -0.055  -0.05
  -0.045  -0.04   -0.035  -0.03   -0.025  -0.02   -0.015  -0.01   -0.005 ]]
```

- 可以看出，eps越小，迭代法得到的解与精确解的误差越小