Probabilistic language models

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Today

Overview

Language model

1. Compute the probability of a sequence of words:

$$P(w_1, w_2, \dots, w_n)$$

2. Predict next word:

$$P(w_n | w_1, w_2, \dots, w_{n-1})$$

LMs help are used to:

- Machine translation: choose best translation
- Spell checking: find incorrect word
- Speech recognition: choose best transcription
- Predict next word in your smartphone
- Generate poems, summaries, answers, etc.

А. А. Марков (1886).

Markov assumptions

1. Chain rule:

$$P(W) = P(w_1, w_2, \dots, w_n) = \prod_i P(w_i | w_1, \dots, w_{i-1})$$

2. Maximum likelihood estimates of probabilities:

$$P(w_i | w_1, \dots, w_{i-1}) = \frac{\text{count}(w_1, w_2, \dots, w_i)}{\text{count}(w_1, \dots, w_{i-1})}$$

3. Markov assumption (k-th order):

$$P(w_i | w_1, \dots, w_{i-1}) \approx P(w_i | w_{i-k}, \dots w_{i-1})$$

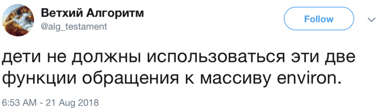
n-gram models

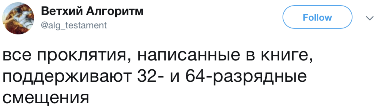
1. Unigram models: $P(W) = P(w_1, \dots, w_n) \approx \prod_i P(w_i)$
2. Bigram models: $P(W) = P(w_1, \dots, w_n) \approx \prod_i P(w_i|w_{i-1})$
3. Perplexity: $PP(W) = P(w_1, w_2, \dots w_n)^{-\frac{1}{N}}$
   The lower perplexity, the better the model predicts an unseen test
4. Smoothing: $P(w_i|w_1, \dots, w_{i-1}) = \frac{\text{count}(w_1, w_2, \dots, w_i)+1}{\text{count}(w_1, \dots, w_{i-1})+\alpha|V|}$, where $|V|$ is the size of dictionary
5. Interpolation: $\hat{P}(w_i|w_{i-1}) = \lambda P_{MLE}(w_i|w_{i-1}) + (1-\lambda)P_{MLE}(w_i)$

n-gram models for text generation

Given w_i:

1. choose the next most probable w_{i+i}
2. randomly select sample from this probability distribution of next words





`https://twitter.com/alg_testament`

n-gram models for IR

Given documents D and query q, estimate the probability of generating the query text from a document language model:

- Rank documents by the probability that the query could be generated by the document model;
- Calculate $P(d|q)$ to rank the documents: $P(d|q) \propto P(q|d)P(d)$
- Assuming prior is uniform, unigram model: $P(q|d) = \prod_i P(q_i|d)$
- MLE: $P(q_i|d) = \frac{\text{count}(q_i,d)}{|d|}$

Today

Overview

Part of speech tagging

Given a sentence or a sequence of words (X), predict its part of speech sequence (Y)

X (words)	the	cat	sat	on	a	mat
Y (POS-tags):	DET	NOUN	VERB	PREP	DET	NOUN

1. Pointwise prediction: choose a POS-tag for a word individually
2. Sequence models:
   - Generative models: $P(y, x)$
   - Discriminative models: $P(y|x)$

Generative sequence models

$$\arg\max_Y P(Y|X) = \arg\max \frac{P(X|Y)P(Y)}{P(X)} \approx \arg\max P(X|Y)P(Y)$$

- $P(X|Y)$ models word/ P OS tag interactions
- $P(Y)$ models POS / POS interactions

А. А. Марков (1886).

Hidden Markov Models

An HMM is specified by the following components:

$Q = q_1, \ldots, q_T$	states (POS-tags)
$A = (a_{ij})$	transition probability matrix: $a_{ij} = P(Q_i \to Q_j)$
$O = o_1, \ldots, o_V$	observations (words)
B	emission probabilities
	$b_i(o_t)$ is the probability of q_i generate o_t
$\pi = \pi_1, \ldots, \pi_N$	initial probability distribution

Probabilities should sum to unity:

$\sum_j a_{ij} = 1$

$\sum_i \pi_i = 1$

Markov assumptions

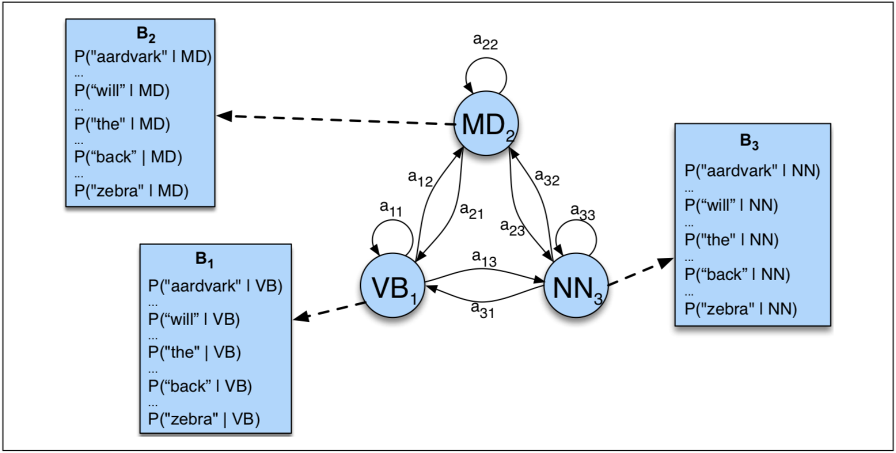
1. The probability of a particular state depends only on the previous state:

$$P(q_i|q_1,\ldots,q_{i-1}) = P(q_i|q_{i-1})$$

2. Output Independence: the probability of an output observation o_i depends only on the state that produced the observation q_i:

$$P(o_i|Q,O) = P(o_i|q_i)$$

Example of HMM (from SLP Book [1])



[1] https://web.stanford.edu/~jurafsky/slp3

Three tasks of HMM

1. Likelihood: given an observation sequence, estimate the likelihood of the observation sequence
2. Decoding: given an observation sequence, discover the best hidden state sequence leading to these observations.
3. Learning: train HMM

Forward-backward algorithm

$O_n = o_1, \dots, o_n$

Forward probabilities: $\alpha_{ij} = P(o_1, \dots, o_i)$

Forward algorithm:

1. $\alpha_{1j} = a_{0j} b_j(o_1), 1 \leq j \leq |Q|$
2. $\alpha_{ij} = \sum_k^Q \alpha_{i-1,k} a_{kj} b_j(o_i), 1 \leq i \leq n, 1 \leq j \leq |Q|$
3. $P(O_n) = \sum_k^Q \alpha_{nk} a_{kF}$

Backward probabilities: $\beta_{oj} = P(o_{i+1}, \dots, o_n)$

Backward algorithm:

1. $\beta_{nj} = a_{jF}, 1 \leq j \leq |Q|$
2. $\alpha_{ij} = \sum_k^Q \beta_{i+1,k} a_{jk} b_k(o_{i+1}), 1 \leq i \leq n, 1 \leq j \leq |Q|$
3. $P(O_n) = \sum_k^Q a_{0k} b_(o_1) \beta$

Decoding

Input: HMM $= (A, B)$, observations $= o_1, \ldots, o_n$
Output: the most probable sequence of states $= q_1, \ldots, q_n$

$$\hat{q}_n = \arg\max_{q_n} P(q_n | o_n) \approx$$

$$\approx \max_{q_n} \prod_{i=1}^{n} P(o_i | q_i) P(q_i | q_{i-1})$$

Viterbi algorithm

Compute path probabilities $V = |n \times T|$. v_{ij} represents the probability that the HMM is in state j after seeing the first i observations.

1. Intialize

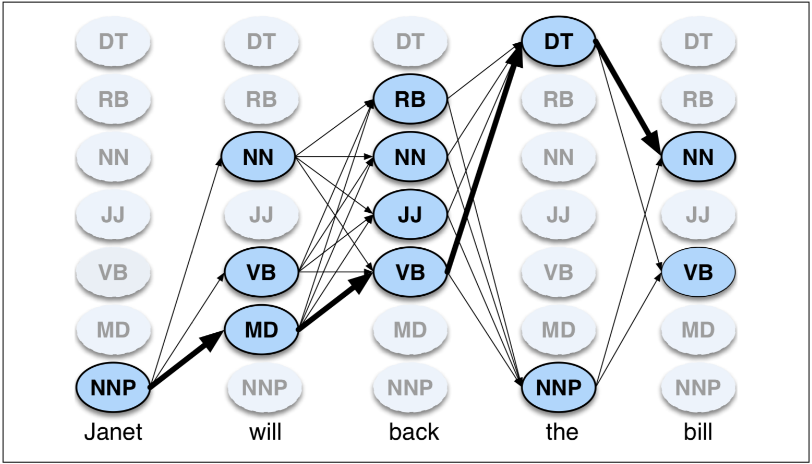
$$v_{1j} = a_{0j} b(o_1), 1 \leq j \leq T$$

2. Recursion

$$v_{ij} = \max v_{i-1,k} a_{kj} b_j(o_i), 1 \leq i \leq n, 1 \leq j \leq T$$

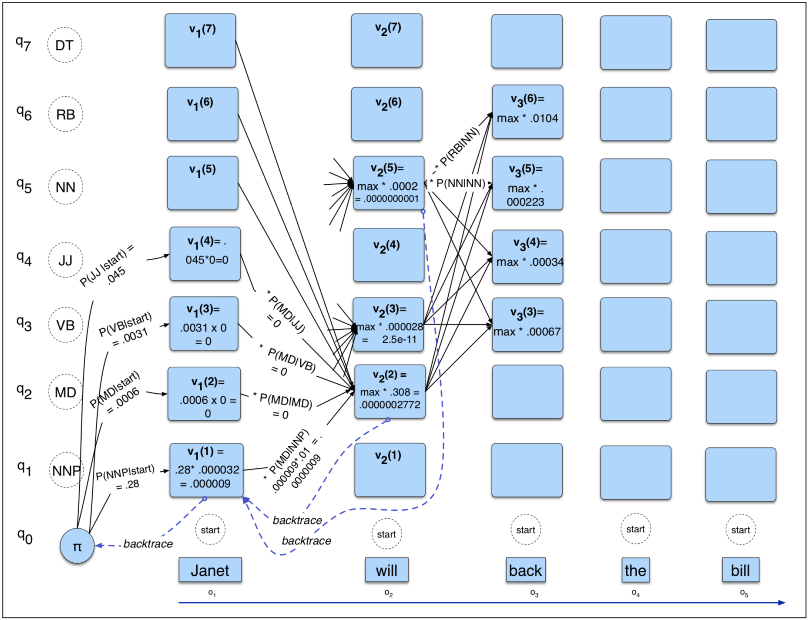
3. End

$$\max_{q \in Q^n} p(o, q) = \max_{1 \leq k \leq T} v_{nk} a_{kF}$$


DT
RB
NN
JJ
VB
MD
NNP
Janet
will
back
the
bill

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

	NNP	MD	VB	JJ	NN	RB	DT
$<s>$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017



q7 DT
q6 RB
q5 NN
q4 JJ
q3 VB
q2 MD
q1 NNP
q0 π
v1(7)
v1(6)
v1(5)
v1(4)= .045*0=0
v1(3)= .0031 x 0 = 0
v1(2)= .0006 x 0 = 0
v1(1) = .28* .000032 = .000009
P(JJ lstart) = .045
P(VBlstart) = .0031
P(MDlstart) = .0006
P(NNPlstart) = .28
v2(7)
v2(6)
v2(5)= max * .0002 = .0000000001
v2(4)
v2(3)= max * .000028 = 2.5e-11
v2(2) = max * .308 = .0000002772
v2(1)
* P(MDIJJ) = 0
* P(MDIVB) = 0
* P(MDIMD) = 0
* P(MDINNP) .000009*.01 = .0000009
v3(6)= max * .0104
v3(5)= max * . 000223
v3(4)= max * .00034
v3(3)= max * .00067
* P(RBINN)
* P(NNINN)
backtrace
backtrace
backtrace
start
start
start
start
start
Janet
will
back
the
bill
o1
o2
o3
o4
o5

TnT POS-tagger [Brants, 2000]

TnT uses second-order HMM for POS-tagging:

$$\arg\max[\prod_j [p(t_i|t_{i-1}, o_{t-2})p(w_i|t_i)]P(t_{T+1}|t_T)$$

The probability of a POS-tag for a given word is computed as a linear interpolation of three LM's:

$$P(t_i|t_{i-1}, t_{i-2}) = l_1 * P(t_i) + l_2 * P(t_i|t_{i-1}) + l_3 * P(t_i|t_{i-1}, t_{i-2})$$

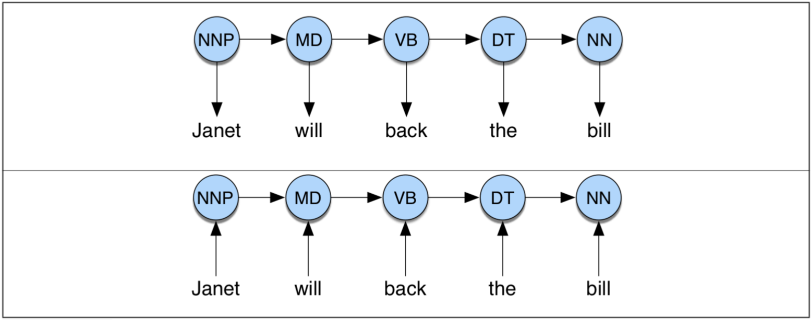
See NLTK examples (English_HMM_POS_tagger).

Today

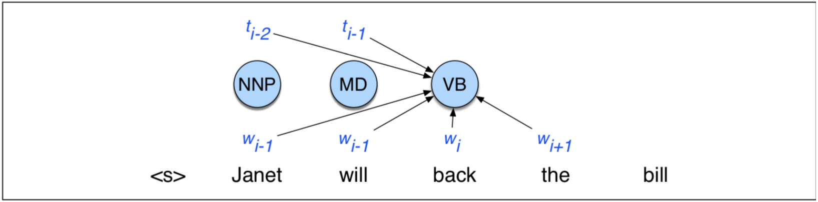
Overview

Maximum Entropy Markov Models (MEMM)

HMM: $\arg\max P(Y|X) = \arg\max_Y P(X|Y)P(Y)$
MEMM: $\arg\max_y P(Y|X) = \arg\max_Y P(y_i|y_{i-1}, x_i)$



Features in a MEMM



- Feature templates: $< t_i, w_{i-2} >, < t_i, t_{i-1} >, < t_i, t_i, w_i, w_{i+1} >$
- Casing, shape, is number?, is string?, has a dash?, has a digit?, etc.

Decoding MEMM

- Locally normalized logistic regression on a sequencer:

$$\hat{Y} = \arg\max P(Y|X) = \arg\max \prod_i P(t_i, w_{i-l}^{i+l}, t_{i-k}^{i-1}) =$$

$$= \arg\max_T \prod_i \frac{exp(\sum_j \theta_j f_j(t_i, w_{i-l}^{i+l}, t_{i-k}^{i-1}))}{\sum_{t' \in T} exp(\sum_j \theta_j f_j(t'_i, w_{i-l}^{i+l}, t_{i-k}^{i-1}))}$$

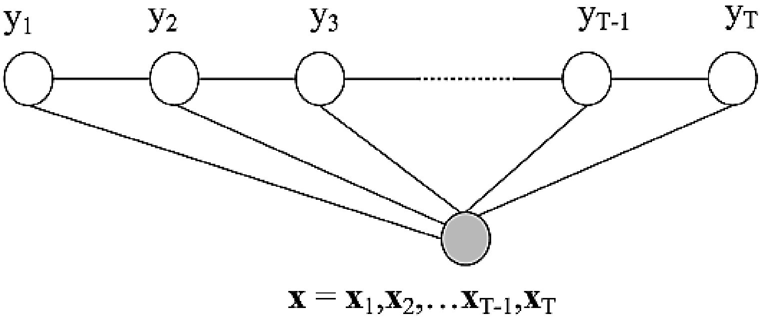
- Viterbi recursion step: $v_{ij} = \max_k v_{i-1,k} P(t_k|t_{k-1}, w_i)$
- Local normalization leads to labels bias: will/NN to/TO fight/VB

Today

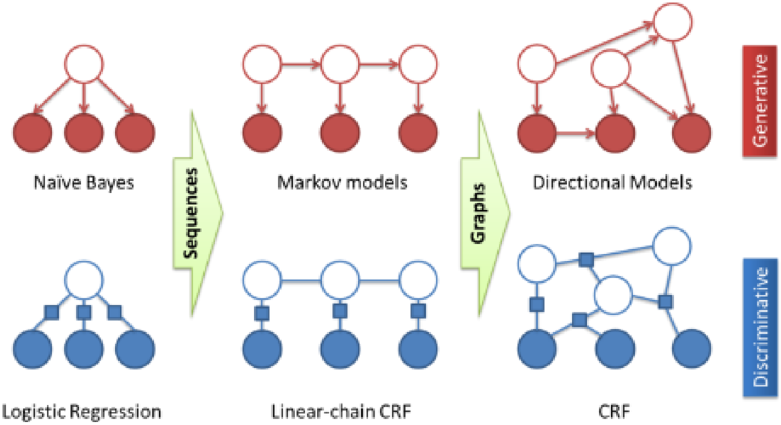
Overview

Conditional Random Field (CRF)

$$p(Y|X) = \frac{e^{\sum_{i=1}^{k} \lambda_i F_i(y,x)}}{\sum_{y' \in C^n} e^{\sum_{i=1}^{k} \lambda_i F_i(y',x)}}$$



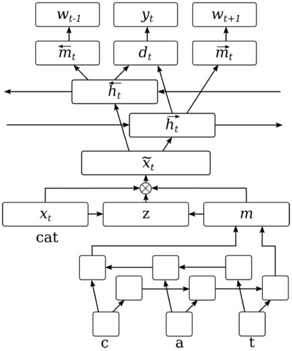
HMM VS CRF



Adapted from C. Sutton, A. McCallum, "An Introduction to Conditional Random Fields", ArXiv, November 2010

DL approaches to sequence modeling

- Neural networks
- Recurrent neural network and its modifications: LSTM, GRU, Highway
- 2D Convolutional Neural Network
- Transformer
- Pointer network



Problems:

1. Training time
2. Amount of training data

Today

Neural language model [1]

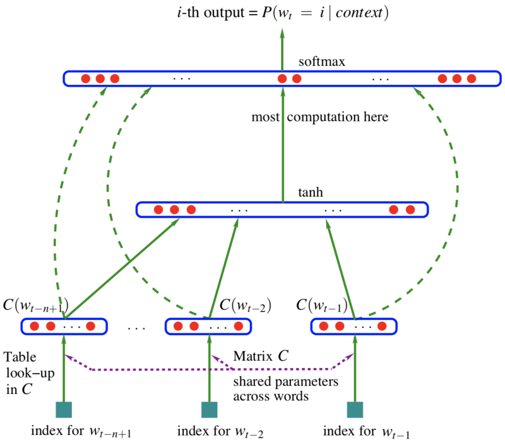


Figure: Neural language model

Recurrent neural network

- Input: sequence of vectors
- $x_{1:n} = x_1, x_2, \ldots, x_n,\ x_i \in \mathbb{R}^{d_{in}}$
- Output: a single vector
  $y_n = RNN(x_{1:n}),\ y_n \in \mathbb{R}^{d_{out}}$
- For each prefix $x_{i:j}$ define an output vector y_i:
  $y_i = RNN(x_{1:i})$
- RNN^* is a function returning this sequence for input sequence $x_{1:n}$:
  $y_{1:n} = RNN^*(x_{1:n}),\ y_i \in \mathbb{R}^{d_{out}}$

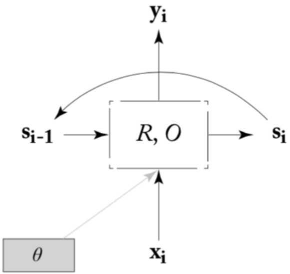


Figure: Goldberg, Yoav. Neural network methods for natural language processing

Sequence modelling with RNN

1. Sequence classification
   Put a dense layer on top of RNN to predict the desired class of the sequence after the whole sequence is processed

$$p(l_j|x_{1:n}) = \texttt{softmax}(RNN(x_{1:n}) \times W + b)_{[j]}$$

2. Sequence labelling
   Produce an output y_i for each input RNN reads in. Put a dense layer on top of each output to predict the desired class of the input

$$p(l_j|x_j) = \texttt{softmax}(RNN(x_{1:j}) \times W + b)_{[j]}$$

More details on RNN

- $RNN^*(x_{1:n}, s_0) = y_{1:n}$
- $y_i = O(s_i)$ – simple activation function
- $s_i = R(s_{i-1}, x_i)$, where R is a recursive function, s_i is a state vector
- s_0 is initialized randomly or is a zero vector
- $x_i \in \mathbb{R}^{d_{in}}$, $y_i \in \mathbb{R}^{d_{out}}$, $s_i \in \mathbb{R}^{f(d_{out})}$
- θ – shared weights

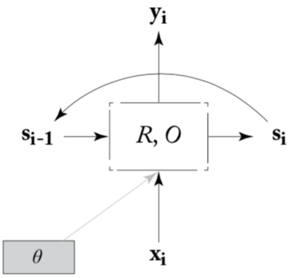


Figure: Goldberg, Yoav. Neural network methods for natural language processing

More details on RNN

- $s_i = R(x_i, s_{i-1}) = g(s_{i-1}W^s + x_iW^x + b)$
- $y_i = O(s_i) = s_i$
- $y_i, s_i, b \in \mathbb{R}^{d_{out}}$, $x_i \in \mathbb{R}^{d_{in}}$
- $W^x \in \mathbb{R}^{d_{in} \times d_{out}}$, $W^s \in \mathbb{R}^{d_{out} \times d_{out}}$

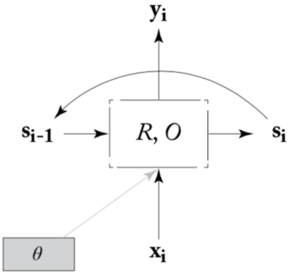
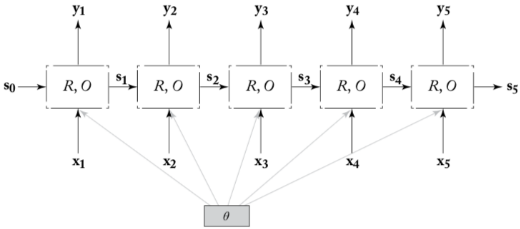


Figure: Goldberg, Yoav. Neural network methods for natural language processing

RNN unrolled

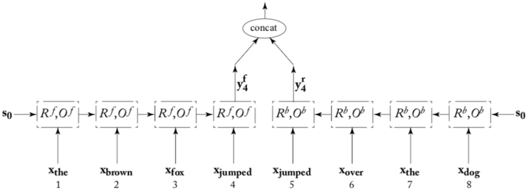


$$s_4 = R(s_3, x_4) = R(R(s_2, x_3), x_4) = R(R(R(s_1, x_2), x_3), x_4) =$$
$$= R(R(R(R(s_0, x1), x_2), x_3), x_4)$$

Figure: Goldberg, Yoav. Neural network methods for natural language processing

Bidirectional RNN (Bi-RNN)

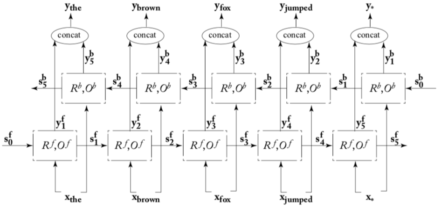
The input sequence can be read from left to right and from right to left. Which direction is better?



$$biRNN(x_{1:n}, i) = y_i = [RNN^f(x_{1:i}); RNN^r(x_{n:i})]$$

Figure: Goldberg, Yoav. Neural network methods for natural language processing

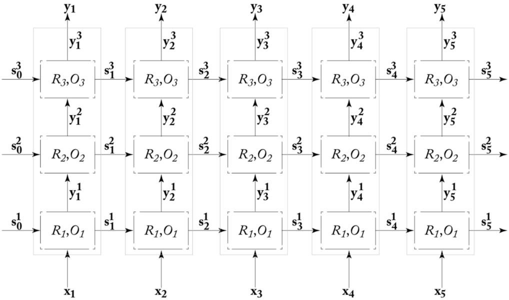
Bi-RNN



$$biRNN^*(x_{1:n}, i) = y_{1:n} = biRNN(x_{1:n}, 1) \ldots biRNN(x_{1:n}, n)$$

Figure: Goldberg, Yoav. Neural network methods for natural language processing

Multilayer RNN



Connections between different layers are possible too: $y_1^2 = \mathtt{concat}(x_1, y_1^1)$

Figure: Goldberg, Yoav. Neural network methods for natural language processing

Sequence classification

- $\hat{y}_n = O(s_n)$
- $\texttt{prediction} = MLP(\hat{y}_n)$
- Loss: $L(\hat{y}_n, y_n)$
- L can take any form: cross entropy, hinge, margin, etc.

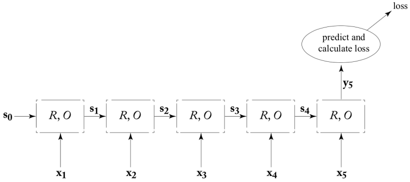


Figure: Goldberg, Yoav. Neural network methods for natural language processing

Sequence labelling

- Output $\hat{t}_i$ for each input $x_{1,i}$
- Local loss: $L_{local}(\hat{t}_i, t_i)$
- Global loss: $L(\hat{t_n}, t_n) = \sum_i L_{local}(\hat{t}_i, t_i)$
- L can take any form: cross entropy, hinge, margin, etc.

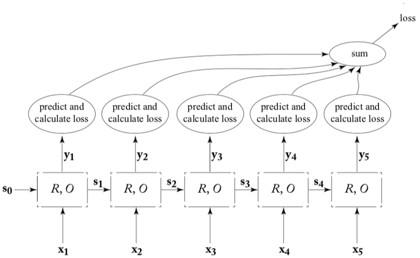
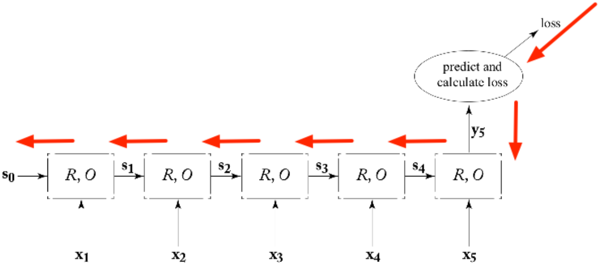


Figure: Goldberg, Yoav. Neural network methods for natural language processing

Backpropogation through time



$s_i = R(x_i, s_{i-1}) = g(s_{i-1}W^s + x_iW^x + b)$

Chain rule: $\frac{\partial L}{\partial w} = \frac{\partial L}{\partial p(\hat{y_5})} \frac{\partial p(\hat{y_5})}{\partial s_4} \left(\frac{\partial s_4}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_w} + \ldots \right)$

Figure: Goldberg, Yoav. Neural network methods for natural language processing

Vanishing gradient problem

Chain rule: $\frac{\partial L}{\partial w} = \frac{\partial L}{\partial p(\hat{y_5})} \frac{\partial p(\hat{y_5})}{\partial s_4} (\frac{\partial s_4}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_w} + \ldots)$

g – sigmoid

1. Many sigmoids near 0 and 1
   - Gradients $\rightarrow 0$
   - Not training for long term dependencies
2. Many sigmoids > 1
   - Gradients $\rightarrow +\inf$
   - Not training again

Solution: gated architectures (LSTM and GRU)

Controlled memory access

- Entire memory vector is changed: $s_{i+1} = R(x_i, s_i)$
- Controlled memory access: $s_{i+1} = g \odot R(x_i, s_i) + (1 - g)s_i$
  $g \in [0, 1]^d, s, x \in \mathbb{R}^d$
- Differential gates: $\sigma(g), g' \in \mathbb{R}^d$
- This controllable gating mechanism is the basis of the LSTM and the GRU architectures

Long short term memory

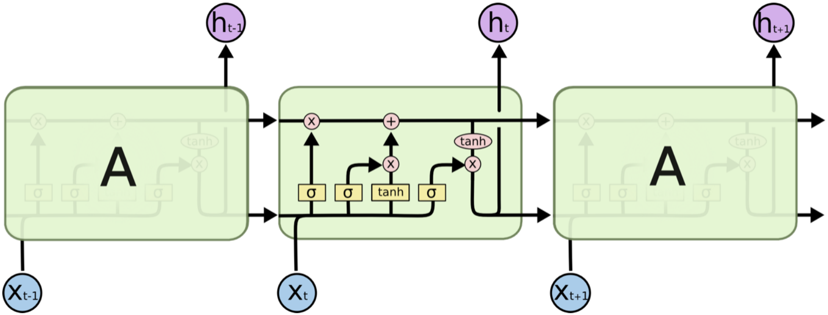
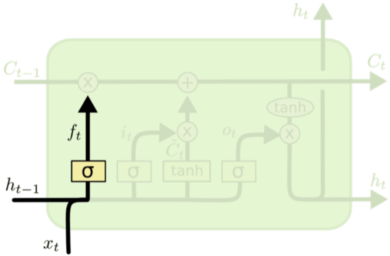


Figure: colahbloghttp://colah.github.io/posts/2015-08-Understanding-LSTMs/

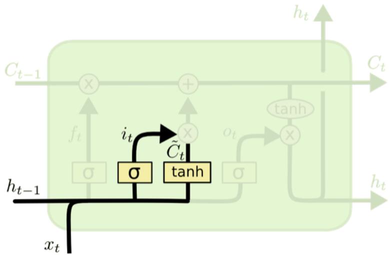
Long short term memory



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Figure: http://colah.github.io/posts/2015-08-Understanding-LSTMs/

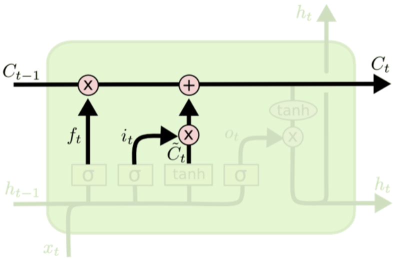
Long short term memory



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Figure: colahbloghttp://colah.github.io/posts/2015-08-Understanding-LSTMs/

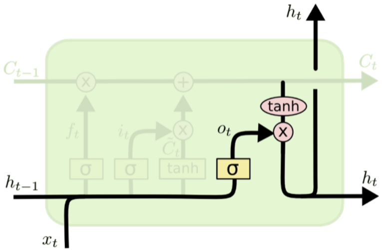
Long short term memory



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Figure: colahbloghttp://colah.github.io/posts/2015-08-Understanding-LSTMs/

Long short term memory

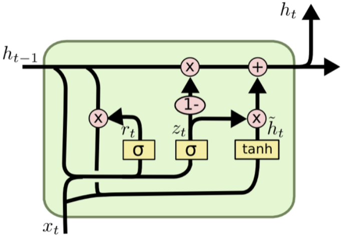


$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

Figure: colahbloghttp://colah.github.io/posts/2015-08-Understanding-LSTMs/

Gated recurrent unit



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Figure: colahbloghttp://colah.github.io/posts/2015-08-Understanding-LSTMs/

Today

Language model

1. Compute the probability of a sequence of words:

$$P(w_1, w_2, \ldots, w_n)$$

2. Predict next word:

$$P(w_n | w_1, w_2, \ldots, w_{n-1})$$

Perplexity

$$2^{H(p)} = 2^{\frac{1}{|V|} - \sum_x \log_2 p(x)}$$

Sequence generation

Teacher forcing: $x := < s > x, y := x < /s >$

$x :< s > x_1 x_2 \dots x_n$

$y : x_1 x_2 \dots x_n < /s >$

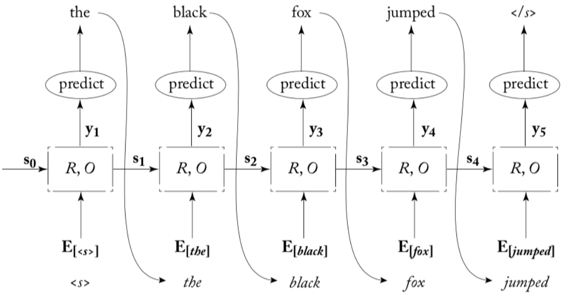


Figure: Goldberg, Yoav. Neural network methods for natural language processing

Sequence generation

- Examples of generated texts:
  http://karpathy.github.io/2015/05/21/rnn-effectiveness/
- Examples of generated MIDI music:
  https://towardsdatascience.com/how-to-generate-music-using-a-lstm-neural-network-in-keras-68786834d4c5

Pros and cons of RNNs

1. Advantages:
   - RNNs are popular and successful for variable-length sequences
   - The gating models such as LSTM are suited for long-range error propagation
2. Problems:
   - The sequentiality prohibits parallelization within instances
   - Long-range dependencies still tricky, despite gating

Take aways

1. POS-tagging is **sequence labelling** task
2. HMMs and CRFs are a generative-discriminative pair
3. MEMM suffer from label bias problem and is rarely used
4. CRF is basically sequential logistic regression
5. Say hi to Andrew Viterbi ¡3!

What is next?

1. Neural language models
2. RNNs and CRFs are best friends
3. Probabilistic context-free grammars (PCFG) and CYK

Reading

1. Sutton, C. An Introduction to Conditional Random Fields. 2012
2. Stuart Russell, Peter Norvig. Artificial Intelligence: A Modern Approach, Ch. 15
3. Dan Jurafsky, James H. Martin. Speech and Language Processing, Ch. 3, Ch. 8