Problem 1.4

$$J_{WLS}(\underline{c},n) = \sum_{k=n-M+1}^{n} g[n-k]\cdot |e[k]|^2 \overset{e\in\mathbb{R}}{=} \sum_{k=n-M+1}^{n} e[k]\cdot g[n-k]\cdot e[k]$$

/ „Laufvariable"

bc $g[n-k]$

$$= \underline{e}^T[n]\cdot \underline{G}\cdot \underline{e}[n] = [e[n], e[n-1], \dots, e[n-M+1]]\cdot \begin{bmatrix} g[M-1] & 0 & \cdots & 0 \\ 0 & g[M-2] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & g[0] \end{bmatrix}\cdot \begin{bmatrix} e[n] \\ e[n-1] \\ 0 \\ \vdots \\ e[n-M+1] \end{bmatrix}$$

P. 1.1

from problem class: $\underline{e}[n] = \underline{d}[n] - \underline{X}\cdot \underline{c}[n]$

$$\underline{d}[n] = \begin{bmatrix} d[n] \\ 0 \\ \vdots \\ d[n-M+1] \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} x[n-M+1] & \cdots & x[n-M+1-N] \\ \vdots & & \vdots \\ x[n] & \cdots & x[n-N] \end{bmatrix}$$

$$J_{WLS}(\underline{c},n) = \underline{e}^T[n]\cdot \underline{G}\cdot \underline{e}[n] = (\underline{d}-\underline{X}\cdot\underline{c})^T\cdot \underline{G}\cdot(\underline{d}-\underline{X}\cdot\underline{c})$$

$$= (\underline{d}^T - \underline{c}^T\cdot \underline{X}^T)\cdot \underline{G}\cdot(\underline{d}-\underline{X}\cdot\underline{c}) =$$

$$= (\underline{d}^T\cdot\underline{G} - \underline{c}^T\cdot\underline{X}^T\cdot\underline{G})\cdot(\underline{d}-\underline{X}\cdot\underline{c}) =$$

$$= (\underline{d}^T\underline{G}\underline{d} - \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{d} - \underbrace{(\underline{d}^T\cdot\underline{G}\cdot\underline{X}\cdot\underline{c})^T}_{\text{Skalar}} + \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{X}\cdot\underline{c})$$

$$= \underline{d}^T\cdot\underline{G}\cdot\underline{d} - \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{d} - \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}^T\cdot\underline{d} + \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{X}\cdot\underline{c} \quad // \ \underline{G}^T = \underline{G}$$

vector is in front

$$= \underline{d}^T\cdot\underline{G}\cdot\underline{d} - \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{d} - \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{d} + \underline{c}^T\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{X}\cdot\underline{c}$$

(const.)

$$\nabla_{\underline{c}}(J_{WLS}) = 0 - \underline{X}^T\cdot\underline{G}\cdot\underline{d} - \underline{X}^T\cdot\underline{G}\cdot\underline{d} + \underline{X}^T\cdot\underline{G}\cdot\underline{X}\cdot\underline{c} \overset{!}{=} 0$$

$$\underline{X}^T\cdot\underline{G}\cdot\underline{X}\cdot\underline{c} = \underline{X}^T\cdot\underline{G}\cdot\underline{d}$$

$$\underline{c} = (\underline{X}^T\cdot\underline{G}\cdot\underline{X})^{-1}\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{d}$$

$$\boxed{\underline{c}_{WLS} = (\underline{X}^T\cdot\underline{G}\cdot\underline{X})^{-1}\cdot\underline{X}^T\cdot\underline{G}\cdot\underline{d}}$$