Analytical Problem 1.2

1) $r_{xx}[k], \underline{R}_{xx} = ?$

$$r_{xx}[k] = E\{x[n+k]\cdot x^*[n]\}$$

$\hookrightarrow x[n] = (u * s_1)[n] = \cancel{\sum_{i=0} s_i \, x[i]} \;\; \underline{s}_1^H \cdot \underline{u}[n]$ impulse respons

$$\underline{s}_1 = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \end{bmatrix} ; \quad \underline{u}[n] = \begin{bmatrix} u[n] \\ u[n-1] \\ u[n-2] \end{bmatrix}$$

$\hookrightarrow s_1 \in \mathbb{R}$

$$r_{xx}[k] = E\{\underline{s}_1^H \cdot \underline{u}[n+k] \cdot \underbrace{(\underline{s}_1^H \cdot \underline{u}[n])^*}_{\text{scalar } ()^T}\}$$

$$= E\{\underline{s}_1^H \cdot \underline{u}[n+k] \cdot \underline{u}^*[n] \cdot \underline{s}_1\}$$

$$= \underline{s}_1^H \cdot \underbrace{E\{\underline{u}[n+k]\cdot \underline{u}^H[n]\}}_{\underline{R}_{uu}} \cdot \underline{s}_1$$

NR

$\underline{R}_{uu}$:
$$\underline{R}_{uu} = E\{\underline{u}[n+k]\cdot \underline{u}^H[n]\} = \begin{bmatrix} u[n+k]\cdot u^*[n] & u[n+k]\cdot u^*[n-1] & u[n+k]\cdot u^*[n-2] \\ u[n+k-1]\cdot u^*[n] & u[n+k-1]\cdot u^*[n-1] & u[n+k-1]\cdot u^*[n-2] \\ u[n+k-2]\cdot u^*[n] & u[n+k-2]\cdot u^*[n-1] & u[n+k-2]\cdot u^*[n-2] \end{bmatrix}$$

$$\underline{R}_{uu}[k] = \begin{bmatrix} \delta[k] & \delta[k+1] & \delta[k+2] \\ \delta[k-1] & \delta[k] & \delta[k+1] \\ \delta[k-2] & \delta[k-1] & \delta[k] \end{bmatrix} \cdot \sigma_u^2 \quad // \; u[n-1] \text{ is uncorrelated to } u[n]$$

$$r_{xx}[k] = \underline{s}_1^H \cdot \underline{R}_{uu}[k] \cdot \underline{s}_1 = [s_0^* \; s_1^* \; s_2^*] \cdot \begin{bmatrix} \delta[k] & \delta[k+1] & \cdots \\ \vdots & \ddots & \\ & & \end{bmatrix} \cdot \begin{bmatrix} s_0 \\ s_1 \\ s_2 \end{bmatrix} \cdot \sigma_u^2$$

~~Annah~~

$$= [s_0^* \; s_1^* \; s_2^*] \cdot \begin{bmatrix} s_0\cdot\delta[k] + s_1\cdot\delta[k+1] + s_2\cdot\delta[k+2] \\ s_0\cdot\delta[k-1] + s_1\cdot\delta[k] + s_2\cdot\delta[k+1] \\ s_0\cdot\delta[k-2] + s_1\cdot\delta[k-1] + s_2\cdot\delta[k] \end{bmatrix} \cdot \sigma_u^2$$

Annahme: $\underline{s}_1 \in \mathbb{R}$

$\hookrightarrow s_1^* = s_1$

$$r_{xx}[k] = \big[(s_0^2\cdot\delta[k] + s_0\cdot s_1\cdot\delta[k+1] + s_0\cdot s_2\cdot\delta[k+2]) + (s_1 s_0\cdot\delta[k-1] + s_1^2\cdot\delta[k] + s_1 s_2\cdot\delta[k+1])$$
$$+ (s_2\cdot s_0\cdot\delta[k-2] + s_2\cdot s_1\cdot\delta[k-1] + s_2^2\cdot\delta[k])\big]\cdot\sigma_u^2$$

$$\underline{R}_{xx} = \begin{bmatrix} r_{xx}[0] & r_{xx}[1] & r_{xx}[2] \\ r_{xx}[-1] & r_{xx}[0] & r_{xx}[1] \\ r_{xx}[-2] & r_{xx}[-1] & r_{xx}[0] \end{bmatrix} = \begin{bmatrix} s_0^2 + s_1^2 + s_2^2 & s_1 s_0 + s_2 s_1 & s_2\cdot s_0 \\ s_0 s_1 + s_1 s_2 & s_0^2 + s_1^2 + s_2^2 & s_1 s_0 + s_2 s_1 \\ s_0 s_2 & s_0 s_1 + s_1 s_2 & s_0^2 + s_1^2 + s_2^2 \end{bmatrix} \cdot \sigma_u^2$$

$\hookrightarrow$ „Probe“: $\underline{R}_{xx} = \underline{R}_{xx}^H$? ✓

2) $\underline{p} = E\{d[n] \cdot \underline{x}[n]\}$ $\qquad \underline{x}[n] = \begin{bmatrix} x[n] \\ x[n-1] \\ x[n-2] \end{bmatrix}$ $\qquad \underline{s}_2 = \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix}$

$\hookrightarrow d'[n] = (u * h)[n] = \underline{h}^H \cdot \underline{u}[n]$

$\underline{w}[n] = \begin{bmatrix} w[n] \\ w[n-1] \\ w[n-2] \end{bmatrix}$

impulse response of 2nd sensor

$\hookrightarrow d[n] = (d'[n] + w[n]) * s_2[n]$

$= (d' * s_2)[n] + (w * s_2)[n]$

$= \underline{s}_2^H \cdot \underline{d}'[n] + \underline{s}_2^H \cdot \underline{w}[n]$

scalar

$\hookrightarrow \underline{d}'[n] = \begin{bmatrix} d'[n] \\ d'[n-1] \\ d'[n-2] \end{bmatrix} = \begin{bmatrix} \underline{h}^H \cdot \underline{u}[n] \\ \underline{h}^H \cdot \underline{u}[n-1] \\ \underline{h}^H \cdot \underline{u}[n-2] \end{bmatrix} = \begin{bmatrix} \underline{u}^T[n] \cdot \underline{h}^* \\ \underline{u}^T[n-1] \cdot \underline{h}^* \\ \underline{u}^T[n-2] \cdot \underline{h}^* \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{u}^T[n] \\ \underline{u}^T[n-1] \\ \underline{u}^T[n-2] \end{bmatrix}}_{\underline{U}} \cdot \underline{h}^* = \underline{\underline{U}[n] \cdot \underline{h}^*}$

$d[n] = \underline{s}_2^H \cdot \underline{U} \cdot \underline{h}^* + \underline{s}_2^H \cdot \underline{w}[n]$

$\underline{p} = E\{(\underline{s}_2^H \cdot \underline{U} \cdot \underline{h}^* + \underline{s}_2^H \cdot \underline{w}[n]) \cdot \underline{x}[n]\}$ // $x[n] = \underline{s}_1^H \cdot \underline{u}[n] \Rightarrow \underline{x}[n] = \underline{U} \cdot \underline{s}_1^*$

$= E\{\underline{s}_2^H \cdot \underline{U} \cdot \underline{h}^* \cdot \underline{U} \cdot \underline{s}_1^*\} + E\{\underline{s}_2^H \cdot \underline{w}[n] \cdot \underline{U} \cdot \underline{s}_1^*\}$ // $\boxed{\underline{s}_1, \underline{s}_2 \in \mathbb{R}} \Rightarrow \underline{s}_1^* = \underline{s}_1$, $\underline{s}_2^* = \underline{s}_2$

$= E\{\underline{s}_2^T \cdot \underline{U} \cdot \underline{h}^* \cdot \underline{U} \cdot \underline{s}_1\} + E\{\underline{s}_2^T \cdot \underline{w}[n] \cdot \underline{U} \cdot \underline{s}_1\}$

NR: $E\{\underline{s}_2^T \cdot \underline{w}[n] \cdot \underline{U} \cdot \underline{s}_1\} =$

$E\{[s_a \ s_b \ s_c] \cdot \begin{bmatrix} w[n] \\ w[n-1] \\ w[n-2] \end{bmatrix} \cdot \underline{U} \cdot \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix}\}$

$= E\{(s_a \cdot w[n] + s_b \cdot w[n-1] + s_c \cdot w[n-2]) \cdot \begin{bmatrix} h_0 \cdot u[n] + h_1 \cdot u[n-1] + h_2 \cdot u[n-2] \\ h_0 \cdot u[n-1] + h_1 \cdot u[n-2] + h_2 \cdot u[n-3] \\ h_0 \cdot u[n-2] + h_1 \cdot u[n-3] + h_2 \cdot u[n-4] \end{bmatrix}\} = \underline{\underline{0}}$

$\hookrightarrow w[n], u[n]$ independent: $E\{u[n] \cdot w[n]\} = 0$

$\boxed{E\{\underline{s}_2^T \cdot \underline{w}[n] \cdot \underline{U} \cdot \underline{s}_1\} = 0}$

$\underline{p} = E\{\underline{s}_2^T \cdot \underline{U} \cdot \underline{h}^* \cdot \underline{U} \cdot \underline{s}_1\} + 0$

$= E\{[s_a \ s_b \ s_c] \cdot \begin{bmatrix} u[n] & u[n-1] & u[n-2] \\ u[n-1] & u[n-2] & u[n-3] \\ u[n-2] & u[n-3] & u[n-4] \end{bmatrix} \cdot \begin{bmatrix} h_0^* \\ h_1^* \\ h_2^* \end{bmatrix} \cdot \begin{bmatrix} u[n] & \cdots & u[n-2] \\ \vdots & & \vdots \\ u[n-2] & \cdots & u[n-4] \end{bmatrix} \cdot \begin{bmatrix} s_0 \\ s_1 \\ s_2 \end{bmatrix}\}$

$\underline{h} = \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/4 \\ 0 \end{bmatrix}$

$$p = E\left\{ [s_a \;\; s_b \;\; s_c] \cdot \begin{bmatrix} u[n] + \frac{1}{4} \cdot u[n-1] \\ u[n-1] + \frac{1}{4} \cdot u[n-2] \\ u[n-2] + \frac{1}{4} \cdot u[n-3] \end{bmatrix} \cdot \underline{U} \cdot \underline{s_1} \right\} =$$

$$= E\left\{ \left(u[n] \cdot s_a + u[n-1] \cdot \frac{1}{4} \cdot s_a + u[n-1] \cdot s_b + u[n-2] \cdot \frac{1}{4} s_b + u[n-2] \cdot s_c + u[n-3] \cdot \frac{1}{4} s_c \right) \cdot \underline{U} \cdot \underline{s_1} \right\}$$

$$= E\left\{ \underbrace{\left(u[n] \cdot s_a + u[n-1] \cdot \left(\frac{s_a}{4} + s_b \right) + u[n-2] \cdot \left(\frac{s_b}{4} + s_c \right) + u[n-3] \cdot \frac{s_c}{4} \right)}_{a} \cdot \begin{bmatrix} u[n] & u[n-1] & u[n-2] \\ u[n-1] & u[n-2] & u[n-3] \\ u[n-2] & u[n-3] & u[n-4] \end{bmatrix} \cdot \begin{bmatrix} s_0 \\ s_1 \\ s_2 \end{bmatrix} \right\}$$

$$= E\left\{ a \cdot \begin{bmatrix} s_0 \cdot u[n] + s_1 \cdot u[n-1] + s_2 \cdot u[n-2] \\ s_0 \cdot u[n-1] + s_1 \cdot u[n-2] + s_2 \cdot u[n-3] \\ s_0 \cdot u[n-2] + s_1 \cdot u[n-3] + s_2 \cdot u[n-4] \end{bmatrix} \right\} =$$

$$= \begin{bmatrix} E\{a \cdot (s_0 \cdot u[n] + s_1 \cdot u[n-1] + s_2 \cdot u[n-2])\} \\ E\{a \cdot (s_0 \cdot u[n-1] + s_1 \cdot u[n-2] + s_2 \cdot u[n-3])\} \\ E\{a \cdot (s_0 \cdot u[n-2] + s_1 \cdot u[n-3] + s_2 \cdot u[n-4])\} \end{bmatrix} = \quad // E\{u[n] \cdot u[n+1]\} = 0 \, ; \; E\{u[n] \cdot u^*[n+k]\} = \underline{\underline{\sigma_u^2 \, \delta[k]}}$$

$$\underline{p} = \begin{bmatrix} s_0 \cdot s_a + s_1 \cdot \left(\frac{s_a}{4} + s_b \right) + s_2 \cdot \left(\frac{s_b}{4} + s_c \right) \\ s_0 \cdot \left(\frac{s_a}{4} + s_b \right) + s_1 \cdot \left(\frac{s_b}{4} + s_c \right) + s_2 \cdot \frac{s_c}{4} \\ s_0 \cdot \left(\frac{s_b}{4} + s_c \right) + s_1 \cdot \frac{s_c}{4} + 0 \end{bmatrix} \cdot \sigma_u^2$$

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3) $\underline{c}_{opt} = \underline{R}_{xx}^{-1} \cdot \underline{p}$ // Wiener Hopf ;

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4) J_{min}

$$J_{min} = J_{MSE}(\underline{c}_{opt}) = E\{|e[n]|^2\} =$$

$$= E\{|d[n] - y[n]|^2\} \quad // \; y[n] = (x * c)[n] = \underline{c}^H \cdot \underline{x}[n] \; ; \; \underline{x}[n] = \underline{U} \cdot \underline{s}_1$$

$$= E\{|\underline{s}_2^T \underline{U} \cdot \underline{h}^* + \underline{s}_2^T \underline{w}[n] - \underline{c}_{opt}^H \underline{x}[n]|^2\} \quad // \; \underline{c}_{opt} = \underline{h}$$

$$= E\{|(\underline{s}_2^T \cdot \underline{U} - \underline{x}_{[n]}^T) \cdot \underline{h}^* + \underline{s}_2^T \cdot \underline{w}[n]|^2\}$$

$$= E\{|(\underline{s}_2^T \cdot \underline{U} - (\underline{U} \cdot \underline{s}_1)^T) \cdot \underline{h}^H + \underline{s}_2^T \cdot \underline{w}[n]|^2\} \quad // \; \underline{U}^T = \underline{U} \; ; \quad (A \cdot B)^T = B^T \cdot A^T$$

$$= E\{|(\underline{s}_2^T \cdot \underline{U} - \underline{s}_1^T \cdot \overset{U}{\underline{U}^T}) \cdot \underline{h}^* + \underline{s}_2^T \cdot \underline{w}[n]|^2\}$$

$$= E\{|\underbrace{(\underline{s}_2^T - \underline{s}_1^T) \cdot \underline{U} \cdot \underline{h}^* + \underline{s}_2^T \cdot \underline{w}[n]}_{e[n]}|^2\}$$

↳ from problem class: $|e[n]|^2 = \underline{e}^T \cdot \underline{e}$

$$= E\{((\underline{s}_2^T - \underline{s}_1^T) \cdot \underline{U} \cdot \underline{h}^* + \underline{s}_2^T \underline{w}[n])^T \cdot ((\underline{s}_2^T - \underline{s}_1^T) \cdot \underline{U} \cdot \underline{h}^* + \underline{s}_2^T \cdot \underline{w}[n])\} \quad // \; \underline{h} \in \mathbb{R}, \; \underline{h}^* = \underline{h}$$

$$= E\{(\underline{h}^T \cdot \underline{U}^T \cdot (\underline{s}_2^T - \underline{s}_1^T)^T + \underline{w}_{[n]}^T \cdot \underline{s}_2) \cdot (\; — \; \| \; — \;)\}$$

$$= E\{\overbrace{\underline{h}^T \cdot \underline{U} \cdot (\underline{s}_2 - \underline{s}_1)}^{scalar} \cdot \overbrace{(\underline{s}_2^T - \underline{s}_1^T) \cdot \underline{U} \cdot \underline{h}}^{scalar} + 2 \cdot \underbrace{\underline{s}_2^T \underline{w}[n]}_{E\{\} = 0} + \overbrace{\underline{w}[n]^T \cdot \underline{s}_2}^{scalar} \cdot \overbrace{\underline{s}_2^T \cdot \underline{w}[n]}^{scalar}\}$$

$$= E\{(\underline{h}^T \underline{U} \cdot (\underline{s}_2 - \underline{s}_1))^T \cdot ((\underline{s}_2^T - \underline{s}_1^T) \cdot \underline{U} \cdot \underline{h})\} + 0 + E\{\underline{s}_2^T \cdot \underbrace{\underline{w}[n] \cdot \underline{w}[n]^T}_{E\{\} = \sigma_w^2 \underline{I}} \cdot \underline{s}_2\}$$

NR: $\underline{s}_2^T \cdot E\{\underline{w}[n] \cdot \underline{w}[n]^T\} \cdot \underline{s}_2 =$

$= \underline{s}_2^T \cdot \underline{I} \cdot \underline{s}_2 \cdot \sigma_w^2 = \underline{s}_2^T \cdot \underline{s}_2 \cdot \sigma_w^2$

$= \underline{\underline{(s_a^2 + s_b^2 + s_c^2) \cdot \sigma_w^2}}$

$$= (\underline{s}_2 - \underline{s}_1)^T \cdot \underbrace{E\{\underline{U} \cdot \underline{h} \cdot \underline{h}^T \cdot \underline{U}\}}_{\rightarrow \text{MatLab: } \underline{h} = \begin{bmatrix} 1 \\ r_4 \\ 0 \end{bmatrix}} \cdot (\underline{s}_2 - \underline{s}_1)^T + (s_a^2 + s_b^2 + s_c^2) \cdot \sigma_w^2$$

$$MR: E\{\underline{u}\cdot\underline{h}\cdot\underline{h}^T\cdot\underline{u}\} \qquad E\{u[n]\cdot u[n-1]\} \to 0$$

$$= E\left\{\begin{array}{c|c} u[n]\cdot\left(u[n]+\frac{u[n-1]}{4}\right)+u[n-1]\cdot\left(\frac{u[n]}{4}+\frac{u[n-1]}{16}\right) & u[n-1]\cdot\left(u[n]+\frac{u[n-1]}{4}\right)+u[n-2]\cdot\left[\frac{u[n]}{4}+\frac{u[n-1]}{16}\right) \\ \hline u[n]\cdot\left(u[n-1]+\frac{u[n-2]}{4}\right)+u[n-1]\cdot\left(\frac{u[n-1]}{4}+\frac{u[n-2]}{16}\right) & u[n-1]^2+u[n-2]^2\cdot\frac{1}{16} \\ \hline 0 & u[n-2]^2\cdot\frac{1}{4} \end{array}\right. \ldots$$

$$\ldots \left.\begin{array}{c} u[n-2]\cdot\left(\frac{u[n]}{4}+\frac{u[n-1]}{4}\right)+u[n-3]\left(\frac{u[n]}{4}+\frac{u[n-1]}{16}\right) \\ \hline u[n-2]^2\cdot\frac{1}{4} \\ \hline u[n-2]^2+u[n-3]^2\cdot\frac{1}{16} \end{array}\right\}$$

terms which get to 0 after E{} left out

$$= \begin{bmatrix} \sigma_u^2+\frac{1}{16}\cdot\sigma_u^2 & \sigma_u^2\cdot\frac{1}{4} & 0 \\ \sigma_u^2\cdot\frac{1}{4} & \sigma_u^2+\sigma_u^2\cdot\frac{1}{16} & \frac{1}{4}\cdot\sigma_u^2 \\ 0 & \sigma_u^2\cdot\frac{1}{4} & \sigma_u^2+\frac{1}{16}\cdot\sigma_u^2 \end{bmatrix} = \sigma_u^2\cdot\begin{bmatrix} \frac{17}{16} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{17}{16} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{17}{16} \end{bmatrix}$$

$$J_{min} = \underbrace{(\underline{s}_2-\underline{s}_1)^T}_{a^T}\cdot\sigma_u^2\cdot\begin{bmatrix} \frac{17}{16} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{17}{16} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{17}{16} \end{bmatrix}\cdot\underbrace{(\underline{s}_2-\underline{s}_1)}_{a} + (s_a^2+s_b^2+s_c^2)\cdot\sigma_w^2 \qquad a=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$$

↳ Matlab

$$J_{min} = \left[a_2\cdot\left(\frac{a_1}{4}+\frac{17\cdot a_2}{16}+\frac{a_3}{4}\right)+a_1\cdot\left(\frac{17\cdot a_1}{16}+\frac{a_2}{4}\right)+a_3\cdot\left(\frac{a_2}{4}+\frac{17\cdot a_3}{16}\right)\right]\cdot\sigma_u^2 + (s_a^2+s_b^2+s_c^2)\cdot\sigma_w^2$$

$$\text{with} \quad \underline{a} = \underline{s}_2-\underline{s}_1 = \begin{pmatrix} s_a-s_0 \\ s_b-s_1 \\ s_c-s_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

Only true if $c_{opt}=\underline{h}$; otherwise wrong (J_{min} for c) and d) is wrong)

$h = \begin{bmatrix} 1 \\ 1/4 \\ 0 \end{bmatrix}$

a) $S_1(z) = S_2(z) = 1$

$\underline{s}_1 = \underline{s}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow$ MATLAB: Analytical_1_2.m

$\underline{R}_{xx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\underline{p} = \begin{bmatrix} 1 \\ 1/4 \\ 0 \end{bmatrix}$; $\underline{c}_{opt} = \begin{bmatrix} 1 \\ 1/4 \\ 0 \end{bmatrix}$; $J_{min} = \sigma_w^2 = \underline{\frac{1}{4}}$

b) $S_1(z) = S_2(z) = 1 + 0{,}5\, z^{-1}$

$\underline{s}_1 = \underline{s}_2 = \begin{pmatrix} 1 \\ 0{,}5 \\ 0 \end{pmatrix}$

$\underline{R}_{xx} = \begin{bmatrix} 5/4 & 1/2 & 0 \\ 1/2 & 5/4 & 1/2 \\ 0 & 1/2 & 5/4 \end{bmatrix}$; $\underline{p} = \begin{bmatrix} 1{,}375 \\ 0{,}8125 \\ 0{,}125 \end{bmatrix}$; $\underline{c}_{opt} = \begin{bmatrix} 1 \\ 0{,}25 \\ 0 \end{bmatrix}$; $J_{min} = \frac{5}{4} \cdot \sigma_w^2 = \underline{\frac{5}{16}}$

c) $S_1(z) = 1$; $S_2(z) = z^{-1}$

$\underline{s}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; $\underline{s}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\underline{R}_{xx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\underline{p} = \begin{bmatrix} 0 \\ 1 \\ 1/4 \end{bmatrix}$; $\underline{c}_{opt} = \begin{bmatrix} 0 \\ 1 \\ 1/4 \end{bmatrix}$; $\cancel{J_{min}} = \frac{13}{8} \sigma_u^2 + \sigma_w^2 = \frac{13}{8} + \frac{2}{8} = \cancel{\frac{15}{8}}$

#h

d) $S_1(z) = 1$; $S_2(z) = 1 + z^{-2}$

$\underline{s}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; $\underline{s}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

#h

$\underline{R}_{xx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\underline{p} = \begin{bmatrix} 1 \\ 1/4 \\ 1 \end{bmatrix}$; $\underline{c}_{opt} = \begin{bmatrix} 1 \\ 1/4 \\ 1 \end{bmatrix}$; $\cancel{J_{min}} = \frac{17}{16} \sigma_u^2 + 2 \cdot \sigma_w^2 = \frac{17}{16} + \frac{2}{4} = \cancel{\frac{25}{16}}$

e)

- If the two sensor responses are the same, the system gets identified correctly.
- — " — but one has a delay (e.g. $S_2(z) = S_1(z) \cdot z^{-1}$), the system won't be identified correctly, unless we introduce some delay on our own (e.g. $S_1(z) \to S_1 z^{-1}$)
- If the sensor response are $S_1(z) = 1$ and $S_2(z) = 1 + z^{-N}$, where N is amount of coefficients, the system can be identified correctly

(7.6)

M.

e) ~~If the two sensor responsen are the same~~, the system

The System in d) cannot be identified correctly.