Signal Analysis: Homework Sheet 1

This is the first out of three homework sheets for the 2019 Signal Analysis problem class. Each homework sheet includes three different examples resulting in a total number of nine homework examples over progress of the course. The presented homework examples are mostly MATLAB examples with a strong real world focus.

The grading of the problem class is based on student presentations (Power Point or Latex presentation) and exercise interviews for five out of the nine examples. For each of the three homework sheets, students have to present one example, which will be selected by the lecturers ("must show"). Students will be informed three days prior to the deadline of the problem sheet, about the number of the example, which has to be presented. The remaining two examples can be selected by the students itself ("will show"). Hereby only one of the remaining two examples per problem sheet can be selected. Note, that the grading is an individual grading.

All presentations and necessary MATLAB-files have to be provided by the deadline to sa@emt.tugraz.at. This holds for all presentations ("must show" and selected "will show"). Please follow the instructions for the upload, which are provided by an additional sheet on the server.

The presentation slides have to consider all necessary derivations, algorithmic considerations, diagrams, conclusions, etc.. A pure presentation of the final results will lead to reduction of your points. Also take care to the representation of necessary diagrams. Slides can be in German of English. Note, that your slides have to include all graphics. A reference to a generating MATLAB-script is not accepted.

Deadline: November 29, 2019

Example 1: Stochastic Processes/ System Identification

The first example deals with the analysis of systems with a damped exponential response, i.e. the homogeneous solution of the system is of form

$$y(t) = e^{-at}\sin(\Omega t). \tag{1}$$

Systems with such a response appear in almost all fields of physics, e.g. in electrical engineering RLC-circuits show this behavior. The mechanical analogon is a damped spring mass system as depicted in figure 1. Damped spring mass systems are also used in many sensor applications, e.g. the behavior of acceleration sensors is modeled by means of a damped spring mass system.

The mathematical relation between the input force F and the output y of the damped spring mass system depicted in figure 1 can be derived using Newton's seconds law. The

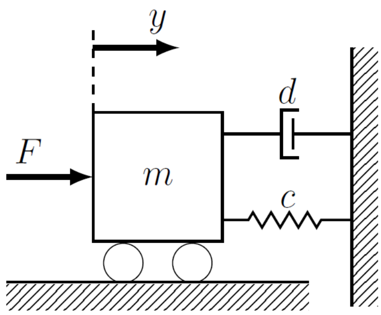

Figure 1: Damped mass spring system.

resulting ordinary differential equation is given by

$$\ddot{y} + \frac{d}{m}\dot{y} + \frac{c}{m}y = \frac{1}{m}F \tag{2}$$

where y is the system elongation in m, m is the mass in kg, c is the spring constant in N/m, and d is an attenuation constant in Ns/m. The force F in N is the excitation of the system.

In this example you have to analyze a damped spring mass system, which is provided by the MATLAB function

```
x_out=mass_spring_system(F,Td,id)
```

The function requires the following inputs

- `F`: vector presenting the force excitation in N.
- `Td`: sampling period in s
- `id`: your student ID (integer number). You can find your student ID on the server.

The parameters of the damped spring mass system are known to be in the range of:

$$m \approx 10\,\text{kg} \tag{3}$$
$$c \approx 450\,\text{N/m} \tag{4}$$
$$d \approx 10\,\text{Ns/m} \tag{5}$$

Note, that the exact values of your system (student ID) are slightly different.

Please provide answers to the following points:

(A) Derive the state space representation and the transfer function $H(s)$ for the differential equation (2). Use MATLAB and plot the frequency response of the system using the values for m, c and d as given by equation (3) to (5). Discuss the properties of the system from the frequency response.

(B) Use the expected frequency response of the previous point and determine the bandwidth Ω_{g} (cut off frequency) of the expected system. Use the following heuristic

$$\frac{\pi}{20\ \Omega_{\text{g}}} < T_d < \frac{\pi}{10\ \Omega_{\text{g}}} \tag{6}$$

and set a proper sampling period T_d for the following tasks.

(C) The steady state solution for a sinusoidal force excitation of form

$$F(t) = \hat{F}\sin(\Omega t) \tag{7}$$

is given by

$$y(t) = \hat{F}|H(j\Omega)|\sin\left(\Omega t + \sphericalangle H(j\Omega)\right) \tag{8}$$

Use the MATLAB function `mass_spring_system` and estimate the frequency response of your system by applying sinusoidal excitation signals and using proper signal analysis methods. The maximum signal power of your force signal is limited by $\mathcal{E}(F^2) < 1$. Before applying your estimation method for the parameters of $H(j\Omega)$, corrupt the simulated signal with white Gaussian noise with a standard deviation of $\sigma = 0.001\,\text{m}$. Reason the choice of your analysis tools.

(D) Now estimate the absolute value of the frequency response by applying white noise as input signal. Again, disturb the simulated output signal with white noise with $\sigma = 0.001\,\text{m}$. Perform the estimation of $|H(e^{j\omega})|$ twice, one time without and one time with considering the additional measurement noise. The signal power of your source is again limited by $\mathcal{E}(F^2) < 1$.

(E) Use the coherence function and analyze the behavior of the system. Give an interpretation and state, whether your previous system identification approaches are valid. You can also perform the coherency analysis using sinusoidal signals.

Hint: To further study the behavior of the system you can apply test signals with different sign at the input, e.g. you can use a positive ramp signal $F_{\text{pos}}(t) = ktu(t)$ and a negative ramp signal $F_{\text{neg}}(t) = -ktu(t)$, where k is a slope parameter.

Example 2: Filter Design

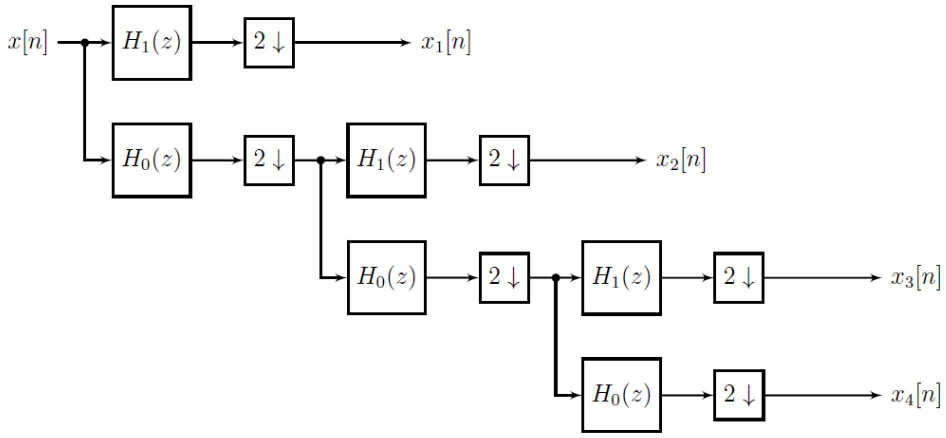

Figure 2: Example for a filter bank. The signal $x[n]$ is splitted into the four signals $x_1[n]$ to $x_4[n]$.

The term filter bank referrers to signal processing structures, which include filters and sample rate converters (up- and/or down-samplers). Figure 2 exemplary illustrates a filter bank, which is referred to as analysis filter bank. They are favourable tools for many signal processing applications, e.g. an efficient implementation of the discrete Wavelet Transform (DWT) is based on a filer structure as depicted in figure 2. Note, that the filter bank of this example is yet not a DWT due to the used filters.

Please provide answers to the following points:

(A) Design FIR filters for $H_0(z)$ and $H_1(z)$ according to figure 3 and plot the frequency response of your filters in one plot.

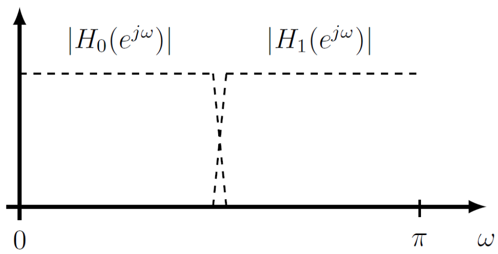

Figure 3: Frequency response of the filters $H_0(z)$ and $H_1(z)$.

(B) Given the filter structure presented in figure 2 and the filters $H_0(z)$ and $H_1(z)$ presented in figure 3. Depict the frequency responses $H_{x\mapsto x_i}(z)$ in one plot and explain the frequency resolution of the filter bank. What are the sample rates of the four outputs with respect to the input frequency?

(C) Provide a MATLAB implementation of the filter bank as depicted in figure 2. Your implementation should be a function of form

```
[x1,t1,x2,t2,x3,t3,x4,t4]=func_filterbank(x,f_S,h0,h1)
```

where `x` is an input vector, f_S is the sampling frequency in Hz, and `h0` and `h1` are the filter coefficients. The output are the signals x_1 to x_4 and the time vectors for these signals.

Hint: You can use the MATLAB command `upfirdwn` for an efficient implementation.

(D) The filter bank will be operated at an input sampling rate of 2 kHz. Determine the frequency range of the different sub-bands of the outputs x_1 to x_4 of the filter bank. Use sinusoidal signals and show the frequency decomposition of the filter bank. How is the input frequency related to the output frequency at a specific channel, e.g. channel x_1?

(E) The `mat`-file `UE2sig.mat` contains a signal vector, which should be analyzed using your filter bank implementation and your designed filters. The signal was sampled with a frequency of 2 kHz. Use subplots for the signals $x_i[n]$, to provide a common representation of the filter bank outputs. Discuss the analysis behavior of the filter bank with respect to the signal. Consider the different time/frequency resolutions of the filter bank in your discussion.

Hint: The MATLAB command `spectrogram(x,256,0,256,f_S)` provides the so called spectrogram as output of an STFT analysis for the signal $x[n]$. The details about the STFT will be explained in a later part of the course, but the result should be self explaining. You can use the spectrogram for the signal $x[n]$, as well as the signals of the filter bank in time domain to investigate the properties of the signal and the analysis filter bank. However, you will observe a distinct difference between the two analysis methods. Note, that the output of the filter bank analysis are the time signals x_1 to x_4, so you have to compare these signals against the spectrogram.

Example 3: Signal Acquisition

The third example deals with the analysis of the effects during the signal acquisition process. To be more explicit, the effects of the sample and hold (S&H) and the quantizer (ADC) will be investigated. The MATLAB function

```
[x,xSH,xQ,xSHQ] = func_ADC(sig,PAR,N,id)
```

simulates an ADC, which provides `N` samples of the signal. The signal is given by `sig` and `PAR`. The ADC is a bipolar 4 Bit ADC with a full scale range of $X_\mathrm{m} = 2$. The sampling frequency is $f_\mathrm{S} = 1\,\mathrm{kHz}$. The aperture jitter depends on your student ID. See the MATLAB file `Demo_UE3.m` for the use of the function.

In contrast to a real world ADC, the function provides access to all intermediate quantities in the signal acquisition stage. E.g. `x` is the continuous time signal (sampled by an ideal C/D converter), `xSH` is the signal after the S&H, `xQ` is the quantized version of `x`, `xSHQ` is the quantized version of `xSH`.

Please provide answers to the following points:

(A) Analyze the behavior of the quantization error for sinusoidal signals with different frequencies (signal `xQ`). Is the white noise assumption for the quantization error always correct? Are there frequencies where the quantization error is again a periodic function?

(B) Dithering is refereed to as a technique where the analog input signal is disturbed by a uniform distributed random signal $w(t)$ with $w(t) \propto \mathcal{U}\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$. Apply dithering to the input and reevaluate the statistics of the quantization error. Discuss the difference.

(C) The effective number of bits (ENOB) of an ADC is defined by

$$\mathrm{ENOB} = \frac{\mathrm{SINAD} - 1.76\,\mathrm{dB}}{6.02}, \tag{9}$$

where the parameter SINAD (signal to noise and distortion ratio) is defined by

$$\mathrm{SINAD} = 10 \log \left(\frac{P_\mathrm{Signal}}{P_\mathrm{Noise}} \right) \tag{10}$$

Use a sinusoidal signal with amplitude 1 and compute the ENOB of the ADC for different frequencies (no dithering).

(D) Analyze the SNR of the S&H as a function over the frequency (`xSH`). Estimate the jitter delay of your ADC.

(E) Analyze the overall SNR and ENOB as a function of the frequency from your ADC (`xSHQ`).

Hint: you can use the MATLAB command `soundsc` to listen to the signals on your computer. We uploaded the document *Aperture Time, Aperture Jitter, Aperture Delay Time-Removing the Confusion by Walt Kester* on the server. You might find it useful for the solution of this example.