

Signalverarbeitung UE

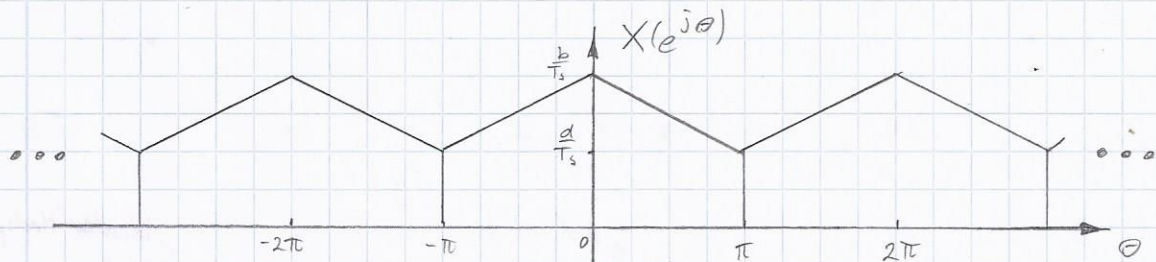
3.1.) a.)

$$\omega_{\max} = 40000\pi$$

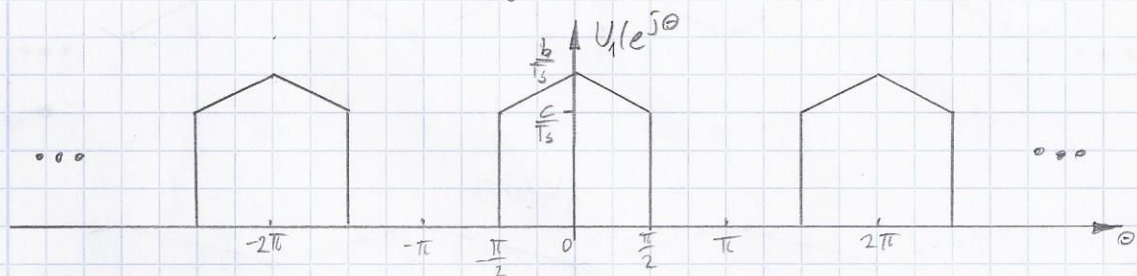
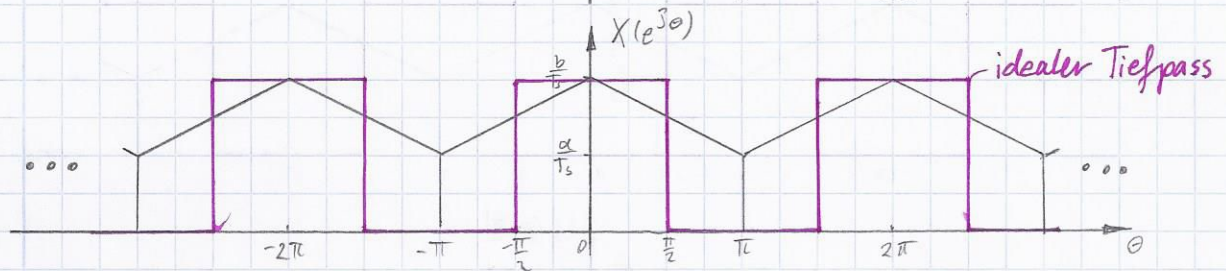
$$\omega = 2\pi f$$

$$f_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{40000\pi}{2\pi} = 20000 \text{ Hz}$$

$$\underline{f_s} \geq 2 \cdot f_{\max} = 2 \cdot 20000 = \underline{40000 \text{ Hz}} \quad \text{Für kritische Abtastung } f_s = 2f_{\max}$$



b.)



Down samplen: $M=2$

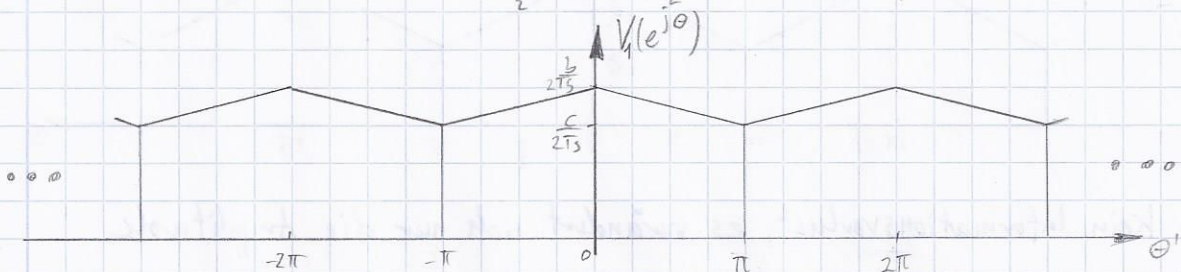
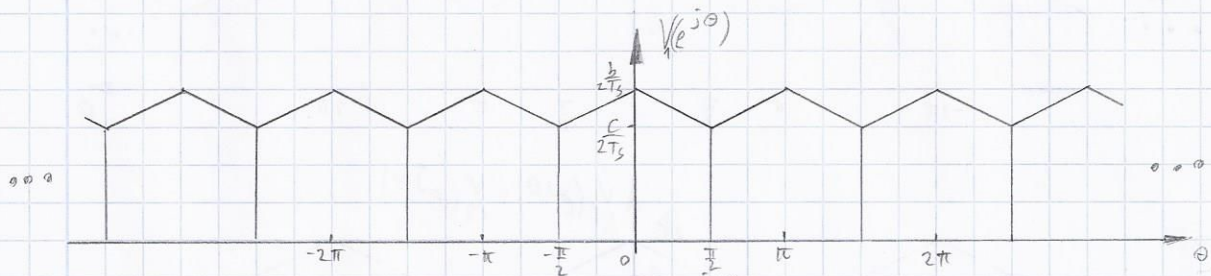
$$y[m] = x[M \cdot m] \quad \xleftrightarrow{\text{FT}}$$

$$Y(e^{j\theta'}) = \frac{1}{M} \sum_{k=0}^{M-1} X(e^{j(\theta' - 2\pi k)/M}) = \frac{1}{2} \sum_{k=0}^1 X(e^{j(\theta' - 2\pi k)/2})$$

$$\theta = \frac{\theta'}{2}$$

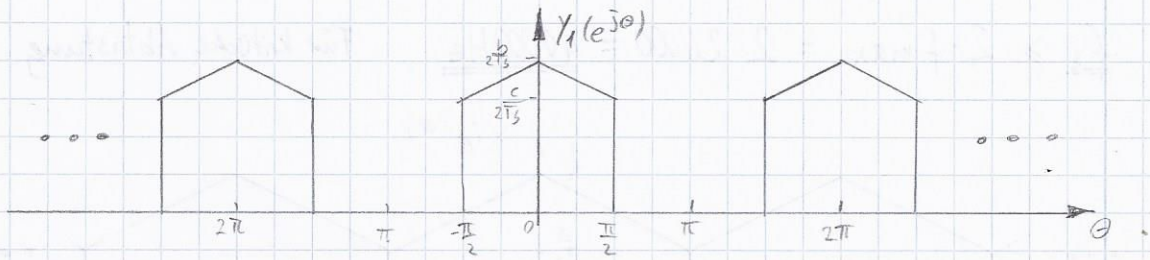
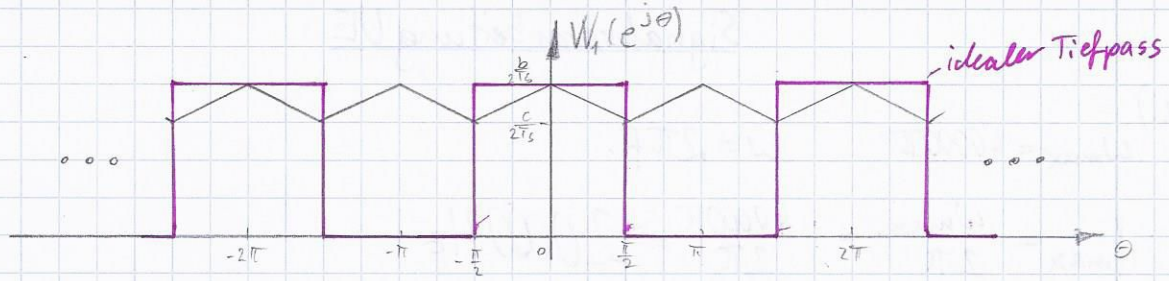
$$Y(e^{j\theta'}) = \frac{1}{2} X(e^{j\theta'/2}) + \frac{1}{2} X(e^{j(\theta'/2 - \pi)}) =$$

$$Y(e^{j\theta'}) = \frac{1}{2} X(e^{j\theta}) + \frac{1}{2} X(e^{j\theta - \pi})$$

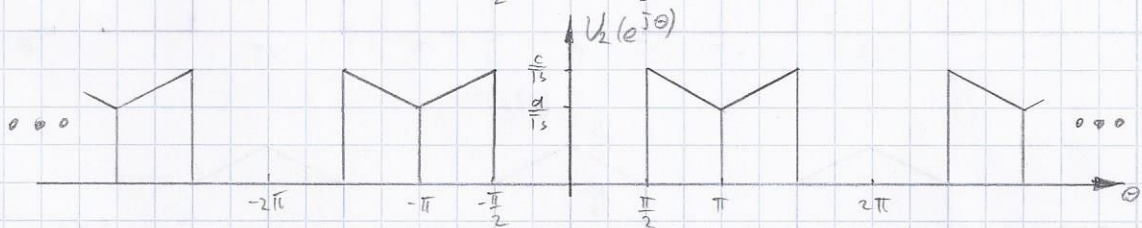
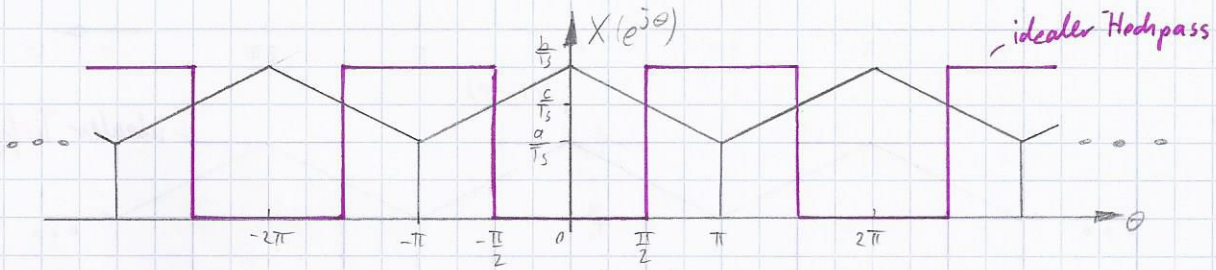


$$\theta' = \theta \cdot 2$$

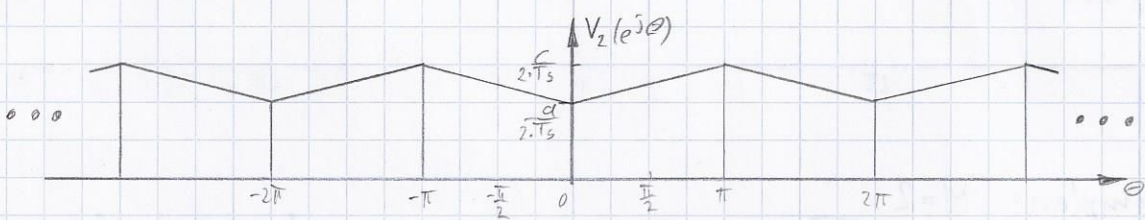
↓ Frequenzachse wird gestauch



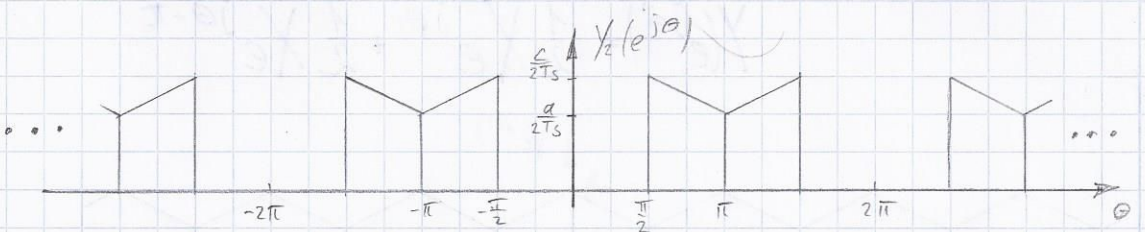
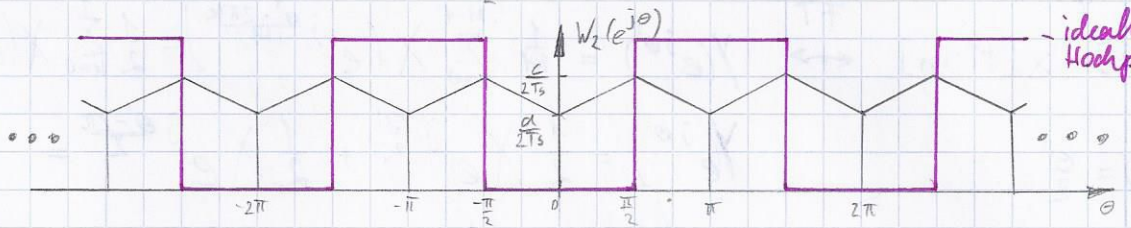
c.)



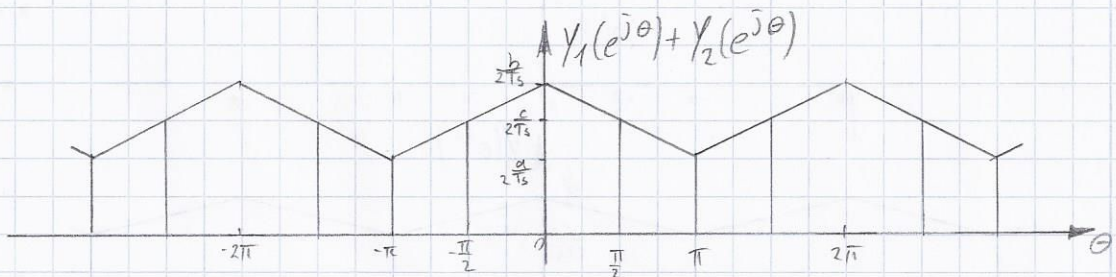
Formel von Punkt b wird angewendet



Frequenzachse wird gestaucht

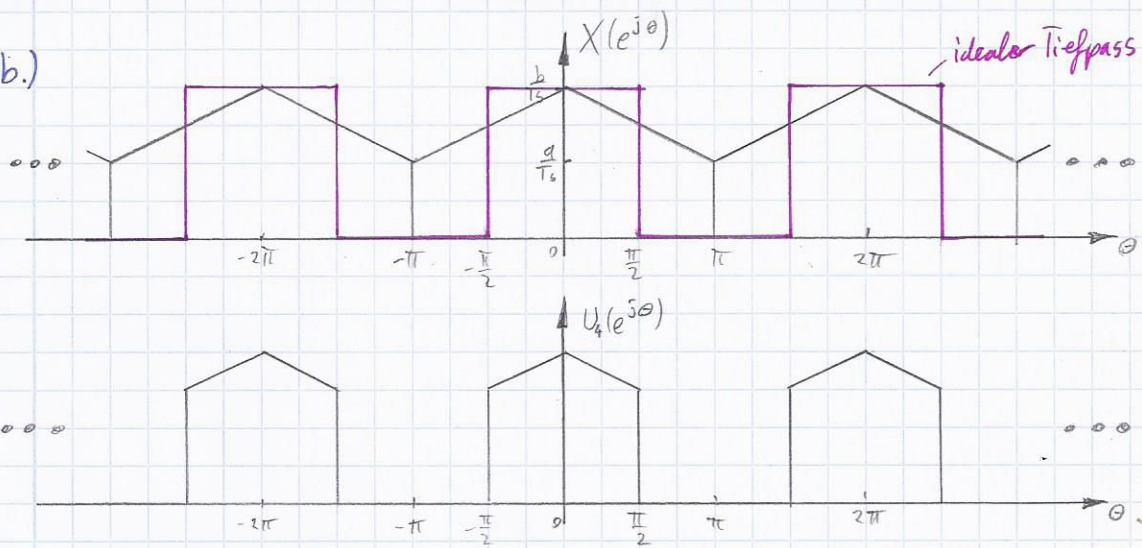


d.)



kein Informationsverlust, es verändert sich nur die Amplitude

e.) mit b.)

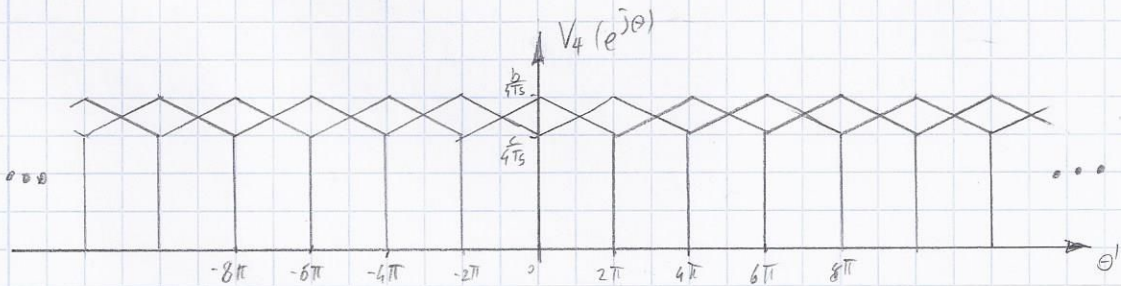
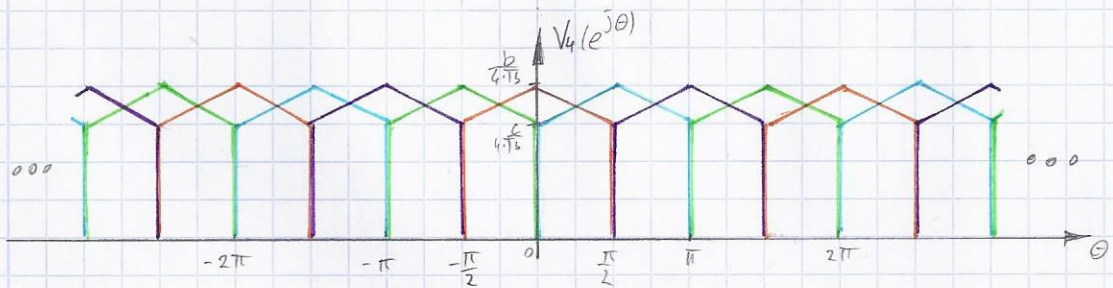


Downsampling: $M=4$

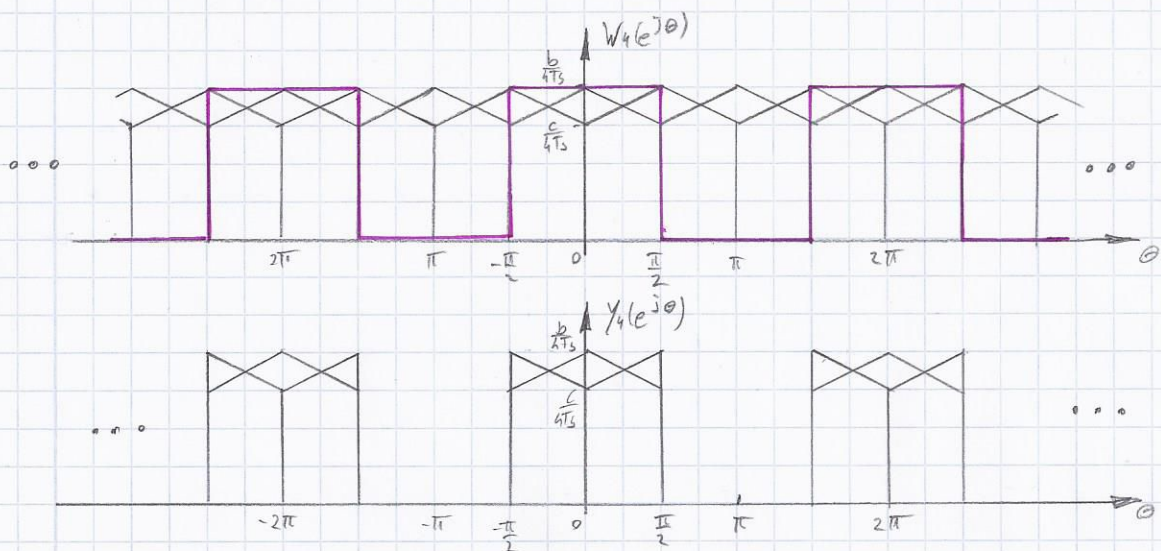
$$y[m] = x[m \cdot M] \xleftrightarrow{\text{FT}} Y(e^{j\theta'}) = \frac{1}{M} \sum_{k=0}^{M-1} X(e^{j(\frac{\theta'}{M} - \frac{2\pi k}{M})}) =$$

$$= \frac{1}{4} \sum_{k=0}^3 X(e^{j(\frac{\theta'}{4} - \frac{2\pi k}{4})}) = \frac{1}{4} [X(e^{j\frac{\theta'}{4}}) + X(e^{j(\frac{\theta'}{4} - \frac{\pi}{2})}) + X(e^{j(\frac{\theta'}{4} - \pi)}) + X(e^{j(\frac{\theta'}{4} - \frac{3\pi}{2})})]$$

$$\frac{\theta'}{4} = \theta \quad = \frac{1}{4} \cdot [X(e^{j\theta}) + X(e^{j(\theta - \frac{\pi}{2})}) + X(e^{j(\theta - \pi)}) + X(e^{j(\theta - \frac{3\pi}{2})})]$$



$$\theta' = 4 \cdot \theta$$



Es kommt zu Aliasing da der Tief-/Hochpassfilter nicht auf die Samplerraten angepasst wurde - Informationsverlust