Mathematics
Tasks
09/08/2023

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2023

Vyet formulas

№ 1-2

$x^2 - 54x + 104 = 0$

$D = (-54)^2 - 4 \cdot 104 = 2916 - 416 = 2500$

$x_1, x_2 = (54 \pm 50) / 2$

$x_1 = 2$

$x_2 = 52$

$x^2 - 10x + 6 = 0$

$\alpha + \beta = -\frac{b}{a} = -\frac{-10}{1} = 10$

$\alpha\beta = \frac{c}{a} = \frac{5}{1} = 5$

$a^2 + b^2 = (\alpha + \beta)^2 - 2ab = 10^2 - 2 \cdot 5 = 100 - 10$

10

$a^2 + b^2 = 90$

$$\frac{a^2 + b^2}{\alpha^2\beta + \alpha\beta^2} = \frac{a^2 + b^2}{\alpha\beta(\alpha + \beta)} = \frac{90}{5 \times 10} = \frac{90}{50} = 1{,}8$$

№ 1-8

$1.x^2 + 5x - 3 = 0$

$D = 5^2 - 4 \cdot 1 \cdot (-3) = 25 + 12 = 37$

$x_1^2 + x_2^2 = x_1^2 + 2x_1x_2 + x_2^2 - 2x_1x_2 = (x_1 + x_2)^2 -$

$- 2x_1x_2$

$x_1 + x_2 = -\frac{b}{a} = -\frac{5}{1} = -5$

$x_1 \cdot x_2 = \frac{c}{a} = \frac{-3}{1} = -3$

$x_1^2 + x_2^2 = \cancel{A^2 + B^2} = (x_1 + x_2)^2 - 2x_1$

$= (1-5+\sqrt{37})/2)^2 + (1-5-\sqrt{37})/2)^2 -$

$\cdot (1(-5)-\sqrt{37}|4 =$

$= 25 - 2 \cdot (25-37)/4 =$

$= 25 - 2 \cdot (-12)/4) = 25 + 6 = 31$

2. $x^2 + 11x + 12 = 0$, найдите $x_1^2 + x_2^2$

$D = 11^2 - 4 \cdot 12 \cdot 1 = 121 - 48 = 73$

~~$x_{1,2} = \frac{-11 \pm \sqrt{73}}{2}$~~

$x_1 = (-11 + \sqrt{73})/2$

$x_2 = (-11 - \sqrt{73})/2$

$x_1^2 + x_2^2 = (1-11+\sqrt{73})/2)^2 + (-11-\sqrt{73})/2)^2 -$

$- 2 \cdot (121 - 22\sqrt{73} + 73)/4 + (121 + 22\sqrt{73} + 73)$

$/4 = 2 \cdot (121 + 73)/4 = 2 \cdot 194/4 = 1242 + 146/4 =$

$x = 388/4 = 97$

3. $x^2 + 9x + 33 = 0$

$D = 9^2 - 4 \cdot 33 \cdot 1$

$x_1 + x_2 = -\frac{b}{a} = -\frac{9}{1} = -9$

$x_1 \cdot x_2 = \frac{c}{a} = \frac{33}{1} = 33$

$$\begin{cases} x_1 + x_2 = -9 \\ x_1 \cdot x_2 = 33 \end{cases}$$

Припустимо $x_1 = 3$, а $x_2 = 11$, або

$x_1 = x_2 - 9$

$x_1(x_2 - 9) - x_2 = 33$

$x_2^2 + 9x_2 = 33$

$$\frac{x_2^2 + 9x_2}{x_2^2 + 9x_2 - x_1} = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2}{x_1 x_2} + \frac{x_1}{x_1 x_2} = \frac{x_1 + x_2}{x_1 x_2}$$

$$\frac{x_1 + x_2}{x_1 x_2} = \frac{-9}{33} = -\frac{3}{11}$$

О: $-\frac{3}{11}$

4. $x^2 - 8x + 11 = 0$

$x_1^3 + x_1^2 + x_1 + x_2^3 + x_2^2 + x_2$

$x_1 + x_2 = \frac{b}{a} = +\frac{-8}{1} = 8$

$x_1 \cdot x_2 = \frac{11}{1} = 11$

$D = (-8)^2 - 4 \cdot 11 = 64 - 44 = 20$

$\sqrt{20}$

$x_1 = (8 + \sqrt{20}) / \sqrt{2}$

$x_2 = (8 - \sqrt{20}) / 2$

$XX = (x_1^3 + x_2^3) + (x_1^2 + x_2^2) + x_1 + x_2 =$

$= (x_1 + x_2)(x_1^2 + x_2^2 - x_1 x_2) +$

$+ (x_1^2 + 2x_1 x_2 + x_2^2 - 2x_1 x_2) + (x_1 + x_2) =$

$= (x_1 + x_2)(x_1^2 + 2x_1 x_2 + x_2^2 - 3x_1 x_2) +$

$+ (x_1 + x_2)^2 - 2x_1 x_2 + (x_1 + x_2) =$

$= (x_1 + x_2)((x_1 + x_2)^2 - 3x_1 x_2) + (x_1 + x_2)^2 + (x_1 + x_2) - 2x_1 x_2.$

$$\begin{cases} x_1 + x_2 = 8 \\ x_1 x_2 = 11 \end{cases}$$

$x_1 = 8 - x_2$

$(8 - x_2) x_2 = 11$

$8 x_2 - x_2^2 = 11$

$x_2^2 - 8x_2 + 11 = 0$

$x_2 = 4 \pm$

$\sqrt{18^2 - 3 \cdot 11} + 8^2 \quad 8 - 2 \cdot 11 = \sqrt{64 - 33} +$

$+ 64 + 8 - 22 = 8 \cdot 31 + 50 = 8 \cdot 31 + 8 \cdot 6 +$

$+ 2 = 8 \cdot 37 + 2 = 298$

$5x^2 - 15x + 36 = 0, \quad |x_1 - x_2|$

$|x| = \sqrt{x^2}$

$|x_1 - x_2| = \sqrt{(x_1 - x_2)^2} = \sqrt{x_1^2 - 2x_1x_2 + x_2^2} =$

$= \sqrt{x_1^2 + 2x_1x_2 + x_2^2 - 4x_1x_2}$

Прибавили $к - 2x_1x_2 + 4x_1x_2$, чтобы получить квадрат суммы, потом отняли $к - 4x_1x_2$, чтобы уравнение осталось истинным

$$\sqrt{(x_1+x_2)^2-4x_1x_2} = \sqrt{15^2-4\cdot 36} = \sqrt{81} = 9.$$

6. $x^2 - 12x + 19 = 0$

$$x_1+x_2 = -\frac{b}{a} = \frac{12}{1} = 12$$

$$x_1 \cdot x_2 = \frac{c}{a} = \frac{19}{1} = 19$$

$$x_1(1-x_1)+x_2(1-x_2) = (x_1-x_1^2)+(x_2-x_2^2) = x_1-x_1^2+x_2-x_2^2 =$$

$$= x_1+x_2-(x_1^2+x_2^2) =$$

$$= x_1^2+2x_1x_2+x_2^2-2x_1x_2 \cdot 9 =$$

$$= x_1+x_2-(x_1+x_2)^2-2x_1x_2) =$$

$$= (x_1+x_2)-(x_1+x_2)^2+2x_1x_2 =$$

$$= (x_1+x_2)((1-(x_1+x_2))+2x_1x_2$$

x_4 $12(1-12)+2\cdot19 = -12\cdot11+38 =$

$= -132+38 = -94$

7. $x^2-4x+1=0$, найти $\left(x_1-\frac{1}{x_1}\right)^2+\left(x_2-\frac{1}{x_2}\right)^2$

По теореме Виета

~~$4^2-4\cdot1$~~

$x_1+x_2 = -\frac{b}{a} = 4$; $x_1 \cdot x_2 = 1$

$\frac{2}{1}\cdot\frac{1}{2} = \frac{2}{2} = 1$

$\frac{2}{1}+\frac{1}{2} = \frac{4}{2}+\frac{1}{2} = \frac{5}{2}$ ✗

$\left(x_1-\frac{1}{x_1}\right)^2+\left(x_2-\frac{1}{x_2}\right)^2 =$

$= (x_1+x_2)^2-\left(\frac{1}{x_1}+\frac{1}{x_2}\right)^2 =$

$= x_1^2+2x_1x_2+x_2^2-\left(\frac{1}{x_1}\right)^2+2\frac{1}{x_1}\cdot\frac{1}{x_2}+\left(\frac{1}{x_2}\right)^2 =$

$= x_1^2+2x_1x_2+x_2^2-\frac{1}{x_1^2}-2\left(\frac{1}{x_1}\cdot\frac{1}{x_2}\right)+\frac{1}{x_2^2}$

$$x_1^2 + 2x_1x_2 + x_2^2 - \frac{1}{x_1^2} + \frac{2}{x_1x_2} + \frac{1}{x_2^2} =$$

$$= x_1^2 + x_2^2 + \frac{x_1^2 + x_2^2}{(x_1x_2)^2} - 4, =$$

-4 | $\to$ по двойное умножение!

$$= 2\left|x_1^2 + x_2^2\right)\left[1 + \frac{1}{(x_1x_2)^2}\right] - 4 =$$

$$= \left|(x_1 + x_2)^2 - 2x_1x_2\right|\left|1 + \frac{1}{(x_1x_2)^2}\right) - 4$$

$$\left(4^2 - 2\right)\left|1+1\right) - 4 = \left|16-2\right)\cdot(2) - 4 =$$

$$= 14 \cdot 2 - 4 = 28 - 4 = 24.$$

8. $x_1, x_2 \to x^2 - 5x + a^2 - 2a + 1 = 0$

$a \in R$. Найти значение a, где x_1, x_2 - минимальны

$$\underbrace{x^2}_{a} \underbrace{- 5x}_{b} \underbrace{+ a^2 - 2a + 1}_{c} = 0$$

Согласно теореме Виета:

$$x_1x_2 = \frac{a^2 - 2a + 1}{x^2(1,1)}$$

$$x_1x_2 = a^2 - 2a + 1 : = a(a-2+1)$$

$$= a^2 - a - a + 1 = a(a-1) - 1(a-1)$$

$$= (a-1)(a-1) \; D; a=1, \text{ мин. знач} = 0.$$