Mathematics Tasks 12/08/2023

12.08.

Arithmetic / Geometric PROGRESSIONS

№ 2, 10

2. ~~Да~~ Нет, Так как каждое последующее число имеет одинаковую разницу с предыдущим, кроме 10 и 12 где разница 2.

10. ~~18 / 9~~ = 3

$a_5 = a_1 + 4d$

$a_2 = a_1 + d$

$a_5 - a_2 = a_1 + 4d - a_1 - d = 3d$

$18 - 9 = 3d$

$d = 9 / 3$

$d = 3$

№ 5,11

$a_1 = 15$; $a_1 + a_2 + a_3 = 102$

$a_2 = a_1 + d$

$a_3 = a_1 + 2d$

$x = a_1 + a_1 + d + a_1 + 2d$

$x = 3a_1 + 3d$

$3a_1 + 3d = 102$

$3 \cdot 15 + 3d = 102$

$45 + 3d = 102$

$3d = 57$

$d = 57/3$

$d = 19$

$\cancel{a\ 15 + 115 + 19 + \ldots}$

$a_{10} = a_1 + 9d = 15 + 9 \cdot 19 = 186$

$a_{10} = a_5 + 5 \cdot d$

$5 + 5d = 15$

$5d = 15 - 5$

$d = 2$

$a_5 = a_1 + 4d$

$a_5\ 5 = a_1 + 8$

$-a_1 = 8 \cancel{+} - 5$

$-a_1 = 3, \quad a_1 = -3$

3. $a_1 = 5, \quad a_3 = 20, \quad q - ?$

$a_3 = a_1 \cdot 2q$

$a_3 = 5 \cdot 2q$

$5 \cdot 2q = 20$

$2q = 20/5$

$2q = 4$

$q = 2$

$5 \cdot 2 = 10 \ (a_2)$

$10 \cdot 2 = 20 \ (a_3)$

O: 2

№ 3, 12

3. $b_1 = 2$; $q = -2$

$b_{10} = b_1 \cdot q^{n-1}$

$b_{10} = b_1 \cdot q^9 = 2 \cdot 2^9 = 2 \cdot 512 = 1024$

$S_n = \frac{b_1(1-q^n)}{1-q} = \frac{2\cdot(1023)}{1-(-2)} = -\frac{2046}{3}$

$S_{10} = -682$

О: -682

$b_1 = 5$; $b_3 = 20$

$b_3 = b_1 \cdot q^{(3-1)}$

$b_3 = 5 \cdot q^2 = 20$

$q^2 = \frac{20}{5}$

$q^2 = 4$

$q = 2$ или $q = -2$

12. $S_4 = \dfrac{a_1 \cdot (1-q^n)}{1-q}$

$40 = \dfrac{1 \cdot (1-q^n)}{1-q}$

$40(1-q) = 1-q^4$

$40 - 40q = 1 - q^4$

$q^4 - 40q + 39 = 0$

$q^4 - q - 39q + 39 = q(q^3-1) - 39(q-1) =$

$= q(q-1)(q^2+q+1) - 39(q-1) =$

$= (q-1)(q^3+q^2+q-39)$

$q^3 + q^2 + q - 39 = 0$

$q^3 - 27 + q^2 - 9 + q - 3 = 0$

$3^3 = 27 \quad 3^2 = 9 \quad 3^1 = 3$

$(q-3)(q^2+3q+9) + (q-3)(q+3) +$

$+ 1(q-3) = 0$

$q^3 - 27 + q^2 - 9 + q - 3 = 0$

$(q-3)(q^2+3q+9) + (q-3)(q+3) +$

$+ 1(q-3) = 0$

$(q-3)(q^2 + 3q + 3 + q + 3 + 1) = 0$

$(q-3)(q^2+4q+7) = 0$

$q^2+4q+7 = (q+2)^2 + 3 > 0$, не имеет решения

7. $a_n = 6 \cdot \left(\frac{1}{3}\right)^n$

$S = \frac{b_1}{1-q}$

$a_1 = 6 \cdot \frac{1}{3}$; $q = \frac{1}{3}$

$S = 6 \cdot \frac{1}{3} \cdot \frac{1}{1-\frac{1}{3}} = \frac{6}{3} \cdot \frac{1}{1-\frac{1}{3}} =$

$= \frac{6}{3 - \frac{3}{3}(1)} = \frac{6}{2} = 3$

$x_1 x_2 \rightarrow x^2 - 3x + a = 0$

$x_1 + x_2 = -\frac{-3}{1} = 3$

$x_1 \cdot x_2 = 1$

$3 \cdot \frac{1}{3} = \frac{3}{3} = 1$, но $3 + \frac{1}{3} = 3\frac{1}{3}$ $3\frac{1}{3}$,

значит эти числа не подходят как корни уравнений

$D = (-3)^2 - 4 \cdot 1 = 9 - 4 = 5$

$x_{1,2} = \left(\frac{3 \pm \sqrt{5}}{2}\right)$

~~12~~. 5. $S_\infty = \frac{b_1}{1-q}$

$S = \frac{1}{1 - \frac{1}{7}} = \frac{1}{\frac{6}{7}} = 1 \cdot \frac{7}{6} = \frac{7}{6}$

$1 + 2x + 3x^2 + 4x^3 + 5x^4 \ldots$ $|x| < 1$

$S = 1 + 2x + 3x^2 + 4x^3 + 5x^4 = 1(1 + x + x^2$

$S = 1 + 2x + 3x^2 + 4x^3 + \ldots =$

$= (1 + x + x^2 + x^3 + x^4 \ldots) + \cancel{x} + 2x\,\cancel{x}$

$+ 3x^2 + 4x^3 + \ldots = (1 + x + x^2 + x^3 + \ldots) +$

$+ \ldots = (1 + x + x^2 + x^3) + (x + x^2 + x^3 + \ldots) +$

$+ x^2 + 2x^3 + 3x^4 + \ldots = 1 \cdot (1 + x + x^2 + x^3$

$\ldots) + x(1 + x + x^2 + x^3 + \ldots) + x^2(1 +$

$+ x + x^2 + x^3 + \ldots) =$

$= (1 + x + x^2 + x^3 + \ldots)^2$

$S = 1 \cdot \frac{1}{1-x} = \left(\frac{1}{1-x}\right)^2 = \left(\frac{1}{1-\frac{1}{7}}\right)^2 = \left(\frac{1}{\frac{6}{7}}\right)^2 =$

$= \left(1 \cdot \frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2 = \frac{49}{36}$