Mathematics Marathon 07/08/2023 Tasks

07.08.23 Quadratic Equations

$x^2 - 5x + 4 = 0$

$D = b^2 - 4ac$

$D = (-5)^2 - 4 \cdot (4)$

$D = 25 - 16$

$D = 9$ $D > 0$, уравнение имеет два корня решения

$$x_1, x_2 = \frac{-b \pm \sqrt{D}}{2a}$$

$x_{1,2} = (5 \pm \sqrt{9}) / 2$.

$x_1 = 8/2 = 4$ $x_2 = 2/2 = 1$

$x^2 - 3x + 2 = 0$ ~~$= (x+1)(x-3)$~~

$D = (-3)^2 - 4(2)$

$D = 1$

$x_{1,2} = (3 \pm \sqrt{1}) / 2$, $x_1 = 2$, $x_2 = 1$

$x^2 - 13x + 12 = 0$

$D = (-13)^2 - 4(12) = 169 - 48 = 121$

$x_{1,2} = (13 \pm \sqrt{121}) / 2 = x_1 = 12$, $x_2 = 1$

$x^2 + 3x - 70 = 0$

$D = 13)^2 - 4 \cdot (-70) = 9 + 280 = 289 = 17^2$

$x_1, x_2 = -3 \pm 17 / 2 = x_1 = 7, \quad x_2 = -10$

$x^2 - 12x + 35 = 0$

$D = (-12)^2 + 4 \cdot 35 = 144 - 140 = 4 = 2^2$

$x_{1,2} = 12 \pm 2 / 2 \quad x_1 = 7, \quad x_2 = 6$

0; 5

№ 6, 7, 9

$$\frac{2x-1}{x-1} = \frac{7x-1}{2x+2}$$

$$\frac{2x-1}{x-1} - \frac{7x-1}{2x+1} = 0$$

$$\frac{2x-1}{x-1}(x-1) - \frac{7x-1}{2x+1} = 0$$

~~$2x - x$~~

$$2x^2 - 2x \cdot 1 - 1 \cdot x + 1 \cdot 1$$

$$\frac{2x^2 - 3x + 1 - 7x - 1 = 0}{2x+1}$$

$$2x^2 - 10x = 0$$

$$D = (-10^2) - 4 \cdot 2 \cdot 0 = 100 = 10$$

$$x_{1,2} = (10 \pm 10)/2, \; x = 0, \; x = 10$$

$2x-1 = |x-1|\,|7x-1|$

$2x-1 = 7x^2 - 8x + 1$

$7x^2 - 10x + 2 = 0$

$D = (-10)^2 - 4 \cdot 7 \cdot 2 = 100 - 56 = 44 = 2\sqrt{11}$

$\frac{2x-1}{x-1} = \frac{7x-1}{2x+2}$

$|2x-1|\,|2x+2| = |7x-1|\,|x-1|$

$4x^2 - 2x + 4x - 2 = 7x^2 - 7x - x + 1$

$4x^2 + 2x - 2 = 7x^2 - 8x + 1$

$7x^2 - 4x - 3 = 0$

$D = (-4)^2 - 4 \cdot 7 \cdot (-3) = 16 + 84 = 100 = 10$

$x_{1,2} = 14 \pm 10 / 2 \cdot 7 = 14/14 = 1,\ -10/14 \quad x_2 = -\frac{5}{7}$ X

$|2x-1|\,|2x+2| = |7x-1|\,|x-1|$

$4x^2 + 4x - 2x - 2 = 7x^2 - 7x - x + 14x^2 + 2x - 2 =$

$= 7x^2 - 8x + 13x^2 - 10x + 30 = 0$

$D = 10^2 - 4 \cdot 3 \cdot 37^2\ 100 - 36\ \ 2 \cdot 4 = 8^2$

$x_{1,2} = \frac{-10 \pm 8}{6} = \frac{2}{6} = \frac{1}{3}$ $= \frac{18}{6} = 3$

$\frac{2x+8}{x} = \frac{x+2}{2}$

$\frac{x+8}{x} - \frac{x+2}{2} = 0$

$2x + 16 - x^2 + 2x = 0$

$-x^2 + 4x + 16 = 0$

$D = (4)^2 - 4 \cdot (-1) \cdot 16 = 16 + 64 = 80$

$\sqrt{80} = \sqrt{2 \cdot 40} = \sqrt{2 \cdot 2 \cdot 20} = 2 \cdot 2 \cdot 2 \cdot 5 = 4\sqrt{5}$

$x_{1,2} = \frac{-4 \pm 4\sqrt{5}}{-2}$

$2(x+8) = x(x+2)$

$2x + 16 = x^2 + 2x$

$2x - 2x; \ 16 = x^2$

$x = 4, \quad x = -4$

7. $x^4 - 3x^2 - 4 = 0$

$x^2 + -3x^2 = -4x^2 + x^2$

$x^4 - 4x^2 + x^2 - 4 =$

$x^2(x^2 - 4) + (x^2 + 1)(x^2 - 4) = 0$

$(x^2 + 1)(x^2 - 4) = 0$

$x^2 + 1 = 0$

$x^2 = -1$

Уравнение не имеет корня решений, так как результат степени не может быть отрицательным

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

Ответ: 2; -2.

$(11 + x)(14 + x) = 304$

$154 + 11x + 14x + x^2$

$x^2 + 25x + 154 = 304$

$x^2 + 25x + 154 - 304 = 0$

$x^2 + 25x - 150 = 0$

$D = 125)^2 - 4(1 \cdot -150) = 625 + 600 = 1225$

$\sqrt{1225} = \sqrt{245 \cdot 5} = \sqrt{49 \cdot 5 \cdot 5} = 7 \cdot 7 \cdot 5 \cdot 5 =$

$= 35.$

$x_{1,2} = (-25 \pm 35) / 2$

$x_1 = 10/2 = 5$

$x_2 = -60/2 = -30$

$O: 5, -30$

$N^0 = 2, 3$

$x^2 - 2ax + a^2 = 0$

$D = (-2)^2 - 4 \cdot (1) \cdot a^2 = 4 - 4a^2$

$D = 4 - 4a^2 = 0$

$4a^2 = 4$

$a^2 = 4/4$

$a^2 = \pm 1$

Задача 3

$\sqrt{6+\sqrt{6+\sqrt{6}+\ldots}}$

$x = \sqrt{6+\sqrt{6+\sqrt{6}+\ldots}}$

$x = \sqrt{6+x}$, где x - подкоренное выражение

$x^2 \geq 6+x$

$x^2 - 6 - x \geq 0$

$x^2 - x - 6 \geq 0$

$D = (-1)^2 - 4 \cdot (1) \cdot (-6) = 1 + 24 = 25 = \sqrt{5}$

$x_{1,2} = (1 \pm 5) / 2 = x_1 = -4/2 = -2 \quad x_2 = 6/2 = 3$

Так как x у нас никогда не может быть отрицательного числа под корнем, следовательно и ответ будет равен 3.