Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science
6.111 - Introductory Digital Systems Laboratory

Problem Set 1

Issued: February 7, 2007 **Due**: February 20, 2007

Boolean Algebra Practice Problems (do not turn in):

Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and re-transform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

1) $a+0=$ ______
2) $\bar{a}\cdot 0=$ ______
3) $a+\bar{a}=$ ______
4) $a+a=$ ______
5) $a+ab=$ ______
6) $a+\bar{a}b=$ ______
7) $a(\bar{a}+b)=$ ______
8) $ab+\bar{a}b=$ ______
9) $(\bar{a}+\bar{b})(\bar{a}+b)=$ ______
10) $a(a+b+c+...)=$ ______

For (11),(12), (13), $f(a,b,c)=a+b+c$

11) $f(a,b,ab)=$ ______
12) $f(a,b,\bar{a}\cdot\bar{b})=$ ______
13) $f[a,b,\overline{(ab)}]=$ ______
14) $y+y\bar{y}=$ ______
15) $xy+x\bar{y}=$ ______
16) $\bar{x}+y\bar{x}=$ ______
17) $(w+\bar{x}+y+\bar{z})y=$ ______
18) $(x+\bar{y})(x+y)=$ ______
19) $w+[w+(wx)]=$ ______
20) $x[x+(xy)]=$ ______
21) $\overline{(\bar{x}+\bar{x})}=$ ______
22) $\overline{(x+\bar{x})}=$ ______
23) $w+(w\bar{x}yz)=$ ______
24) $\bar{w}\cdot\overline{(wxyz)}=$ ______
25) $xz+\bar{x}y+zy=$ ______
26) $(x+z)(\bar{x}+y)(z+y)=$ ______
27) $\bar{x}+\bar{y}+xy\bar{z}=$ ______

Problem 1: Karnaugh Maps and Minimal Expressions

For each of the following Boolean expressions, give:

i) The truth table,
ii) The Karnaugh map,
iii) The minimal sum of products expression. (Show groupings)
iv) The minimal product of sums expression. (Show groupings)

1) $(\bar{a}+b\cdot\bar{d})\cdot(c\cdot b\cdot a+\bar{c}\cdot d)$
2) $\overline{(w+\bar{x})(z\bar{y}+x)}$

Problem 2: Karnaugh Maps with “Don’t Cares”

Karnaugh Maps are useful for finding minimal implementations of Boolean expressions with only a few variables. However, they can be a little tricky when “don't cares” (X) are involved. Using the following K-Maps:

cd \ ab	00	01	11	10
00	X	0	0	1
01	1	0	0	X
11	0	X	0	1
10	0	0	0	1

(1)

cd \ ab	00	01	11	10
00	1	X	0	1
01	1	1	1	0
11	0	0	X	0
10	X	0	1	1

(2)

i) Find the minimal sum of products expression. Show your groupings.
ii) Find the minimal product of sums expression. Show your groupings.
iii) Are your solutions unique? If not, list and show the other minimal expressions.
iv) Does the MPS = MSP?

Problem 3: DeMorgan’s Theorem

Use DeMorgan's Theorems to simplify the following expressions:

1) $\overline{\overline{(a+d)}\cdot\overline{(\bar{b}+c)}}$

2) $\overline{\overline{(a\cdot b\cdot \bar{c})}+\overline{(\bar{c}\cdot d)}}$

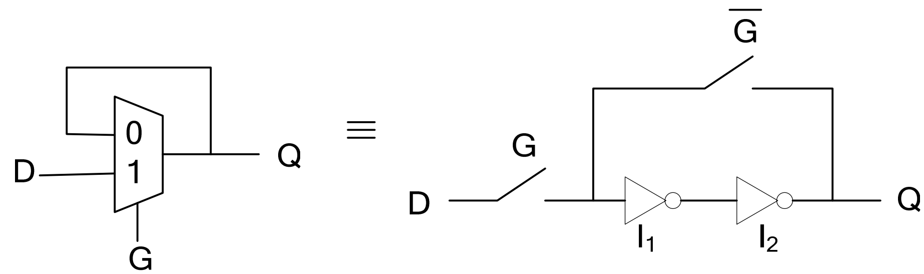
3) $\overline{\bar{a}+d}\cdot\overline{b+\bar{c}}\cdot\overline{\bar{c}+d}$

Problem 4: Transistor/Gate Level Synthesis

1) Construct a transistor level circuit with inputs *A*, *B*, and *C*, and output *F* of the following function using NMOS and PMOS devices: $F = \overline{A + B\cdot C}$
2) Construct a gate level circuit of the same function with inputs *A*, *B*, and *C*, and output *F* only using NAND gates.

Problem 5: Setup and Hold Times for D Flip-Flop *(Flip-flops will be covered in lecture 4)*

1) Let a D latch be implemented using a mux and realized as follows:

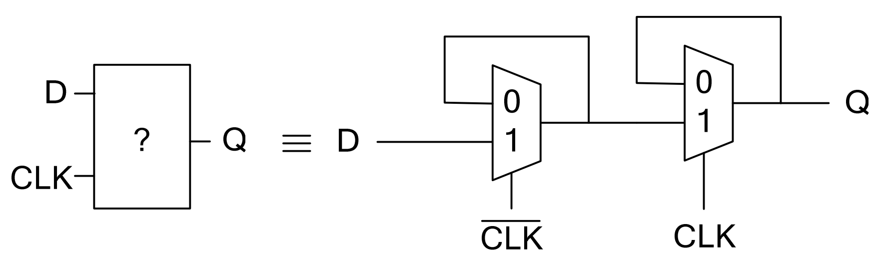


You may assume the following:

a) G and $\overline{G}$ are complements and have zero skew, i.e. when G is 1, $\overline{G}$ is exactly 0, and vice versa.
b) Assume the switches are ideal, with no delay. E.g. when G is 0, the switch is open.
c) The propagation delay of the inverters is t_{inv} (assume that the contamination delay or minimum delay is equal to the propagation delay).

What is the setup and hold time of this latch?

2) What memory element is created when two muxes are cascaded as in the figure below? Assume that CLK and $\overline{CLK}$ are complements with zero skew.



3) What is the setup time, hold time, and clock to Q delay of the above memory element?

Solutions to the Boolean Algebra Practice Problems

1) $a + 0 = a$
2) $\overline{a} \cdot 0 = 0$
3) $a + \overline{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \overline{a}b = (a + \overline{a})(a + b) = a + b$
7) $a(\overline{a} + b) = a\overline{a} + ab = ab$
8) $ab + \overline{a}b = b(a + \overline{a}) = b$
9) $(\overline{a} + \overline{b})(\overline{a} + b) = \overline{a}\overline{a} + \overline{a}b + \overline{b}\overline{a} + \overline{b}b = \overline{a} + \overline{a}b + \overline{a}\overline{b} = \overline{a}(1 + b + \overline{b}) = \overline{a}$
10) $a(a + b + c + ...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \overline{a} \cdot \overline{b}) = a + b + \overline{a}\overline{b} = a + b + \overline{a} = 1$
13) $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \overline{a} + \overline{b} = 1$
14) $y + y\overline{y} = y$
15) $xy + x\overline{y} = x(y + \overline{y}) = x$
16) $\overline{x} + y\overline{x} = \overline{x}(1 + y) = \overline{x}$
17) $(w + \overline{x} + y + \overline{z})y = y$
18) $(x + \overline{y})(x + y) = x$
19) $w + [w + (wx)] = w$
20) $x[x + (xy)] = x$
21) $\overline{(\overline{x} + \overline{x})} = x$
22) $\overline{(x + \overline{x})} = 0$
23) $w + (w\overline{x}yz) = w(1 + \overline{x}yz) = w$
24) $\overline{w} \cdot \overline{(wxyz)} = \overline{w}(\overline{w} + \overline{x} + \overline{y} + \overline{z}) = \overline{w}$
25) $xz + \overline{x}y + zy = xz + \overline{x}y$
26) $(x + z)(\overline{x} + y)(z + y) = (x + z)(\overline{x} + y) = xy + \overline{x}z$
27) $\overline{x} + \overline{y} + xy\overline{z} = \overline{x} + \overline{y} + \overline{z}$