

Lecture 02

Date: 4 Jan 24

▷ Create an array $\text{Fib}[0 \dots n]$

$\text{Fib}[0] \leftarrow 0$

$\text{Fib}[1] \leftarrow 1$

for $i \leftarrow 1$ to n

do $\text{Fib}[i] \leftarrow \text{Fib}[i-1] + \text{Fib}[i-2]$

▷ Create $T[1 \dots n][0 \dots t]$

for $i \leftarrow 1$ to n

do $T[i][0] \leftarrow \text{True}$

for $j \leftarrow 1$ to t

do if $A[1] = j$

then $T[i][j] \leftarrow \text{True}$

else $T[i][j] \leftarrow \text{False}$.

for $i \leftarrow 2$ to n

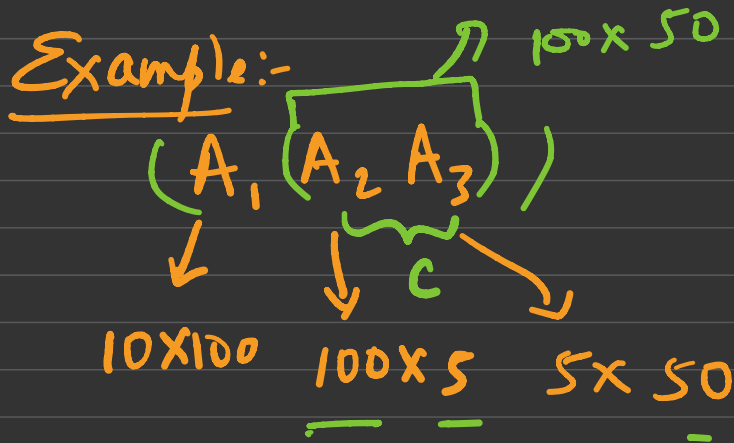
do for $j \leftarrow 1$ to t

do if $A[i] \leq j$

then $T[i][j] \leftarrow T[i-1][j-A[i]]$
 $\vee T[i-1][j]$

else $T[i][j] \leftarrow T[i-1][j]$

Matrix Chain Multiplication



$$C = A B$$

$\downarrow \quad \quad \downarrow$
 $s \times t \quad t \times r$

$$c_{ij} = \sum_{k=1}^t a_{ik} b_{kj}$$

$$100 \times 5 \times 50 = 25000 \quad \text{Total \# mult}^n = 8tr.$$

$$A_1 C \rightarrow 10 \times 100 \times 50 = 50000$$

$$\text{Total } 75000$$

$(A_1 A_2) A_3$

7500

Input:- A sequence of matrices A_1, \dots, A_n

Find:- The "best" way to multiply these
matrices

The one that
minimizes # mult

in order

$$\prod_{i=1}^n A_i$$

A_i has dimension $p_{i-1} \times p_i$

p p_0, \dots, p_n

A_i

$P(n)$ = # ways to multiply n matrices

$$P(1) = 1$$

$$P(3) = 2$$

$$P(2) = 1$$

$$P(4) = 5$$

$$((A_1 A_2) A_3 A_4)$$

$$(?) ((A_1 (A_2 A_3)) A_4)$$

$$(A_1 ((A_2 A_3) A_4))$$

$$(A_1 (A_2 (A_3 A_4)))$$

$$(((A_1 A_2) A_3) A_4)$$

$$P(n) = \sum_{k=1}^{n-1} P(k) P(n-k)$$

$P(n)$

Catalan number

↓

$$\frac{1}{n} \binom{2n}{n}$$

$$P(n) = \sum \left(\frac{4^n}{n^{3/2}} \right)$$

$$P(n) > 2^n$$

$m[i, j]$ = optimal cost of multiplying
 A_i, \dots, A_j

$$m[i, i] = 0 \quad A_i$$

$$p_{i-1} \times p_u \quad \overbrace{A_i \dots A_k}^{p_{i-1} \times p_u} \quad \overbrace{A_{k+1} \dots A_j}^{p_k \times p_j}$$

\swarrow

$$p_{i-1} \times p_i$$

$$m[i, k] + m[k+1, j] + p_{i-1} p_k p_j$$

$$\underbrace{m[i, j]} = \min_{i \leq k < j} \left\{ \underbrace{m[i, k]} + \underbrace{m[k+1, j]} + p_{i-1} p_k p_j \right\}$$

HW:- ① Find the optimal multiplication strategy
② Implement.

Optimal Substructure property:-

$A_1 \dots A_n$

$A_1 \dots A_k \mid A_{k+1} \dots A_n$
↑

$(A_1 (A_2 A_3)) \mid (A_4 (A_5 A_6))$

An optimal solⁿ to a problem contain within it the optimal solⁿ to subproblems.

