DSA Problems

January 2024

Greedy Algorithms

Question 1

You want to collect sorted lists from different databases and merge them into a single sorted list. The cost to merge a list of size l_1 with one of size l_2 is $l_1 + l_2$ and creates a new list of size $l_1 + l_2$, replacing the old ones. You are given as input the sizes of n lists $l_1, l_2, \ldots, l_n$ and need to schedule merges in order to unite them into a single list. You wish to minimize the total cost for all merges. Give an efficient algorithm that, given $l_1, l_2, \ldots, l_n$, finds the lowest cost schedule to merge lists of those sizes.

Question 2 (CLRS)

Describe an efficient algorithm that, given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determines the smallest set of unit-length closed intervals that contains all the given points. Argue that your algorithm is correct.

Question 3

You are driving along a highway with refueling stations. Each station is at distance $D[i]$ from your starting point. Your car holds enough gas to travel up to 100 miles before refueling. Assume that you start out with an empty tank, but there is a refueling station at your starting point (i.e., $D[0] = 0$). Assume, likewise, that there is a fueling station at your destination, $D[n]$. Design and analyze an efficient algorithm that computes the minimum number of refueling stops you have to make to reach your destination or return ∞ if this is impossible.

Question 4 (CLRS)

Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the i-th element of set A, and let b_i be the i-th element of set B. You then receive a payoff of $\Pi_{i=1}^{n} a_i^{b_i}$. Give an algortihm that will maximize your payoff, and state its running time.

Question 5 (CLRS)

You are given a set of activities to schedule among a large number of lecture halls, where any activity can take place in any lecture hall. You wish to schedule all the activities using as few lecture halls as possible. Give an efûcient greedy algorithm to determine which activity should use which lecture hall.

(This problem is also known as the *interval-graph coloring problem*. It is modeled by an interval graph whose vertices are the given activities and whose edges connect incompatible activities. The smallest number of colors required to color every vertex so that no two adjacent vertices have the same color corresponds to ûnding the fewest lecture halls needed to schedule all of the given activities.)