DSA Problems

January 2024

Dynamic Programming

Question 1: Longest Increasing Subsequence (CLRS)

You are given a sequence of distinct integers. You need to find a longest increasing subsequence of the sequence. For example, 3, 1, 2, 5, 4 has 1, 2, 4 as a longest increasing subsequence. a) Design a dynamic programming algorithm to find a longest increasing subsequence of a sequence that runs in time $O(n^2)$ where n is the length of the sequence. b) Design an $O(n \log n)$ algorithm for the problem.

Question 2: Weighted Interval Scheduling

Consider a set of intervals with weights $\{(s_1, f_1, w_1), (s_2, f_2, w_2), \ldots, (s_n, f_n, w_n)\}$, where s_i is the start time, f_i is the finish time, and w_i is the weight of the i-th interval. The objective is to find a subset of non-overlapping intervals that maximizes the total weight. Design an efficient algorithm for the above problem using dynamic programming.

Question 3: Palindrome Partitioning

Given a string s, the goal is to partition s into palindromic substrings with the minimum number of cuts. A palindrome is a sequence of characters that reads the same forward as backward. For example:

$s =$ "aab", then optimal partitioning is "aa"|"b". Cut required is 1.

$s =$ "$racecar$" then s is already palindrome. Cut required is 0.

Design an algorithm to find the minimum number of cuts.

Question 4: Maximal Square in Binary Matrix

Given a binary matrix with entries 0's and 1's, provide an efficient algorithm to compute a square submatrix of maximum size with all 1's. See Figure 1 for illustration.

1	0	1	1	1
0	1	1	1	1
1	1	1	1	1
0	0	1	0	1

1	0	1	1	1
0	1	1	1	1
1	1	1	1	1
0	0	1	0	1

Figure 1: **Left:** Input matrix. **Right:** The shaded region indicates the maximal square with all 1's

Question 5: Bitonic Sequence (JE)

Call a sequence $X[1 \dots n]$ of numbers bitonic if there is an index i with $1 < i < n$, such that the prefix $X[1 \dots i]$ is increasing and the suffix $X[i \dots n]$ is decreasing. Design an algorithm to find the length of the longest bitonic subsequence of an arbitrary array A of integers.

Question 6: Breaking a String (CLRS)

A certain string-processing language allows a programmer to break a string into two pieces. Because this operation copies the string, it costs n time units to break a string of n characters into two pieces. Suppose a programmer wants to break a string into many pieces. The order in which the breaks occur can affect the total amount of time used. For example, suppose that the programmer wants to break a 20-character string after characters 2, 8, and 10 (numbering the characters in ascending order from the left-hand end, starting from 1). If she programs the breaks to occur in left-to-right order, then the first break costs 20 time units, the second break costs 18 time units (breaking the string from characters 3 to 20 at character 8), and the third break costs 12-time units, totaling 50 time units. If she programs the breaks to occur in right-to-left order, however, then the first break costs 20 time units, the second break costs 10 time units, and the third break costs 8 time units, totaling 38 time units. In yet another order, she could break first at 8 (costing 20), then break the left piece at 2 (costing 8), and finally the right piece at 10 (costing 12), for a total cost of 40.

Design an algorithm that, given the numbers of characters after which to break, determines a least-cost way to sequence those breaks. More formally, given a string S with n characters and an array $L[1 \dots m]$ containing the break points, com- pute the lowest cost for a sequence of breaks, along with a sequence of breaks that achieves this cost.

Question 8: Planning a company party (CLRS)

Professor Stewart is consulting for the president of a corporation that is planning a company party. The company has a hierarchical structure; that is, the supervisor relation forms a tree rooted at the president. The personnel office has ranked each employee with a conviviality rating, which is a real number. In order to make the party fun for all attendees, the president does not want both an employee and his or her immediate supervisor to attend.

Professor Stewart is given the tree that describes the structure of the corporation, using the left-child, right-sibling representation. Each node of the tree holds, in addition to the pointers, the name of an employee and that employee's conviviality ranking. Describe an algorithm to make up a guest list that maximizes the sum of the conviviality ratings of the guests. Analyze the running time of your algorithm.

Question 9: Printing neatly (CLRS)

Consider the problem of neatly printing a paragraph with a monospaced font (all characters having the same width) on a printer. The input text is a sequence of n words of lengths $l_1, l_2, \ldots, l_n$, measured in characters. We want to print this paragraph neatly on a number of lines that hold a maximum of M characters each. Our criterion of "neatness" is as follows. If a given line contains words i through j, where $i \leq j$, and we leave exactly one space between words, the number of extra space characters at the end of the line is $M - j + i - \sum_{k=i}^{j} l_k$ which must be nonnegative so that the words fit on the line. We wish to minimize the sum, over all lines except the last, of the cubes of the numbers of extra space characters at the ends of lines. Give a dynamic-programming algorithm to print a paragraph of n words neatly on a printer. Analyze the running time and space requirements of your algorithm.