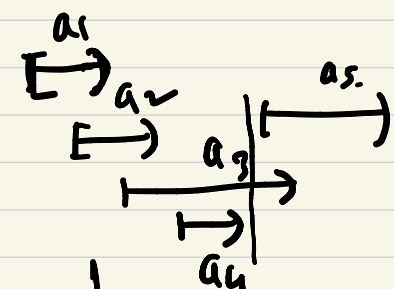
Lecture 07 Date: 23 Jan 2024

Greedy Algorithms

Activity Selection

Intervals $[s_i, t_i)$ $\quad 0 \leq s_i < t_i$

$$[s_i, t_i) = \{x \mid s_i \leq x < t_i\}$$



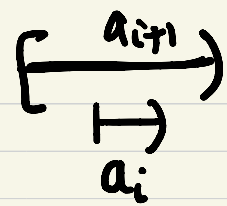
$i = 1, \dots, n$ $\quad a_1, \dots, a_n$

Input: $a_i = [s_i, t_i)$ $\quad S = \{a_1, \dots, a_n\}$ $\quad a_1, a_3 \mid a_1, a_4, a_5.$

Sorted based on their finish time $t_1 \leq t_2 \leq \cdots \leq t_n$

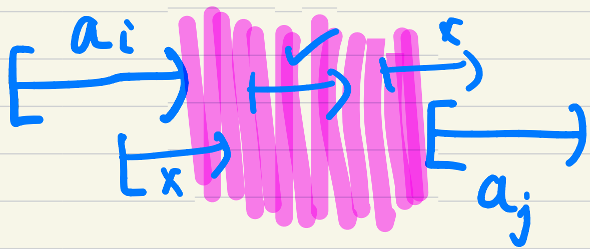
$s_i \leq s_{i+1}$?

$\times$



$a_0 = [-2, -1)$ $\quad$ $a_{n+1} = [t_n + 1, t_n + 2)$

$S_{ij} = \{a_k \mid t_i \leq s_k < t_k \leq s_j\}$



We want sol^n to $S_{0,n+1}$ ←

* Find:- $X \subseteq S$, s.t. ① $a, b \in X$

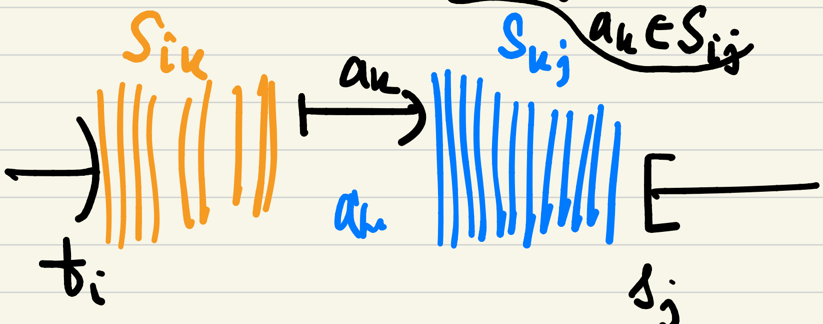
$a \cap b = \phi$

② $|X|$ is maximum.

A DP sol^n

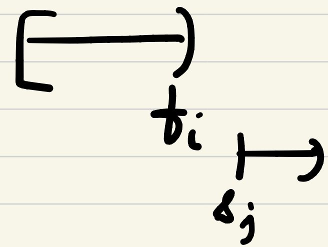
$c[i,j]$ = size of sol^n to S_{ij}

$$c[i,j] = \begin{cases} 0 & S_{ij} = \emptyset \\ \underbrace{\max_{i<k<j}}_{a_k \in S_{ij}} c[i,k] + c[k,j] + 1 & S_{ij} \neq \emptyset \end{cases}$$



$S_{ij} = \{a_{k_1}, a_{k_2}, \ldots\}$

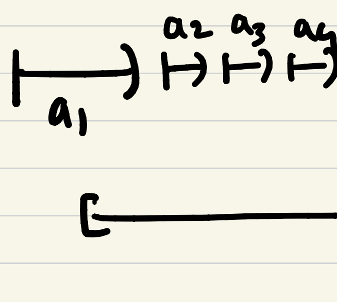
① Is this true: $t_i \geq s_j \quad \forall j > i$



Not true

② If $S_{1n} = \emptyset$ then is it possible to have $S_{ij} \neq \emptyset$ for some $i, j \in \{1, \dots, n\}$?

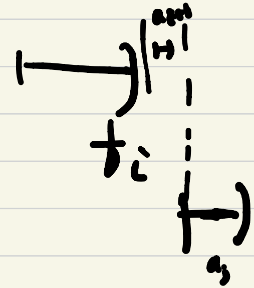
Yes



$S_{14} = \{a_2, a_3\}$

Q:- Can we have $S_{ij} \neq \emptyset$ $\forall j > i$?

$i, \; j = i+1$



No

Thm:- Consider a nonempty subproblem S_{ij}. and let a_m be the activity in S_{ij} with earliest finish time. Then

① Activity a_m is in some optimal sol^n.

② $S_{im} = \emptyset$. So choosing a_m leaves subproblem S_{mj} as the only subproblem that may be nonempty.

Proof:- Let Opt be an optimal solⁿ to S_{ij}.

If $a_m \in Opt$ done.

Suppose $a_m \notin Opt$.

Let $a_r \in Opt$ be the activity with the earliest finish time (in Opt).

$f_m \leq f_r$

$$N = (\underbrace{Opt \setminus \{a_r\}}_{\text{remove}}) \cup \underbrace{\{a_m\}}_{\text{add}}$$

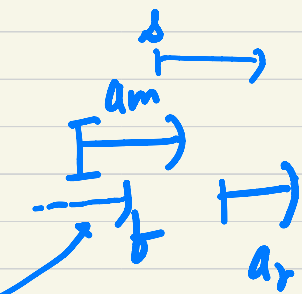
We want to prove that N is an opt sol^n.

$|N| = |Opt|$.

Enough to prove that $a_m \cap a = \emptyset \ \forall \ a \in N \setminus \{a_m\}$

$\times$

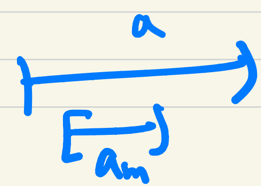
$[s, t)$



Then $s_m < t < t_m$ or $s_m < s < t_m$.

$\times$

or $a_m \subseteq a$



$t_r \le t$

$t_r < s$

$t_m \le t_r < s$

a

a^*

a

a_r

a_m

② $S_{im} = \emptyset$

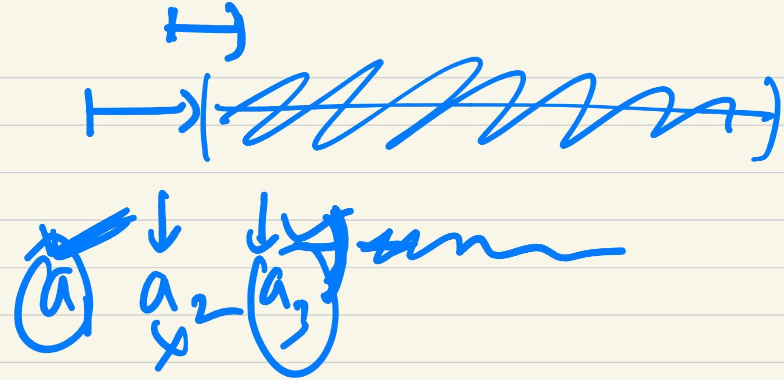
s_i

$a_k \to t_i \le s_k < t_k \le s_m$

t_j

$Sol \leftarrow \{a_1\}$

$\underbrace{S_{1(n+1)}}$ =



$f_1 \not\leq s_2$

$Sol \leftarrow \{a_1\}$

$i \leftarrow 1$

for $j \leftarrow 2$ **to** n

  do **if** $s_j \geq f_i$

    do $A \leftarrow A \cup \{a_j\}$

$i \leftarrow j$

$O(n)$ Runtime.