PHYS 2310H, Assignment 5

Due Thursday, March 19.

1. In this question, you are asked to make a bifurcation diagram, like the one in Fig. 3.11 of G & N.

   (a) You will need to re-read the last part of Sec. 3.3 on Poincaré sections in order to complete this problem. The challenge for this problem is that you want to keep values of the array $\theta(t)$ at regularly spaced time intervals T_D *and* you want to exclude the first part of the array where transients are contributing to the motion. Imagine you have generated two arrays: `theta[:]` and `t[:]`, which hold the angle θ and time t. Explain what the following line of code does and how you would use it:

   ```
   strobe = np.logical_and(np.abs(np.remainder(t, TD)) < dt/2, t>10*TD)
   ```

   In particular

   i. What are the type, size, and shape of `strobe`?

   ii. What does the line `(np.abs(np.remainder(t, TD)) < dt/2` do? Why `dt/2` and not `dt`?

   iii. What is the point of `t>10*TD`?

   iv. How would you use this to make a plot like Fig. 3.11 in G & N?

   (b) For this problem, I want you to have exposure to a new algorithm beyond the Euler-Cromer method. Write your code so that it uses the Verlet algorithm to calculate `theta[:]`. Be careful how you handle the first couple of time steps.

   (c) Now write a code that allows you to generate Fig. 3.11 yourself. Be careful in the region between $F_D = 1.45$ and $F_D = 1.5$! Note that because the Verlet method is more accurate than the Euler-Cromer method, the bifurcations will appear in slightly different places.

   (d) Where, if anywhere, do you believe the motion of the pendulum is chaotic? This requires care to answer, and you need to justify your answer. to do this well, you probably need fairly long simulation times and you need to be sure to eliminate transients.