PHYS 2310H: Computational Physics
Assignment 2
Due: Tuesday, Feb. 11

1. This question is based on the bike-racing lab from Week 04. Generalize the bike-racing model to include the possibility that the bike is travelling up a hill. Do this in steps:

   - Draw a free-body diagram showing all the forces on the bicycle, and obtain the component of the gravitational force that is parallel to the motion.
   - Write out the modified equation of motion.
   - Then obtain the modified equation for `v[i+1]` using the Euler method.

   Your final answer should consist of a single figure showing $v(t)$ for both a level surface, and a 10% grade (i.e. $\tan\theta = 0.1$, where θ is the angle the slope makes with the horizontal.) The caption must include a discussion of the two curves. What are the key points you want the reader to take away from your results? If you have time, you might also consider the terminal speed as a check of your code. Note that in this case, you'll need to solve a cubic equation, which can be done using `numpy.roots()`.

2. This question is based on G & N Question 2.11. The goal is to make a single figure and a table that shows the sensitivity of the final position of the shell to the initial conditions.

   (a) To get an accurate estimate of the landing position, this problem requires you to interpolate between two data points to find where the height of the shell is zero. This requires using Eq. (2.11) in G & N. Derive it.

   (b) It is important that your code be accurate. Make a table showing, for a single set of parameters, how the final landing position depends on `dt`. Based on this table, what do you consider a reasonable value of `dt` and why?

   (c) Because you will be calculating the range for multiple sets of parameter values, you need to define a function that calculates the trajectory, and call it for each set of parameters. The rule of thumb is that, if you are going to perform a calculation more than once, you should put it in a function.

   (d) Run your code including both air resistance and the adiabatic model for air density, Eq. (2.24). How sensitive is the final position of the shell to a 1% change in the initial speed of the cannon shell (both increased and decreased)? Make a single figure showing your results and make a table showing the ranges you obtain. Remember the guidelines for making figures that I gave you in Lab 01.