3a) $E = \frac{mL^2\omega^2}{2} + mgL(1-\cos\theta)$

$\omega_{i+1} = \omega_i - \frac{g\theta_i \Delta t}{L}$

$\theta_{i+1} = \theta_i + \omega_i \Delta t$

Assuming low θ: $\cos\theta \cong 1$

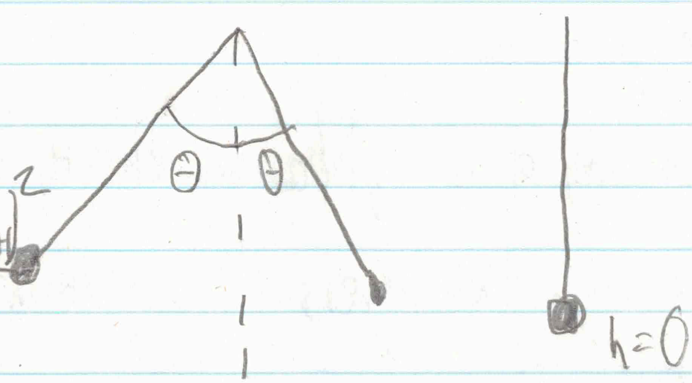
$E = \frac{mL^2\omega^2}{2}$

$\downarrow$

$E_{i+1} = \frac{mL^2(\omega_{i+1})^2}{2}$

$\downarrow$

$= mL^2\left(\omega_i - \frac{g\theta_i\Delta t}{L}\right)^2$



Crt:

$\Omega^2 = \frac{q^2}{4}$

$q = 2\Omega$

In essence, the rightmost term in Eq. 3.8 can never be 0. This means energy is always added to the system. or negative.

$mL^2\left(\omega_i^2 - \frac{2g\theta_i\omega_i\Delta t}{L} + \left(\frac{g\theta_i\Delta t}{L}\right)^2\right)$

(36) This question was tackled assuming gravity, the pendulum mass, and the pendulum length would not change. They are global variables, set to 9.81 m/s^2, 5.0 kg, and 2.5 m respectively.

(In the program, g is a global variable as well, to avoid having to pass it through two layers of arguments. This will change in Question 4.)

I'm also assuming a start from rest, to keep the exact solution simple. The pendulums will also start at one radian, to avoid having to round π.

Update: g has been made an argument of the EC and Fnet functions.

3d $\theta(t) = \theta_0 e^{\frac{-qt}{2}} \sin(\alpha t + \phi) \qquad \alpha = \sqrt{\frac{g}{L} - \frac{q^2}{4}}$

$$\omega(t) = \theta_0 \left(\frac{-q}{2}\right) e^{\frac{-qt}{2}} \sin(\alpha t + \phi) + \theta_0 e^{\frac{-qt}{2}} (\alpha) \cos(\alpha t + \phi)$$

$$\omega(t) = \theta_0 e^{\frac{-qt}{2}} \left(\frac{-q}{2} \sin(\alpha t + \phi) + \alpha \cos(\alpha t + \phi)\right)$$

$\downarrow$ start from rest

$$\omega(0) = 0 = \theta_0 \left(\frac{-q}{2} \sin(\alpha t + \phi) + \alpha \cos(\alpha t + \phi)\right)$$

$\downarrow \theta_0 \neq 0$

$$0 = \frac{-q}{2} \sin(\phi) + \alpha \cos\phi$$

$$\frac{q}{2} \sin\phi = \alpha \cos\phi$$

$$\tan\phi = \frac{2\alpha}{q}$$

$\theta(t) = (\theta_0 + Ct) e^{\frac{-qt}{2}} \qquad \theta(0) = \theta_0$

$$\omega(t) = C e^{\frac{-qt}{2}} + (\theta_0 + Ct)\left(\frac{-q}{2}\right)\left(e^{\frac{-qt}{2}}\right)$$

$$\omega(0) = C + (\theta_0)\left(\frac{-q}{2}\right)$$

$$0 = C + -\frac{q\theta_0}{2}$$

$$C = \frac{q\theta_0}{2}$$

3e) 03/18/20: As of right now, the error is not dependent on dt. Until larger sources of error are found it shall remain this way.
03/22/20: Thanks to the prof's help I've fixed the error, and it's neglegible now.

3f) $F_D = A\sin(\Omega_D t)$ Note that this is actually for 3d

$$\Theta(t) = \Theta_1 e^{-\left(\frac{q}{2}+\sqrt{\frac{q^2}{4}-\Omega^2}\right)t} + \Theta_2 e^{-\left(\frac{q}{2}-\sqrt{\frac{q^2}{4}-\Omega^2}\right)t}$$

$$\omega(t) = \Theta_1(-\alpha^+)e^{-\alpha^+ t} + \Theta_2(-\alpha^-)e^{-\alpha^- t}$$

$$= \Theta_2\left(\sqrt{\frac{q^2}{4}-\Omega^2}-\frac{q}{2}\right)e^{-\alpha^- t} - \Theta_1 \alpha^+ e^{-\alpha^+ t}$$

$$\Theta(0) = \Theta_1 + \Theta_2$$

$$\omega(0) = \Theta_2\left(\sqrt{\frac{q^2}{4}-\Omega^2}-\frac{q}{2}\right) - \Theta_1\left(\frac{q}{2}+\sqrt{\frac{q^2}{4}-\Omega^2}\right)$$

$$\Theta_1\left(\frac{q}{2}+\sqrt{\frac{q^2}{4}-\Omega^2}\right) = \Theta_2\left(\sqrt{\frac{q^2}{4}-\Omega^2}-\frac{q}{2}\right)$$

$$\Theta_1 = \underbrace{\frac{\left(\sqrt{\frac{q^2}{4}-\Omega^2}-\frac{q}{2}\right)}{\left(\frac{q}{2}+\sqrt{\frac{q^2}{4}-\Omega^2}\right)}}_{\beta}\Theta_2 \qquad \Theta(0) = (\beta+1)\Theta_2$$

3f $F_D = A\sin(\Omega_D t)$

Googling also nets the result

$$\Theta(t) = A_h e^{-\gamma t}\sin(\omega' t + \varphi_h) + A\cos(\omega t - \varphi)$$

$$\therefore \omega(t) = A_h(-\gamma)e^{-\gamma t}\sin(\omega' t + \varphi_h) + A_h e^{-\gamma t}\omega'\cos(\omega' t + \varphi_h) - A_h \sin(\omega t - \varphi)$$

where:

$$A_h = \frac{x_0 - A\cos\varphi}{\sin\varphi_h} \qquad \gamma = \frac{c}{2m} \qquad \omega' = \sqrt{\omega_0^2 - \gamma^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\varphi_h = \arctan\left(\frac{\omega'(x_0 - A\cos\varphi)}{v_0 + \gamma(x_0 - A\cos\varphi) - A\omega\sin\varphi}\right)$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \qquad \varphi = \arctan\left(\frac{c\omega}{k - m\omega^2}\right) - \varphi d$$

c is the damping coefficient

However

if the force is instead $F_0\cos(\Omega_D t)$

the solution is $x(t) = X_0\cos(\omega t + \phi)$

where $X_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + (\omega\gamma)^2}}$, $\phi = \arctan\left(\frac{\gamma\omega}{\omega^2 - \omega_0^2}\right)$

$\omega_0 = \frac{k}{m}$ $\gamma = q$

$$\Theta = \Theta_D + \Theta_H$$

$$= A_D \sin(\Omega_D t) + A_H e^{-\frac{qt}{2}} \sin(\alpha t + \phi)$$

$$\omega = A_D \cos(\Omega_D t) + A_H \left(\frac{-q}{2}\right) e^{-\frac{qt}{2}} \sin(\alpha t + \phi)$$

$$+ A_H (\alpha) e^{-\frac{qt}{2}} \cos(\alpha t + \phi)$$

$$\Theta_{(0)} = A_H \sin \phi$$

$$\omega_{(0)} = A_D + A_H \left(\frac{-q}{2}\right) \sin \phi + A_H (\alpha) \cos \phi$$

$$= A_D + \Theta_{(0)} \left(\frac{-q}{2}\right) + A_H \alpha \cos \phi$$

4a $\Delta\theta \cong e^{\lambda t}$

$$\frac{\ln\left(\frac{\Delta\theta}{c}\right)}{} = \lambda t$$

$$\frac{\ln\left(\frac{\Delta\theta}{c}\right)}{t} = \lambda$$

Assuming $C = 1$

$$\frac{\ln(\Delta\theta)}{t} = \lambda$$