Understanding Teleparallel Gravity

Your Name

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Introduction to Teleparallelism

Teleparallel Gravity is an alternative formulation of gravity where the curvature of spacetime is replaced by torsion. In this framework:

- Gravity is encoded in torsion rather than curvature.
- We work within flat Minkowski spacetime.
- This simplifies the mathematical treatment by using a torsion-based description of gravity.

Levi-Civita Connection

The traditional Levi-Civita connection in General Relativity is given by:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left(\frac{\partial g_{\sigma\mu}}{\partial x^{\nu}} + \frac{\partial g_{\sigma\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} \right)$$

where:

- $g_{\mu\nu}$ is the metric tensor.
- $g^{\lambda\sigma}$ is the inverse metric tensor.
- $\frac{\partial g_{\sigma\mu}}{\partial x^{\nu}}$ are the partial derivatives of the metric components.

Weitzenböck Connection

In Teleparallel Gravity, we use the Weitzenböck connection, which involves torsion and is defined as:

$$\overset{\star}{\Gamma}{}^{\lambda}_{\mu\nu} = \frac{1}{2}\left(\overset{\star}{T}{}^{\lambda}_{\mu\nu} + \overset{\star}{T}{}^{\lambda}_{\nu\mu} - \overset{\star}{T}{}_{\mu\nu}{}^{\lambda}\right)$$

where $\overset{\star}{T}{}^{\lambda}_{\mu\nu}$ represents the torsion tensor components.

Connecting Levi-Civita and Weitzenböck Connections

The Levi-Civita connection ($\Gamma^{\lambda}_{\mu\nu}$) involves curvature and is used in General Relativity. In contrast, the Weitzenböck connection ($\overset{\star}{\Gamma}{}^{\lambda}_{\mu\nu}$) is used in Teleparallel Gravity and involves torsion.
Specifically, the contortion tensor, which relates these connections, is given by:

$$K^{\rho}_{\mu\nu} = \frac{1}{2}\left(T^{\rho}_{\nu\mu} + T^{\rho}_{\mu\nu} - T^{\rho}_{\mu\nu}\right)$$

where:

- $K^{\rho}_{\mu\nu}$ is the contortion tensor.
- $T^{\rho}_{\mu\nu}$ represents the components of the torsion tensor.

This formula is cited from Page 9 of Aldrovandi and Pereira's book [1].

Physical Interpretation

- In General Relativity, the Christoffel symbols describe how vectors change in curved spacetime.
- In Teleparallel Gravity, the Weitzenböck connection in flat spacetime describes gravitational effects using torsion.
- This formulation simplifies the equations and helps to visualize gravity as a property of flat spacetime with torsion.

Stereographic Projection

Stereographic projection can be used to illustrate the effects of torsion:

- This projection maps a spherical surface to a plane, helping to demonstrate the difference between curvature and torsion.
- Geodesic and non-geodesic paths on the sphere can represent the effects of torsion, illustrating how torsion affects trajectories differently compared to curvature.

R. Aldrovandi and J. G. Pereira, *An Introduction to Teleparallel Gravity*, pg. 9.