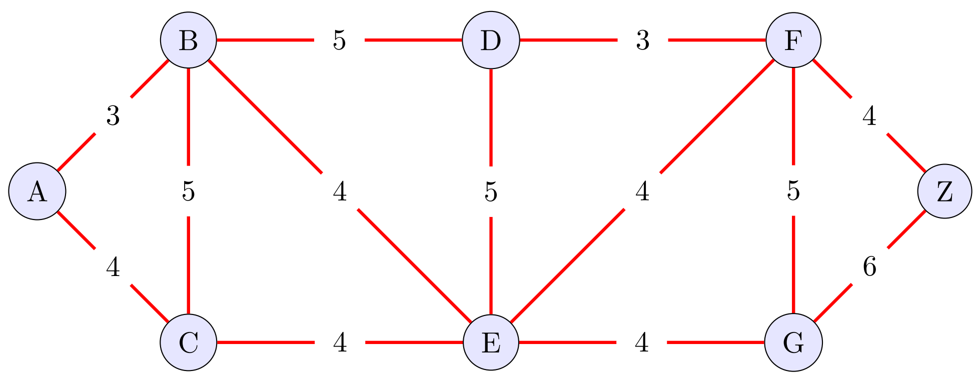
Data Sheet

do not submit

Graph K



Union by Rank

The current state S of the `union by rank` representation of disjoint subsets of the set of vertices {A, B, C, D, E, F, G, H} is given by

pi

A	**B**	**D**	**D**	**A**	**F**	**D**	**A**	**F**
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**

rank

1	**0**	**0**	**1**	**0**	**1**	**0**	**0**	**0**
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**

Sequence of edges: `union(B, C)` `union(F, G)` `union(A, H)` `union(A, F)`

CIS 3223 HW 7

Dr Anthony Hughes

Name:

Temple ID (last 4 digits:

1 (20 pts) For the graph K construct a hash table, then sort the edges in each bucket using alphabetical ordering.

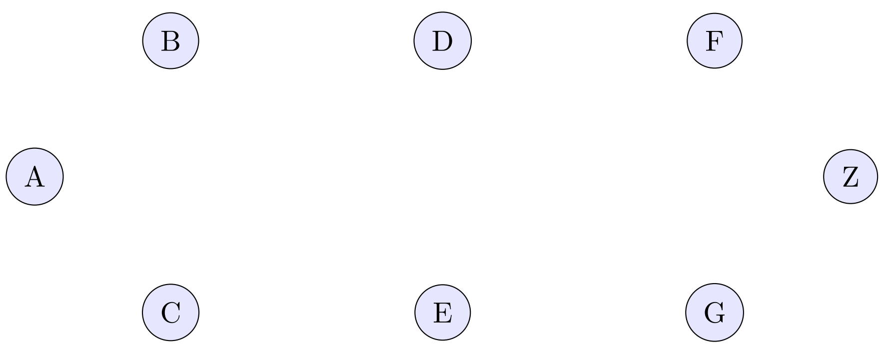
Edge List (Hash Table)

3	AB				

Sorted Edge List (Hash Table)

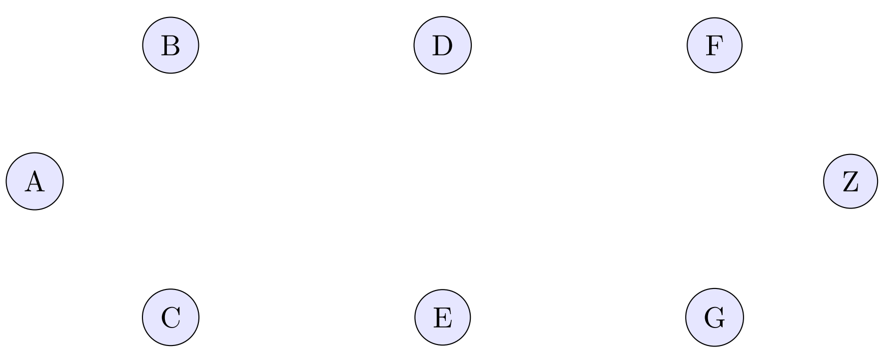
3					

(a) Construct a minimum cost spanning tree by using the edges in each bucket moving from left to right starting with bucket with the lowest value:



Minimum Cost

(b) Construct a maximum cost spanning tree by using the edges in each bucket moving from left to right starting with bucket with the highest value:



Maximum Cost

2 (15 pts) (a) For the state S, draw the corresponding trees representing the sets.

(b) Consider the following **SEQUENCE** of operations. Draw the corresponding trees representing the sets after each of the operations (use alphabetical order.

Specify the current state of `pi` and `rank` after the sequence has been executed.

`union(B, C)`: (B, C) $\xRightarrow{\text{pi}}$ (B, D) $\xRightarrow{\text{rank}}$ (0, 1) Set pi(B) = D

`union(F, G)`

`pi`

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**

`rank`

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**

3 Use induction to show that if a subtree T constructed in the rank by union procedure has rank m, then T contains at least 2^m nodes.

First verify this result using the trees drawn after the specified sequence specified in the previous question has been completed.

`rank`									
2^{rank}									
`|nodes|`									
`Verified`									
`root`	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**

Base case: $m = 0$:

T has one node

$2^k = 2^0 = 1$

So true in this case.

Inductive case: Assume true for for $m = k$. Show true for $m = k + 1$

[Hint: Trace back to when the rank of a subtree is increased?]

4 (a) Sort the following edge list of the undirected graph G given in s-t-w format (scan from left to right and place edges in buckets, then sort buckets alphabetically).

$$
\begin{array}{lcl}
s & = & [\,B\ \ A\ \ D\ \ B\ \ A\ \ C\ \ D\ \ B\ \ E\ \ E\ \ F\ \ F\ \ G\,] \\
t & = & [\,E\ \ C\ \ F\ \ D\ \ B\ \ E\ \ E\ \ C\ \ F\ \ G\ \ G\ \ H\ \ H\,] \\
w & = & [\,4\ \ 3\ \ 5\ \ 3\ \ 4\ \ 4\ \ 5\ \ 3\ \ 4\ \ 5\ \ 3\ \ 4\ \ 5\,]
\end{array}
$$

Edge List (Hash Table)

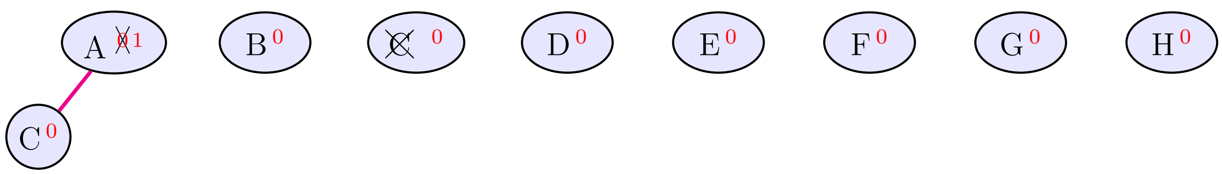
3	AC					
4	BE					
5						

Buckets sorted alphabetically

3	AC					
4						
5						

(b) Implement the union by rank algorithm to process the edges of the sorted list.

[Delete a node and join to parent – redraw when joining two roots with non-zero rank]



pi

A	**B**	**C**	**D**	**E**	**F**	**H**	**G**

rank

A	**B**	**C**	**D**	**E**	**F**	**H**	**G**

Which edges were discarded? ______________________________