

CIS 3223 HW 2

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Name:**Temple ID (last 4 digits:**

You may use a simple non-graphing calculator (make): _____

1 Complete the following table by writing 1 or 0 in each box, where 1 represents “true ” and 0 represents “false ”. No justification required. For each row, enter the number represented (1010 gives 10).

f	g	$f = O(g)$	$f = \Omega(g)$	$f = o(g)$	$g = o(f)$	number
\sqrt{n}	$\log(1 + n^2)$					
$(\log n)^{10}$	$n^{0.1}$					
3^{n+2}	5^n					
$2n + \log n$	$n + (\log n)^2$					
2^n	$5^{n/2}$					
n	$\sum_{k=1}^n \log k$					

What is the most dominant function in the table?

Give as good big- θ estimate for each of the following functions.

(a) $f(n) = (n^4 + 2^n)(n^3 + \log(n^4 + 1))$

(b) $f(n) = (4n^2 \log(2^n + 1) + n^4)(n! + n2^n)$

2 Give a big- θ bound for the solutions of the following recurrence relations.

(a) $T(n) = 8T(n/3) + 1000n^2$

(b) $T(n) = 5T(n/4) + 7n$

(c) $T(n) = 2T(n/2) + \sqrt{n}$

(d) $T(n) = T(n-1) + n^2$

3 Use strong induction to prove the following: $F_n \leq 2^{0.7n}$, $n \geq 1$

Base cases: Show true for $n = 1$ and 2 :

$n = 1$:

$$\text{lhs} = F_1 = 1$$

$$\text{rhs} = 2^{0.7} = 1.6245$$

So $\text{lhs} \leq \text{rhs}$. True for $n = 1$

$n = 2$:

$$\text{lhs} = F_2 = 1$$

$$\text{rhs} = 2^{1.4} = 2.6390$$

So $\text{lhs} \leq \text{rhs}$. True for $n = 2$

Inductive case: Assume true for $n = s$, $1 \leq s \leq k$, $k \geq 2$

Show true for $n = k + 1$:

lhs =

(Bonus 2 pts) Can you improve on 0.7 using 2 decimal precision?

$$F_n \leq 2^{n} \quad n \geq \underline{\hspace{1cm}}$$