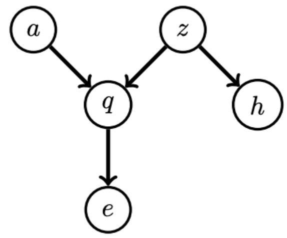
COMP64101 - Lecture 3 - Exercise sheet

Exercise 1 **[Directed graph concepts]** For this exercise, consider the directed graph shown below.



(a) List all trails in the graph (of maximal length).

(b) List all directed paths in the graph (of maximal length).

(c) What are the descendants of z?

(d) What are the non-descendants of q?

(e) Which of the following orderings are topological with respect to the graph?

- (a, z, h, q, e)
- (a, z, e, h, q)
- (z, a, q, h, e)
- (z, q, e, a, h)

Exercise 2 **[Ordered and local Markov properties, d-separation]** For this exercise, we continue considering the directed graph shown in Exercise 1.

(a) The ordering (z, h, a, q, e) is topological to the graph. What are the independencies that follow from the ordered Markov property?

(b) What are the independencies that follow from the local Markov property?

(c) The independency relations obtained via the ordered and local Markov property include $q \perp\!\!\!\perp h \mid \{a, z\}$. Verify the independency using d-separation.

(d) Use d-separation to check whether $a \perp\!\!\!\perp h \mid e$ holds.

(e) Assume all variables in the graph are binary. How many numbers do you need to specify, or learn from data, in order to fully specify the probability distribution?

Exercise 3 **[More on ordered and local Markov properties, d-separation]** For this exercise, we continue considering the directed graph shown in Exercise 1.

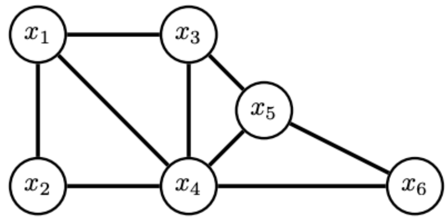
(a) Why can the ordered or local Markov property not be used to check whether $a \perp\!\!\!\perp h \mid e$ may hold?

(b) The independency relations obtained via the ordered and local Markov property include $a \perp\!\!\!\perp \{z, h\}$. Verify the independency using d-separation.

(c) Determine the Markov blanket of z.

(d) Verify that $q \perp\!\!\!\perp h \mid \{a, z\}$ holds by manipulating the probability distribution induced by the graph.

Exercise 4 **[Factorisation and independencies for undirected graphical models]** Consider the undirected graphical model defined by the following graph.



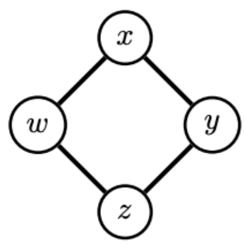
(a) What is the form of Gibbs distributions induced by the graph?

(b) Let p be a pdf that factorises according to the graph. Does $p(x_3|x_2, x_4) = p(x_3|x_4)$ hold?

(c) Explain why $x_2 \perp\!\!\!\perp x_5 \mid \{x_1, x_3, x_4, x_6\}$ holds for all distributions that factorise over the graph.

(d) Assume you would like to approximate $\mathbb{E}(x_1x_2x_5|x_3, x_4)$ with a sample average. Do you need to have joint observations for all five variables $x_1, \ldots, x_5$? Explain.

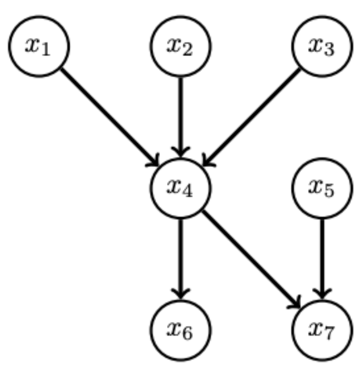
Exercise 5 **[Factorisation and independencies for undirected graphical models]** Consider the undirected graphical model defined by the following graph.



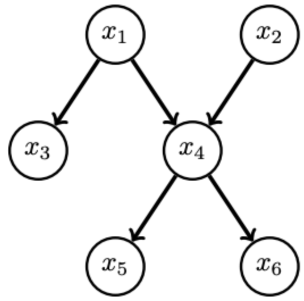
(a) How do the pdfs/pmfs corresponding to this undirected graphical model factorise?

(b) List all independencies that hold for this undirected graphical model.

Exercise 6 **[Moralisation]** For the DAG below, call it G, find the minimal undirected I-map for $I(G)$.



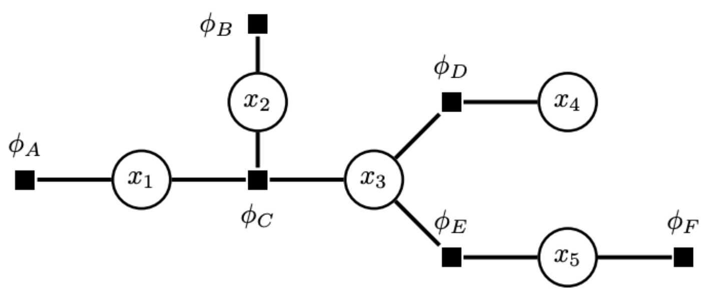
Exercise 7 **[Conversion to factor graphs for polytrees]** Consider the directed graph shown below. (This kind of graph is called a polytree: there are no loops but a node may have more than one parent.)



(a) Draw the moralised graph for this directed graph.

(b) Draw an undirected factor graph for this directed graph.

Exercise 8 **[Message passing]** For this exercise, consider the following factor tree:



(a) Mark the graph with arrows indicating all messages that need to be computed for the computation of $p(x_1)$ via sum-product message passing.

(b) Mark the graph with arrows indicating all messages that need to be computed for the computation of $\arg\max_{x_1,\ldots,x_5} p(x_1, \ldots, x_5)$ via max-sum message passing with x_1 as root.

(c) Mark the graph with arrows indicating all messages that need to be computed for the computation of $\arg\max_{x_1,\ldots,x_5} p(x_1, \ldots, x_5)$ via max-sum message passing with x_3 as root.