Matte standardprosjekt

1)

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$f(x) = e^x \rightarrow f'(1,5) = 4,48168907$

$h = 0,01$:

$$f'(1,5) \approx \frac{e^{1,51} - e^{1,5}}{0,01} = 4,50417$$

ca. $2 \cdot 10^{-2}$ unna faktisk svar.

$h = 0,001$:

$$f'(1,5) \approx \frac{e^{1,501} - e^{1,5}}{0,001} = 4,48393$$

ca. $2 \cdot 10^{-3}$ unna svar

$h = 0,0001$

$$f'(1,5) \approx \frac{e^{1,5001} - e^{1,5}}{0,0001} = 4,4819$$

ca. $2 \cdot 10^{-4}$ unna svaret

$h = 0,00001$:

$$f'(1,5) \approx \frac{e^{1,50001} - e^{1,5}}{0,00001} = 4,481711$$

ca. $2 \cdot 10^{-5}$ unna svaret

$h = 1 \cdot 10^{-6}$

$$f'(1,5) \approx \frac{e^{1,5+1\cdot 10^{-6}} - e^{1,5}}{1 \cdot 10^{-6}} = 4,48169131$$

dersom vi går videre og setter $h = 1 \cdot 10^{-8}$ og mindre, begynner datamaskinen å avrunde, og blir mer og mer feil. Når vi fått hit $1 \cdot 10^{-12}$ vil svaret bli helt feil

2)

$$f'(x) = \frac{f(x+h) - f(x-h)}{2 \cdot h}$$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2}h^2 + \frac{f'''(x)h^3}{6} + O(h^4)$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2}h^2 - \frac{f'''(x)}{6}h^3 + O(h^4)$$

Setter inn i formel

$$\frac{f(x) + f'(x)h + \frac{f''(x)}{2}h^2 + \frac{f'''(x)h^3}{6} + O(h^4) - \left(f(x) - f'(x)h + \frac{f''(x)}{2}h^2 - \frac{f'''(x)}{6}h^3 + O(h^4)\right)}{2 \cdot h}$$

$$= \frac{2f'(x)h + 2h^3 \frac{f'''(x)}{6}}{2h} = \underbrace{h^2 \frac{f'''(x)}{6}}_{\text{feil}} + f'(x)$$

Altså er feilen her proporsjonal med h^2

3)

Gjort i Python

4)

Eksplisitt:

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Vi vet at $u_{i,0} = \sin(x)$

Løser for $u_{i,j+1}$:

$$u_{i,j+1} = \frac{(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) \cdot k}{h^2} + u_{i,j}$$

der $\frac{k}{h^2} = r$

5)

Implisitt:

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2}$$

$$u_{i,j+1} \cdot h^2 - u_{i,j}h^2 = u_{i+1,j+1}k - 2u_{i,j+1} \cdot k + u_{i-1,j+1} \cdot k$$

$$h^2 u_{i,j+1} + 2k\,u_{i,j+1} = u_{i+1,j+1}k + u_{i-1,j+1} \cdot k + u_{i,j}h^2$$

$$u_{i,j+1} = \frac{u_{i+1,j+1}k + u_{i-1,j+1} \cdot k + u_{i,j}h^2}{h^2 + 2k}$$

6)