Portfolio Optimization Using Monte Carlo Simulations

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Abstract

This project investigates the construction of an optimal portfolio using modern portfolio theory (MPT) principles. The aim is to identify the optimal combination of assets that maximizes the Sharpe ratio by focusing on the generation and evaluation of all possible asset subsets to identify the configuration with the highest Sharpe ratio. The project implements a backtesting framework with quarterly rebalancing to simulate realistic investment conditions.

1. Introduction

Portfolio optimization aims to allocate capital among assets to maximize expected return for a given level of risk. Traditional mean–variance optimization (Markowitz, 1952) assumes known covariances and normally distributed returns. However, real-world markets exhibit non-normalities, fat tails, and changing correlations.

Monte Carlo simulations relax these assumptions by exploring the full weight space through random sampling, enabling discovery of robust high-performing portfolios under realistic conditions.

2. Theory

The central goal of portfolio optimization is to find weight vector $\mathbf{w}$ that maximizes the Sharpe ratio:

$$\max_{\mathbf{w}} \; SR = \frac{E[R_p - R_f]}{\sigma_p}$$

subject to:

$$\sum_{i=1}^{N} w_i = 1, \quad w_i \geq 0$$

where $R_p = \mathbf{w}^\top \mathbf{r}$ is the portfolio return, R_f the risk-free rate, and $\sigma_p = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}$ the portfolio volatility.

Monte Carlo simulation samples thousands of weight combinations $\mathbf{w}$ and evaluates each portfolio's annualized return, volatility, and Sharpe ratio empirically using historical returns rather than assuming analytical distributions.

3. Data

Historical daily closing prices were collected for five ETFs via Yahoo Finance (2001–2025):

- SPY — S&P 500 ETF
- QQQ — NASDAQ-100 ETF
- VTI — Total US Stock Market ETF
- EFA — MSCI EAFE ETF (developed international equities)
- TLT — Long-term Treasury ETF
- SHY — Short-term Treasury ETF
- TIP — Treasury Inflation-Protected Securities ETF

Daily returns are computed as:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where P_t is the closing price on day t. The risk-free rate is assumed at 4% annually.

4. Methodology

4.1. Monte Carlo Simulation

For each possible subset of ETFs, random weight vectors are generated:

$$w_i = \frac{x_i}{\sum_j x_j}, \quad x_i \sim U(0,1)$$

Each portfolio undergoes quarterly rebalancing, simulating practical management over 20+ years. For each simulated portfolio, we compute:

$$R_{ann} = (1 + R_{tot})^{252/T} - 1 \tag{1}$$

$$\sigma_{ann} = \text{Std}(\Delta \ln V_t)\sqrt{252} \tag{2}$$

$$SR = \frac{R_{ann} - R_f}{\sigma_{ann}} \tag{3}$$

4.2. Rolling-Window Optimization

To reduce overfitting, a rolling-window Monte Carlo approach is used:

- Training window: 10 years (2520 days)
- Step size: 1 year (252 days)
- Best Sharpe portfolio from each window is applied out-of-sample

Median weights across windows are used to form a stable, time-robust "median-weight" portfolio.

5. Results and Discussion

5.1. Optimal Quarterly-Rebalanced Portfolio

The Monte Carlo simulation identifies the portfolio with the highest Sharpe ratio over the full sample period. The optimal weights are:

Asset	Weight (%)
QQQ	63.05
SHY	36.95

Table 1: Optimal weights (quarterly rebalanced).

Performance summary:

$$R_{ann} = 11.68\%, \quad \sigma_{ann} = 13.24\%, \quad SR = 0.580$$

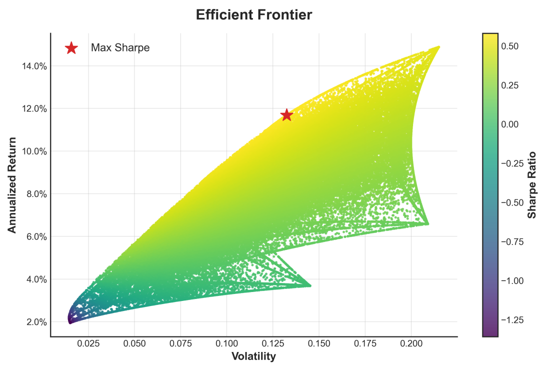

Figure 1: Efficient frontier of simulated portfolios. Star marks maximum Sharpe ratio.

5.2. Rolling Median Portfolio

To evaluate robustness over time, we compute the median weights across all rolling windows. Table 2 shows the resulting median weights.

Applying these median weights with quarterly rebalancing produces a stable portfolio whose performance can be compared to SPY, as summarized in Table 3.

Asset	Median Weight (%)
QQQ	53.18
TLT	46.82

Table 2: Median portfolio weights across all windows.

Portfolio	Ann. Return	Ann. Volatility	Sharpe
Median Portfolio	10.76%	11.86%	0.57
SPY	11.11%	19.38%	0.37

Table 3: Performance metrics of the quarterly-rebalanced median portfolio versus SPY (Oct 2005 – Oct 2025).

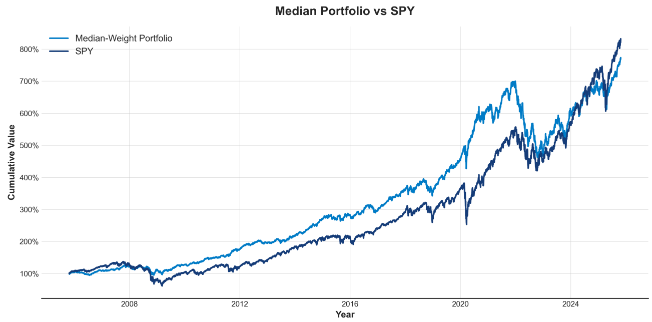

Figure 2: Cumulative growth: median portfolio vs SPY.

5.3. Discussion

- Monte Carlo identifies diversified, non-intuitive allocations, e.g., the 53% QQQ / 47% TLT median portfolio.
- Quarterly rebalancing lowers volatility: 11.86% for the median portfolio vs 19.38% for SPY.
- Rolling-window results show stability over time, reducing regime-specific bias.
- The median portfolio achieves a higher Sharpe ratio (0.57) than SPY (0.37), reflecting better risk-adjusted performance.

6. Limitations

- Monte Carlo relies on historical returns, which may not reflect future conditions.
- The risk-free rate is static, though real rates vary over time.
- Transaction costs are excluded, potentially overstating returns.
- Enumerating all subsets grows exponentially with the number of assets.
- Maximum drawdown can exceed SPY despite lower volatility, due to concentration and timing of losses (see Figure 2, 2022).

7. Conclusion

This project demonstrates the practical application of quantitative portfolio optimization, providing a full pipeline from data processing to performance visualization. Quarterly rebalancing combined with rolling-window median weighting produces stable, high Sharpe portfolios that outperform SPY over the 20-year horizon. Future enhancements include dynamic risk-free rates, transaction cost modeling, and machine learning extensions for predictive weight selection.

8. References

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