1. In propositional logic, we say that a formula ϕ_2 can be inferred from a formula ϕ_1 if for every assignment that makes ϕ_2 true also makes ϕ_1 true. The number of semantically distinct propositional logic formulae (including ψ itself) over the three variables p, q, r, that can be inferred asssuming the formula $\psi = (p \Rightarrow (q \Rightarrow r))$ is:

   (a) 0

   (b) 1

   (c) 2^7 (correct answer)

   (d) 7

   The given formula ψ has seven satisfying assignments, as can be easily verified. Thus a formula that can be inferred from it must evaluate to true whenever ψ evaluates to true, and can evaluate to any value when ψ is false. Thus it can take any truth value for the only assignment that makes ψ false, and thus there are only two such formulae (ψ itself and the tautology, which is true on all assignments). Thus the correct answer as provided by the course instructor to the exam platform is wrong. The evaluation will be revised accordingly.

2. Let ψ be a formula over variables p_1, p_2, p_3, p_4 and let $\phi = (p_1 \Rightarrow \psi)$. The number of satisfying assignments of this formula ϕ cannot be:

   (a) 6 (correct answer)

   (b) 8

   (c) 10

   (d) 12

   An implication is always true when the premise is false. Here the premise is the propositional variable which is false on 8 of the 16 assignements. Thus the number of assignments that make ϕ true cannot be less than 8. The only number less than 8 provided in the options is 6, corresponding to option (a).

3. Let ψ be a formula over variables p_1, p_2, p_3, p_4 and let $\phi = (p_1 \vee \psi)$. The number of satisfying assignments of this formula ϕ cannot be:

   (a) 6 (correct answer)

   (b) 8

   (c) 10

   (d) 12

   A disjunction is always true when at least one of the disjuncts is true. Here we have a formula where the highest level operator is an or (disjunction) and one of the disjuncts is the propositional variable p_1. Thus the 8 assignments where it is set to true make the formula true so the number of satisfying assignments cannot be less than 8. The only such option provided is 6 and is option (a).

4. Let ψ be a formula over variables p_1, p_2, p_3, p_4 and let $\phi = (p_1 \wedge \psi)$. The number of satisfying assignments of this formula ϕ can be:

   (a) 6 (correct answer)

   (b) 10

   (c) 12

   (d) 14

   A formula which is a conjunction is false when at least one of the conjuncts is false. Here we have a formula where the highest level operator is and (conjunction) and one of the two conjuncts is the propositional variables p_1. Thus the 8 assignments where it is set to false make the formula false. Thus the number of satisfying assignments is at most 8. The only such option provided is 6 which corresponds to option (a).

5. Let ϕ be a propositional logic formula on 6 variables $p_1, \ldots, p_6$. Suppose ϕ is such that every time we change an assignment by altering the truth of just one propositional variable, the truth value of the formula changes. Given this information, which among the below assignment of truth values makes ϕ evaluate to a different value from the other three assignments?

p_1	p_2	p_3	p_4	p_5	p_6
0	0	1	1	1	1

(a)

p_1	p_2	p_3	p_4	p_5	p_6
1	1	1	1	1	1

(b)

p_1	p_2	p_3	p_4	p_5	p_6
0	0	1	0	0	1

(c)

p_1	p_2	p_3	p_4	p_5	p_6
0	1	1	0	0	1

(d) (correct answer)

The correct answer is obtained by seeing which of them has a different parity (odd-even) number of true/1 in comparison with the other three. The correct option is (d) which has three variables set to true,. Options (a), (b) and (c) have 4,6 and 2 variables set to true respectively. Since every change of one variable's truth value changes the formula's truth value an even number of changes will bring us back to the original truth value.

6. Let ϕ be a propositional logic formula on 5 variables $p_1, \ldots, p_5$. Suppose ϕ is such that every time we change an assignment by altering the truth of just one propositional variable, the truth value of the formula changes. Given this information, How many satisfying assignments does ϕ have?

   (a) 1
   (b) 2
   (c) 16 (correct answer)
   (d) 32

   By the fact that that the truth value of the formula changes for every single change of a propositional variable's truth value, we can say that the original truth value is restored after every two changes. Thus all assignments with an odd number of variables set to true lead to the same truth value of the formula and all assignemnts with an even number of variables set to true lead to the opposite evaluation of the formula (but again same as each other). Thus exactly half the assignments cause the formula to evaluate to true. This number is 16 since there are five variables.

7. Suppose a boolean formula ϕ is such that its truth value does not change when we change the truth value of just one of its propositional variables. Then:

   (a) ϕ is definitely a tautology
   (b) ϕ is definitely a contradiction
   (c) ϕ is either a tautology or a contradiction (correct answer)
   (d) ϕ is neither a tautology nor a contradiction

   Correct answer is (c) either a tautology or a contradiction. Since a single change of propositional variable truth value doesnt change the formula's truth value it is easy to see by iteration/induction/transitive closure that the formula evaluates to the same truuth value always. If that evaluation is true it is a tautology and if it is false it is a contradiction.

8. The boolean function $\phi = (\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$ is equivalent to:

   (a) $(p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$ (correct answer)
   (b) $(p_1 \vee p_2 \vee p_3) \wedge (p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$
   (c) $(p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$
   (d) $(p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee p_2 \vee p_3)$

   The function given looks **BIG** but really it has a pattern. The firast two clauses have p_1 and the last two have $\neg p_1$. All four clauses have p_2 and p_3 in synchronisation. Thus it is easy to figure out using the idea of proof by cases/counting by cases. Take first p_1 as true then as false. Then look at the symmetry of the reduced clauses and it becomes evident.

9. Which of the following formulae has more thas one satisfying assignment:

   (a) $p_1 \wedge p_2 \wedge p_3$
   (b) $\neg(p_1 \vee p_2 \vee p_3)$
   (c) $(p_1 \vee p_2 \vee p_3) \Rightarrow (\neg p_1 \wedge \neg p_2 \wedge \neg p_3)$
   (d) $(p_1 \Rightarrow (p_2 \Rightarrow p_3))$ (correct answer)

   It is the last option. It is an implication where the highest level operator is an implies ($\Rightarrow$) and one of the operands is a propositional variable (not a complicated formula) and that already means at least half the assignments are satisfying.

10. The number of assignments of truth values on which the formulae $\phi_1 = ((p_1 \Rightarrow p_2) \Rightarrow p_3)$ and $\phi_2 = (p_1 \Rightarrow (p_2 \Rightarrow p_3))$ evaluate to the same truth value is:

(a) 3
(b) 4
(c) 5
(d) 6 (correct answer)

11. For the pairs of formulae given below, determine for which pair the second formula is implied by the first.

    (a) $\phi = p_1 \wedge p_2$, $\psi = p_1 \oplus p_2$
    (b) $\phi = p_1 \oplus p_2$, $\psi = p_1 \vee p_2$ (correct answer)
    (c) $\phi = p_1 \oplus p_2$, $\psi = p_1 \wedge p_2$
    (d) $\phi = p_1 \vee p_2$, $\psi = p1 \oplus p_2$

    The correct option is given. This follows from the truth tables, if you are unable ot see it quicker.

12. $\psi = \exists x(P(x) \Rightarrow Q(x))$ is equivalent to

    (a) $\neg(\forall x((\neg(P(x))) \vee (Q(x))))$
    (b) $\neg(\forall x((P(x)) \vee (\neg(Q(x)))))$
    (c) $\neg(\forall x((\neg(P(x))) \wedge (Q(x))))$
    (d) $\neg(\forall x((P(x)) \wedge (\neg(Q(x)))))$ (correct answer)

    Follows from De Morgan's Laws for first order logic ($\exists$ and $\forall$) and the rule for replacing $\Rightarrow$ by $\vee$.

13. Which of the following is equivalent to $p \Rightarrow q$? (This is called the contrapositive and is used as a proof technique)

    (a) $q \Rightarrow p$
    (b) $p \Rightarrow \neg q$
    (c) $\neg p \Rightarrow q$
    (d) $\neg q \Rightarrow \neg p$ (correct answer)

    This is obvious fronm the syntactic or semantic rules covered in class.

14. Let $\phi = (p \Rightarrow (q \Rightarrow r))$ and $\psi = ((p \Rightarrow q) \Rightarrow r)$. Then which of the following is true?

    (a) $\phi \Rightarrow \psi$
    (b) $\psi \Rightarrow \phi$ (correct answer)
    (c) $\phi \Leftrightarrow \psi$
    (d) $\psi \oplus \phi$

    We want to justify the correct answer. This happens if whenever ψ is true then ϕ is also true. Let us do this using **proof by cases** and **simplification**. One scenario is that r is true, which in ϕ means the **conclusion** is true, which makes the truth or falsity of the premise irrelevant. If r is false then for ψ to be true we need $p \Rightarrow q$ to be false which means p is true and q is false. These lead immediately to the formula ϕ being true. This is the proof.

15. $p \oplus q$ is equivalent to:

    (a) $\neg(p \wedge q) \wedge (p \vee q)$ (correct answer)
    (b) $(p \wedge q) \wedge \neg(p \vee q)$
    (c) $\neg(p \wedge q) \vee (p \vee q)$
    (d) $(p \wedge q) \vee \neg(p \vee q)$

    It is XOR. So at least one must be true and at most one must be true. It follows that it is **or** and **not and**.

16. A set of formulae in propositional logic over the same set of propositional variables is called consistent if there is at least one assignement, that makes all the formulae true simultaneously. For the following set of formulae, written in semicolon separated form, which formula needs to be removed, so that the remaining three are consistent?

$$(p \Rightarrow q) \Rightarrow (r \Rightarrow \neg p); p \oplus q \oplus r; \neg p \wedge \neg q \wedge \neg r; p \vee q \vee r$$

    (a) $p \Rightarrow q) \Rightarrow (r \Rightarrow \neg p)$
    (b) $p \oplus q \oplus r$
    (c) $\neg p \wedge \neg q \wedge \neg r$ (correct answer)
    (d) $p \vee q \vee r$

    Evident from truth tables. With p, q, r all false we ger (c). None of the others are true with this assignment. It may be verified that $p = True$, $q = False$, $r = False$ satisfies the remaining formulae.

17. Which of the following propositional logic connectives is not commutative?

    (a) $\wedge$ (AND)
    (b) $\vee$ (OR)
    (c) $\oplus$ (XOR)
    (d) $\Rightarrow$ (Implication) (correct answer)

    Evident from truth tables.

18. Which of the following propositional logic connectives is not associative? Meaning, if it is used in a chain, then the order of bracketing influences the final truth value evaluation.

    (a) $\wedge$ (AND)
    (b) $\vee$ (OR)
    (c) $\oplus$ (XOR)
    (d) $\Rightarrow$ (Implication) (correct answer)

    This is self-evident from the truthe tables.

19. Which of the following have the most number of satisfying assignments?

    (a) $p \wedge q \wedge r$
    (b) $(p \rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow p)$
    (c) $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$
    (d) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee p)$ (correct answer)

    (a) has exactly one satisfying assignment.
    (b) has exactly two satisfying assignments (those where all three variables are assigned the same truth values).
    (c) is true for all those assignments with at least two variables set to true. Thus the number that do not satisfy is 4 (all false or exactly one true). Thus the number of satisfying assignments is also 4.
    (d) If $q = False$ then $r = False$ and $p = True$. If $q = True$ then $p = True$ and r can take either value. Thus the number of satisfying assignments is 3.

    From this it is evident that the answer provided by the instructor to the evaluation platform is wrong. This needs to be reevaluated and students re-marked and I will do it in due course.

20. Which of the following have the fewest number of satisfying assignments?

    (a) $p \wedge q \wedge r$ (correct answer)
    (b) $(p \rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow p)$
    (c) $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$
    (d) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee p)$

    The answer to this is correct as provided. The explanation is same as the previous question.