Euler Paths and Circuits

→ **Euler Path:** Traverse each edge exactly once (only once)
(vertex can be repeated here)

→ **Euler Circuit:** first & last vertex are same.

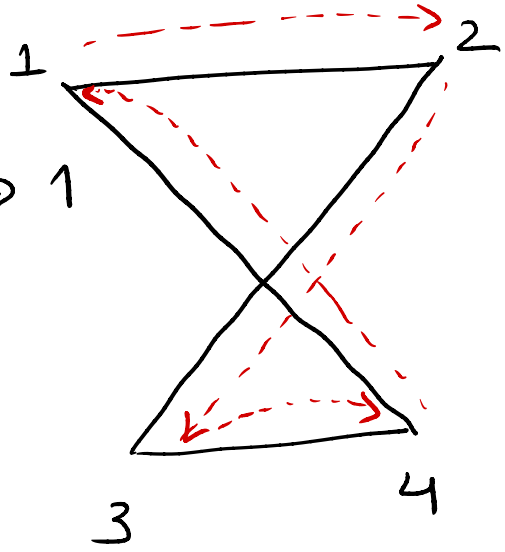
* circuit that traverses each edge exactly one time.

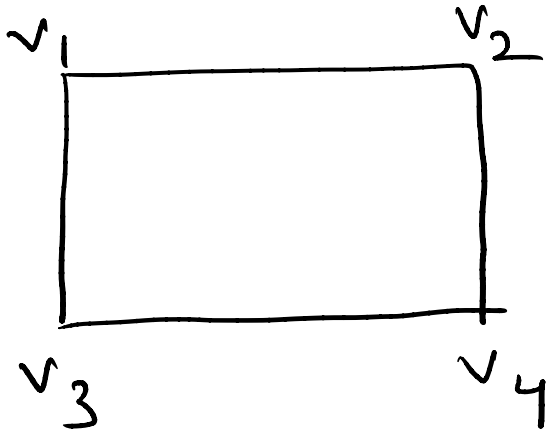
→ **Euler Graph:** A graph containing Euler path is an Euler Graph.

* It is always connected as it contains Euler path containing all edges.

Example :

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$



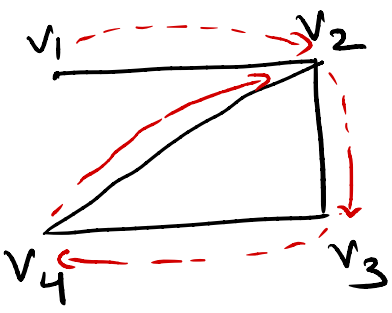


It is not connected, so NOT possible

→ A connected graph is Euler Graph iff it has atmost 2 odd degree vertices.

→ Euler Circuit: Each vertex is of even degree

→ Euler Path: Maximum two vertices of odd degree



Euler Path ✓

Euler circuit X

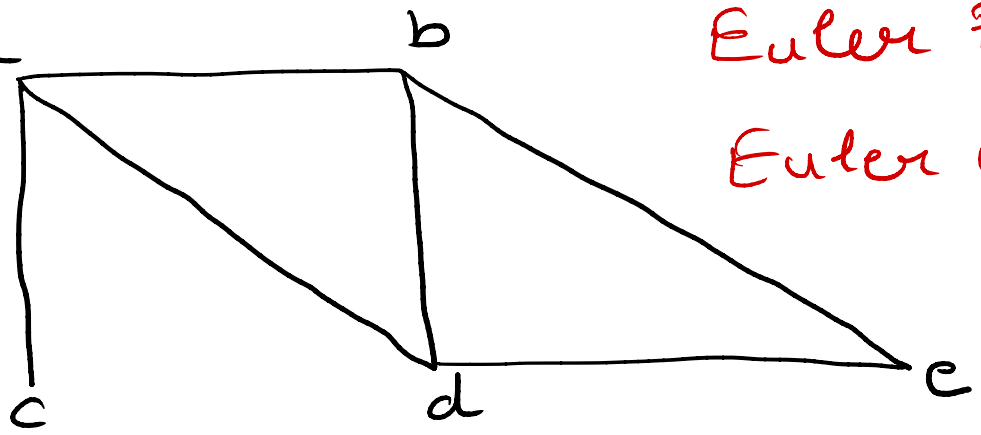
$\deg(v_1)$: 1
$\deg(v_2)$: 3
} 2 odd degrees

$\deg(v_3)$: 2

$\deg(v_4)$: 2

More Examples:

1.



Euler Path: —

Euler circuit: —

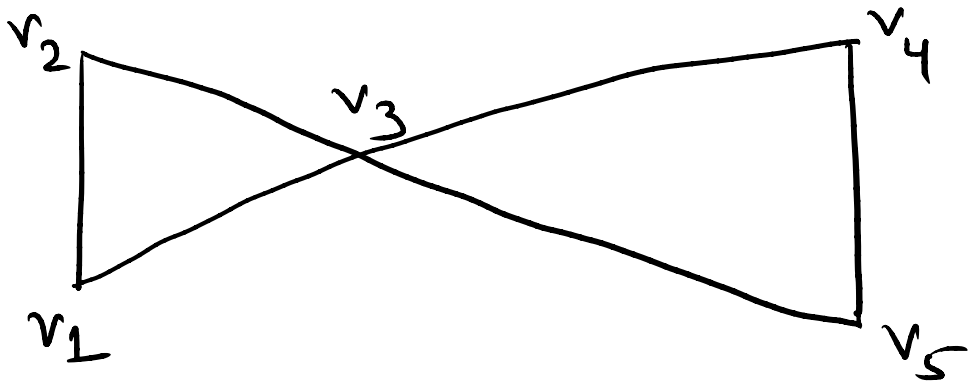
$\deg(a)$:

$\deg(b)$.

$\deg(c)$:

$\deg(d)$:

$\deg(e)$



Euler Path: _____

Euler circuit: _____

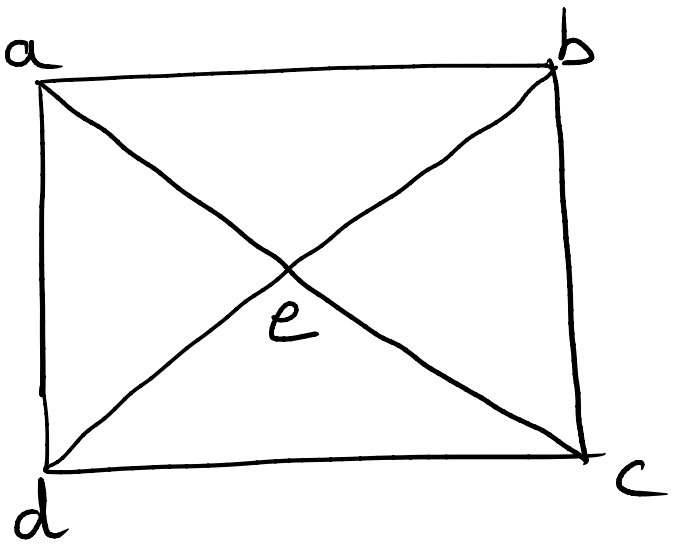
$\deg(v_1)$:

$\deg(v_2)$:

$\deg(v_3)$:

$\deg(v_4)$:

$\deg(v_5)$:



Euler Path: _____

Euler Circuit: _____

deg (a):

deg (b):

deg (c):

deg (d):

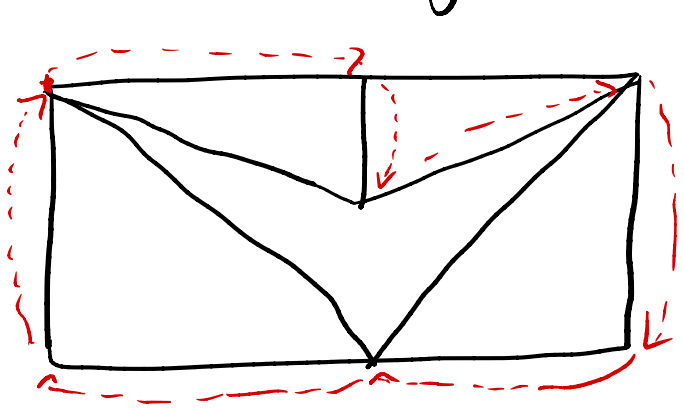
deg (e):

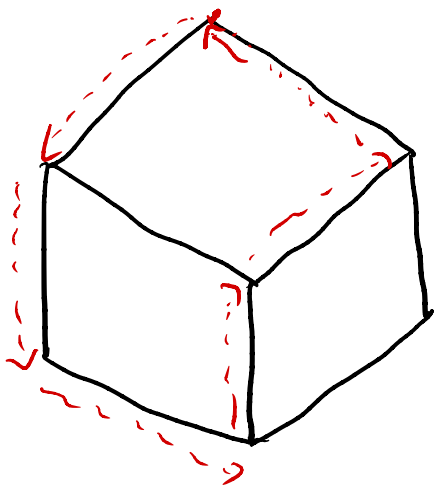
Hamiltonian Path & Circuit

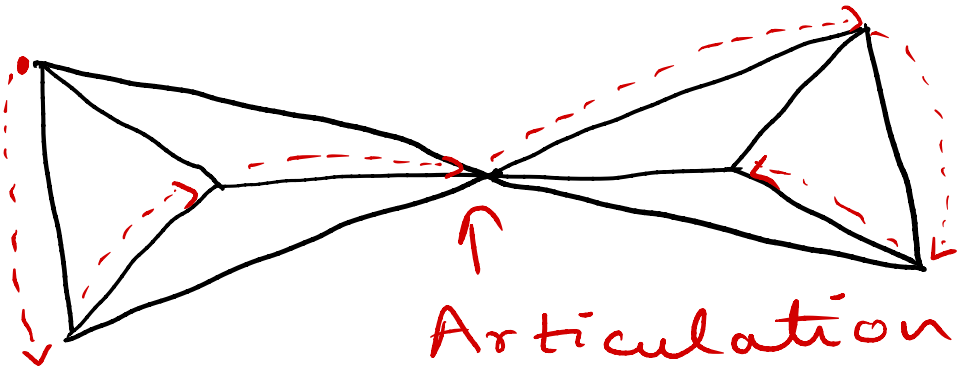
→ **Hamiltonian Path:** A path visiting every vertex exactly once.

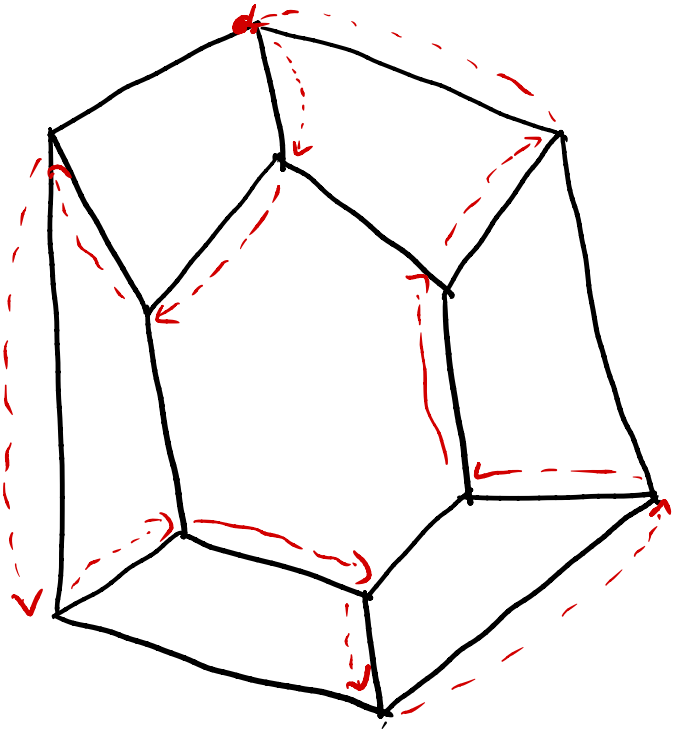
→ **Hamiltonian Circuit:** In a connected graph, each vertex must be traversed exactly once except the starting/ending vertex

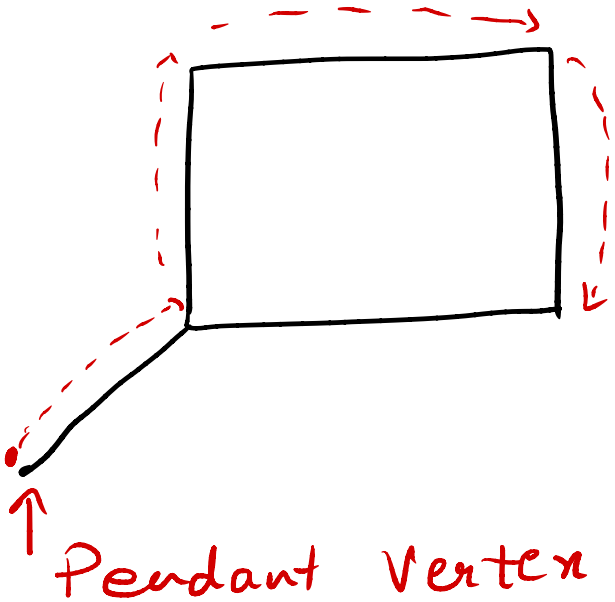
→ **Hamiltonian Graph:** A graph containing Hamiltonian circuit.



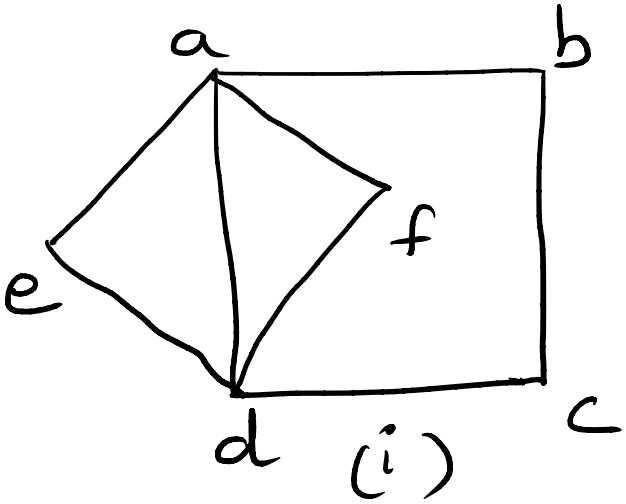



Articulation
Point (Connecting
vertex)




Pendant Vertex

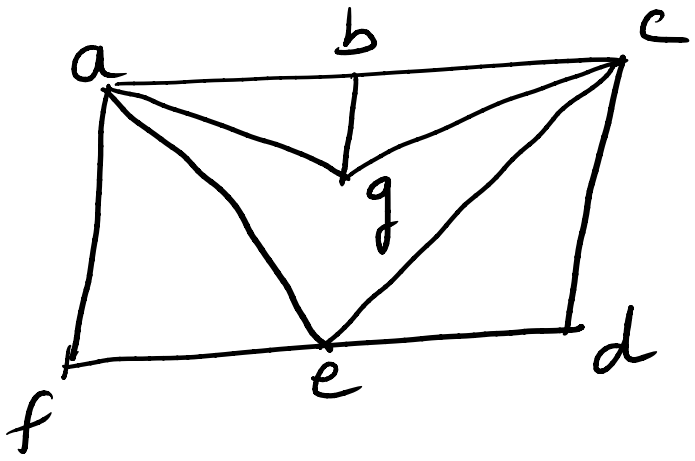
More Examples on Euler & Hamiltonian Graphs



(i)

Euler: ✓

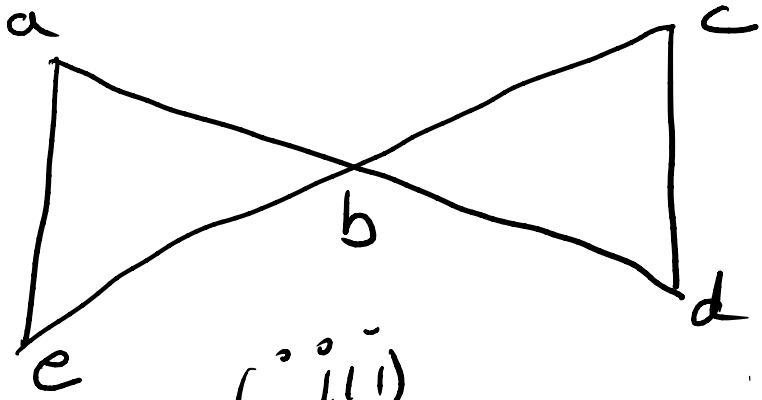
Hamiltonian: ✗



(ii)

Euler: ✗

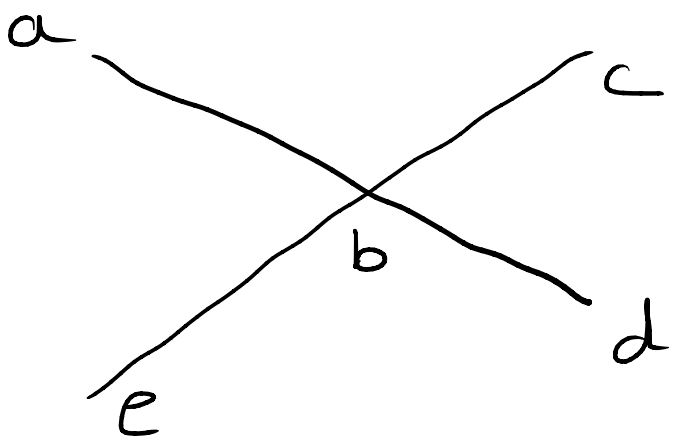
Hamiltonian: ✓



(iii)

Euler: ✓

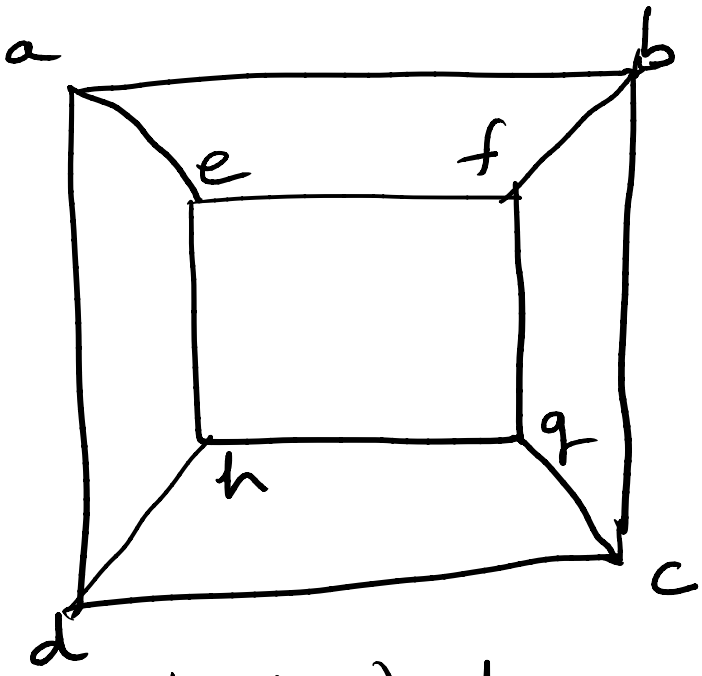
Hamiltonian: ✗



(iv)

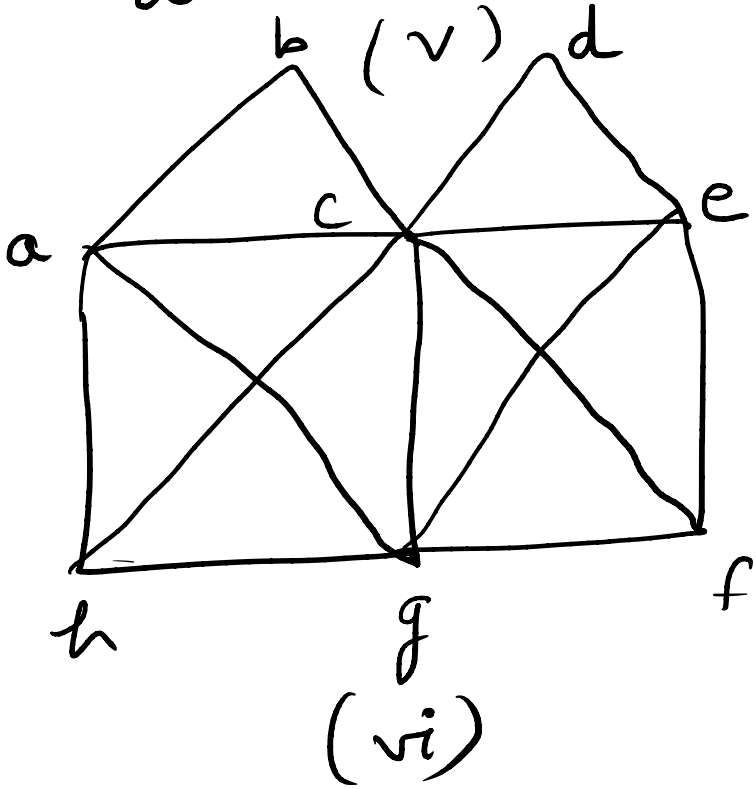
Euler: X

Hamiltonian: X



Euler: X

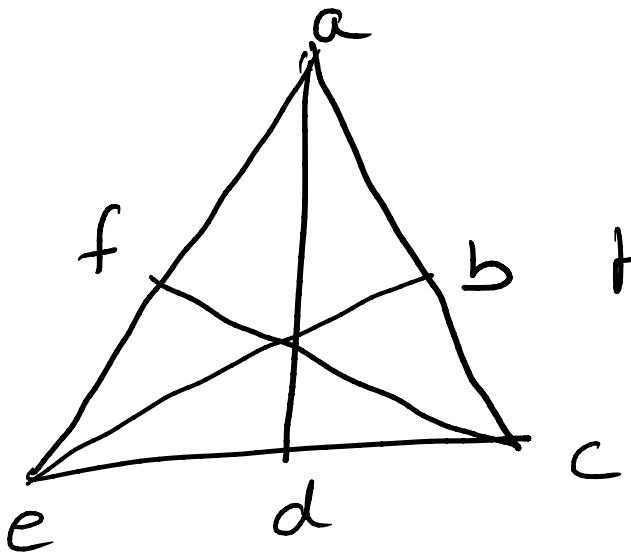
Hamiltonian: ✓



(vi)

Euler: X

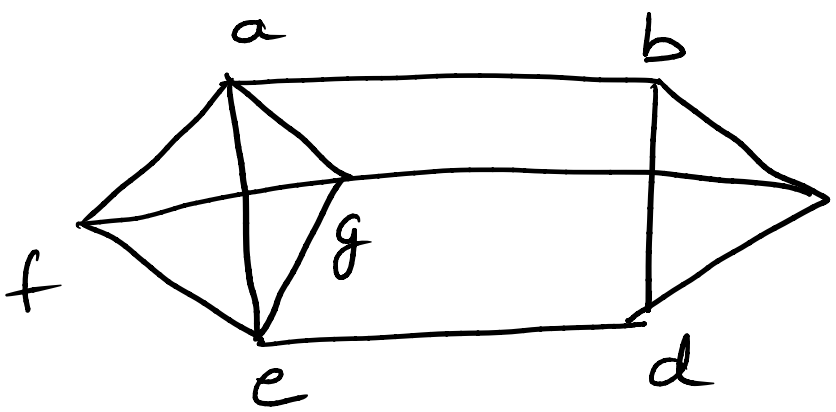
Hamiltonian: ✓



Euler: ✗

Hamiltonian: ✓

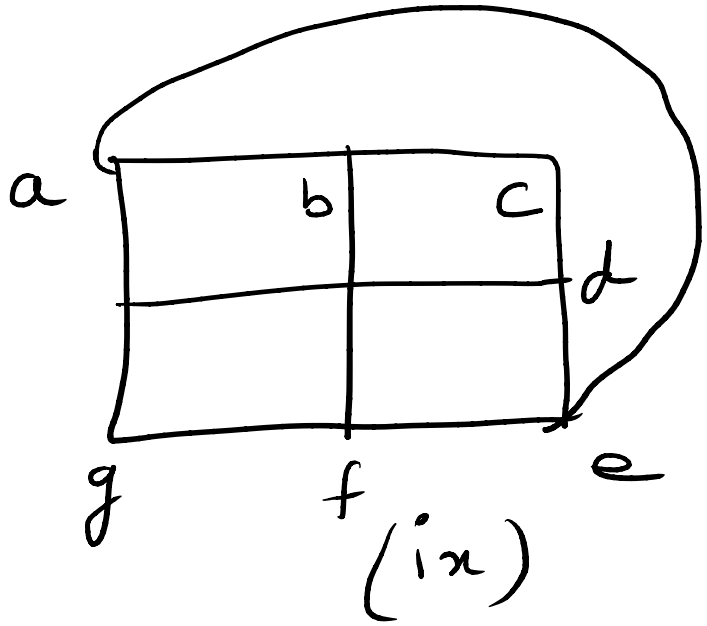
(vii)



Euler: ✗

Hamiltonian: ✓

(viii)



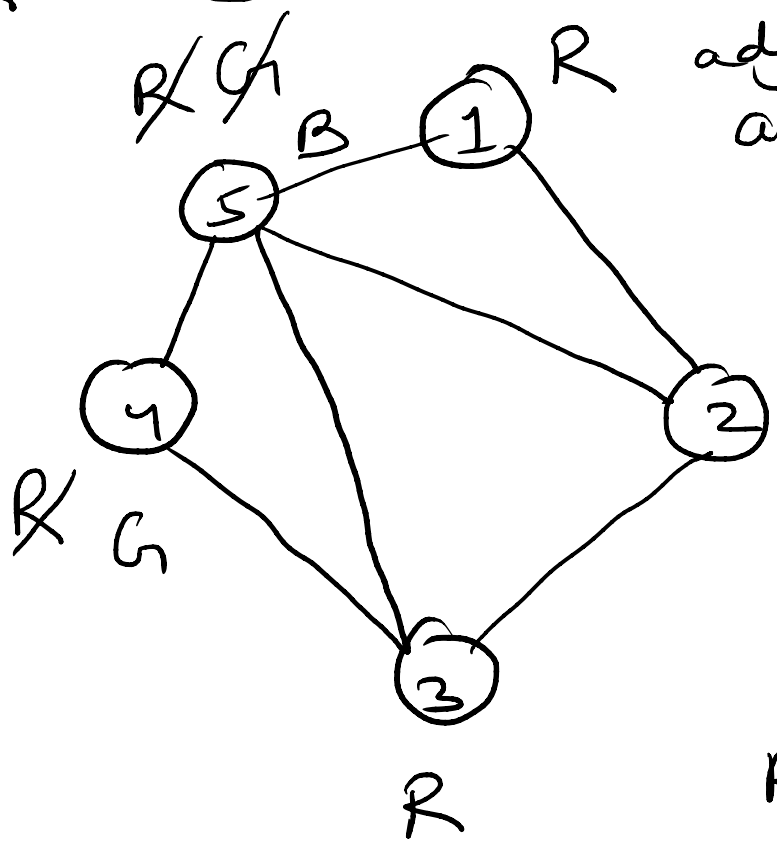
Euler: ✗

Hamiltonian: ✓

(ix)

Graph Coloring

→ Color the vertices so that no two adj. vertices are of same color



$m = 3$

R, G, B

Vertex	1	2	3	4	5
Color	R	G	R	G	B

One possible solution

Chromatic Number: The smallest number of colors needed to color a graph.