Discrete Mathematics (SC612)
Tutorial 5
12^{th} November, 2021

1. (a) Convert 2148 in decimal system to:
    i. Base 2 (binary)
    ii. Base 3 (Ternary)
    iii. Base 7
    iv. Base 8 (Octal)
   (b) Which decimal system integer is represented by the base 7 number 632?
   (c) Which decimal system integer is represented by the base 8 (octal) number 577
   (d) Which decimal system number is represented by the base 3 (ternary) number 21022
   (e) Which decimal system number is represented by the base 2 (binary) number 10101110?

2. For each of the languages described below, construct a finite automaton where possible, and prove mathematically that none eists if that be the case.
   (a) The language of strings over $\Sigma = \{a\}$ consisting of exactly those words of prime number length.
   (b) The language of all words over $\Sigma = \{a, b, c\}$ where the last character is a.
   (c) The set of all words over $\{a, b, c\}$ which are palindromes (that is the word is the same when read from left-to-right or right-to-left).

(d) The set of all words over $\Sigma = \{a, b, c\}$ consisting of all those words where no two identical characters in the word are separated by a different character.

3. (a) Suppose L is a language over a finite alphabet $\Sigma = \{a, b\}$, such that $|L|$ is a finite number. Show that it has a finite automaton recognising it.

(b) Suppose L is a language over $\Sigma = \{0, 1, 2, 3\}$ which is recognised by a finite automaton. Prove that the complement language $\Sigma^* \setminus L$ is also accepted by a finite automaton.

(c) Suppose L_1 and L_2 are languages over the same finite alphabet that can both be recognised by finite automata (obviously different machines for the two). Show that the language $L_1 \cup L_2$ can also be recognised by a finite automaton.

(d) Suppose L_1 and L_2 are languages over the same finite alphabet that can both be recognised by finite automata (obviously different machines for the two). Show that the language $L_1 \cap L_2$ can also be recognised by a finite automaton.

HINT: Use the concept of cartesian product for parts (c) **and** (d)