Discrete Mathematics

Scribed Notes 7

Relation:

- A relation is any subset of of **s1 * s2**
  s * s

Binary Relation:

A binary relation describes **a relationship between the elements of 2 sets**. Set may be same or different. i.e **S1 x S2 or S X S**

This type of relation is called Binary Relation.

Example - s1 * s2 * s3 -> ternary relation

Relation Databases :

Example –

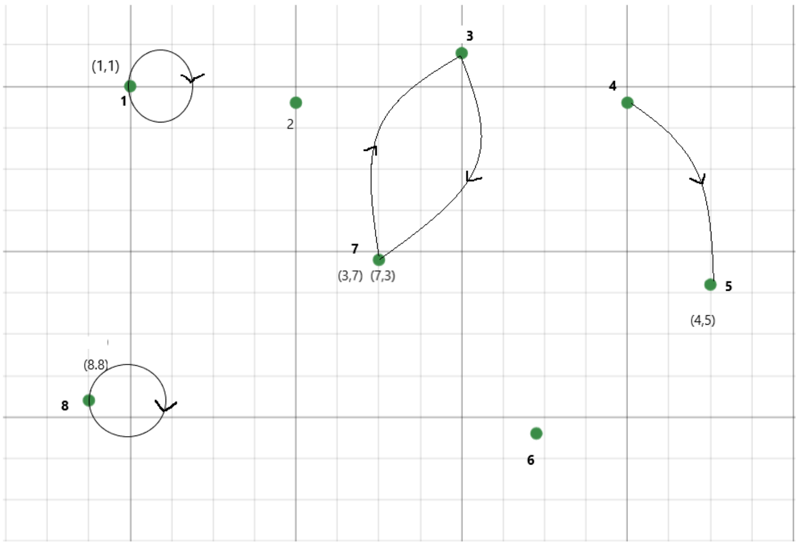
Name	*	Country	*	Age	*	Event

Here, columns are called fields and records are called tuples.

Diagrammatic Representation:

- It is via dots for elements of the set and curves between points to represent ordered pairs.

Example :

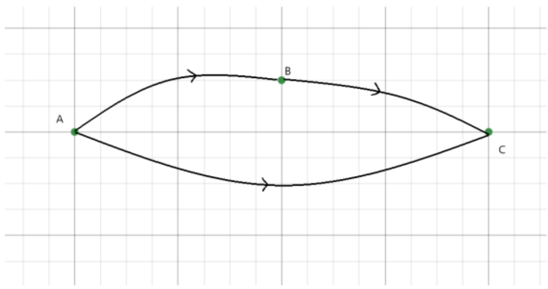


Transitive relations :

In transitive relation indirect path implies direct path.

(a, b) ϵ R & (b, c) ϵ R => (a, c) ϵ R

Example :



S = {a, b, c, d, e}

R = {(a, a), (b, c), (c, d), (e, d), (a, e), (e, b)}

R^2 = {(a, b), (a, a), (b, c), (c, d), (b, d), (a, d), (e, b), (e, c), (e, d)}

Here, R represents Original Relation and R^2 represents Iterated Relation.

Iteration :

f(x) = (3x+1)^2

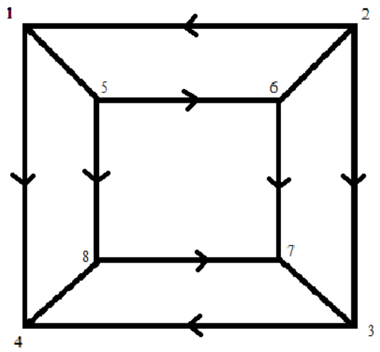
f(f(x)) = (3(3x+1)^2 + 1)^2

Transitive Closure:

It can be achieved by applying transitivity rule again and again until applied iterative relation doesn't result in any change.

Rule of Transitivity :

Indirect path implies direct path



Types of Relation :

a. Reflexive
b. Irreflexive
c. Symmetric
d. Anti-symmetric

e. Transitive

Categories of Relation :

a. Equivalence Relation
b. Partial Order

Equivalence Relation :

- Equivalence relation is a binary relation that is reflexive, symmetric and transitive
- Reachability relation in a graph is always reflexive
- Undirected graphs are symmetric
- In undirected graphs, the reachability relation is an equivalence relation. The point reachable from each other are called connected components.

Equivalence Relation Partition :

Partition of a set is a grouping of its elements into non-empty subsets in such a way that empty element is included in exactly one subset.

$$S = \cup \bigcup_{i=1}^{k} S$$

For $1 \leq i < j \leq k \quad Si \cap sj = \emptyset$

Example –

S = {1, 2, 3, 4, 5, 6, 7, 8}

S1 = {1, 3, 5}

S2 = {2, 4, 7}

S3 = {6}

S4 = {8}

R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (1, 3), (3, 1), (1, 5), (1, 5), (5, 1), (3, 5), (5, 3), (2, 4), (4, 2), (4, 7), (7, 4), (2, 7), (7, 2)}

Relation is Reflexive, Symmetric and Transitive

Theorem: The set of equivalence relations on a set S has a one-to-one correspondence (one-to-one and onto) with the set of partitions of the underlying set S.