Discrete Mathematics

Scribed Notes-11

Counting:

Counting is method of count the successful outcomes of experiments and all the possible outcomes of experiments to determine probabilities of discrete events

counting techniques are used extensively when probabilities of events are computed

we already using few counting techniques in different context

The Basics Counting Techniques

- Rule of product
  - Suppose that a procedure can be broken down into a sequence of two tasks. If there are n1 ways to do the first task and for each of these ways of doing the first task, there are n2 ways to do the second task, then there are n1n2 ways to do the procedure.

  E.g:

  The combined dresses(pant-shirts) of a shop are to be purchased with 7 shirts followed by a 5 pants. What is the largest number of combined dresses(pant-shirts) you can purchased differently?

  -> The product rule shows that there are $5 \cdot 7 = 35$ different ways that combined dress can be purchased. Therefore, the largest number of ways that combined dress can be purchased differently is 35

- Rule of sum
  - If a task can be done either in one of n1 ways or in one of n2 ways(choices), where none of the set of n1 ways is the same as any of the set of n2 ways, then there are n1 + n2 ways to do the task

  E.g:

  A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

  -> The student can choose a project by selecting a project from the first list, the second list, or the third list. Because no project is on more than one list, by the sum rule there are $23 + 15 + 19 = 57$ ways to choose a project

- Principle of Inclusion & exclusion
  - If a task can be done in either n1 ways or n2 ways, then the number of ways to do the task is n1 + n2 minus the number of ways to do the task that are common to the two different ways.

    E.g:

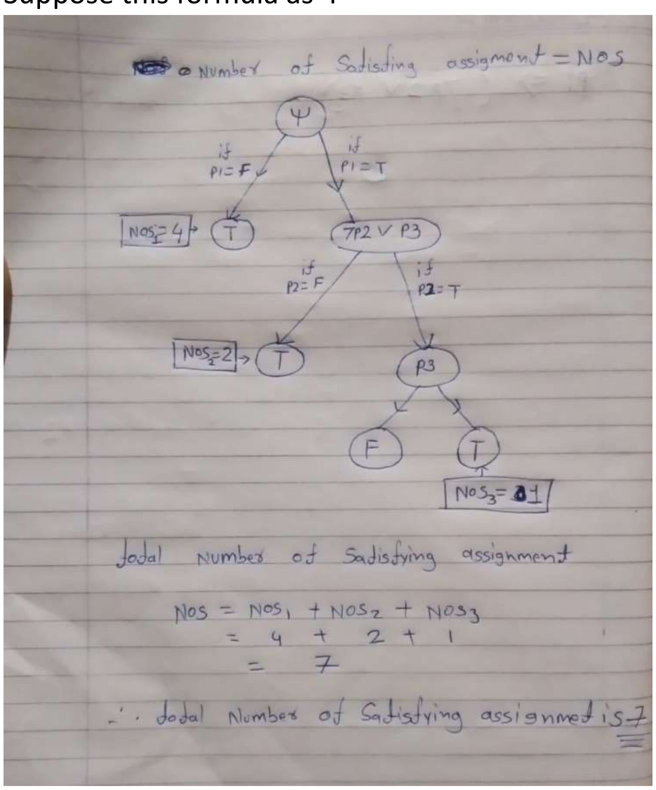
    suppose we have two subject A and B in which few topics are similar , then the number of total new topic we learned by attending both subject is
    $|A \cup B|=|A|+|B|-|A \cap B|$

- Counting by cases (modelled using a tree diagram)
  - Counting problems can be solved using tree diagrams. A tree consists of a root, a number of branches leaving the root, and possible additional branches leaving the endpoints of other branches.
  - To use trees in counting, we use a branch to represent each possible choice. We represent the possible outcomes by the leaves, which are the endpoints of branches not having other branches starting at them.

    E.g: Suppose we have to find number of satisfying assignment for formula which contains binary operator(A **binary operator** is an operator that operates on two operands) using tree diagram

    Formula is :P1->(˥p2 V p3)

    Suppose this formula as Ψ



- There are more techniques of counting
  - Pigeon-hole principle and its generalization
  - Ramsey number
  - Permutations
  - Combinations
  - Permutations with repetitions
  - Combinations with repetition
  - Binomial Theorem and Binomial Coefficients
  - Geometric and Binomial Distributions
  - Identities involving binomial coefficients
  - Double counting
  - Symmetry

Advance counting techniques

- Recurrence relations
- Divide & Conquer recurrences
- Generating Functions

NOTE: Counting is much faster than enumeration

Let's see some **Counting problems**

1. Derrangement

Derangement can be simply defined as a permutational arrangement with no fixed points. In other words, derangement can be explained as the permutation of the elements of a certain set in a way that no element of that set appears in their original positions.

So , there are n! choice that people get mismatch there things to each other.

So, Let's take an example

There are 5 people and their have own caps. So, find derrangement of 5 caps among the 5 people.

=> So cardinality of 5 caps is 0,1,2,3,4,5

Total Numbers of derrangement is 5! = 120

Number of Subset	Number of derrangement
0	44
1	45
2	20
3	10
4	0
5	1

- So ,for 5 people get mismatch possibility will be 1 only.

- For 4 people the mismatch possibility is 0 because if 4 person mismatch their cap to each other but there is still one person who get their own cap.

- For 3 people the mismatch possibility is 10 because if out of five people three are mismatch there caps possibility is $5c_3$ and possibility of mismatch in one person out of two is 1 so apply product of rule answer will be 10 *1 = 10.

- For 2 people the mismatch possibility is 20 because if out of five people two are mismatch so for there caps possibility is $5c_2$ and possibility of mismatch in two person out of three is 2 and for 2 person it will be 1 so apply product of rule answer will be 10 *2*1 = 20.

- For 1 people there will we can divide into two parts for half part and for full part.
  So , in first part we divide 4 person for 2 Like

  1)1 and 2 for 1 and 2 derrangement is 2 and 1 only

  2)3 and 4 for 3 and 4 derrangement is 4 and 3 only

  Finally we get $4c_2$ and it's divide by 2 = 3

  For full part take 1 , 2 , 3 ,4 . So ,for 1 possibility for derrangement is 3 and for 2

  Is 2 and for 3 is 1 and for 4 is also 1. In end we get 3*2*1*1 = 6.

  And for total 5 persons it will be 5.

  So ,final answer for 1 person is 5*[3+6] = 45.

- For 0 person who get derrangement will divide into three parts
  1)For 5 persons.

  2) For 3 persons.

  3)For 3 persons.

So, for 5 persons for 1 person derrangement is 4 , for 2 persons it will be 3 , for3 it will be 2 and for 4 it will be 5 and for also 1 so total is 4! = 4*3*2*1* = 24

For 3 persons for 1 person is 2 and for other 2 it will be 1 only. In final get 2*1*1 = 2

For 2 persons will be only 1 and adding also for final answer for 3 and 2 persons will also have $5c_2$ Finally we get 10*1*2 = 20.

And total derrangement for 0 persons will be 20 +24 = 44.

2 Fischer-Random Configurations

This is a chess variant, also called CHESS 960. The approach allows you to arrangethe pieces in a non-standard way within some restrictions.

Basically, we have a pair of Rooks, a pair of Knights, a pair of Bishop, a Queen, and aKing.

Two Rules to follow such:

Rule 1 -

The king must be in between two rooks.

Rook - King - Rook

Rule 2 -

The bishops are placed on the opposite-colored squares.

Bishop 1 placed in a3 then bishop 2 must be placed in a6

Starting with rule 2-
The possible arrangements are as follows:
There are 4 dark squares and 4 light squares.
For Bishop 1 four possible places to choose followed by the rule of sum.
Similarly for Bishop 2 four possible ways to arrange are there.
Therefore, to arrange the bishops on the chessboard there are total4 x 4
= 16 possible ways to do so. (rule of product)

Thus we are left with 6 empty squares on the chessboard to arrange the remainingones.

Following rule 1, when we take the R-K-R arrangement. It can be arranged in $6c_3$ ways.

Therefore, possible ways to arrange R-K-R
$=6c_3$
=(6*5*4)/(3*2*1)
=20

Now we're left with one Queen and 2 knights.

We can arrange the queen in $3c_1$= 3 ways for the remaining 3 spots on the chessboard.And the remaining knights take positions automatically on the remaining spots.

Therefore, to arrange the 8 pieces on the chessboard,

total configurations

= 16 * 20 * 3

= 960.

Thus, we have a total of 960 configurations.