❖ Theorem:

- Every partition is uniquely associated to an equivalence relation and vice versa.
- If two elements are in same part/partition, they must be related.
- If two elements are not in same partition, they must not be related.

❖ Function

- Definition: - A function is a special class of relation from a domain to co-domain where every point of domain has exactly one image in the co-domain.
- Every element in the domain has to appear in exactly one pair.
- A Function is a special subset of a Relation.

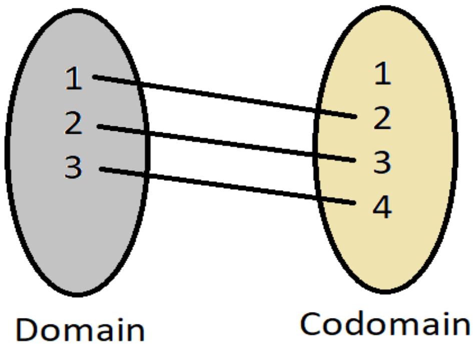


fig. 1

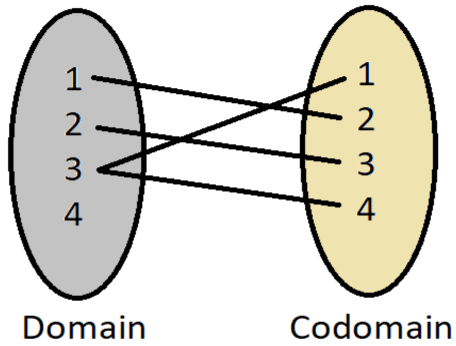


Fig. 2

- As shown in the fig.1 is a relation has a one and only one image for every point of domain in the codomain so that this relation is called function.
- but in fig. 2 point 3 has two images in codomain 3 and 1 also point 4 does not have any image in codomain so this relation is not a function.
- Number of pair in a function is equal to the Number of elements in a Domain set.

EXAMPLE 1 :-

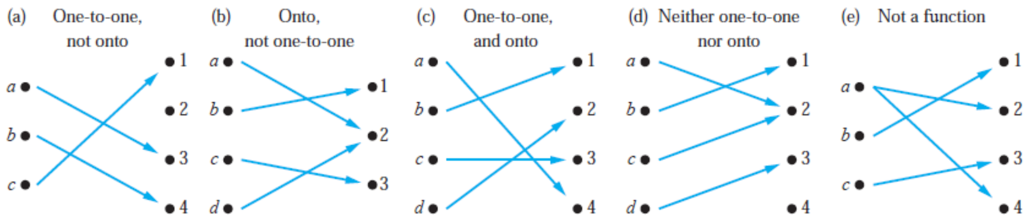
For D = {a, b, c, d} and C = {x, y}

R1 = {(a, x), (a, y)} is Relation but NOT a function because ‘a’ doesn’t have a unique image.

R2 = {(a, x), (b, x), (c, y)} is Relation but NOT a function because ‘d’ doesn’t have an image.

R3 = {(a, x), (b, x), (c, y), (d, y)} is a function, because there is no restriction on how many times an element in the co-domain can be an image.

❖ TYPES OF FUNCTIONS: -



❖ Cardinality of Function

|D|=No. of element in a set D = m

|C|=No. of element in a set C = n.

Here, every element of set D has a **n** number of choices.

So, the total no function f:D->C is n^m = (size of codomain)$^{\text{(size of domain)}}$

Cardinality of Function = Number of element in Domain.

❖ Restriction of a Function

- Definition: - Suppose function f: A->B is a function from a set A to a set B. now if a set C is subset of set A, then restriction of function f to A is the function $f_{|C}$: C->B.
- Restriction of a function means you simply restrict the domain by ignoring some elements.
- Those terms which are not ignored, must retained the same ordered pair as appears in the original function.

Example 1

A = {a, b, c, d} and B = {x, y, z}

f: A->B = {(a, x), (b, x), (c, y), (d, z)}

now restriction of a function g = f $_{\{b, c\}}$: A- {a, d} -> C = {(b, x), (c, y)}.

Here g is restricted function of f to the sub domain {b, c}.

h= f $_{\{a, c, d\}}$: A-{b}->C = {(a, x), (c, y), (d, z)}

here h is restricted function of f to the sub domain {a, c, d}.

Example 2

D = {a, b, c, d} and C = {a, b, c, d}

f: D->C = {(a, c), (b, c), (c, d), (d, a)} now restriction of a function g = f $_{\{a, c\}}$ = {(a, c), (c, d)}.

Here g is restricted function of f to the sub domain {a, c}.

❖ Restriction of a Relation

- In restriction of a relation restriction is applied on both Domain and Co-domain.

EXAMPLE 3

For D = {a, b, c, d} and C = {a, b, c, d} Now, Relation R is defined from set D to set C as shown below.

Relation R = {(a, c), (b, c), (c, d), (d, a)}
Relation R restricted to {a, c} = g = $R_{\{a, c\}}$ = {(a, c)}.

❖ PARTIAL ORDER

- Definition: - A relation R on a set A is called partial order if it satisfies reflexive, anti-symmetric, and Transitive properties.
- In the partial order if (x, y) is present in R then (y, x) must be absent in R.
- It is often used for Sorting.
- If we change "NOT AND" with "XOR" then we get special case of partial order. i.e., Total order.

Example 3

Show whether the relation (x, y) ∈ R, if, $x \geq y$ defined on the set of Positive integers is a partial order relation.

Solution:

Consider the set A = {1, 2, 3, 4} containing four integers. Find the relation for this set such as R = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (1, 1), (2, 2), (3, 3), (4, 4)}.

- **Reflexive:** The relation is reflexive as for every a ∈ A. (a, a) ∈ R, i.e. (1, 1), (2, 2), (3, 3), (4, 4) ∈ R.
- **Antisymmetric:** The relation is antisymmetric as whenever (a, b) and (b, a) ∈ R, we have a = b.
- **Transitive:** The relation is transitive as whenever (a, b) and (b, c) ∈ R, we have (a, c) ∈ R.

Above Relation satisfies all three properties so that relation is Partial order relation.

NOTE: -

A binary relation on a non-empty set S is equivalence relation if and only if the Relation is Reflexive, Symmetric and Transitive.

A Binary relation on non-empty set S is Partial Order if Relation is Reflexive, Anti-symmetric and Transitive.