



closure algorithm

= ABCDE

= ABFCDE

CFBADE

$\{AB\}^+$

CF<sup>+</sup>

ABF<sup>+</sup>

ABF,  
CF

Input: X, F.  
Output: X<sup>+</sup>

```
X+ := X;
repeat
  oldX+ := X+
  for each fd Y → Z in F do
    if X+ is superset of Y then
      X+ := X+ ∪ Z;
until (X+ = oldX+);
```

yes / no

AB → F ✓

AB<sup>+</sup>

F

- 1 R(ABCD),  $F = \{AB \rightarrow C, AC \rightarrow D\}$
  - 2 R(ABCDE),  $F = \{AB \rightarrow C, CD \rightarrow E\}$
  - 3 R(ABCDE),  $F = \{A \rightarrow B, C \rightarrow D, AC \rightarrow E\}$
  - 4 R(A,B,C,D,E,F),  $F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
  - 5 R(CourseNo, Sem, AcadYear, InstructorID, StudentID, Grade).
- Identify FDs yourself.
- 

R1

$AB \rightarrow C$

$AC \rightarrow D$

$$\boxed{AB^+ = ABCD} \quad \checkmark$$

$$\cdot \quad \boxed{AC}^+ = ACD$$

$R(ABCD), F = \{AB \rightarrow C, AC \rightarrow D\}$

$R(ABCDE), F = \{AB \rightarrow C, CD \rightarrow E\}$

$R(ABCDE), F = \{A \rightarrow B, C \rightarrow D, AC \rightarrow E\}$

$R(A,B,C,D,E,F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$

$R(\text{CourseNo}, \text{Sem}, \text{AcadYear}, \text{InstructorID}, \text{StudentID}, \text{Grade})$ .

Identify FDs yourself.

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$AB \rightarrow C$

$CD \rightarrow E$

ABC

CDE

$ABD^+ = ABCDE$

$R(ABCD E)$

$A \rightarrow BC$

$B \rightarrow D$

$AE^+ = AEBCD$

$R(ABCD), F = \{AB \rightarrow C, AC \rightarrow D\}$

$R(ABCDE), F = \{AB \rightarrow C, CD \rightarrow E\}$

$R(ABCDE), F = \{A \rightarrow B, C \rightarrow D, AC \rightarrow E\}$

$R(A,B,C,D,E,F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$

$R(\text{CourseNo}, \text{Sem}, \text{AcadYear}, \text{InstructorID}, \text{StudentID}, \text{Grade}).$

Identify FDs yourself.

$\{\text{CNO}, \text{AcadYr}, \text{Sem}\} \rightarrow \text{InstructorID}$

$\{\text{CN}, \text{AcadYr}, \text{Sem}, \text{SID}\}^+ \rightarrow \text{Grade}$

Key: <sup>minimal</sup> Set of attribute  
X,

$$X^+ = R$$

$A \rightarrow B$   
 ~~$A \rightarrow C$~~  ✓  
 $B \rightarrow C$   
 ~~$A \rightarrow B$~~   
 ~~$AB \rightarrow C$~~

$\Rightarrow$

$A \rightarrow B$   
 $B \rightarrow C$

$\{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$   
 $\{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$   
 $\{AB \rightarrow C, B \rightarrow D, D \rightarrow A\}$   
 $\{AB \rightarrow CD, B \rightarrow C, C \rightarrow D\}$   
 $\rightarrow \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$   
 $\therefore \{ABC \rightarrow D, A \rightarrow B\}$

Key  
 $A$   
 $Ae^+ = AeBD$   
 $\checkmark$   
 $Ae$