Laws of propositional logic:

1. **Identity Law:**
- P v F = P
- P ^ T = P

2. **Domination Law:**
- P v T = T
- P ^ F = F

3. **Idempotent Law:**
- P v P = P
- P ^ P = P

4. **Double Negation Law:**
- **¬ (¬p) = P**

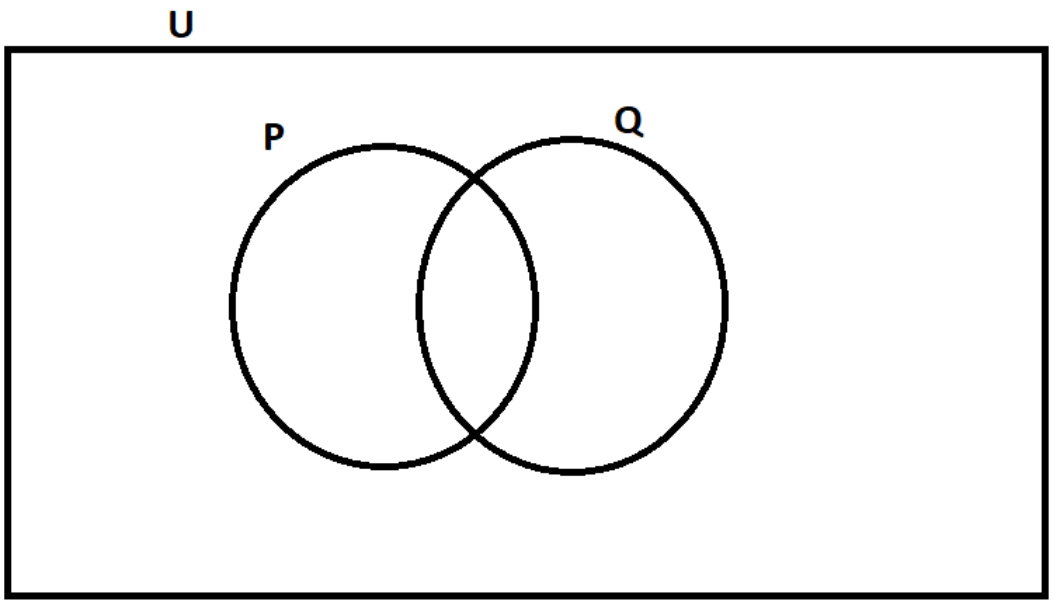
5. **Commutative Law:**
- P v Q = Q v P
- P ^ Q = Q ^ P

6. **Associative Law:**
- (P v Q) v R = P v (Q v R)
- (P ^ Q) ^ R = P ^ (Q ^ R)
- **(P v Q) ^ R = (P v Q) ^ R** **This is incorrect**

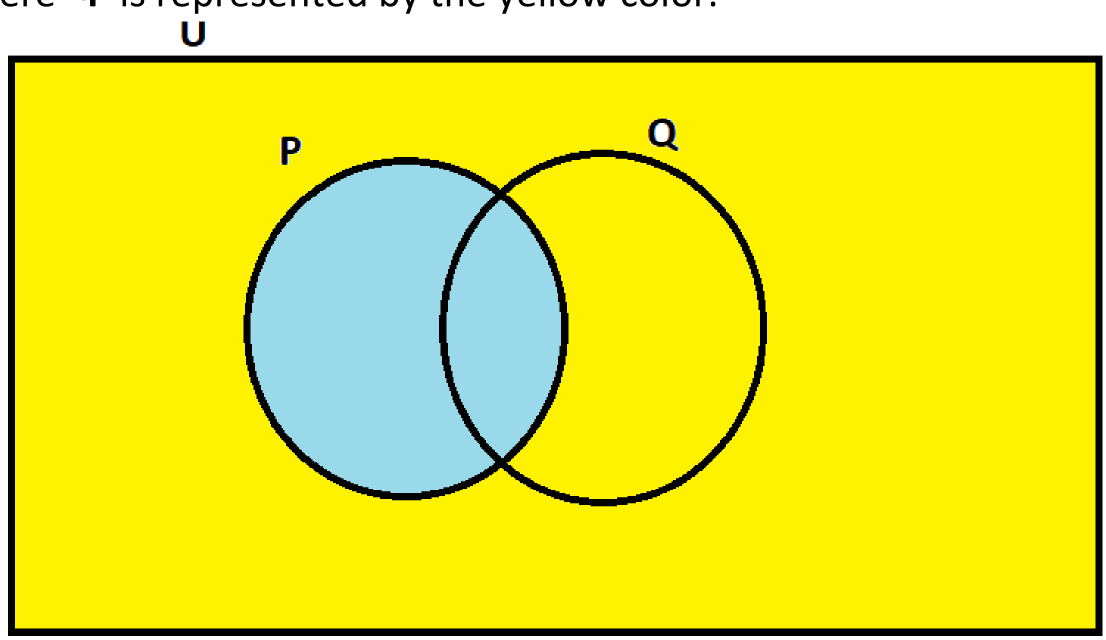
7. **Distributive Law:**
- P v (Q ^ R) = (P v Q) ^ (P v R)
- P ^ (Q v R) = (P ^ Q) v (P ^ R)

8. <u>**De-Morgan's Law:**</u>

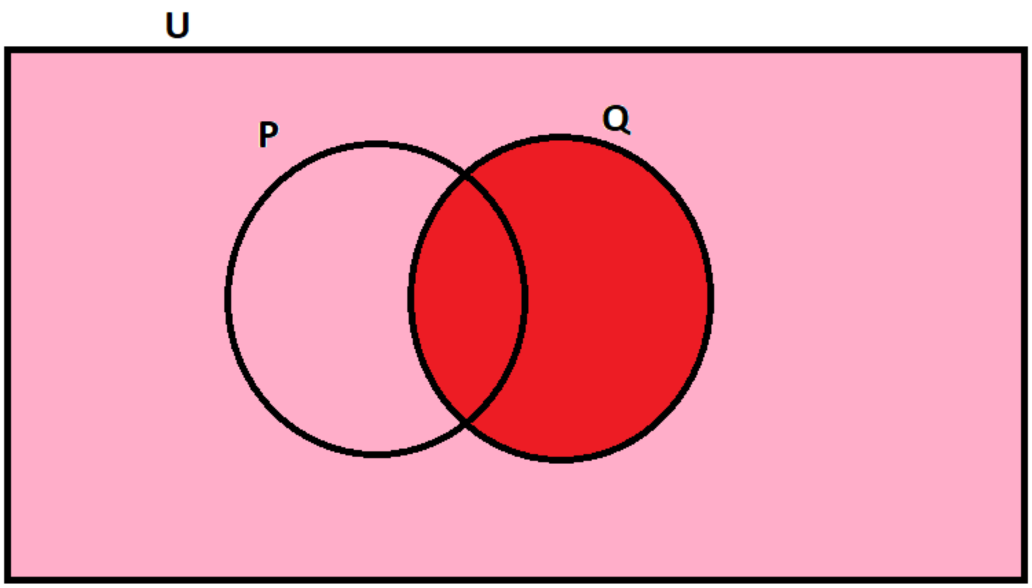
- Here U (Rectangle) represents a Universal set.
- P represents a set where P is TRUE.
- Q represents a set where Q is TRUE.



Here ¬**P** is represented by the yellow color.



Here ¬**Q** is represented by the pink color.



- (P ^ Q) **=** ¬ (¬ (P ^ Q)) **=** ¬ ((¬P) v (¬Q))
- (P v Q) **=** ¬ (¬ (P v Q)) **=** ¬ ((¬P) ^ (¬Q))

9. <u>Absorption Law:</u>

- P v (P ^ Q) = P
- P ^ (P v Q) = P

10. <u>Negation Law:</u>

- P v ¬P = T
- P ^ ¬P = F

Some Definitions

- **Axioms:**

Axioms are the statement which are not true but assume to be true.

- **Inference Rules:**

Rules of inference can be used to draw some conclusion from a premise based on the syntax and semantics of propositional logic.

The rules of inference are a logical form consisting of premises and draws a conclusion.

E.g.: Premise

$$
\begin{array}{l}
P \Rightarrow (Q \wedge (\neg R)) \\
P \\
\hline
Q \wedge (\neg R) \\
\hline
Q \\
\neg R
\end{array}
$$

Conclusion

In the above example, by applying rules of implication and assuming premise is true we can say conclusion is true i.e., if $P \Rightarrow (Q \wedge (\neg R))$ is true and if P is true then we can infer that $Q \wedge (\neg R)$ is true.

- **Complete System**

A system of inference rules and axioms which allows you to derive any correct theorem or any theorem that has proof.

- **Sound System**

Logical system is sound if and only if every formula that can be proved in the system is logically valid with respect to semantics of the system.

First Order Predicate Logic

Predicates: Predicates are the statements which are neither true nor false Until and unless the values of the variable are specified.

E.g.: X is a prime no.

Here we cannot define that the above statement is true or false until and unless the subject (X) is defined. The subject can be referred as variable and the rest of the sentence is known as predicate. Predicate refers to the property that the subject can have.

The above statement and some related examples, are not propositions. because the above example is neither true or false (the necessary definition for proposition).

Quantifiers: these words refer to the words which describes the quantity. Such as for all, such all, some or all etc. these words are used to express quantities without expressing the exact number.

Ex: I had some bananas.

Here some is showing quantity without referencing the exact number.

The 2 types of quantifiers are:

1. Universal Quantifier [for all **∀ ()]**
2. Existential Quantifier [there exists **∃ ()**].

E.g., for Universal Quantifier: x+1>x

This will satisfy with every positive value of x.

So, this predicate or function can be said as, **∀** (For all) +ve integers the function is true.

E.g., for Existential Quantifier:

F(x) = x<2

The above sentence will satisfy only with value 1, as the value will exceed the value 1, it becomes invalid sentence.

So this predicate or function can be said as, **∃** (there exists) some x for which the above function is true.

Negated Quantified Expression

- **¬ (∀x, P(x)) ≡ (∃x, ¬P(x))**
- In the above expression, (∀x, P(x)) implies that for all (x) Property (P(x)) is true. Negation of it (¬ (∀x, P(x))) can only be true if (∀x, P(x)) is false. And now (∀x, P(x)) can only be false if and only if there exists an (x) for which P(x) is false. This further means that - if and only if there exists an (x) for which ¬P(x) is true i.e. (∃x, ¬P(x)).
- Similarly,

  ¬ (∃x, P(x)) ≡ (∀x, ¬P(x))

Nested Quantifiers.

Example:

Lim $(x^2 + 4x - 5)/x - 1$

x→1

For the limit we can factorize it as:

Lim $\frac{(x + 5)(x - 1)}{x - 1}$

x→1

Lim (x + 5)

x→1

The limit of the above as (x) tends to 1 will be equal to 6.

So, if we go close to 6, let's say:

5.5 - 6.5 $\varepsilon = 0.5$

5.9 - 6.1 $\varepsilon = 0.1$

5.999 - 6.001 $\varepsilon = 0.001$

So, as we go closer to 6 with reducing the value of ε , we need to go closer and closer to 1.

This states that,

$\forall\varepsilon, \exists\delta$ such that | 6 - ε | <= F (x +- t) <= | 6 + ε | where t <= δ