

## **Scribed Notes 21 (10<sup>th</sup> November, 2021)**

### **★ Finite State Automata:**

#### **Definition:**

A finite automaton is a collection of three things:

1. A finite set of states, one of which is designated as the initial state, called the start state, and some (maybe none) of which are designated as final states.
2. An alphabet  $\Sigma$  of possible input letters.
3. A finite set of transitions that tell for each state and for each letter of the input alphabet which state to go next.

- **Deterministic Finite Automata:**

**Definition:**

A DFA is a mathematical model of a simple computational device that reads a string of symbols over the input alphabet  $\Sigma$ , and either accepts or reject the input string.

Deterministic Finite Automaton (DFA) is defined as a 5-tuple  $(Q, \Sigma, \delta, s, F)$  consisting of

A finite set  $Q$  (the set of states)

A finite set of symbols  $\Sigma$  (the input alphabet)

A transition function  $\delta: Q \times \Sigma \rightarrow Q$  mapping the current state  $q \in Q$  and input symbol  $a \in \Sigma$  to a new state  $\delta(q, a) \in Q$

An initial states  $s \in Q$  (the start state)

A set of accepting states  $F$  (the final states)

**Example:**

$\Sigma = \{0,1,2\}$  words where the remainder on division by 3 of the number of 0's, 1's and 2's are all distinct.

$Q = \{000, 001, 002, 010, 020, 200, 100, 111, 110, 112, 101, 121, 011, 211, 222, 220, 221, 202, 212, 022, 122, 201, 012, 021, 210, 102, 120\}$

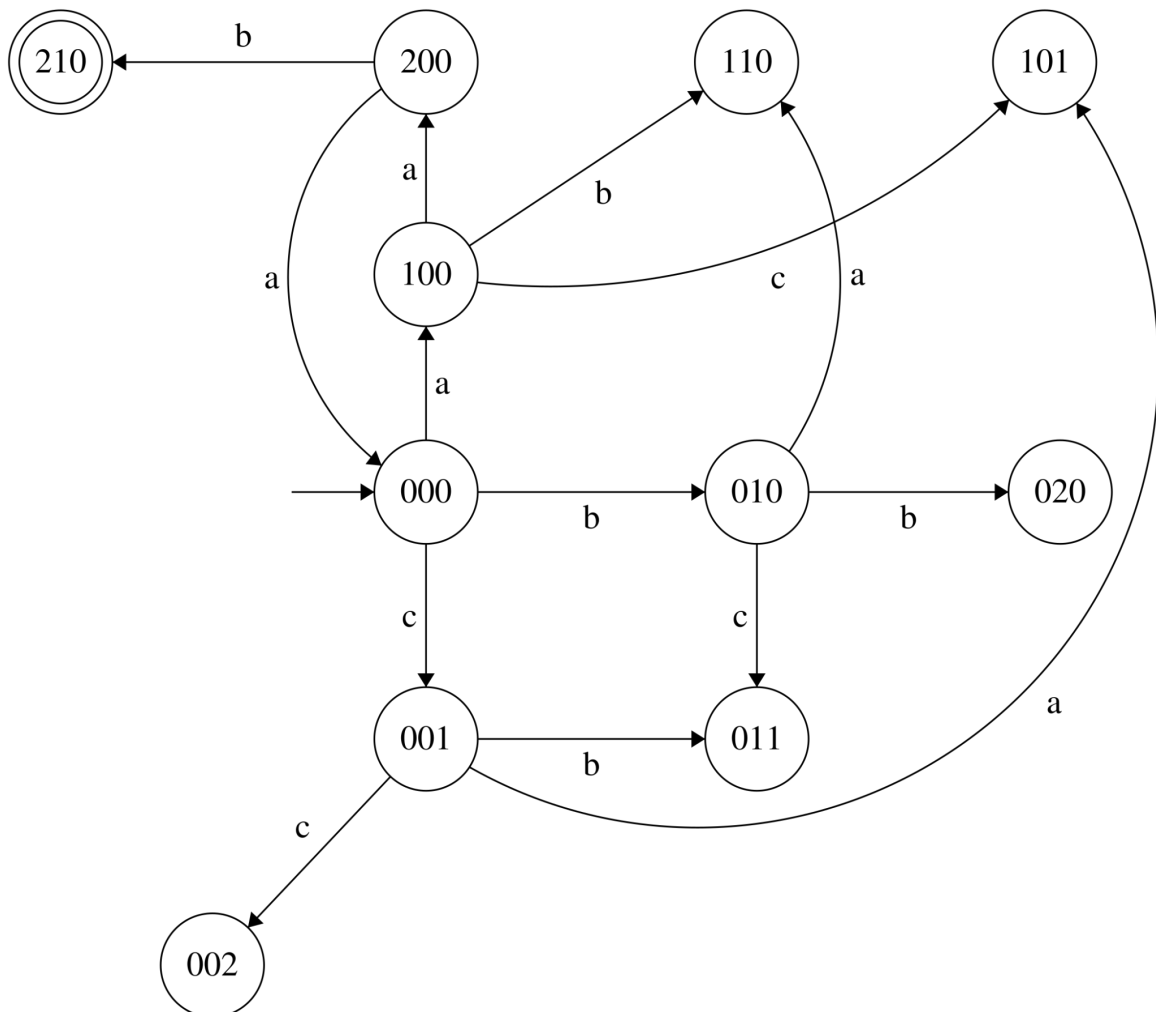
$\Sigma = \{a, b, c\}$

$s = \{000\}$

$F = \{012, 021, 102, 120, 201, 210\}$

$\delta = Q * \Sigma$

aaa, aab, aac, aba, aca, caa, baa, bbb, bba, bbc, bab, bcb, abb, cbb, ccc, cca, ccb, cac, cbc, acc, bcc, cab, abc, acb, cba, bac, bca



Note : As there are 27 numbers, therefore it will be difficult to draw the whole diagram so it is just a proper example.

$$\delta(011,c) = 012$$

$$\delta(121,c) = 122$$

$$\delta(022,b) = 002$$

- **Ternary Number System:**

- **Decimal to Ternary Conversion :**

|   |     |   |
|---|-----|---|
| 3 | 101 | 2 |
| 3 | 33  | 0 |
| 3 | 11  | 2 |
| 3 | 3   | 0 |
|   | 1   |   |

$$101_{10} = 10202_3$$

- **Ternary to Decimal Conversion :**

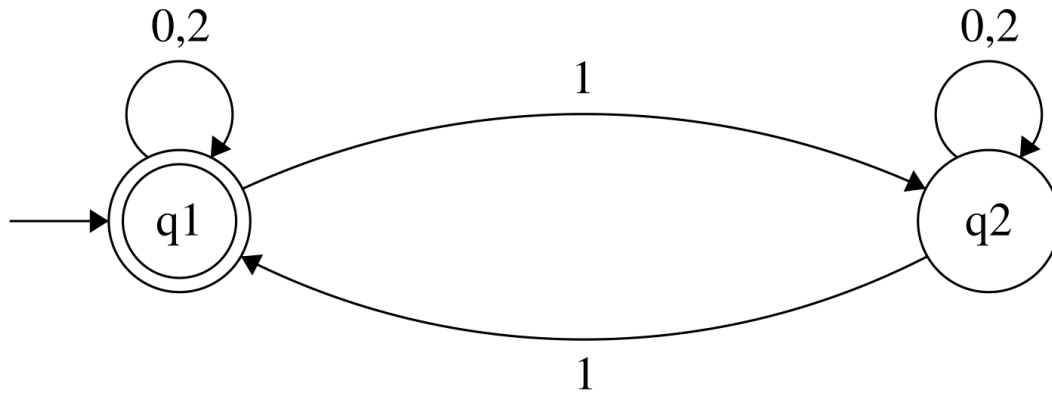
|       |       |       |       |       |            |
|-------|-------|-------|-------|-------|------------|
| 4     | 3     | 2     | 1     | 0     | ← Position |
| 1     | 0     | 2     | 0     | 2     |            |
| ↑↓    | ↑↓    | ↑↓    | ↑↓    | ↑↓    |            |
| $3^4$ | $3^3$ | $3^2$ | $3^1$ | $3^0$ |            |

$$\Rightarrow 3^4 \cdot 1 + 3^3 \cdot 0 + 3^2 \cdot 2 + 3^1 \cdot 0 + 3^0 \cdot 2$$

$$\Rightarrow 81 + 0 + 18 + 0 + 2$$

$$\Rightarrow 101_{10}$$

**Example:**



( $\because$   $q1 = \text{Qeven}$ ,

$q2 = \text{Qodd}$ )

$Q = \{q1, q2\}$

$\Sigma = \{0, 1, 2\}$

$s = \{q1\}$

$F = \{q1\}$

$\delta = Q * \Sigma$

00111221  $\rightarrow$  Qeven Qeven Qeven Qodd Qeven Qodd Qodd Qodd  
Qeven

## Conclusion :

$\nexists$  an FSA | FSM | DFA

( $\because$  FSA = Finite State Automata,

FSM = Finite State Machine,

DFA = Deterministic Finite Automata )

For  $L = [a^n b^n \mid n \in \mathbb{Z}^+]$