Scribed Notes 13 (11th October)

Subsequence

A subsequence of a given sequence is a sequence that can be derived from the given sequence by deleting some or no elements without changing the order of the remaining elements.

It is different from subarrary or substring which are contiguous, but in subsequence we can jump/skip elements without changing the order.

Increasing/decreasing subsequence in a list of distinct integers

Here we are finding the longest, either increasing or decreasing, monotonic subsequence.

Example 1,

13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1

Longest increasing subsequence = 1

Longest decreasing subsequence = 13.

The longest increasing/decreasing subsequence for n number distinct integers will always be greater than 0. So even for extreme cases where the elements are in decreasing order the length of increasing is 1 as if you pick any element it would be a subset of the list and there would be nothing to compare this element with so it's increasing. This is also called as **vacuously** true which means the case does not arise where the length will be zero.

Problem Statement

If you have (n^2 + 1) length sequence of distinct integers implies a monotonic subsequence of length greater than or equal to n+1.

Monotonic means there must be **either** an increasing /decreasing subsequence.

For n = 5, $n^2 + 1 = 26$,

There must be either increasing subsequence of 5 + 1=6 integers or decreasing subsequence of 6 integers.

Proof for this problem is given by the PigeonHole Principle.

(L_i, L_d)

L_i => Longest Increasing Subsequence

L_d => Longest Decreasing Subsequence

Example 2,

7,6,4,11,3,1,8,10,2,9,5

We take one element at a time and find L_i and L_d for that element, then add one element to the existing list and based on the new list find L_i and L_d of the new element **such that subsequence ends with the new element.** That means we need to use the new element in the subsequence.

So as per our example let's take sequence till 11

	A	B	C	D
1	(1,1)	(1,2)	(1,3)	(2,1)
2	7	6	4	11

For 11 Li is 2 as the longest increasing subsequence including that element is 2 ie 7,11 or 6,11 or 4,11. Longest decreasing subsequence is 1 as there are no other element

greater than 11. Here we can not take 7,6,4 as decreasing subsequence as stated earlier element 11 should be used while creating subsequence.

Now with this logic we can determine other Li and Ld. If we want to check Li, Ld of element 10 which appears later in the sequence, we know for 11 its (2,1) and we know 11>10 hence we can say that for 10 Ld will increase at least by 1

This way we can say that no two points will have the same coordinates.

Using Proof by contradiction to prove that monotonic subsequence of length greater than or equal to n+1.

Assumption : Longest monotonic subsequence has length <= n

The greatest value of Li or Ld will be n and the smallest value will be 1.

As seen before all the coordinates will be distinct i.e no two position will have same (Li,Ld).

So Li,Ld such that 1<= Li <=n and i<=Ld<=n , number of ordered pairs = n^2 (By product rule).

Number of elements are $n^2 + 1$.

So by PigeonHole Principle at least 1 element will have duplicate value which negates our argument that ordered pairs are distinct.

Since our arguments are logically correct our assumption is wrong.

Hence proved that the longest monotonic subsequence has length = n + 1.