SCRIBED NOTES 27

Eulerian Graph Theorem: A graph is Eulerian if it is possible to start at a vertex, traverse all the edges exactly once(not more not less), and return to the starating vertex.

Not all graphs are Eulerian graph. If a graph is a Eulerian trail than we can start at any vertex for a trail.

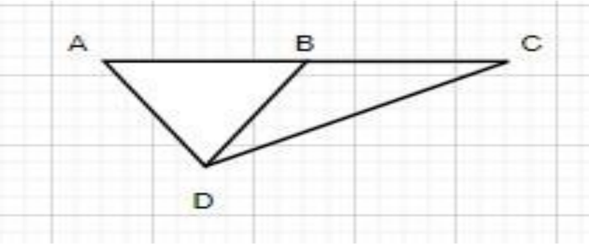

FIG:1

This I not eulerian graph because we didn't have trail here because we have to cover all edges and have to come back at the starting point (B and D are in odd degree, for trail we should have every vertex in even degree).

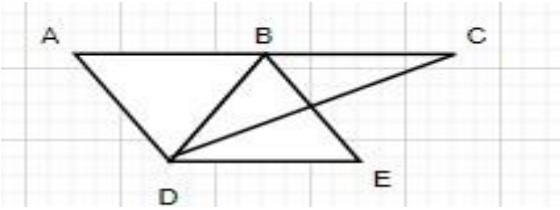

FIG:2

This is Eulerian graph because we can make trail here because all the vertex are in even degree.

D,A,B,C,D,B,E,D OR D,A,B,C,D,E,B,D

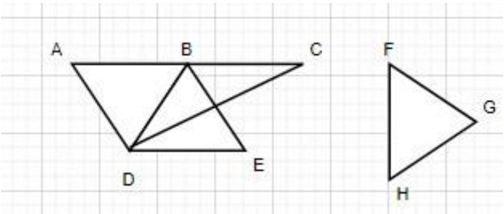

FIG:3

This Is no Eulerian graph because we can not make trail here.

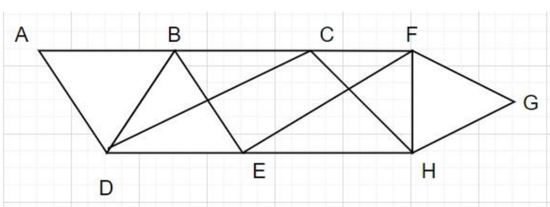

FIG:4

This is Eulerian graph because we can make a trail here.

Trail: A,B,C,F,G,H,F,E,H,C,D,E,B,D,A

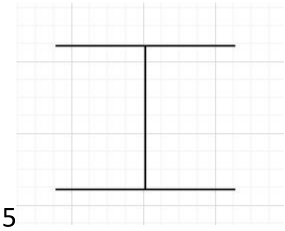

FIG:5

This is not Eulerian graph because trail is not coming to starting vertex.

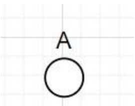

FIG:6

This is Eulerian graph because the starting vertex is also the ending node and vertex is also traversed.

Theorem: A graph is Eulerian if and if has only one non-trival component and every vertices have even degree.

1. Greedy algorithm: Your trail has to keep moving on the vertices, only on the starting point the trail can stop.

2. Mathematical induction on the number of edges: We assume that all for all graphs on upto k edges the theorem holds. And also for more than k edges.

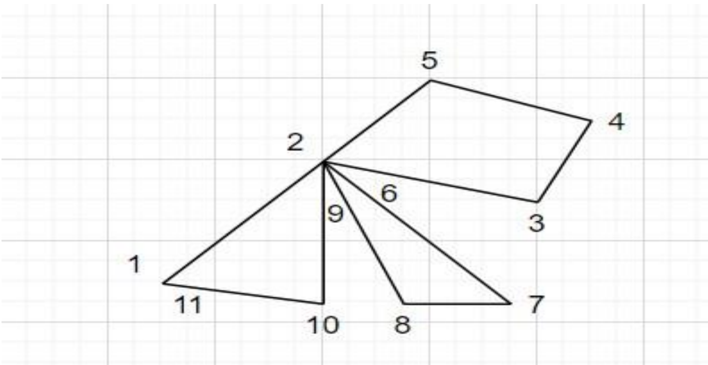

FIG: 7

Note: For Eulerian graph we can traverse vertices as many times as we want but we have to traverse edges on single times. And have to make a trail.

Bipartite graph: A graph is bipartite if its vertices can be patitioned into two sets such that all edges go from one part to the other.

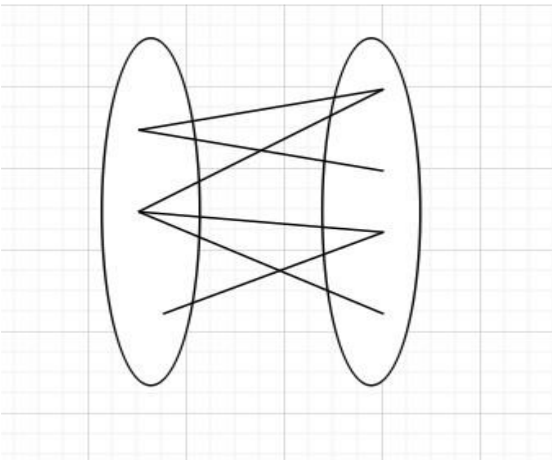

FIG: 8

- Every subgraph of a bipartite graph is also bipartite.

P: G is bipartite.

Q: G contains an odd cycle.

- If a subgraph H is non-bipartite, then the graph G is also non-bipartite.
- Characterising theorem : A graph is bipartite if and only If it contains no odd cycle.

(P<====> (-Q))

(Q ==== > (-P))

(P===> (-Q))

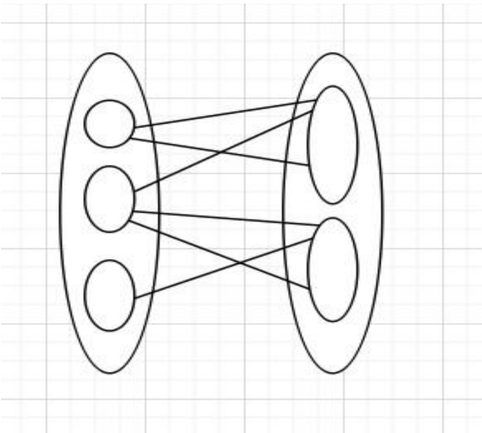

FIG:9

- Every closed odd walk contains a odd cycle.

To be proved:

((-Q)====>P)

((-P)=====>Q)

- Every connected bipartite graph has a unique bipartition.