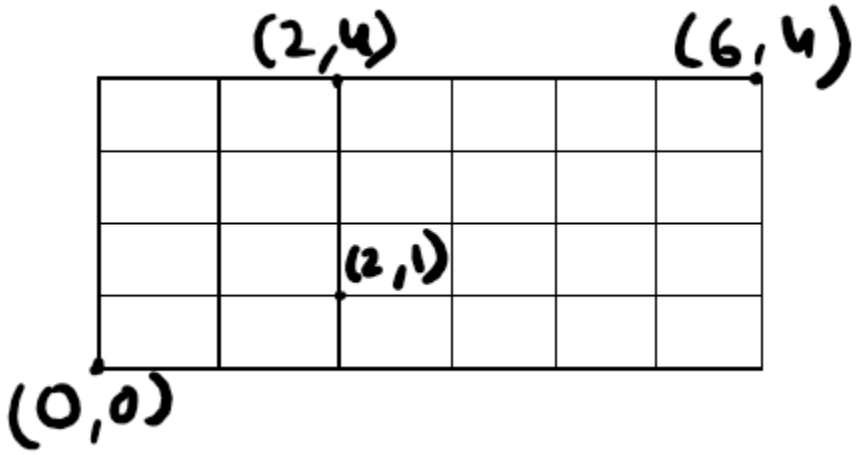
Scribed Notes - 12

Number of ways of going along a grid by a shortest path.

Definition: To find the shortest path from point A to reach point B.



- ➔ If there are n numbers of path from (0, 0) & (2, 1)
- ➔ If we want to go from (0,0) to (2, 1) then the shortest path will consist of 3 points.((0, 0) →(1, 0)→ (2, 0)→ (2, 1)).
- ➔ Similarly, If we want to go from (0, 0) to (6,4) then the shortest path will be 10 points.

Important Terminology

- **Computational Problem:** It is a mathematical function from a domain of legal inputs to a co-domain of corresponding expected outputs
- **Algorithm:** An algorithm is a systematic step-by-step procedure to solve a computational problem
- **Problem instance:** A specific point in the domain of definition of the problem

Example :

- ➔ $f(x) = 2^x$ → This is a problem
- ➔ f(5) = 32 → This is a problem instance
- ➔ f(8) = 256

Sorting Example :

A => 11, 33, 18, 16, 51, 54, 25

B =>	1	2	3	4	5	6	7

If :

1. (B[1] < B[2]) ^ (B[2] < B[3]) ^ (B[3] < B[4]) ^ (B[4] < B[5]) ^ (B[5] < B[6]) ^ (B[6] < B[7])

Then ⇔ B is Sorted .

2. (B[1] = A[min1] ^ B[2] = A[min2] ^ B[3] = A[min3] ^ B[4] = A[min4] ^ B[5] = A[min5] ^ B[6] = A[min6] ^ B[7] = A[min7])

Then ⇔ B is Sorted .

Recurrence relation :

If m, n > 0 => #(m, n) = #((m-1), n) + #(m, (n-1))

Terminating Condition:

If m * n = 0 => #(m, n) = 1

Example :

Find the number of shortest path from (0, 0) to (m, n) → This is a problem

Given that m = 6 & n = 4 → This is a problem instance

#(6, 4) = #(5, 4) + #(6, 3) → Recurrence relation => #(m, n) = #((m-1), n) + #(m, (n-1))

= #(5, 3) + #(4, 4) + #(5, 3) + #(6, 2)

= 2 #(5, 3) + #(4, 4) + #(6, 2)

#(6, 2) = #(5, 2) + #(6, 1)

#(6, 1) = #(5, 1) + #(6, 0)

#(6, 0) = 1 Path → Terminating Condition => (m * n =0)

Pseudocode for Grid Shortest Path

1. If (m * n) = 0
2. Then return 1
3. Else
4. Horizontal ← Grid Shortest Path (m-1, n)
5. Vertical ← Grid Shortest Path (m, n-1)
6. Return (Horizontal + Vertical)

Continuation to finding the shortest path in the grid, we have a grid of 4 rows and 6 columns. So if we want to go from (0, 0) to (6,4) then the shortest path will be 10 points. We can represent the path using 0s and 1s. For example, we can say 0 for going straight to the next point and 1 for going up.

So if the 10 points are: 0111001000, then the path will be as below: