Discrete Mathematics (SC612)
Tutorial 1
27^{th} August, 2021

1. Find the number of satisfying assignments for each of the following formulae.

   (a) $p_1 \wedge p_2 \wedge p_3$

   (b) $\neg(p_1 \vee p_2 \vee p_3)$

   (c) $(p_1 \vee p_2) \Rightarrow (\neg p_1 \wedge \neg p_2 \wedge p_3)$

   (d) $(p_1 \Rightarrow (p_2 \Rightarrow p_3))$

2. On how many assignments of truth values do the formulae $\phi_1 = ((p_1 \Rightarrow p_2) \Rightarrow p_3)$ and $\phi_2 = (p_1 \Rightarrow (p_2 \Rightarrow p_3))$ evaluate to the same truth value?

3. Let a, b and c be three atomic propositions in propositional logic. Suppose you are told that

   (i) $a \vee (b \wedge c)$ is true, and

   (ii) $(a \vee b) \wedge c$ is true.

   Which of the individual truth values can be inferred from the given information?

4. Assume the truth of the statement "Every country has at least citizen that knows at least one citizen of all other countries." Assume that every country has more than one citizen. Iceland and Norway are countries.

   (a) There is a person in Iceland who knows everyone in Norway.

   (b) There is a person in Iceland who knows no one in Norway.

   (c) There is a person in Iceland who knows someone in Norway.

   (d) Every person in Iceland knows at least one person from Norway.

5. A boolean function is said to be symmetrical with respect to a propositional variable if substituting true or false for that propositional variable leads to an identical reduced formula in the remaining variables. On which propositional variables is the formula $\phi = (\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$ symmetrical?

6. Translate the boolean function $\phi = (\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$, which is in Disjunctive Normal Form (DNF) into its equivalent Conjunctive Normal Form (CNF):

7. A boolean function is said to be symmetrical with respect to a propositional variable if substituting true or false for that propositional variable leads to an identical reduced formula in the remaining variables. For the following formulae determine in which variables they are symmetrical.

   (a) $(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$

   (b) $(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (p_1 \wedge \neg p_2 \wedge p_3) \vee (p_1 \wedge p_2 \wedge p_3)$

   (c) $(\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge \neg p_2 \wedge p_3)$

   (d) $(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge \neg p_2 \wedge p_3) \vee (\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$

8. Construct a boolean formula over 3 propositional variables such that it has exactly 5 assignments that make it true. Write it down in the form of a truth table. How many such possible formulae are there? Convert the truth table into a formula with only $\wedge$ and $\neg$. Do the same with only $\vee$ and $\neg$.

9. Write a long formula with $\wedge$, $\neg$ and $\vee$ and convert it into a formula involving only $\neg$ and $\Rightarrow$.

10. Statement 1: $\exists x$, such that $\forall y$, $x - y = 5$, where $x \in \mathcal{R}$ and $y \in \mathcal{R}$.
Statement 2: $\forall x$, $\exists y$, such that $x - y = 5$, where $x \in \mathcal{R}$ and $y \in \mathcal{R}$.

    Which of the two statements is correct?

11. $\phi = ((p_1 \Rightarrow (p_2 \vee (\neg p_1 \wedge (p_3 \vee \neg(p_1 \Rightarrow p_3))))) \vee (\neg p_1 \wedge (p_2 \vee \neg p_3)))$. Write the cascading set of formulae involving fewer variables by setting first $p_1 = T$ and next, $p_1 = F$. Reduce these formulae further by setting p_2 to the two values in succession. If you get ϕ to be true at some stage make a note of it. This means the formula is true regardless of the truth values of unassigned variables.

12. Consider the 2-player game where the players A and B play alternately. The initial configuration is a list $1, 3, 4, 2$. The play begins with player A. A each step the player to move is allowed to interchange the position of any two elements. However a move is illegal if it leads to the same configuration as has already occurred earlier in the game. If at any point the configuration 1,2,3,4 is reached then player A is the winner and if 4,3,2,1 is reached then player B is the winner (it doesn't matter which player made the move leading to the winning configurations, only the configurations matter). If a player to move doesnt have a legal move then the game ends in a draw. For the given configuration determine the outcome with best play (each player trying to win) by both players.

    Write down the description of a win for a player in a 2-player strategy game in the form of a predicate logic formula.