Continue with Graph Theory

Graph :

- A graph G = (V , E) consists of V , a nonempty set of vertices and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Sub-graph :

- Suppose we have G=(V,E) such that V is a vertex set and E is a Edge Set.
- Here H=(V',E') is a Subgraph of G=(V,E) if $V' \subseteq V$, $E' \subseteq E$ and H is a Graph.

Induced Sub-graph :

- (x,y) element of E(H)⇔(x,y) element of E(g) and x element of v(h) and y element of v(h) and xεv(g) and yεv(g).
- Any induced subgraph is a subgraph but not every subgraph is an induced sub-graph.

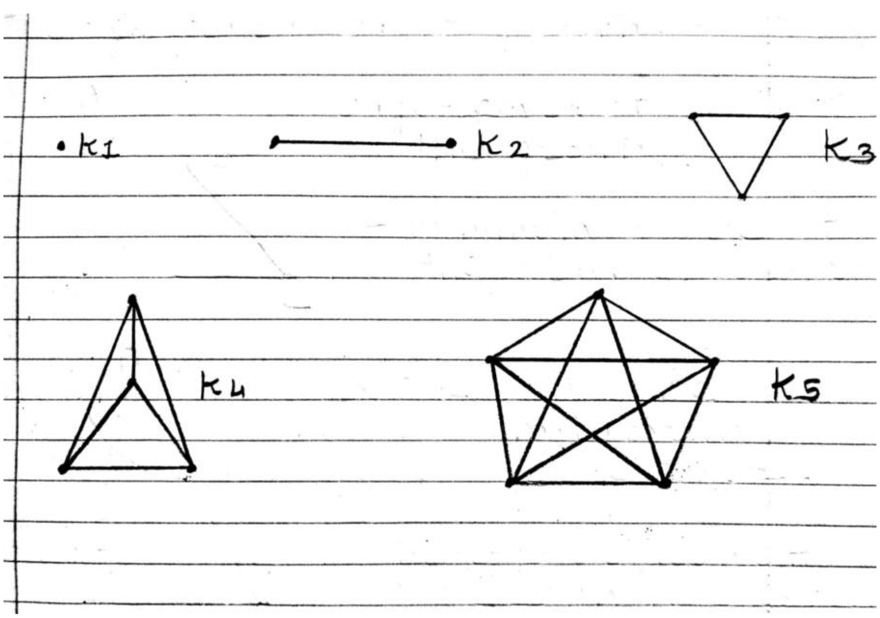
Spanning Sub-Graph :

- Spanning subgraph is again a subgraph but V’ - For the subset of the vertex set to the subgraph such that V’=V.
- You have to select all vertices but it has no restriction of what edges are present or absent.
- So, every edge has a choice for being present or absent in spanning subgraph but you must include all the vertices.
- The no of edges is usually denoted by m or cardinality of edge is |E|.

Complete Graph :

- Denoted By Kn for every n vertices.
- All the pairs are adjacent basically.
- E={(x,y) | xεv,yεv}.

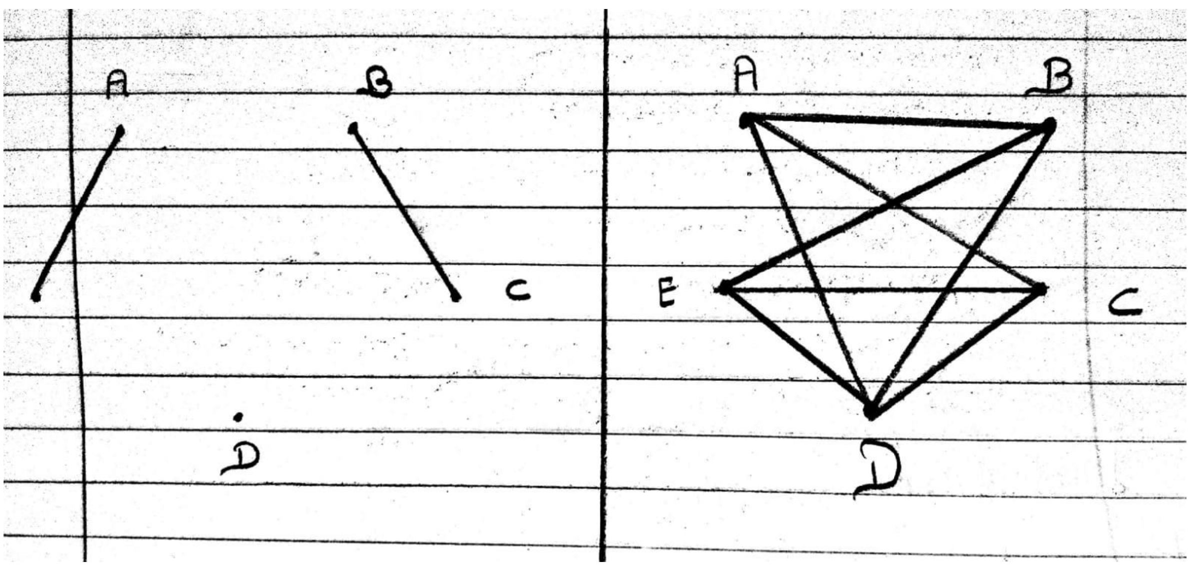
Example :



Graph Complement :

- It is a spanning subgraph where you include all edges which are absent exclude all edges which are present.
- It is a special case of graph partition.
- G=(V,E)
- $\bar{G}$ = (V,E bar = Kv\ E). (Set difference).

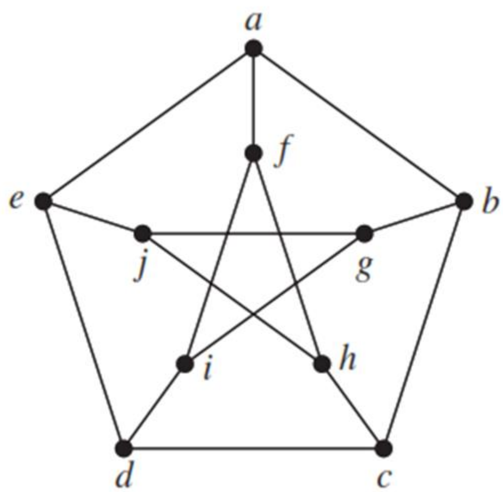
Diagram :



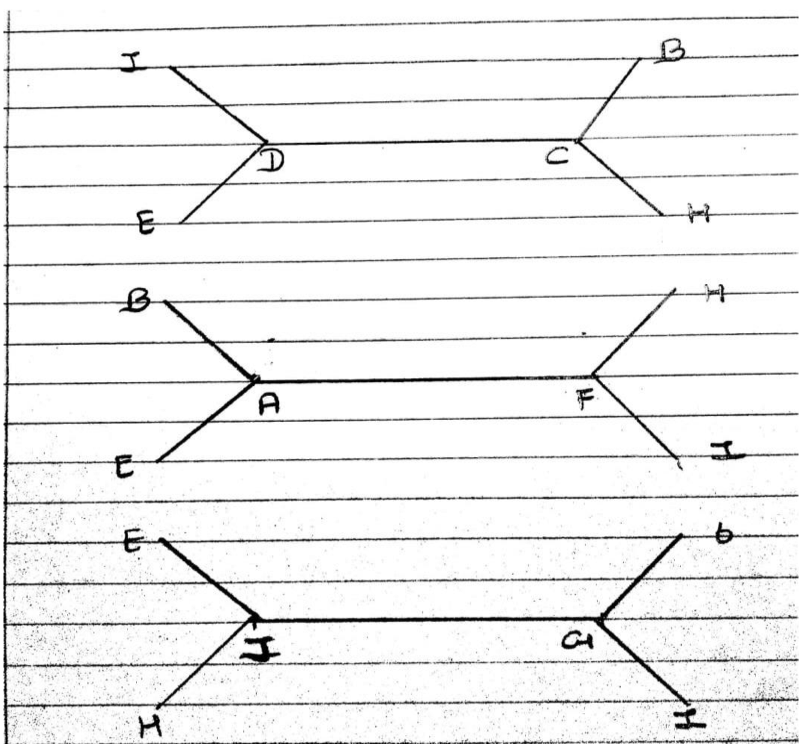
Partition Of Graph :

- Collection of subsets of a set whose union is a whole set and whose intersection is empty.
- Let us now partition of **petersen Graph :**

petersen Graph :



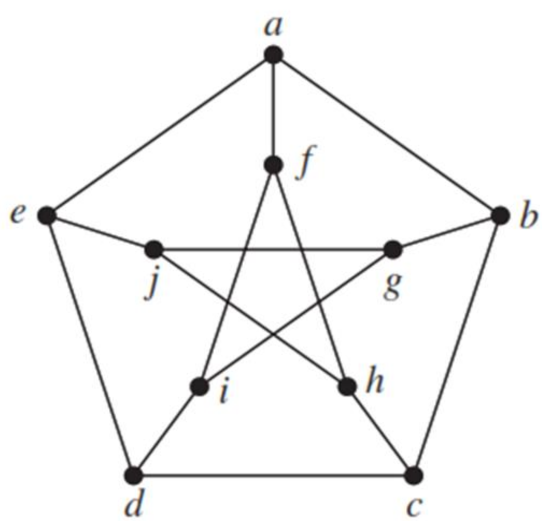
Partition of the petersen graph :



This is called Graph Decomposition.

Vertex Cover :

- A set of vertices that cover all the edges is called a vertex cover.
- Vertex cover is a subset of vertices that cover all edges.
- Vertex cover on **petersen graph.**



- We can not cover petersen graph with less then 5 vertices.
- Because every vertex cover has 3 edges and you want to cover 15 edges.
- If pick a vertex A it can cover AF,AE,AB.
- 5 vertices maybe insufficient, because some 2 vertices may covering some common edge in which case it doesn't help.

Minimum size vertex Cover :-

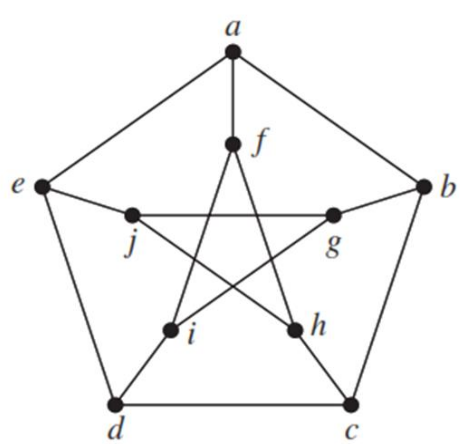
- A set of minimum no of vertices that cover all the edges is called a vertex cover.

- Here by using {a,c,e} we can cover all the outer edge.
- {g,i,h} we can cover all the inner edges.
- 6 vertices cover all the 15 edges this is called minimum size vertex cover.

Dominating Set :-

- A set of vertices but they must cover all vertices.
- Dominating set is a subset of vertices such that every vertex not in a subset is adjacent to vertex is a subset.

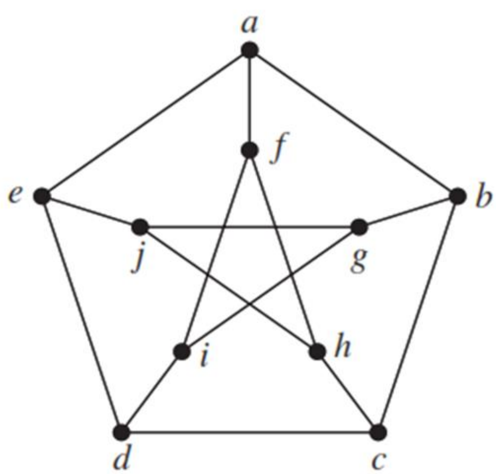
Dominating set on petersen graph:-



- Here If we pick **a** we are dominating **e,b,f.**
    - **a-e,b,f.**
    - **d-c,i.**
    - **j-g,h.**
- a,d,j cover all other vertices.
- A vertex cover is always a dominating set , but a dominating set need not to be a vertex cover.

Independent Set :-

- Independent set is a subset of vertices such that the induce subgraph has 0 edges.
- Independent set on petersen graph.

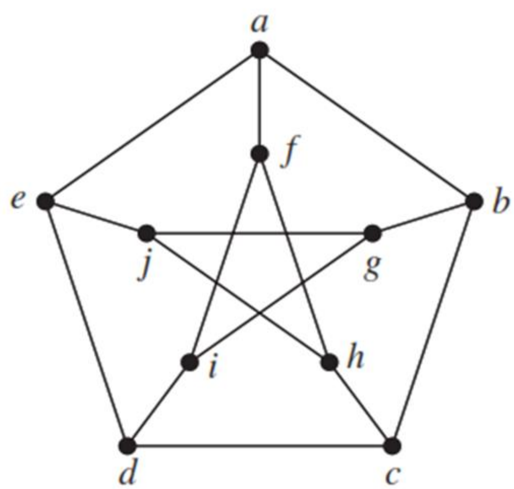


- If you pick {A} you can not pick e,f,b.
- Here {a,j,i,c} is an independent set.
- Maximum independent set of petersen graph is exactly 4. Denoted by $\alpha(4)$.

Clique :-

- Which is a largest subset which induces complete graph.
- Denoted by $\omega()$.
- Biggest clique in petersen graph :- 2

- Under graph complementation every clique becomes an independent set and every independent set becomes a clique when you do graph complement.



- {i,h,b} - Independent set.
- {a,b} - clique.
- {a,b,i} - neither clique nor independent set.
- Every subset of independent set is also independent.
- Every subset of clique is also a clique because it is complete graph.

- If $I \subseteq V$ and $k \subseteq v$ and $I \cup k \subseteq V$,
  $| I \cup k | <= |V|$,

Applying inclusion and exclusion :-

- $| I \cup k | = | I | + | k | - | I \cap k |$.

- What is the upperbound of | I | + | k | ?
  - | I | + | K | = | I U K | + | I ∩ k|.

- What is the maximum value of I ∩ k ?
  1.

- What is the maximum value of sum ?
  N+1.

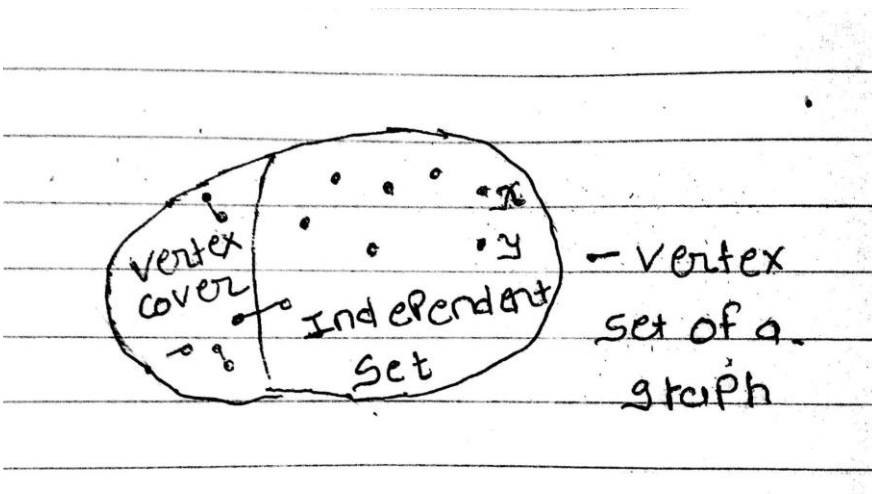
For any graph ,

- | I | + | k | <= n+1
  Where I = α , k = ω
  $\alpha + \omega$ <= n+1
  $\alpha(G) = \omega(\overline{G})$
  $\omega(G) = \alpha(\overline{G})$

- Since every clique become independent set and every independent set becomes clique so , The largest independent set and largest independent set becomes a largest clique.

- Let us now co-relate α & β.
- $\alpha - vertex\ cover.$
- $\beta - independent\ set.$

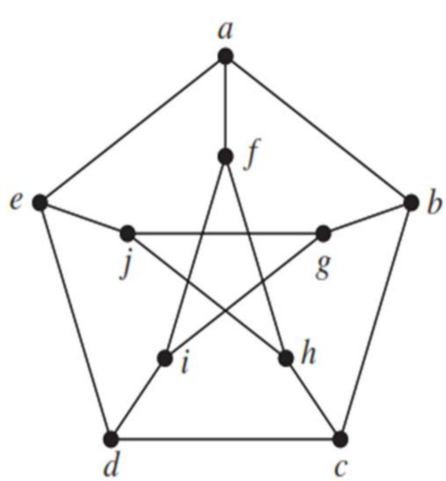
Vertex set of the graph :



- $\alpha+\beta=n$.
- The smaller the vertex cover larger the independent set.

Edge - cover :

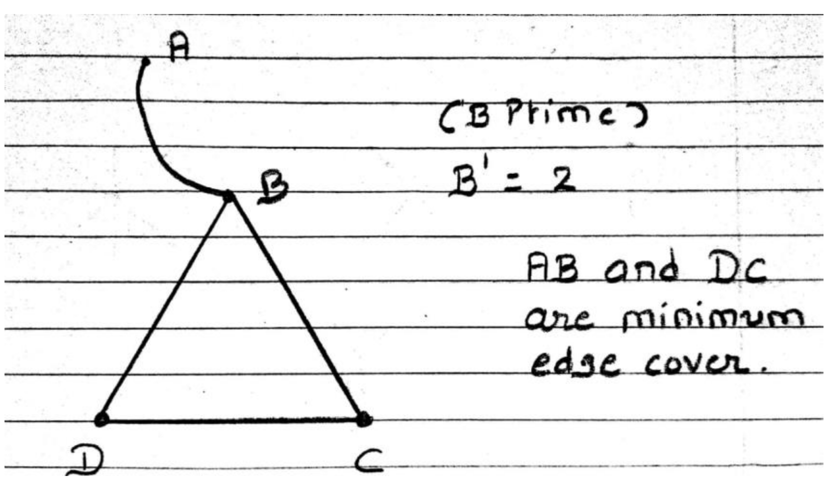
- it is a subset of edges that cover all vertices.
- Edge cover on petersen graph.



- Every edge contain utmost 2 vertices.
- But sometimes we get duplicate covers. Ab and af. Then a is covers twice.
- If there are n vertices every edge can cover atmost 2 vertices.
- Trivial example of a edge cover is 1,2,3,4,5 this is crossing edges.can you see all 10 vertices are covered.

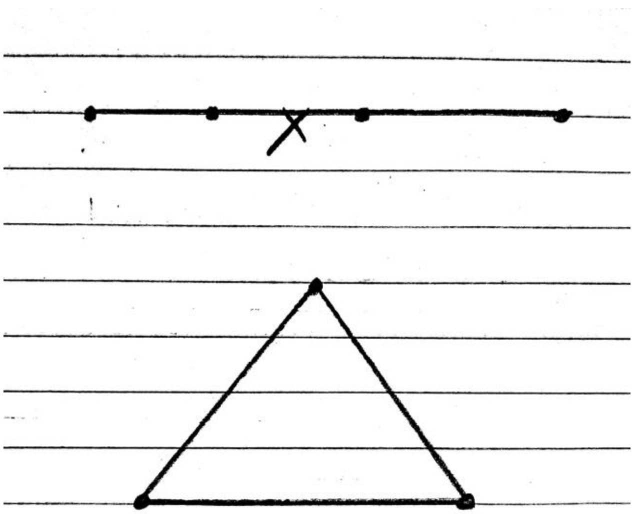
Minimum Edge cover : (β’)

- Minimum edge cover is a no of edges in the smallest edge cover of a graph.
- Minimum degree of the vertices is atleast 1.



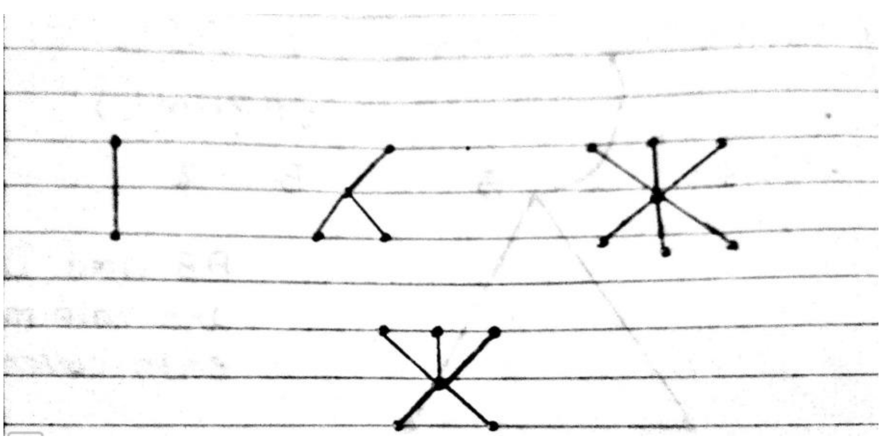
Not allowed diagram :-

- There is no path of 3 edges in the sub graph induced by edges minimum edge cover.



Allowed Diagram :

- Not part of minimum edge cover.
- Minimum Edge cover looks like this :-



- The kind of graph called stars special class of trees.

- A minimum edge cover is always a collection of vertex disjoint stars.