Scribed Notes 22 (15th November, 2021)

- **MATRIX :**

  Matrix is a type of 2D array. It is a collection of numbers.
  They could represent states or symbols they can also have real number entries.

- **Scalar Multiplication :**

$$3\begin{bmatrix} 7 & 9 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 21 & 27 \\ 12 & 0 \end{bmatrix}$$

(Multiply each element by 3)

$$\boldsymbol{k} \times \boldsymbol{A} = \boldsymbol{k} \times \begin{bmatrix} a & \mathrm{b} & \mathrm{c} \\ d & \mathrm{e} & \mathrm{f} \\ g & \mathrm{h} & \mathrm{i} \end{bmatrix} = \begin{bmatrix} k \times a & k \times \mathrm{b} & k \times \mathrm{c} \\ k \times \mathrm{d} & k \times \mathrm{e} & k \times \mathrm{f} \\ k \times g & k \times \mathrm{h} & k \times \mathrm{i} \end{bmatrix}$$

- **Addition :**

$$\begin{bmatrix} 7 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 17 \\ 11 & 22 \end{bmatrix} = \begin{bmatrix} 4 & 17 \\ 15 & 23 \end{bmatrix}$$

(Addition term by term)

$$\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} a & \mathrm{b} & \mathrm{c} \\ d & \mathrm{e} & \mathrm{f} \\ g & \mathrm{h} & \mathrm{i} \end{bmatrix} + \begin{bmatrix} j & \mathrm{k} & \mathrm{l} \\ \mathrm{m} & \mathrm{n} & \mathrm{o} \\ p & \mathrm{q} & \mathrm{r} \end{bmatrix} = \begin{bmatrix} a + j & \mathrm{b} + \mathrm{k} & \mathrm{c} + \mathrm{l} \\ \mathrm{d} + \mathrm{m} & \mathrm{e} + \mathrm{n} & \mathrm{f} + \mathrm{o} \\ g + p & \mathrm{h} + \mathrm{q} & \mathrm{i} + \mathrm{r} \end{bmatrix}$$

- **Matrix Multiplication :**

M x N = M rows, N cols.
(M x N) x (N x P) ≡ M x P **(TRUE)**
Square Matrix = (m = n)

Multiplication of 3×3 matrix

If $A = \begin{bmatrix} a & \mathrm{b} & \mathrm{c} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} \\ g & \mathrm{h} & \mathrm{i} \end{bmatrix}$ and $B = \begin{bmatrix} j & \mathrm{k} & \mathrm{l} \\ \mathrm{m} & \mathrm{n} & \mathrm{o} \\ p & \mathrm{q} & \mathrm{r} \end{bmatrix}$ then the multiplication of $A \times B$ will be:

$$\boldsymbol{A \times B} = \begin{bmatrix} \boldsymbol{aj + bm + cp} & \mathbf{ak + bn + cq} & \mathbf{al + bo + cr} \\ \mathbf{dj + em + fp} & \mathbf{dk + en + fq} & \mathbf{dl + eo + fr} \\ \boldsymbol{gj + hm + ip} & \mathbf{gk + hn + iq} & \boldsymbol{gl + ho + ir} \end{bmatrix}$$

Upper Triangular Matrix : A square matrix $A = [a_{ij}]$ is said to be upper triangular matrix, if $a_{ij} = 0$ whenever $i > j$.

Example : $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}_{3\times3}$

Lower Triangular Matrix : A square matrix $A = [a_{ij}]$ is said to be lower triangular matrix, if $a_{ij} = 0$ whenever $i < j$.

Example : $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}_{3\times3}$

❖ **Examples**

1. $\begin{bmatrix} 4 & 0 & 3 \\ 2 & 5 & 2 \end{bmatrix}_{2*3}$ x $\begin{bmatrix} 3 & 6 \\ 11 & 1 \\ 5 & -2 \end{bmatrix}_{3*2}$ = $\begin{bmatrix} 27 & 18 \\ 71 & 13 \end{bmatrix}_{2*2}$

2. Identity Matrix (square):

$$\begin{bmatrix} 3521 & 233 & 79 \\ 290 & 368 & 44 \\ 21 & 111 & 2048 \end{bmatrix}_{3*3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3*3} = \begin{bmatrix} 3521 & 233 & 79 \\ 290 & 368 & 44 \\ 21 & 111 & 2048 \end{bmatrix}_{3*3}$$

❖ Practical use of Matrix :

- How much of each raw material is used per unit of the product ?

$$(\text{Product 1 to n}) \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix} \text{x} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{(per unit cost of raw material)}$$

Here a product is made using different units of different things the final matrix comprises of cost of the product .Here there can be various column vectors as there can be various product vendors.

❖ Linear Equations :

$3x_1 + 7x_2 - 43x_3 = 22$

$x_1 + 2x_2 - x_3 = 77$

$7x_1 - x_2 + 21x_3 = 16$

Matrix form of this equations is

$$\begin{bmatrix} 3 & 7 & -43 \\ 1 & 2 & -1 \\ 7 & -1 & 21 \end{bmatrix} \text{x} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 77 \\ 16 \end{bmatrix}$$

- **Elementary Row Operation**:

- There are some legit operations that do not change the course of solution.
- We can interchange any 2 rows and multiply by scalar value (NON ZERO)

❖ **Augmented Matrix :** (using above example)

$$\begin{bmatrix} 3 & 7 & -43 & 22 \\ 1 & 2 & -1 & 77 \\ 7 & -1 & 21 & 16 \end{bmatrix}$$

Swap R_1 & R_2

$$\begin{bmatrix} 1 & 2 & -1 & 77 \\ 3 & 7 & -43 & 22 \\ 7 & -1 & 21 & 16 \end{bmatrix}$$

($R_2 \leftarrow R_2 - 3R_1$)

($R_3 \leftarrow R_3 - 7R_1$)

$$\begin{bmatrix} 1 & 2 & -1 & 77 \\ 0 & 1 & -40 & -209 \\ 0 & -15 & 28 & -523 \end{bmatrix}$$

($R_3 \leftarrow R_3 + 15R_2$)

$$\begin{bmatrix} 1 & 2 & -1 & 77 \\ 0 & 1 & -40 & -209 \\ 0 & 0 & -572 & -3658 \end{bmatrix}$$

($R_1 \leftarrow R_1 - 2R_2$)

{similarly if we reduce the entire matrix to identity matrix, the 4^{th}column is the values of X_1,X_2,X_3 respectively}

Matrix multiplication is Non-Commutative($AB \neq BA$) but Associative($(AxB)xC = Ax(BxC)$).

- **INVERSE OF MATRIX:-**

  {$A\times A^{-1} = A^{-1}\times A = I$ OR $A\times I = A^{-1}$}

$$\begin{bmatrix} 1 & 7 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

($R_2 \leftarrow R_2 - 3R_1$)

$$\begin{bmatrix} 1 & 7 & 1 & 0 \\ 0 & -16 & -3 & 1 \end{bmatrix}$$

($R_1 \leftarrow R_1 + \frac{7}{16}R_2$)

$$\begin{bmatrix} 1 & 0 & \frac{-5}{16} & \frac{7}{16} \\ 0 & -16 & -3 & 1 \end{bmatrix}$$

$(^{R2}/_{-16})$

$$\begin{bmatrix} 1 & 0 & \frac{-5}{16} & \frac{7}{16} \\ 0 & 1 & \frac{-3}{16} & \frac{-1}{16} \end{bmatrix}$$

So here we get $A^{-1} = \begin{bmatrix} \frac{-5}{16} & \frac{7}{16} \\ \frac{-3}{16} & \frac{-1}{16} \end{bmatrix}$

Now if we do $A \times A^{-1}$,we get identity matrix(I).

$$A \times A^{-1} = \begin{bmatrix} 1 & 7 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} \frac{-5}{16} & \frac{7}{16} \\ \frac{-3}{16} & \frac{-1}{16} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence $\{A \times A^{-1} = A^{-1} \times A = I\}$ Proved…