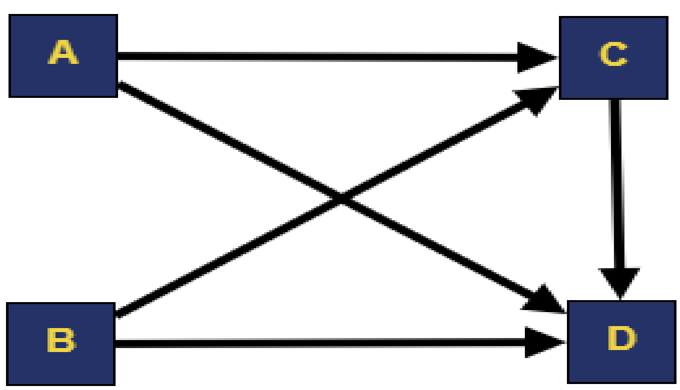
Scribed Notes 9

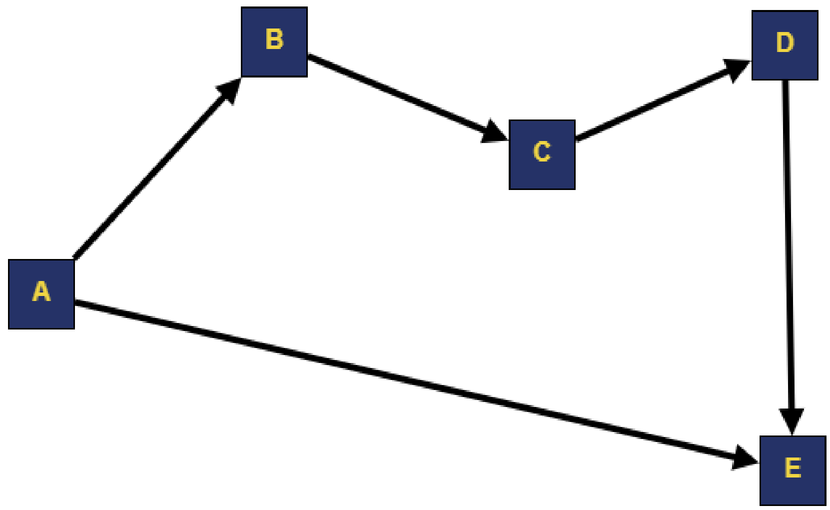
➢ Directed Acyclic Graph

- A directed acyclic graph is a directed graph with no directed cycles. That is, it consists of vertices and edges, with each edge directed from one vertex to another, such that following those directions will never form a closed loop.

- All directed acyclic graphs need not necessarily be partial order.

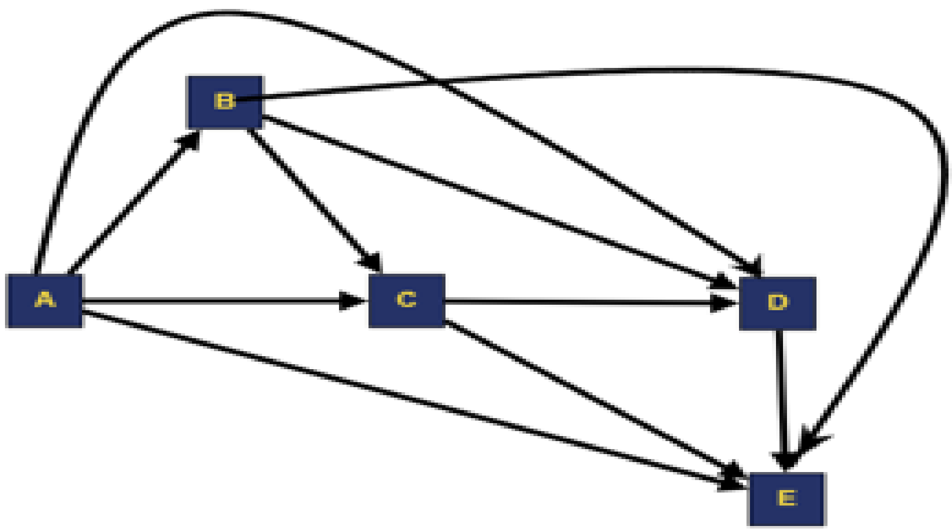


Acyclic Directed Graph (Partial Order) Example 1



Acyclic Directed Graph

(Not a partial Order)

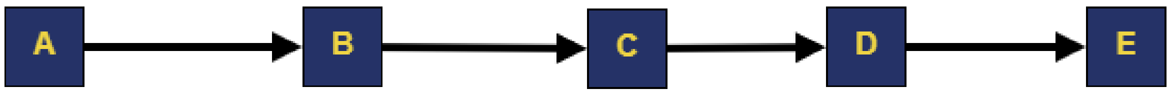


Acyclic Directed Graph

(Partial Order) Example 2

Hasse Diagram:-

- Associated with every partial order, there is a minimal representation where we only put those edges whose presence can't be determined by transitive closure.



Hasse Diagram of Example 2

Note:- Partial Order and Hasse Diagram are special cases of Directed Acyclic

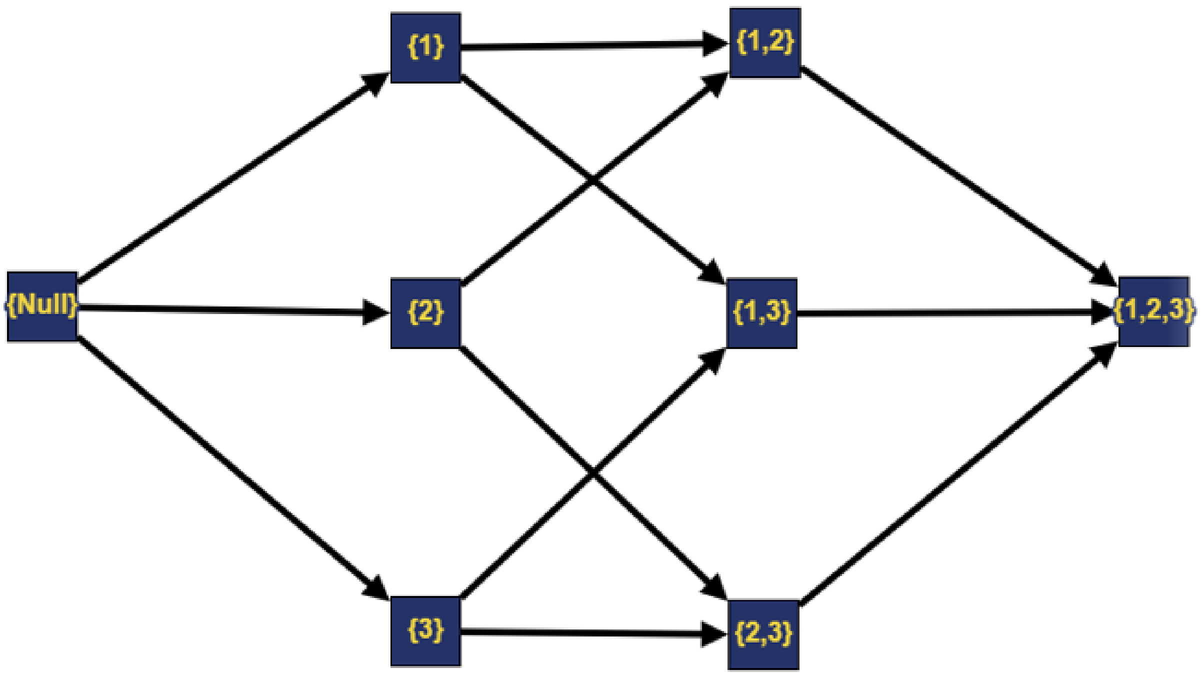
Source and Sink:-

- A local source is a node of a directed graph with no entering edges, and a global source (often simply called a source) is a node in a directed graph which reaches all other nodes.
- A local sink is a node of a directed graph with no exiting edges, also called a terminal. A global sink (often simply called a sink) is a node in a directed graph which is reached by all directed edges

Topological Sort:-

- It gives a linear ordering of vertices in a directed acyclic graph such that, for every directed edge a->b, vertex 'a' comes before vertex 'b'
- In topological sort we have to select vertices where **in-degree is zero** and then we go further by not considering the vertices selected earlier so that we get new vertices where in-degree is zero. This process is continued further till the end.

- Since we are doing it for a partial order, while doing this process we may get only one source or multiple sources and similarly one sink or multiple sinks. If we get only one source, it will be unique and will be known as **lower lattice.** If we get only one sink it will be unique and will be known as **upper lattice.**



Hasse diagram of the power set partial order where (a,b) belongs to R if and only if A is a subset of B

✓ **In terms of Propositional logic, we can denote it as follows:**

- p : exactly one source

- q : exactly one sink

1. $\neg p \wedge \neg q$:- Not upper/lower lattice
2. $p \wedge \neg q$:- Lower lattice
3. $\neg p \wedge q$:- Upper lattice
4. $p \wedge q$:- Lattice

- In the Hasse diagram of the power set partial order mentioned above, we can depict it without using the arrows by putting binary bits in place of the sets.
- In our example, we have taken {1,2,3}; so the bit value of this set would be 111. Wherever an element is absent, we put a 0 in place of it. So a set {1,2} would be 110, {2,3} would be 011 and so on.
- Changing an element from 0 to 1 is the same as adding an element to a subset to get a bigger subset. This family of graphs is known as Hypercubes.

Hypercubes:-

- In hypercube graph Q(n), n represents the degree of the graph. Hypercube graph represents the maximum number of edges that can be connected to a graph to make it an n degree graph, every vertex has same degree n and in that representation, only a fixed number of edges and vertices are added as shown in the figure below :

- The directed version of Qn is the same as Hasse diagram of the power set partial order of a set of n elements.

Q(0) with vertex 1

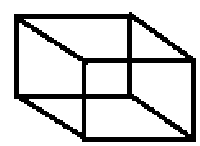
(Single vertex)

Q(1) with vertex 2

(Complete graph)

Q(2) with vertex 4

(Cycle of length 4)



Q(3) with vertex 8

(Cubical graph)

- So, going from one small subset to a bigger subset of the next order, a change of only one bit is required.
- So, for Q3, it would be 000,001,010,100,011,110,101,110,111.

- From 000, by changing one bit we can go to 001 or 010 or 100. Similarly, we can go on till we reach 111.
- For Q4, we just have to put a 0 once and 1 once at the end of each element in Q3 to get a new set of bit values. Again for Q5, we will do the same that we did to get Q4 from Q3. Therefore, we are taking 0 and 1 and just placing it at the end of each element of Q4 to get another 16 elements.