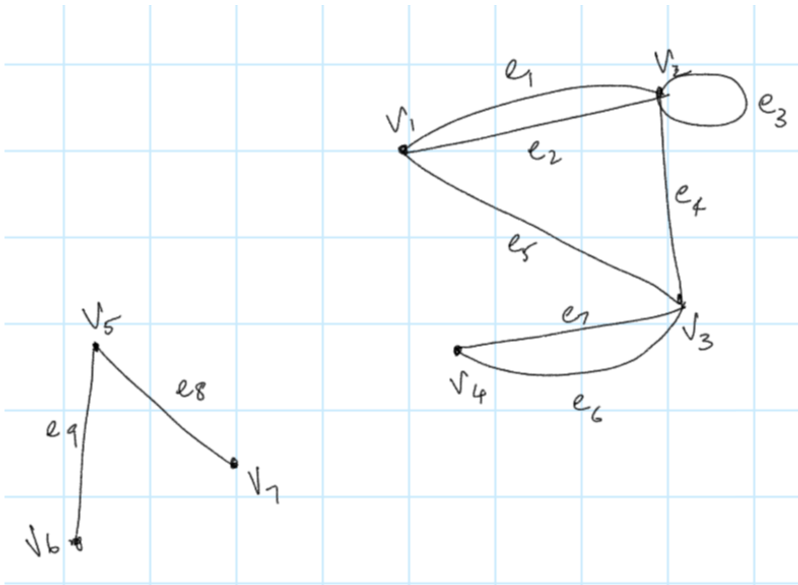
Scribed Notes 26

Walk:

A walk in an undirected graph is an alternating sequence of vertices and edges of the graph, beginning with a vertex and ending with a vertex such that for every edge in the sequence, the vertices immediately before and after it, are its two (not necessarily distinct) endpoints.



V1 e1 e2 V2
The above sequence is not a walk as the vertices and the edges are not alternating.

V1 e1 V3 e4 V2
The above sequence is not a walk as vertices before and after e1 are not its endpoints.

e1 V1 e2 V2 e3
The above sequence is not a walk as the sequence is not starting with a vertex.

V1 e1 V2 e3 V2 e4 V3
The above sequence is a walk.

Open Walk:
A walk that begins and ends at different vertices.
Eg: V1 e1 V2 e3 V2 e4 V3

Closed Walk:
A walk that begins and ends at the same vertex.
Eg: V1 e1 V2 e3 V2 e4 V3 e5 V1

From the above definition and examples, we can say in a walk,
Number of Vertices = Number of Edges + 1

Length of Walk :

The number of edges in a walk (not necessarily distinct) as a sequence (repeated edges are not ignored).
Eg: V1 e1 V2 e3 V2 e3 V2 e4 V3
Length of the above-given walk = 4

Note:

- Simple graphs do not contain self-loops or parallel edges. Often, it also assumes unweighted implicitly.
- An unweighted graph is a special case of a weighted graph, where all the edges have weight 1.
- Maximum walk length is unbounded in any graph with at least one edge, as the edges and vertices can occur as many times as we want.

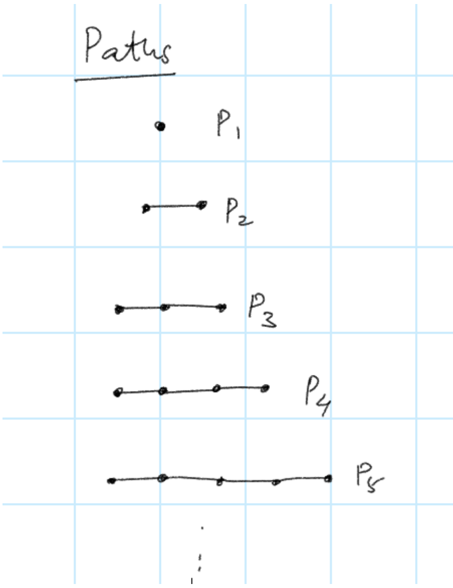
Trail :

A walk where repeated edges are forbidden.
Trail is a special case of a walk.

Eg : V1 e1 V2 e3 V2 e4 V3 e5 V1 e2 V2
Length : 5
Degree of vertices in the trail :
V1 = 3
V2 = 5
V3 = 2
The above trail can be called a **Maximal Trail** as it cannot be further extended at its endpoints.
It is also an open trail as the starting and ending points are different.

For an open trail, the degree of the **vertex at the beginning** is always going to be **odd**. Similarly, the degree of the **vertex at the ending** is always going to be **odd** for an open trail. But the vertices in the **middle** will have an **even degree**.

Path:

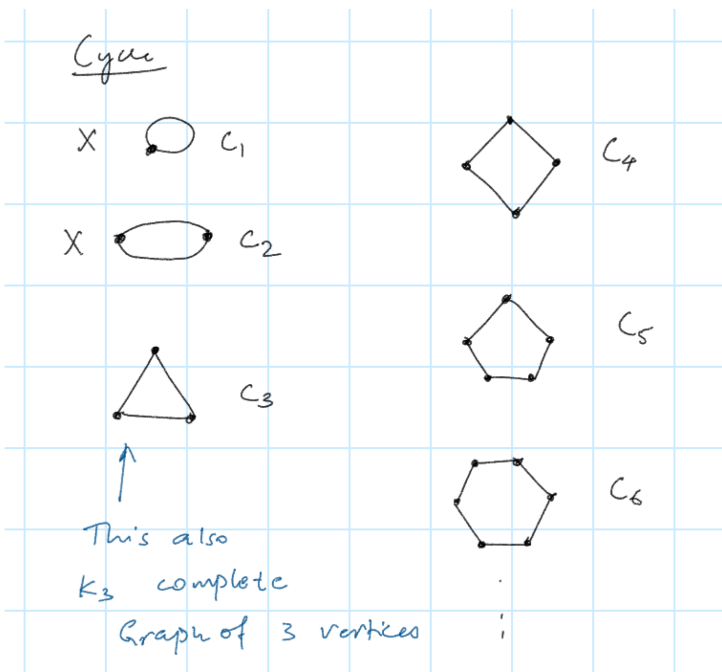


A walk where repeated vertices are forbidden.
If a vertex cannot be repeated in a walk then an edge also cannot be repeated as an edge is a combination of 2 vertices.

Degree Sequence for a path P_k = 1, 1, 2, 2, … ,2 ((k-2) times)

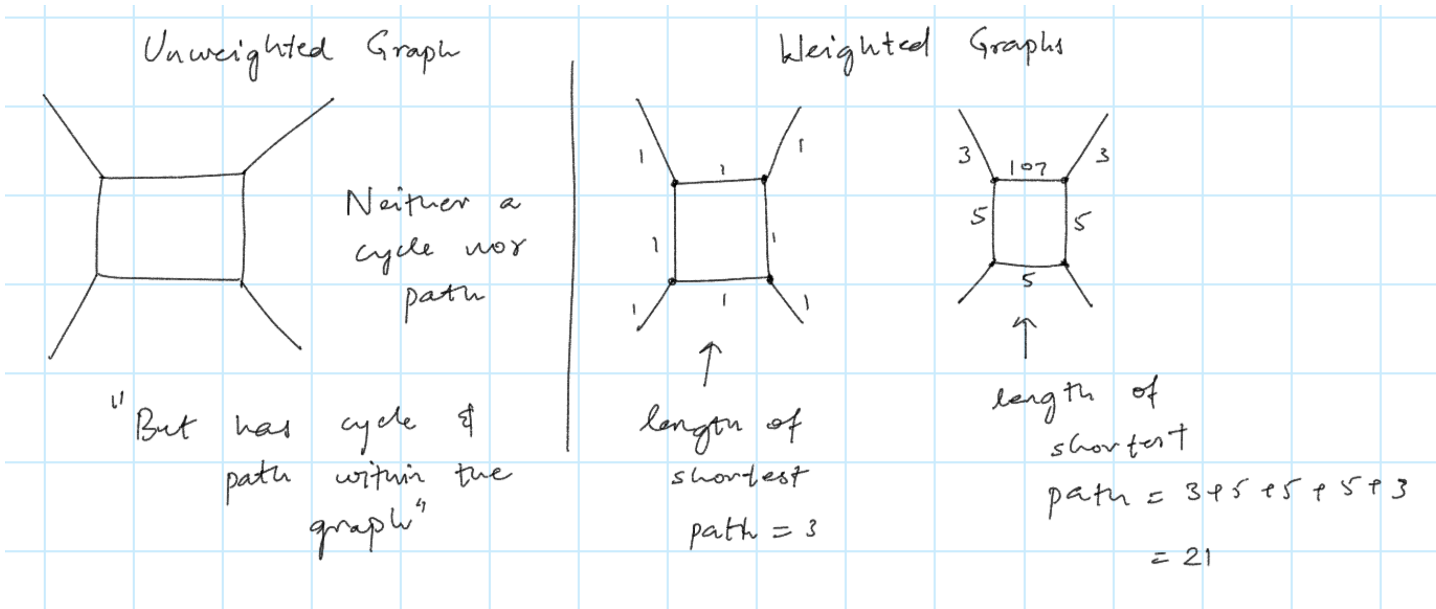
Path is a special case of a Trail and therefore a walk.

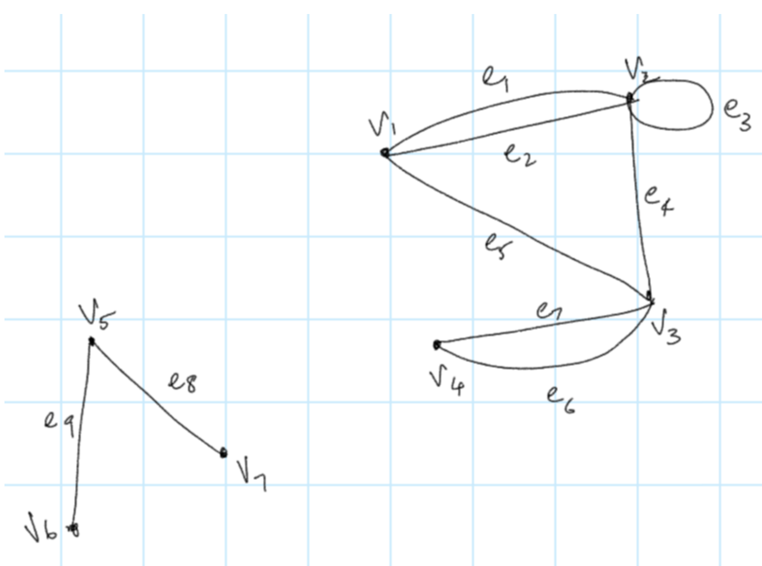
Cycle :



A closed walk where exactly one vertex is repeated.

Weighted and Unweighted Graphs





Connected Graphs:

$$\forall x, y \in V, \exists P: x \to y$$

Disconnected Graphs:

Negation of Connected graphs :

$$\exists x, y\ ,\ \nexists P: x \to y$$

The above given graph is not connected or is a disconnected graph as there no path from vertex V1 to V5.

In simple graphs, walks (and therefore trails/ paths/ cycles) are representable as sequences of **only vertices**.
As it doesn't contain any self loops or parallel edges. Hence there is no ambiguity about the edge being chosen to move from one vertex to another.

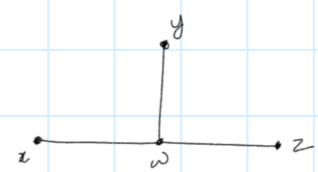
Results

1. Every $x \to y$ walk contains an $x \to y$ path.

If we have a walk as given below in the image, then even if we truncate the part in the brackets, the sequence will continue to be a walk. So, if we remove the sequence till the repeated vertex then we get a path which is a part of the original walk.

(v_1 . - - .) v_1 . . - -

2. If $\exists P_1: x \to y$ and $\exists P_2:\ y \to z$ then $\exists P_3:\ x \to z$



P_1 : x w y

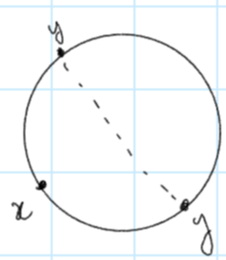
P_2 : y w z

P_3 : x w y w z

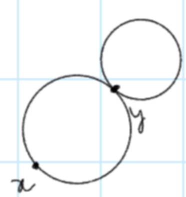
→ since we can remove the repeated vertex (i.e the diversion)

P_3 : x w z

3. Every closed odd walk contains an odd cycle.



odd walk



The given image is an odd walk and let say there is a repeated vertex 'y' then we can short circuit it and make a smaller cycle so either the cycle one has odd number of vertices or the cycle two either way the above statement holds true.