Unit:2

SETS-

- What is a set?

  - A set is a collection of **distinct**, **well-defined** objects; usually called **elements** (of the set).

  - Here Well-defined states that membership of any element should not be ambiguous.

- **Cardinality** of set-

  - Cardinality refers to the number of elements in the set.
  - Example:
    - Let set S = {1, BROWN, WOLF, A, 79, HELLO}
  - In the above example the cardinality of S will be denoted as |S| and will be equal to 6.
  - |S|=6

- Various Operations on Sets-
  - Union - $\cup$
  - Intersection - $\cap$
  - Difference - $\setminus$
  - Symmetric Difference – $\triangle$
  - Complement - A^c or A' or $\bar{A}$
  - Universal Set – U
  - Cartesian Product – $A \times B$

- Powerset - P(S)
- Subset - ⊆
- Proper Subset
- Venn-Diagram
- Finite Sets
- Infinite Sets

- Understanding Concepts by examples-

Let –

S1 = {1,3,4,8,10}

S2 = {3,4,7,9,10,11,13}

U = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}

$\overline{S1}$ = {2,5,6,7,9,11,12,13,14,15}

$\overline{S2}$ = {1,2,5,6,8,12,14,15}

S1 U S2 = {1,3,4,7,8,9,10,11,13}

S2 ∩ S2 = {3,4,10}

S1 \S2 = {1,8}

S2 \ S2 = {7,9,11,13}

S1∆ S2 = {1,7,8,9,11,13}

Eg: attendance sheet-

e_1	e_2	-	-	-	e_n
0	1	-	-	-	1

Present=1

Absent=0

- **<u>Characteristic Vector:</u>**

$|\bar{S}1| = |U| - |S1|$

Subsets of S can be mapped uniquely to a string over {0,1} of length |S|.

e.g.

U = {1 2 3 4 5 6 7 8 9 10 11 12 13 14 15}
S1 = {1 0 1 1 0 0 0 1 0 1 0 0 0 0 0}
S2= {0 0 1 1 0 0 1 0 1 1 1 0 1 0 0}

A set of n elements has 2^n subsets.

Empty set is denoted by ϕ.
|ϕ|=0

S is a subset, these two are called **improper subsets (**ϕ, S), other are called **proper subsets.**

Powerset is the collection of all subsets (proper + improper)

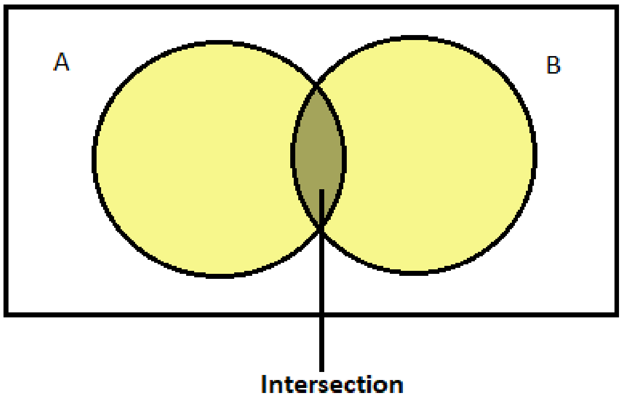
Example:

S = {1,2,3}

P(S) = {ϕ, {1},{2},{3}, {1,2}, {2,3}, {1,3}, {1,2,3}}

P(S) = {000,100,010,001,110,011,101,111}

Venn Diagram:

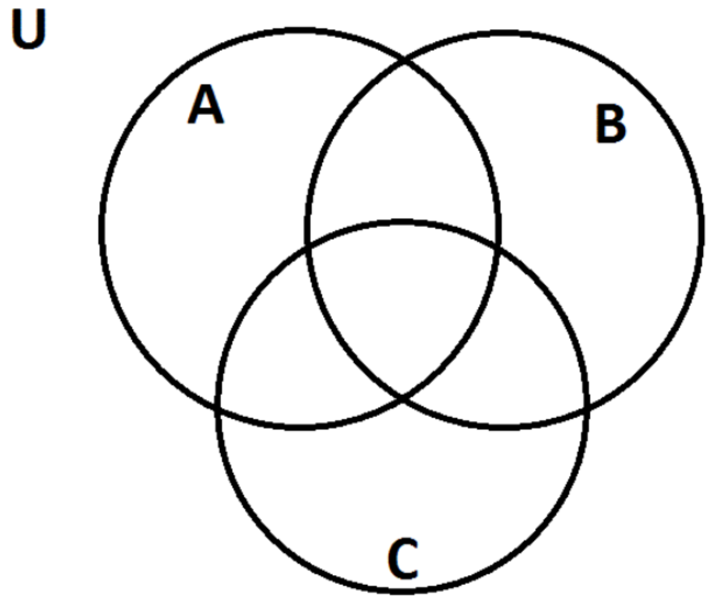


A U **B:** Union

A ∩ **B:** Intersection

| A U **B | = |A| + |B| - | A** ∩ **B |**

PRINCIPLE OF INCLUSION AND EXCLUSION:



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap D| - |A \cap D| - |B \cap D| - |A \cap C| + |A \cap B \cap C| + |A \cap D \cap C| + |A \cap B \cap D| + |D \cap B \cap C| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$$

The formula for the Inclusion-Exclusion Principle becomes:

(Next Page)

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{1\le i<j\le n} |A_i \cap A_j| + \sum_{1\le i<j<k\le n} |A_i \cap A_j \cap A_k| - \ldots + (-1)^{n-1} \left|\bigcap_{i=1}^{n} A_i\right|$$

- **Cartesian Product-**

R= {a, b}
S= {1,2,3}

R X S= {(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)}
S X R= {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)}

|R X S|=6=|R| X |S|

H's not commutative

S X S= {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}

A relation from domain(D) to co-domain(C) is any subset of the cartesian product (D X C)

Relation = $2^{|D|*|C|}$

$2^{|s|*|s|}$ or 2^{n*n}