Discrete Mathematics

Scribed Notes 15 (18th October)

Recurrence Relation :

A Recurrence relation of the sequence $\{a_n\}$ is a question that expresses an a_n in term of one or more of the previous term of the sequence, namely, a_0, a_1, a_2, a_3, …a_{n-1} for all integer n with $n \geq n_0$, where n_0 is non negative integer.

Generating Function :

Generating functions are used to represent sequences efficiently by coding the terms of a sequence as coefficients of powers of a variable *x* in a formal power series.

Generating functions can be used to solve recurrence relations by translating a recurrence relation for the terms of a sequence into an equation involving a generating function.

Definition of Generating Function:

The *generating function for the sequence* $a_0, a_1, \ldots, a_k, \ldots$ of real numbers is the infinite series.

$$G(x) = a_0 + a_1x + \cdot\cdot\cdot + a_kx^k + \cdot\cdot\cdot = \sum_{k=0}^{\infty}(a_kx^k)$$

- **A quick solution of Q6(Tutorial - 3) using recurrence relation**.

Base Case:

f (1) = 1; (if 1 element)

f (2) = 2; (if 2 element)

f (3) = 4; (if 3 element)

for example

- 1,2,3

number of consecutively increasing or decreasing Sequence are 4

1. 2,1,3 (dec)
2. 2,3,1 (inc)
3. 1,3,2 (inc)
4. 3,1,2 (dec)

here (1) and (4) are Dual such that (2) and (3) are Dual.

Move form (n-1) to n position (like induction).

1,7,4,10,2,9,3,8,5,6

10,4,7,1,9,2,8,3,6,5

In This Example Dual are (1,7), (4,10), (2,9), (3,8), (5,6)

here Domain and Co-Domain are equal so it is similar as Bijective Function.

Let's Solve f (4) using Recurrence:

- 4 at Position 1

  4, x, y, z

2 Solution For decrease

so; Total * $\frac{1}{2}$ $\binom{3}{3}$ * $\frac{1}{2}$ f(3)

- 4 at last (Position 4)

  x, y, z, 4

  decrease

  $\binom{3}{0}$ * $\frac{1}{2}$ f(3)

- 4 at Position 2

  x, 4, y, z $\binom{3}{1}$ * $\frac{1}{2}$ f(3)

- 4 at Position 3

  x, y, 4, z $\binom{3}{2}$ * $\frac{1}{2}$ f(3)

So, F (4) = 2+2+3+3 = 10

Now, solving F (5) using a recurrence.

F (5) = $\binom{4}{0}$* $\frac{1}{2}$ f(4) + $\binom{4}{1}$* $\frac{1}{2}$ f(3) + $\binom{4}{2}$* $\frac{1}{2}$ f(2) + $\binom{4}{3}$* $\frac{1}{2}$ f(3)

+$\binom{4}{4}$* $\frac{1}{2}$ f(4)

So, F (5) = 5 +16 +6 +16 +5= 48

Now, solving F (6) using a recurrence

F (6) = 2 * $\frac{1}{2}$ f(5) $\binom{5}{0}$ + 2 * $\frac{1}{2}$ f(4) $\binom{5}{1}$ + 2 * $\frac{1}{2}$ f(2) $\binom{5}{2}$

$+ \frac{1}{2}$ f(3)

So, F (6) = 122

for f(n)

$$\therefore f(n) = \sum_{i=0}^{n-1} \binom{n-1}{i} * \frac{1}{2} f(i) * \frac{1}{2} (f(n-i-1))$$