Discrete Mathematics (SC612)
Tutorial 2
17^{th} September, 2021

1. Consider a generic relation defined over any list of positive integers, which says that $(x, y) \in R$ if and only if,

   $$(((x < y) \wedge ((y - x) \leq 200)) \vee ((x - y) \geq 150))$$

   This can be instantiated by considering any finite subset of the integers of your choice. When considered over all positive integers determine whether this relation is:

   (a) reflexive/irreflexive or neither

   (b) symmetric/anti-symmetric or neither

   (c) transitive

2. Suppose the size of a finite set $|S|$ divides the size of its power set $\mathcal{P}(S)$. What are the possible values of $|S|$?

3. Construct a relation over a set of 8 elements that takes exactly 5 iterations to reach transitive closure.

4. Consider the relation $R = \{(a, b), (b, c), (c, d), (d, e), (e, f)\}$ over the set $S = \{a, b, c, d, e, f\}$. What is the minimum number of ordered pairs to be added to R to transform it into a transitive relation? What it the minimum number of pairs to be deleted from R to transform it into a transitive relation?

5. Suppose an equivalence relation over a set S contains exactly 79 ordered pairs. Give the minimum and maximum possible value of $|S|$ and also state which values in this range are possible.

6. We know that a set is a collection of **well defined, distinct** objects, and there is no further restriction. Thus we could have a set of sets (that is the elements of the set are each sets, which are distinct). Consider such a set $\mathcal{A}$ of sets $\{S_1, \ldots, S_n\}$. Let us define a relation R over $\mathcal{A}$ where $(S_i, S_j) \in R$ if and only if $S_i \subseteq S_j$. Is the relation R:

   (a) Reflexive, irreflexive or neither?

   (b) Symmetric, anti-symmetric or neither?

   (c) Transitive?

7. Consider a set of sets. We define a relation over this set where two elements are related if and only if their intersection is of size atleast 5.

   (a) This relation is reflexive if and only if _____

   (b) Is this relation symmetric, anti-symmetric or neither, in general?

   (c) Is this relation transitive, in general?

8. Let $S = \{1, \ldots, 100\}$, the set of the first 100 positive integers. Define a relation R where $(x, y) \in R$ if and only if

$$((x = y) \vee ((|x - y| \leq 15) \wedge (|x - y| \geq 5)))$$

   Find the cardinality of the largest subset X of S, such that the relation R restricted to the subset X is an equivalence relation. How many such subsets are there in S?