Mathematical Logic and Proofs

1. Mathematical Logic:

- It is a precise mathematical language used in much of mathematics.
- PURPOSE: to eliminate ambiguities (situation in which something has more than one possible meaning) that arise in communication in natural language.
- Applicability in limited in scope but it brings greater precision.

  1) Propositional Logic: Boolean Logic
  2) Predicate Logic

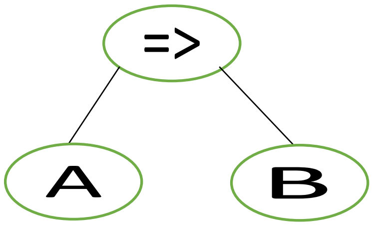
2. Propositional Logic:

- A proposition is a statement that is either true or false. It cannot be **neither** nor **both**. A propositional logic formula consists of propositional variables and connectives. We denote the propositional Variables by capital letters (A, B, etc.) and the Connectives connects the variables.
  - It helps to interpret complex statements.
  - Building block called "Atomic Value".
- Truth Tables: A Tabular way to list standard (and non-standard) Boolean functions.
- Syntax: It is the structure by which formula are constructed.
- Examples of Propositional Logic :
  a. May I drink water (True or False / Yes Or No)
  b. Math is difficult or not (True of False)
- Standard Connectives:
  - AND ($\wedge$)
  - OR (V)
  - Negation/NOT ($\neg$)
  - Implication ($\rightarrow$)
  - Equivalence ($\Leftrightarrow$)
- Semantics :
  - In P => Q , the P is Premise , Q is Conclusion
    - If P is True then Q must be True
    - If P is False then there is no guarantee , Therefore it is True

A	B	$A \rightarrow B$
True	True	True
True	False	False
False	True	True

False	False	True

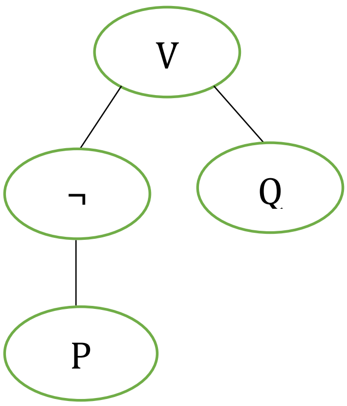
- Formula Tree



- ¬P V Q

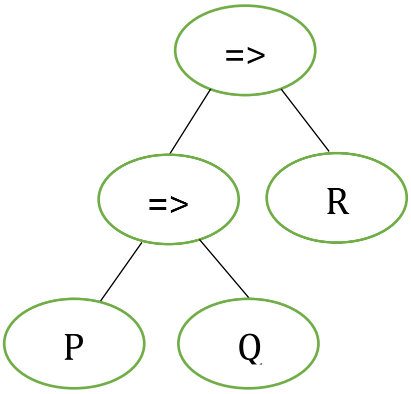
P	Q	¬P	¬PVQ
True	True	False	True
True	False	False	False
False	False	True	True
False	True	True	True

- Formula Tree

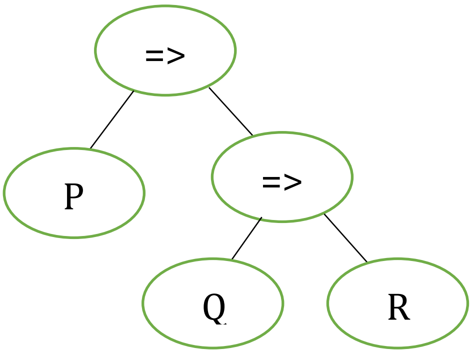


- This both are syntactically distinct but semantically identical.

- <u>**Note**</u>: AND (Λ) is Associative but Implies (=>) is not.
- <u>**Another Example Of Formula tree :**</u>
  - ((P=>Q)=>R)



- (P=>(Q=>R))



- ((P1 V P2) => (¬P3 => P1)) Λ (P2 ⇔ ¬P1)