DISCRETE MATHEMATICS

Scribed Notes 18 (25th October)

Binomial Theorem

Monomial means only one term and Binomial means having two terms.

$(x + y)^n$; where n is a positive integer.

$$(x + y)^{-a} = \frac{1}{(x + y)^a}$$

Where a is a non-negative integer.

$(\mathrm{x} + \mathrm{y})(x + y) = x^2 + 2xy + \ y^2$

In the above equation, there are 2 terms in each factor i.e x and y.

There will be 2^n distinct terms for every n number of terms. As every time while multiplying the terms, there will be various combinations for multiplying the terms x and y with each other.

It is similar to,
Cartesian Product of n Sets=Cardinality of A_1 x Cardinality of A_2x…x Cardinality of A
So, if A has n elements and B has n elements, therefore their cardinality will be n^2.

<u>For example:</u>
N=2
$(\mathrm{a} + \mathrm{b})(a + b)$
The above equation will be substituted as:-

$(a\ x\ a)(a\ x\ b)(b\ x\ a)(b\ x\ b)$ = 4 distinct terms

∴ $2^n = 2^2$ = 4 distinct terms

<u>Equation for Binomial Theorem:</u>

$$(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

Addition of power of x and power of y must be n , therefore if x is having power i, y will have (n - i) power on the basis of rule of product.
i.e. (I + (n-i))=n

In the above equation, x will be picked from i groups (from 0 to n), while y will be picked from (n-i) groups (from n-0 to n-n).

The most important thing in counting is symmetry. All the orderings formed in the above Binomial Theorem equation are symmetric.

For example:
There are 6 T-shirt of Red, Green, Blue, Orange, Violet, Yellow colours each costing 300Rs. If the customer is having only 900Rs for the purchase than he/she will be selecting any 3 from the 6 available t-shirt

Choice of Customer 1 = Blue, Red, Violet
Choice of Customer 2 = Red, Violet, Blue

Both the customers have ${}^{6}C_{3}$ choices in this case.

In the above example both customers have picked the same coloured t-shirts irrespective of their priorities. First customer has highest priority for blue colour and then for red and violet. While the second customer has highest priority for red colour and then for violet and blue.So it doesn't matter that they have picked different pairs but end of the day they got the same combination.

Choice of selection = $\frac{(n(n-1)(n-2)\ldots(n-r+1))}{r!}$

$$n(n-1)(n-2)\ldots(n-r+1) = n!$$

Extending multiplication by multiplying (n-r)! in denominator

$$\frac{n!}{r!\,(n-r)!}$$

Binomial Co-efficients:

$\binom{n}{i}$ are called binomial co-efficients

Theorems for Binomial Co-efficients:-

1) $\binom{n}{r} = \binom{n}{n-r}$

${}^{n}C_{r} = {}^{n}C_{n-r}$

The number of combinations from n elements taken r at a time will be ${}^{n}C_{r}$. Now if we take out a group of r things, we are left with a group of (n-r) things. Hence the number of combinations of n things taken r at a time is equal to the number of combinations of n things taken (n-r) at a time.

$\therefore \; {}^nC_r = {}^nC_{n-r}$

Arithmetic method :

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nC_{n-r} = \frac{n!}{(n-r)!(n-n+r)!} = \frac{n!}{r!(n-r)!}$$

$\therefore \; {}^nC_r = {}^nC_{n-r}$

2) $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$\frac{(n+1)!}{r!\,(n+1-r)!}\left(\frac{(n-r+1)}{(n+1)} + \frac{r}{(n+1)}\right)$$

$$\frac{(n+1)!}{r!\,(n+1-r)!}\left(\frac{(n-\cancel{r}+1+\cancel{r})}{(n+1)}\right)$$

$$\frac{(n+1)!}{r!\,(n+1-r)!}\left(\frac{(\cancel{n+1})}{(\cancel{n+1})}\right)$$

$$\frac{(n+1)!}{r!\,(n+1-r)!}$$

$$\therefore \frac{(n+1)}{r}$$

Fibonacci series

F(1)=1
F(2)=1
F(3)=2
F(4)=3
F(5)=5
F(6)=8
F(7)=13

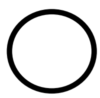
Fibonacci series follows Linear recurrence.
Fibonacci series is defined as $a_n = a_{n-2} + a_{n-1}$ where $a_0 = 1$ and $a_1 = 1$.

Recurrence Relation for Binary Tree

Recurrence relation for n nodes in binary tree.

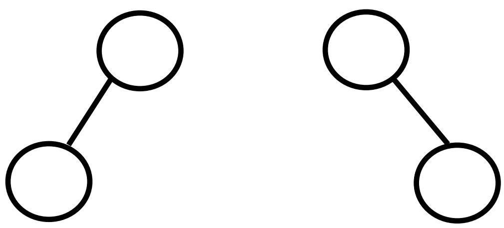
$$t(\mathrm{n}) = \sum_{i=1}^{n} \mathrm{t(i-1)t(n-i)}$$

If number of node n =1



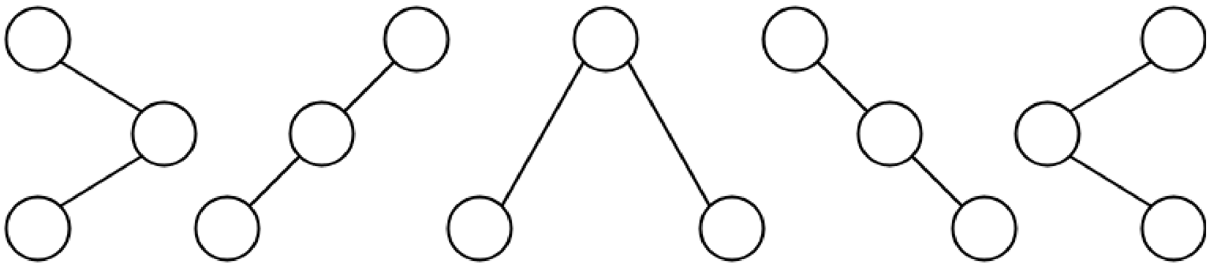
∴ t(1) = t(0) + t(1) = 1

If number of node n = 2



∴t(2) = t(0)t(1) + t(1)t(0) = 2

If number of node n = 3



∴t(3) = t(0)t(2) + t(1)(1) + t(2)t(0) = 2 + 1 + 2 = 5

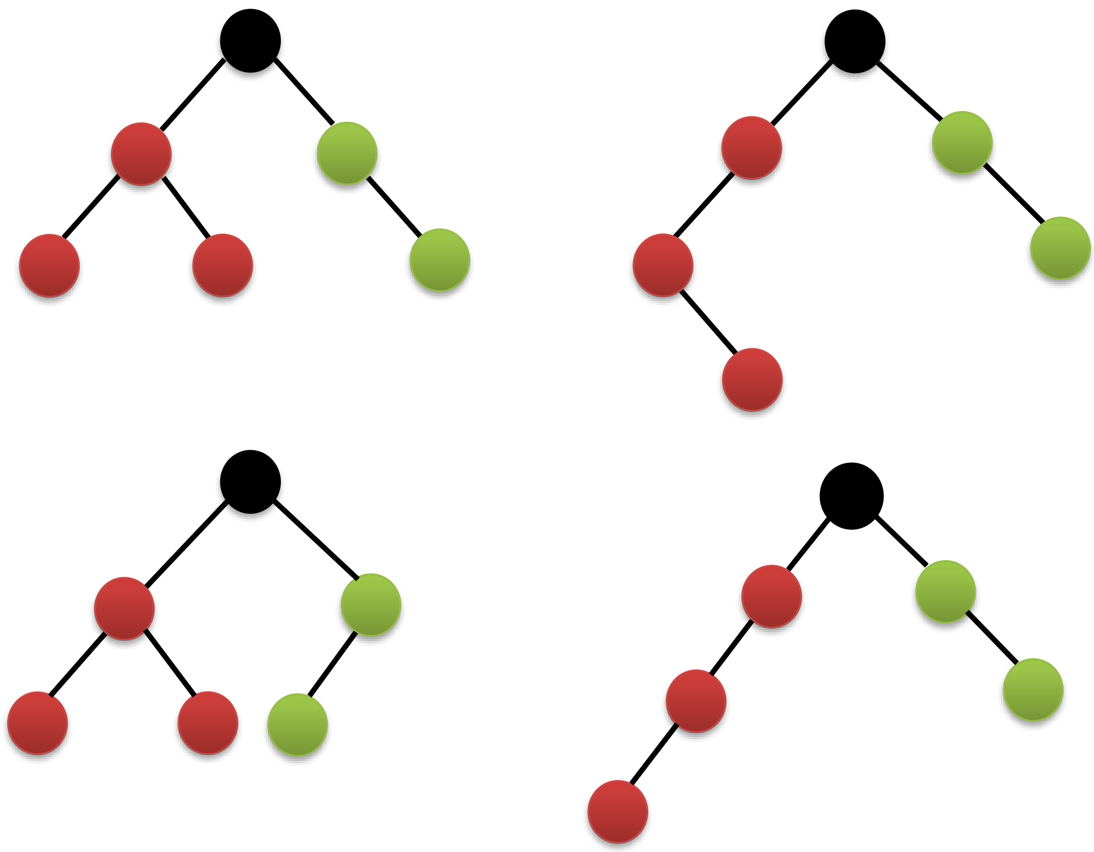
For n nodes binary tree, left subtree will be binary tree with k nodes and right subtree will be binary tree with n-1-k nodes.

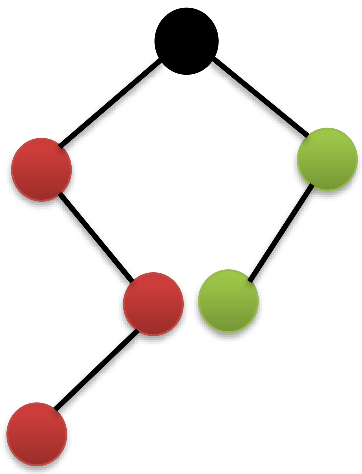
Both the subtree can be represented as individual binary trees.

<u>For example :</u>

Left subtree has 3 nodes and it can create 5 different trees.

Right subtree has 2 nodes and it can create 3 different trees.





$$BTn = \sum_{k=0}^{n-1} BT_k \times BT_{n-1-k}$$

$$G(x) = \sum_{i=0}^{\infty} BT_i \ x^i$$

For finding out number of trees obtained from n nodes, following equation is used:-

$$BTn = \frac{1}{n+1}\binom{2n}{n}$$

<u>For example</u>

If n=3

$$BTn = \frac{1}{n+1}\binom{2n}{n}$$

$$= \frac{1}{3+1}(\ 6C_3)$$

$$= \frac{1}{4}\left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1}\right)$$

$$= \frac{20}{4}$$

$$= 5$$

$\therefore$ 3 nodes = 5 trees