Systems of Equations

Systems of equations are 2 or more equations that have 2 or more variables that can be solved using various methods.

In Algebra 1, you will only deal with 2 equations that have 2 variables to solve using 3 methods:

Graphing, Substitution, and Elimination

Let's go through each method and practice!

Graphing:

Solving systems of equations using graphing requires us to graph both equations and find the INTERSECTION point (where the 2 equations cross). This intersection point is the solution of the system.

Example:

$8x - 2y = 8$

$x - 4y = 4$

The first step is to write both equations in slope-intercept form so we can graph both equations.

Move the x terms to the RIGHT side of the equation and then DIVIDE by the coefficient in front of y to get both equations in slope-intercept form.

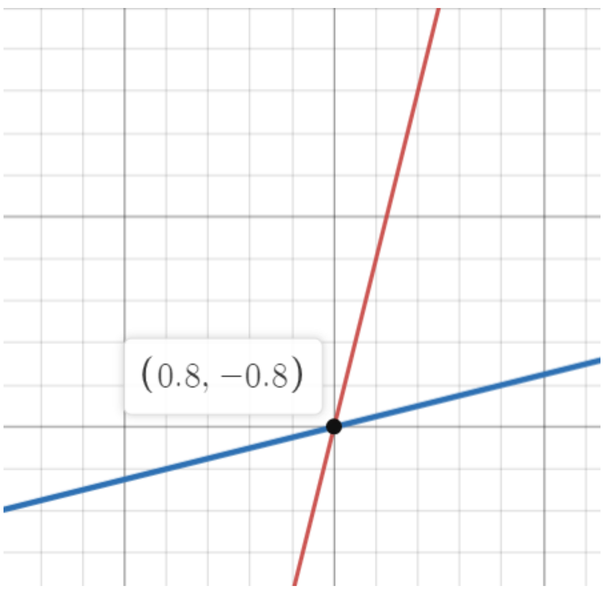
$$\begin{array}{rcl} 8x - 2y & = & 8 \\ -8x & & -8x \\ \hline \frac{-2y}{-2} & = & \frac{-8x}{-2} + \frac{8}{-2} \\ y & = & 4x - 4 \end{array}$$

$$\begin{array}{rcl} x - 4y & = & 4 \\ -x & & -x \\ \hline \frac{-4y}{-4} & = & \frac{-x}{-4} + \frac{4}{-4} \\ y & = & \frac{1}{4}x - 1 \end{array}$$

Graph these functions by first starting at the Y-INTERCEPT of both functions - (0, -4) and (0, -1) respectively.

Then follow the SLOPE to draw the line (up 4 and right 1 for the first line and up 1 and right 4 for the second line).

After graphing, your lines should look like this when you focus on the intersection point:



The intersection point, (0.8, -0.8), is the solution to this system of equations.

Substitution:

Solving systems of equations using substitution requires us to replace a variable with another expression that is solved for that particular variable.

Use this method when one of the equations in your system is ALREADY SOLVED for one variable.

Example:

$y = 4x + 2$

$5x + 2y = 30$

First, plug in the expression y is equal to (4x + 2) into the second equation where you see the variable y.

$$5x + 2(4x + 2) = 30$$

Solve for x using the order of operations and remember that variables and constants need to be on opposite sides of the equation.

$$5x + 8x + 4 = 30$$

$$13x + 4 = 30$$

$$\underline{\quad -4 \quad -4}$$

$$\frac{13x}{13} = \frac{26}{13}$$

$$x = 2$$

Substitute x = 2 into the first equation that was already solved for y.

$y = 4x + 2$

$y = 4(2) + 2$

$y = 8 + 2$

$y = 10$

So, the solution in coordinate form (x, y) is:

$$(2, 10)$$

Coordinate form is the way we want to record our solution to systems of equations. This form allows us to plot the intersection point of the 2 equations on a coordinate grid.

If you are unsure whether your solution is correct, plug it back into the 2nd equation:

$$5x + 2y = 30$$

$$5(2) + 2(10) = 30$$

$$10 + 20 = 30$$

$$30 = 30$$

Since 30 = 30, the solution for this system is (2, 10).

Elimination:

Solving systems of equations using elimination requires us to MANIPULATE the system so we can "cancel" one variable in order to solve for the other.

We do this by manipulating the system so that one variable has the SAME numerical coefficient in each equation but they are OPPOSITE signs. When we add the 2 systems, they will equal 0 or cancel out, allowing us to solve for the other variable.

There are 3 kinds of elimination problems:

Easy, Medium, and Hard. There will be an example of each in this lesson.

Easy Example:

$7x + 2y = 10$

$3x - 2y = 30$

These are called Easy problems because we can already cancel out a variable (y) WITHOUT having to manipulate the system.

Add the 2 equations together to cancel y.

$$\begin{array}{rr} & 7x + 2y = 10 \\ + & 3x - 2y = 30 \\ \hline & 10x = 40 \end{array}$$

Now we have a simple one-step equation. Divide both sides by 10 to get x by itself.

$$\frac{10x}{10} = \frac{40}{10}$$

$$x = 4$$

Plug in x = 4 to one of the 2 equations in the original system.

Preferably pick the equation where the coefficient on y is POSITIVE so we do not have to divide by a negative number.

In this case, pick the 1st equation and solve for y:

$7x + 2y = 40$

$7(4) + 2y = 40$

$28 + 2y = 40$

$- 28 \qquad\quad - 28$

$$\frac{2y}{2} = \frac{12}{2}$$

$y = 6$

So, the solution to this system is (4, 6).

Medium Example:

$$2x + 3y = 15$$

$$4x + 7y = 35$$

The problem mentioned above is considered a Medium problem because we have to manipulate ONE of the equations before we can cancel out a variable.

Look at the coefficients of x (2 and 4) and the coefficients of y (3 and 7). *Is it easier to cancel x or y?*

Think about LEAST COMMON MULTIPLE between each pair.

Canceling x is easier because we just have to multiply the top equation by -2 (remember coefficients need to be the same number but opposite sign) before adding the equations

$$-2\ (2x + 3y = 15)$$

$$4x + 7y = 35$$

Distribute -2 to the top equation before adding:

$$\begin{array}{r} -4x - 6y = -30 \\ +\ 4x + 7y = 35 \\ \hline y = 5 \end{array}$$

Plug $y = 5$ into the first original equation to get:

$$2x + 3y = 15$$

$$2x + 3(5) = 15$$

$$2x + 15 = 15$$

$$2x = 0$$

$$x = 0$$

Solution: $(0, 5)$

Hard Example:

$3x - 2y = 2$

$5x - 5y = 10$

This problem is considered a Hard problem because BOTH equations have to be manipulated in order to cancel a variable.

When determining what variable to eliminate, pick the variable that requires us to multiply both equations by the smaller numbers (find the smallest LCM between the 2 variables).

In this case, let's multiply the top equation by 5 and the bottom equation by -2 (to get +10y and -10y so y can cancel out).

$$5\ (3x - 2y = 2)$$

$$-2\ (5x - 5y = 10)$$

Distribute and add the resulting equations to get:

$$
\begin{array}{r}
15x - 10y = 10 \\
+\ -10x + 10y = -20 \\
\hline
5x = -10
\end{array}
$$

Divide by 5 to get x by itself and then plug into top equation to get y:

$$x = -2$$

$$-6 - 2y = 2$$

$$y = -4$$

Solution: (-2, -4)

Tips for Solving Problems:

1. Remember to write the solution to your systems of equations, regardless of what method you are using, as an ordered pair. This allows us to plot out the solution if needs be on a coordinate grid.

2. To determine what number you should multiply equations by when doing a HARD elimination problem, pick the smallest numbers for multiplication. For instance, if your x terms have coefficients 2 and 3 and your y terms have coefficients 5 and 7, use 2 and 3 because that will make it easier when multiplying the equations.

3. When doing elimination problems, make sure to manipulate the system so one variable has the SAME coefficient but DIFFERENT sign in both equations so they can add up to 0 and cancel out.