Multiplying and Dividing Radicals

In addition to adding and subtracting radicals, it is also possible to multiply and divide them.

This lesson will first start with multiplying radicals (as it is easier) before going into dividing radicals.

Multiplying Radicals:

The only "rule" for multiplying radicals is to multiply normally with terms underneath the radicals.

Examples:

1) $\sqrt{8} \cdot \sqrt{10}$

Simply do $8 \cdot 10$ underneath the radical for the multiplication part of this problem.

$$\sqrt{8} \cdot \sqrt{10} = \sqrt{80}$$

It is **ALWAYS** important to simplify our answer if possible. Write out the factors of 80 and find the **HIGHEST PERFECT SQUARE** factor of 80.

Factor Pairs of 80: 1 and 80, 2 and 40, 4 and 20, 5 and 16, 8 and 10

The highest perfect square factor of 80 is 16. Our next step is to take the other number in its factor pair and write them out as 2 terms under radicals with a multiplication sign between them.

$$\sqrt{16} \cdot \sqrt{5}$$

Simplify $\sqrt{16}$ (which is equal to 4) and attach it to $\sqrt{5}$ to get:

$$4\sqrt{5}$$

Let's try a slightly harder problem that still involves the same idea of multiplying radicals by multiplying the numbers inside the radicals.

2) $\sqrt{6} \cdot 2\sqrt{5}$

The first step in this problem is to actually CHANGE $2\sqrt{5}$ to only a radical with NO coefficient in front of it.

To do this, we convert 2 into an equivalent square root ($\sqrt{4}$) and then multiply it by $\sqrt{5}$.

$$\sqrt{4} \cdot \sqrt{5} = \sqrt{20}$$

Now, substitute $\sqrt{20}$ into the original problem where it says $2\sqrt{5}$ to get:

$$\sqrt{6} \cdot \sqrt{20} = \sqrt{120}$$

Find the highest perfect square factor of 120 and the other number in its factor pair and write them as 2 separate radicals separated by a multiplication sign.

Factor Pairs of 120: 1 and 120, 2 and 60, 3 and 40, 4 and 30, 6 and 20, 8 and 15, 10 and 12

The highest perfect square factor is 4, so write it and 30 in separate radicals to get:

$$\sqrt{4} \cdot \sqrt{30}$$

Simplify $\sqrt{4}$ (which is equal to 2) and attach it to $\sqrt{30}$ to get:

$$2\sqrt{30}$$

Let's move on to dividing radicals!

Dividing Radicals:

The process for dividing radicals is a bit harder than multiplying radicals.

This is because the step of simplifying radicals before dividing them (like we did in the last problem with multiplication) helps us simplify coefficients or cancel radicals.

Example:

1) $\frac{\sqrt{54}}{\sqrt{24}}$

We need to first simplify both radicals to where they both have **COEFFICIENTS** and **SMALLER** numbers underneath the radicals.

First deal with $\sqrt{54}$. Find the highest perfect square factor and the other number in its factor pair. Write them in 2 separate radicals with a multiplication sign between them.

Factor Pairs of 54: 1 and 54, 2 and 27, 3 and 18, 6 and 9

The highest perfect square factor is 9, so take it and 6 and write them under 2 radicals to get:

$$\sqrt{9} \cdot \sqrt{6}$$

Simplify $\sqrt{9}$ (which is equal to 3) and attach it to $\sqrt{6}$ to get:

$$3\sqrt{6}$$

Now follow the exact same steps for $\sqrt{24}$.

Factor Pairs of 24: 1 and 24, 2 and 12, 3 and 8, 4 and 6

The highest perfect square factor of 24 is 4, so write it and 6 under 2 separate radicals with a multiplication sign between them.

$$\sqrt{4} \cdot \sqrt{6}$$

Simplify $\sqrt{4}$ (which is equal to 2) and attach it to $\sqrt{6}$ to get:

$$2\sqrt{6}$$

Substitute $3\sqrt{6}$ and $2\sqrt{6}$ for $\sqrt{54}$ and $\sqrt{24}$ respectively in the original problem so they can be divided.

$$\frac{3\sqrt{6}}{2\sqrt{6}}$$

Now, we divide normally.

Remember you can only divide coefficients by coefficients and radicals by radicals.

Although the coefficients of 3 and 2 cannot be reduced, the radicals can. They are both the same, so cancel them out to get:

$$\frac{3}{2}$$

Example:

2) $\frac{\sqrt{75}}{\sqrt{2}}$

Simplify the radicals first since 75 does not divide evenly into 2. Focus on $\sqrt{75}$ first before going to $\sqrt{2}$.

Factor Pairs of 75 : 1 and 75, 3 and 25, 5 and 15

The highest perfect square factor is 25, so rewrite it and 3 in 2 separate radicals:

$\sqrt{25} \cdot \sqrt{3}$ which simplified becomes $5\sqrt{3}$

We cannot simplify $\sqrt{2}$, so rewrite the original problem as:

$$\frac{5\sqrt{3}}{\sqrt{2}}$$

This is not our final answer though because we **CANNOT** have a radical in the denominator.

We have to eliminate the radical in the denominator by Rationalizing the Denominator.

We do this by multiplying the numerator and denominator of our fraction by the radical in the denominator.

So here, multiply the fraction by $\sqrt{2}$ in the numerator and $\sqrt{2}$ in the denominator.

$$\frac{5\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{6}}{2}$$

Tips for Solving Problems:

1. When multiplying radicals, remember to just multiply the numbers underneath the radicals and then simplify the product by pulling out the square root of the highest perfect square factor from the radical (for instance, for $\sqrt{75}$, it simplifies to $\sqrt{25} \cdot \sqrt{3}$ and then $5\sqrt{3}$ - 5 being the square root of 25).

2. When dividing radicals, first try to divide the 2 radicals if there is a common factor between the numerator and denominator (Like $\sqrt{120}$ divided by $\sqrt{6} = \sqrt{20}$ ($120/6 = 20$)). If there is no common factor, like $\sqrt{45}$ and $\sqrt{2}$, then simplify both radicals and then divide coefficients by coefficients and radicals by radicals.

3. If you have a radical in the denominator after division, you **HAVE** to do Rationalizing the Denominator (multiply the numerator and denominator of the fraction by the radical in the denominator).