

# **Different Forms of Quadratics**

***There are 3 different forms of quadratics you will learn in Algebra 1:***

***Standard Form, Factored Form and Vertex Form***

It is important to know the significance of each form and how to convert between the forms!

## **1) Standard Form**

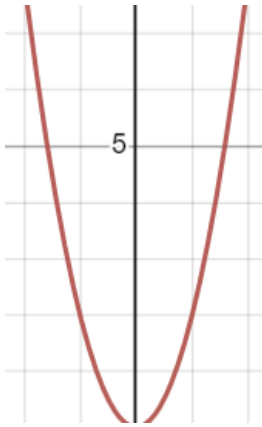
$$y = ax^2 + bx + c$$

***Standard form is a trinomial arranged from the term with the highest exponent (2) to the term with the lowest exponent (0).***

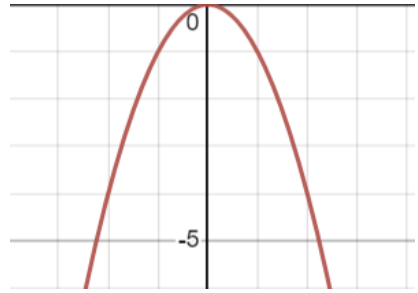
Let's break down this equation by letter!

The  $a$  value tells us the DIRECTION the parabola opens and whether it is more narrow or more stretched out. Depending on the SIGN of our  $a$  value, our parabola can look like:

Positive a value:



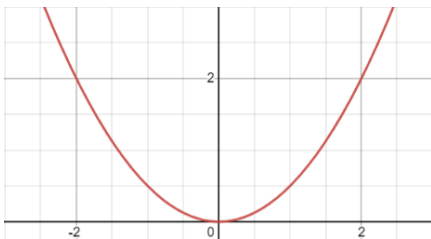
Negative a value:



The SMALLER a is, the WIDER the parabola is.

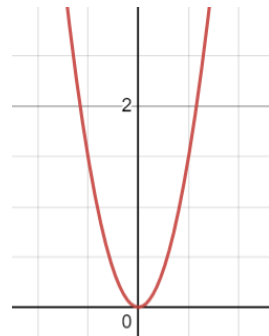
The LARGER a is, the NARROWER the parabola is.

$$y = 0.5x^2$$



SMALLER = WIDER

$$y = 6x^2$$



LARGER=NARROWER

The  $b$  value tells us the  $x$ -coordinate of the VERTEX (or AXIS OF SYMMETRY) of a parabola when its sign is switched and it is divided by  $2a$ .

You can figure out the  $y$ -coordinate of the vertex by PLUGGING in the  $x$ -coordinate to standard form and finding  $y$ .

The  $c$  value tells us the  $y$ -intercept (where the parabola INTERSECTS the  $y$ -axis) of the parabola.

Let's practice writing a parabola in standard form given certain information.

Write the equation of a parabola in standard form that opens up with a value of 1, has a  $y$ -intercept of  $(0, 20)$  and has a vertex of  $(-5, -5)$ .

From this question alone, we can already determine our  $a$  and  $c$  values.

$$a = 1$$

$$b = ?$$

$$c = 20$$

The vertex,  $(-5, -5)$  is the MINIMUM of the parabola (if the  $a$  value is NEGATIVE, the vertex would represent the MAXIMUM of the parabola).

Let's write what we have in standard form before plugging in the vertex to find b.

$$y = x^2 + bx + 20$$

$$-5 = (-5)^2 - 5b + 20$$

$$-5 = 25 - 5b + 20$$

$$-5 = -5b + 45$$

$$-50 = -5b$$

$$b = 10$$

Final Equation:  $y = x^2 + 10x + 20$

## 2) Factored Form

$$y = (x - r_1)(x - r_2)$$

*Factored form is 2 binomials that show the x-intercepts of the quadratic.*

$r_1$  and  $r_2$  represent the x-intercepts of the quadratic while  $x$  and  $y$  are the variables.

For instance, in:

$$y = (x - 4)(x - 7)$$

*4 and 7 represent the x-intercepts of the quadratic.*

Let's practice converting between standard and factored form!

## Standard Form to Factored Form:

Write  $y = x^2 + 5x - 24$  in factored form.

To convert from standard to factored form, we need to remember our FACTORING rules!

Since there is no GCF greater than 1 to factor out from the equation, we are looking for 2 factors that have a DIFFERENCE of 5 and MULTIPLY together to get 24.

Factor Pairs of 24: 1 and 24, 2 and 12, 3 and 8, 4 and 6

Since the only factor pair that has a difference of 5 is 8 and 3, those are our factor pairs!

Since the value of the MIDDLE term of the quadratic is POSITIVE, the larger FACTOR (8) will be POSITIVE and the smaller FACTOR (3) will be NEGATIVE.

So, the factored form for this quadratic is:

$$y = (x + 8)(x - 3)$$

## **Factored Form to Standard Form:**

Write  **$y = (x + 10)(x - 4)$**  as a quadratic in standard form.

To convert from factored form to standard form, we are going to do the inverse of factoring, which is FOIL!

Remember FOIL stands for First Outer Inner Last and each word represents the MULTIPLICATION of 2 numbers in the factored binomials.

**First (F) -  $x * x = x^2$**

**Outer (O) -  $x * (-4) = -4x$**

**Inner (I) -  $10 * x = 10x$**

**Later (L) -  $10 * (-4) = -40$**

Put the 4 terms together with their respective signs to get:

$$**y = x^2 - 4x + 10x - 40**$$

Combine the middle 2 terms to get the quadratic in standard form:

$$**y = x^2 + 6x - 40**$$

If you want a more in-depth explanation on FOIL and/or factoring trinomials, make sure to check out those lessons after this one!

### 3) Vertex Form

$$y = a(x - h)^2 + k$$

Vertex form allows us to determine the vertex  $(h, k)$  of the quadratic.

For instance, in:

$$y = (x - 10)^2 + 18$$

$(10, 18)$  represents the vertex of the quadratic. Let's practice converting between standard and vertex forms now!

#### Standard Form to Vertex Form:

Convert  $y = x^2 - 10x + 24$  to vertex form.

To convert from standard form to vertex form, we need to find the vertex from standard form before plugging it into vertex form.

Remember the  $a$  value of the quadratic is 1,  $b$  is -10 and  $c$  is 24!

To find the x-coordinate of the vertex, we have to FLIP the sign of b and DIVIDE it by 2a.

$$\frac{-b}{2a} = \frac{-(-10)}{2(1)} = \frac{10}{2} = 5$$

Now that we have the x-coordinate of the vertex, PLUG 5 into the equation for x and solve for y.

$$y = (5)^2 - 10(5) + 24$$

$$y = 25 - 50 + 24$$

$$y = -1$$

Now that we know our vertex is (5, -1), we can put this vertex into vertex form:

$$y = (x - 5)^2 - 1$$

## **Vertex Form to Standard Form:**

Convert  $y = (x - 5)^2 + 8$  into a quadratic in standard form.

To convert from vertex form to standard form, we need to follow order of operations by squaring the binomial  $x - 5$  before adding 8 to the result.



First rewrite  **$(x - 5)^2$**  as 2 binomials with  **$x$**  - **5** in each of them.

$$y = (x - 5)(x - 5) + 8$$

Remember the special case of the  
NEGATIVE perfect square trinomial!  
Multiply  $x * x$ , double the product of  $-5 * x$   
and multiply  $-5 * -5$  to get the 3 terms for  
the expanded trinomial!

$$y = x^2 - 10x + 25 + 8$$

Combine like terms to get the answer:

$$y = x^2 - 10x + 33$$

*If you want a more in-depth explanation on  
the negative perfect square trinomial and  
the other special cases of trinomials, make  
sure to check out that lesson after this one!*

## Tips for Solving Problems:

1. Remember for the vertex form equation, it is  $(x - h)^2 + k$ , meaning that  $h$  or the  $x$ -coordinate of the vertex is the opposite sign of how it is in the equation (for instance, if it is  $(x + 5)^2 + 2$ , the vertex is  $(-5, 2)$ ).

2. To go from vertex form to factored form or vice versa, convert to standard form first before converting between standard form and the desired form.

3. This lesson is very important and the skills needed for this lesson (FOIL, factoring trinomials and special cases) are also very important too! Make sure to review and practice any of these lessons if you need to!