

Completing the Square

Completing the square is a technique that you can use to help convert quadratics from standard form, $y = ax^2 + bx + c$, to vertex form, $y = a(x - h)^2 + k$, and it can help you solve quadratic equations.

We will go through the steps of completing the square with an example.

$$x^2 - 12x + 32 = 0$$

1) After setting the quadratic equal to 0, move the constant (in this case 32) to the other side of the equation, leaving a space where 32 was on the left side of the equation.

$$x^2 - 12x + \underline{\hspace{1cm}} = -32$$

2) Now divide the b value (in this case -12) by 2, square the value you get and add it to both sides (for this problem, $-12/2 = -6$, square it to get 36, so add 36 to both sides).

$$x^2 - 12x + 36 = -32 + 36$$

3) Simplify both sides of the equation. Simplify the left to $(x - 6)^2$ because of the NEGATIVE perfect square trinomial and simplify the right to 4.

$$(x - 6)^2 = 4$$

4) To solve for **x**, take the square root of both sides, keeping in mind for the square root of 4, you are including the POSITIVE (2) and NEGATIVE (-2) root to set equal to **x - 6**.

$$\mathbf{x - 6 = \pm 2}$$

5) Split the above equation into 2 equations (**x - 6 = 2** and **x - 6 = -2**) and solve for **x** in both to get the solutions.

$$\mathbf{x - 6 = -2}$$

$$\mathbf{x - 6 = 2}$$

$$\mathbf{x = 4}$$

and

$$\mathbf{x = 8}$$

Try this next problem on your own before looking at the solution!

$$\mathbf{x^2 - 4x - 32 = 0}$$

First add 32 to both sides, making sure to leave a space where -32 was so we can do the next step of taking half of the b value (**-4/2 = -2**) and squaring that value (**(-2)² = 4**) to add to the right side where 32 is. Simplify the equation **to (x - 2)² = 36** and take the square root of both sides (**x - 2 = 6** and **x - 2 = -6** as the remaining equations). After solving the equations, we get:

$$\mathbf{x = -4 \text{ and } x = 8 \text{ as solutions}}$$

Tips for Solving Problems:

1. Remember to leave a SPACE after the bx term on the LEFT side of the trinomial after moving the constant to the right side of the equation! You need this for the next step where you will create a PERFECT SQUARE trinomial!

2. The main step in completing the square is taking half of the b value in the bx term, SQUARING that value and ADDING that value to both sides. If you have done this process correctly, the left side of the trinomial should become a PERFECT SQUARE trinomial.

3. Quick reminder of the perfect square trinomial! If you have a trinomial with leading coefficient 1, a b value that is DOUBLE a certain number and a c value that is the SQUARE of that certain number, you can simplify the trinomial to that **$x +$** **or $x -$** that certain number squared.