Converting between Rational Exponents and Radicals

To convert between rational exponents and radicals and vice versa, there is a specific rule that works to help convert these into their other forms.

$$x^{\frac{2}{3}} = \sqrt[3]{x^2} \quad or \quad (\sqrt[3]{x})^2$$

The denominator of the rational exponent corresponds with the index (top-left number that determines what type of root the radical is).

For instance, the index of the radical/denominator of the rational exponent in the problem above is 3.

*If you see a radical with no index, assume the index is 2 because we do not write an index of 2 for **SQUARE ROOTS** (it is understood to be 2).*

The numerator of the rational exponent corresponds with the exponent on the radicand (term inside the radical) or the exponent of the entire term.

For instance, the exponent on the radicand (or entire radical)/numerator of the rational exponent in the problem above is 2.

Let's practice with a couple of examples.

Examples:

1) Convert $2^{\frac{5}{3}}$ to equivalent radicals.

$$2^{\frac{5}{3}} = \sqrt[3]{2^5} \quad or \quad \sqrt[3]{32} \quad or \quad (\sqrt[3]{2})^5$$

Remember that the numerator of the rational exponent is the exponent of the number in the radical (2 raised to the 5th power, which can be simplified to 32) and the denominator of the rational exponent is the index of the radical (3 or cube root).

Let's convert the other way now!

2) Convert $\sqrt{2^4}$ to a base with a fractional exponent before simplification.

Remember square roots have an index of 2!

$$\sqrt{2^4} = 2^{\frac{4}{2}}$$

When the fractional exponent is simplified, the base and exponent become:

$$2^2 = 4$$

These are all the steps you need to know, so keep practicing and you will understand it more!

Tips for Solving Problems:

1. The **NUMERATOR** of the rational exponent corresponds with the **EXPONENT** of the radicand (the term under the radical). The **DENOMINATOR** of the rational exponent corresponds with the **INDEX** of the radical (the number at the top left of the radical sign).

2. If you can, simplify after you follow the initial steps of converting between rational exponents and radicals. For instance, simplify the square root of 2^2 by writing 2 as the answer.

3. Remember if there is no index on the radical, it is assumed to be 2 and it is called a square root.