Absolute Value Equations

Absolute value equations are similar to regular equations with 2 major differences:

The number on the side of the equation OPPOSITE the variable must be POSITIVE and when solving for solutions, you will have a solution where that number is POSITIVE or one where that number is NEGATIVE.

Let's practice a few examples for these conditions to make more sense!

1) $|x+5| = 13$

Since the number on the side of the equation OPPOSITE the variable is POSITIVE (13), we can find our 2 solutions with the equations being equal to 13 and -13.

$$|x+5| = 13 \qquad\qquad |x+5| = -13$$
$$-5 \quad -5 \qquad\qquad -5 \quad -5$$
$$x = 8 \qquad\qquad x = -18$$

The solutions to this equation are -18 and 8.

Why do we have 2 solutions for these equations?

Remember absolute value can only be positive, so the number that sums up to -13 counts because absolute value makes it 13.

2) |5x| = 25

Since 25 is positive, let's find the solution that satisfies 25 and -25.

$$\frac{|5x|}{5} = \frac{25}{5} \qquad \frac{|5x|}{5} = \frac{-25}{5}$$

$$x = 5 \qquad x = -5$$

The 2 solutions to this equation are -5 and 5.

3) |2x| - 5 = 17

Follow the same steps as you would solving any multi-step equation with the bonus of finding the 2 solutions (Add 5 and divide by 2).

$$|2x| - 5 = 17$$
$$+5 \quad +5$$
$$|2x| = 22$$

$$\frac{|2x|}{2} = \frac{22}{2} \qquad \frac{|2x|}{2} = \frac{-22}{2}$$

$$x = 11 \qquad x = -11$$

The two solutions to this equation are 11 and -11.

Tips for Solving Problems:

1. The number on the side of the equation OPPOSITE the variable has to be POSITIVE. If it is not, then there are NO solutions to the equation because the absolute value will never be negative.

2. When solving absolute value equations, you are trying to find 2 solutions, one that sets the equation equal to the number opposite the variable and the other that sets the equation equal to the negative version of the number opposite the variable.

3. Remember to always do ADDITION/SUBTRACTION first for solving absolute value equations UNLESS you are dealing with distribution and need to MULTIPLY/DIVIDE first.