Special Cases for Factoring Trinomials

After learning the regular way to factor trinomials in Algebra 1, special cases are introduced. These are specific instances where the trinomial or the factors do not follow the conventional method of factoring.

There are 3 specific ones that you will learn in this lesson that are taught in Algebra 1.

1) Difference of Squares

$$x^2 - c$$

The first case is called difference of squares where we actually have a binomial with a first term and a last term, but no middle term.

Why is that the case?

Remember that the b value is the SUM of our factors. The only way our b value could be 0 (or not included in the polynomial) is if our factors were the same BUT they differed in sign.

This special case works when both terms in the "trinomial" are perfect squares and there is no middle term.

Let's practice with a couple examples.

$x^2 - 49$

The first step is to always look for a GCF other than 1 that can be factored out. Since there is not a GCF other than 1 we can factor out, we need to figure out what 2 numbers multiply to give us 49 and have a difference of 0.

Those numbers would be 7 and 7. Remember since the last term in the trinomial is negative, one factor is positive and the other is negative. Writing it out, we get:

$(x + 7)(x - 7)$

Now try this example with the steps above:

$2x^2 - 32$

First, factor out the GCF of 2.

$2(x^2 - 16)$

Then, determine what 2 numbers multiply to 16 and have a difference of 0 (4 and 4) and write out the factors in parentheses.

$2(x - 4)(x + 4)$

2) The Positive Perfect Square Trinomial

$$\mathbf{a^2 + 2ab + b^2}$$

The second case is called the positive perfect square trinomial and it is in the format of the first term in the factored binomials squared, double the product of the first and last term in each factored binomial and the last term in the factored binomials squared.

Why is this a special case?

The factored form of this trinomial is:

$$\mathbf{(a + b)(a + b)}$$

This can also be written as:

$$\mathbf{(a + b)^2}$$

Let's practice with some examples!

$$\mathbf{x^2 + 8x + 16}$$

When you are first doing these problems, follow the same steps you would factoring any trinomial. Determine if there is a GCF other than 1 before finding 2 numbers that sum up to 8 and multiply together to get 16.

Since there is no GCF that can be factored, our factors would be 4 and 4. Since both factors are the same, we only need to write one set of parentheses, raising it to the 2nd power.

$$(x + 4)^2$$

Let's try another one, using the trend mentioned on the last page.

$$x^2 + 10x + 25$$

You should hopefully be able to recognize that x^2 and 25 are perfect squares (meaning that x and 5 are our terms that go in the binomial) and 10x is double the product of x and 5, so this is a positive perfect square trinomial.

$$(x + 5)^2$$

3) The Negative Perfect Square Trinomial

$$a^2 - 2ab + b^2$$

This trinomial is very similar to the last special case, but it differs in 1 way.

It is called the negative perfect square trinomial because the middle term is negative, meaning that your factors will both be negative.

Let's practice!

$$\mathbf{x^2 - 16x + 64}$$

Since there is no GCF that can be factored out of this trinomial, we need to find 2 factors of 64 that sum up to 16.

Those factors would be 8 and 8. Since the factors are the same, use just 1 set of parentheses and raise it to the 2nd power to get:

$$\mathbf{(x - 8)^2}$$

Let's do one more example:

$$\mathbf{2x^2 - 20x + 50}$$

If the negative perfect square trinomial does not stand out at first glance, that is okay.

Remember the first step when factoring trinomials is to factor out the GCF!

Factor out a 2 to give us:

$$\mathbf{2(x^2 - 10x + 25)}$$

Now, the perfect square trinomial should stand out! x^2 and 25 are both perfect squares and -10 is double the product of x and -5. Use 1 set of parentheses and raise it to the 2nd power, not forgetting the 2 at the front.

$$\mathbf{2(x - 5)^2}$$

Tips for Solving Problems:

1. Remember difference of squares can be done on any binomial whose 2 terms are perfect squares and whose 2nd term is negative (like $x^2 - 25$).

2. Remember that the positive perfect square trinomial only differs from the negative perfect square trinomial by the SIGN OF THEIR MIDDLE TERM. In both, you are still looking for the first and last terms to be positive perfect squares and the middle term needs to be the product of the square roots of the first and last terms in the trinomial.

3. Remember if you factor out a GCF during the first step, you need to include it in the final factored form. This applies in regular factoring as well, but especially here with special cases not automatically standing out when looking at them.