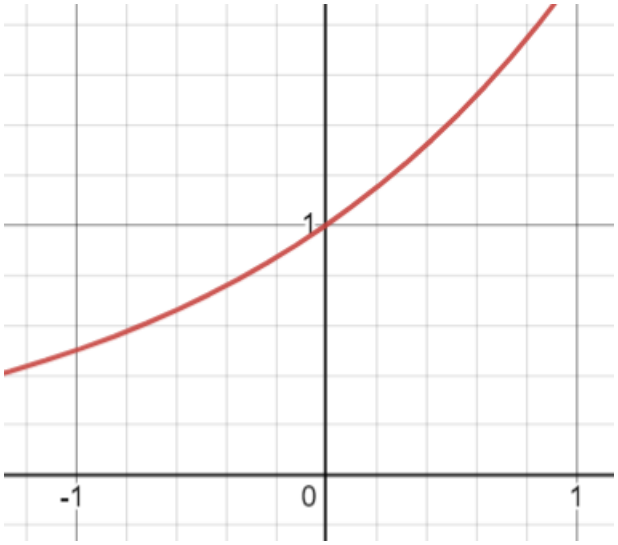
Exponential Growth and Decay

This is the most important lesson in exponential functions! We will first start with exponential growth before going into exponential decay. This lesson will go over the main graph, main equation, and word problems for both concepts.

Exponential Growth:



$y = 3^x$

The equation for any exponential growth function is $y = b(a^x)$ where $a > 1$ and b is the y-intercept/initial value. Exponential growth functions as graphs always start from the left side and go towards POSITIVE infinity.

You will also typically see word problems related to exponential growth so let's practice some of those!

A town doubles its population every 13 years. The current population of the town is 1,800 people. What will the population be in 52 years?

The first step is to determine how many times the population will be doubled. Since the population doubles every 13 years and we want to find out the population in 52 years, it doubles 52/13 = 4 times.

Now, we can use the formula: $y = b(a^x)$. Use 1,800 for b (initial population), 2 for a (because you are doubling) and 4 for x.

$$y = 1800\ (2)^4$$

$$y = 1800\ (16)$$

$$y = 28{,}800 \text{ people}$$

The population of rabbits was 210 in a town. The population triples every year. How many rabbits are in the town in 5 years?

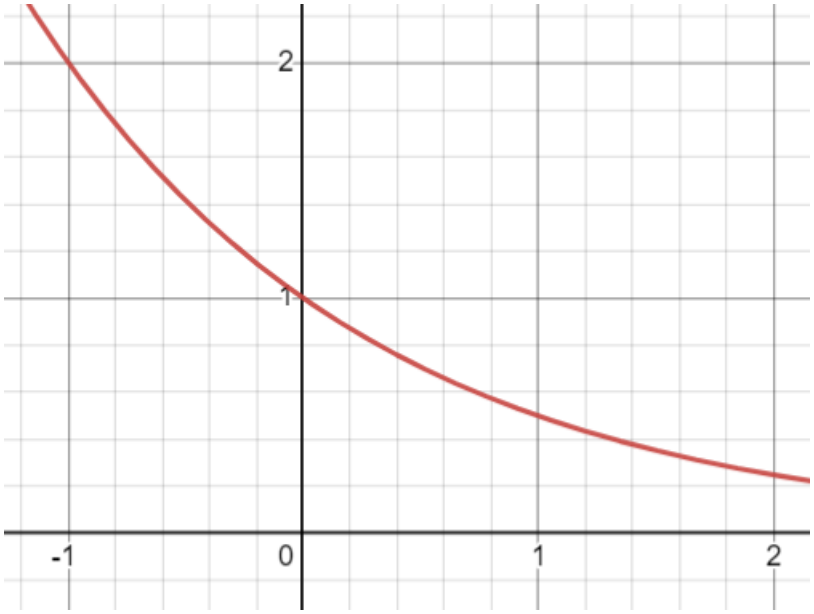
To solve this problem, use the formula $y = b(a^x)$. Use 210 for b, 3 for a (because you are tripling) and 5 for x (5 years).

$$y = 210\ (3^5)$$

$$y = 210\ (243)$$

$$y = 51{,}030 \text{ rabbits}$$

Exponential Decay:



$y = 0.5^x$

The general formula for exponential decay problems is the same as exponential growth: $y = b(a^x)$. The difference is that a is BETWEEN 0 and 1. This makes the graph of exponential decay functions start from POSITIVE or NEGATIVE infinity on the left to a HORIZONTAL asymptote on the right.

Just like with exponential growth, you will use $y = b(a^x)$ to solve word problems related to exponential decay, so let's practice some word problems!

A new car has a value of $20,000. It depreciates 30% of its value every year. How much is the car worth in 3 years?

The key to exponential decay problems is to figure out what a is. It is NOT 0.3 (remember a needs to be a decimal between 0 and 1). Instead, subtract 0.3 from 1 to get 0.7 as our a. b is $20,000 and x is 3.

$$y = 20{,}000(0.3^3)$$

$$y = 20{,}000(0.027)$$

$$y = \$540$$

A new phone is worth $500. It loses 20% of its value every year. How much is the phone worth in 4 years?

Use the formula $y = b(a^x)$ where b is $500, a is 0.8 and x is 4.

$$y = 500\ (0.8^4)$$

$$y = 500\ (0.4096)$$

$$y = \$204.80$$

Make sure to keep practicing these problems - You will understand them more!

Tips for Solving Problems:

1. You use the same formula - $y = b(a^x)$ for exponential growth and exponential decay problems. Remember b is the initial value, a is the base of the exponent (it is between 0 and 1 for decay and greater than 1 for growth) and x is the exponent.

2. Exponential growth functions start from a horizontal asymptote on the LEFT and go towards positive or negative infinity on the RIGHT. Exponential decay functions start from positive or negative infinity on the LEFT and go toward a horizontal asymptote on the RIGHT.

3. Remember for exponential decay word problems, the a is the percentage (converted to a decimal) in the problem subtracted from 1 (for instance, if the problem says a 30% depreciation (0.3), a for this problem would be 1 - 0.3 = 0.7.