Different Forms of Linear Functions

There are 3 different forms of linear functions that you will learn in Algebra 1:

Slope-intercept form, Standard form, and Point-slope form

Recognizing and going between the different forms is important, so pay attention in this lesson!

Slope-Intercept Form:

$$\mathbf{y = mx + b}$$

Let's break down this equation by letter!

y - *Represents the y coordinates of the line*

m - *Represents the slope of the line*

x - *Represents the x coordinates of the line*

b - *Represents the y-intercept of the line*

This form of the line helps us identify the slope and the y-intercept of a line.

Example:

Write the equation of a line in slope-intercept form that passes through (5, 2) and (-2, -3)

The first step is to find the slope of this line. Use the formula for slope with (5, 2) as x_1 and y_1 respectively and (-2, -3) as x_2 and y_2 respectively.

$$\text{Slope} = \frac{-3-2}{-2-5}$$

$$\text{Slope} = \frac{-5}{-7}$$

$$\text{Slope} = \frac{5}{7}$$

Plug in the slope into slope-intercept form.

$$y = \frac{5}{7}x + b$$

Plug in 1 of the points given in the question to solve for b, the y-intercept.

Let's use (5, 2) as the point since it is easier to deal with positive numbers. Plug in 5 for x and 2 for y.

$$2 = \frac{5}{7} . \frac{5}{1} + b$$

$$2 = \frac{25}{7} + b$$

Convert 2 on the left side of the equation to a fraction with a denominator of 7 so it is easier to subtract.

$$\frac{14}{7} = \frac{25}{7} + b$$

Now subtract the constant from the right side of the equation on both sides.

$$\frac{14}{7} \quad = \quad \frac{25}{7} + b$$

$$-\frac{25}{7} \quad - \quad \frac{25}{7}$$

$$\frac{-11}{7} \quad = \quad b$$

Plug the y-intercept into slope-intercept form to get the correct equation.

$$y = \frac{5}{7}x - \frac{11}{7}$$

Parallel and Perpendicular Lines:

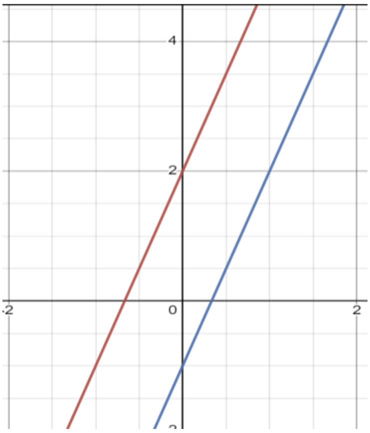
Slope-intercept form allows us to understand the concept of parallel and perpendicular lines.

Parallel lines have the SAME slope. This means that these lines will never intersect and will be an equal distance from each other.

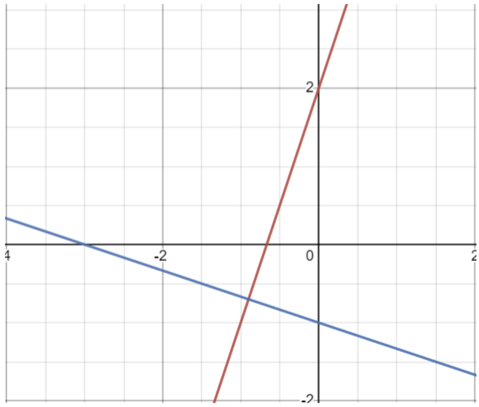
Perpendicular lines have OPPOSITE RECIPROCAL slopes. OPPOSITE RECIPROCAL means that the 2nd line's slope is the opposite sign and the reverse of the 1st slope (4 and -1/4 for example). These lines intersect each other at right angles.

Examples of how they both look on a graph:

Parallel Lines



Perpendicular Lines



Horizontal and Vertical Lines:

A horizontal line is ANY line in the form:

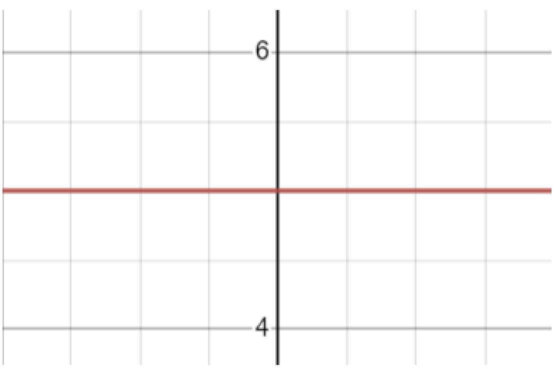
y = a number

A vertical line is ANY line in the form:

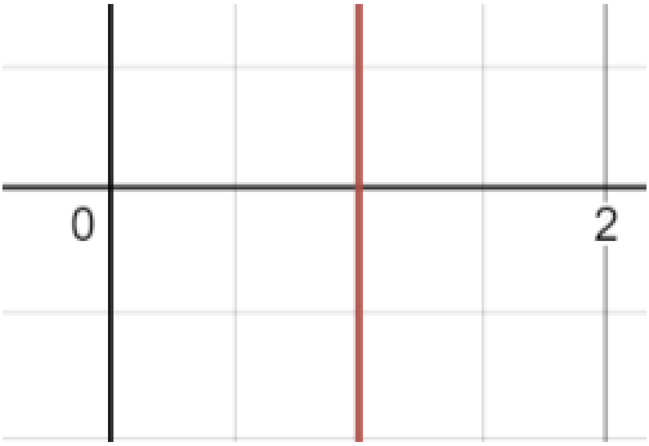
x = a number

An example of each with their graphs:

$y = 5$



x = 1



Standard Form:

$$Ax + By = C$$

where A, B and C are integers, have no other common factors except 1 and A is non-negative

This form of a line allows us to identify the x and y-intercepts of a line (where the line intersects the x-axis and y-axis respectively).

The x-intercept can be found by setting y = 0 and dividing C by A.

The y-intercept can be found by setting x = 0 and dividing C by B.

Examples:

What are the x and y-intercepts of the line 4x + 6y = 12?

X-intercept	Y-intercept
1) 4x + 6 (0) = 12	1) 4 (0) + 6y =12
2) 4x + 0 = 12	2) 6y + 0 = 12
3) $\frac{4x}{4} = \frac{12}{4}$	3) $\frac{6y}{6} = \frac{12}{6}$
4) x = 3	4) y = 2

The x-intercept of the line is (3, 0) and the y-intercept of the line is (0, 2).

Write the equation of the line with slope 5 and y-intercept (0, 6) in standard form.

The key to solving these types of problems is to first write the line in slope-intercept form before manipulating the equation to get it in standard form.

The equation of this line in slope-intercept form is:

$$y = 5x + 6$$

Remember the standard form is Ax + By = C, so subtract 5x from both sides to get the equation closer to standard form.

$$\begin{array}{rcl} y & = & 5x + 6 \\ -5x & & -5x \\ \hline -5x + y & = & 6 \end{array}$$

We are not done yet! Remember one of the conditions of standard form is that the Ax term CANNOT be negative. Multiply every term by -1 to get this equation in standard form.

$$-1\ (-5x + y = 6)$$

$$5x - y = 6$$

This line is in standard form because A, B and C are integers, have no other factors in common except 1 and A is positive.

Point-Slope Form:

$$y - y_1 = m\ (x - x_1)$$

Let's break down this equation by letter!

y - Represents the y coordinates of the line

y_1 - Represents the y coordinate of a point on the line

m - Represents the slope of the line

x - Represents the x coordinates of the line

x_1 - Represents the x coordinate of a point on the line

This form of the line allows us to determine the slope and a point on the line. We can plug the point given on a coordinate grid and follow the slope to plot the line.

Examples:

Write the equation of the line in a point-slope form that passes through (-2, 1) and (9, 8)

First, find the slope of the line before plugging it into point-slope form (We have everything for point-slope form except the slope at the moment).

$$\text{Slope} = \frac{8-1}{9-(-2)}$$

$$\text{Slope} = \frac{8-1}{9+2}$$

$$\text{Slope} = \frac{7}{11}$$

Now that we have everything for point-slope form (use (9, 8) as the point to plug in as it keeps the signs the same as in the formula), plug it in to get:

$$y - 8 = \frac{7}{11}(x-9)$$

Change y + 5 = 6(x - 6) from point-slope form to slope-intercept form.

Since x and y are already on the correct sides, it comes down to manipulating the equation so all of the constants are on the right side of the equation.

First, distribute 6 to x - 6 on the right side of the equation.

$$y + 5 = 6x - 36$$

Now subtract 5 from both sides to move all of the constants to the right side of the equation and we will get this line in slope-intercept form.

$$\begin{array}{rcl} y + 5 & = & 6x - 36 \\ -5 & & -5 \\ \hline y & = & 6x - 41 \end{array}$$

To convert to standard form, subtract 6x from both sides and multiply all terms by -1.

Tips for Solving Problems:

1. Each form of a linear function (point-slope, standard and slope-intercept) help us determine various characteristics of that line. Make sure to know what form the linear function needs to be in and the information that is needed to write the function in that form.

2. Remember the criteria that equations in Standard Form need to be written in! A, B and C have to ALL be integers, have no factors in common except 1 and A CANNOT be negative.

3. In point-slope form, the sign of x_1 and y_1 are the opposite of the sign that they are on the coordinate grid (for instance, if the point is (-5, -9) and the line has a slope of 3, point-slope form would be $y + 9 = 3(x + 5)$ - Since the coordinates were negative, they are positive in point-slope form).