Complete search - Backtracking

Le Gia Khang 21522189
Nguyen Hoang Tan 21521413

University of Information Technology

April 6, 2023

References

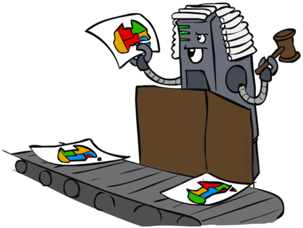
The contents of this document are taken mainly from the follow sources:

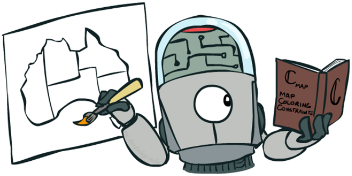
- Rina Dechter and Daniel Frost, Backtrackging algorithms for constraint satisfaction problems
- Pter van Beek, Chapter 4 - Backtracking Search Algorithms

Table of Contents

Table of Contents

- A special subset of search problems.
- State is defined by variables X_i with values from a domain D (sometimes D depends on i)
- Goal test is a set of constraints specifying allowable combinations of values for subsets of variables





Constraint satisfaction problems (CSPs)

Definition

A constraint satisfaction problem (CSP) is a tuple (X,D,C) where:

- $X = \{x_1, x_2, \ldots, x_n\}$ is the set of variables.
- $D = \{d_1, d_2, \ldots, d_n\}$ is the set of domains.
- $C = \{c_1, c_2, \ldots, c_n\}$ is a set of constraints.

For example, $x, y, z \in \{0, 1\}, x + y = z$ is a CSP where:

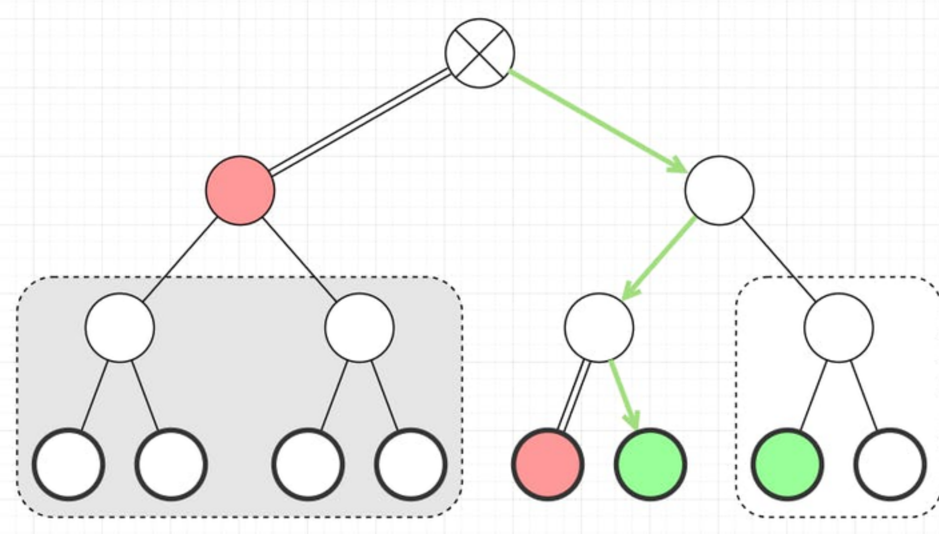
- Variables are: x, y, z
- Domains are: $d_x = d_y = d_z = \{0, 1\}$
- There is a single constraint: $x + y = z$

Table of Contents

Brute-force Approach

- Brute-force is a simple and naive algorithmic approach to solving problems that involves exhaustively checking all possible solutions.
- It is typically used when the problem size is small and the search space is manageable.
- It can also be used in combination with other techniques, such as pruning or heuristics, to improve their efficiency and effectiveness.

Complete Search - Backtracking



- The basic idea behind backtracking is to recursively build a partial solution by making choices from a set of available options, and then backtrack if the solution fails to satisfy the constraints.
- The algorithm explores the search space depth-first, which means that it goes as far as possible along each branch of the search tree before backtracking to the previous decision point and exploring another branch.
- if a node in the search tree does not lead to a solution, it is considered a deadend and its subtree can be pruned.

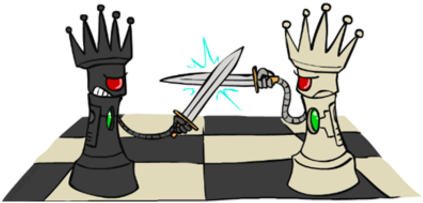
Complete Search - Backtracking

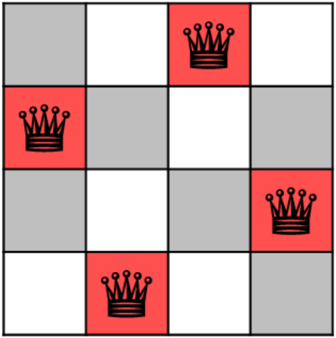
```
CSP-BACKTRACKING(PartialAssignment a)
     If a is complete then return a
     X <- select an unassigned variable
     D <- select an ordering for the domain of X
     For each value v in D do
         If v is consistent with a then
              Add (X = v) to a
              result <- CSP-BACKTRACKING(a)
              If result <> failure then return result
              Remove (X = v) from a
     Return failure
```

Table of Contents

N-queens problem

The N-Queens problem is a classic problem in computer science and mathematics that involves placing N chess queens on an N x N chessboard such that no two queens attack each other.





Knapsack

The Knapsack problem involves packing a knapsack with items of different weights and values. The goal is to maximize the value of the items in the knapsack while keeping the total weight of the knapsack below a certain limit.

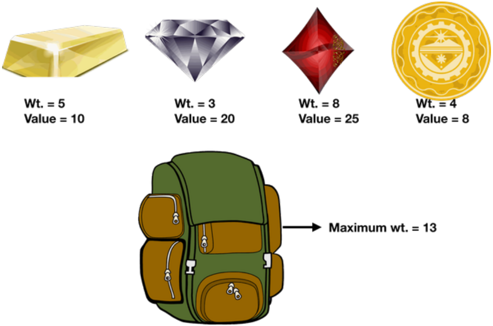


Table of Contents

Benefits and Drawbacks of Backtracking

Benefits

- Backtracking is a more intelligent and efficient way of searching through large solution spaces than brute-force;
- Backtracking problems are very intuitive to code;

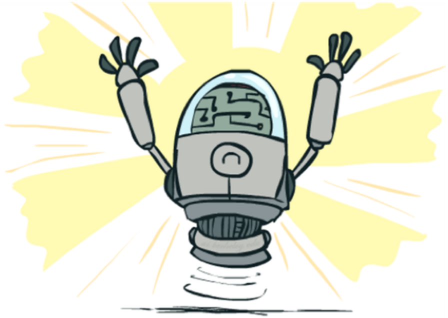
Drawbacks

- In the worst case where all possible solutions must be explored, Backtracking has the same worst-case time & space complexity as Brute-force;
- Backtracking is complete but not guaranteed to find optimal solution, or just sub-optimal;

Improvements to backtracking

- Backtracking usually suffers from thrashing, namely, rediscovering the same inconsistencies and the same partial successes during search.

- Efficient cures for such behavior in all cases are unlikely, since the problem is NP-hard.



Filtering

Ordering