Complete search - Backtracking

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References

The contents of this document are taken mainly from the follow sources:

- Rina Dechter and Daniel Frost, Backtrackging algorithms for constraint satisfaction problems
- Pter van Beek, Chapter 4 - Backtracking Search Algorithms

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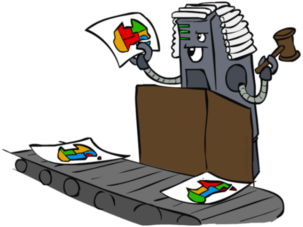
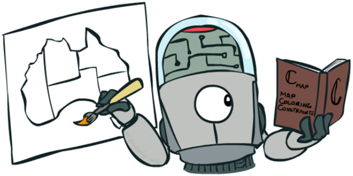
Constraint satisfaction problems (CSPs)

- A special subset of search problems.
- State is defined by variables X_i with values from a domain D (sometimes D depends on i)
- Goal test is a set of constraints specifying allowable combinations of values for subsets of variables





Constraint satisfaction problems (CSPs)

Definition

A constraint satisfaction problem (CSP) is a tuple (X,D,C) where:

- $X = \{x_1, x_2, \ldots, x_n\}$ is the set of variables.
- $D = \{d_1, d_2, \ldots, d_n\}$ is the set of domains.
- $C = \{c_1, c_2, \ldots, c_n\}$ is a set of constraints.

For example, $x, y, z \in \{0, 1\}, x + y = z$ is a CSP where:

- Variables are: x, y, z
- Domains are: $d_x = d_y = d_z = \{0, 1\}$
- There is a single constraint: $x + y = z$

Brute-force Approach

- Brute-force is a simple and naive algorithmic approach to solving problems that involves exhaustively checking all possible solutions.
- It is typically used when the problem size is small and the search space is manageable.
- It can also be used in combination with other techniques, such as pruning or heuristics, to improve their efficiency and effectiveness.

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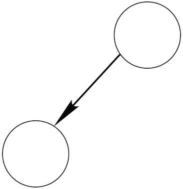

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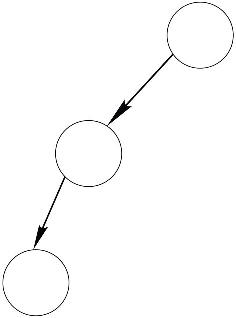

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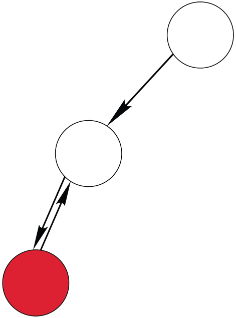

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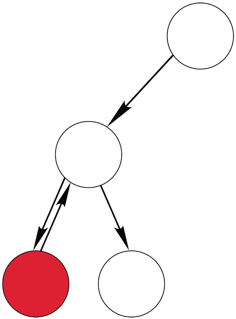

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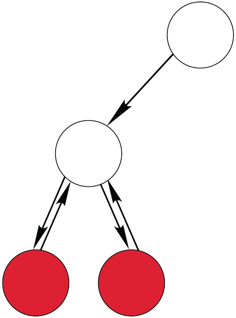

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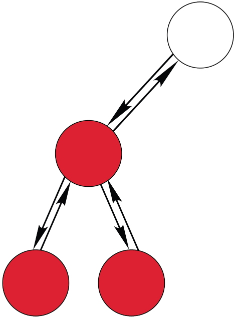

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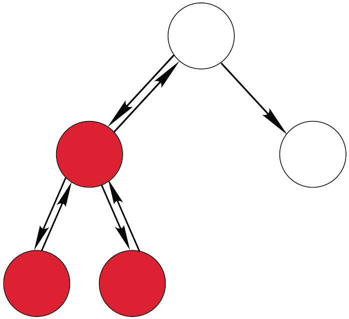

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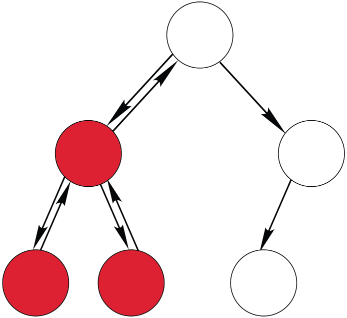

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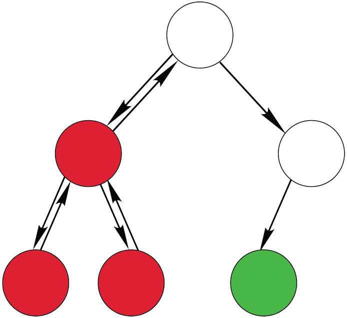

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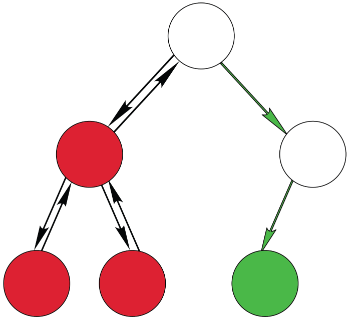


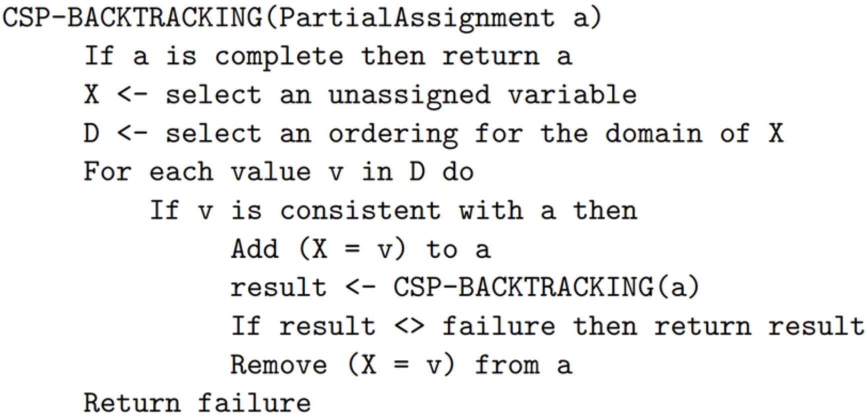
Complete Search - Backtracking

- The basic idea behind backtracking is to recursively build a partial solution by making choices from a set of available options, and then backtrack if the solution fails to satisfy the constraints.

- The algorithm explores the search space depth-first, which means that it goes as far as possible along each branch of the search tree before backtracking to the previous decision point and exploring another branch.

- if a node in the search tree does not lead to a solution, it is considered a deadend and its subtree can be pruned.

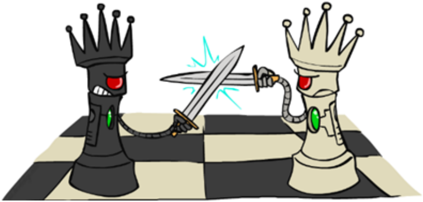
Complete Search - Backtracking

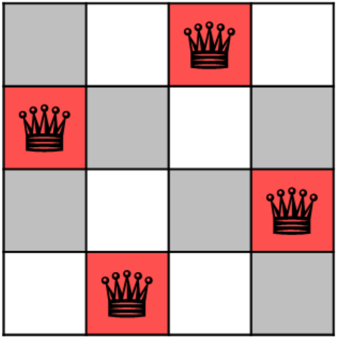


```
CSP-BACKTRACKING(PartialAssignment a)
    If a is complete then return a
    X <- select an unassigned variable
    D <- select an ordering for the domain of X
    For each value v in D do
        If v is consistent with a then
            Add (X = v) to a
            result <- CSP-BACKTRACKING(a)
            If result <> failure then return result
            Remove (X = v) from a
    Return failure
```

N-queens problem

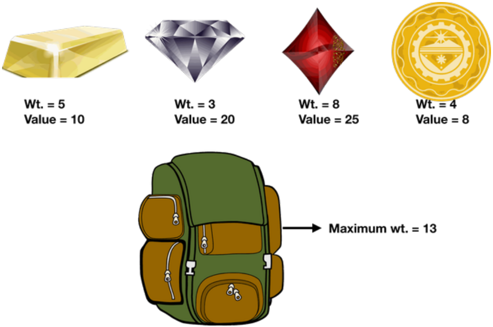
The N-Queens problem is a classic problem in computer science and mathematics that involves placing N chess queens on an N x N chessboard such that no two queens attack each other.





Knapsack

The Knapsack problem involves packing a knapsack with items of different weights and values. The goal is to maximize the value of the items in the knapsack while keeping the total weight of the knapsack below a certain limit.


Wt. = 5
Value = 10
Wt. = 3
Value = 20
Wt. = 8
Value = 25
Wt. = 4
Value = 8
Maximum wt. = 13