CHAPTER 3: N-gram Language Models

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Group 5: 3.4 - 3.5

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5.4 Generalization and Zeros

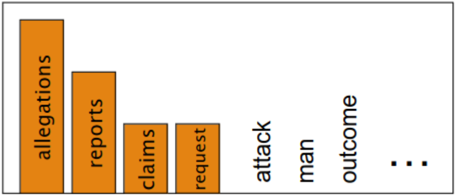
3.5 Smoothing

The intuition of smoothing

When we have sparse statistics:

$P(w \mid \text{denied the})$

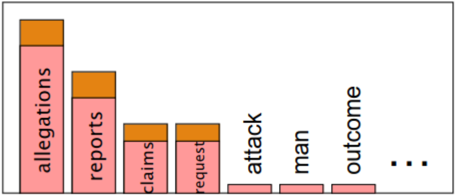
3 allegations
2 reports
1 claims
1 request
7 total



Steal probability mass

$P(w \mid \text{denied the})$

2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 request
7 total



Add-one estimation

Unsmoothed maximum likelihood estimate

$$P(w_i) = \frac{c_i}{N}$$

Laplace smoothing merely adds one to each count:

$$P_{Laplace}(w_i) = \frac{c_i + 1}{N + V}$$

Add-one estimation

It is convenient to describe how a smoothing algorithm affects the numerator, by defining an adjusted count c^*:

$$c_i^* = (c_i + 1)\frac{N}{N + V}$$

A related way to view smoothing is as discounting (lowering) some non-zero counts in order to get the probability mass that will be assigned to the zero counts:

$$d_c = \frac{c^*}{c}$$

Add-one estimation

Also called Laplace smoothing

Pretend we saw each word one more time than we did Just add one to all the counts!

- MLE estimate:

$$P_{MLE}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

Berkeley Restaurant Corpus

Raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Figure: Bigram counts for eight of the words (out of V = 1446) in the Berkeley Restau- rant Project corpus of 9332 sentences. Zero counts are in gray.

Berkeley Restaurant Corpus

Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{c(w_{n-1}w_n) + 1}{c(w_{n-1}) + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n) + 1] \times C(w_{n-1})}{C(w_{n-1}) + V}$$

	i	**want**	**to**	**eat**	**chinese**	**food**	**lunch**	**spend**
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Figure: Add-one reconstituted counts for eight words (of V = 1446) in the BeRP corpus of 9332 sentences. Previously-zero counts are in gray

Berkeley Restaurant Corpus

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	**want**	**to**	**eat**	**chinese**	**food**	**lunch**	**spend**
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-k smoothing

Instead of adding 1 to each count, we add a **fractional** count k (.5? .05? .01?).

$$P^*_{Add-k}(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + k}{C(w_{n-1} + kV)}$$

Add-k smoothing requires that we have a method for choosing k; this can be done, for example, by optimizing on a **devset**.

Add-1 estimation is a blunt instrument

So add-1 isn't used for N-grams:

- We'll see better methods

But add-1 is used to smooth other NLP models

- For text classification
- In domains where the number of zeros isn't so huge.

Backoff and Interpolation

Sometimes it helps to use **less** context

- Condition on less context for contexts you haven't learned much about

Backoff

- Use trigram if you have good evidence
- Otherwise bigram or unigram

Interpolation

- Mix unigram, bigram, trigram

Simple interpolation

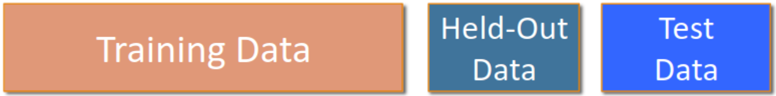
$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1 P(w_n|w_{n-2}w_{n-1}) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n)$$

$$\sum_i \lambda_i = 1$$

Lambdas conditional on context:

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1(w_{n-2}^{n-1}) P(w_n|w_{n-2}w_{n-1}) + \lambda_2(w_{n-2}^{n-1}) P(w_n|w_{n-1}) + \lambda_3(w_{n-2}^{n-1}) P(w_n)$$

Use a **held-out** corpus



Choose λs to maximize the probability of held-out data:

- Fix the N-gram probabilities (on the training data)
- Then search for λs that give largest probability to held-out set:

$$\log P(w_1 \ldots w_n | M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i | w_{i-1})$$

Backoff

In a **backoff** n-gram model

- If the n-gram we need has zero counts, we approximate it by backing off to the (n-1)-gram.
- We have to discount the higher-order n-grams to save some probability mass for the lower order n-grams.
- If we don't, the total probability assigned to all possible strings by the language model would be greater than 1.

Katz Backoff

We'll need a function alpha to distribute this probability mass to the lower order n-grams.

$$P_{BO}(w_n|w_{n-N+1:n-1}) = \begin{cases} P^*(w_n|w_{n-N+1:n-1}), & \text{if } C(w_{n-N+1:n}) > 0 \\ \alpha(w_{n-N+1:n-1})P_{BO}(w_n|w_{n-N+2:n-1}), & \text{otherwise} \end{cases}$$

Reference

Speech and Language Processing (3rd ed. draft)
Dan Jurafsky and James H. Martin
Part I: Fundamental Algorithms, *Chapter 3: N-gram Language Models*

Thanks for listening!

Q&A section