IMPORTANT INSTRUCTIONS

1- The folder contains book; physics 132
2- The book is composed in **MS-Word 2007**. The print out must be taken in MS-Word 2007.
3- The book is composed for four color printing.

Chief Writer

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SYLLABUS (PHY-132)

COMMON WITH COMPUTER, TELECOMMUNICATION,

COMPUTER INFORMATION & CRITICAL HEALTH CARE TECHNOLOGIES

MEASUREMENTS

1.1 Fundamental units and derived units
1.2 Systems of measurement and S.I units
1.3 Concept of dimensions, dimensional formula
1.4 Conversion from one system to another
1.5 Significant figures

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PREFACE

No nation can flourish without scientific and technical education. Realizing this fact, TEVTA, Government of the Punjab decided to prepare the books on science as well as on technical subjects for DAE students.

This book has been written according to the new curriculum. Some of the salient features of this book are as follows.

1. The International system of units (SI units) and nomenclature have been used.
2. Important statements, definitions and numbered equations have been made prominent.
3. The subject matter has been made more effective by adding figures and diagrams.
4. Summary of the chapter is given at the end of each chapter for a quick revision.
5. At the end of each chapter, important short as well as long questions have been included.
6. Multiple choice questions (MCQ's) have also been included at the end of each chapter.

As there is always a room for improvement, new ideas, information, fresh knowledge, continuous research and national demands always expect the curricula to be revised and updated regularly so MDC hope that the teachers, students and experts will continuously keep the committee informed of their opinion about the textbook.

Above all, members MDC and MEC thank Almighty Allah, who provided them the opportunity and gave the courage to undertake this task of national importance. It is prayed that this effort might be regarded as a fresh waft of air in this new era of Science and Technology for DAE students. (Amin!)

Members
Manual Development Committee &
Manual Evaluation Committee

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TECHNICAL EDUCATION AND VOCATIONAL TRAINING AUTHORITY
PUNJAB

A TEXT BOOK OF

APPLIED PHYSICS

PHY-132

FOR

DAE STUDENTS

OF

TECHNICAL EDUCATION

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Chapter 01

MEASUREMENTS

Course contents:

Learning Objectives:

At the end of this chapter the students will be able to:

- Write dimensional formulae for physical quantities.
- Derive units using dimensional equations
- Convert a measurement from one system to another
- Use concepts of measurement and significant figures in problem solving.

FUNDAMENTAL UNITS AND DERIVED UNITS

All physical quantities have their magnitudes, which can be measured experimentally. These magnitudes help the scientists to formulate the laws.

Fundamental Units:

The measurement of various physical quantities requires a suitable unit for each. A certain system of units is formed by arbitrarily defining the units of some suitable quantities. These units are known as fundamental units.

For example meter, kilogram and second are the fundamental units of length, mass and time respectively in MKS system.

Derived Units:

The units of physical quantities expressed in terms of fundamental quantities are known as derived units.

Following are the few examples of derived units

Meter per second ($m\ s^{-1}$), which is unit of speed.

Meter per square second ($m\ s^{-2}$), which is unit of acceleration.

Newton ($N = kg\text{-}m/s^2$), which is the unit of force, weight & effort.

Joule ($J = N\text{-}m$), which is the unit of work and energy.

SYSTEMS OF MEASUREMENT AND S.I. UNITS

M.K.S System

M.K.S System or meter-kilogram-second has the following fundamental units.

Meter	Unit of length
Kilogram	Unit of mass
Second	Unit of time

C.G.S System

C.G.S system or centimeter -gram-second system has the following fundamental units

Centimeter	Unit of length
Gram	Unit of mass
Second	Unit of time

F.P.S System

F.P.S system or foot-pound-second system or British Engineering system has the following fundamental units.

Foot	Unit of length
Pound	Unit of force (Pound is also used for the unit of mass)
Second	Unit of time

SYSTEM INTERNATIONAL UNITS

In 1875 A.D. the scientists of all over the world established an international Bureau of Weights and measures near Paris. The international system proposed by this Bureau is known as S.I units which operated all over the world.

S.I units or system international units has the following fundamental units.

Quantity	Symbol (for quantity)	Unit	Symbol (for unit)
Length	L	Meter	m
Mass	M	Kilogram	kg
Time	T	Second	s
Electric Current	I	Ampere	A
Temperature	T	Kelvin	K
Quantity of substance	N	Mole	mol
Luminous Intensity	I	Candela	cd

There are two supplementary units; radian for plane angle and steradian for solid angle.

DEFINITIONS:

Meter

"The distance between two finely drawn lines on metal bar placed in the International Bureau of Weights and Measures, at Paris."

Currently, meter is defined as the distance travelled by a light in vacuum in $\frac{1}{299,792,458}$ second."

Kilogram

"The mass of a cylinder of specific dimensions of platinum - iridium alloy kept in the international Bureau of Weights and Measures near Paris is taken to be the standard one kilogram."

Second

"Second is defined as $\frac{1}{86400}$ of a mean Solar day of the year 1900."

"Second was redefined as that time during which 9,192,631,770 vibrations of Cesium-133 atom take place."

MULTIPLES AND SUB MULTIPLES OF UNITS

Length

1 kilometer	=	1000 meters(m)
1 meter	=	100 centimeter(cm)
1 centimeter	=	10 millimeter(mm)
1 Meter	=	10^{6}micrometer(μm)
1 Meter	=	10^{9}nanometer(nm)

Mass

1 kilogram	=	1000 grams(g)
1 gram	=	1000 milligrams(mg)
1 gram	=	10^{6} microgram(μg)
1000 kilogram	=	1 tonne

Time

1 Hour	=	60 minutes
1 Minute	=	60 seconds
1 Hour	=	3600 seconds
1 millisecond	=	10^{-3} seconds
1 microsecond	=	10^{-6} seconds
1 nanosecond	=	10^{-9} seconds

SOME IMPORTANT EQUATIONS FOR CONVERSION OF UNITS

There are some important relations which are useful for the conversion of units from one system of units to the other.

1 meter	=	3.28 feet
1 mile	=	1.61 kilometers

1 *lb* (pound)	=	4.45 newton
1 inch	=	2.54 cm
1 slug	=	14.5 kg
1N	=	10^5 dyne
1 kg	=	2.21 *lb*

SOME IMPORTANT STANDARD PREFIXES FOR THE S.IUNITS

Multiples			Fractions		
Name	**Prefix**	**Factor**	**Name**	**Prefix**	**Factor**
deca	da	10^1	deci	d	10^{-1}
hecto	h	10^2	centi	c	10^{-2}
kilo	k	10^3	milli	m	10^{-3}
mega	M	10^6	micro	μ	10^{-6}
gega	G	10^9	nano	n	10^{-9}
tera	T	10^{12}	pico	p	10^{-12}
peta	P	10^{15}	femto	f	10^{-15}

EXAMPLE: Speed of a cycle is 2ft/s, find its speed in m/s.

SOLUTION:

$$V = 2 \text{ ft/s} = 2 \text{ ft/s} \times \frac{1\,m}{3.28\,ft}$$

$$= 0.61 \text{ m/s}$$

CONCEPT OF DIMENSION AND DIMENSIONAL FORMULA

In physics the word dimension represents the physical nature of a quantity. A distance may be measured in feet or in meters but it is still a distance. We say its dimension is length. In the same way irrespective of the system of units the dimension of mass is represented by M and the dimension of time is represented by T.

The dimensions of three fundamental quantities of MKS system are represented by the first letterof their name in capital alphabets.

The dimensional formula of a derived physical quantity is obtained by defining its relation with other fundamental or derived physical quantities and then expressing these quantities in terms of mass [M], length [L] and time [T].

For example;

Velocity

Velocity is defined as displacement traveled per unit time.
The dimension of velocity is:

$$Velocity = \frac{Displacement}{time} = \frac{L}{T} = LT^{-1} \text{ or } M^{o}LT^{-1}$$

This is the dimensional formula for the velocity. It shows that velocity depends upon length and mass but it is independent of mass. It can also be inferred from the dimensional formula of velocity that the in the unit of velocity the power of L is 1 and T is -1 i.e. in S.I units the unit of velocity is ms^{-1}.

Density

Density is defined as mass per unit volume.

The dimension of density is: $density = \frac{mass}{volume} = \frac{M}{L^3} = ML^{-3}$

or $ML^{-3}T^{o}$

This is the dimensional formula for the density. It shows that density depends upon mass and length and it is independent of time. It also indicates that in the unit of density the power of M is 1 and L is -3 i.e. in S.I units the unit of density is $kg\text{-}m^{-3}$.

Dimensional equation

Dimensional equation is obtained when a physical quantity is equated with its dimensional formula.
For example in dimension equation [Acceleration] =[LT^{-2}], the left hand side shows the physical quantity and right hand side shows its dimension formula in terms of mass, length and time.

DIMENSIONAL FORMULAE AND UNITS OF

IMPORTANTPHYSICAL QUANTITIES

QUANTITY	FORMULA	DIMENSIONAL FORMULA	UNIT
Area	$Length \times width$	L^2	m^2
Volume	$Length \times width \times height$	L^3	m^3
Density	$\frac{mass}{volume}$	ML^{-3}	$kg\ m^{-3}$
Velocity	$\frac{displacement}{time}$	LT^{-1}	ms^{-1}
Acceleration	$\frac{Velocity}{time}$	LT^{-2}	ms^{-2}

Force	$Mass \times acceleration$	MLT^{-2}	N
Momentum	$Mass \times velocity$	MLT^{-1}	Ns
Work	$Force \times displacement$	ML^2T^{-2}	J
Energy	$Mass \times gravitational\ acceleration \times height$	ML^2T^{-2}	J
Pressure	$\frac{Force}{Area}$	$ML^{-1}T^{-2}$	Nm^{-2}
Power	$\frac{Work}{time}$	ML^2T^{-3}	$watt$
Stress	$\frac{Force}{Area}$	$ML^{-1}T^{-2}$	Nm^{-2}
Strain	$\frac{Change\ in\ dimesion}{Original\ dimension}$	$Dimensionless$	$No\ unit$
Torque	$Force \times perpendicular\ distance$	ML^2T^{-2}	Nm

USES OF DIMENTIONAL EQUATIONS

Dimensional equation can be used to:
Convert one system of units into another.
Derive a correct relationship between different physical quantities.
Dimensional analysis: It is the procedure for analyzing the units of equations.

Example: Check by dimension that the following equation of motion iscorrect.

$$S = v_i t + \frac{1}{2} a t^2$$

$$\text{Dimension of L.H.S} = L^1$$

$$\text{Dimension of R.H.S} = [L^1T^{-1}][T^1] + \frac{1}{2}[L^1T^{-2}][T^2]$$

$$= L^1 + \frac{1}{2} L^1$$

$$= L^1$$

As Dimension of L.H.S = Dimension of R.H.S

So equation is correct.

SIGNIFICANT FIGURES

"In any measurement, the accurately known digits and the first doubtful digit are called significant figures."

In other words, a significant figure is the one which is known to be reasonably reliable. Significant figures of a number are those digits that carry meaning contributing to its precision.

Whenever a physical quantity is measured, there is inevitable some uncertainty about its determined value. One of the reasons of this uncertainty is the instrument used for the measurement. Every measuring instrument is calibrated to a certain smallest division. This fact put a limit to the degree of accuracy which may be achieved while taking measurements with this instrument. Foot rules are usually calibrated in millimeters. Suppose that we want to measure the length of a matchstick with this foot rule. Let end point of the matchstick lies between 2.3 and 2.4 cm marks. By convention, if the end of the matchstick does not touch or cross the midpoint of the smallest division, the reading is confined to 2.3 cm. In case the end of the matchstick seems to be touching or have crossed the midpoint, the reading is extended to 2.4 cm.

So in this measurement digit 2 is accurate and next digit 3 or 4 is doubtful one. So the number of significant digits in this measurement is 2.

RULES FOR IDENTIFYING SIGNIFICANT FIGURES

- All non-zero digits are considered significant. For example 92 has two significant figures 9 and 2, while 123.73 has five significant figures (1,2,3,7 and 3)
- Zeros appearing anywhere in between two non-zero digits are significant. For example 102.203 has six significant figures (1,0,2,2,0 and 3).
- In any value leading zeros are not significant. For example in 0.00042 only two digits are significant (4 and 2, leading zeros are not significant).
- Trailing zeros in a number with decimal point are significant.For example in number 18.200 five digits are significant (1,8,2,0 and 0)
- The trailing zeros in a number having no decimal point may be ambiguous. In 12000 the trailing zeros may or may not be significant. It depends upon the accuracy of the measuring instrument.
- When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example in 8.32×10^3 kg only three digits (8,3 and 2) are significant.
- When numbers are multiplied or divided the number of significant figures kept in the result must not be more than the significant figures of the number having least significant digits. For example if 3.456×10^5 and $2.25 \times 10^6$are multiplied the result is 7.776×10^{11}. In second number there are only 3 significant digits but the result shows 4

significant digits. So the result must be expressed in three significant digits for that purpose it must be rounded to three significant digits.

SUMMARY

- The foundation of physics rests upon physical quantities. The laws of physics are expressed in terms of these physical quantities. These physical quantities are divided into two categories.
- The base quantities are minimum number of those physical quantitates in terms of which other physical quantities can be defined. The examples of base quantities are length, mass and time.
- Derived physical quantities are those physical quantities whose definitions are based on other physical quantities.
- The units of base physical quantities are arbitrarily chosen and are called fundamental units.
- The units of derived quantities are derived from fundamental units and are called derived units.
- There are 7 base units in International System of Units.
- Each base quantity is considered a dimension denoted by a specific symbol written within square bracket. It shows the nature of the physical quantity. The dimensions of length, mass and time are [L], [M] and [T] respectively.
- The significant figures or digits in a measured or calculated quantity are those digits that are known to be reasonably reliable. The accurately known digits and the first doubtful digit in a measurement are called significant digits.

EXERCISE

LONG QUESTIONS

1- Write fundamental S.I quantities and their units.

2- What do you mean by significant figures? Describe rules for identification of significant figures with examples.

3- Prove by dimensions that the equation for the time period of simple pendulum is correct. Equation for time period of simple pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

4- Show that the expression V_f^2 - V_i^2 = 2 a s is dimensionally correct.

SHORT QUESTIONS

1- Define fundamental and derived units.
2- Write basic quantities of M.K.S system and write their fundamental units.
3- Write basic quantities of C.G.S system and write their fundamental units.
4- Define "meter".
5- Define "second".
6- Write names of basic quantities in system international.
7- Write fundamental S.I units.
8- Convert 72 km/h into m/s.
9- Convert 25 m/s into km/h.
10- Convert density of 1000 kg/m^3 into g/cm^3.
11- Write dimension of work.
12- Write dimension of Force.
13- Define significant figures.

MULTIPLE CHOICE QUESTIONS (MCQ's)

Encircle the correct answer.

1- **The fundamental S. I. Unit of temperature is:**
a) Centigrade b) Fahrenheit c) Kelvin d) Joule

2- **The fundamental unit of length in C. G. S. system is:**
a) Foot b) Centimeter c) Mile d) Meter

3- **1 pound is equal to:**
a) 4.45 N b) 10^5 N c) 2.54 N d) 100 N

4- **One inch is equal to:**
a) 2.54 cm b) 2.45 cm c) 0.9 cm d) 12 cm

5- **1 N is equal to:**
a) 10^5 dynes b) 10^3 dynes c) 10^7 dynes d) 4.45 dynes

6- **1 milligram is equal to:**
a) 10^3 gram b) 10^6 gram c) 10^{-6} gram d) 10^{-3} gram

7- **1 meter is equal to:**
a) 10^3 mm b) 10^6 mm c) 10^{-6} mm d) 10^{-3} mm

8- **Fundamental S.I. units are:**
a) 3 b) 7 c) 4 d) 5

9- **Unit of current is:**
a) ampere b) Kelvin c) newton d) joule

10- **S.I Unit of plane angle is:**
a) Centigrade b) Degree c) Radian d) Steradian

11- The dimension of momentum is:

a) $M\,L\,T^{-1}$ b) $M\,L\,T^{-2}$ c) $M\,L^{2}\,T^{-1}$ d) $M\,L^{3}$

12- The dimension of force is:

a) $M\,L\,T^{-1}$ b) $M\,L\,T^{-2}$ c) $M\,L^{2}\,T^{-1}$ d) $M\,L^{3}$

13- The numbers of significant figures in 2.80×10^{5} are:

a) 3 b) 2 c) 1 d) 5

PROBLEMS

1.1 Convert 90 Km/hr into m/sec. [Ans. 25 m/s]

1.2 Convert 25 m/sec into Km/h. [Ans. 90km/hr]

1.3 Convert 5 hours 30 minutes into seconds. [Ans. 19800 s]

1.4 Height of a student is 5ft and 9 inches. What will be his height in meters? (hint: 1 inch = 2.54 cm) [Ans. 175.26cm]

1.5 Maximum speed limit at motor way in Pakistan is 120 km/hr. What will be this limit in m/s? [Ans. 33.33 m/s]

1.6 What will be the weight of a 50 kg bag of cement in newton? (Take value of $g = 10\ m/s^2$) [Ans. 500N]

1.7 Speed of light is 3×10^{8} m/s. Express it in km/s and cm/s. [Ans. 3×10^{5}km/s, 3×10^{10}cm/s]

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Chapter 02

SCALARS AND VECTORS

Course contents:

2.1. Revision of head to tail rule.
2.2. Laws of parallelogram, triangle and polygon of forces.
2.3. Resolution of a vector.
2.4. Addition of vectors by rectangular components.
2.5. Multiplication of two vectors, dot product and cross product.

Learning Objectives:

At the end of this chapter the students will be able to:

- Define physical quantities, i.e. scalars and vectors.
- Define the kinds of vectors.
- Explain the Multiplication of a vector by a number, subtraction of vectors.
- Describe addition of vectors by head-to-tail rule.
- Define law of parallelogram of forces, law of triangle of forces and law of polygon of forces.
- Explain resolution of a vector into rectangular components.
- Explain addition of vectors by rectangular components method.
- Describe multiplication of vectors, i.e. scalar product and vector product.

PHYSICAL QUANTITIES

The quantities which can be measured are known as physical quantities. They are divided into the following two types.

1. SCALARS:

Physical quantities that can be completely specified by their magnitude and suitable unit are known as scalars. They can be added, subtracted, multiplied and divided by ordinary mathematical rules.

Examples:

Mass, distance, speed, energy, work, area, volume, temperature, time, money, electric potential etc.

2. VECTORS:

Physical quantities that can be completely specified by their magnitude, suitable unit and direction are known as vectors. They cannot be added, subtracted and multiplied by ordinary mathematical rules; but we use methods of vector addition, vector subtraction and vector multiplication for this purpose.

Examples

Displacement, velocity, acceleration, force, momentum, torque, angular velocity etc.

REPRESENTATION OF A VECTOR:

Generally, a vector is represented by a bold-faced type, letter. e.g., **A** or by putting an arrow or a bar above or below the letter such as $\vec{A}$,$\overline{A}$ or $\underline{A}$. The magnitude of the vector is represented in light-faced italics or by its modulus. For example, magnitude of **A** is represented by *A*, or $|A|$.

Graphically:

Graphically, a vector is represented by an arrow of certain length. According to the chosen scale, the length of the arrow represents the magnitude of the vector. Thus the length and direction of arrow represents the magnitude and direction of the vector. For example, a force of 10N is acting in the direction of North. To represent it, we chose a suitable scale.

Let 5N = 1 cm

and 10N = 2 cm

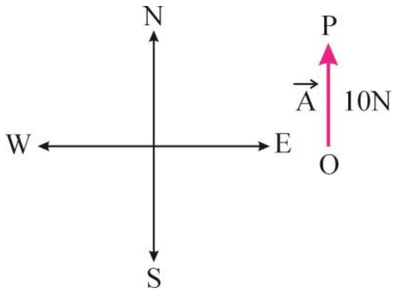

Fig 2.1

Here, 10N force is represented by a 2 cm line $\overline{OP}$ as shown in Figure 2.1

KINDS OF VECTORS:

1. Unit Vector:

A vector whose magnitude is one is called a unit vector. It is generally used to represent the direction of a vector. It is represented by a letter with a cap over it as $\hat{A}$. Any vector **A** can be written in terms of unit vector as

$$\mathbf{A} = A\hat{A} \qquad \text{or} \qquad \mathbf{A} = |\mathbf{A}|\hat{A}$$

Here A or | A | is the magnitude of the vector **A** and $\hat{A}$ is the unit vector along the vector **A** which shows the direction. Unit vector can also be written as

$$\hat{A} = \frac{\mathbf{A}}{|A|} \quad \text{or} \quad \hat{A} = \frac{\mathbf{A}}{A}$$

or Vector = magnitude × direction

Unit vector = 1 × direction = Direction of the vector

Note: $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$are also the unit vectors usually taken along x , y and z–axis respectively

2. Null Vector:

It is a vector having zero magnitude and arbitrary direction. It is written as

Null vector = **O**

It is an imaginary vector and cannot be represented along any axis.

3. Resultant Vector:

When we add two or more than two vectors, we get a single vector which has the same effect as the combined effect of all the vectors to be added. The single vector is known as the resultant vector. For example, we add three vectors $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$ acting on a body at O, then the resultant vector **A** is written as

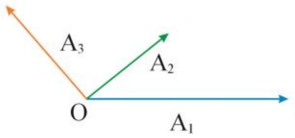

Fig 2.2(a)

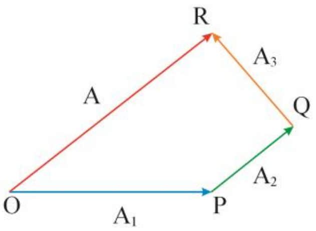

Fig 2.2(b)

$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3$

4. Negative of a Vector:

A vector having the same magnitude as that of a given vector **A**but opposite in direction is called the negative of **A** and is denoted by – **A**as shown in Fig. 2.3.

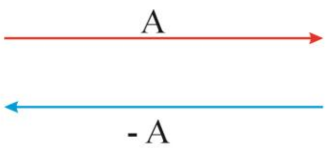

Fig 2.3

5. Component Vector:

A vector can be resolved into two or more than two such vectors which have the same combined effect as that of the effect of original vector, then each of these vectors is called a component vector. In the above Fig. 2.2, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$ are the component vectors.

Multiplicationofa Vectorbya Number:

When a certain vector **A** is multiplied by a number n then the magnitude of the resultant vector becomes n times the magnitude of the original vector,

Fig 2.4

i.e., $|n\,\mathbf{A}| = n\,|\mathbf{A}|$. When n is positive, its direction remains the same and it is reversed when n is negative.

Fig. 2.4 shows the multiplication of a vector **A** by $\frac{1}{2}$, 2 , and –3.

Addition of Vectors by Head-to-Tail Rule:

To add two or more than two vectors, we draw them in such a way that the head of the first vector coincides with the tail of the second vector; the head of the second vector coincides with the tail of the third vector and so on. Then the resultant vector is obtained by joining the tail of the first vector with the head of the last vector. The magnitude and the direction of the resultant vector are found by measuring rod and protractor.

In Fig. 2.5(a) we are given two vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, acting at point O, making angles θ_1 and θ_2 respectively with x-axis and are represented by **OA** and **OB**. To add these vectors by head-to-tail rule, we draw their representative vectors $\overline{CD}$ and $\overline{DE}$ such that the head of $\overline{CD}$ coincides with the tail of $\overline{DE}$, then the resultant vector F is obtained by combining the tail of $\overline{CD}$ with the head of $\overline{DE}$ and is represented by the line $\overline{CE}$ as shown in Fig. 2.5(b)

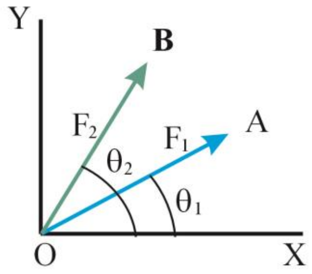

Fig 2.5(a)

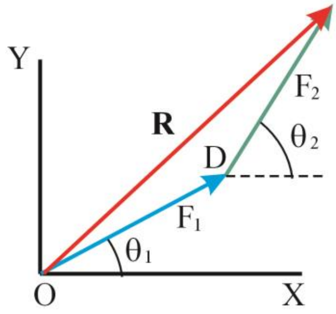

Fig 2.5(b)

LAW OF PARALLELOGRAM OF FORCES:

When two forces are acting at a point such that they can be represented by the adjacent sides of a parallelogram then their resultant will be equal to that diagonal of the parallelogram which passes through the same point.

Explanation:

(Analytical method to find resultant of two vectors): Let us have two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at the point O which are represented by the lines $\overline{OA}$ and $\overline{OB}$ with angle θ between them as shown in Fig. 2.6(a). They are forming the two adjacent sides $\overline{OA}$ and $\overline{OC}$ of the parallelogram OABC. Their

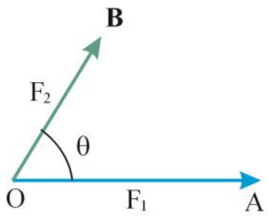

Fig 2.6(a)

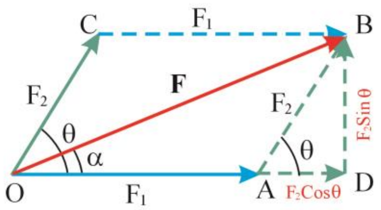

Fig 2.6(b)

resultant **R** will be equal to the diagonal $\overline{OB}$ which is passing through the same point O. the magnitude of this resultant force is calculated by considering the right angled triangle OBD.

Here

$$\overline{AB} = \overline{OC} = \mathbf{F}_2$$

$$\overline{AD} = F_2\cos\theta \; ; \qquad \overline{DB} = F_2\sin\theta$$

In right angled triangle ODB:

$$F_x = \overline{OD} = \overline{OA} + \overline{AD} = F_1 + F_2 \, Cos\,\theta$$

$$F_y = \overline{DB} = F_2 \, Sin\,\theta$$

$$|\,F\,| = \sqrt{F_x^2 + F_y^2} = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

$$= \sqrt{F_1^2 + F_2^2 cos^2\theta + 2\,F_1F_2\,cos\theta + F_2^2 sin^2\theta}$$

$$= \sqrt{F_1^2 + F_2^2(cos^2\theta + sin^2\theta) + 2\,F_1F_2\,cos\theta}$$

$$\text{Magnitude} = |\,F\,| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

This is the magnitude of the resultant vector**F**. To find its direction, let α be the angle which the resultant vector makes with x-axis, then in right angled triangle ODB.

$$\tan\alpha = \frac{\overline{BD}}{\overline{OD}} = \frac{F_y}{F_x} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

or

$$\alpha = tan^{-1}\left(\frac{F_2\,Sin\,\theta}{F_1 + F_2\,Cos\,\theta}\right)$$

Note: To find the unknown angle, we always take the tangent of the angle.

LAW OF TRIANGLE OF FORCES:

If two forces are acting on a body such that they can be represented by the two adjacent sides of a triangle, taken in the same order, then their resultant will be equal to the third side (enclosing side) of that triangle taken in the opposite order.

We are given two forces F_1 and F_2 actingat point O represented by lines $\overline{OA}$ and $\overline{OB}$ as shown in the Fig.2.7(a). If we represent these forces by the two sides $\overline{CD}$ and $\overline{DE}$ of a triangle, taken in the same order, then their resultant force F will be equal to the third side $\overline{CE}$ of the triangle, taken in the opposite order, as shown in Fig. 2.7(b). Mathematically it is written as

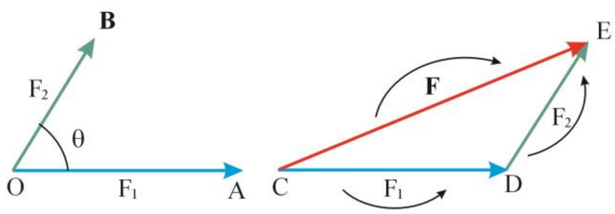

Fig 2.7(a) Fig 2.7(b)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

Triangleofforces (converseoflawoftriangleofforces:

If three forces acting on a body are such that they can be represented by the three sides of a triangle, taken in thesame order, then their resultant will be equal to zero and the body will be in equilibrium as shown in Fig. 2.8. Mathematically it is written as

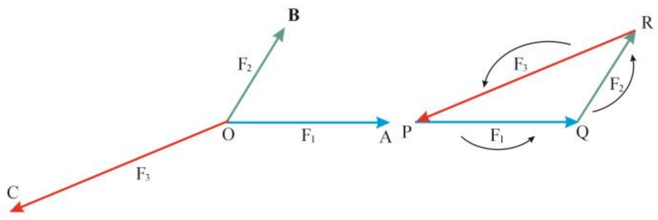

Fig 2.8(a) Fig 2.8(b)

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$

LAW OF POLYGON OF FORCES:

If several forces are acting on a body such that they can be represented by the adjacent sides of a polygon, taken in the same order, then their resultant will be equal to that side of the polygon which completes the polygon (closing side), taken in the opposite order.

We are given five forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$ and $\mathbf{F}_5$ as shown in Fig. 2.9 (a).They can be represented by the sides of a polygon, taken in the same order, then their resultant $\mathbf{F}$, represented by line $\overline{OE}$, which completes the polygon will be taken in the opposite order as shown in Fig. 2.9 (b). Mathematically,

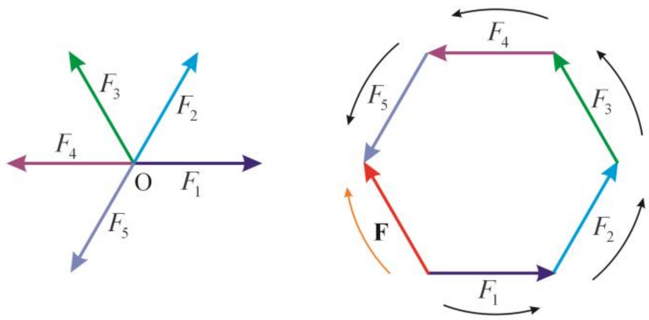

Fig 2.9(a) Fig 2.9(b)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \mathbf{F}_5$$

In the case of n forces $\mathbf{F}_1$,$\mathbf{F}_2$, $\mathbf{F}_3$,- - - , $\mathbf{F}_n$ acting on the body we shall have

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + - - - + \mathbf{F}_n$$

Polygon of forces (converse of law of polygon of forces:

If several forces are acting on a body such that they can be represented by the sides of a closed polygon, taken in the same order, then their resultant will be equal to zero and the body will be in equilibrium.

If we are given six forces $\mathbf{F}_1$,$\mathbf{F}_2$, $\mathbf{F}_3$, - - - , $\mathbf{F}_6$ acting at point O as shown in Fig. 2.10

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + - - - + \mathbf{F}_6 = 0$

In the case of n forces $\mathbf{F}_1$,$\mathbf{F}_2$, $\mathbf{F}_3$,- - -, $\mathbf{F}_n$ acting on the body O, we shall have

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_2 + - - - + \mathbf{F}_n = 0$

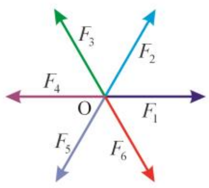

Fig 2.10(a)

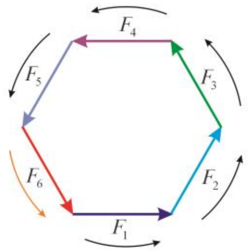

Fig 2.10(b)

RESOLUTION OF A VECTOR INTO COMPONENTS:

The process of splitting up of single vector into two or more vectors is called resolution of the vector.

A vector can be resolved into two or more vectors which have the same combined affect as that the effect of original vector. Then each of these vectors is called a component vector. In Fig. 2.11, a vector **F** has been resolved into three component vectors $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and can be written as:

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

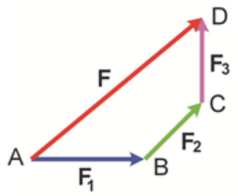

Fig 2.11

Resolution of a Vector into Rectangular Components:

If a vector is resolved into such components which are at right angle (perpendicular) to each other then they are called the rectangular components of that given vector. In Fig. 2.12, we are given a position vector **F** represented by line $\overline{AC}$ and making angle θ with the x-axis. To resolve it into rectangular components, draw a normal from the point C on the x-axis and we get a point B as shownin Fig. 2.12. Now join $\overline{AB}$and $\overline{BC}$by head- to-tail rule then

$$\mathbf{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

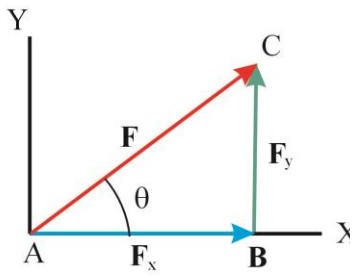

Fig 2.12

Here the vectors $F_x\hat{\imath}$ and $F_y\hat{\jmath}$ are the horizontal and vertical components of the vector **F**. As they are normal to each other, so they are called the rectangular components. They can be found as:

In the right-angled triangle ΔABC we see that

$$\frac{\overline{AB}}{\overline{AC}} = \cos\theta \quad \Rightarrow \quad \overline{AB} = \overline{AC}\cos\theta$$

or $F_x = F\cos\theta$ - - - - - - (1)

$$\frac{\overline{BC}}{\overline{AC}} = \sin\theta \Rightarrow \overline{BC} = \overline{AC}\sin\theta$$

or $F_y = F\sin\theta$ - - - - - - (2)

In vector form

$F_x = F\cos\theta\hat{\imath}$ & $F_y = F\sin\theta\hat{\jmath}$

Here, $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors along x-axis and y-axis respectively.

EXAMPLE 2.1: A force of 15N is acting at an angle of 30° with x–axis. Find its rectangular components F_x and F_y

SOLUTION:

$F_x = F\cos\theta$

$= 15\cos 30^o$

$= 15 \times .866 = 12.99N$

$F_y = F\sin\theta = 15\sin 30^o$

$= 15 \times 0.5 = 7.5N$

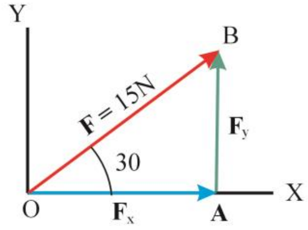

Fig 2.13

How to find a Vector from its Rectangular Components:

Let us have two vectors $\mathbf{F_x}$ and $\mathbf{F_y}$ which are acting along x-axis and y-axis respectively and are represented by the lines $\overrightarrow{AB}$ and $\overrightarrow{AD}$ as shown in Fig. 2.14. In order to get the resultant vector, we complete the rectangle ABCD. Join the point C with point A, then $\overrightarrow{AC}$ represents the resultant vector **F**.

As $\overrightarrow{BC} = \overrightarrow{AD}$ so $\overline{BC} = F_y$

Applying Pythagoras Theorem in right-angled triangle ΔABC

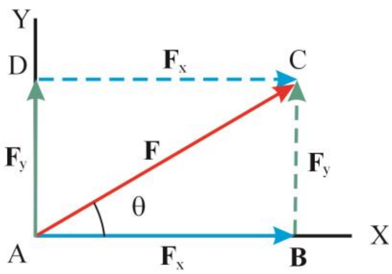

Fig 2.14

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$|\overline{AC}| = \sqrt{(\overline{AB})^2 + (\overline{BC})^2}$$

or $$F^2 = F_x^2 + F_y^2$$

$$\Rightarrow F = \sqrt{F_x^2 + F_y^2}$$

This equation gives the magnitude of the resultant vector F.

To find the direction of the resultant **F** which is making angle θ with the x-axis, we have

$$\tan\theta = \frac{\overline{BC}}{AB} = \frac{F_y}{F_x}$$

$$\Rightarrow \quad \theta = tan^{-1}\left(\frac{F_y}{F_x}\right)$$

EXAMPLE 2.2: The x and y–components of a force Fare 3N and 4N respectively. Find magnitude and direction of F

SOLUTION:

Here $F_x = 3N$ and $F_y = 4N$

Then magnitude of the resultant force is given by

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5N$$

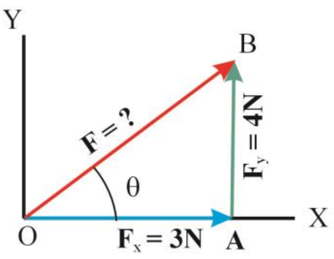

Fig 2.15

To determine the direction, we have

$$\tan\theta = \frac{F_y}{F_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}(1.33) = 53^o \text{ with x-axis}$$

ADDITION OF VECTORS BY RECTANGULAR COMPONENTS METHOD[TrigonometricMethod]:

Consider two vectors $\mathbf{F_1}$ and $\mathbf{F_2}$ which are making angles θ_1 and θ_2 with the x-axis respectively and are shown by lines $\overline{AB}$ and $\overline{AC}$ as shown in Fig 2.16. To find the resultant vector **F**, we add $\mathbf{F_2}$ in $\mathbf{F_1}$ byhead to tail rule.

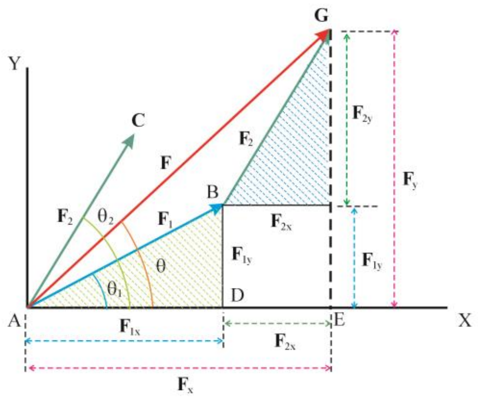

Fig 2.16

i.e. $\mathbf{F} = \mathbf{F_1} + \mathbf{F_2}$

In fig 2.16 $\overline{AE} = \overline{AD} + \overline{DE}$

$$\mathbf{F_x} = \mathbf{F_{1x}} + \mathbf{F_{2x}}$$

This shows that the horizontal component of the resultant vector **F** is equal to the sum of the horizontal components of the component vectors $\mathbf{F_1}$ and $\mathbf{F_2}$.

Similarly $\overline{EG} = \overline{EU} + \overline{UG}$

$$\mathbf{F_y} = \mathbf{F_{1y}} + \mathbf{F_{2y}}$$

This shows that the vertical component of the resultant vector **F** is equal to the sum of the vertical components of the component vectors $\mathbf{F_1}$ and $\mathbf{F_2}$

As $\mathbf{F}_x$ and $\mathbf{F}_y$ are the rectangular components of the resultant vector **F**

$$\mathbf{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

$$= (F_{1x} + F_{2x})\hat{\imath} + (F_{1y} + F_{2y})\hat{\jmath}$$

The magnitude of the resultant vector **F** is written as

$$|F| = \sqrt{(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2}$$

If the resultant vector makes an angle θwith the x-axis then its direction is found as:

$$\tan\theta = \left(\frac{F_{1y} + F_{2y}}{F_{1x} + F_{2x}}\right)$$

$$\theta = tan^{-1}\left(\frac{F_{1y} + F_{2y}}{F_{1x} + F_{2x}}\right)$$

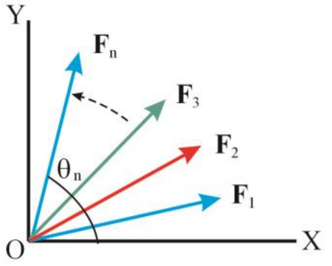

Fig 2.17

If we have n vectors $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, ……..,$\mathbf{F}_n$ as shown in the Fig. 2.17 then

$$\mathbf{F}_x = \mathbf{F}_{1x} + \mathbf{F}_{2x} + \mathbf{F}_{3x} + ---- + \mathbf{F}_{nx}$$

$$\mathbf{F}_y = \mathbf{F}_{1y} + \mathbf{F}_{2y} + \mathbf{F}_{3y} + ---- + \mathbf{F}_{ny}$$

$$|F| = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(F_{1x} + F_{2x} + ... + F_{nx})^2 + (F_{1y} + F_{2y} + ... + F_{ny})^2}$$

And the direction of the resultant vector is found as

$$\tan\theta = \frac{F_y}{F_x} = \frac{[F_{1y} + F_{2y} + ... + F_{ny}]}{[F_{1x} + F_{2x} + ... + F_{nx}]}$$

How to Solve the Problems:

If we have been given n vectors $\mathbf{F}_1$,$\mathbf{F}_2$, $\mathbf{F}_3$, ..., $\mathbf{F}_n$ and we want to find their resultant vector, then

1. Add the x-components of all the vectors and denote it by $\mathbf{F}_x$,
   i.e., $\mathbf{F}_x = \mathbf{F}_{1x} + \mathbf{F}_{2x} + \mathbf{F}_{3x} + --- + \mathbf{F}_{nx}$
2. Add the y-components of all the vectors and denote it by $\mathbf{F}_y$,
   i.e., $\mathbf{F}_y = \mathbf{F}_{1y} + \mathbf{F}_{2y} + \mathbf{F}_{3y} + --- + \mathbf{F}_{ny}$
3. Find the magnitude of the resultant vector F by using the formula

$$|F| = \sqrt{F_x^2 + F_y^2}$$

4. Find the direction of the resultant vector. First find the absolute angle θ, $\theta = tan^{-1}\left(\left|\frac{F_y}{F_x}\right|\right)$,then find the standard angle (ϕ) i.e. the angle which the resultant vector makes with the positive x-axis, using the following formulas :

(a) If F_x and F_y are both positive, then take

$$\phi = \theta = \tan^{-1}\left|\frac{F_y}{F_x}\right|$$

(b) If F_x is negative and F_y is positive, then take

$$\phi = 180^\circ - \theta$$

$$= 180^\circ - \tan^{-1}\left|\frac{F_y}{F_x}\right|$$

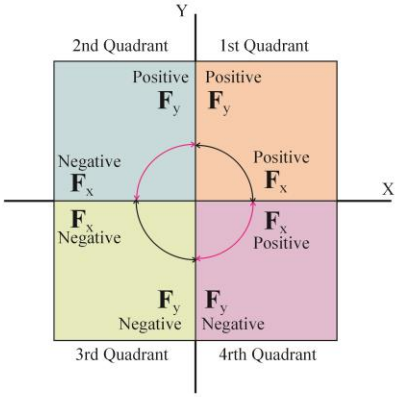

Fig 2.18

(c) If F_x and F_y are both negative, then take

$$\phi = 180^\circ + \theta$$

$$= 180^\circ + \tan^{-1}\left|\frac{F_y}{F_x}\right|$$

(d) If F_x is positive and F_y is negative, then take

$$\phi = 360^\circ - \theta$$

$$= 360^\circ - \tan^{-1}\left|\frac{F_y}{F_x}\right|$$

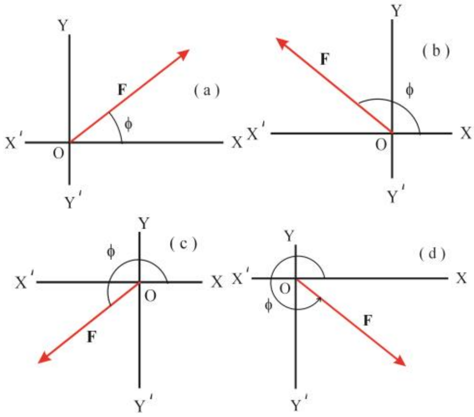

Fig 2.19

MULTIPLICATION OF VECOTRS:

Two vectors can be multiplied with each other by the following two ways:

SCALAR PRODUCT OR DOT PRODUCT:

When the product of two vectors isa scalar quantity, then it is called the scalar product or the dot product because a dot (**.**) is put between themultiplying vectors. The dot product of the

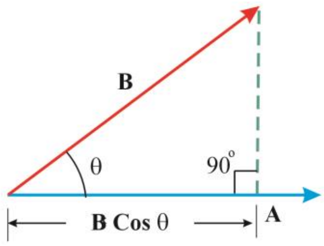

Fig 2.20

two vectors is equal to the product of magnitudes of the two given vectors and the cosine of the angle between them. Thus the dot product of two vectors A and B with angle θ between them is written as

$\mathbf{A} \cdot \mathbf{B}$ = (magnitude of **A**)(magnitude of component of **B** in the direction of **A**)

= $(A)(B\cos\theta)$

$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

Examples of scalar product:

1. Power = P = $\mathbf{F} \cdot \mathbf{v}$
2. Kinetic energy = K.E. = $\frac{1}{2}m(\mathbf{v} \cdot \mathbf{v})$
3. Electric flux = Φ_e = $\mathbf{E} \cdot \Delta\mathbf{A}$
4. Magnetic flux = Φ_B = $\mathbf{B} \cdot \Delta\mathbf{A}$

Here F is the force, d is the displacement, E is the electric intensity, B is the magnetic induction and ΔA is the vector area.

PROVE THAT A.B = B.A

Proof:

If we have two vectors A and B with angle θ between them then

$\mathbf{A} \cdot \mathbf{B} = (A)(B\cos\theta) = AB\cos\theta$ - - - - - (1)

Similarly, we have

$\mathbf{B} \cdot \mathbf{A} = (B)(A\cos\theta)$

$= AB\cos\theta$ - - - - - -(2)

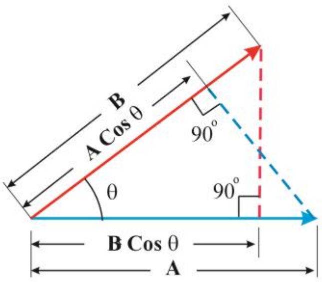

Fig 2.21

Comparing (1) and (2), we have

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

Characteristics of Scalar Product:

1. Commutative Law:

The scalar product of two vectors is commutative, i.e.

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

2. Associative Law:

The scalar product of two vectors is associative, i.e. , if m and n are scalars, then

$$(m\mathbf{A}) \cdot (n\mathbf{B}) = mn\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot mn\mathbf{B} = n\mathbf{A} \cdot m\mathbf{B}$$

3. Distributive Law:

The scalar product of vectors is distributive, i.e., if**A** , **B**, **C**, Dare vectors, then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C} + \mathbf{A} \cdot \mathbf{D}$$

4. Collinear vectors:

If two vectors A and B are collinear then their dot product is

$$\mathbf{A}\cdot\mathbf{B} = \pm AB$$

Case I. If **A** and **B** are in the same direction, i.e., $\theta = 0^\circ$, then

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = AB\cos 0^\circ = AB \qquad \{\because \cos 0^\circ = 1\}$$

Case II. If **A** and **B** are anti-parallel, i.e., $\theta = 180^\circ$, then

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = AB\cos 180^\circ = -AB \; \{\because \cos 180^\circ = -1\}$$

5. Perpendicular or Orthogonal Vectors:

If **A** and **B** are non-zero vectors and they are mutually perpendicular then the angle between them is 90˚, therefore

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = AB\cos 90^\circ = 0 \qquad \{\because \cos 90^\circ = 0\}$$

VECTOR PRODUCT OR CROSS PRODUCT:

When the product of two vectors is a vector quantity then it is called the Vector Product or the Cross Product because a cross (×) is put between the multiplying vectors. The cross product of two vectors is equal to the product of magnitudes of the two given vectors and the sine of the angle between them. Thus the cross product of two vectors A and B with angle θ between them is written as

$$\mathbf{A}\times\mathbf{B} = \mathbf{C} = |C|\hat{n} \qquad \text{where } |C| = AB\sin\theta$$

Thus $\mathbf{A}\times\mathbf{B} = AB\sin\theta\;\hat{n}$

Where $\hat{n}$ is the unit vector which shows the direction of the vector **C**. It is perpendicular to the plane of **A** and **B**. The direction of **C** is found by the right hand rule which is stated as

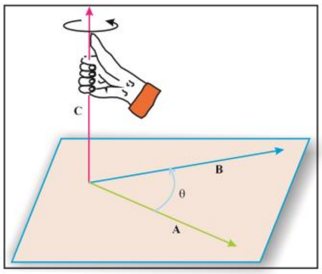

Fig 2.22

Right-hand Rule:

1. Join the tails of the two multiplying vectors **A** and **B**. In this way we get a plane. The vector **C** will be perpendicular to this plane.
2. Stretch the fingers of your right hand along the first vector and curl them towards the second vector through smaller angle such that $0^\circ < \theta < 180^\circ$. Then the erected thumb will show the direction of the vector **C** as shown in Fig. 2.22
3. Velocity = $\mathbf{V} = \mathbf{r}\times\boldsymbol{\omega}$

Here ω is angular velocity and r is the position vector.

4. Acceleration $= \mathbf{a} = \mathbf{r} \times \boldsymbol{\alpha}$

Here α is angular acceleration and r is the position vector.

5. Force on charge q moving with velocity V in magnetic field of strength B is: $\mathbf{F} = q(\mathbf{V} \times \mathbf{B})$

To Prove that: A × B = - B × A

Proof:

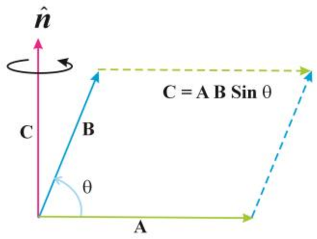

Fig 2.23

If we have two vectors A and B with angle θ between them then

$$\mathbf{A} \times \mathbf{B} = \mathbf{C} = |C|\,\hat{n} \quad \text{- - - -} \quad (1)$$

Here $|C| = AB \sin\theta$.

This shows that the area of a parallelogram with adjacent sides **A**and**B** gives the magnitude of the vector **C** and $\hat{n}$is a unit vector, shows the direction of the vector **C** which is along positive z-axis as shown in Fig. 2.23

Similarly

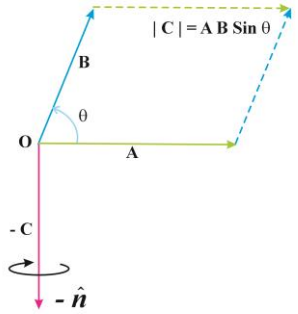

Fig 2.24

$$\mathbf{B} \times \mathbf{A} = -\mathbf{C} = |C|(-\hat{n}) \quad \text{- - - -} \quad (2)$$

Here $|C| = AB \sin\theta$

This shows that the area of a parallelogram with adjacent sides A and B gives the magnitude of vector **C** and $-\hat{n}$is a unit vector, shows the direction of the vector **C** along negative z- axis as shown in Fig. 2.24. Equation (2) can be written as

$$-\mathbf{B} \times \mathbf{A} = \mathbf{C} = |C|\,\hat{n} \quad \text{- - - -} \quad (3)$$

Comparing (1) and (3), we get

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

CHARACTERISTICS OF VECTOR PRODUCT:

1. Commutative Law:

The vector product of two vectors is non-commutative, i.e.

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad \Longrightarrow \quad \mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

2. Associative Law:

The vector product of two vectors is associative, i.e., if m and n are scalars, then

$$(m\mathbf{A}) \times (n\mathbf{B}) = mn\mathbf{A} \times \mathbf{B} = \mathbf{A} \times mn\mathbf{B} = n\mathbf{A} \times m\mathbf{B}$$

3. Distributive Law:

The scalar product of vectors is distributive, i.e., if A, B, C and D are vectors, then

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C} + \mathbf{D}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} + \mathbf{A} \times \mathbf{D}$$

4. **Collinear vectors:**

If two vectors **A** and **B** are collinear then their cross product is

$$\mathbf{A} \times \mathbf{B} = 0$$

Case I. If **A** and **B** are in the same direction, i.e., $\theta = 0^\circ$, then

$$\mathbf{A} \times \mathbf{B} = AB \sin\theta = AB \sin 0^\circ \hat{n} = 0 \qquad \{\because \sin 0^\circ = 0\}$$

Case II. If **A** and **B** are anti-parallel, i.e., $\theta = 180^\circ$, then

$$\mathbf{A} \times \mathbf{B} = AB \sin\theta\, \hat{n} = AB \sin 180^\circ\, \hat{n} = 0 \quad \{\because \sin 180^\circ = 0\}$$

SUMMARY

- In scalars, we need magnitude and proper unit to specify them while in case of vectors; we need magnitude, proper unit and direction for their complete specification.
- A unit vector has magnitude equal to one. It is used to show the direction and is obtained by dividing a vector by its magnitude. Mathematically

$$\hat{A} = \frac{A}{|A|}$$

- **Law of Parallelogram of Forces:**
  When two forces are acting at a point to form the two adjacent sides of a parallelogram, then their resultant will be equal to that diagonal of the parallelogram which passes through the same point.
- **Law of Triangle of Forces:**
  When two forces form the two sides of a triangle in the same order then their resultant will be equal to the third side of the triangle in opposite order.
- **Law of Polygon of Forces:**
  When several forces form the sides of a polygon in the same order, then their resultant will be equal to the closing side of the polygon in opposite order.
- If a vector A is making an angle θ with the x-axis, then its horizontal component is $A_x = A\cos\theta$ and vertical component $\mathbf{A_y} = A\sin\theta$.
- The magnitude of a vector in terms of its rectangular components is:

$$|A| = \sqrt{A_x^2 + A_y^2}$$

and its direction is $\tan\theta = \frac{A_y}{A_x} \Rightarrow \theta = tan^{-1}\frac{A_y}{A_x}$.

- The scalar product of two vectors with angle θ between them is written as:

A . B = ABcos θ while their vector product is written as:

A × B = AB sin θ $\hat{n}$.

EXERCISE

LONG QUESTIONS

2.1. State and explain the law of parallelogram of forces.

2.2. Explain the resolution of a vector in to is rectangular components.

2.3. Describe how a vector is found from its rectangular components.

2.4. Explain the addition of vectors by rectangular components method.

2.5. Describe scalar product with examples.

2.6. Explain vector product with examples. Also write its characteristics.

SHORT QUESTIONS

2.1. Define a unit vector

2.2. Define a component vector.

2.3. State law of parallelogram of forces.

2.4. Describe law of triangle of forces

2.5. State law of polygon of forces.

2.6. Define scalar product.

2.7. Write the characteristics of scalar product

2.8. Describe vector product.

2.9. Write three examples of scalar product.

2.10. Write three examples of vector product.

2.11. Write the characteristics of vector product

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1- **Which of the following is a scalar quantity?**

a) Energy *b)*Velocity *c)* Force *d)*Torque

2- **Which one is the vector quantity?**

a) Density *b)*Length *c)*Force *d)* Work

3- **A vector which has magnitude (one) is called**

a) Null vector *b)* Position vector

c) Unit vector *d)*Negative vector

4- **The angle between two rectangular components of a vector is**

a) 30° *b)* 60° *c)* 90° *d)* 180°

5- A force of 6N is acting along x–axis. Its y–component is
a) 6N *b)* Zero *c)* 12N *d)* 3N

6- Cos 60° has the value:
a) 1 *b)*0.866 *c)* 0.5 *d)* 0.707

7- If a vector A makes an angle θ with x–axis, its x-component is given by
a) A Sinθ *b)* A tanθ *c)* A Cosθ *d)*A Cotθ

8- If two vectors make an angle of90°with each other, their scalar product is:
a) 0 *b)* 1 *c)* -1 *d)* 2

9- The scalar product of two vectors A and B at an angle θ with each other is
a) AB *b)* AB Sinθ *c)* AB Cosθ *d)* AB tanθ

10- If vectors A and B are parallel to each other then
a) **A. B** = 0 *b)* **A . B** = 1 *c)* **A . B** = AB *d)* **A . B** = - AB

11- The cross product of vector A with itself (A × A) is
a) A *b)* 1 *c)* Zero *d)*A^2

12- In a right angle triangle, if one of its angle is 30° then the other will have value
a) 0° *b)* 30° *c)* 60° *d)* 90°

13- Which of these statements is correct?
a) **A . B = – B. A** *b)* **A × B = B × A**
c) **A . B = B. A** *d)* **A . B ≠ B . A**

14- Time is example of
a) Vector *b)* Scalar *c)* Negative of a vector *d)*Null vector

PROBLEMS

2.1. Four forces of magnitude 10, 20, 30 and 40 newton act upon a body in directions making angles of 30˚, 45˚, 60˚ and 90˚ with *x*–axis. Find the resultant force.[*Ans: 93.13N,θ = 66˚*]

2.2. A crow flies northwards and covers a distance of 8km. It then flies eastwards and covers a distance of 6km. Find the net displacement and the direction of this flight.[*Ans: 10km, 36.87˚ from North to East.*]

2.3. Two forces F_1 and F_2 are acting on a body. The angle between F_1 and F_2 is 60˚. Assume F_1 is along *x*–axis. Find their resultant.

$$[Ans:\ Magnitude\ = \sqrt{F_1^2 + F_2^2 + F_1F_2} \,\&\, \tan\theta\ = \frac{\sqrt{3}F_2}{2\,F_1 + F_2}]$$

2.4. Calculate the dot product of force and displacement if a force of 10N displaces the body through 4 meters in the direction of force.
[*Ans: 40*]

2.5. A river is flowing eastwards with a velocity of 8km/h. A boat starts to row in it northwards. The velocity of the boat in still water is 6km/hour. Find the velocity of the boat in the river.
[*Ans: 10km/h* $\theta = 37^\circ$ *from East to North*]

2.6. Three forces of 40, 30 and 90 newton act on a body making angles of 0°, 30° and 135° with the *x*–axis. Find the resultant force.
[*Ans: 78.66N* $\theta = 88.28^\circ$ *with x–axis*]

2.7. A force of 100N makes an angle of 30° with *x*–axis. Find its horizontal and vertical components.[*Ans: 86.6.N, 50N*]

2.8. A force of 50N acts on a body at a distance of 0.5m from the axis of rotation. Find the torque.[*Ans: 25N.m*]

***

Chapter 03

MOTION

Course contents:

3.1 Review of laws and equations of motion.
3.2 Law of conservation of momentum.
3.3 Angular motion.
3.4 Relation between linear and angular motion.
3.5 Centripetal acceleration and force.
3.6 Equations of angular motion.

Learning Objectives:

At the end of this chapter the students will be able to:

- Describe the difference between distance and displacement.
- Define velocity and acceleration.
- State and explain Newton's laws of motion.
- Define momentum and explain law of conservation of momentum.
- Describebeangular motion.
- Explain angular velocity and angular acceleration.
- Describe the difference between linear motion and angular motion.
- Derive the relation for centripetal acceleration and centripetal force.

REST AND MOTION

Rest:

If a body does not change its position with respect to its surroundings, it is said to be at rest. For example, a book lying on a table is at rest with respect to the table.

Motion:

If a body changes its position with respect to its surroundings, it is said to be in motion. For example, a running bicycle.

DISTANCE AND DISPLACEMENT

Distance:

The length of the path followed by a body between two points is called the distance as shown by ACB in the Fig. 3.1

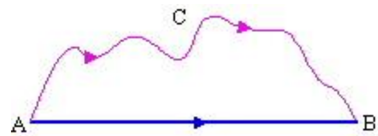

Fig. 3.1

Displacement:

The shortest distance between two points is called the displacement as shown by the line $\overline{AB}$ in Fig 3.1. It is a vector quantity. Its unit in S.I. system is meter.

SPEED AND VELOCITY

Speed:

It is defined as the distance covered by a body in unit time. If ΔS is the distance covered in time Δt, then

$$\text{Speed} = \frac{Distance}{Time} = \frac{\Delta s}{\Delta t}$$

It is a scalar quantity. It gives the magnitude of velocity. Its unit in S. I. system is m/sec.

VELOCITY OR LINEAR VELOCITY:

It is defined as "**The rate of change of displacement of a body**". It can also be defined as "**The distance covered by a body in a unit time in a particular direction**". If the change in displacement of a body is Δ**d** in time Δt, then velocity

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$v = \frac{\Delta d}{\Delta t}$$

It is a vector quantity. Its unit in S.I. system is m/sec.

ACCELERATION OR LINEAR ACCELERATION:

It is defined as the rate of change of velocity of a body, or change in velocity per unit time. If the change in velocity is Δv in time Δt, then acceleration is given by

$$a = \frac{\Delta v}{\Delta t}$$

Acceleration is a vector quantity. Its unit in S.I. system is meter /sec^2

Uniform Acceleration:

When the change in velocity of a body is equal, in equal intervals of time then its acceleration is uniform. At this point, the average and the instantaneous accelerations become equal.

Force:

Any action that changes or tends to change the state of a body of rest or of uniform velocity is called force. It is a vector quantity. In S. I. system, its unit is newton.

newton:

One newton is that force which produces an acceleration of one meter/sec^2 in a body of mass one kilogram.

NEWTON'S FIRST LAW OF MOTION

Statement:

A body remains at rest or continues to move with uniform velocity unless acted upon by an external unbalanced force. This is also known as **Law of Inertia**.

Explanation:

Newton's first law of motion consists of two parts.

1. According to the first part, a body at rest remains at rest, e.g., a book lying on a table remains lying on the table unless acted upon by an external unbalanced force.

2. According to the second part, a body in motion continues its motion with constant velocity. We see that a rolling ball stops its motion after covering some distance. Apparently, it is against the first law of motion. In fact, several forces are acting on the body which stops it. Main of these is friction of the ground, air resistance and gravity. If we remove all of these forces, the body will continue its motion with constant velocity for maximum time.

Law of Inertia:

The property of the body that opposes any change in its state of rest or of uniform motion is called inertia.

Example:

When brakes are suddenly applied on a fast moving bus, the passengers will fall in the forward direction. Its reason is that the passengers are moving with the same velocity as that of the bus. On applying brakes, the lower parts of their bodies come to rest with the bus but upper part of the bodies is still moving. So the passengers will fall in the forward direction. Similarly, if we apply a gear suddenly on a bus at rest, the passengers will fall in the backward direction.

NEWTON'S SECOND LAW OF MOTION

Statement:

When an unbalanced force acts on a body, it produces acceleration in the body in the direction of the force.The magnitude of the acceleration is directly proportional to the applied force and inversely proportional to the mass of the body.

Explanation:

Consider a force **F** acting on a body of mass **m** and producing acceleration **a** in it. Then

$$\mathbf{a} \propto \mathbf{F} \quad \text{-----} \quad (1)$$

$$\mathbf{a} \propto \frac{1}{m} \quad \text{-----} \quad (2)$$

Combining these two, we get $\mathbf{a} \propto \frac{F}{m} \implies F \propto ma$

or $F = k\,m\,a \quad \text{-----} \quad (3)$

Here k is the constant of proportionality. Its value can be calculated as:

When m = 1 kg, a = 1 m/sec^2 then F = 1 N, then putting these values in Eq.(3), we get

$$1\text{ N} = k \,.\, 1\text{ kg} \times 1\text{ m/sec}^2 \implies k = 1$$

Hence, (3) takes the form

$$a = \frac{F}{m} \implies \mathbf{F} = m\,\mathbf{a}$$

NEWTON'S THIRD LAW OF MOTION:

Statement:

To every action, there is always an equal and opposite reaction. Moreover, action and reaction act upon two different bodies.

Explanation:

Consider the interaction between two bodies A and B as shown in Fig 3.2

The force exerted by A on B is $\mathbf{F}_{AB}$ and the force exerted by B on A is $\mathbf{F}_{BA}$. Since these two forces are equal and opposite, so we have

$$\mathbf{F}_{AB} = -\,\mathbf{F}_{BA}$$

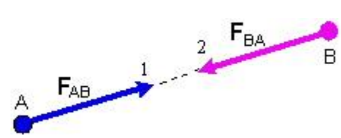

Fig.3.2

It is important to note that two bodies are necessary for action and reaction. Action and reaction cannot act on thesame body. Also action and reaction forces never balance each other because they act on two different bodies. But the forces acting on the same body can balance each other.

Examples:

1. When a bullet is fired from the gun, the bullet moves in the forward direction due to the force of action. As a reaction, the gun moves in the backward direction.
2. In a rocket, the hot gases move downward is the force of action. As a reaction the rocket moves upward.

EQUATIONS OF MOTION

There are three basic equations of motion for bodies moving with uniform acceleration. These equations relate initial velocity, final velocity, acceleration, time and the distance covered by a moving body.

I. First Equation of Motion:

This equation explains the relation between the initial velocity v_i of the body, its acceleration a and its final velocity v_f after time t. It is stated as

$$v_f = v_i + at$$

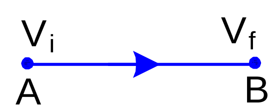

Fig. 3.3

Proof:

Consider a body moving with initial velocity v_i at point A. If a force **F** acts on the body and produces an acceleration a in it, then after time t seconds, its final velocity becomes v_f at point B. Then change in velocity of the body is $v_f - v_i$, and the rate of change of velocity of the body is equal to the acceleration acting on the body, i.e.,

$$a = \frac{v_f - v_i}{t}$$

or $$v_f - v_i = at$$

or $$v_f = v_i + at$$

II. Second Equation of Motion:

This equation explains the relation between the initial velocity V_i of the body, its acceleration a and the distance S covered by the body after time t. It is stated as

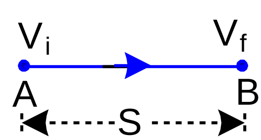

Fig. 3.4

$$s = v_i t + \frac{1}{2} at^2$$

Proof:

Consider a body with initial velocity vi, moving with an acceleration a. It will cover a distance S, in time t and its final velocity becomes v_f then its average velocity is given as

$$v_{av} = \frac{v_f + v_i}{2} \qquad \text{- - - - - (1)}$$

Distance covered by the body in time t is given by

$$S = V_{av} \times t \qquad \text{- - - - - (2)}$$

Putting the value of V_{av} from Eq.(1) in Eq.(2), we get

$$S = \frac{v_f + v_i}{2} \times t \qquad \text{- - - - - (3)}$$

According to the first eq of motion

$$v_f = v_i + at \qquad \text{- - - - - (4)}$$

Putting the value of v_f from Eq.(4) in Eq.(3), we get

$$S = \frac{v_i + at + v_i}{2} \times t$$

$$S = \frac{2v_i t + at^2}{2}$$

$$\Rightarrow \quad S = v_i t + \frac{1}{2} a t^2$$

III. Third Equation of Motion:

This equation explains the relation between the initial velocity v_i of the body, its acceleration *a* and the distance s covered by the body after time t and the final velocity v_f. It is stated as

$$2aS = v_f^2 - v_i^2$$

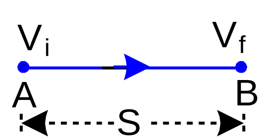

Fig. 3.5

Proof:

Consider a body with initial velocity v_i moving with acceleration a. After time t its final velocity becomes v_f and the body covers a distance s. Then the average velocity of the body is given as

$$v_{av} = \frac{v_f + v_i}{2} \qquad \text{- - - - - (1)}$$

From the first equation of motion,

$$v_f = v_i + at \implies t = \frac{v_f - v_i}{a} \qquad \text{- - - - - (2)}$$

Distance covered by the body is given as; AB = s

$$S = V_{av} \times t \qquad \text{- - - - - (3)}$$

Putting the value of v_{av} and t from Eqs.(1) and (2), in Eq.(3),

$$s = \left(\frac{v_i + v_f}{2}\right)\left(\frac{v_f - v_i}{a}\right)$$

or $\quad 2as = (v_i + v_f).(v_f - v_i)$

or $\quad 2as = v_f^2 - v_i^2$

Some rules to be used in the above equations of motion:

1. If a body starts its motion from rest, then use $v_i = 0$.
2. If a moving body stops its motion, then $v_f = 0$.
3. If the velocity of the body is increasing then its acceleration is positive and its direction is the same as that of the velocity.
4. If the velocity of the body is decreasing then its acceleration is negative and its direction is opposite to that of the velocity.

Equations of Motion for a Free Falling Body:

If a body is moving freely upward or falling downward then its acceleration **a** will be the acceleration due to gravity, i.e, g = 9.8 m/sec^2 and its distance S will show the height *h* of the body. Then the equations of motion will take the form:

Linear Motion	**Motion under Gravity**	
	Downward Motion	**Upward Motion**
$v_f = v_i + at$	$v_f = v_i + gt$	$v_f = v_i - gt$
$s = v_i t + \frac{1}{2}at^2$	$h = v_i t + \frac{1}{2}gt^2$	$h = v_i t - \frac{1}{2}gt^2$
$2aS = v_f^2 - v_i^2$	$2gS = v_f^2 - v_i^2$	$-2gS = v_f^2 - v_i^2$

Some rules to be used in the above equations of motion:

1. If a body is dropped downward, then; $v_i = 0$.
2. If we throw a body upward, then; $v_f = 0$.
3. When a body is moving downward, then g will be positive.

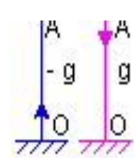

Fig. 3.6

4. When a body is moving upward, then g will be negative.

Example 3.1: A body of mass 10 kg is moving with an acceleration of $2 m/sec^2$. Find the force acting on the body.

Solution:

Mass of the body = m = 10 kg

Acceleration = a = $2\ m/sec^2$

Force acting on it = F = ?

We know that F = ma

$\therefore$ F = 10 × 2 = 20N

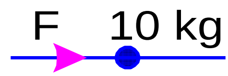

Fig. 3.7

MOMENTUM

The quantity of motion of a moving body is known as momentum. In other word, ***the product of mass and velocity of a body is known as momentum.***

If a body has mass m and is moving with velocity **v** then its momentum is

$$\mathbf{P} = m\,\mathbf{v}$$

In S.I.System its unit is kg-m/sec or N-sec.

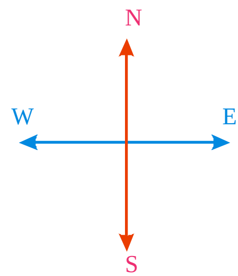

Fig. 3.8

Example 3.2: A body of mass 3 kg is moving towards east with a velocity of 9 m/sec. Find its momentum.

Solution:

Mass of the body = m = 3 kg

Velocity of the body = v = 9 m/sec

Momentum = P = ?

We know that P = mv = 3 × 9 = 27 kg m/sec

Fig. 3.9

LAW OF CONSERVATION OF MOMENTUM

Statement:

"In the absence of an unbalanced external force, the total momentum of asystem of colliding bodies remains constant in a particular direction".In other words***"The total momentum of an isolated system of interacting bodies remains constant in a particular direction."***

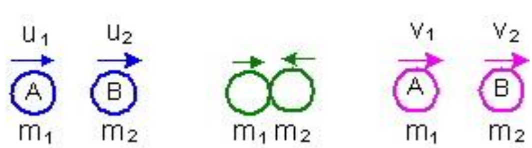

Fig. 3.10

Explanation:

Consider a system of two bodies A and B of masses m_1 and m_2 moving with velocities U_1 and U_2 respectively as shown in Fig. 3.10. Therefore,

Total momentum of the system before collision = $m_1U_1 + m_2U_2$

If U_1 is greater than U_2 and the bodies collide, then after collision, let their velocities become V_1 and V_2, then

Total momentum of the system after collision = $m_1V_1 + m_2V_2$

By the law of conservation of momentum, we have

Total momentum before collision = Total momentum after collision

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$$

Proof of Law of Conservation of Momentum:

Consider a system of two bodies A and B of masses m_1 and m_2 moving along the same line with initial velocities U_1 and U_2 respectively. They collide with each other and after collision their velocities become V_1 and V_2 as shown in the figure 3.10. Therefore,

Total initial momentum of the system = $m_1U_1 + m_2U_2$

Total final momentum of the system = $m_1V_1 + m_2V_2$

During collision the two bodies exert equal and opposite forces on each other. We know that the force acting on a body is equal to the rate of change of momentum of the body, therefore

Total force acting on body A by B = $F_{BA} = \dfrac{m_1v_1 - m_1u_1}{\Delta t}$

Total force acting on body B by A = $F_{AB} = \dfrac{m_2v_2 - m_2u_2}{\Delta t}$

According to Newton's Third law of motion,

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

$$\frac{m_2v_2 - m_2u_2}{\Delta t} = -\frac{m_1v_1 - m_1u_1}{\Delta t}$$

or $\quad m_2\mathbf{v}_2 - m_2\mathbf{u}_2 = -m_1\mathbf{v}_1 + m_1\mathbf{u}_1$

or $\quad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{u}_1 + m_2\mathbf{u}_2$

or $\quad m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

$$\begin{bmatrix} \textit{Total initial momentum of the} \\ \textit{system before collision} \end{bmatrix} = \begin{bmatrix} \textit{Total final momentum of the} \\ \textit{system after collision} \end{bmatrix}$$

Example 3.3: ***A gun of mass 50 kg fires a bullet of mass 100 gram with a velocity of 100 m/sec. Find the velocity with which the gun recoils.***

Solution:

Mass of gun = m_1 = 50 kg
Velocity of gun = v_1 = ?
Mass of bullet = m_2 = 100gm
= 0.1 kg
Velocity of bullet = v_2 = 100 m/sec

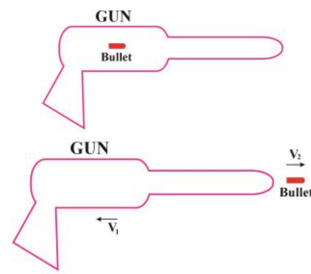

Fig 3.11

By law of conservation of momentum

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

As $u_1 = 0$ & $u_2 = 0$

Therefore $m_1v_1 + m_2v_2 = 0$

$$m_1v_1 = -m_2v_2$$
$$50 \times v_1 = -0.1 \times 100$$
$$v_1 = -\frac{0.1 \times 100}{50}$$
$$= -0.2 \text{ m/sec}$$

The negative sign shows that the gun moves in the backward direction.

CIRCULAR OR ANGULAR MOTION

When a body moves in a circle, its motion is called circular motion. For example, a planet revolves round the sun.

Angular Displacement:

The angle,through which a particle moves while revolving in a circle, is called angular displacement. Its unit is revolution, degree or radian. It is easy to measure it in radian because when the angle is measured in radian, we can multiply it with other quantities.

$$1 \text{ radian} = 57.3^o$$

Prove that s = r θ

Consider a circle with centre o and radius r.
Let S = AB, be the arc of the circle with an angle θ radian at the centre.
Let us take another arc AC = r

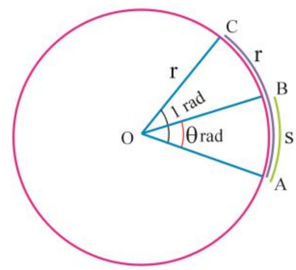

Fig. 3.12

so that ∠AOC = 1 radian. Then

$$\frac{\text{Angle AOB}}{\text{Angle AOC}} = \frac{\text{Arc AB}}{\text{Arc AC}} \quad \text{- - - - - (1)}$$

But $\angle AOB$ = θ radian and $\angle AOC$ = 1 radian,

Arc AB = S, and Arc AC = r

so, Eq.(1) becomes

$$\frac{\theta \text{ rad}}{1 \text{ rad}} = \frac{S}{r} \quad \Rightarrow \quad S = r\,\theta$$

Angular Velocity:

It is defined as ***the rate of change of angular displacement***. It is a vector quantity. Mathematically, it is written as

$$\omega = \frac{\theta}{t}$$

Its units are rev/sec, deg/sec, rad/sec or rev/min.

Relation between Linear and Angular Velocity (v = r ω):

Let a body is moving in a circle of radius r. After time t, it covers an arc s and subtends an angle θ at the centre as shown in Fig.3.11, As

$$s = r\,\theta$$

If the time interval Δt is very small, then the body covers an arc Δs and subtends angle Δθ at the centre then

$$\Delta s = r\,\Delta\theta \qquad \text{- - - - (1)}$$

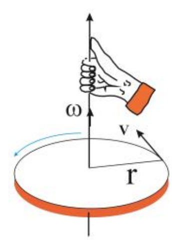

Fig. 3.13

Dividing both sides by Δt and taking limit Δt → 0, then

Eq. 1 becomes

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

As $\frac{\Delta s}{\Delta t} = v$ and $\frac{\Delta \theta}{\Delta t} = \omega$

So $v = r\,\omega$

In vector form $v = r \times \omega$

Direction of Angular Velocity:

Angular velocity is a vector quantity. Its direction is found by right-hand rule that is stated as: "Curl the fingers of the right hand around the axis of rotation in the direction of rotation, the thumb will show the direction of angular velocity ω.

ANGULAR ACCELERATION:

"*The angular acceleration is defined as the rate of change of angular velocity*". If the change of angular velocity is Δω in time interval Δt, then

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

The unit of angular acceleration is rad/sec^2, rev/sec^2 or deg/sec^2.

Relation between Linear and Angular Acceleration (a = r α):

We know that the acceleration of a body is given as

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad \text{- - - - (1)}$$

If the body is moving in a circle of radius r with angular velocity ω, then

$$\Delta v = r\,\Delta\omega$$

Putting its value in Eq.(1), we get

$$a = r \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

As $\lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \alpha$ so $a = r\,\alpha$

In vector form, $\mathbf{a} = \mathbf{r} \times \boldsymbol{\alpha}$

Direction of Angular Acceleration (α):

The angular acceleration is a vector quantity. It can have positive or negative value. If the angular velocity ω increases then **α** is in the direction of ω as shown in the Fig. 3.14 (a). But if angular velocity decreases then the direction of **α** is opposite to that of ω as shown in Fig. 3.14(b).

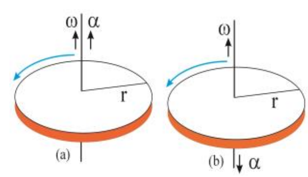

Fig. 3.14

Centripetal Acceleration:

Let a body moves in a circular path with uniform speed, its velocity continuously changes due to change in direction. This produces acceleration in the body which is directed towards the centre of the circular path. This is called the centripetal acceleration.

Centripetal Force:

When a body moves in a circular path, there is always a force which compels the body to move in the circular path. This force is called centripetal force. Mathematically it is written as

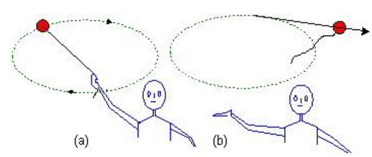

Fig. 3.15

$$F_c = \frac{mv^2}{r}$$

This equation shows that the centripetal force is directly proportional to the mass of the body as well as the square of its velocity and is inversely proportional to the radius of the circle.

Example:

A man is moving a ball of mass *m* in a circular path with the help of a string. The man is providing the centripetal force to the ball as shown in the Fig. 3.15(a). If he leaves the string to remove the centripetal force, then the ball will move in a straight line instead of the circular path as shown in Fig. 3.15(b).

Derivation of formula for Centripetal Acceleration and Centripetal Force:

Consider a body of mass m moving in a circle of radius r with constant angular speed ω. Its linear speed v is also constant. In a small interval of time Δt, the body moves from P to Q as shown in figure 3.16. In this way, the length of the arc PQ = v Δt. Let the velocity of the body at point P and Q be v and v′. The magnitudes of these velocities are the same, i.e., v = v′.
They differ only in direction. The change in velocity ΔV is calculated as follows:

Take a point O outside the circle. Draw vectors

$$v' = v + \Delta v.$$

The instantaneous acceleration will be:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

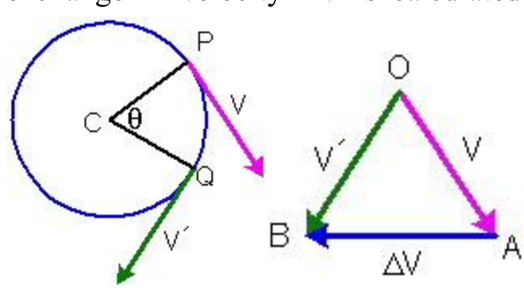

Fig. 3.16

The direction of this acceleration will be the same as that of Δv. If time is very small, then the acceleration will be directed towards the centre. In figure 3.16, we see that ΔOAB and ΔCPQ are similar, so

$$\frac{PQ}{CP} = \frac{AB}{OA} \quad \Rightarrow \quad \frac{PQ}{r} = \frac{\Delta v}{v} \quad \text{- - - - - (1)}$$

When time is very small then arc PQ = chord PQ = V Δt
Putting these values in Eq.(1), we get

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v} \Rightarrow \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

or $$a_c = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Using F = ma, the centripetal force is given by

$$F_c = \frac{mv^2}{r}$$

In Vector form $$\mathbf{F}_c = \frac{mv^2}{\boldsymbol{r}}$$

Comparison of Linear Motion and Angular Motion:

Liner Equations	*Angular Equations*
1. $S = vt$	$\theta = \omega t$
2. $v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
3. $v_{av} = \frac{v_i + v_f}{2}$	$<\omega> = \frac{\omega_i + \omega_f}{2}$
4. $2as = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$
5. $s = v_i t + \frac{1}{2} at^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
6. $F = ma$	$\tau = I\alpha$

EXAMPLE 3.3 A body of mass 2 Kg is moving in a circle with velocity 2 m/sec. If the radius of the circle is 100 cm, find centripetal acceleration and centripetal force acting on the body?

SOLUTION: Mass of the body = m = 2 kg

Velocity of the body = v = 2 m/ sec

Radius of the circle = r = 100 cm = 1 m

Centripetal acceleration $= a_c = \frac{v^2}{r} = \frac{2^2}{1} = 4 \text{ m/sec}^2$

Centripetal force $= F_c = \frac{mv^2}{r}$

$$F_c = \frac{2\times2^2}{1} = 8 \text{ N}$$

SUMMARY

- The minimum distance between two points is called **displacement**.
- The rate of change of displacement is called **velocity** and the rate of change of velocity is called **acceleration**.
- **Force** is that agency which changes or tends to change the state of rest or uniform velocity of a body.
- One **newton** is that force which produces an acceleration of 1 m/sec^2 in a body of mass 1 kg.
- **Newton's First Law of motion** states that in the absence of an external force, a body continues its state of rest or uniform velocity.
- **Newton's Second Law of motion** states that the acceleration produced by an unbalanced force is directly proportional to the applied force and inversely proportional to the mass of the body.
- **Newton's Third Law of motion** states that to every action there is an equal and opposite reaction.
- **Momentum** is equal to the product of mass and velocity of a body.
- **Centripetal acceleration**: When a body moves in a circular path, there is always an acceleration which is directed towards the centre of the circle. This is called the centripetal acceleration.
- **Centripetal force** is that force which compels a body to move in a circular path.

EXERCISE

LONG QUESTIONS

1. State and explain Newton's second law of motion.
2. Derive third equaion of motion
3. State and explain the law of conservatin of momentum.
4. Prove that $s = r\,\theta$
5. Show that $v = r\,\omega$
6. Diferentiate linear and angular acceleration. Also prove that $a = r\,\alpha$
7. Define centripetal force. Derive formula for centripetal force.

SHORT QUESTIONS

1. Differentisate distance and displacement.
2. Define uniform acceleration.

3. Define force. Als write its S.I unit.
4. Descibe law of inertia.
5. Describe Newton's first law of motion.
6. State Newton's third law of moion.
7. Prove first equation of motion.
8. State law of conservation of momentum?
9. A stone is droped from the top of a building, his the ground after 6 sec. Find the height of the building.
10. Define momentum.
11. Define angular velocity.
12. Define centripetal force.

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1- A mass of 20 kg is at rest. Its momentum is

a) Zero *b)* 200 kg m/sec *c)* 196 N.sec *d)* 20 kg m/sec

2- As we go away from the surface of earth, the value of g

a) Increases *b)* Decreases
c) Becomes zero *d)* Remain constant

3- How many feet are in a yard.

a) One *b)* Two *c)* Three *d)* Four

4- S.I is also called

a) M.K.S. System *b)* C.G.S. System
c) F.P.S. System *d)*System International

5- Two forces of 5N and 3N are acting on a body making an angle of 180° with each other. Their resultant force is

a) 6 N *b)* 15 N *c)* 35 N *d)* 2 N

6- Motion of molecules of a body at absolute zero is

a) Maximum *b)* Minimum
c) Zero *d)* Random

7- newton is the unit of

a) work *b)*force *c)*energy *d)* torque

8- Force per unit area is called

*a)*density *b)* viscosity *c)* pressure *d)* energy

9- The weight of a body is 9.8 N. It mass is

a) 9.8 kg *b)* 98 kg *c)* 1 kg *d)* 10 kg

10- A 5 kg mass is falling freely. Its weight will be

a) 5 N *b)* Zero *c)* 19.6 N *d)* 49 N

PROBLEMS

3.1. A force of 20 N acts on a body of mass 4k g. What is the acceleration in the body. [*Ans : 5 m/sec*^{2}]

3.2. The mass of body of 25 kg is initially at rest. If a force of 25 N acts on it for one minute, find the acceleration produced in it and the distance covered by the body. [*Ans : 1m/sec*^{2}*, 1800 m*]

3.3. A body of mass 3 kg is at rest. How much force will move it to a distance of 200 m in 10 sec. [*Ans : 12 N*]

3.4. A car has a weight of 10,000 N starts from rest at a uniform acceleration. If it covers a distance of 300m in 2 minutes, find the force, the engine applying on the car. [*Ans : 42.5 N*]

3.5. A gun of 10 kg fires a bullet of 20 g with a velocity of 50 m/sec. Find the velocity with which the gun recoils.[*Ans : – 0.1 m/sec*]

3.6. A body of mass 50g is moving with a speed of 200 cm/sec in a circle. If the radius of the circle is 100 cm. Find the centripetal acceleration and centripetal force acting on the body.

[*Ans : 4 m/sec*^{2}*, 0.2 N*]

3.7. A body of mass 5 kg is moving due east with a velocity of 10 m/sec. Find its momentum. [*Ans : 50 N.sec*]

3.8. A motor is travelling at a speed of 30 m/sec. If its wheel has a diameter of 1.5 m. Find its angular speed in rad/sec.

[*Ans : 40 rad/sec*]

3.9. A mass of 4 kg is kept moving in a circle of radius 1 m with constant speed of 10 m/sec. Find the centripetal force applied.[*Ans : 400 N*]

3.10. Find the force required to keep a car of mass 2000 kg in an arc of radius 5 m when it is turning round a corner at 10 m/sec.

[*Ans : 40,000 N*]

***

Chapter 04

TORQUE,EQUILIBRIUM AND ROTATIONAL INERTIA

Course contents:

4.1. Torque
4.2. Centre of Gravity and centre of mass
4.3. Equilibrium and its conditions
4.4. Torque and angular acceleration
4.5. Rotational inertia

Learning Objectives:

At the end of this chapter the students will be able to:

- Explain concept of torquein daily life.
- Differentiate center of mass and center of gravity.
- Describe equilibrium and its conditions.
- Find the moment of inertia of a rigid body.
- Describe the relation between torque and angular acceleration

TORQUE

The turning effect of a force produced in a body about its axis of rotation is called torque. In other words, *the cross product of force and its position vector is called torque*. If **F** is the force and **r** is the position vector then torque is given as

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

Magnitude:

If θ is the angle between **F** and **r** then the magnitude of torque is

$$\tau = r\,F\sin\theta$$

Direction:

Torque is a vector quantity. The direction of $\boldsymbol{\tau}$ is perpendicular to the plane containing **r** and **F**. It is found by right-hand rule. If we curl the fingers of the right hand along the direction of rotation then the erected thumb will show the direction of the torque.

Example:

Consider a door which is capable of rotation about its hinge as shown in the Fig.4.1. When we apply force at point A, the door rotates easily. But when we apply this force at point B, the door does not rotate so easily. It is because the point B is closer to the hinge than the point A and has small position vector. Hence it does not rotate easily.

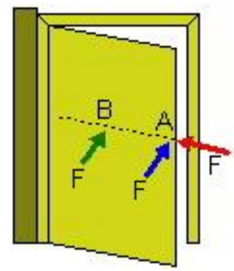

Fig. 4.1

Now if the same force is applied in the direction of the door, i.e., in such a way that the line of action of the force passes through the hinge then the door does not rotate at all.

Unit of Torque:

The unit of torque in S. I. system is Nm.

Factors Upon Which Torque Depends:

Torque depends upon the following quantities:

1. The force F.
2. The position vector **r**, i.e., the perpendicular distance between the line of action of force and the axis of rotation.

Positive andNegative Torque:

The torque which rotates the body in anti-clockwise direction is termed as positive while the torque which rotates the body in clockwise direction is termed as negative.

EXAMPLE 4.1: ***A force of 25N is applied to open a door. If the point of application of the force is 25cm from the axis of rotation, thencalculate the torque applied on the door*****:**

SOLUTION: Force applied = F = 25N

Perpendicular distance of force = OA = r = 25 cm = 0.25 m

$$\text{Torque} = \tau = F \times r = 25 \times 0.25 = 6.25 \text{ N.m}$$

Centre of Mass:

The centre of mass of a body is that point at which the whole mass of the body is concentrated. In other words, *it is that point in the body which has only translational motion*.


Fig. 4.2

Explanation:

Consider a body which consists of a large number of particles. If the body has translational motion as well as it has rotational or vibrational motion. There is only a single point in the body that moves along the straight line. This point is called the centre of mass of the set of particles as shown by the stroboscopic photograph of a wrench sliding across a smooth surface.

Centre of Gravity:

The centre of gravity of a body is that point at which the whole weight of the body acts. Through this point the line of action of the weight of the body passes.

Explanation:

Consider an extended object which consists of a large number of particles such that the mass of each particle is m. The weight of each particle is mg which is directed towards the centre of earth as shown in the Fig. 4.3

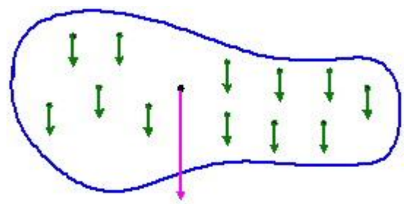

Fig. 4.3

As the size of the extended object is small as compared to that of the earth, therefore, the acceleration due to gravity is considered as uniform over it. Each particle of the object experiences the same force mg towards the centre of the earth. The resultant force on the object is sum of all these forces and it is given as:

$$\sum mg = Mg = w$$

Which is equal to the weight of the body. So the point within the object through which this resultant force passes is called the **center of gravity**.

Conclusion:

Since g is uniform over the extended object so the mass of the constituent particles can be replaced by the total mass of the object at its centre of gravity in a uniform gravitational field. This point is known as the centre of mass. So it is concluded that

1. There is no difference between centre of mass and centre of gravity if g remains uniform.
2. If g is not uniform over the extended object then centre of mass will not coincide with the centre of gravity.

EQUILIBRIUM AND ITS CONDITIONS

Equilibrium:

A body is said to be in equilibrium if it remains at rest or moves with uniform velocity. In other words, if a body has zero acceleration (linear as well as angular acceleration), it is said to be in equilibrium.

KINDS OF EQUILIBRIUM:

There are two kinds of equilibrium.

1. Static Equilibrium:

If a body does not change its position with respect to its surroundings, it is said to be at rest or in static equilibrium. For example, a book lying on a table in fig.4.4(a)

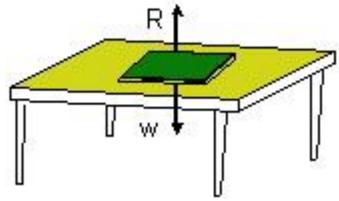

Fig.4.4(a)

Fig.4.4(b)

2. Dynamic Equilibrium:

If a body is moving with uniform velocity, it is said to be in dynamic equilibrium. For example, the motion of a paratrooper jumped from an airplane in fig. 4.4(b)

Conditions of Equilibrium:

There are two conditions of equilibrium.

First Condition of Equilibrium:

If the algebraic sum of the several forces acting on a body is equal to zero then the body will be in translational equilibrium. If the body is acted upon by n forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, - - - $\mathbf{F}_n$ and under these forces the body is in translational equilibrium, then

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots + \mathbf{F}_n = 0$$

$$\text{or} \qquad \sum \mathbf{F} = 0$$

Explanation:

Consider a ring upon which two equal and opposite forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are acting as shown in Fig. 4.5

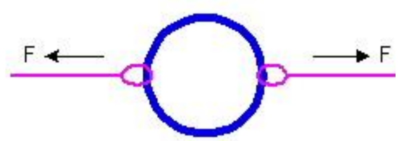

Fig. 4.5

These two forces cancel each other's effect and the ring is in equilibrium. This is only possible when $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal and opposite and both are passing through the same line to give zero resultant. If the forces are not equal in magnitude then the ring will begin to move in the direction of the greater force.

Second Condition of Equilibrium:

If the algebraic sum of all the torques acting on a body about its axis of rotation is equal to zero then the body will be in rotational equilibrium, i.e.,

$$\text{Clockwise torque} = \text{Anti-clockwise torque}$$

If the body is acted upon by n torques τ_1, τ_2, τ_3, - - - , τ_n, and under these torques, the body is in rotational equilibrium, then

$$\tau_1 + \tau_2 + \tau_3 + \text{- - -} + \tau_n = 0 \quad \Rightarrow \quad \sum \tau = 0$$

Explanation:

We can make a rod on the wedge in rotational equilibrium by equating clockwise torque and anticlockwise torque as shown in the fig. 4.6

Anticlockwise torque = Clockwise torque

$$w \times CG = w_1 \times AG + w_2 \times BG$$

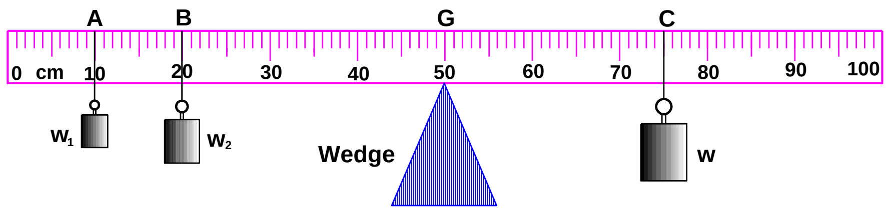

Fig.4.6

NOTE: The body will be in complete equilibrium when it is in both translational and rotational equilibrium.

EXAMPLE 4.2: ***A force of 40N is acting on a body towards North. What would be the magnitude and direction of the other force required to keep the body in equilibrium. Draw the necessary diagram*:**

SOLUTION:

$F_2 = 40N$, towards South

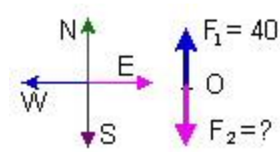

Fig. 4.7

TORQUE AND ANGULAR ACCELERATION:

Here we shall find the relation between the torque and angular acceleration of a rigid body which rotates about an axis. At first, we find the relation between torque and angular acceleration in case of a very small body round an axis. For this, consider a light rod of length r whose one end is fixed at O, acting as the axis of rotation and the other end is free to rotate, attached by a small mass m as shown in Fig 4.8. Let a force F be applied tangentially at the mass m on the free end of the rod. The rod starts rotating in a circle of radius r about the axis O. Hence the torque acting on the rod is given as:

$$\tau = \mathbf{F} \times \mathbf{r}$$

As the force is acting perpendicular to r, therefore, the magnitude of the torque is given as:

$$\tau = Fr \sin 90^\circ = F\,r \qquad \text{- - - - (1)}$$

But according to Newton's second law of motion, the force acting is given as

$$F = ma$$

So, eq. (1) becomes

$$\tau = mar \quad \text{- - - - (2)}$$

As $a = r\,\alpha$

∴ Eq.(2) becomes

$$\tau = mr^2\,\alpha \quad \text{- - - - (3)}$$

But $mr^2 = I$

∴ Eq.(3) becomes

$$\tau = I\,\alpha \quad \text{- - - - (4)}$$

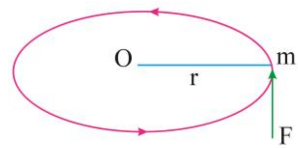

Fig. 4.8

Here I is the moment of inertia of the rod.

Now consider a rigid body rotating about its axis O. If the body consists of n small particles of masses m_1, m_2, m_3, - - - , m_n having distances r_1, r_2, r_3, - - - , r_n from the axis of rotation O as shown in Figure 4.9, then torques acting at m's are given by

$$\tau_1 = m_1 r_1^2\,\alpha_1$$

$$\tau_2 = m_2 r_2^2\,\alpha_2$$

$$\tau_3 = m_3 r_3^2\,\alpha_3$$

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$$\tau_n = m_n r_n^2\,\alpha_n$$

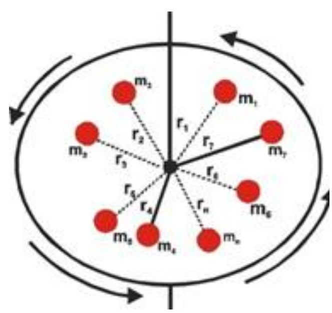
Fig. 4.9

The total torque of the rigid body is given as

$$\tau = \tau_1 + \tau_2 + \tau_3 + \text{- - -} + \tau_n$$

$$= m_1 r_1^2\alpha_1 + m_2 r_2^2\alpha_2 + m_3 r_3^2\alpha_3 + \text{- - -} + m_n r_n^2\alpha_n$$

As the body is rigid so all the particles will have the same angular acceleration,

So $\alpha_1 = \alpha_2 = \alpha_3 = \text{- - -} = \alpha_n = \alpha$

∴ $\tau = \tau_1 + \tau_2 + \tau_3 + \text{- - -} + \tau_n$

$$= (m_1 r_1^2\alpha + m_2 r_2^2\alpha + m_3 r_3^2\alpha + \text{- -} + m_n r_n^2\alpha) \quad \text{- - - (5)}$$

$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \text{- - -} + m_n r_n^2)\,\alpha$$

$$\tau = \left(\sum_{i=1}^{n} m_i\, r_i^2 \right) \alpha$$

Here, $$I = \left(\sum_{i=1}^{n} m_i\, r_i^2 \right)$$

= moment of inertia of the whole body about the axis of rotation O.

$$\therefore \quad \tau = I\,\alpha \qquad ---- \quad (6)$$

This is the total torque of the rigid body.

ROTATIONAL INERTIA *OR* MOMENT OF INERTIA:

Definition:

"The resistance of a rotating body to change its state of rotation or rest is called Moment of Inertia".

If we want to stop a rotating wheel, we have to apply force. Similarly, a stationary wheel resists to be put into motion. The resistance that a wheel shows against the change of its state of rotation or of rest is called moment of inertia. Its unit is kg-m^2.

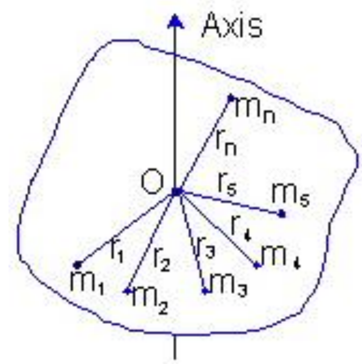

Fig. 4.10

Prove that: $I = MK^2$

Consider a body made up of a large number of small masses $m_1, m_2, m_3, ---, m_n$ having distances $r_1, r_2, r_3, ---, r_n$ respectively from the axis of rotation, then the moment of inertia of the body about the axis of rotation is given by:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + --- + m_n r_n^2 = \left(\sum_{i=1}^{n} m_i\, r_i^2 \right)$$

So the moment of inertia of the body is equal to the product of the mass and the square of the distance from the axis of rotation. Now if $m_1 = m_2 = m_3 = --- = m_n = m$, then

$$I = m\,(r_1^2 + r_2^2 + r_3^2 + --- + r_n^2)$$

Multiply and divide by n , then

$$I = \frac{mn\,(r_1^2 + r_2^2 + r_3^2 + ---- + r_n^2)}{n}$$

But mn = M and let $\frac{(r_1^2 + r_2^2 + r_3^2 + --- + r_n^2)}{n} = K^2$ then

$$I = MK^2$$

Here $K = \sqrt{\frac{(r_1^2 + r_2^2 + r_3^2 + --- + r_n^2)}{n}}$

K is called the Radius of Gyration.

Radius of Gyration:

Radius of gyration is defined as the square root of the mean of the squares of the distances of the various mass distributions from the axis of rotation. In other words, *the distance from the axis of rotation of the body to a point where the whole mass may be supposed to be concentrated is called the radius of gyration*. It is written as

$$K = \sqrt{\frac{I}{M}}$$

EXAMPLE 4.3: A flywheel of mass 25 kg has radius of gyration 2 m. Calculate its moment of inertia?

SOLUTION: Mass of fly wheel = M = 25 kg

Radius of gyration = K = 2 m

Moment of inertia = I = ?

$$I = MK^2$$

$$= (25)(2)^2 = 100 \text{ kg-m}^2$$

SUMMARY

- ***Torque*** is the cross product of the force and the position vector from the axis of rotation.
- ***Centre of gravity*** of a body is that point at which the whole weight of the body acts.
- When a body has zero acceleration, it is said to be in equilibrium.
- ***First Condition of Equilibrium***: A body will be in translational equilibrium if the algebraic sum of all the forces acting on the body is zero, i.e. $\sum \mathbf{F} = 0$.
- ***Second Condition of Equilibrium***: A body will be in rotational equilibrium if the algebraic sum of all the torques acting on the body is zero, i.e., $\sum \boldsymbol{\tau} = 0$.
- The relation between torque acting on the body and its angular

acceleration is given as $\tau = I\alpha$.

- ***Moment of inertia*** of a rotating body is its resistance against its change in acceleration.

EXERCISE

LONG QUESTIONS

1. Define torque. How it is calculated?
2. State and explain the conditions of equilibrium.
3. Prove that: $I = mr^2$
4. Define moment of inertia of a rigid body. Show that $I = MK^2$.
5. Derive relation between torque and angular acceleration.

SHORT QUESTIONS

Answer the following questions.

1. Define torque.
2. Describe the factors on which torque depends upon.
3. Differentiate center of mass and center of gravity.
4. Define equilibrium.
5. State the two conditions of equilibrium.
6. Differentiate static and dynamic equilibrium.
7. Define moment of inertia of a rigid body.
8. Define radius of gyration of a rigid body.
9. Write relation between torque and angular acceleration.
10. A fly wheel of mass 10 kg has radius of gyration 3 m. Find its moment of inertia.

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1- Torque is the product of

a) Mass and acceleration
b) Force and mass
c) Mass and velocity
d) Force and vertical distance of force from axis of rotation.

2- The relation between linear and angular acceleration is

a) $a = \frac{1}{\alpha}$ *b)* $\alpha = ar$ *c)* $a = r\alpha$ *d)* $a = \alpha^2$

3- One revolution is equal to

a) 360° *b)* 180° *c)* 90° *d)* 270°

4- **The unit of angular velocity is**

a) m/sec *b)* rad/sec *c)* meter *d)* radian

5- **The conditions of equilibrium are**

a) One *b)* Two *c)* Three *d)* Four

6- **Torque has maximum value if angle between r and F is**

a) 0° *b)* 30° *c)* 45° *d)* 90°

7- **Torque has zero value if angle between F and r is**

a) 0° *b)* 45° *c)* 60° *d)* 90°

8- **The centre of gravity of a body is**

a) The centre of the body

b) The point at which the mass of the body acts

c) The point at which the whole weight of the body acts

d) None of these

9- **The point at which an applied force produces a linear acceleration but no rotation is called**

a) Centre of body *b)* Centre of mass

c) Weight of body *d)* Fulcrum

10- **A body will be in translational equilibrium, if**

a) $\Sigma\vec{F} = \mathbf{0}$ *b)* $\Sigma\vec{P} = \mathbf{0}$ *c)* $\Sigma\vec{L} = \mathbf{0}$ *d)* $\Sigma\vec{\tau} = 0$

11- **A body will be in rotational equilibrium, if**

a) $\Sigma\vec{L} = \mathbf{0}$ *b)* $\Sigma\vec{P} = \mathbf{0}$ *c)* $\Sigma\vec{F} = \mathbf{0}$ *d)* $\Sigma\vec{\tau} = 0$

PROBLEMS

4.1. A force of 2N with positive vector of 3m acts on a body with an angle of 30°. Find the torque acting on the body.[*Ans : 3 N.m*]

4.2. How much force should be applied on the long arm of the lever when a mass of 10 kg is suspended to the short arm of the lever. The ratio in the length of its arms is 5 **:** 1. Take $g = 9.8 m/sec^2$.[*Ans : 19.6 N*]

4.3. A 25kg flywheel has a radius of gyration of 2m. What is its moment of inertia? [*Ans : 100Kg . m*^{2}]

4.4. If the torque acting on a body (by a force) is 20N**.**m and its moment of inertia is 4 kg**.**m^2. Find its angular acceleration.[*Ans : 5 rev/sec*^{2}]

***

Chapter 05

WAVEMOTION

Course contents:

5.1. Review Hooke's law of elasticity.
5.2. Motion under an elastic restoring force.
5.3. Characteristics of simple harmonic motion.
5.4. S. H. M. and circular motion.
5.5. Simple pendulum.
5.6. Wave form of S. H. M.
5.7. Resonance.
5.8. Transverse vibration of a stretched string.

Learning Objectives:

At the end of this chapter the students will be able to:

- Explain Hooke' law of elasticity.
- Explain motion under an elastic restoring force.
- Describe characteristics of simple harmonic motion.
- Explain S. H. M. and circular motion.
- Describe simple pendulum.
- Illustrate, wave form of S. H. M.
- Explain the phenomenon of resonance.
- Explain transverse vibration of a stretched string.

Elasticity:

Elasticity is that property of a body which opposes any deforming force. When we apply a force on a body, it produces a change in the body. On removing the force, the body tends to recover its original condition. This power of recovery is called elasticity.

STRESS:

The elastic force developed in a body per unit area is called stress. It is also defined as the resistive force per unit area against the deforming force is called stress.

Mathematically, Stress $= \frac{Force}{Area} = \frac{F}{A}$

Unit of stress:

The unit of stress in S. I. system is N/m²

TYPES OF STRESS:

1. Longitudinal Stress:

The force which acts on the unit area of a body and produces a change in its length is called the longitudinal stress. It has the following two types.

a) Tensile Stress:

The stress which increases the length of a body is called the tensile stress.

b) Compressive stress:

The stress which decreases the length of a body is called the compressive stress.

2. **Volumetric Stress:**

The forces which acts on the unit area of a body and changes its volume is called the volumetric stress.

3. **Shearing Stress:**

The force which acts on the unit area of a body and changes the shape of the body is called shearing stress.

STRAIN:

The fractional change produced in a body due to stress is called strain.

Mathematically, Strain $= \frac{Change}{Original\ Measurement}$

It is of the following three types.

1. Longitudinal Strain:

It is defined as the change in length per unit length of the body. Let a wire has original length L. On applying a force, let its length increases by ΔL. Then its longitudinal strain is given by.

Longitudinal strain $= \frac{Change\ in\ length\ of\ wire}{Original\ length\ of\ wire} = \frac{\Delta L}{L}$

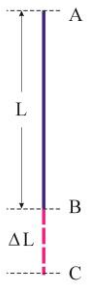

Fig 5.1

2. Volumetric Strain:

It is defined as the change in volume per unit volume of the body. If a cylinder of gas has original volume V. On decreasing pressure, let its increase in volume be ΔV. Then

Volumetric strain $= \frac{Change\ in\ volume}{Original\ volume} = \frac{\Delta V}{V}$

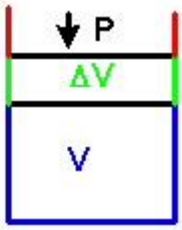

Fig 5.2

3. Shearing Strain:

Shearing strain changes the shape of a body without changing its volume. Suppose, we fix the lower surface of a body and apply a force on its upper surface such that its vertical edge turns through an angle θ then it is shearing strain. Shearing strain is expressed in radians.

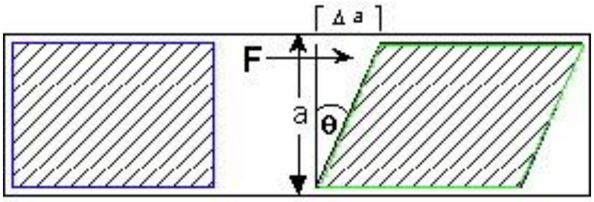

Fig 5.3

Shearing strain is defined as change in area per unit area of a body. Let ametallic sheet has original area a. On applying force, its area changes by Δa, then

$$\text{Shearing Strain} = \frac{Change\ in\ area}{Original\ area} = \frac{\Delta a}{a}$$

But $\frac{\Delta a}{a}$ = tan θ, For small θ, tan θ = θ, Therefore Shearing Strain = θ

HOOKE'S LAW:

Within elastic limits, the change produced in a body is proportional to the force acting on it

$$\text{Stress} \propto \text{Strain}$$

$$\Rightarrow \frac{Stress}{Strain} = \text{E}$$

Here E is constant, called the *modulus of elasticity*.

Explanation:

If we draw a graph between stress and strain for an iron wire, we get a straight line from o to x. In this region, the strain is directly proportional to the stress and the body remains elastic. If we remove the stress, the body recovers its original position. Point x is called the elastic limit. If we apply stress more than the elastic limit, the graph does not remain a straight line. In this case, the stress will produce a permanent change in the body and the body beyond point x (the elastic limit) becomes plastic.

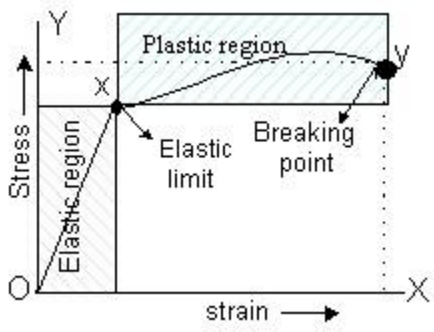

Fig 5.4

Here the length of the body increases rapidly. At last the body will break at y. It is always considered that the stress may not exceed the elastic limit.

MODUL I OF ELASTICITY:

(i) Young's Modulus:

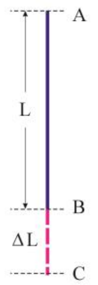

Fig 5.5

The ratio of stress to longitudinal strain is called Young's Modulus. Let F be the force acting on a wire of length L and cross-sectional area A. If increase in length of the wire is ΔL, then

$$\text{Young's Modulus} = \frac{Stress}{Longitudinal\ Strain}$$

$$= \frac{F/A}{\Delta L/L} = \frac{F\,L}{A\,\Delta L}$$

Its unit in S I. system is: N/m^2

(ii) Bulk Modulus:

The ratio of stress to volumetric strain is called Bulk Modulus. Let F be the force acting on a gas cylinder of volume V and cross-sectional area A. If the force changes its volume by ΔV, then

$$\text{Bulk Modulus} = \frac{Stress}{Volumetric\ Strain}$$

$$= \frac{F/A}{\Delta V/V} = \frac{F\,V}{A\,\Delta V}$$

Its unit in S I. system is N/m^2.

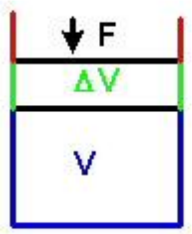

Fig 5.6

(iii) Rigidity Modulus:

Suppose a force F acts on a body at surface area A. If the shear produced is θ radian, then

$$\text{Rigidity Modulus} = \frac{Stress}{Shearing\ Strain}$$

But $\quad \text{Shearing Strain} = \frac{\Delta a}{a}$

$$\text{Rigidity Modulus} = \frac{F/A}{\Delta a/a}$$

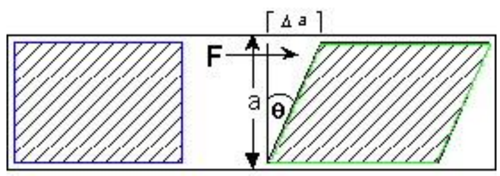

Fig 5.7

But $\quad \frac{\Delta a}{a} = \tan\theta$,

For small value of θ, $\tan\theta = \theta$

Therefore $\quad \text{Rigidity Modulus} = \frac{F/A}{\theta} = \frac{F}{A\theta}$

Its unit in S I. system is N/m^2

TYPES OF MOTION:

1. Translatory Motion:

The motion of a body along a straight line is called translatory motion. For example, the motion of the bullet fired from a gun. Zig-zag also falls in it.

2. Rotatory Motion:

The motion of a body along a circular path around an axis of rotation is called rotatory motion. For example, the motion of a stone in a circular path with the help of a string and the motion of a fly-wheel are rotatory motion.

3. Vibratory Motion:

The to and fro motion of a body around its mean position is called vibratory motion. For example, the motion of a pendulum, the motion of a violin string and the motion of a body attached to a string.

4. Periodic Motion:

It is that motion of a body which repeats itself after equal intervals of time. For example, the motion of a pendulum is periodic motion.

Conditions for vibratory motion:

1. There should be an elastic restoring force.
2. There should be inertia in the vibrating body

MOTION UNDER ELASTIC RESTORING FORCE:

Consider a ball of mass m rolling on a frictionless surface, attached to a fixed support with the help of a spring as shown in Fig. 5.8. If we apply a force F on the ball which displaces it through distance x from the mean position, then according to Hooke's law, this applied force is directly proportional to the displacement, i.e.

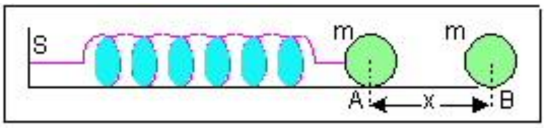

Fig 5.8

$$F \propto x$$

or $$F = kx$$

Where k is the constant of proportionality and is called the **spring constant**. Due to elasticity, the spring applies an equal restoring force in the opposite direction.

Applied force $= F = k\,x$

Restoring force $= F = -k\,x$

The negative sign shows that the restoring force is directed towards the mean position.

Restoring Force:

It is that force which tends to move a body back to its original position (mean position) when the applied force is removed.

Motion under elastic restoring force:

Consider a ball at point O, attached to a fixed support with the help of a spring on a frictionless horizontal surface. If we apply a force F on the ball, it is displaced through a distance x_o to the point A. If the applied force is within elastic limits then an equal restoring force will act on the ball in the opposite direction. This restoring force acting on the ball at point A is given by

$$F = -k x_o$$

To bring the ball from point O to A, we do some work on it which is stored in it in the form of potential energy. Now if the ball is released at point A, the restoring force brings it towards the point O. When it reaches O, all its P. E. is converted into K. E. Due to inertia; the ball continues to move towards left and reaches B. At point B, the spring is compressed and all its kinetic energy is converted into potential energy. Again, the restoring force moves the ball towards right and it reaches the point A. In this way, the ball continues to vibrate between the points A and B.

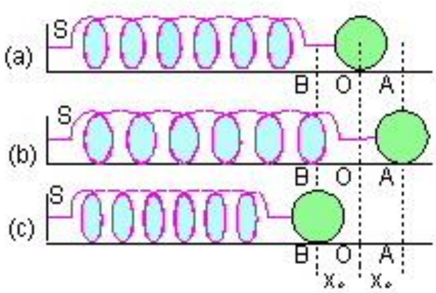

Fig 5.9

During its vibratory motion, when the ball is at distance x from the mean position O, the restoring force acting on it is given by

$$F = -kx \qquad \text{- - - - - - - (1)}$$

Let 'a' be the acceleration produced in the ball by the force *F* at that instant. Then by Newton's second law of motion.

$$F = ma \qquad \text{- - - - - - - (2)}$$

Comparing Eqs.(1) and (2), we get, $ma = -k x$

$$\Rightarrow \quad a = -\frac{k}{m} x \qquad \text{- - - - - - - (3)}$$

Here $\frac{k}{m}$ = constant

$$\therefore \quad a = -(\text{constant})\, x$$

$$\Rightarrow \quad a \propto x$$

This equation shows that acceleration of the ball is directly proportional to its distance from the mean position and the negative sign shows that the acceleration is directed towards the mean position O. This shows that the motion of the spring-mass system is a Simple Harmonic Motion.

IMPORTANT TERMS ABOUT VIBRATORY MOTION:

1. Vibration:

One complete round trip of a body about its mean position is called a vibration.

2. Time Period:

The time required to complete one vibration is called the time period. It is denoted by T. Its S. I. unit is second.

3. Frequency:

The number of vibrations completed by a body in one second is called frequency. It is denoted by f. Its S. I. unit is cycle/sec or Hertz. The relation between frequency and the time period is given as

$$f = \frac{1}{T}$$

4. Displacement (x):

It is the distance of a vibrating body from the mean position at any instant of time

5. Amplitude (x_o):

The maximum displacement of a vibrating body from the mean position is called amplitude

Simple Harmonic Motion:

It is that vibratory motion in which the acceleration is directly proportional to the displacement from the mean position and it is always directed towards the mean position

Characteristics of Simple Harmonic Motion:

The Simple Harmonic Motion has the following characteristics

a. Vibratory motion:

The Simple Harmonic Motion is a vibratory motion, i.e., it repeats itself after equal intervals of time.

b. Nature of motion:

The motion of the body performing S.H.M. is always directed towards the mean position

c. Direction of acceleration:

The acceleration of the body performing S.H.M. is always directed towards the mean position

d. Magnitude of acceleration:

The magnitude of the acceleration of a body performing S.H.M. is directly proportional to the displacement from the mean position. It is maximum at the extreme points and is zero at the centre

e. Velocity:

The velocity of the body performing S.H.M. is zero at the extreme points and is maximum at the mean position

f. Kinetic Energy:

In a S. H. M., the K. E. of the body is zero at the extreme positions and maximum at the mean position.

g. Potential Energy:

In a S.H.M., the P. E. of the body is proportional to the square of the distance from the mean position. It is maximum at the extreme positions and is zero at the centre

h.Total Energy:

In S. H. M., the total energy of the body remains constant

S.H.M AND CIRCULAR MOTION:

Consider a body moving in a horizontal circular path of radius r with constant angular velocity ω. It has its projection on the nearby wall. If the body moves from point A to point P, it covers an angular displacement θ (θ = ωt). At point P, its linear velocity is given by $v_P = r\omega$

From the point P, draw perpendicular PQ on the diameter AB of the circle. The point Q will be the projection of P. As the body moves along the circular path with constant angular velocity ω, so its projection moves to and fro along the diameter AB. We have to prove that the motion of Q is S. H. M. As the body moves in the circular path due to centripetal force, so the acceleration of Q is always directed towards the centre. At point P, the centripetal acceleration of the body is given by

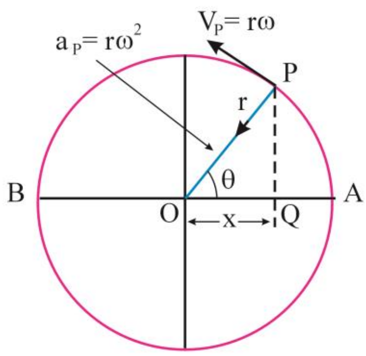

Fig 5.10

$$a_P = \frac{v_P^2}{r} = \frac{r^2\omega^2}{r} = r\,\omega^2 \qquad \text{-------} \quad (1)$$

The acceleration of the point Q will be equal to the horizontal component of the centripetal acceleration of the body at point P. If we take it as a, then

$$a = a_P\cos\theta = r\,\omega^2\cos\theta \qquad \text{-------} \quad (2)$$

But according to the Fig.5.10, $\cos\theta = \frac{x}{r}$

Putting in (2) we have $a = r\,\omega^2\left(\frac{x}{r}\right) = \omega^2 x$

As the acceleration is directed towards the mean position, so we write it with negative sign and have

$$a = -\omega^2 x \quad \Rightarrow \quad a \propto -x$$

This equation shows that the projection of the body moving in a circular path performs S. H. M.

Time Period of the point Q:

The time period of the motion of the point Q is the time in which the body completes one cycle. In this way it covers an angle of 2π radian.

As $\omega t = \theta$

When $t = T$ (Time period of motion) and $\theta = 2\pi$,

Therefore $\omega t = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$

Velocity of the point Q:

The velocity of Q is equal to the horizontal component of the velocity of the point P. From the figure, we see that

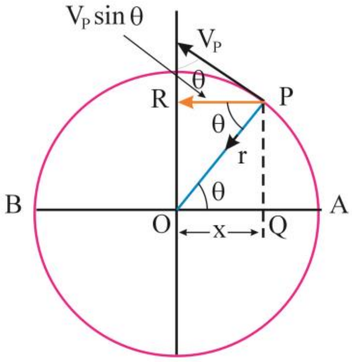

Fig 5.11

$$v = v_p \sin\theta = r\omega\sqrt{sin^2\theta}$$

$$= r\,\omega\sqrt{1 - cos^2\theta}$$

But according to the Fig.5.11,

$\cos\theta = \frac{x}{r}$, putting it, we have

$$v = r\omega\sqrt{1 - \frac{x^2}{r^2}}$$

But $r = x_o$ (the amplitude), therefore

$$v = \omega\, x_o\sqrt{1 - \frac{x^2}{x_o^2}}$$

SIMPLE PENDULUM:

Construction:

The simple pendulum consists of a small bob suspended from a frictionless support by a light inextensible string

Working:

Consider a pendulum of length l with its mean position at O. When we bring the bob to the point A and leave it, it starts its vibration between two extreme points A and B.

At A, all its energy is the P. E. which is equal to the amount of work done in

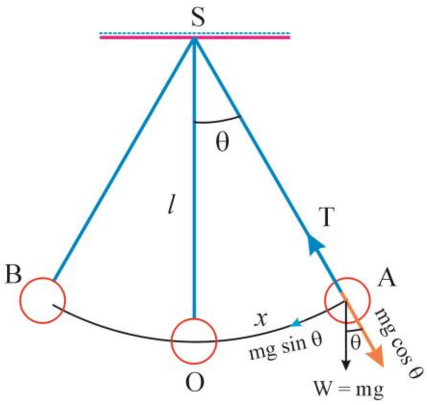

Fig.5.12

bringing the bob from point O to point A. At point A, the restoring force acts on it, and it starts its motion towards the mean position O. At O, all its P. E. is converted into K. E. Due to inertia, it continues its motion towards left and reaches the point B. At B, all its K. E. is again converted into P. E. At B, restoring force again acts on it and it starts its motion towards the point A. In this way, the bob of the pendulum continues to vibrate between two extreme points A and B.

Second's Pendulum:

It is that pendulum whose time period is equal to 2 seconds

Show that the Motion of a Simple Pendulum is S. H. M:

Consider the pendulum at the point B during its vibratory motion. Let m be the mass of the bob and x is its displacement from the mean position. Two forces are acting on the bob at this position.

1. The weight mg of the bob acting vertically downward.
2. The tension T of the string acting along BO.

   Weight w = mg can be resolved into two rectangular components as follows:

(i) Component of weight along the string = $mg\cos\theta$

(ii) Component of weight perpendicular to string = $mg\sin\theta$

As there is no motion of the bob along the string, so they cancel the effect of each other. i.e

$$T = mg\cos\theta$$

Thus, the component $mg\sin\theta$ is the only responsible force for the motion of the bob towards the mean position. This component will be equal to the restoring force acting on the bob and is written as

$$F = -mg\sin\theta \qquad \text{------ (1)}$$

The negative sign shows that this force is directed towards the mean position. If 'a' is the acceleration of the bob at point B, then according to Newton's Second Law of Motion,

$$F = ma \qquad \text{------ (2)}$$

Comparing eqs. (1) and (2), we get

$$ma = -mg\sin\theta$$

$$\Rightarrow \quad a = -g\sin\theta$$

When θ is very small, then $\sin\theta = \theta$

$$\therefore \quad a = -g\theta \qquad \text{------ (3)}$$

We know that $S = r\theta \implies \theta = \dfrac{S}{r}$

Here $S = x$ and $r = l$. Therefore, $\theta = \dfrac{x}{l}$ (in radians)

Putting this value in Eq.(3), we get

$$a = -g\left(\frac{x}{l}\right) = -\left(\frac{g}{l}\right)x \qquad \text{------ (4)}$$

But $\frac{g}{l}$ is constant

Therefore a = – (constant) x $\Longrightarrow a \propto -x$

This equation shows that the motion of the simple pendulum is S. H. M.

Time Period:

We know that the time period of S. H. M. is

$$T = \frac{2\pi}{\omega} \qquad \text{------ (1)}$$

But the acceleration of a body performing S. H. M. is given as

$$a = -\omega^2 x \qquad \text{------ (2)}$$

In case of a simple pendulum,

$$a = -\left(\frac{g}{l}\right)x \qquad \text{------ (3)}$$

Comparing Eqns. (2) and (3), we get

$$\omega^2 = \frac{g}{l} \quad \Longrightarrow \quad \omega = \sqrt{\frac{g}{l}}$$

Putting this value of ω in Eq.(1), we get

$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}} \quad \Longrightarrow \quad T = 2\pi\sqrt{\frac{l}{g}}$$

RESONANCE:

Natural Frequency & Natural Time Period:

When a simple pendulum is displaced from its mean position, it vibrates with a certain time period T and frequency f. Whenever this pendulum is disturbed, it always vibrates with the same time period. This time period is known as the natural time period of that pendulum.

As the time period depends upon the length of the pendulum at certain place, therefore, the simple pendulum of the same length will always have the same natural time period.

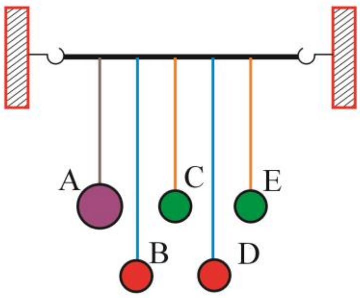

Fig 5.13

Definition of Resonance:

When we apply such force on a vibrating body whose time period is equal to the natural time period of the body, then its amplitude of vibration increases. This is called Resonance.

Experiment: Consider a stretched rubber cord tied horizontally between two fixed points. Five pendulums are suspended with the cord.

Three of these pendulums A, C and E have length l_1 each and two of these B and D have length l_2 each.

Set the pendulum A to vibrate. It will be seen that C and E will also begin to vibrate with same amplitude as that of A, while B and D will almost remain stationary. It is because that A, C and E have the same length and time period but B and D have different time period and they remain almost stationary.

Now stop the motion of all the pendulums. Make the length of "A" equal to that of B and D, i.e., l_2. Again, set A to vibrate. It will be seen that the pendulums B and D will also begin to vibrate with the same amplitude as that of A while C and E will almost remain stationary. This experiment explains the principle of resonance.

EXAMPLES OF RSONANCE:

1. When troops are marching on a bridge which can vibrate with certain natural frequency. If the frequency of the steps of the troops coincides with the natural frequency of the bridge, then bridge may break due to resonance. Therefore, the soldiers are ordered to break the steps while crossing the bridge.
2. If we apply such force on a swing whose time period is equal to the natural time period of the swing, then the amplitude of the swing can be made quite large.
3. Tuning of a radio is an example of electrical resonance. By tuning a radio, its frequency can be made equal to the frequency of the waves of desired broadcasting station.
4. When the sound waves from a tuning fork are made to enter an air column, the frequency of this air column is equal to the frequency of the tuning fork. Then the resonance takes place and loud sound is heard.

Applications of Resonance:

1. It is used to find the natural frequencies of different bodies.
2. It is used to find the speed of the sound

STATIONARY OR STANDING WAVES:

Definition:

When two waves of the same amplitude, time period and speed travel along the same straight line in the opposite direction, these waves superimpose to give stationary or standing waves. This is a particular form of interference of waves.

Some terms about transverse waves:

Nodes:

Those points in a wave where the amplitude of the wave is zero are called Nodes.

Antinodes:

Those points in a wave where the amplitude of the wave is maximum are called Antinodes.

Loop:

The shape produced by a curve crossing itself is called a loop.

TRANSVERSE VIBRATIONS ON A STRETCHED STRING:

Consider a string of length L , is stretched by clamping its two ends, we shall see what will happen if we pluck the string at different points.

1. Vibrations of the String in One Loop. (First mode):

When the string is plucked at its middle point, two transverse waves will start from that point and move towards the ends in opposite directions. They are reflected back at the ends and superimpose to form the stationary waves. The string will vibrate in one loop as shown in the Fig. 5.14. Let λ_1 be the wavelength and f_1 be the frequency of vibration in this mode. Then

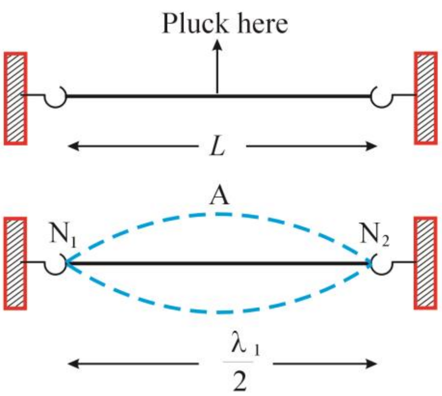

Fig 5.14

$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L \quad ---- (1)$$

If v is the speed of the waves, then

$$v = \lambda_1 f_1 \Rightarrow v = 2L f_1$$

$$\Rightarrow \quad f_1 = \frac{v}{2L} \quad ------ (2)$$

2. Vibrations of the String in Two Loop: (Second mode)

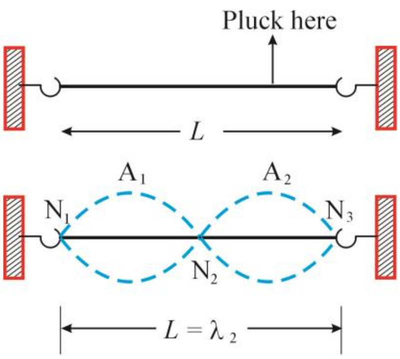

Fig 5.15

When the same string is plucked from one-quarter (i.e.1/4 of the length), the string will vibrate in two loops as shown in Fig. 5.15.

Let λ_2 be the wavelength and f_2 be the frequency of vibration in this mode. Then

$$L = \lambda_2 \qquad \text{------ (3)}$$

Here the wavelength has decreased and the frequency has increased in the same ratio.

So the speed of the wave will remain the same. Now

If v is the speed of the waves, then

$$V = \lambda_2 f_2 \Rightarrow \qquad v = L f_2$$

$$f_2 = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2 f_1$$

or $\qquad f_2 = 2 f_1$

3. Vibrations of the String in Three Loop. (Third mode):

When the string is plucked from one-sixth (i.e.1/6 of its length), the string will vibrate in three loops as shown in Fig. 5.16.

Let λ_3 be the wavelength and f_3 be the frequency of vibration in this mode. Then

$$\frac{3}{2}\lambda_3 = L$$

$$\Rightarrow \quad \lambda_3 = \frac{2L}{3}$$

If v is the speed of the waves, then

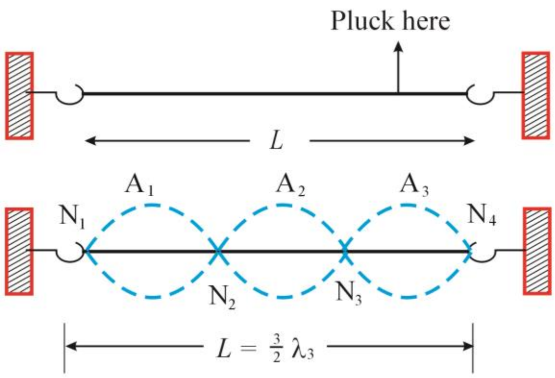

Fig 5.16

$$v = \lambda_3 f_3 \Rightarrow v = \frac{2L}{3} f_3$$

$$f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right) = 3 f_1$$

or $\qquad f_3 = 3 f_1$

Similarly, if the string will vibrate in n loops then

$$f_n = n\, f_1$$

Here f_1 is called the fundamental frequency and the other frequencies which are integral multiples of the fundamental frequency are called **overtones** or **harmonics**

SUMMARY

- **Hooke's Law:**
  Under elastic limits, stress is directly proportional to the strain produced in the body.
- **Restoring force:**
  It is that force which tends to restore the original condition of the body when the applied force is removed.
- **S. H. M:**
  Simple harmonic motion is the vibratory motion in which the acceleration produced in the body is directly proportional to the displacement from the mean position and is directed toward the mean position.
- **Resonance:**
  When we apply such force on a vibrating body whose time period is equal to the natural time period of the body, then its amplitude is increased. This is called resonance.
- **Stationary waves:**
  When two identical waves travel along the same straight line in opposite directions, they superimpose to give stationary waves.

EXERCISE

LONG QUESTIONS

1. State and explain the Hooke's law of elasticity.
2. Discuss motion under elastic restoring force.
3. Show that the motion of the projection of a particle moving in a circular path is S.H.M.
4. Describe the construction and working of a simple pendulum. Show that its motion is SHM.
5. Write a note on the phenomenon of resonance..
6. Define stationary waves. Show that the harmonics produced in stationary waves are integral multiple of the Fundamental frequency.

SHORT QUESTIONS

Answer the following questions.

1. Define elasticity.
2. Define stress and write its types.
3. Define strain and write its types.

4. State Hooke's law of elasticity.
5. Describe young modulus of elasticity.
6. Write types of motion.
7. Write conditions for vibratory motion.
8. Describe elastic restoring force.
9. Define time period in vibratory motion.
10. Define simple harmonic motion. Also write its characteristics.
11. Define seconds pendulum.
12. Write formula for time period of a simple pendulum.
13. Define stationary waves.
14. Define harmonics in stationary waves.

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1- Time period of simple pendulum is

a) $\pi\sqrt{\frac{2l}{g}}$ b) $\pi\sqrt{\frac{l}{2g}}$ c) $2\pi\sqrt{\frac{l}{g}}$ d) $\pi\sqrt{\frac{l}{g}}$

2- Which of the following substance has relatively high value of elasticity?

a) Rubber *b)* Steel *c)* Air *d)* Gold

3- The waveform of S.H.M is similar to

a) Cosine wave *b)* Sine wave *c)* Pulsed waves *d)* Square wave

4- The maximum distance of a vibrating body from mean position is called

a) Time period *b)* Displacement *c)* Amplitude *d)* Frequency

5- Time period of a second's pendulum is

a) Zero *b)* 1 Seconds *c)* 2 Seconds *d)* 3Seconds

6- The frequency of second's pendulum

a) 1Hertz *b)* 2Hertz *c)* 0.5Hertz *d)* 0.25Hertz

7- The length of second's pendulum is

a) 1 m *b)* 0.5 m *c)* 0.6 m *d)* 0.992 m

8- In a vibrating string, the points where the amplitude is maximum is called.

a) Crest *b)* Trough *c)* Antinode *d)* Node

9- The distance between two consecutive nodes is

a) $\frac{\lambda}{4}$ *b)* $\frac{\lambda}{2}$ *c)* λ *d)* 2λ

10- The wavelength of the fundamental mode of vibration in a string is

a) l *b)* $2l$ *c)* $4l$ *d)* $\frac{l}{2}$

11- The unit of stress is

a) N/m *b)* N/m^2 *c)* newton *d)* joule

12- The example of S.H.M is

a) Motion of a train *b)* Motion of a fan

c) Motion of a simple pendulum *d)* Motion of a wheel

13- In S.H.M, the acceleration is always towards

a) right side *b)* left side *c)* the center *d)* None

14- In S.H.M which form of energy is maximum at the center

a) Rotational energy *b)* Vibrational energy

c) K.E *d)* P.E

15- The time period of simple pendulum depends upon:

a) Mass *b)* Weight *c)* Length *d)* Amplitude

16- The frequencies other than Fundamental frequency are called:

a) Harmonics *b)* Triangle *c)* Crest *d)* Amplitude

17- The distance between a node and an anti-node is

a) 0.25λ *b)* 0.5λ *c)* λ *d)* 2λ

PROBLEMS

5.1. A simple pendulum has length 150cm. Find its time period if g = 9.8 m/sec^2. [*Ans: 2.46 sec.*]

5.2. A body of mass 0.5Kg is attached to a spring of spring constant 8N/m (which is vibrating). Find its time period. [*Ans: 1.57 sec.*]

5.3. A body of mass 1Kg is hanging with a spring. The body is pulled down 2cm from its equilibrium position, its tension increases by 0.98N. Find the spring constant and the time period.[*Ans: 0.89 sec., 49 N/m*]

5.4. The buffer of a railway wagon is compressed 5cm when a force of 200 N is applied to it. Calculate the spring constant.[*Ans: 4000 N/m*]

5.5. A wire 1 meter long plucked at its centre, vibrates with a frequency of 250 Hertz. Calculate the wavelength and the speed of the wave in the wire.[*Ans: 2m, 500m/sec*]

***

Chapter 06

SOUND

Course contents:

6.1 Longitudinal waves
6.2 Intensity, loudness, pitch and quality of sound
6.3 Unit of Intensity level and frequency response of ear
6.4 Interference of sound waves, silence zones, beats
6.5 Acoustics
6.6 Doppler's Effect

Learning Objectives:

At the end of this chapter the students will be able to:

- Describe longitudinal waves and their propagation.
- Explain the concepts: Intensity, loudness, pitch and quality of sound.
- Explain units of Intensity level of sound and frequency response of ear.
- Explain phenomena of silence zones, beats.
- Explain Acoustics of buildings
- Explain Doppler's Effect giving mathematical expressions.

WAVES

A **mechanism by which energy is transferred from one place to another without the movement of the particles is known as wave motion".**

There are three kinds of waves.

1. MECHANICAL WAVES

The waves which require a material medium for their propagation are known as mechanical waves. For example sound waves, waves on the surface of water and waves along a string.

2. ELECTROMAGNETIC WAVES

The waves which do not require a material medium for their propagation are called electromagnetic waves. For example light, heat and radio waves.

3. MATER WAVES

The waves associated with particles are called matter waves. For example a fast moving electron is associated with waves.

TYPES OF MECHANICAL WAVES

With reference to the movements of the material particles through which the waves are transmitted waves are classified into two types i.e. transverse waves and longitudinal waves.

TRANSVERSE WAVES

The waves in which the particles of the medium execute simple harmonic motion along lines at right angles to the direction of propagation of waves are known as transverse waves. For example water waves, light waves, waves along the string.

Consider waves in a string as shown in the fig. 6.1, the portion of the wave above its mean level is called a crest and the portion of the wave below its mean level is called a trough. The distance between two consecutive crests or troughs is called wavelength of the transverse wave and is denoted by 'λ'

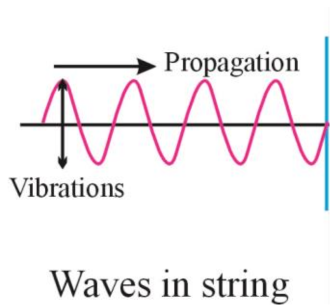

Fig 6.1

LONGITUDINAL OR COMPRESSIONAL WAVES

The waves, in which the various particles of the medium vibrate about their mean positions along the direction of the waves, are known as longitudinal waves.

To examine the nature of the longitudinal waves consider waves produced in air by a vibrating strip 'A' in air whose lower end is clamped by a vice as shown in the Fig 6.2.

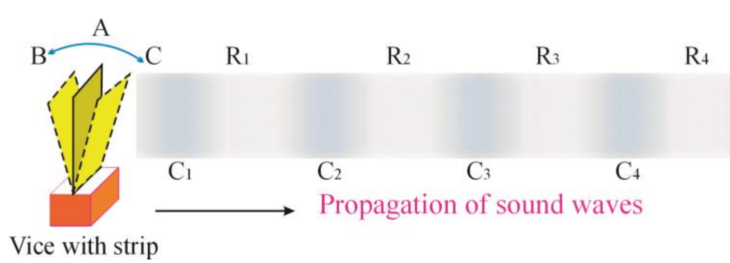

Fig 6.2

The body vibrates between the extreme positions 'B' and 'C'. When it moves towards 'C', it compresses the layer of air immediately in front of it. This layer then expands and in this way it compresses the second layer. The second layer then expands and in this way it compresses the third layer. The third layer compresses the fourth layer and so on. In this way a compression moves from layer to layer. When the body moves toward 'B' the first layer is rarefied. The second layer at once expands to restore the first one to its normal pressure, and in doing so it is rarefied. Then the third layer expands to restore the second layer to its normal pressure and so it is itself rarefied. In this way, a rarefaction also moves from layer to layer behind the first compression.

As the body goes on vibrating we get a series of alternate compressions and rarefactions. In Fig. 6.2, C_1, C_2, C_3 and C_4 etc. are the centers of compressions and R_1, R_2, R_3 and R_4 etc. are the centers of the rarefactions.

The compression means higher pressure as compared to normal pressure of the air and rarefaction means lower pressure as compared to normal pressure of the air.

"The distance between two consecutive centers of compressions or rarefactions is called wavelength."

If the frequency of vibration be 'f' then 'f' waves are produced in one second. If the wavelength be 'λ' then the disturbance travels through a distance '$f\lambda$' in one second. The velocity of longitudinal waves is thus given by

$$v = f\lambda$$

SOUND WAVES

Sound may be defined psychologically and physically.

Psychologically it is the sensation produced in ear.

Physically it is said to be the stimulus, due to a sounding body, capable of producing sensation.

Three things are necessary for the hearing sound.

1. **Vibrating body**
   A body emits sound only when it vibrates. Therefore a vibrating body is essential to produce sound.
2. **Material medium**
   Whenever a body vibrates it produce compressional waves in air. These waves which are called sound waves travel through air and reached to the ear. Thus a material medium is essential for the propagation of sound.
3. **Receiver**
   A receiver that is ear is essential to hear sound because sound waves produce the sensation of sound on the ear. Therefore a receiver of the sound waves is also essential.

SPEED OF SOUND

The speed of sound in dry air at 0 °C is about 331.3m/s.

Sound travel faster in solids. Its speed is medium in liquids and slow in gases. At standard temperature and pressure its speed is 331 m/s. At 20 ^oC the speed of sound in air is 334 m/s. Sound cannot travel through vacuum.

INTENSITY OF SOUND

Sound waves carry energy away from the source. The intensity of sound is determined by the energy transferred.

"The energy transmitted per second through a unit area (held perpendicular to the direction of propagation of the waves) by the sound waves is called the intensity of the sound waves".

The unit of intensity of sound will be power unit divided by area unit i.e. watt/m^2.

Mathematically intensity of sound is given by

$$I = \frac{E}{A \times t} \quad ----- \quad (1)$$

Where E = Sound energy

A = Area of the surface

t = Time

In M.K.S system the unit of intensity is $\frac{J}{m^2\, s}$ or Wm^{-2}.

LOUDNESS OF SOUND

The magnitude of auditory sensation produced in ear by sound is called loudness of sound. It is denoted by 'L'. It depends upon intensity of sound and the sensitivity of ear.

WEBER-FECHNER LAW

This law states that loudness of sound is directly proportional to the logarithm of intensity.

$$L \propto \log_{10} I$$

$$L = k \log_{10} I$$

Where 'K' is a constant of proportionality and its value depend upon system of units.

INTENSITY LEVEL

The difference in loudness of two sounds where one sound is faintest audible sound is called intensity level.

If the intensities of the two sounds are **I** and **I_0** and loudness L and L_0 respectively, then

According to Weber Fechner law

$$L = klog_{10} I$$

$$L_0 = klog_{10} I_o$$

Where I_o is the intensity of faintest audible sound.

According to the definition of intensity level, we can write

Intensity Level = L − L$_o$

$= klog_{10}I - klog_{10}I_o$

$= k(log_{10}I - log_{10}I_o)$

Intensity Level $= klog_{10}\frac{I}{I_o}$ - - - - - (1)

Where I is the intensity of any given sound and I_o is intensity of faintest audible sound which is considered as 10^{-12} Wm^{-2}.

UNIT OF INTENSITY LEVEL

The unit of intensity level id Bell.

'If the intensity of sound is 10 I_o (ten times of I_o), then the intensity level of the given sound is called one bell'.

We know that

Intensity Level $= klog_{10}\frac{I}{I_o}$

Putting $I = 10I_o$ in above equation we get

Intensity Level $= klog_{10}\frac{10\,I_o}{I_o}$

$= k\ log_{10}(10)$

$= k\times\ 1$ [as$k\ log_{10}(10) = 1$]

If we measure intensity level in bel, then $k = 1$. Thus

Intensity Level = 1bel.

Decibel (dB) is a smaller unit of Bel and is equal to $\frac{1}{10}$ bel.

So Intensity level of sound $= 10\ log_{10}\frac{I}{I_o}$ dB

FREQUENCY RESPONSE OF EAR

A normal human ear can hear those sound frequencies which lie between 20 hertz and 20,000 hertz. If the frequency of sound is higher than 20,000 hertz, it cannot be heard. Sounds of frequencies higher than 20,000 is called ultrasonic. The sensitiveness of ear for high frequency sounds decreases with age. Children can generally hear sound of 20 kHz while elderly people cannot hear anything above 15000 hertz. The sensitivity of an average human ear is different in different frequency ranges. Normal ear is most sensitive in the frequency range 2000 to 5000 hertz. There is a threshold value of intensity level below which we cannot hear. There is also an upper limit of intensity above which we feel pain rather than of hearing.

PITCH OF SOUND

The property or characteristic of sound by which a shrill sound can be distinguished from a grave one is called pitch of sound.

Pitch of sound depends upon the frequency of the sound. The greater is the frequency, the higher is the pitch and the lower is the frequency the lower is the pitch.

The sound produced by children and birds etc. are of high pitch i.e. of high frequency, while the sound produced by man , frogs etc. are of low pitch i.e. of low frequency.

QUALITY OF SOUND

It is the property of sound by which the sounds of same pitch and loudness produced by different sources can be distinguished.

It is a common experience that a note of given pitch and loudness sounded on Piano is easily distinguished from one of exactly the same pitch and loudness played, for example, on Violin. It is because the quality of the two notes is different.

The quality of sound depends upon the wave-form of the resultant and is controlled by the number and relative intensities and phase of harmonics that are present; the resultant wave-forms have different effects on the ear even though they have the same pitch and loudness.

MUSICAL SOUND AND NOISE

Audible sounds are classified into two groups, namely musical sounds and noise. A musical sound is that in which the vibrations of the sounding body are periodic, follow each other regularly and rapidly, so as to produce a pleasing effect on the ear without any sudden change in loudness.

The waveform of musical sounds is smooth. Musical sounds can be produced using a flute, sitar or harmonium, etc. Periodicity, regularity and continuity are their characteristics. The waveform of noise, on the other hand, is irregular.

“Sound which produces a pleasing sensation in the ear is called a musical sound”

“Sound which produces a jarring or a displeasing effect is called a noise”.

The conditions necessary for the production of a musical sound are that sounds should follow each other at regular intervals in quick succession and without sudden changes in loudness.

Noise on the other hand is sounds of very short duration having no periodicity and their character is changing. Some familiar examples of noise are the roaring of traffic, sound of hammer etc.

The curve of musical sound is uniform and regular while the curve of noise is irregular and shows sudden changes in loudness.

INTERFERENCE OF SOUND WAVES

"When two sound waves of the same frequency, wavelength and amplitude superimpose on each other, they cancel each other at some point and reinforce at other points, this phenomenon is known as interference".

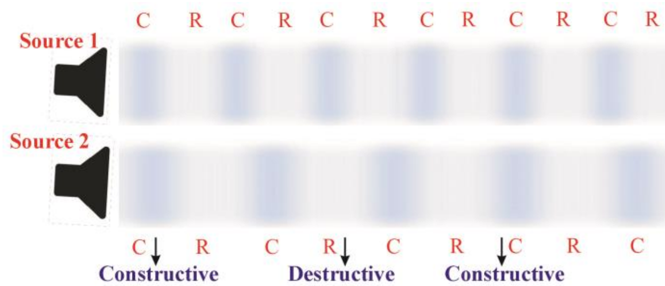

Fig 6.3

There are two types of interference.

CONSTRUCTIVE INTERFERENCE

"In this case two sound waves arrive at a point in the same phase. The compression falls on compression and rarefaction falls on rarefaction. As a result the intensity of the resultant sound wave increases which in turn increases the loudness of the sound.

For constructive interference the path difference between two sound waves is given by

$$S = 0, \lambda, 2\lambda, 3\lambda, \text{-----}$$

or $$S = n\lambda$$

where $n = 0, 1, 2, 3, \text{----}$

DESTRUCTIVE INTERFERENCE

In this case two sound waves arrive at a point in opposite phase. That is compression of one falls on the rarefaction of the other and they cancel each other. As a result, intensity decreases which in turn decreases the loudness of sound.

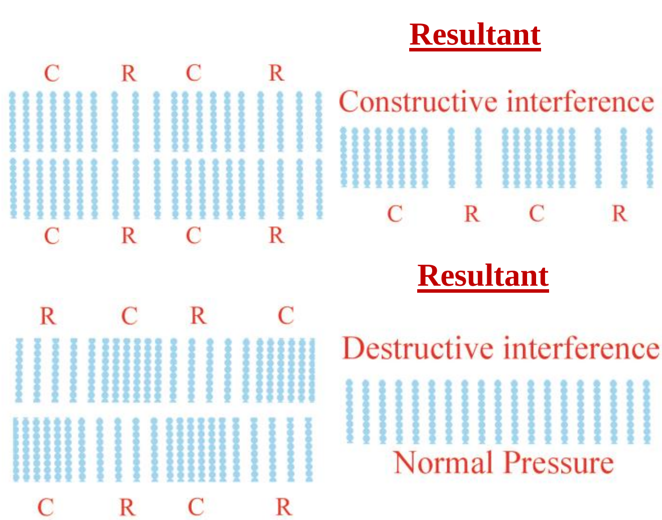

Fig 6.4

For destructive interference the path difference between two sound waves is given by $S = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, - - - - -$

or $S = (2n + 1)\frac{\lambda}{2}$

where $n = 0, 1, 2, 3, - - - -$

DEMONSTRATION OF INTERFERENCE BY AN EXPERIMENT

A vibrating tuning fork is placed in front of the opening A. the sound waves on entering A will split, half the intensity goes through the tube 'C' and the remaining half through the tube 'D'. The two sound waves reunite at the outlet 'B' and can be heard by a sound detector such as ear placed at 'B'.

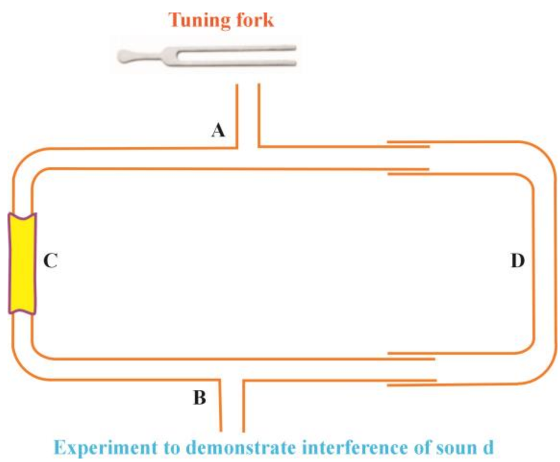

Fig 6.5

If the sliding tube 'D' is adjusted so as to make the two paths ACB and ADB equal, the waves arrive at 'B' in phase i.e. without path difference. Then these waves interfere at 'B' constructively and a loud sound is heard at 'B'. If the sliding tube 'D' is now drawn out, path ADB becomes longer than the path ACB. The sound waves arriving at 'B' via 'D' will fall more and more behind those coming via 'C'. When the difference of path between the waves is half a wave length, they interfere destructively and consequently no sound is heard at 'B'. If the rubber portion of the tube is pinched so as to stop the sound waves coming through 'C' the ear will again hear the sound. This proves that the silence is due to the destructive interference of the two sound waves.

SILENCE ZONES

Consider a fog siren situated on a high cliff. A ship approaching it may find itself in a zone of silence where the siren is inaudible. When the ship moves towards or away from the cliff, the sound again becomes audible. This zone of silence can be explained as being due to

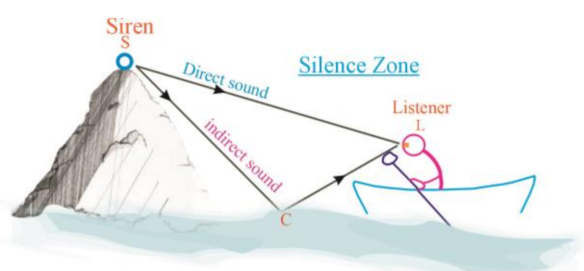

Fig 6.6

interference. The sound of a siren 'S' can reach the listener 'L' on the ship through two paths; one by the direct path SL and the other by the path SCL after suffering reflection from the surface of the sea. If these two paths differ by half a wave length or any odd integral multiple of half the wave length, then the compression due to one wave will find a rarefaction due to the other wave at the point 'L'. Since the compression tends to increase the pressure while the rarefaction tends to decrease the pressure at 'L', the two will cancel the effect of each other. As a result no sound will be heard at 'L'.

BEATS

"When two bodies having slightly different frequencies are sounded simultaneously, the periodic alterations of sound between maximum and minimum loudness are produced, which are known as beats".

The two sound waves from two sources of slightly different frequencies interfere constructively as well as destructively. When they interfere constructively maximum loudness is produced and when they interfere destructively minimum loudness is produced. Hence we can say that the production of beats is special type of interference.

Consider two tuning forks 'A' and 'B' vibrating with frequencies 32 Hertz and 30 Hertz respectively placed at equal distance from the ear. Then the following cases will arise.

1. Let us suppose that at a certain time $t = 0$, the two forks are in phase that is right hand prongs, of both the forks are moving towards right and are thus sending compressions. These two compressions will reach at the ear together and thus a loud sound is heard.
2. When $t = \frac{1}{4}$ sec. The fork 'A' completes 8 vibrations and 'B' completes $7\frac{1}{2}$ vibrations. The fork 'A' is sending compression while 'B' is sending rarefaction. They will cancel each other and no sound is heard.

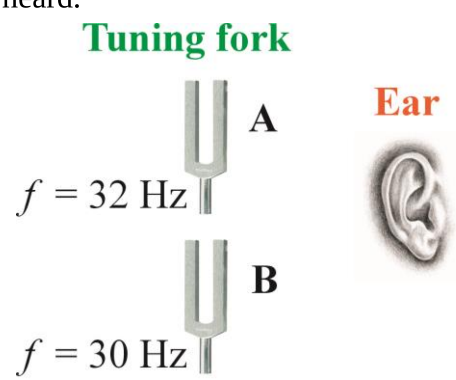

Fig 6.7

3. When $t = \frac{1}{2}$ sec, the fork 'S' and 'B' completes 16 and 15 vibrations respectively. Both the forks are sending compressions which reinforce each other and thus a loud sound is heard.

4. After t $= \frac{3}{4}$ sec. Fork 'A' will complete 24 vibrations and fork 'B' will complete 22 $\frac{1}{2}$ vibrations. at this instant fork 'A' will be sending a compression while fork 'B' will be sending rarefaction. Thus no sound will be heard.
5. After t = 1 sec. Fork 'A' will complete 32 vibrations and fork 'B' will complete 30 vibrations. Both the forks will be sending compressions and a loud sound will be heard.

From this discussion we can conclude that**;**

"The number of beats per second is equal to the difference between the frequencies of the two forks i.e. sounding bodies".

Mathematically

$$\text{Number of beats} = f_1 - f_2$$
$$\pm n = f_1 - f_2$$

Uses of Beats:

1. The phenomenon of beats is used in finding the unknown frequencies. The two bodies are sounded simultaneously and the number of beats is counted. The number of beats will be equal to the difference of the frequency of the sounding bodies. So if the frequency of one body is known, we can find the unknown frequency of the other body.
2. The phenomenon of beats is used to tune the musical instruments.

EXAMPLE: Two tuning forks A and B give 4 beats per second. On loading tuning fork A slightly we get 3 beats per second. What is the frequency of tuning fork A before and after loading it that of B is 256?

SOLUTION: Before Loading

Number of beats = n = 4
Frequency of tuning fork B = f_B = 256 Hertz
Frequency of tuning fork A = f_A = ?

$$f_A - f_B = \pm n$$
$$f_A = f_B \pm n$$
$$= 256 \pm 4 = 260 \text{ or } 252$$

After Loading

Number of beats = n = 3
Frequency of tuning fork B = f_B = 256 Hertz
Frequency of tuning fork A = f_A = ?

$$f_A - f_B = \pm n$$
$$f_A = f_B \pm n$$
$$= 256 \pm 3 = 259 \text{ or } 253$$

252 and 253 answer could not be correct as frequency could not increase after loading. Therefore 260 and 259 is correct answer.

ECHO

Echo is produced when the sound waves reflect from a hard plane surface for example wall or cliff. The sound which we hear after 50 to 100ms of the original sound is called echo. If we clap at some distance from a high wall we hear the sound of our clapping almost at the same moment, but after a few moments we hear the sound of our clapping once again, it is due to the reflection of the sound from the wall and is called echo. To hear echo the obstacle must be at least 17 meters away from the source of sound and listener.

REVERBERATIONS

The persistence of sound after a sound is produced is called reverberation. Reverberation is created when a sound is reflected causing large number of reflections to build up and then decay as the sound is absorbed by the surfaces of objects present in surroundings including air. Reverberation is most noticeable when the sound source stops sounding but the reflections continue, decreasing in amplitude, until they reach zero amplitude. The length of decay of reverberation is called reverberation time.

ACOUSTICS

Acoustics is the science that deals with the production, control, transmission, reception, and effects of sound. The study of acoustics revolves around the generation, propagation and reception of mechanical waves and vibrations.

When we discuss the properties of buildings such as theaters and auditoriums, the qualities that determine the ability of such buildings to reflect sound waves in such a way as to produce distinct hearing are considered. The acoustic properties of buildings include the discussion of interference, echoes and reverberation etc. The branch is called architectural acoustics.

An auditorium is said to have good acoustics when speech can be heard almost equally well throughout the space, without troublesome echoes and reverberations. The dais and stage should be so designed that speech sounds are projected out into the audience.

Multiple echoes from the ceiling and walls of the room should not be entirely absent or the room will be acoustically dead, as if the speaker were addressing a crowd in the open air.

DOPPLER'S EFFECT

"The apparent change in the pitch of sound caused by the relative motion of either the source of sound or the listener is called the Doppler Effect".

For example a train while whistling passes near a listener, a considerable change in the pitch of the note, produced by the whistle, will occur. When the train is approaching, the pitch of the note increases and when the train is receding, the pitch decreases.

Doppler Effect can be considered under following cases.

1. WHEN SOURCE IS MOVING TOWARDS A STATIONARY LISTENER

Consider a source of sound producing sound waves of frequency 'f' and wavelength 'λ'. The source and listener are at rest. In this case the listener should receive 'f' waves in one second which is occupied in a space of length 'V'. The wavelength 'λ' can be given as

$$\lambda = \frac{Distance\ occupied\ by\ f\ waves}{number\ of\ waves}$$

$$\lambda = \frac{V}{f} \qquad ----\ (1)$$

Now suppose the source is moving with velocity 'V_S' towards the listener at rest as shown in the figure, the 'f' waves will be contained in length '$V - V_S$' and the apparent wavelength λ' is given by

$$\lambda' = \frac{V - V_S}{f} \qquad ----\ (2)$$

The apparent or changed frequency of the sound waves can be given by

$$f' = \frac{V}{\lambda'} \qquad ----\ (3)$$

Putting the value of λ' in equation (3) we get

$$f' = \frac{V}{\frac{V - V_S}{f}}$$

$$f' = \frac{V}{V - V_S} f \quad ---\ (4)$$

Equation (4) shows that

$$f' > f$$

Therefore, the apparent frequency or pitch of the sound increases when the source moves towards a stationary listener.

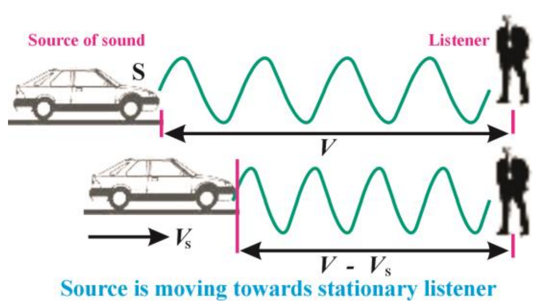

Fig 6.8

2. WHEN THE SOURCE MOVES AWAY FROM THE STATIONARY LISTENER

When the source is moving away from the stationary listener with speed 'V_S', then after one second f waves will occupied in a distance '$V + V_S$' as shown in the figure. It means that the apparent wavelength will increase i.e.

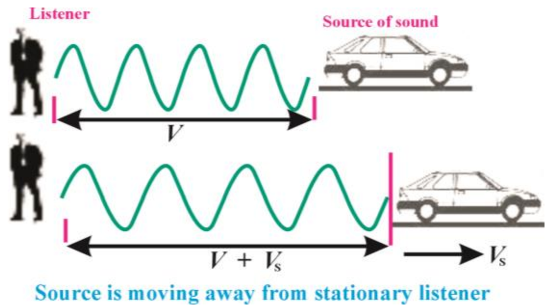

Fig 6.9

$$\lambda' = \frac{V + V_S}{f} \quad \text{- - - (1)}$$

The apparent or changed frequency of the sound waves can be given by

$$f' = \frac{V}{\lambda'} \quad \text{- - - - (2)}$$

Putting the value of λ' in equation (2) we get

$$f' = \frac{V}{\frac{V + V_S}{f}}$$

$$f' = \frac{V}{V + V_S} f \quad \text{- - - - (3)}$$

Equation (3) shows that $f' < f$, therefore, the apparent frequency or pitch of the sound decreases when the source moves away from a stationary listener.

3. WHEN THE LISTENER IS MOVING TOWARDS A STATIONARY SOURCE

When source is at rest, the wavelength of the sound waves is given by

$$\lambda = \frac{V}{f} \quad \text{- - - - (1)}$$

Let the source is at rest and the listener is moving towards him with a velocity 'V_L'. In this case the speed of the waves will appear to the listener equal to '$V + V_L$' because the waves and the listener move in opposite directions. Since wavelength remains same and the apparent frequency can be given as.

$$f' = \frac{V + V_L}{\lambda} \quad \text{- - - - (2)}$$

$$= \frac{V + V_L}{\frac{V}{f}}$$

$$f' = \frac{V + V_L}{V} f \quad \text{- - - - (3)}$$

This equation shows that $f' > f$ i.e. the pitch of the sound increases when the listener moves towards a stationary source.

4. WHEN THE LISTENER MOVES AWAY FROM A STATIONARY SOURCE

When source is at rest, the wavelength of the sound waves is given by

$$\lambda = \frac{V}{f} \qquad \text{- - - - (1)}$$

When the listener moves with speed 'V$_L$' away from the source at rest, the speed of waves will appear to the listener equal to 'V − V$_L$'.

$$f' = \frac{V - V_L}{\lambda} \qquad \text{- - - - (2)}$$

$$= \frac{V - V_L}{\frac{V}{f}}$$

$$f' = \frac{V - V_L}{V} f \qquad \text{- - - - (3)}$$

This equation shows that $f' < f$ i.e. the pitch of the sound decreases when the listener moves away from a stationary source.

Example: A caris moving towards a building with the speed of 30 ms^{-1}.Its horn is producing sound of the frequency of 750 Hz. What will be the apparent frequency for a listener in the building? Assume that the speed of sound is 334 ms^{-1}.

Solution:

In this case the source is moving towards stationary listener.

So, $f' = \left(\frac{V}{V - V_S}\right) \times f$

$f = 750$ Hz

$V = 334$ ms^{-1}

$V_S = 30$ ms^{-1}

$f' = ?$

$$f' = \frac{V}{V - V_S} f = \frac{334}{334 - 30} \times 750$$

$$f' = \left(\frac{334}{304}\right) \times 750 = 824.01\text{Hz}$$

APPLICATIONS OF DOPPLER'S EFFECT

1. Doppler's effect has been successfully applied to light. The frequency of light from certain stars is found to be slightly different from the frequency of same light on earth. This is due to the motion of stars. By measuring the difference of frequency, the velocity of these stars can be determined.
2. The waves sent out from radar are reflected from the airplane. If the plane is moving away from the radar then the frequency of reflected waves is decreased. If the airplane is moving towards the radar then frequency of the reflected waves is increased. By calculating frequency difference the direction and speed of an airplane is determined.

3. When sonar (sound waves under water) is reflected from moving submarine, the change in frequency of sonar give the speed and direction of the submarine.

SUMMARY

- Sound waves are mechanical waves and they are longitudinal waves. They require a material medium for their propagation.
- Sound waves travel through air in the form of compression and rarefaction. The wavelength of a sound wave is the distance between two consecutive compressions or rarefactions
- Energy transferred per second through unit area held perpendicular to the direction of the sound is called intensity of sound and its unit is Wm^{-2}.
- The intensity level of sound is measured in dB.
- The characteristic of sound by which a shrill sound is distinguished from a grave one is called the pitch of sound.
- When two sound waves of same frequency superimpose on each other it is called interference of sound waves. If these waves reinforce each other's effect such interference is called the constructive interference and if they cancel each other's effect then it is called the destructive interference.
- Audible frequency range for a normal human ear is 20 Hz to 20 kHz.
- Pleasant sound is called musical sound but unpleasant sound is called noise.
- Beats are produced when we hear simultaneously two sounding bodies having slightly different frequencies.
- When reflected sound is heard separately from the original one heard directly, it is called echo.
- The apparent change in the frequency (or pitch) of sound for the listener due to relative motion between the source of sound and the listener is called Doppler's Effect.

EXERCISE

LONG QUESTIONS

1. Distinguish between longitudinal and transverse waves.
2. Define interference of sound waves. Explain constructive and destructive interference.
3. Describe the phenomenon of Beats.
4. Write note on the acoustic requirements of an auditorium.
5. What do you mean by Doppler's Effect? Find relationship for the apparent frequency when source of sound is moving towards the stationary listener.

SHORT QUESTIONS

1. Differentiate between mechanical and electromagnetic waves.
2. Define transverse waves and give two examples of transverse waves.
3. Define longitudinal waves and give one example.
4. Define intensity of sound. What is its unit?
5. Define Loudness of sound.
6. Define intensity level of sound.
7. Define Pitch of Sound.
8. Define quality of sound.
9. Differentiate between musical sound and noise.
10. Define interference of sound waves.
11. Define constructive and destructive interference.
12. Define Silence Zones?
13. Define phenomenon of Beats.
14. Describe two uses of Beats.
15. Define Doppler's effect.
16. Describe any two application of Doppler's effect.

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1- **The velocity of sound in air at 0 ºC is:**
   a) 3×10^8 m/s b) 330 km/s c) 331.3 m/s d) 3300 m/s

2- **Sound waves are:**
   a) Transverse waves b) Longitudinal waves
   c) Electromagnetic waves d) Matter waves

3- **The speed of sound is greatest in:**
   a) Iron b) Water c) Air d) Vacuum

4- Unit of intensity level of sound is:

a) Graham Bel b) Hertz c)Wm^{-2} d) Decibel

5- Sounds of frequencies higher than 20,000 hertz are called:

a) Harmonic b) Ultrasonic c) Supersonic d) Beats

6- The property or characteristic of sound by which a shrill sound can be distinguished from a grave one is called:

a) Pitch of sound b) Quality of sound
c) Wavelength of sound d) Intensity of sound

7- The unit of intensity of sound is:

a) watt/m b) watt/m^2 c) N/m d) Bell

8- Ear of normal human being response to the frequencies of sound between:

a) 20 to 200 Hz b) 20 to 2000 Hz
c) 20 to 20,000 Hz d) 2000 to 30,000 Hz

9- For destructive interference of sound waves the condition for path difference is:

a) $S = n\lambda$ b) $S = 2n\lambda$

c) $S = (2n+1)\lambda$ d) $S = (2n+1)\dfrac{\lambda}{2}$

10- Two tuning forks of frequencies 250 and 252 hertz are heard simultaneously. The number of beats will be:

a) 1 b) 2 c) 4 d) 502

11- The change in the pitch of sound caused by the relative motion of either the source of sound or the listener is called the:

a) Doppler's Effect b) Beats c) Echo d) Acoustics

12- The relationship between velocity 'V', frequency 'f' and wavelength 'λ' of a wave is:

a) $f = V\lambda$ b) $V = f\lambda$ c) $\lambda = Vf$ d) $V = \dfrac{f}{\lambda}$

PROBLEMS

6.1 A man is at a distance of 30 meter from a wall. Find the time interval after which he will hear echo of his sound.
(Take velocity of sound = 334 m/s) [Ans. 0.179 s]

6.2 The sound of the thunder is heard after 2 seconds of the flash. Find the distance of the clouds if velocity of sound is 340 ms^{-1}. [Ans. 780 m]

6.3 The frequency of the siren of a car is 2000 Hz. What will be its apparent frequency for a listener at rest, while the car is approaching to him with a velocity of 20 m/sec? (Take velocity of sound = 334 m/s) [Ans. 2127.38 Hz]

6.4 Two tuning forks A and B give 3 beats per second. On loading tuning fork A slightly we get 2 beats per second. What is the frequency of tuning fork A before and after loading it that of B is 250?
[Ans. Before loading 253 Hz, after loading 252 Hz]

6.5 Velocity of sound in air is 340 ms^{-1} and its frequency is 15 kHz. Find its wavelength. [Ans. 0.0226 m]

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Chapter 07

LIGHT

Course contents:

7.1 Review laws of reflection and refraction
7.2 Image formation by mirrors and lenses
7.3 Optical Instruments
7.4 Wave theory of light
7.5 Interference, diffraction, polarization of light waves
7.6 Applications of polarization in sunglasses, optical activity and stress analysis

Learning Objectives:

At the end of this chapter the students will be able to:

- Explain laws of reflection and refraction.
- Use mirror and lens formula to solve problems
- Use the concepts of image formation by mirrors and lenses to describe working of optical instruments, e.g. microscopes, telescopes, camera and sextant.
- Explain wave theory of light.
- Explain phenomena of interference, diffraction and polarization of light waves.
- Describe important applications of polarization.

REFLECTION OF LIGHT

When light travelling in one medium meets the surface of another medium, a part of light is sent back. The turning back of the light from the boundary of a medium is called reflection of light.

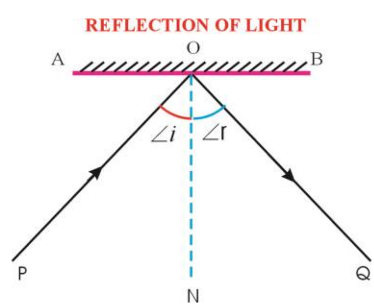

Fig 7.1

Reflection of light is shown in the Fig. 7.1. AB is a plane mirror. A ray of light travelling along PO is reflected at the point O and is sent back along a new direction OQ. NO is a perpendicular to the surface AB at point O which is called *point of incidence*.PO is called the *incident ray*. OQ is called the *reflected ray*. The angle between incident ray and the normal to the surface at point of incidence is called *angle of incidence*

The angle between reflected ray and the normal to the surface at point of incidence is called *angle of reflection.* Angle of incidence is represented by $\angle$i and angle of reflection is represented by $\angle$r.

KINDS OF REFLECTION

There are two kinds of reflection.

1- Regular reflection

If parallel rays of light after reflection from a surface remain parallel to each other then it is called regular reflection. Regular reflection takes place from smooth surfaces like mirrors.

2- Irregular or diffuse reflection

If parallel rays of light after reflection from a surface do not remain parallel to each other then it is called irregular or diffuse reflection. Irregular reflection takes place from rough surfaces like wall.

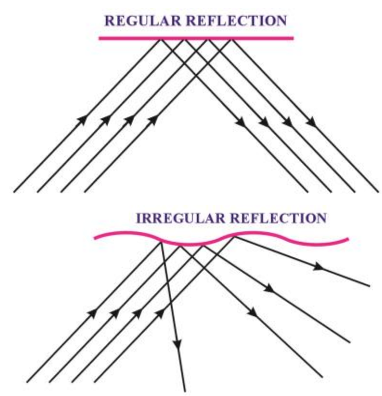

Fig 7.2

LAWS OF REFLECTION

There are two laws of reflection.

1. The incident ray, the reflected ray, and the normal to the surface at the point of incidence, all lies in the same plane.
2. The angle of reflection is equal to the angle of incidence.

i.e. $\angle i = \angle r$

REFRACTION OF LIGHT

When a beam of light is incident upon the boundary (making some acute angle) separating two transparent media such as air and glass, some of the light is reflected in the same medium while the remaining portion enters the second medium and its path undergoes a change in direction.

The bending of light when it enters from one medium to the other is called refraction of light.

Or

The phenomenon is which the path of light undergoes a change in its direction while it enters from one medium to the other is known as the refraction of light.

Or

The change in direction of light when it passes from one medium to another is called refraction of light.

In figure the refraction of light from air to glass is shown. AO is the incident ray, O is point of incidence, OB is the refracted ray. OC is the reflected ray. NON´ is the normal to the separating boundary at the point of incidence. The angle between normal and the incident ray is called angle of incidence and is represented by $\angle i$, while the angle between the refracted ray and the normal is called angle of refraction and is denoted by $\angle r$.

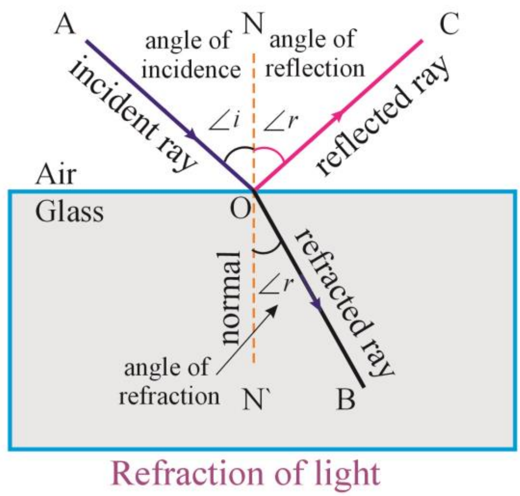

Fig 7.3

When light travels from water to air, the refracted ray bends away from the normal and air is said to by optically rare than water. When light travels from air to glass, the refracted ray bends towards the normal and the glass is said to be optically denser than air. In general

LAWS OF REFRACTION

1. The incident ray, the refracted ray, and the normal to the surface at the point of incidence, all lie in the same place.
2. For any two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for all angles of incidence. This law is also known as Snell's Law.
   Mathematically

$$n = \frac{\sin \angle i}{\sin \angle r}$$

Here 'n' is constant and is called *refractive index* of the second medium with respect to the first. Usually the refractive index of any medium is determined with respect to air, the refractive index of which is approximately equal to 1. The refractive index of glass with respect to air is found 1.5 while the refractive index of water with respect to air is 1.33. In general the refractive index of any medium is the ratio of speed of light in vacuum (which is 3×10^8 m/s) to the speed of light in that medium

$$\text{Refractive index of a medium} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in that medium}}$$

Example: Light passes from air into diamond at an angle of incidence of 45^o. Calculate the angle of refraction if the refractive index of diamond is 2.42.

Solution:

Angle of incidence = $\angle i$ = 45^o
Refractive index = n = 2.42
Angle of refraction = $\angle r$ = ?

Refractive index = $n = \frac{Sin \angle i}{Sin \angle r}$

$$2.42 = \frac{Sin\ 45^o}{Sin \angle r}$$

$$Sin \angle r = \frac{0.707}{2.42}$$

$$Sin \angle r = 0.292$$

$$\angle r = Sin^{-1}\ 0.292 = 17^o$$

Example: A ray of light traveling in air strikes the glass surface at an angle of 49^o with the normal while the angle of refraction is 30^o. What is refractive index of glass?

Solution:

Angle of incidence = $\angle i$ = 49^o
Refractive index = n = ?
Angle of refraction = $\angle r$ = 30^o

Refractive index = $n = \frac{Sin \angle i}{Sin \angle r}$

$$n = \frac{Sin\ 49^o}{Sin\ 30^o} \Longrightarrow n = \frac{0.7547}{0.5000}$$

$$n = 1.509$$

MIRROR

If the reflecting surface is plane the mirror is called a plane mirror.A plane mirror makes an image of objects placed in front of it.It is made by using a straight piece of glass and some highly reflecting and polished surface such as silver surface. The reflecting surface almost reflects all the light coming on it. The mirrors used in bathrooms and in dressing tables are examples of plane mirrors.

IMAGE FORMATION BY A PLANE MIRROR

Rays from the object placed in front of a plane mirror reflect according to laws of reflection. These rays after reflection from the mirror appear to be coming from behind the mirror and the image is formed behind the mirror.

CHARACTERISTICS OF THE IMAGE FORMED BY A PLANE MIRROR

The image formed by the plane mirror has following characteristics.

1. The image formed by a plane mirror is as far behind the mirror as the object is in front of the mirror.

2. The image formed by the plane mirror is laterally inverted. It means that it is not upside down but left and right inverted.
3. The image formed is imaginary.

Virtual or Imaginary image:

An image is called virtual or imaginary if rays of light do not actually go at the place of image but only appears to go to or come from the point of formation of the image. Secondly an imaginary or virtual image cannot be taken on a screen.

SPHERICAL MIRRORS

If the reflecting surface is a part of a hallow sphere then it is called spherical mirror.

A portion of a polished hallow sphere is called a spherical mirror.

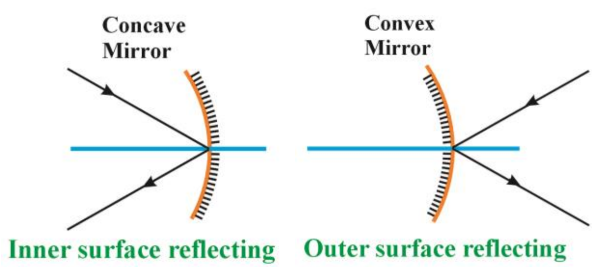

Fig 7.4

Spherical mirrors are of two types depending upon whether the inner or the outer surface of the sphere is polished i.e. reflecting. If the inner surface of the sphere is reflecting then the mirror formed by its part is called *concave mirror* and if the outer surface of the sphere is reflecting surface then the mirror formed by its portion is called *convex mirror*.

TERMS RELATED TO SPHERICAL MIRROR

Center of curvature

The center of the sphere of which the spherical mirror is made is known as the center of curvature of the spherical mirror. It is represented by ‘C’.

Aperture

The boundary of a spherical mirror is usually circular. The diameter of this circular boundary is called the aperture.

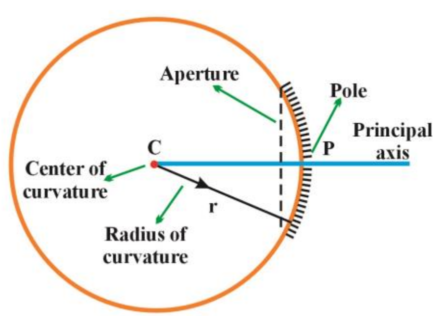

Fig 7.5

Pole

The center of the mirror is called its pole and it is denoted by ‘P’.

Principal axis

The straight line passing through the center of curvature and the pole of the mirror is called the principal axis.

Radius of curvature

The radius of the sphere of which spherical mirror is a part is called radius of curvature or the distance between the center of curvature and the mirror is called the radius of curvature. It is denoted by ‘r’.

PRINCIPAL FOCUS AND FOCAL LENGTH

When a parallel beam of light, parallel to the principal axis of a concave mirror incident on it, all the rays are reflected so as to converge to a point on the principal axis of the mirror. This point is called principal focus and is denoted by ‘F’. The concave mirror is also called converging mirror.

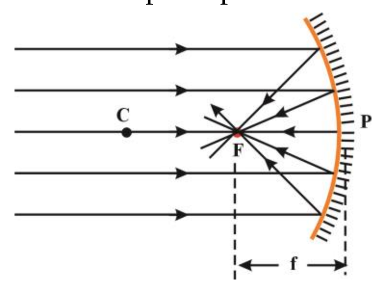

Fig 7.6

If light from a distant source (i.e. parallel rays of light), parallel to the principal axis of a convex mirror incident on it, all the rays are reflected so as to diverge and appear to come from a point on the principal axis , behind the mirror. This point is called the principal focus of the convex mirror. The convex mirror is also called diverging mirror.

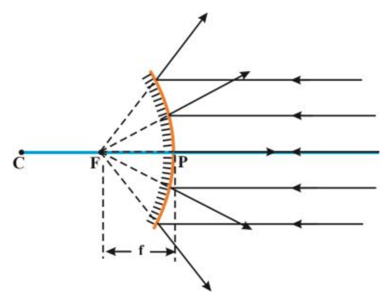

Fig 7.7

FOCAL LENGTH

The distance between the principal focus and the pole of the mirror is called its focal length and is denoted by ‘f’.

Relation between focal length and radius of curvature

The focal length of a spherical mirror is one half of its radius of curvature. Focal length is represented by ‘f’ and radius of curvature is represented by ‘R’. Therefore

$$f = \frac{1}{2} r$$

GRAPHICAL CONSTRUCTION OF IMAGES FORMED BY SPHERICAL MIRRORS

We have the following rules for concave mirrors

1. A ray parallel to the principal axis after reflection passes through the focus.
2. A ray passing through the focus is reflected parallel to the principal axis.

3. A ray passing through the center of curvature is reflected back along its own path.
4. A ray incident at the pole of the mirror reflected making an equal angle on the other side of the principle axis.

We have the following rules for convex mirrors.

1. A ray parallel to the principal axis after reflection diverges so as to appear coming from the principal focus.
2. A ray going towards the principal focus of the mirror becomes parallel to the principal axis after reflection from the mirror.
3. A ray going towards the center of curvature of the mirror reflects back on the same path.
4. A ray incident at the pole is reflected, making the same angle with the principal axis.

By using any of the three rays or even two we can draw the image of the object as shown in the diagram.

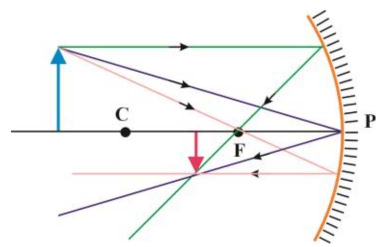

Fig 7.8

IMAGE FORMATION BY A CONCAVE MIRROR

The nature of an image of an object formed by a concave mirror depends upon the distance of the object from the mirror. There may be following different cases.

1. THE OBJECT IS AT INFINITE DISTANCE FROM THE CONCAVEMIRROR

Suppose an object is placed at an infinite distance from a concave mirror.

The rays coming from the object will be parallel to each other. These rays after reflection from the concave mirror will cross each other and form the image of the object at principal focus of the mirror. Nature of the image:

- Image is formed at ‘F’
- It is real.
- It is inverted.
- It is smaller than the object.

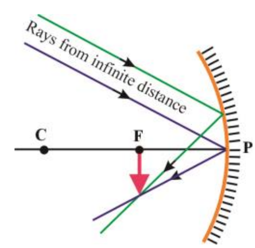

Fig 7.9

2. THE OBJECT IS BEYOND CENTER OF CURVATURE.

An object is placed beyond center of curvature of a concave mirror. Image formation is shown in Fig. 7.10.

Nature of the image:

- Image is formed in between ‘F’ and ‘C’,
- It is real.

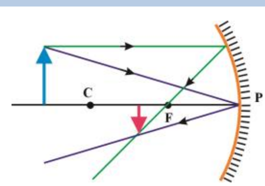

Fig 7.10

- It is inverted.
- It is smaller than the object.

3. THE OBJECT IS AT CENTER OF CURVATURE.

An object is placed at center of curvature of a concave mirror. Image formation is shown in Fig 7.11.

Nature of the image:

- Image is formed at ‘C’,
- It is real.
- It is inverted.
- It is of the same size as the object.

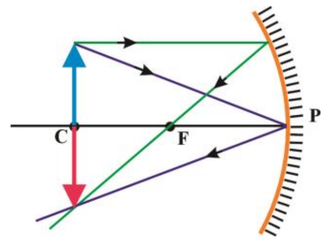

Fig 7.11

4. THE OBJECT IS IN BETWEEN F AND C

An object is placed in between the principal focus and the center of curvature of a concave mirror. Image formation is shown in Fig. 7.12.

Nature of the image:

- Image is formed beyond ‘C’,
- It is real.
- It is inverted.
- It is larger than the object.

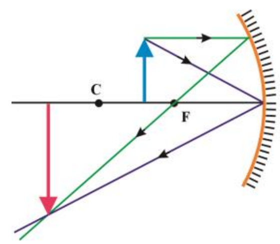

Fig 7.12

5. THE OBJECT IS AT PRINCIPAL FOCUS

An object is placed at the principal focus of a concave mirror. A ray coming from the object parallel to the principal axis of the mirror will pass after reflection through the principal focus of the mirror. Another ray which is incident at the pole will reflect making the same angle with the principal axis. These rays after reflection from the mirror will become parallel to each other and the image of the object is supposed to form at an infinite distance.

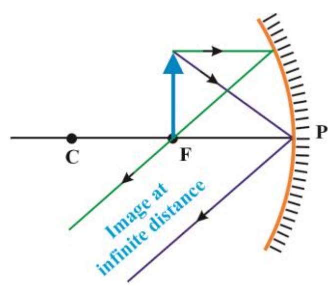

Fig 7.13

Nature of the image:

- Image is formed at infinite distance.
- It will be real.
- It will be inverted.
- It will be very large in size.

6. THE OBJECT IS IN BETWEEN THE PRINCIPAL FOCUS AND POLE OF THE CONCAVE MIRROR

Suppose an object is placed in between the principal focus and the pole of a concave mirror. A ray coming from the object parallel to the principal axis of the mirror will pass after reflection through the principal focus of the mirror. Another ray which is incident at the pole will reflect so that it will make the same angle with the principal axis as the incident ray makes. These rays after reflection from the mirror will diverge and never meet each other in front of the mirror though they appear to be coming from a point behind the mirror and the image of the object will be formed at that point.

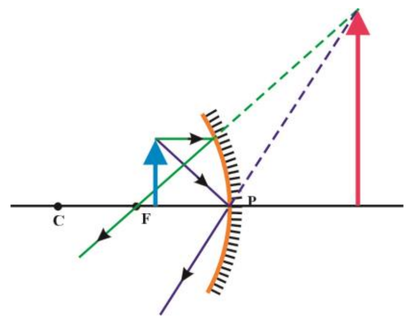

Fig 7.14

Nature of the image:

- Image is formed behind the mirror.
- The image is virtual or imaginary.
- It is erect.
- It is larger in size than the object.

IMAGE FORMATION BY A CONVEX MIRROR

Convex mirror gives virtual images only. The nature of any image formed by a convex mirror can be described as

- It is always erect.
- It is always formed in between ‘P’ and ‘F’.
- It is always virtual.
- It is always smaller in size than the object

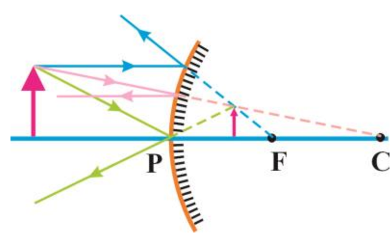

Fig 7.15

THE MIRROR EQUATION

The distance of an object from a spherical mirror is denoted by ‘p’ and the distance of the image formed by the spherical mirror from the pole of the mirror is denoted by ‘q’. The distance between pole and the principal focus of a spherical mirror is called its focal length and is denoted by ‘f’. The relationship between these three quantities i.e. p, q and f is called Mirror Equation.

The Mirror Equation is

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

DERIVATION OF MIRROR EQUATION FOR A CONCAVE MIRROR

Consider an object OB **is** placed in front of a concave mirror. IM is the image of the object. The distance of the object from the mirror is ‘p’ and the distance of the image from the mirror is ‘q’. The focal length of the mirror is ‘f’.

From the geometry of the figure

Triangles OBP and IMP are similar because angle $\angle BPO \text{ and } \angle MPI$ are equal being angle of incidence and angle of reflection respectively. Further $\angle BOP \text{ and } \angle MIP$ are also equal as both are right angles.

There fore

$$\frac{OB}{IM} = \frac{OP}{IP} \qquad \text{------} \quad (1)$$

Also the triangles EPF and IMF are similar

There fore

$$\frac{EP}{IM} = \frac{FP}{IF}$$

Because EP = OB, therefore

$$\frac{OB}{IM} = \frac{FP}{IF}$$

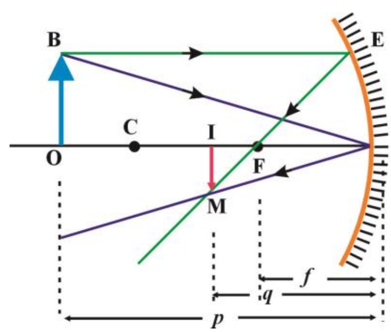

Fig 7.16

From figure IF = IP - FP,

Therefore,

$$\frac{OB}{IM} = \frac{FP}{IP - FP} \qquad \text{----} \quad (2)$$

Comparing equation (1) and (2), we get

$$\frac{OP}{IP} = \frac{FP}{IP - FP}$$

By putting values, we get

$$\frac{p}{q} = \frac{f}{q - f}$$

By cross multiplication, we get

$$pq - pf = fq$$

Dividing above equation by, ‘pqf’, we get

$$\frac{1}{f} - \frac{1}{q} = \frac{1}{p}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \quad \text{-----} \quad (3)$$

Equation (3) is the mirror equation for a concave mirror.

REAL AND VIRTUAL IMAGE

"An image formed by the actual intersection of rays is called real image"

"An image which is formed by the apparent intersection of rays when their directions have been produced backwards is called virtual image".

SIGN CONVENTION

To solve the problems using mirror equation all distances are measured from the mirror as origin.Distances of real objects and images are taken as positive whereas the distances of virtual or imaginary objects and images are taken as negative.

On this convention a concave mirror has a real principal focus and hence a positive focal length and a convex mirror have a virtual principal focus and a negative focal length.

DERIVATION OF MIRROR EQUATION FOR A CONVEX MIRROR

Consider an object OB isplaced in front of a convex mirror. IM is the image of the object. The distance of the object from the mirror is 'p' and the distance of the image from the mirror is 'q'. The focal length of the mirror is 'f'. From the geometry of the figure triangles OBP and IMP are similar.

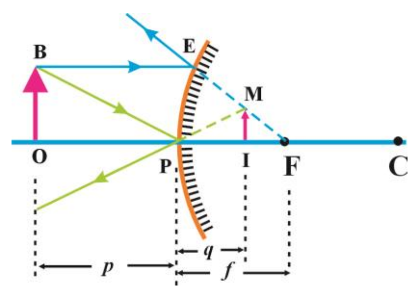

Fig 7.17

There fore

$$\frac{OB}{IM} = \frac{OP}{IP} \quad \text{----} \quad (1)$$

Also the triangles EPF and MIF are similar, therefore

$$\frac{EP}{IM} = \frac{PF}{IF}$$

Because EP = OB, therefore

$$\frac{OB}{IM} = \frac{PF}{IF}$$

From figure IF = PF - PI

$$\frac{OB}{IM} = \frac{PF}{PF - PI} \quad \text{----} \quad (2)$$

Comparing equation (1) and (2), we get

$$\frac{OP}{IP} = \frac{PF}{PF - PI}$$

Putting values, we get

$$\frac{p}{-q} = \frac{-f}{-f - (-q)} \quad ---- \quad (3)$$

(as the image is imaginary and mirror is convex the distance of image is taken as negative and focal length is as taken as negative)

By cross multiplication, we get

$$-pf + pq = fq$$

Dividing above equation by, 'pqf', we get

$$-\frac{1}{q} + \frac{1}{f} = \frac{1}{p}$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \quad ---- \quad (4)$$

EXAMPLE 1: The height of an object is 6 cm and it is placed in front of a concave mirror at a distance of 10 cm. The radius of curvature of the mirror is 10 cm, find the height and position of the image formed.

SOLUTION:

p = 10 cm, r = 10 cm, q = ?

r = 10 cm $\Rightarrow$ $f = \frac{r}{2} = \frac{10}{2}$ = 5 cm

The Mirror Equation is

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Putting value in this equation

$$\frac{1}{5} = \frac{1}{10} + \frac{1}{q}$$

$$\Rightarrow \quad \frac{1}{q} = \frac{1}{5} - \frac{1}{10}$$

$$\frac{1}{q} = \frac{2-1}{10}$$

$$\Rightarrow \quad q = 10 \text{ cm}$$

Now

$$\frac{q}{p} = \frac{Size\ of\ image}{Size\ of\ object}$$

$$\frac{10\ cm}{10\ cm} = \frac{6\ cm}{Size\ of\ object}$$

$\Rightarrow$ Size of the image = 6 cm

EXAMPLE 2: An object is placed in front of a convex mirror at a distance of 10 cm. Its image is formed at a distance of 5 cm behind the mirror. Find the focal length of the mirror.

SOLUTION:

$p = 10$ cm, $q = -5$ cm, $f = ?$

The Mirror Equation is

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Putting value in this equation

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{-5}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{10} - \frac{1}{5}$$

$$\frac{1}{f} = \frac{1-2}{10} = \frac{-1}{10}$$

$$\Rightarrow \quad f = -10 \text{ cm}$$

Focal length is negative because the mirror is convex.

LENSES

Lenses or magnifying glass have been in use for centuries. Lenses of different types play an important part in our everyday life. Lenses are used in cameras, spectacles, projectors, microscopes and telescopes etc.

"A lens is a piece of transparent medium having two opposite surfaces either both curved or one curved and one plane.

Or

"A lens is a piece of refracting medium bounded by either two spherical surfaces or one spherical and one plane surface".

CLASSIFICATION OF LENSES

There are two types of lenses.

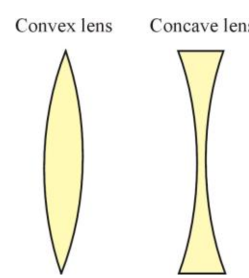

Fig 7.18

1. CONVEX OR CONVERGING LENS

Those lenses which converge a pencil of parallel rays are called converging lenses. Such lenses are thicker in the middle and thinner on the edges.

2. CONCAVE OR DIVERGING LENS

Those lenses which diverge a pencil of parallel rays are called diverging lens. Such lenses are thinner in the middle and thicker on the edges.

TERMS RELATED TO LENSES

PRINCIPAL AXIS

The straight line joining the centers of curvature of the two spherical surfaces of a lens is called the principal axis.

APERTURE

The diameter of the lens is called its aperture.

PRINCIPAL FOCUS

When a beam of light parallel to principal axis is incident on a convex lens, the beam of light after refraction converges to a point 'F' on the principal axis on the other side of the lens. This point is known as principal focus of convex lens.

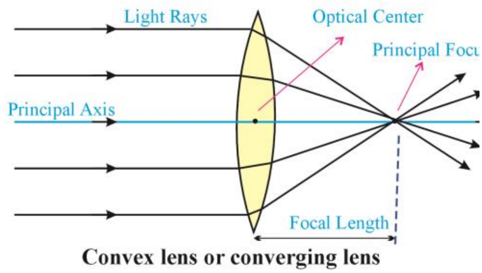

Fig 7.19

When a beam of light parallel to principal axis is incident on a concave lens, the beam after refraction through the lens diverges and appears to diverge from a point 'F' on the principal axis on the same side of lens. This point is known as principal focus of concave lens.

CENTER OF CURVATURE

A simple lens is usually a piece of glass bounded by spherical surfaces. These spherical surfaces are the portion of spheres. The centers of these spheres are called the center of curvature and are denoted by 'C'.

OPTICAL CENTER

The optical center of a lens divides the line joining the centers of curvature of the two surfaces in the ratio of their radii of curvatures. It is a fixed point for a given lens. The ray of light passing through the optical center of a lens are not deviated but only slightly displaced parallel to their original direction. When the lens is thinthis displacement is sufficiently small to be ignored, so that in all our diagrams rays going through the center of curvature of the lens are drawn straight. *The center of lens is thus called the optical center*. It is denoted by 'O'.

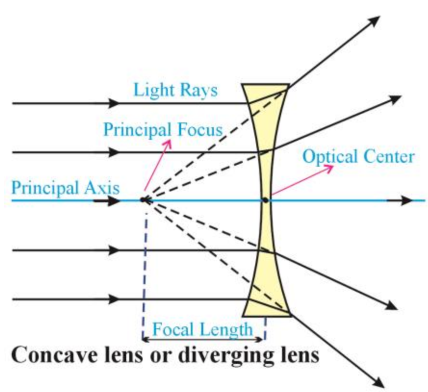

Fig 7.20

FOCAL LENGTH

Focal length of a lens is the distance between the optical center and the principal focus. It is denoted by ‘f’.

A lens has two principal foci and center of curvature

Light may pass through a lens in either direction; there will be two principal foci equidistant from the optical center, one on either side of the lens. These are denoted by ‘F’ and ‘F´ ‘. Similarly a lens has two center of curvature, one on either side of the lens.

POWER OF A LENS

The reciprocal of the focal length of a lens (taken in meters) is called the power of the lens.

The unit of power of a lens is dioptre.

$$\text{Power of a lens} = \frac{1}{f}$$

DIOPTRE

It is the power of the lens of one meter focal length.

$$\text{Power in Dioptres} = \frac{1}{\textit{Focal length in meters}}$$

CONSTRUCTION OF RAY DIAGRAM

The following three classes of rays are used in geometrical constructions to locate the image formed by a converging lens.

1. Rays parallel to the principal axis will pass through the principal focus on the other side of the lens after refraction through the lens. As shown by Ray 1 in Fig. 7.21.

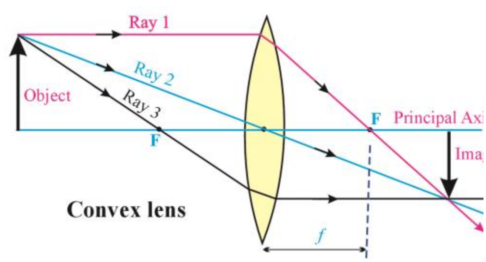

Fig 7.21

2. Rays passing through the principal focus will become parallel to the principal axis after refraction through the convex lens. As shown by Ray 3 in Fig. 7.21.
3. Rays passing through the optical center of the convex lens will pass through the lens without deviation. As shown by Ray 2 in Fig. 7.21.

The following three classes of rays are used in geometrical constructions to locate the image formed by a diverging lens.

1. Rays parallel to the principal axis after refraction through the lens will appear to be diverging from the principal focus. As shown by Ray 1 in Fig. 7.22.
2. Rays going towards the principal focus on the other side of the lens will become parallel to the principal axis after passing through the concave lens. As shown by Ray 2 in Fig. 7.22.
3. Rays through the optical center will pass through the lens without deviation. As shown by Ray 3 in Fig. 7.22.

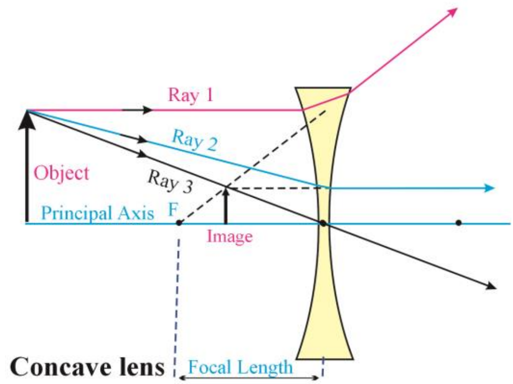

Fig 7.22

IMAGE FORMED BY A CONVEX LENS

The nature and position of the image formed by a convex lens depends upon the position of the object from the lens. The nature of the image formed in different cases is as follows.

WHEN THE OBJECT IS AT AN INFINITE DISTANCE FROM THE LENS

When the object is at an infinite distance from the lens the rays coming from the object will be parallel to oneanother and after refraction from the lens they will form an image as show in the diagram. The nature of the image will be as follows.

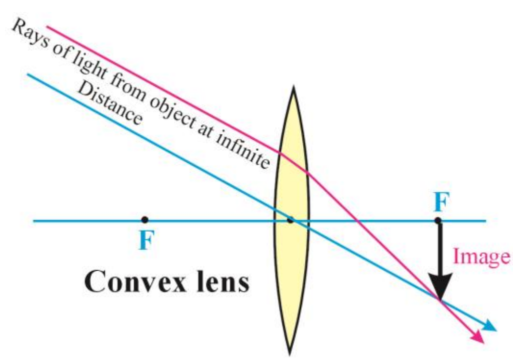

Fig 7.23

- The image is real
- The image is inverted
- The image formed is at ‘F’
- The image is smaller in size than the object.

WHEN THE OBJECT IS IN BEYOND THE CENTER OF CURVATURE

When the object is beyond the center of curvature of the convex lens the rays coming from the object will intersect each other after refraction from the lens and form an image as show in the diagram. The nature of the image will be as follows.

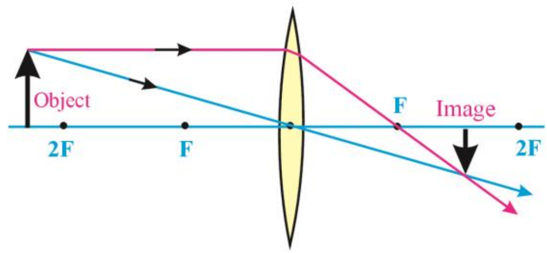

Fig 7.24

- The image is real
- The image is inverted
- The image formed is in between 'F' and 'C'.
- The image is smaller in size than the object.

WHEN THE OBJECT IS AT 2F OF A CONVEX LENS

When the object is at the center of curvature or '2F' of the convex lens the rays coming from the object will intersect each other after refraction from the lens and form an image as show in the diagram.

The nature of the image will be as follows.

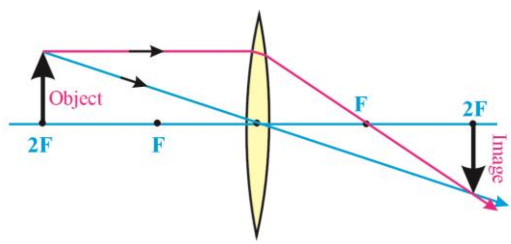

Fig 7.25

- The image is real
- The image is inverted
- The image formed is at '2F' or 'C' on the other side.
- The image is same in size as that of the object

WHEN THE OBJECT IS IN BETWEEN 'F' AND '2F'

When the object is in between 'F' and '2F' of the convex lens the rays coming from the object will intersect each other after refraction from the lens and form an image on the other side of the lens as show in Fig 7.26.

The nature of the image will be as follows

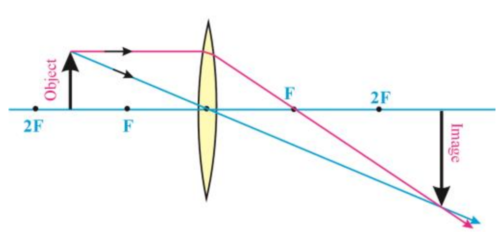

Fig 7.26

- The image is real
- The image is inverted
- The image formed is beyond '2F' on the other side of the lens
- The image is larger in size than the object.

WHEN THE OBJECT IS AT 'F'

When the object is at 'F' of the convex lens the rays coming from the object will become parallel to each other after refraction from the lens and will intersect each other at infinity, where the image will form as show in the diagram.

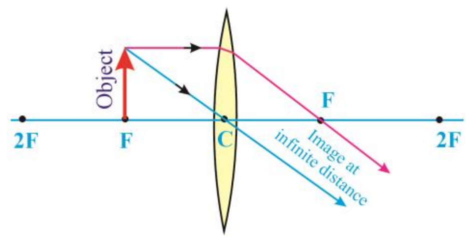

Fig 7.27

The nature of the image will be as follows.

- The image will be real
- The image will be inverted
- The image formed will be at an infinite distance on the other side of the lens
- The image is larger in size than the object.

WHEN THE OBJECT IS IN BETWEEN 'F' AND OPTICAL CENTER

When the object is in between 'F' and optical center of the convex lens the rays coming from the object will diverge after refraction from the lens and will not intersect each other. They will appear to diverge from a point on the same side as that of the object, where the image will be formed as show in the diagram.

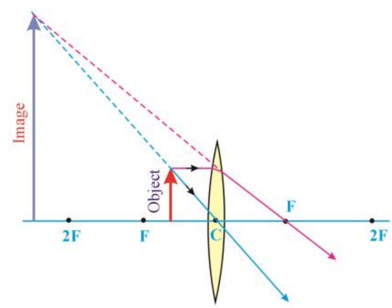

Fig 7.28

The nature of the image will be as follows.

- The image is virtual.
- The image is erect
- The image formed will be on the same side of the lens as that of the object and is behind the object.
- The image is larger in size than the object.

IMAGE FORMED BY A CONCAVE LENS

The nature of the image formed by a concave lens does not depend upon the distance of the object from the lens as it is in the case of a convex lens. The nature of the image formed by a convex lens is always as follows.

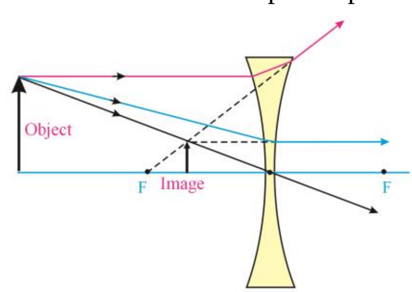

Fig 7.29

- It is always virtual.
- It is always erect
- It is always smaller in size than the object
- It always formed in between theprincipal focus andthe optical center of the lens on

the same sides as that of the object.

SIGN CONVENTIONS

An image is said to be real if the rays of light actually pass through it and a real image can be thrown on a screen. An image is said to be virtual if the rays of light do not pass through it, but only appear to diverge from it. Therefore the focal length of the convex lens is real and that of a concave lens is virtual. According to sign conventions the distances of real object and images are taken positive and the distances of the virtual images and objects are taken negative. The distances are measured along the principal axis from the optical center of the lens.

LENS FORMULA

The distance of the object from the lens is denoted by ‘p’, the distance of the image formed is denoted by ‘q’ and the focal length of the lens is denoted by ‘f’. The relationship between ‘p’, ’q’ and ‘f’ is called lens formula. Lens formula is

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

DERIVATION OF LENS FORMULA FOR A CONVEX LENS

Consider an object OB is placed in front of a convex lens. IM is its real image. ‘*p*’ is the distance of the object from the lens and ‘*q*’ is the distance of the image from the lens. ‘*f*’ is the focal length of the lens.

From the geometry of the figure (7.30) Triangles OBC and IMC are similar

There fore

$$\frac{OB}{IM} = \frac{OC}{IC} \quad ---- \quad (1)$$

Similarly triangles CEF and IMF are also similar.

There fore

$$\frac{CE}{IM} = \frac{FC}{IF}$$

Because CE = OB , therefore

$$\frac{OB}{IM} = \frac{FC}{IF}$$

Because IF = CI - CF,

Therefore

$$\frac{OB}{IM} = \frac{FC}{CI-CF} \quad ---- \quad (2)$$

$$\frac{OB}{IM} = \frac{f}{q-f}$$

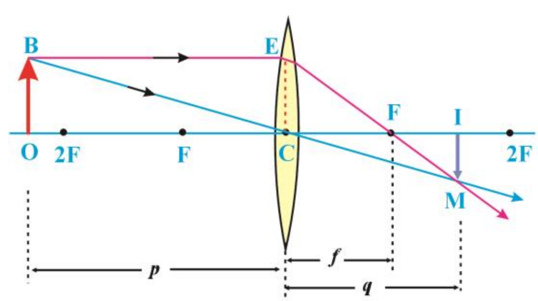

Fig 7.30

Comparing equation (1) and (2)

$$\frac{OC}{IC} = \frac{FC}{CI-CF}$$

By putting values, we get

$$\frac{p}{q} = \frac{f}{q-f}$$

By cross multiplication

$$pq - pf = qf$$

Dividing by "pqf"

$$\frac{1}{f} - \frac{1}{q} = \frac{1}{p}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \quad ------ \quad (3)$$

Equation (3) is the lens formula for a convex lens.

DERIVATION OF LENS FORMULA FOR A CONCAVE LENS

Consider an object OB placed in front of a concave lens. IM is the image of this object formed by the lens. The distance of the object from the lens is 'p' and the distance of the image from the lens is 'q'. The focal length of the lens is 'f'.

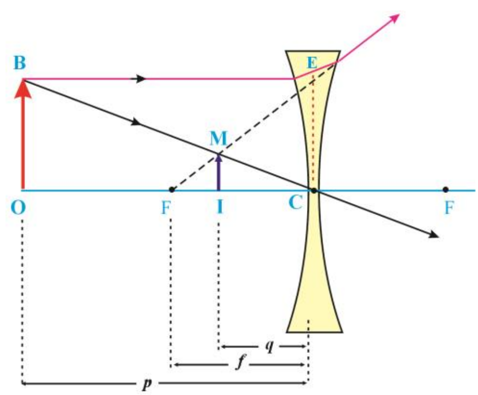

Fig 7.31

From the geometry of the figure (7.31), triangles OBC and IMC are similar

There fore

$$\frac{OB}{IM} = \frac{OC}{IC} \quad --- \quad (1)$$

Similarly triangles CEF and IMF are also similar.

There fore

$$\frac{CE}{IM} = \frac{FC}{IF}$$

Because CE = OB, therefore

$$\frac{OB}{IM} = \frac{FC}{IF}$$

Because IF = CF – CI, therefore

$$\frac{OB}{IM} = \frac{FC}{CF-CI} \quad ---- \quad (2)$$

Comparing equation (1) and (2)

$$\frac{OC}{IC} = \frac{FC}{CF-CI}$$

Putting values in above equation, we get

$$\frac{p}{-q} = \frac{-f}{-f-(-q)}$$

(As the lens is concave, so its focal length is taken as negative. Image is imaginary so its distance is also taken as negative)

By cross multiplication

$$-pf + pq = qf$$

Dividing by "pqf"

$$-\frac{1}{q} + \frac{1}{f} = \frac{1}{p}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad ----(3)$$

So, equation (3) is used as lens equation for a concave lens.

MAGNIFICATION

It is defined as the ratio of the linear dimensions of the image to those of the object.

Or

The ratio between the size of the image and the object is called magnification.

It is given as

$$\text{Magnification} = \frac{\text{Size of the Image}}{\text{Size of the Object}}$$

$$M = \frac{h_i}{h_o}$$

It is found that the ratio between the size of the image and the object is same as the ratio between their respective distances from the lens, there fore

$$M = \frac{h_i}{h_o} = \frac{q}{p}$$

So, $$\frac{\textit{Size of the image}}{\textit{Size of the object}} = \frac{\textit{Distance of image from lens}}{\textit{Distance of object from lens}}$$

EXAMPLE : An object of 12 cm height is placed at 24 cm from a convex lens of focal length 12 cm. Find the nature of the image.

SOLUTION:

$p = 24$ cm, $f = 12$ cm, $q = ?$

$h_o = 12$ cm, $h_i = ?$

Lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{12} = \frac{1}{24} + \frac{1}{q}$$

$$\Rightarrow \quad \frac{1}{q} = \frac{1}{12} - \frac{1}{24}$$

$$\frac{1}{q} = \frac{2-1}{24} = \frac{1}{24}$$

$$\Rightarrow \quad q = 24 \text{ cm}$$

Now $\frac{h_i}{h_o} = \frac{q}{p}$

$$\frac{h_i}{12} = \frac{24}{24}$$

$$\Rightarrow \quad h_i = 12\, cm$$

EXAMPLE: An object of 2 cm height is placed at a distance of 20 cm from a concave lens. Its image is formed at 4 cm from the lens. Find the focal length of the lens and the height of the image.

SOLUTION:

p = 20 cm, q = – 4 cm, f = ? ,
h_o = 2 cm, h_i = ?
Lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{20} - \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1-5}{20} = \frac{-4}{20}$$

$$\Rightarrow \quad f = -\frac{20}{4} = -5cm$$

Now $\frac{h_i}{h_o} = \frac{q}{p}$

$$\frac{h_i}{2} = \frac{4}{20}$$

$$\Rightarrow \quad h_i = \frac{4}{20} \times 2$$

$$\Rightarrow \quad h_i = 0.4\, cm$$

OPTICAL INSTRUMENTS

In order to see our surrounding objects more clearly, we need the help of some devices, called optical instruments. The device to see very small objects is called microscope. Distant objects are seen through a telescope. There are many other such instruments used in our daily life.

In optical instruments suitable arrangements of lenses, mirrors and prisms are used. Our eye is a natural optical instrument which functions like a convex lens.

Here we discus some of the optical instruments.

MAGNIFYING GLASS OR SIMPLE MICROSCOPE

A convex lens of short focal length is called a simple microscope.

If an object is placed between the lens and its principal focus, an erect, virtual and magnified image of the object is formed as shown in Fig 7.32. A convex lens is thus also called a magnifying glass.

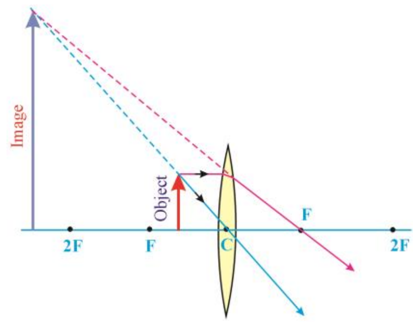

Fig 7.32

Visual Angle:

The apparent size of an object depends upon the angle subtended by it at the eye. This angle is called the visual angle. The greater is the visual angle, the greater is the apparent size of the object.

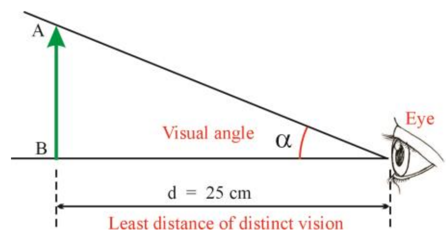

Fig 7.33

Least distance of distinct vision:

If we wish to see the fine details of a small object, we bring it as close to the eye as possible, thus increasing the visual angle and getting a large and real image on the retina of the eye. But a normal person cannot see clearly an object if it is closer than the least distance of distinct vision, which is 25 cm as shown in Fig 7.33.

A convex lens helps us to see the details of an object by bringing it closer than 25 cm; such a convex lens is known as a magnifying glass. As it is shown in the Fig 7.34, the object OB which when viewed by an unaided eye cannot be seen distinctly. A convex lens is then interposed between the eye and the object so that the image is set at least distance of distinct vision from the eye and the image becomes most distinct. The rays of light forming the virtual and magnified image IM of the small object AB as seen through the lens by placing the eye very close to it are shown in Fig 7.34.

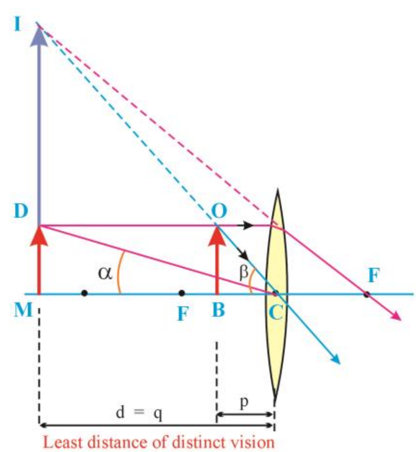

Fig 7.34

Magnifying Power:

The magnifying power of a magnifying glass is defined as **the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye when both the image and the object are situated at the distance of distinct vision.**

Thus if β is the angle which the image IM subtends at the eye and α is the angle which the object OB (which is equal to DM) subtends at the eye when both are placed at the least distance of distinct vision 'd' then,

$$\text{Magnifying Power} = M = \frac{\beta}{\alpha}$$

Since angles are small $\alpha = \tan\alpha$

and $\beta = \tan\beta$

Therefore

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{OB}{CB}\Big/\frac{DM}{CM} = \frac{OB \times CM}{DM \times CB}$$

$$M = \frac{CM}{CB} \qquad (\text{Because } OB = DM)$$

$$\Rightarrow \quad M = \frac{d}{p} \qquad \text{------ (1)}$$

Since the image is virtual and at the least distance of distinct vision from the lens, therefore

$$q = -d$$

Putting values in lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{-d}$$

Multiplying both sides by 'd'

$$\frac{d}{f} = \frac{d}{p} + \frac{d}{-d}$$

$$\Rightarrow \quad \frac{d}{f} = \frac{d}{p} - 1$$

$$\Rightarrow \quad \frac{d}{p} = \frac{d}{f} + 1 \qquad \text{----- (2)}$$

From equation (1) & (2)

$$M = 1 + \frac{d}{f} \qquad \text{----- (3)}$$

Equation (3) shows that the magnification of a simple microscope depends upon the focal length of the convex lens. The lens of smaller focal length will have greater magnification.

COMPOUND MICROSCOPE

The magnification produced by a simple microscope is generally not sufficient. Therefore a combination of two convex lenses of different focal lengths is employed to have a larger magnification. The instrument so designed is known as a compound microscope.

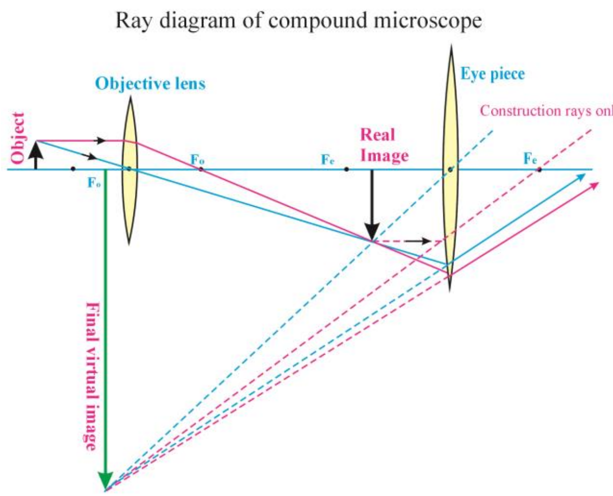

Fig 7.35

The lenses are arranged as shown in the Fig 7.35. The lens in front of which object is placed is called objective lens. The lens through which the image is viewed by the eye is called eye piece. The focal length of the objective lens is shorter than as compared to the eye piece.

The two lenses are placed in two metal tubes so as to have common principal axis. The eye piece fitted in a draw tube can slide within the main tube.

The object is placed at a distance slightly greater than the focal length of the objective lens at its principal axis. A real, inverted and magnified image is formed by the objective lens within the focal length of the eye piece. This intermediate image serves as an object for the eye piece. Eye piece acts as a magnifying glass and the final, virtual, inverted and magnified image is formed as shown in the diagram.

The magnification of a compound microscope is equal to the product of the magnifications of both the lenses.

$$M = M_1 \times M_2 \qquad \text{- - - - - (1)}$$

Now

$$M_1 = \frac{\text{Distance of the image}}{\text{Distance of the object}} = \frac{q}{p}$$

As in this case the '*p*' is nearly equal to the focal length of the objective i.e. 'f_o' and '*q*' is nearly equal to the length of the tube of the microscope, there fore

$$M_1 = \frac{L}{f_o} \qquad \text{- - - -} \quad (2)$$

and

$$M_2 = 1 + \frac{d}{f_e} \qquad \text{- - - -} \quad (3)$$

Where ‘d’ is the least distance of distinct vision and ‘f_e’ is the focal length of the eye piece.

From equations 1, 2 and 3

$$\text{Magnification of microscope} = M = \frac{L}{f_o}\left(1 + \frac{d}{f_e}\right)$$

TELESCOPES

When an object is at a great distance, it subtends a small angle at the eye**. The function of a telescope is to increase the angle which a distant object appears to subtend at the eye, and therefore produces the same effect as if the object were either larger or else closer to the eye.**

The telescopes are of two types.

1. REFRACTING TELESCOPES

In refracting telescopes, a real image of a distant object is formed at the focus of the objective or object glass. This image is them seen through a magnifying glass, called the eye piece or eye lens. Following are the important kinds of refracting telescopes.

- The Astronomical Telescope
  This is a kind of telescope which is used to see the astronomical objects. The final image formed by it is imaginary and inverted.
- The Terrestrial Telescope
  This is a kind of telescope which is used to see distant objects at the earth. Final image formed by this telescope is imaginary and erect. To get the erect image an additional convex lens called field lens or erecting lens is used.
- The Galilean Telescope
  This telescope makes an erect and imaginary image. Objective lens of this telescope is convex but eye piece is concave.

2. REFLECTING TELESCOPES

In a reflecting telescope, concave mirrors of very large diameter are used to collect more light coming from the distant object. A real image of a distant object is formed at the focus of a concave mirror. This image is then seen through an eye-piece, as in the case of refracting telescope.

CAMERA

It is an optical instrument which is used to form a real and diminished image of an object on a screen.

It consists of a lens or lens system placed in front of a dark box. The film is at the screen at opposite side of the lens in the dark box. The distance between the lens and the screen can be changed to get a sharp and well-defined image of the object on the screen. The screen is a sensitive plate or film which is exposed to light only for a short time. The light is controlled by a shutter which is a screen having a circular aperture at its center. The diameter of this aperture can be varied to control the light. The time of exposure of light is usually a small fraction of a second.

The exposed film is carefully taken to a dark room where it is washed in a solution called the developer. Then it is washed in another solution called the fixer. In this way we get what is called the negative as bright parts of the object appear dark in the image formed on the plate and dark parts of the object appear bright. Another sensitive plate is placed below the negative in a dark room and is exposed to light for a suitable time. The new exposed plate is also treated with the developer and fixer. In this way we get the print. What is dark in the negative becomes bright and bright becomes dark in the positive or print. In this way the original colours of the object or landscape are exposed on the print or positive.

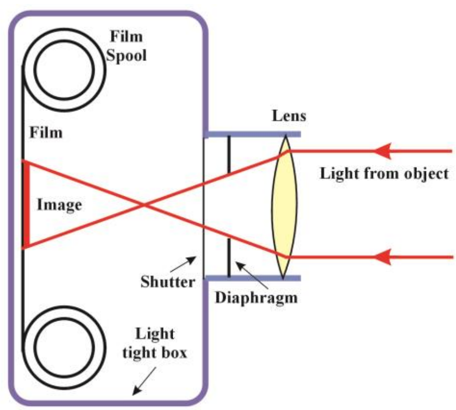

Fig 7.36

SEXTANT

A sextant is a device for measuring the angle between two objects with great precision. Most commonly it is associated with navigation at sea. A sextant can also be used to help calculate the height of trees, buildings, flag poles or any other vertical object.

It uses two mirrors and a lever which arcs across the instrument at slightly over 60 degrees, hence the name sextant.

Important parts of a sextant are shown in Fig. 7.37.

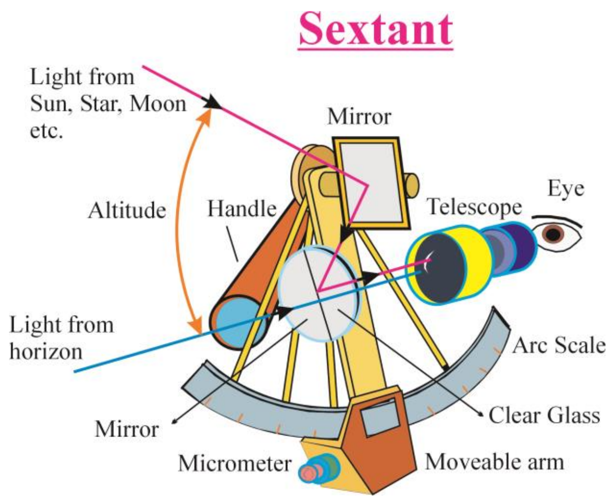

Fig 7.37

The working principle of a sextant is that a given ray of light is reflected from two mirrors in succession, then the angle between the first and last direction of the ray is twice the angle between the mirrors. This angle is read off on the arc of the sextant whileshooting an object (taking an altitude).

Working steps are as follows:

1. Choose an observation point from which you can clearly see both the top and the bottom of the object you wish to measure. Determine the exact distance between the observation point and the base of the object. Let it is "D".
2. Set the sextant to zero and look at the object through the eyepiece, adjusting your view until it is in the center of the frame.
3. Adjust the sextant arm to split the screen in two halves. Continue moving the arm until the top half of the object on one side of the image is aligned with the bottom half of the object on the other side of the image.
4. Read the angle from the arc of the sextant. Let this angle is "θ".
5. Use a following formula to calculate the height of the object.

$$\textit{Height of the object} = \textit{Distance of object} \times \tan\theta$$

$$h_o = D\tan\theta \quad ------ \quad (1)$$

NATURE OF LIGHT

Light is radiant energy, usually referring to electromagnetic radiation. Human eye is sensitive to this radiation; hence this is responsible for the sense of sight. Visible light is usually defined as having a wavelength in the range of 400 nm to 700 nm. Its speed in vacuum is 3×10^8 m/s approximately.

1. Corpuscular theory of light

Isaac Newton stated in his Hypothesis of Light that light was composed of corpuscles (particles of matter) which were emitted in all directions from a source. Newton's theory could be used to predict the reflection and refraction of light. It also explains the rectilinear motion of light. This theory could not explain interference, diffraction and polarization of light.

2. Wave theory of light

Huygens proposed that light travels in the form of waves in a medium. The wave theory predicted that light waves could interfere with each other like sound waves. The phenomenon of reflection and refraction can also be explained on the wave theory of light.

Huygens presented a principle about the motion of light waves. According to him, when a wave passes through a medium, the particles of the medium begin to perform the simple harmonic motion. The locus of all the points in the medium with the same phase is called a wave front.

In case of a point source of light the wave fronts will be concentric spheres with center at the source 'S' such a wave front is known as *spherical wave front.*

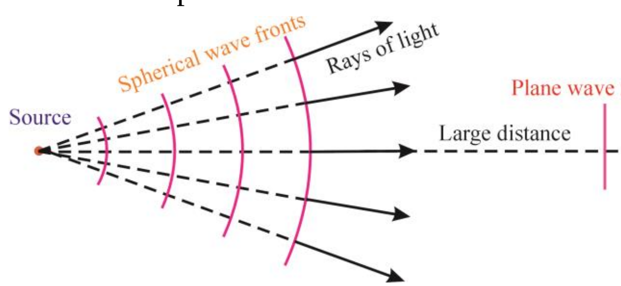

Fig 7.38

At a very large distance from the source a small portion of a spherical wave front will become very nearly plane, as shown in Fig. 7.38. This type of wave front is called as plane wave front.

The direction in which wave moves, is always normal to the wave front. Thus a ray of light means the direction in which a light wave propagates and it is always along the normal to the wave front.

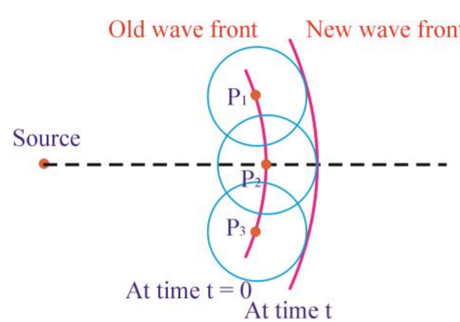

Fig 7.39

HUYGEN'S PRINCIPLE

According to Huygens principle

1. **Every point on a wave front can be considered as a source of secondary spherical wave fronts.**
2. **The new position of the wave front after time 't' can be found by drawing a plane tangential to the secondary wave lets.**

As shown in the figure at the old wave front points P_1, P_2, P_3, P_4 and P_5 acts as secondary sources and after time 't' the position of the new wave front is shown in Fig 7.39.

Thomas Young showed by means of a diffraction experiment that light behaved as waves. He also proposed that different colours were caused by different wavelengths of light. By the year 1821, Fresnel was able to show that polarization could be explained only by the wave theory of light and only if light was entirely transverse.

3. Maxwell's Electromagnetic Theory

Faraday proposed that light was a high-frequency electromagnetic vibration, which could propagate even in the absence of a medium such as the ether.

Maxwell discovered that self-propagating electromagnetic waves would travel through space at a constant speed, which happened to be equal to the previously measured speed of light. From this, Maxwell concluded that light was a form of electromagnetic radiation. Soon after, Hertz confirmed Maxwell's theory experimentally by generating and detecting radio waves in the laboratory, and demonstrating that these waves behaved exactly like visible light.

4. Quantum theory

In 1900 Max Planck, attempting to explain black body radiation suggested that although light was a wave, these waves could gain or lose energy only in finite amounts related to their frequency. Planck called these packets of light energy "quanta".

In 1905, Albert Einstein used the idea of light quanta to explain the photoelectric effect, and suggested that these light quanta had a "real" existence. These packets of energy later on named as photons.

Eventually according to the modern theory of quantum mechanics light is considered to have dual nature i.e. both a particle and a wave.

INTERFERENCE OF LIGHT WAVES:

Light waves are transverse waves. Like sound waves and waves produced in water, light waves also show the property of interference. The combination or interference of two trains of similar light waves produces dark and bright bands called interference bands or fringes. When light waves of the same amplitude are held close to each other, then at some places the two waves reinforce and at some other points they cancel each other's effect.

Definition:

When two light waves from different coherent sources meet together, then the distribution of energy due to one wave is disturbed by the other.

This modification in the distribution of light energy due to super-position of two light waves is called "Interference of light".

CONDITIONS FOR INTERFERENCE

In our daily life although we observe that light from different light sources reach at a point simultaneously but we do not observe interference of light. The reason is that for interference there are essential conditions without which the phenomenon of interference cannot take place. These conditions are as follows.

1. **Phase coherence**
   The sources must be phase coherent, that is, they must maintain a constant phase with respect to each other. Two sources of waves are said to be coherent if there is a fixed phase relationship between the waves they emit.
2. **Monochromatic sources**
   The sources of light must be monochromatic i.e. they must be emitting light of same frequency and wavelength.
3. **Principle of superimposition**
   The principle of superimposition must apply. The light of two sources must superimpose.
4. **Distance between the sources**
   The two sources of light should be very close to each other.

The light from a source comes from a large number of individual atoms, each of which sends out bursts of light only during a short interval at instants which may be different for different atoms. When we turn on the light source, we start the overall process of the emission of light but we do not have any control on the instants at which the atoms emit light, that is, the various atoms emit light just at random. Thus there is no question of having phase coherence between two separate light sources. Hence, we can expect to get an interference pattern of light only when the two interfering sources of light have phase coherence.

Those sources of light which emit light waves continuously of same wavelength, and time period, frequency and amplitude and have zero phase difference or constant phase difference are coherent sources.

One method for producing two coherent light sources is to use a single source of light and divide its light into two parts by small openings i.e. slits.

CONSTRUCTIVE INTERFERENCE

When two waves superimpose or combine so as to produce a resultant wave of amplitude greater than that of either of the individual waves the interference is called constructive one.

- In constructive interference, two waves of light reinforce each other.

- In constructive interference, a bright fringe is obtained on the screen.

Condition for constructive interference

For constructive interference, path difference 'S' between two waves is mλ.

$$\text{Path difference} = m\lambda$$

$$\text{where} \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

DESTRUCTIVE INTERFERENCE

When two waves combine so as to produce a resultant wave of amplitude less than the amplitude of the individual waves, the interference s called destructive one.

- In destructive interference, two waves cancel the effects of each other.
- Due to destructive interference a dark fringe is obtained on the screen.

Condition for destructive interference

For destructive interference, path difference 'S' between two waves is:

$$\text{Path difference} = S = \left(m + \frac{1}{2}\right)\lambda$$

$$\text{where} \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

DIFFRACTION OF LIGHT

The phenomenon of bending of incident light towards the geometrical shadow of a sharp wedge is called diffraction of light.

Or

The bending and spreading of light waves around sharp edges or corner or through small openings is called diffraction of Light.

Condition for diffraction

Diffraction effect depends upon the size of obstacle. Diffraction of light takes place if the size of obstacle is comparable to the wavelength of light.

Light waves are very small in wavelength, i.e. from 4×10^{-7} m to 7×10^{-7} m. If the size of opening or obstacle is near to this limit, only then we can observe the phenomenon of diffraction.

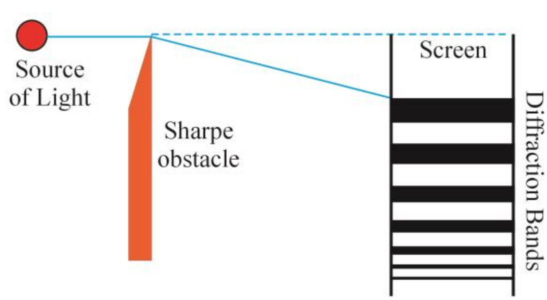

Fig 7.40

Ordinary obstacles, such as tables, chairs, doors, walls etc. are too big for the light waves. That is why the shadows of these objects have sharp edges and light seems to travel in straight lines. But, when the obstacle is of small width, the shadow does not show sharp edges and light waves do bend round its edges.

The space outside the geometrical shadow is seen bordered by alternate bright and dark fringes of rapidly falling intensity and decreasing width.

The phenomenon of diffraction can be explained on the basis of Huygens principle, which tells us that for any kind of wave motion we may regard each point on a wave front as a new source. Diffraction is a special type of interference.

POLARIZATION OF LIGHT

A wave may be transverse or longitudinal. Polarization is a phenomenon associated only with transverse waves.

Light waves are electromagnetic waves that is produced by vibrating electric charges. Like all electromagnetic waves light has both an electric and a magnetic component.

The transverse nature of light waves is quite different from any other type of wave as the vibrations of the electric and magnetic fields occur in more than one plane of vibration.

A light wave that is vibrating in more than one plane is referred to as un-polarized light. Light emitted by the sun, by a lamp, or by a candle flame is un-polarized light.

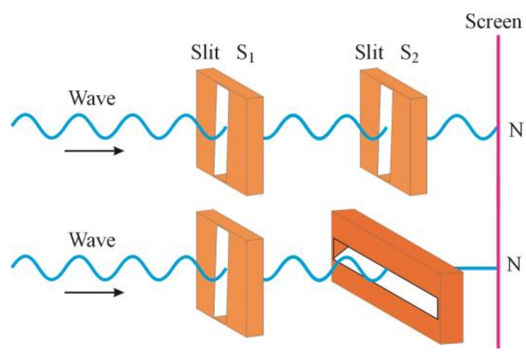

Fig 7.41

Polarized light waves are light waves in which the vibrations occur in a single plane. The process of transforming un-polarized light into polarized light is known as polarization.

There are a variety of methods to polarize light. For example:

- Polarization by Transmission
- Polarization by Reflection
- Polarization by Refraction
- Polarization by Scattering

Suppose we produce transverse waves in a long stretched string passing through two slits S_1 and S_2 in two different cards. The transverse waves produced in the string may have any direction perpendicular to the string. But the slit S_1 will transmit only those vibrations which are parallel to its axis. If S_2 is placed parallel to S_1 then it will also transmit waves. Now rotate slit S_2 to

make it perpendicular to the slit S_1, we shall get no vibrations at point 'N'. The transverse waves transmitted by S_1 are said to be polarized.
Light waves are also transverse waves because they also show the property of polarization. When a ray of light falls on two tourmaline crystals placed with their crystallographic axes parallel, the beam transmitted. If, however, one of the crystals is rotated with respect to the other, the emergent beam becomes dimmer and dimmer and ultimately the light is totally cut of when the axes of the two crystals become perpendicular to each other as shown in Fig 7.42. On further rotation, the light appears and becomes brightest when the axes are again parallel.

Light consists of transverse wave in which the vibrations take place in all directions in a plane perpendicular to the direction in which the light is travelling. When such a beam passes through a crystal, one component of the vibrations is absorbed, and the other component is transmitted. Consequently the emerging beam differs from incident light in the sense that all the vibration are in one direction. Such a beam of light is said to be plane polarized.

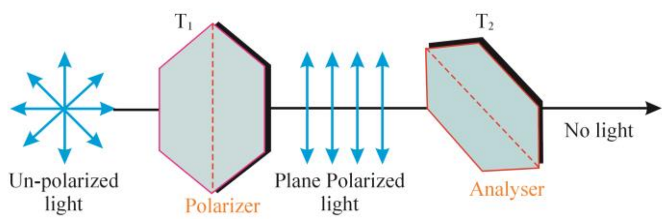

Fig 7.42

According to electromagnetic theory a light wave consists of a periodic variation of electric field vector accompanied of a magnetic field vector at right angle to it. Tourmaline and many other materials have their internal molecular structure such that by their interactions, the electric vibrations are confined in a particular plane and are executed parallel to only one direction. Therefore, when light passes through such a crystal, it becomes plane polarized. This interaction which makes light polarized may be one of reflection from a transparent surface, refraction through a crystal, selective absorption in certain crystals or scattering by small particles.

POLAROID

Polaroid was invented in 1929, and is used to produce plane polarized light. A Polaroid is a transparent plastic sheet in which special needle-like crystals of Herapathite (or iodoquinine sulfate is a chemical compound whose crystals are dichroic and thus can be used for polarizing light) have been embedded and orientated. This sheet allows light to pass through it only if the electric vector is vibrating in a specific direction.

APPLICATIONS OF POLARISATION

Polarization of light has many technical and scientific applications.

- One use of the polarized light is the determination of the concentration of optically active substance such as sugar.
- Polaroids are used to make curtain less windows. An outer polarizing disc is fixed in position and an inner one may be rotated to adjust the amount of light admitted.
- Control of headlight glare in night driving is possible if each car has light polarizing head lights and a light polarizing viewer. Polaroid glasses eliminate the glare of light because it is partly polarized by reflection from water and pavements.
- In photography it is often desirable to enhance the effect of sky and clouds. Since light from the blue sky is partially polarized by scattering, suitable orientated polarizing disc in front of the camera lens will serve as a sky filter.
- Polarization is used in glare-reducing sunglasses.
- Polaroid filters are used to perform stress analysis tests on transparent plastics.
- Polarization is also used in the entertainment industry to produce and show 3-D movies.
- Some optical measurement techniques are based on polarization.

SUMMARY

- The turning back of the light from the boundary of a medium is called reflection of light.
- The bending of light when it enters from one medium to the other is called refraction of light.
- For any two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for all angles of incidence. This law is known as Snell's Law.

  Mathematically $n = \frac{Sin\,\angle i}{Sin\,\angle r}$

  Here '*n*' is constant and is called *refractive index* of the second medium with respect to the first.
- A mirror is a plane, convex or concave surface which form image of objects placed in front of them by reflection of light.
- A lens is a piece of transparent material bounded by two surfaces. If rays of light parallel to the principal axis of the lens converge at a point after passing through a lens it is called converging or convex lens and if they

diverge after passing through the lens it is called diverging or concave lens.

- The equation $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ is called mirror equation for spherical mirrors and lens formula for lenses. Where 'f 'is the focal length of mirror or lens, 'p' is the distance of object from mirror or lens and 'q' is the distance of image from mirror or lens.
- A microscope is an optical instrument used to observe very small objects which cannot be seen clearly with naked eye and a telescope is an optical instrument used to observe distant objects clearly.
- A camera is used to take photo graphs. It consists of a light tight body having opening at one face to enter light in it through the lens and a light sensitive film at opposite face of the light tight box. Light entering it falls on the photo sensitive film which is then developed into a photo graph.
- A sextant is a device for measuring the angle between two objects with great precision. Most commonly it is associated with navigation at sea. A sextant can also be used to help calculate the height of trees, buildings, flag poles or any other vertical object.
- Corpuscular theory of light was propounded by Sir Isaac Newton and Wave theory of light was propounded by Christian Huygens. According to quantum theory of light it has dual nature. It behaves like particles as well as waves.
- Every point on a wave front can be considered as a source of secondary spherical wave fronts. This statement is called Huygens' Principle.
- When two light waves from different coherent sources meet together, then the distribution of energy due to one wave is disturbed by the other. This modification in the distribution of light energy due to super-position of two light waves is called "Interference of light".
- The bending and spreading of light waves around sharp edges or corner or through small openings is called diffraction of Light.
- Light consists of transverse wave in which the vibrations take place in all directions in a plane perpendicular to the direction in which the light is travelling. When such a beam passes through a crystal, one component of the vibrations is absorbed, and the other component is transmitted. Consequently the emerging beam differs from incident light in the sense that all the vibration are in one direction. Such a beam of light is said to be plane polarized.

EXERCISE

LONG QUESTIONS

1. Derive mirror equation for a concave mirror.
2. Derive a mirror equation for a convex mirror.
3. Derive lens formula for a concave lens.
4. Derive lens formula for a convex lens.
5. Explain the working of a simple microscope. Derive relation for its magnification.
6. Explain the construction and working of a compound microscope.
7. Define interference of light. Describe constructive and destructive interference.
8. Describe working of a camera.
9. Define polarization of light. Describe any three uses of polarization.

SHORT QUESTIONS

1- Describe laws of reflection?
2- Describe laws of refraction?
3- What are the characteristics of the image formed by a plane mirror?
4- Define Principal focus and focal length of a concave mirror.
5- Draw ray diagram to show the image of an object placed in between F and C of a concave mirror.
6- Differentiate between real and virtual images.
7- Define focal length of a convex lens.
8- Define power of the lens. Write its unit.
9- An object is in between F and 2F of a convex lens. Draw its ray diagram and describe the nature of the image.
10- Define least distance of distinct vision.
11- Define interference of light.
12- Describe constructive and destructive interference of light.
13- Write conditions for the interference of light.
14- Define diffraction of light.
15- Describe two applications of polarized light.
16- Describe use of sextant.
17- Describe wave nature of light.

MULTIPLE CHOICE QUESTIONS (MCQ's)

Encircle correct answers.

1- **The velocity of light is:**
a) 3×10^8 m/s b) 330 km/s c) 330 m/s d) 3×10^5 m/s

2- **Light waves are:**
a) Mechanical waves b) Longitudinal waves
c) Electromagnetic waves d) Matter waves

3- **Corpuscular theory of light was propounded by:**
a) Newton b) Huygens c) Einstein d) Maxwell

4- **Wave theory of light was proposed by:**
a) Newton b) Huygens c) Maxwell d) Einstein

5- **The turning back of a part of light from the boundary of a medium is called:**
a) Refraction b) Rarefaction c) Polarization d) Refraction

6- **The bending of light when it enters from one medium to the other is called:**
a) Reflection b) Refraction c) Dispersion d) Diffraction

7- **The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for all angles of incidence. This law is known as:**
a) Snell's law b) Hook's law c) Lenz's law d) Law of reflection

8- **The relationship between focal length '*f*' and radius of curvature 'r' of a spherical mirror is:**
a) $f = 2\,r$ b) $f = \frac{r}{2}$ c) $f = 2\,r^2$ d) $R = 2\,f^2$

9- **If an object is placed at C of the concave mirror, then the image formed will be:**
a) At F b) Behind the mirror c) At C d) In between C and F

10- **If an object is placed within the focal length of a concave mirror then the image formed will be:**
a) Real and erect b) Virtual and inverted
c) Real and behind the mirror d) Virtual and behind the mirror

11- **The radius of curvature of a concave mirror is 40 cm, its focal length will be equal to:**
a) 80 cm b) 20 cm c) 40 cm d) 4 cm

12- **The unit for the power of lens is:**
a) Watt b) cm c) meter d) dioptre

13- The focal length of a lens is 20 cm; its power in dioptre will be equal to:

a) 20 D b) 0.5 D c) 2 D d) 5 D

14- If an object is placed in front of a convex lens in between its F and 2F then the image will be:

a) Virtual b) Erect c) Real and inverted d) Real and erect

15- The least distance of distinct vision is:

a) 20 cm b) 25 cm c) 15 cm d) 5 cm

16- A convex lens of short focal length is used as:

a) Simple microscope b) Compound microscope
c) Simple telescope d) Sextant

17- Magnifying power 'M' of a simple microscope is equal to:

a) $1 + \frac{q}{f}$ b) $1 + \frac{d}{f}$ c) $1 - \frac{d}{f}$ d) $1 - \frac{q}{f}$

18- The phenomenon of bending of incident light towards the geometrical shadow of a sharp wedge is called:

a) Dispersion of light b) Interference of light
c) Polarization of light d) Diffraction of light

19- To measure the height of distant objects we use:

a) Camera b) Telescope c) Microscope d) Sextant

PROBLEMS

7.1 Light enters from air to another medium. Find the refractive index of the medium if speed of light in it is 1.75×10^8 m/s. Speed of light in air is 3×10^8 m/s. [Ans: 1.71]

7.2 Light enters from one medium to the second. If the angle of incidence is $29°$ and angle of refraction is $20°$, then find the refractive index of the second medium. [Ans: 1.417]

7.3 An object of 3 cm height is placed in front of a concave mirror of focal length 7 cm at a distance of 15 cm. Find the position and height of the image formed. [Ans: q = 13.125 cm, h_i = 5.625 cm]

7.4 An object is placed at a distance of 10 cm from a concave mirror and its real image is formed at a distance of 15 cm from the mirror, find the focal length of the mirror. [Ans: 6 cm]

7.5 How much apart an object be placed in front of a concave mirror of focal length 25 cm to get its real image of double size? [Ans: 37.5 cm]

7.6 Focal length of a concave mirror is 10 cm. A virtual image of an object is formed at a distance of 15 cm form the mirror. Find the distance of the object form the mirror. [Ans: 6]

7.7 Focal length of a convex lens is 20 cm. An object is placed at a distance of 25 cm from the lens. Find the position of the image. [Ans: 100 cm]

7.8 How much apart an object be placed in front of a convex lens of focal length 10 cm to get a virtual image of double size? [Ans: 5 cm]

7.9 Focal length of a concave lens is 10 cm. An object is placed at a distance of 20 cm from the lens. Find the distance of the image formed from the lens. [Ans: - 6.66 cm]

7.10 Focal length of a simple microscope is 5 cm, find its magnification. [Ans: 6]

7.11 Power of a convex lens is 2.5D. Find its focal length in centimeters. [Ans: 40 cm]

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Chapter 08

OPTICAL FIBER

Course contents:

8.1 Optical communication and problems
8.2 Review of total internal reflection of light and critical angle
8.3 Structure of optical fiber
8.4 Fiber material and manufacture
8.5 Uses of optical fiber

Learning Objectives:

At the end of this chapter the students will be able to:

- Explain the structure of the optical fiber.
- Explain principle of working of optical fiber
- Describe use of optical fiber in industry and medicine

TOTAL INTERNAL REFLECTION

When light rays enter from a denser medium into a rare medium, the light rays bend away from the normal.

Suppose light rays from glass are entering into air, as shown in the fig 8.1. In this case the angle of refraction θ_r is always greater than the angle of

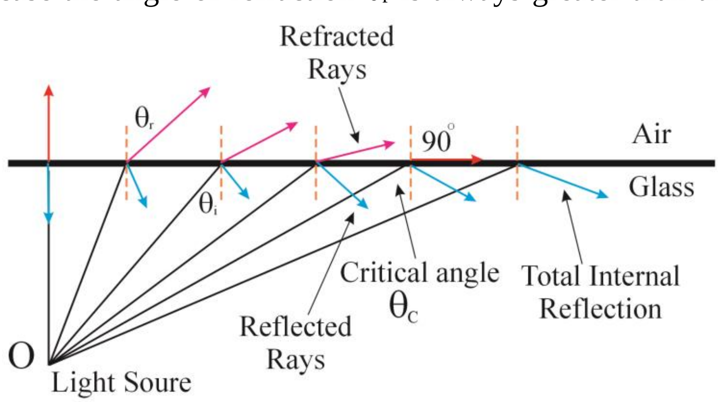


Fig 8.1

incidence θ_i. As the angle of incidence is increased, the corresponding angle of refraction is also increased till a stage is reached when the refracted ray makes an angle of refraction equal to 90°.

The angle of incidence for which the angle of refraction is 90° is called critical angle. Critical angle is denoted by θ_c.

If angle of incidence is greater than θ_c, the ray is totally reflected into the denser medium at the interface. **This phenomenon is called the *total internal reflection*.**

Total internal reflection takes place only when a beam of light travels from a denser medium to a rare medium and the angle of incidence is greater than the critical angle.

When angle of incidence is greater than the critical angle the light instead of refraction is refracted back into the same medium making an angle equal to the angle of incidence. Then it is called total internal reflection of light.

RELATION BETWEEN CRITICAL ANGLE AND REFRACTIVE INDEX

According to Snell's law

$$n_1 \, Sin \, \theta_i \quad = \quad n_2 \, Sin \, \theta_r$$

Where n_1 is the refractive index of the first medium and n_2 is the refractive index of the second medium.

When angle of incidence is equal to critical angle the angle of refraction will be equal to 90^o. The refractive index of air is equal to 1. Putting values in above equation we get:

$$n_1 \, Sin \, \theta_C \quad = \quad 1 \times Sin \, 90^o \; = \; 1$$

$$n_1 \quad = \quad \frac{1}{Sin \, \theta_C} \qquad \text{- - - -} \; (1)$$

Equation (1) shows that the refractive index of a medium is equal to the reciprocal of the sine of its critical angle.

Refractive Index:

The refractive index is a way of measuring the speed of light in a material. Light travels fastest in a vacuum. The speed of light in a vacuum is about 300,000 km/s. The refractive index of a medium is calculated by dividing the speed of light in a vacuum by the speed of light in that medium. The refractive index of vacuum is therefore 1, by definition.

Material	Refractive Index
Air	1.000293
Hydrogen	1.000132
Water	1.33
Ice	1.309
Crown glass	1.52
Flint glass	1.62
Plexiglas	1.49
Diamond	2.42

EXAMPLE 1: Find critical angle of glass,if its refractive index is 1.5.

SOLUTION:

Refractive index of glass = n = 1.5

Critical angle of glass = θ_c = ?

We know that the relationship between critical angle and refractive index is

$$n = \frac{1}{Sin\,\theta_C} \quad \Rightarrow \quad 1.5 = \frac{1}{Sin\,\theta_C}$$

$$\Rightarrow \quad \text{Sin}\,\theta_c = \frac{1}{1.5} = 0.666$$

$$\Rightarrow \quad \theta_c = Sin^{-1}(0.666)$$

$$\Rightarrow \quad \theta_c = 42°$$

OPTICAL FIBER

An optical fiber (or optical fibre) is a flexible, transparent fiber made of extruded glass (silica) or plastic, slightly thicker than a human hair. It can function as a waveguide, or "light pipe" to transmit light between the two ends of the fiber. The principle of its working is based on the total internal reflection of light as shown in the fig 8.2.

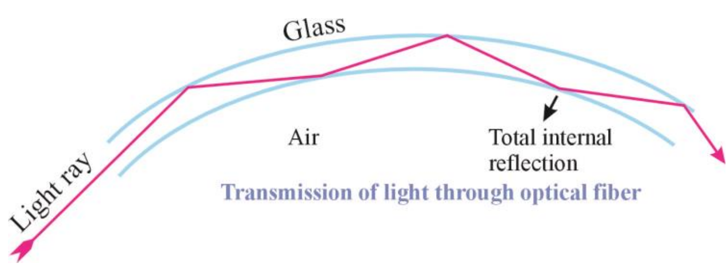
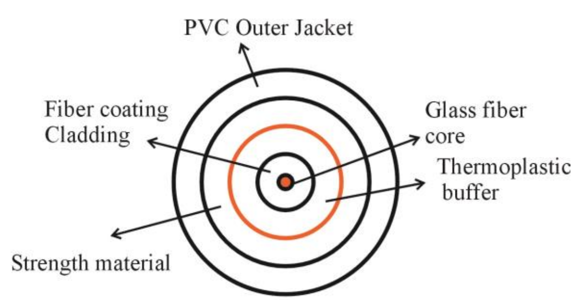


Fig 8.2

Applied science and engineering concerned with the design and application of optical fibers is known as fiber optics.

Optical fibers are widely used in fiber-optic communications where they permit transmission over longer distances and at higher bandwidths (data rates) than wire cables. Optical Fiber is rapidly replacing conventional method of communication by using metal wire.



Tight buffered cable

Fig 8.3

CONSTRUCTION OF OPTICAL FIBER

The internal part of the fiber is called core and it is made up of transparent material usually of silica. The signal is transmitted through this core. It is enveloped with an outer part having high refractive index called cladding. This fiber is covered by a plastic layer called buffer. Buffer is used for the purpose of providing such functions as mechanical isolation, protection from physical damage and fiber identification.

Then there is a layer of material to strengthen it also called Kevlar. Then PVC outer jacket covers it. A number of fibers are enveloped in a TVC Nylon to form a cable.

The construction of an optical fiber is shown in the fig 8.3. Kevlar is made of carbon and it is light weight, having ability to absorb light. The purpose of Kevlar is to strengthen the fiber to avoid its breakage when pulled through a tube. The outer diameter of the wire is normally 100 to 300 μm and the inner diameter is 50 to 100 μm. The diameter of the cable depends upon the number of optical fiber passing through it.

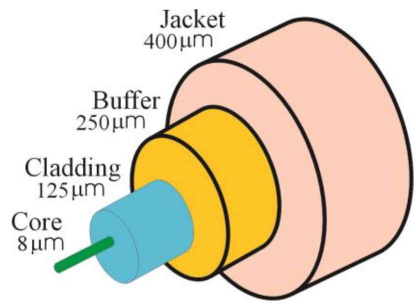


Fig 8.4

FIBER MATERIAL AND MANUFACTURE

The raw material for optical fiber is sand from which silica glass is made. To make the refractive index of the material varying Germanium, Phosphorus, Boron and Florence are added. From core to the outer surface, Germanium and Phosphorus increases the refractive index whereas Boron and Florence decreases the refractive index.

Standard optical fibers are made by first constructing a large-diameter "preform" with a carefully controlled refractive index profile, and then "pulling" the preform to form the long, thin optical fiber.

KINDS OF THE OPTICAL FIBER

Optical waves are propagated through the core of the optical fiber. The propagation of the optical signal through the fiber is due to the phenomenon of total internal reflection within the fiber. Many signals can be transmitted through the fiber at the same time. The number of signals which can be transmitted through the fiber depends upon the wave length of the wave and the difference of the refractive index between the core and the outer surface of the fiber.

Mono-mode Optical Fiber

If only one wave can be transmitted through a fiber then it is called *Mono mode fiber or Single mode fiber* or SMF. Mono mode optical fiber has very narrow core as compared to multi-mode optical fiber.

Multi-mode Optical Fiber

The fiber through which a number of signals can be transmitted simultaneously is called *Multi mode optical fiber* or MMF. Multimode fiber is of two types; Step index fiber and Graded index fiber.

Step Index Fiber

In step index fiber the refractive index of the fiber is constant. So the total internal reflection takes place at the edge of the core. It is also used for the transmission of the single signal.

Graded Index Fiber

In a graded index fiber the refractive index is not constant. It decreases gradually from core to the outer surface. Therefore the signals instead of reflecting from the outer surface of the fiber refract inside the fiber as shown in the fig 8.5.

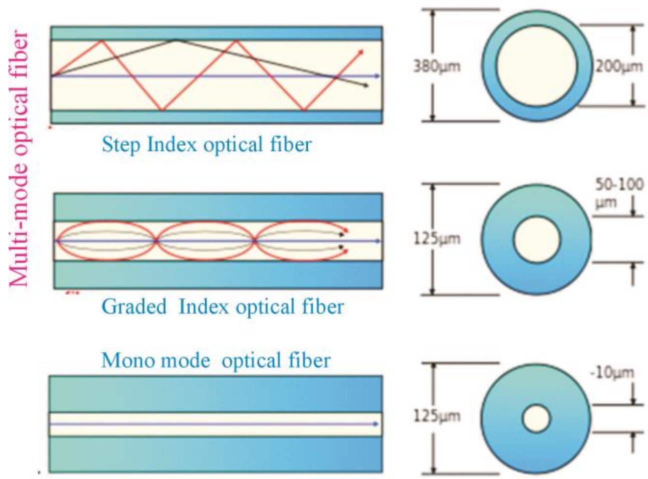


Fig 8.5

ADVANTAGES OF OPTICAL FIBER

The advantages and disadvantages of an optical fiber can be estimated by its comparison with the metal wire used for the same purpose. The advantages of optical fiber are as follows.

1. In a metal wire electrical signal is transmitted where as in optical fiber the light signal is transmitted. In metal wire the losses of energy is greater than those in the optical fiber. Attenuation in modern optical cables is far less than in electrical copper cables.
2. Optical fiber can transmit wide frequency range than the metal wire.
3. Communication through metal wire is effected by electric and magnetic fields where as in optical fiber the communication is not affected by these fields.
4. Metal wires due to greater density of copper as compared to silica are heavier than the optical fiber. Optical fiber is light weight.
5. Raw material of optical fiber is abundant and easily available.
6. On optical fiber the effect of temperature is comparatively less.

7. Over hearing in metal wires is possible where as it is not possible in case of optical fiber.
8. In metal wire cross talk is possible if they are in touch with each other whereas there is no possibility of over talk in optical fiber, as the signal does not leave the fiber.

DISADVANTAGES OF OPTICAL FIBER

Following are the disadvantages of the optical fiber as compared to the metal wire.

1. Though the raw material of an optical fiber is abundant, but the manufacturing cost of the optical fiber is very high.
2. Strength of the metal wire is much greater than the optical fiber, which is easily breakable. There are many problems of transportation.
3. Remote power feeding is not possible in case of optical fiber.
4. Its splicing (joining) is very difficult.

USES OF OPTICAL FIBER

- Optical fibers are widely used in fiber –optic communications, where they permit transmission over longer distances and at higher bandwidths (data rates) than wire cables.
- Fibers are used instead of metal wires because signals travel along them with less loss and are also immune to electromagnetic interference.
- Fibers are also used for illumination, and are wrapped in bundles so that they may be used to carry images, thus allowing viewing in confined spaces.
- Optical fiber is replacing metal wire in telecommunication. It is now a day's used in telephone cables, T.V channels, video phone, fax and telex etc.
- Optical fiber is used in surgery to transmit laser into the internal parts of the body. For example to crush stone inside the kidney optical fiber is used. A coherent bundle of fibers is used, sometimes along with lenses, for a long, thin imaging device called an endoscope, which is used to view objects through a small hole.
- Optical fiber is used in industry to transmit laser into the internal parts of the machinery to find faults.
- They are used as light guides in medical and other applications where bright light needs to be shone on a target without a clear line-of-sight path. In some buildings, optical fibers route sunlight from the roof to other parts of the building.

SUMMARY

- Optical fiber is made up of transparent material usually of glass. It is thin and flexible and is used as light guide for its transmission.
- The basic principle used for the transmission of light through optical fiber is total internal reflection of light.
- It has an inner part of transparent material called core and an outer part of transparent medium of higher refractive index than the core called cladding.
- Optical fiber is now a days replacing conventional metal wire in the field of telecommunication and information technology as it has number of advantages over the metal wire.
- Mono-mode and multimode optical fiber are the main kinds of optical fiber. The multimode optical fiber is further divided into step index multimode optical fiber and graded index multimode optical fiber.
- The major parts of an optical fiber are core, cladding, buffer, strengthen material and outer jacket.
- It has vast uses in fields other than communication. For example in medical it is used to image internal organs by using instrument called endoscope.

EXERCISE

LONG QUESTIONS

1- Explain total internal reflection.
2- Find the relationship between critical angle and refractive index.
3- Write uses of optical fiber.
4- Write note on the material, structure and types of optical fibers.
5- Describe advantages and disadvantages of optical fiber as compared to the metal wire.

SHORT QUESTIONS

1- Define critical angle
2- Define refractive index?
3- Write relationship of critical angle and refractive index.
4- What is optical fiber?
5- Define total internal reflection.
6- Describe three advantages of optical fiber over metal wire.
7- Describe difference between mono-mode and multimode optical fiber.
8- Differentiate between step index and graded index optical fiber.

9- Describe two uses of optical fiber.
10- Describe principle of transmission of light through optical fiber.

MULTIPLE CHOICE QUESTIONS (MCQ's)

Encircle the correct answers.

1- The angle of incidence for which the angle of refraction is 90° is called:
a) Critical angle b) Crucial angle
c) Right angle d)Visual angle

2- The relationship between critical angle 'C' and refractive index '*n*' is:
a) $n = \frac{1}{Cos\,\theta_C}$ b) $n = \frac{Sin\,i}{Sin\,r}$ c) $n = \frac{1}{Sin\,\theta_C}$ d) $\theta_C = \frac{1}{Sin\,n}$

3- The refractive index of glass is:
a) 1.33 b) 1.52 c) 2.5 d) 1.4

4- The refractive index of water is:
a) 1.5 b) 1.33 c) 2.33 d) 0.33

5- The raw material for optical fiber is:
a) Copper b) Silver c) Plastic d) Silica

6- The signals are transmitted through optical fiber by:
a) Refraction b) Total internal Reflection
c) Dispersion d) Diffraction

7- The signals transmitted through optical fiber are:
a) Electrical signals b) Light signals
c) Sound waves d) Gamma rays

8- The optical fiber through which only one signal is transmitted at a time is called:
a) Mono mode b) Multi mode
c) Multi mode Step index d) Multi mode Graded index

9- The diameter of the core of single mode optical fiber is approximately equal to:
a) 5 mm b) 8μm c) 5 m d) 5 cm

10- The diameter of the core of multi-mode fiber is approximately equal to:
a) 50 mm b) 50 μm c) 5 μm d) 50 cm

11- The central part of the optical fiber is called
a) Cladding b) Core c) Optical center d) Kevlar

PROBLEMS

8.1. Refractive index of a material is 1.52, if the angle of incidence is 30°, find the angle of refraction.(Ans: 19.2°)

8.2. The angle of incidence is 60° and angle of refraction is 30°, find the refractive index of the material.(Ans: 1.73)

8.3. Find critical angle of water. The refractive index of water is 1.33. (Ans: 48.7°)

8.4. Refractive index of water is 1.33 if the angle of refraction is 30°, find the angle of incidence. . (Ans: 41.6°)

8.5. Refractive index of diamond is 2.42, find its critical angle. . (Ans: 24.4°)

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Chapter 09

LASERS

Course contents:

9.1. Corpuscular theory of light.
9.2. Emission and absorption of light.
9.3. Absorption and stimulated emission of light.
9.4. Laser principle.
9.5. Structure and working of lasers.
9.6. Types of lasers with brief description.
9.7. Applications (basic concepts).
9.8. Material processing.
9.9. Laser welding.
9.10. Laser assisted machining.
9.11. Micro machining.
9.12. Drilling, scribing and marking.
9.13. Printing.

Learning Objectives:

At the end of this chapter the students will be able to:

- Describe corpuscular theory of light.
- Differentiate between emission and absorption of light.
- Describe absorption, spontaneous emission and stimulated emission of light.
- Illustrate the principle of laser.
- Explain the structure and working of lasers.
- Describe applications of laser.

CORPUSCULAR THEORY OF LIGHT:

According to Newton, light consists of tiny particles which emit from the source of light and travel along straight lines with great speed like water flowing in the stream. These particles are called corpuscles. When these particles enter the eye, they create sensation of sight on striking with the retina of eye. This is called the corpuscular theory of light. This theory explains the rectilinear motion of light and reflection and refraction of light. According to this theory, velocity of light in denser medium like glass is greater than in rare medium like air which is wrong. This theory could not explain the interference, diffraction and polarization of light

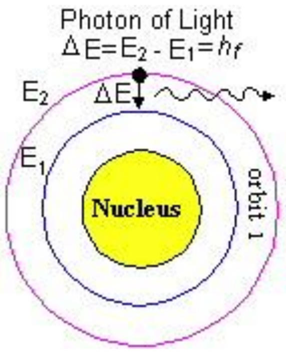


Fig. 9.1

Bohr's Atomic Model:

In 1913, Neil Bohr presented his theory of atomic structure using the quantum theory of Max Planck. According to him an atom is a neutral particle. All its positive charge lies in the nucleus and negatively charged electrons revolve around the nucleus in different orbits

According to Bohr's atomic model

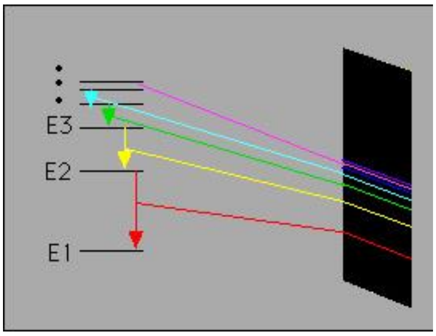


Fig. 9.2

1. An electron moves around the nucleus only in particular stable orbit. As long as the electron remains in this orbit it will not absorb or emit any energy.
2. Each orbit has a definite amount of energy associated with it.
3. When an electron absorbs energy from an external source, it jumps from lower energy orbit E_1 to higher energy orbit E_2 and absorbs energy equal to the difference in energies of the two orbits i.e.

$$E_2 - E_1 = \Delta E = hf_{12}$$

Similarly when an electron jumps from higher energy orbit E_2 to lower energy orbit E_1, it emits energy hf_{12} in the form of a photon of light.

4. The angular momentum of an electron in an orbit is given as

$$mVr = \frac{nh}{2\pi}$$

Here m is the mass of the electron, r is the radius of the orbit, V is the linear velocity of the electron, and n is the orbit number and h is the Planck 's constant with value:

$$h = 6.63 \times 10^{-34} \text{ J. sec.}$$

ABSORPTION AND EMISSION OF LIGHT:

Absorption of Light:

Suppose we have an electron with energy E_1, moving in orbit No. 1. We strike it with a photon of light having energy $E_2 - E_1 = hf$. The electron absorbs this energy and goes to the higher energy orbit No. 2 with energy E_2. In this way after absorbing energy from the photon of light, the electron goes into the excited state. This is called the absorption of light

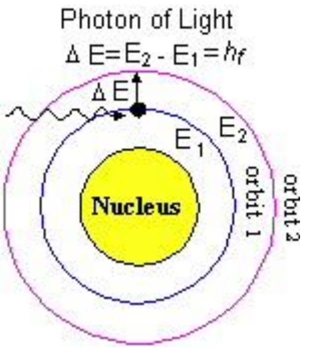


Fig. 9.3

Emission of Light:

When an excited electron is revolving higher energy orbit No.2 with energy E_2. It jumps to the lower energy orbit No.1 with energy E_1 and emits energy $E_2 - E_1$ in the form of a photon of light. This is called the emission of light

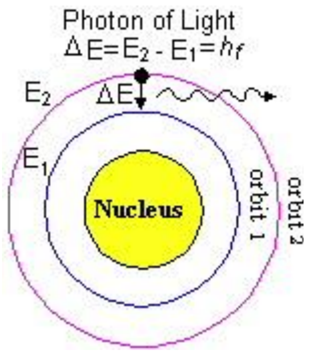


Fig. 9.4

Ground State:

The least energy level of a particle is called the ground state. In other words, the least energy stable state of a particle is called the ground state of that particle

Excited State:

If a particle goes from ground (stable) state to the higher energy state (unstable state),then this higher energy state is called the excited state.

ABSORPTION, SPONTANEOUS EMISSION & STIMULATED EMISSION OF LIGHT:

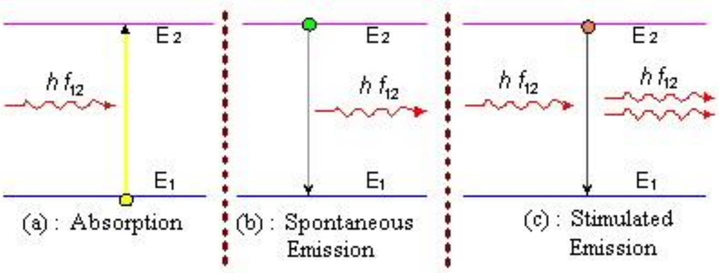


Fig 9.5(a)

Suppose we have an atom in ground state with energy E_1. When we strike it with a photon of light with energy difference $E_2 - E_1 = hf_{12}$. Here h is Planck's constant and f_{12} is the frequency difference of ground state and excited state. The electron absorbs this energy and goes into the excited state with energy E_1 as shown in the Fig.9.5(a). This process is called the absorption of light.

Now the atom can go from excited state to the ground state by two ways.

1. If the atom goes by itself from excited state to the ground state with the emission of a photon of light of energy hf_{12}. Then, it is called the spontaneous emission of light as shown in Fig. 9.5 (b).
2. If we strike the atom in excited state E_2 with a photon of light with energy hf_{12}, then the atom goes to the ground state and we get two photons of light with energy hf_{12}. One is the emitted photon and the other is the incident photon. Both the photons are of same frequency and amplitude and are phase coherent i.e. of same phase. This is called the stimulated emission of light as shown in Fig. 9.5 (c).

LASER:

Laser is the abbreviation of "*Light Amplification by Stimulated Emission of Radiations*". It means to amplify the waves of light by stimulated emission of radiation. Laser amplifies only the visible part of the electromagnetic spectrum. In other words, *laser amplifies only rays of light.* Laser light has following properties.

1. Laser light is monochromatic i.e. of single wavelength.
2. Laser light is bright.
3. Laser light is phase coherent.

Laser Principle:

The basic principle of laser is to amplify the light waves by the stimulated emission of radiations

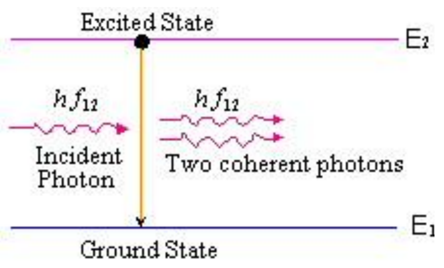


Fig. 9.6

This principle of laser is based on the 'Quantum theory of light". When we strike an excited atom with energy E_2 by such a photon of light whose energy is hf_{12} (Heref_{12} is the frequency difference of ground state and excited state). Then stimulation of light produces in the excited atom and the atom comes into the ground state as shown in Fig. 9.6. Two photons of light are also produced which are monochromatic and phase coherent. These two photons can produce stimulated emission in two other excited atoms. In this way the stimulated emission can be produced in a large number of atoms in a very short interval of time and we get a laser light as shown in

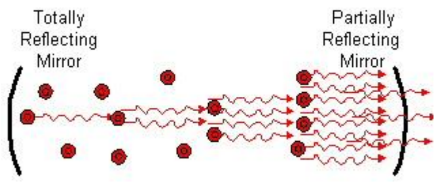


Fig.9.7

Fig.9.7

CONDITIONS FOR LASER:

The following conditions are necessary for the production of laser light.

1. Collimated Light:

To produce laser, we must have collimated light i.e. in which all light rays are parallel. It is achieved by placing the light source at the principal focus of a double convex lens as shown in the Fig. 9.8

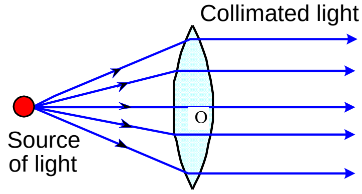


Fig. 9.8

2. Population Inversion:

When an atom absorbs photon and go to higher state E_3 it is called excited state. It produces a rapidly decaying photon and comes to excited level E_2, which is a meta-stable state. If we have more number of atoms in meta-stable excited state than in ground state then it is called the population inversion as shown in Fig. 9.9

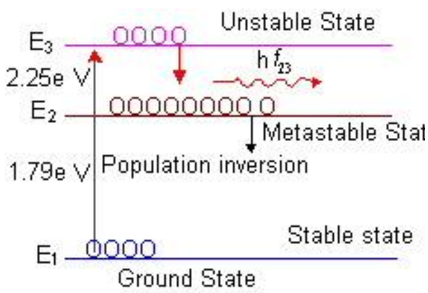


Fig. 9.9

3. Meta-stable State:

Meta-stable state is necessary to produce laser. It is that excited state in which the atoms can stay more than the usual excited state. Usually an atom can stay in excited state for 10^{-8}seconds. But in meta-stable state, an atom can stay in excited state for 10^{-3} seconds or more. For Ruby laser, the energy for meta-stable state E_2 is 1.79 eV and for unstable state E_3 it is 2.25 eV as shown in the Fig. 9.9

CONSTRUCTION (STRUCTURE) OF LASER:

Laser consists of following three things.

1. Pumping Source:

It is a device to keep more atoms in the excited state than in the ground state i.e. to produce population inversion. It is usually consists of a Xenon flash lamp as shown in Fig. 9.10.

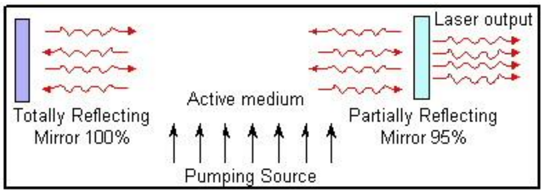


Fig. 9.10

2. Active Medium:

To produce a laser beam, we require some material medium with particular characteristics by which we can have the atoms in the excited state. In ruby laser, the impure ruby rod with chromium atoms acts as active medium as shown in Fig. 9.12. In fact the function of active medium is to provide raw material for laser action to occur.

3. Resonator (Resonant Cavity):

It consists of a cylindrical tube with reflecting mirrors at the two ends. One of the mirrors is totally reflecting and the other is partially reflecting. The mirrors reflect the light again and again and thus amplify the light as shown in Fig.9.10

WORKING (OPERATION) OF LASER:

Consider an atom which has three energy states with E_1 as ground state and E_2 E_3 are excited states as shown in figure 9.11. In ground state if we strike an atom with a photon of light with energy ($E_3 - E_1$). The atom absorbs this energy and goes to the temporary excited state E_3. Here the atom spontaneously emits a rapidly decaying photon and goes to the lower energy state E_2. It is called the permanent excited state or meta-stable state. Here more atoms are collected than the ground state as shown in Fig. 9.11. Thus we get population inversion. In energy E_2 we strike atom with such a photon of light with energy $E_2 - E_1$. Then it brings the atom in the ground state E_1 with the emission of two photons which are phase coherent. This is called the stimulated emission of light. These two photons can produce stimulated emission of light in two more excited atoms. In a small interval of time, we get a large number of photons. The mirrors at the ends of the laser tube reflect the laser light again and again and amplify it making light more intense. Then this light is emitted through the partial reflecting mirror as shown in Fig. 9.10

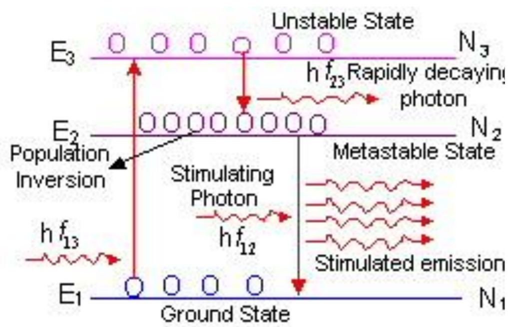


Fig. 9.11

TYPES OF LASER:

Laser is found in solid, liquid or gaseous state. Its frequency depends upon the material used. Its power varies from one milliwatt to one megawatt.

1. Solid Laser:

If the laser material is in the form of solid, then it is called the solid laser. For example, ruby laser, glass or semiconductor laser.

Ruby Laser:

Ruby is the crystalline form of aluminum oxide (Al_2O_3) and it can emit light of all colors. In ruby, a small amount of chromium is mixed. The color of laser depends upon the amount of chromium added in it as shown in Fig. 9.12

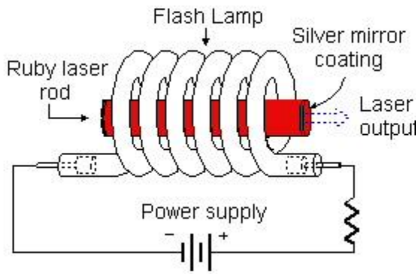


Fig.9.12

Construction and Working:

The ruby crystal is in the form of a cylinder with parallel ends. One end of the ruby cylinder is totally reflecting while other is partially reflecting. A xenon flash tube surrounds the ruby cylinder as shown in Fig. 9.12. Chromium ions in ruby absorb light energy and get temporary excited state E_3. Here they emit a rapidly decaying photon and go to lower excited state or meta-stable state E_2. Here, when we fall a photon on the atom with energy $E_2 – E_1$ then laser is emitted.

2. Gas Laser:

When the laser material is in the form of a gas then it is called a gas laser for example Helium–Neon gas laser.

3. Liquid Laser:

In liquid laser, a color is mixed in Methanol or some other liquid of this kind and thus laser is produced.

LASER APPLICATIONS:

Laser is an intense, monochromatic and coherent light. It is successfully being used in many fields of life. Some of its uses are given below

1. Material Processing:

A material is produced after passing through different steps. This is called material processing. During preparation, characteristics and quality of a material can be changed by using laser. The absorption of laser radiations in a material depends upon the thickness of the material and its density. Using laser, we can control the quality of the products. For example, by using laser, we can change the thickness of the metals. Similarly the thickness and density of plastic and paper can be checked by using laser. Similar method is used in the preparation of steel, rubber, aluminum and cloth

2. Laser Welding:

The power of laser light varies from one milliwatt to one megawatt. So laser can be used for very small and large welding. Laser can weld those things that cannot be welded by usual methods. We can weld even 0.0006 inch thick wire with the help of laser. With laser, we can do micro welding and small thermo couples can also be welded. Low energy laser is used to weld eye retina. Carbon dioxide laser is used for metal welding. Laser welding has following advantages.

1. Laser welding can be done in room with less pollution.
2. Complicated materials can be cut and welded with laser.
3. We can do very thin and correct welding.
4. During welding, we do not need to grip the material

3. Laser Assisted Machining:

Machining is a process in which different things are cut to form different parts of the machine. Then these different parts and components are joined to form the machine. Different things can be cut and joined to form a machine with the help of laser. In this way laser is helpful in the construction of a machine

4. Micro Machining:

Machine–work with very small bodies is called micro machining. A well-focused laser beam can trace and mend a printed circuit. Laser beam can be used to mend the mask used in photography. It can be used to develop the microcircuit. For example, in the beginning computers are only placed in big rooms due to their large size. But now the size of a computer is so small that it can be lifted by hand. Small components of machines can be made and after combining them small machines can be framed. This is only possible by laser

5. Laser Drilling:

Laser is a powerful controlled light which can cut hard things such as steel and can drill in diamond. It can drill a hole of 20 micrometer only in one millisecond. Holes can also be drilled in soft things such as rubber and plastic with the help of laser. Carbon dioxide laser is used to drill a hole in baby nipple, paper of cigarette and capsule so that medicine can dissolve in the body soon. Carbon dioxide laser is also used to drill a hole in glass, electronic chipboard and other electronic instruments. Laser drilling has the following advantages.

1. We can drill a hole at correct place.
2. It is possible to drill quickly.
3. Hole can be drilled in hard as well as in soft things.
4. The drilling process can be seen with the help of a monitor.

Scribing:

Scribing means to write on a hard material (such as brick, stone or wood) by scratching. First we mark on the body then we write on the body by scratching. Laser has made it easy to scratch a hard material

Marking:

It is easy to mark on the brittle thing such as glass and ceramic with the help of laser

6. Laser Printing:

We can use laser printer with the computer for printing. It takes data from the computer and makes its image in its memory system. This image is then transformed to a light sensitive drum with the help of laser beam. The drum prints that image on the paper. A very fine quality print is obtained by laser printer

7. Medical use of Laser:

1. Kidney stone can be broken with the help of laser and can be removed out from the body without operation.
2. Hairs can be planted on head and extra hair from the face can be removed by laser.
3. Eye retina can be attached with laser.
4. Laser is used to cure cancer.

   Small tumors can be removed from the body without bleeding by laser

8. Laser in Medicine:

Laser is used to study the structure of a cell. Laser is used to change the tissues and to sterile the thing. It is used to diagnose different diseases and to prepare medicines for the diseases. Hair can be planted, cancer can be treated, eye retina can be joined, kidney stone can be broken and tumors can be uprooted without bleeding with the help of laser. Moreover,

- Laser is used to find the composition of medicines.
- Laser is used to increase the durability of medicines.
- Laser controls the chemical reaction in medicines.
- Laser is used in the production of herbal medicines.

SUMMARY

- **Corpuscular Theory of Light:**
  Light consists of small particles which move in straight lines like water flowing in the stream. These particles are called corpuscles.
- When a particle receives energy from an outside source, it goes from inner orbit to the outer orbit.
- When an atom goes from excited state to the ground state. It emits light in the form of a photon.
- In spontaneous emission of light, an atom goes from excited state to the ground state by itself with the emission of a photon of light.

- In stimulated emission of light, an excited atom is bombarded with a photon of light. As a result the atom goes to the ground state with the emission of two photons of light which are phase coherent.
- Laser acts on the principle of stimulated emission of radiations.
- In population inversion, more atoms are in excited state than the number of atoms in the ground state.
- Meta stable state is that excited state in which an atom can stay for 10^{-3} sec or more.

EXERCISE

LONG QUESTIONS

1. Write a note on Bohr's atomic model.
2. Discuss absorption, spontaneous emission and stimulated emission of light.
3. Define laser. Describe its principle.
4. Describe the working of laser.
5. Write a note on gas laser.
6. Write note on any three applications of laser.

SHORT QUESTIONS

Answer the following questions.

1. Describe Newton's corpuscular theory of light.
2. Write main points of Bohr's atomic model.
3. Describe absorption of light.
4. Define ground state and excited state.
5. De-abbreviate the word LASER.
6. Define population inversion in laser.
7. Describe meta-stable state in laser.
8. Define pumping source in laser.
9. Describe active medium in laser.
10. Describe the use of laser in medicine.

MULTIPLE CHOICE QUESTIONS (MCQ's)

Encircle the correct answer.

1- The corpuscular theory of light was put forward by

a) Einstein *b)* Max Planck *c)* Young *d)*Newton

2- Neil Bohr put forward his theory in

a) 1887 *b)* 1902 *c)* 1913 *d)* 1917

3- When an electron goes from higher energy level to lower energy level it

a) Emits light *b)* Does not emit light

c) Absorbs light *d)* None of these

4- Laser is a device which can produce

a) An intense beam of light *b)* Coherent light

c) Monochromatic light *d)* All of these

5- The most widely used types of gas lasers are

a) Helium *b)* Neon

c) Carbon dioxide *d)* All of these

6- The meta-stable states of atoms are

a) Excited atomic states of long period

b) De–excited atomic state *c)* Ground states of atom

d) None of these

7- The excited atoms return to their ground state in

a) 10^{-8}sec *b)* 10^{-5}sec *c)* 10^{-3}sec *d)* 10^{15}sec

8- An atom can exist in meta-stable state for

a) 10^{-5}sec *b)* 10^{-3}sec *c)* 10^{-8}sec *d)* 10^{-10}sec

***

Chapter 10

ELECTROMAGNETIC WAVES

Course contents:

Learning Objectives:

At the end of this chapter the students will be able to:

- Define electromagnetic waves.
- Describe the magnetic field around a current carrying conductor.
- Illustrate electric field induced around a changing magnetic flux.
- Describe moving fields.
- Explain the mechanism of electromagnetic waves.
- Illustrate the types of electromagnetic waves.
- Explain the generation of radio waves.
- Describe the spectrum of electromagnetic waves.

ELECTROMAGNETIC WAVES:

The waves which do not require a material medium for their propagation from one place to the other place are known as electromagnetic waves. For example, radio waves, microwaves, light waves, x-rays etc.

MAGNETIC FIELD AROUND A CURRENT CARRYING CONDUCTOR:

When we pass current through a wire, a magnetic field is induced around the wire. This field is perpendicular to the direction of current and it can be found by right hand rule.

Right Hand Rule:

If we grip the current carrying wire in our right hand such that the thumb points in the direction of the current then the curled fingers show the direction of the magnetic field.

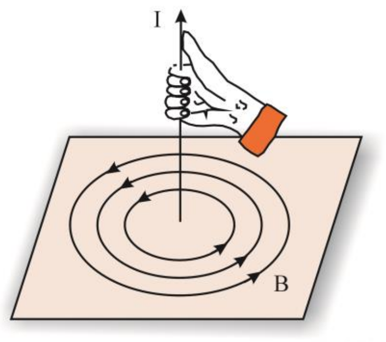


Fig 10.1(a)

In Fig.10.1(a), magnetic lines of force are shown by concentric circles which are produced due to current flowing in the wire. The magnetic field has the following properties.

1. The magnetic field exists as long as the current continues to flow.
2. The lines of force are circular.
3. The strength of the magnetic field is greater near the wire and decreases as the distance increases from the current carrying wire.
4. The direction of the magnetic lines of force can be found by right hand rule.
5. The value of magnetic field at a distance r from the wire can be found by the relation

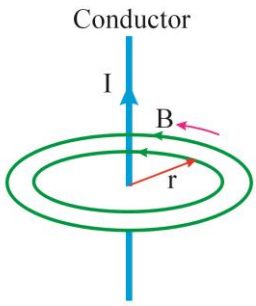


Fig 10.1(b)

$$B = \frac{\mu_o}{2\pi}\frac{I}{r}$$

where $\frac{\mu_o}{2\pi}$ = constant

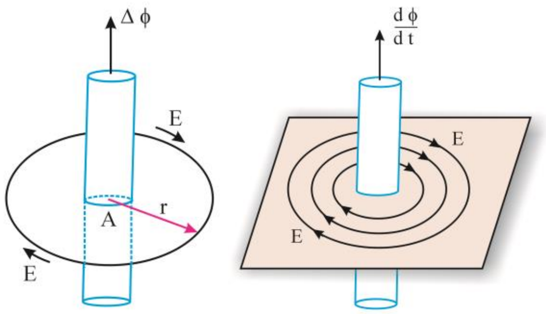
This equation shows that the magnetic field "B" is proportional to the amount of current "I" passing through the wire and is inversely proportional to the distance from the wire.

ELECTRIC FIELD INDUCED AROUND A CHANGING MAGNETIC FLUX:

In 1831, Michal Faraday told that when a magnetic is moved towards or away from the loop of a wire or when the loop of a wire is moved towards or away from a magnetic, a current is produced in the loop of the wire as shown in Fig. 10.2. This current can be detected by connecting a galvanometer with the loop of wire. It is due to the fact that magnetic lines of force, emitting from the magnet changes (magnetic flux) with the motion of either the magnet or the loop of the wire. Due to this motion, current passes through the loop of wire. This current passes due to the e.m.f produced in the coil. This shows that an electric field is produced due to a changing magnetic flux.

According to Faraday's law of electromagnetic induction, a changing magnetic flux produces an induced electromotive force.

Consider a circular wire loop of radius r and the magnetic flux through it is increased by an amount $\Delta\phi$ in time Δt. This change of magnetic flux is produced by changing the magnetic field at the centre of the loop through a circular area *A* as shown in the Fig.13.2.



(a) **(b)**

Fig 10.2

According to Faraday's law, an e.m.f. (ε) or potential difference will be induced in the loop.

$$\varepsilon = \frac{d\varphi}{dt} \qquad \text{----} \quad (1)$$

Due to this induced e.m.f, an induced current flows in the loop. As the charges (electrons) move, they will experience some force. This force experienced by a unit charge is called ***electric intensity***. The electric intensity will have same value at all points of circular loop with same radius.

The direction of intensity is determined by Lenz's law.

It means that a unit positive charge if free to move all the way around the loop is pushed by the electric field of constant strength E through the whole length $2\pi r$ of the loop. It receives energy equal to $E \times 2\pi r$ from the induced electric field. This energy per unit positive charge is the induced *emf*, that is,

$$\varepsilon = 2\pi r E = \frac{d\varphi}{dt} \quad \text{or} \quad E = \frac{1}{2\pi r}\frac{d\varphi}{dt}$$

This shows that the induced electric field around the changing magnetic flux must run around in circles and the magnitude of this induced field must decrease inversely with the distance from the centre.

NOTE: It is important to note that if a current flows, it will be in such a direction that the magnetic field it produces tends to counteract the change in flux that induces the *emf*.

MOVING FIELDS:

A changing magnetic flux creates an electric field and a changing electric flux creates a magnetic field. This shows that, if a change of electric or magnetic flux is taking place through any region, the electric and magnetic fields will propagate out of this region in the surrounding space.

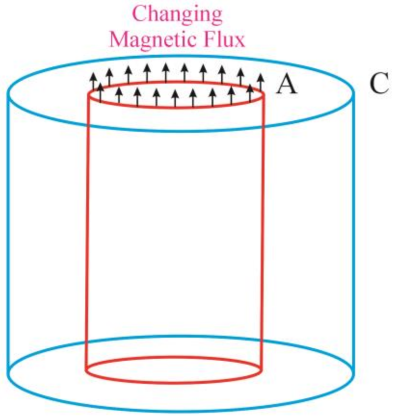


Fig 10.3

Consider a region *A* through which magnetic flux is changing as shown in Fig. 10.3 This change of magnetic flux will set up an electric field in the surrounding region C. This electric field in the region C will change electric flux through it and this will set up a magnetic field around it. Thus each field generates the other field and the package of electric and magnetic fields will move through space. These moving fields are called electromagnetic waves.

Mechanism of Electromagnetic Waves:

Consider a slab of magnetic field which is pointing towards x-axis and moving parallel to y-axis as shown in Fig. 10.4 (a). Imagine a certain region ad of space which is just adjacent to AD. When the magnetic field moves into this region, it will create a change of flux in it.

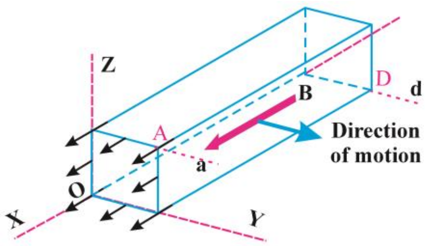


Fig 10.4(a)

The change of magnetic flux in ad will create an electric field in the surrounding region. When the flux through ad increases, the direction of $\Delta\phi$ will be the same as that of the field, i.e. towards the x-axis. Right hand rule and Lenz's law show that electric field will be generated parallel to z-axis in the region where

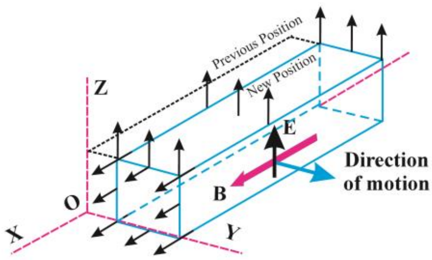


Fig 10.4(b)

the magnetic field is present as shown in figure (10.4) (b). The magnetic field and the generated electric field both will move with the slab along y-axis.

Now we consider the effect of motion of this electric field as shown in Fig. 10.5 (a). When the moving slab of electric field sweeps into the region ad, adjacent to AD, an increase of electric flux $\Delta\phi_e$ takes place in this region. The direction of $\Delta\phi_e$ will be the same as that of the field, i.e., along the z-axis. Due to this increase of electric flux in the region ad, a magnetic field will be induced in the surrounding region. This magnetic field will be along x-axis as shown in Fig. 10.5 (b).

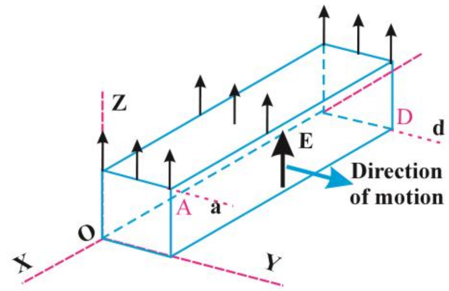


Fig 10.5(a)

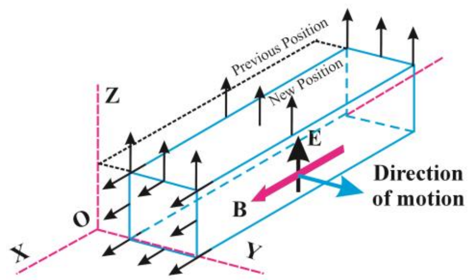


Fig 10.5(b)

So a moving magnetic field generates a moving electric field and this electric field generates a moving magnetic field. The motion of these fields will be possible only when they move with a certain definite speed. Maxwell showed that in free space, this speed is

$$C = \frac{1}{\sqrt{\epsilon_o \mu_o}}$$

Here ε_o is permittivity of free space = 8.85×10^{-12} F/m and μ_o is the permeability of free space = 1.26×10^{-6} H/m and c is the velocity of electromagnetic waves = 3×10^{-8} m/sec.

TYPES OF ELECTROMAGNETIC WAVES:

1. Radio waves:

These are the electromagnetic waves with wavelength from few decimeter to several meters and frequency up to few giga (10^9) Hz. These are generated from electronic devices. These are used the transmission of radio signal.

2. Microwaves:

Microwaves are electromagnetic waves with wavelength from 0.1mm to 1 decimeter and frequency from few giga (10^9) Hz to few tera (10^{12}) Hz.

These are generated from electronic devices. There are used in radar and microwave even.

3. Infrared waves:

These are the electromagnetic waves with wavelength from 1 micrometer to 0.1 millimeter and frequency from 10^{12} hertz to 10^{14} hertz. These are emitted from hot bodies. Their energy varies from 0.01 eV to 2 eV.

4. Visible Waves:

Visible waves have wavelength from 400 nanometers to 750 nanometers and frequency of 10^{14} hertz. They have energy of 2eV to 3eV. We can see things around us in the presence of unable waves.

5. Ultraviolet waves:

Ultraviolet waves are electromagnetic waves with wavelength of 1 nanometer to 400 nanometer and frequency of 10^{14} hertz to 10^{17}. This energy varies from 3 eV to 1000 eV. They are emitted from atomic sparks and arcs. These are used in photoelectric effect.

6. X-Rays:

X-Rays are electromagnetic waves with wavelength from 1 nanometer to 1000 nanometers and frequency from 10^{17} hertz to 10^{19} hertz. They have energy from 10^{3}eV to 10^{5}eV. They are emitted from electron impact on solid. They are used to detect broken bones in a human body.

7. γ–Rays:

γ–Rays are electromagnetic wave with wavelength from less than 10^{-11}meter and frequency greater than 10^{20} hertz. They are emitted from radioactive nuclei.

GENERATION OF RADIOWAVES:

Radio waves are generated by a radio transmitting antenna. An antenna is a loop of wire charged by an alternating voltage of frequency ν and time period T as shown in the figure.

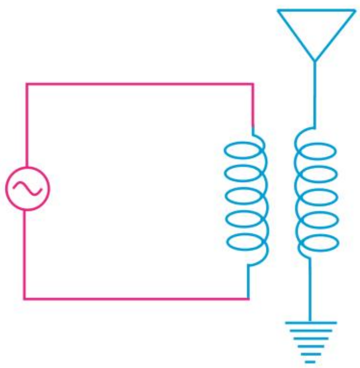


Fig 10.6

Due to alternating supplied voltage, charge on the antenna constantly reverses its direction. So if charge on the top of the antenna is +q at any time, it will be –q after time T/2. This change is due to the accelerated motion of electrons and it creates a changing magnetic field around it. In this way, electromagnetic waves are generated which propagate away from the antenna. The propagation of electromagnetic waves is shown in Fig. 13.6

If such a wave is intercepted by conducting wire, the oscillating electric field will develop an oscillating voltage in it.

The electromagnetic waves are transmitted in the form concentric spherical wave fronts that are as shown in Fig. 10.7.

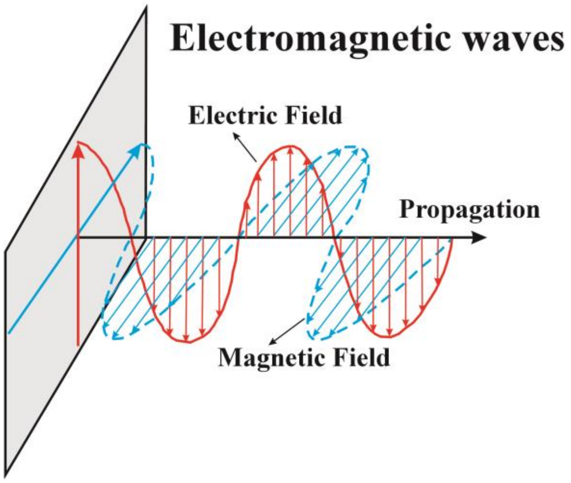


Fig 10.7

SPECTRUM OF ELECTROMAGNETIC WAVES:

Radio waves are not the only example of electromagnetic waves. We have a large variety of them according to the effect that they produce. This effect depends upon the value of their frequencies. For example, they produce the sensation of light, if their frequencies range from 3×10^{12} cps to 8×10^{12} cps. These frequencies are of red and violet colours respectively. The electromagnetic waves between these frequencies are visible to eye. Special instruments are used to detect waves of higher and lower frequencies. The electromagnetic waves are periodic waves. The speed of electromagnetic waves in free space is $c = 3 \times 10^8$m/sec. The wavelength λ of an electromagnetic wave of frequency ν is given by $\lambda = \frac{c}{\nu}$

Electromagnetic Spectrum:

The arrangement of all the electromagnetic waves in a pattern showing their frequency, wave length and energy etc. is known as spectrum of electromagnetic waves. Electromagnetic waves spectrum is shown in the Fig. 10.7 indicating all the important electromagnetic waves in ascending frequency.

The spectrum shows that gamma rays are high energy waves. Their frequency is also very high, but their wave length is low. The radio waves are of low energy and low frequency waves. But their wave length is large as compared to other electromagnetic waves.

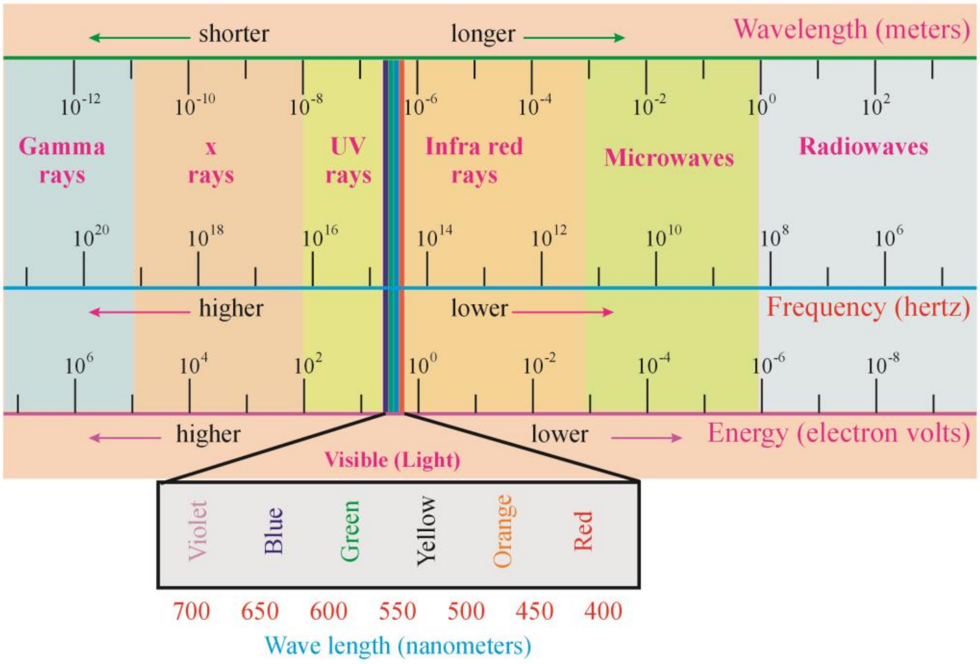


Fig 10.7

Visible light is a very small portion of electromagnetic wave spectrum. The visible light spectrum is highlighted. The wavelength is shown in nanometers. Violet light has high frequency and low wave length but red light has low frequency and high wave length.

SUMMARY

- The magnetic field induced around a wire by passing current through it is always perpendicular to the direction of the current.
- According to Faraday's law of electromagnetic induction, a changing magnetic flux produces an induced electric field which is given by

$$E = \frac{1}{2\pi r}\frac{d\phi}{dt}$$

- A changing magnetic flux creates an electric field and a changing electric flux creates a magnetic field. These fields will move in a direction which is perpendicular to both of these fields. These are called moving fields.
- The electromagnetic waves include radio waves, microwaves, infrared waves, visible waves, ultraviolet rays, X-rays, gamma rays, cosmic rays, etc.
- The radio waves are generated by a transmitting antenna and are received by the receiving antenna of a radio receiver. Their frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

EXERCISE

LONG QUESTIONS

1. Explain magnetic field produced around a current carrying conductor.
2. Describe electric field induced around a changing magnetic flux.
3. Explain the mechanism of electromagnetic waves.
4. Illustrate briefly the types of electromagnetic waves.
5. Explain the generation, transmission and reception of radio waves.
6. Write a note on electromagnetic wave spectrum.

SHORT QUESTIONS

Answer the following questions.

1. Define electromagnetic waves.
2. Define a magnetic field.
3. Describe the right hand rule.
4. Define magnetic flux.
5. Describe moving fields
6. Define micro waves.
7. Define ultraviolet waves.
8. Describe radio waves.

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1- **The field around a moving charge is called**
   a) Gravitational field *b)* Electric field
   c) Magnetic field *d)* Atomic field

2- **The magnetic effect near the current carrying conductor was discovered by**
   a) Coulomb *b)* Faraday *c)* Ampere *d)* Hans Oersted

3- **Which one of the following metals is non–magnetic substance?**
   a) Brass *b)* Cobalt *c)* Nickel *d)* Iron

4- **The magnetic field at a point due to a current carrying conductor is proportional to**
   a) Thickness of the conductor *b)* Resistance of the conductor
   c) Distance from the conductor *d)* Current through conductor

5- **Two lines of magnetic force**
   a) Can cross each other *b)* Always cross each other
   c) Can never cross each other *d)* None of the above

6- **A changing magnetic flux produces around itself an induced.**
   a) Magnetic field *b)* Gravitational field
   c) Electromotive force *d)* Electric field

7- **The electric field induced around a changing magnetic flux varies as**
   a) r *b)* r^{-1} *c)* r^{2} *d)* r^{3}

8- **The electromagnetic waves travel in free space with the speed of**
   a) Sound *b)* Light *c)* Positive rays *d)* Cathode rays

9- **The average temperature of earth atmosphere is**
   a) 250 K *b)* 300 K *c)* 350 K *d)* 200 K

10- **The minimum frequency of radio waves is**
   a) 1 M Hz *b)* 5 M Hz *c)* 50 M Hz *d)* 10 M Hz

11- **In electromagnetic waves, the electric and magnetic field are**
   a) Parallel *b)* Opposite *c)* Perpendicular *d)* All are correct

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Chapter 11

ARTIFICIAL SATELLITES

Course contents:

11.1. Review law of gravitation.
11.2. Escape velocity.
11.3. Orbital velocity.
11.4. Geosynchronous and geostationary satellites.
11.5. Use of satellites in data communication.

Learning Objectives:

At the end of this chapter the students will be able to:

- Describe the law of gravitation.
- Define and explain Escape velocity.
- Explain Orbital velocity.
- Describe Geosynchronous and geostationary satellites.
- Illustrate the use of satellites in data communication.

NEWTON'S LAW OF GRAVITATION:

Statement:

This law states that *everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.*

Explanation:

If we have two bodies A and B of mass m_1 and m_2 respectively are placed such that the distance between their centers is r as shown in Fig. 11.1. The force of attraction between them is given as

$$F \propto m_1 m_2 \quad ---- \quad (1)$$

and

$$F \propto \frac{1}{r^2} \quad ---- \quad (2)$$

Combining eq. (1) and (2) we get

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{r^2}$$

"G" is the constant of proportionality which is known as universal gravitational constant.

Its value in S.I system is 6.673×10^{-11} N.m^2/kg^2.

In vector form, this force is written as:

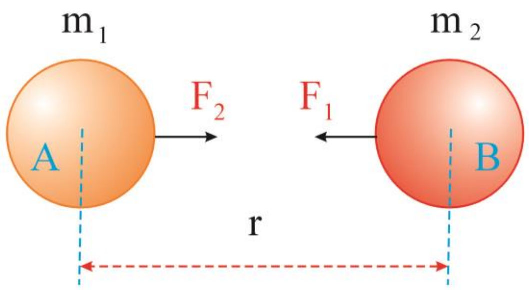


Fig 11.1

$$\mathbf{F} = -G\frac{m_1 m_2}{r^2}\hat{r}$$

Negative sign shows that force between these two bodies is attractive.

DETERMINATION OF UNIVERSAL GRAVITATIONAL CONSTANT G, BY CAVENDISH METHOD:

In 1798, Cavendish found the value of gravitational constant G, using the apparatus called torsion balance. The apparatus consists of a light rod which is suspended at the centre by a fine quartz fiber. The other end of the fiber is attached to a fixed support. Two similar balls each of mass m are attached at the ends of the rod. A lamp–scale arrangement is attached to the fiber to see the deflection in the rod.

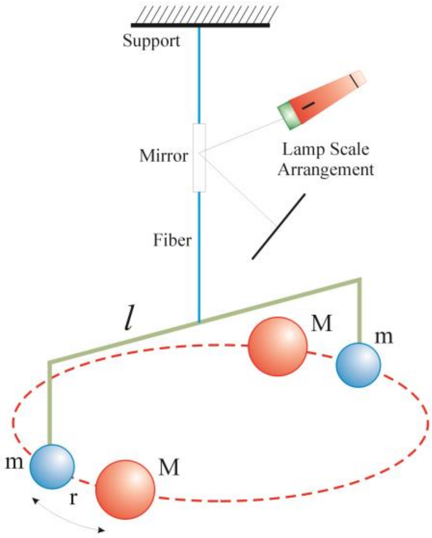


Fig 11.2

Now two similar heavy balls, each of mass M are brought near the small balls on opposite sides of the rod. They exert a force of attraction on the suspended balls. According to Newton's law of gravitation, the magnitude of the force at each end of the rod is given as in Fig. 11.2

$$F = G\frac{m\,M}{r^2}$$

Here 'r' is the distance between the center of ball M and m. This force produces torque and twists the fiber due to the deflection of rod through an angle θ. The twist produced in the fiber is proportional to the magnitude of the torque. Mathematically, it is written as:

$$\tau \propto \theta$$

or $$\tau = c\,\theta \quad \text{- - - -} \quad (1)$$

Here ‘c’ is the constant known as torsion constant which depends upon the nature of the fiber. If ‘l’ is the length of the rod then magnitude of the torque ‘τ’ on the right side of the rod is given as:

$$\tau_1 = G\frac{m\,M}{r^2} \times \frac{l}{2}$$

Here τ = Force × Distance from center

and $\frac{l}{2}$ = distance of each of the mass m from the center of the rod.

Similarly torque ‘τ_2’ on the left side of the rod is given as

$$\tau_2 = G\frac{m\,M}{r^2} \times \frac{l}{2}$$

Since both torques are anti–clock wise. Therefore total torque on the rod is equal to the sum of these two torques.

So $$\tau = \tau_1 + \tau_2 = G\frac{m\,M}{r^2} \times \frac{l}{2} + G\frac{m\,M}{r^2} \times \frac{l}{2}$$

$$\tau = G\frac{m\,M\,l}{r^2} \quad \text{- - - -} \quad (2)$$

Comparing Eq. (1) and (2) we get

$$G\frac{m\,M\,l}{r^2} = c\,\theta$$

$$\Rightarrow \quad G = \frac{c\,\theta\,r^2}{m\,M\,l} \quad \text{- - - -} \quad (3)$$

Knowing the values of c, θ, r, M, m and l we can find the value of G which is equal to 6.673×10^{-11} N.m^2/kg^2.

MASS OF THE EARTH:

We can find the mass of the earth using law of Universal Gravitation. The mass of Earth is M_e and its radius is R_e. Let a body of mass m be placed at the surface of the Earth. The radius of the body is very small so the distance between the body and the center of the Earth is taken as R_e. Then the gravitational force with which the Earth attracts the body towards its center is

$$F = \frac{G\, m\, M_e}{R_e^2} \qquad ---- \quad (1)$$

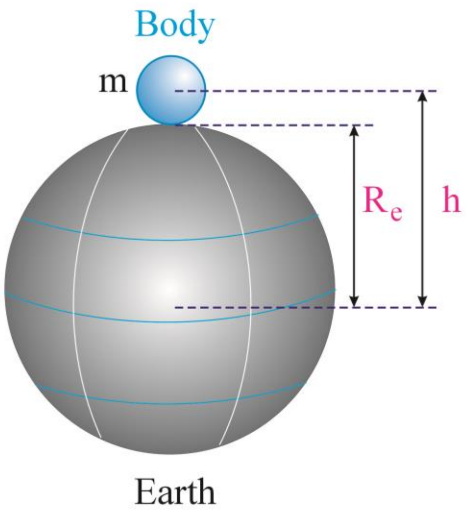


Fig 11.3

But the force with which the earth attracts the body towards its center is equal to the weight of the body

$$F = w = mg \qquad ---- \quad (2)$$

As these two forces are equal so comparing eq. (1) and (2)

$$mg = \frac{G\, m\, M_e}{R_e^2}$$

$$\Rightarrow \quad g = \frac{G\ M_e}{R_e^2}$$

$$\Rightarrow \quad M_e = \frac{g\ R_e^2}{G}$$

Putting value of $R_e = 6.4 \times 10^6$m

$g = 9.8$ m/sec^2

$G = 6.673 \times 10^{-11}$ N.m^2/kg^2

$$M_e = \frac{9.8 \times (6.4 \times 10^6)^2}{6.673 \times 10^{-11}} = 6 \times 10^{24}\ \text{kg.}$$

ESCAPE VELOCITY

"The minimum velocity that is required to project a particle from the earth's surface to an infinite distance is called the escape velocity".

Let we throw a ball of mass 'm' upward in the air. It returns back on the earth from some height. If we throw the ball with larger velocity, it turns back from higher point. If the ball is thrown upward with such velocity that it goes out of the pull of the earth, it is called the escape velocity. For this, the K.E of the ball is equal to the absolute P.E of the ball. We know that gravitational force is given as

$$F = \frac{G\, M\, m}{R^2}$$

Here, m. is the mass of the body at the surface of earth, M is the mass of the earth, G is the gravitational constant and R is the radius of the earth. Potential energy is given as

$$P.E = Work = F.S = \frac{G\,M\,m}{R^2} \times R = \frac{G\,M\,m}{R}$$

When K. E = P. E

Then $$\frac{1}{2}\,m\,V_{esc}^2 = \frac{G\,M\,m}{R^2} \quad ---- \quad (1)$$

$$V_{esc} = \sqrt{\frac{2\,G\,M}{R}} \quad ---- \quad (2)$$

In case of equilibrium, the gravitational force acting on the body is equal to the weight of the body.

$$W = F$$

$$mg = \frac{G\,M\,m}{R^2}$$

$$GM = gR^2 \quad ---- \quad (3)$$

Putting value of GM from Eq (3) in Eq (2)

$$V_{esc} = \sqrt{\frac{2\,g\,R^2}{R}} = \sqrt{2\,g\,R}$$

As $g = 9.8\ m/sec^2$ and $R = 6.4 \times 10^6 m$

$$V_{esc} = \sqrt{2 \times 9.8 \times 10^6}$$

$$= 11200\ m/sec = 25000\ mile/hour$$

This shows that escape velocity of the body at the surface of earth is 11200 m/sec or 25000 miles/hour.

ORBITAL VELOCITY:

It is the velocity required by a body to move it around the earth in a certain orbit.

Let we have a planet with mass m. It is revolving around the earth in an orbit which is at a distance “r” from the center of the earth. If the velocity of the planet in that particular orbit is V_o, then the gravitational force between the earth and the planet is given as:

$$F = \frac{G\,M\,m}{r^2} \qquad ---- \quad (1)$$

But to move around the earth in an orbit, the planet requires a centripetal force which is provided by the earth.

$$F_C = \frac{m\,V_o^2}{r} \qquad ---- \quad (2)$$

As these two forces are equal so comparing eq. (1) and (2) we get

$$\frac{m\,V_o^2}{r} = \frac{G\,M\,m}{r^2}$$

or $$V_o^2 = \frac{G\,M}{r}$$

or

$$V_o = \sqrt{\frac{G\,M}{r}}$$

But $\sqrt{G\,M}$ is constant, therefore

$$V_o = \text{constant} \times \frac{1}{\sqrt{r}}$$

or $$V_o \propto \frac{1}{\sqrt{r}}$$

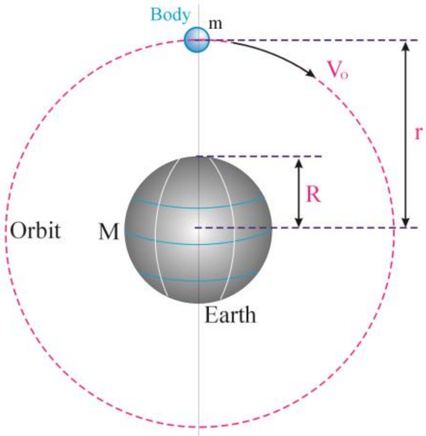


Fig 11.4

So the *orbital velocity is inversely proportional to the square root of the radius of the orbit*.

GEOSYNCHRONOUS *OR* GEOSTATIONARY SATELLITES:

In 1945, Arther C. Clark gave the concept of geostationary satellite which appears to remain stationary with respect to a certain point on the earth. It can cover about 40% area of the earth. According to him, three such satellites at a distance of 120° around the earth can cover the whole world as shown in fig. a. The data communication and T.V. transmission can be sent anywhere in the world through these satellites. The Olympic matches and world cup can also be telecast live through these satellites.

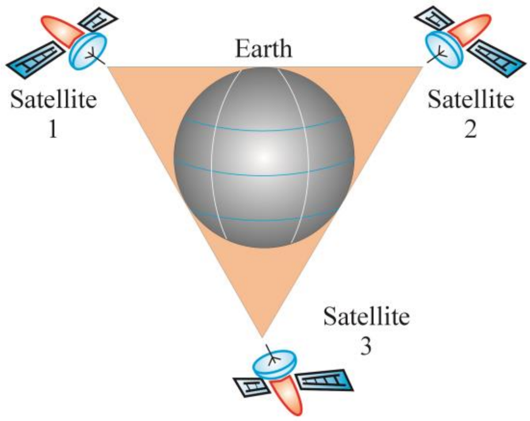
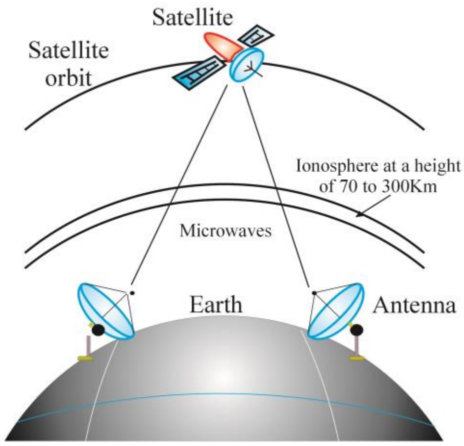


Fig 11.5(a)

A geostationary satellite is at a height of 35800 km from the earth's surface and it moves around the earth in such a way that appears to be stationary from a certain point at the earth i.e. it covers a complete circle (360°) around the earth in 24 hours. Its distance from the center of earth is (6400 + 35800) = 42200 and the circumference of the orbit is 42200 × 2π = 265000 km . So it covers a distance of 26500 km in 24 hours and in this way its speed is 11000 km/hour.

The PTV transmissions are sent to 32 countries in Asia and many countries in Europe and America through satellite. We see T.V. programs by using dish antennas through these satellites. These satellites are also used to broadcast radio programs and weather forecast through internet.



Basic concept of telecommunication
Fig 11.5(b)

To find the orbital radius of a geostationary satellite:

The orbital velocity of a satellite is found by the following formula:

$$V_{ob} = \sqrt{\frac{GM}{r}} \qquad ---- \quad (1)$$

This orbital velocity is equal to the average speed of the satellite.

$$V = \frac{S}{t} = \frac{2\pi r}{T} \qquad ---- \quad (2)$$

Comparing Eq (1) & (2)

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring both sides

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

or

$$r^3 = \frac{GMT^2}{4\pi^2} \qquad ---- \quad (3)$$

Here $G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$

$M = 5.98 \times 10^{24}$ kg

$T = 24 \text{ hours} = 86400 \text{ sec}$

Putting these values we get

$$r^3 = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (86400)^2}{4 \times \left(\frac{22}{7}\right)^2}$$

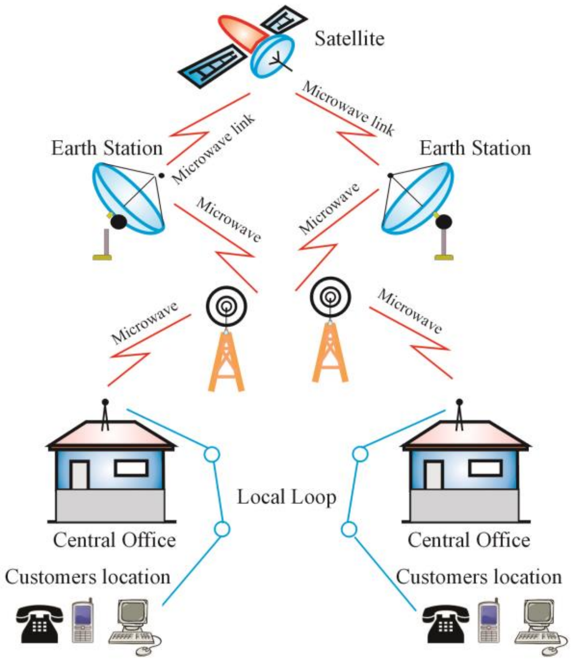
$$r = 42250 \times 10^3 \text{m} = 42250 \text{ km}$$

This orbital radius of satellite is from the center of earth. Therefore, the distance of satellite from the surface of earth is

$$h = r - R = 42250 - 6400 = 35850 \text{ km}$$

USE OF SATELLITE IN DATA COMMUNICATION:

The use of satellites in data communication is shown in the figure. First, a local microwave system is used for data transmission in telephone and T.V in a country then the information are sent to the Earth station in that country as shown in the fig. The earth station sends the information to the satellite. The satellite sends this information (signal) to another earth station in the other country from where the information is sent to the destination. This is done by the following three ways.



Satellite communication system

Fig 11.6

1. Ground – Ground Communication:

In this method, the signal information is sent to the earth station from the central office through microwave system. The earth station sends this signal to the satellite through uplink and in return, the satellite sends this signal to another earth station through down link. Then this signal is sent to the required central office or to the required destination. This system is called ground–ground system.

In this system an electromagnetic modulated carrier signal is sent to the satellite through uplink. The satellite receives a part of this signal and sends it back towards the required earth station through down link. In this way the satellite acts as a repeater and is called a repeater satellite, a transponder or a processing satellite.

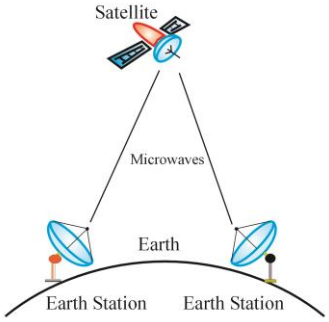


Fig 11.7

2. Ground - Cross Link - Ground Communication:

In this system, a satellite receives a signal from the earth station and sends it to another satellite in space. The other satellite provides this signal to the required earth–station. Therefore it is called ground–cross link–ground system. In it, a cross-link of signal is taken place between two such satellites which are not seen by each other. Communication is possible by it to remote areas which are not possible by usual methods.

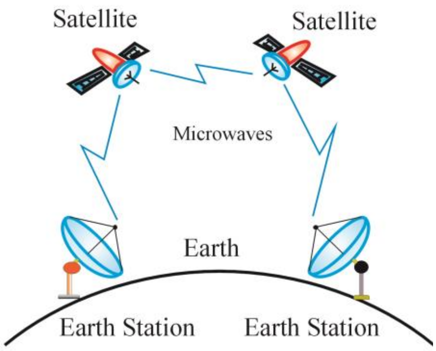


Fig 11.8

3. Ground- User Relay:

In this system, an earth station sends a signal to a satellite through uplink. From the satellite, the signal is sent to an airplane a ship or other services. In this way the receiving body is not stationary and it does not take help from the earth station. In it, the satellite works as a relay and sends the earth signal to an airplane or a ship. Sometimes, the user which receives the signal sends a message to the satellite which is received by the earth station through this satellite. The link from

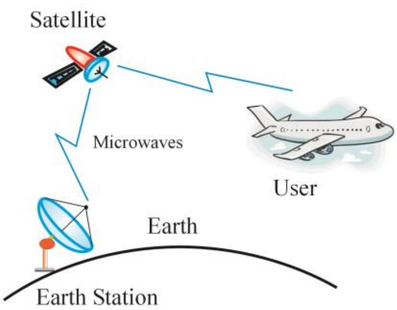


Fig 11.9

satellite to the user is called forward link and from user to satellite is called return link.

Advantages:

- The transmission of information is possible across a wide area through satellite communication and only three satellites are enough to cover the whole world.
- These provide continuous and uninterrupted service to the specific areas.
- Their signal quality is better than a microwave link.
- More power can be transmitted and received through satellite.
- They have the ability of more bands, therefore a microwave link have the capacity of more channels than a coaxial cable.
- As they cover a large area, so they are cheap as compared to other sources.

Disadvantages:

- They are costly.
- Transmission is not possible at poles through satellites.
- As the satellites are at much height there may be a time delay between the sending signal and the receiving signals.

SUMMARY

- **Newton's law of Gravitation:** In the universe, everybody attracts every other body with a force which is directly proportional to the product of masses of the two bodies and inversely proportional to the square of the distance between their centers.
- **Escape Velocity:** It is the velocity acquired by a body in order to escape from the gravitational field of the earth.
- **Orbital Velocity:** It is the velocity acquired by a body to move it around the earth in a certain orbit.
- **Geosynchronous Satellite:** It is that satellite which appears to be stationary from a certain point at the surface of earth.

EXERCISE

LONG QUESTIONS

1. State and explain Newton's law of gravitation.
2. Describe Cavendish method to determine the gravitational constant.

3. Find the value of escape velocity on the surface of earth.
4. Show that orbital velocity is inversely proportional to the square root of the radius of the orbit.
5. Describe a Geostationary satellite. Find the radius of its orbit from the surface of earth
6. Explain the transmission of data through satellite.

SHORT QUESTIONS

1. State Newton's law of gravitation.
2. Define escape velocity.
3. Define orbital velocity.
4. Define geostationary satellite.
5. Define ground-ground communiation.
6. Define ground-cross link-ground communiation.
7. Define ground-user relay.

MULTIPLE CHOICE QUESTIONS (MCQs)

Encircle the correct answer.

1. The value of universal gravitational constant G is

a) 6.2×10^{-11}N .m^2/kg^2 *b)* 1.6×10^{-11} Coulomb
c) 9.1×10^{-31}kg^2 *d)* 10 N

2. The unit of force in C.G.S system is

a) Newton *b)* Erg *c)* Dyne *d)* Pound

3. A satellite can cover the area on earth

a) 25% *b)* 30% *c)* 35% *d)* 40%

4. A synchronous satellite can complete its cycle around the earth in

a) 12 hours *b)* 24 hours *c)* 36 hours *d)* 48 hours

5. The communication system of a satellite is like

a) Microwave repeater *b)* X–rays
c) γ–rays *d)* Ultraviolet rays

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