Matrices

A Matrix is rectangular array of number.
A matrix with m row & n column is said to have diamension, m×n & may be represented as follows :-

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = a_{ij}$$

We often use capital letters to represent matrices & into the enclosed array of no. in bracket & ( ).

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

We do not use simply straight line in place of brackets. when writing matrices because the notation $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ has a special meaning in linear algebra.

$A = [a_{ij}]$ is short hand notation often used when one wishes to specify how the element are to be represented. Where the 1st subscript [i] denotes the 'Row no.' & the [j] denotes the 'column no.' of the array.

Thus if one writes a_{34} one is referring to the element 3rd row & 4th column.

* Types Of Matrix :-

1) Unit Or Identity Matrix :-

A unit or Identity Matrix is diagonal Matrix with all the elements in the principle diagonal equal to 1. For any arbitary matrix A following relationship hold through :-

$AI = A$

$IA = A$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AI = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} (1+0+0) & (0+2+0) & (0+0+3) \\ (4+0+0) & (0+5+0) & (0+0+6) \\ (7+0+0) & (0+8+0) & (0+0+9) \end{bmatrix}$$

2) **Null Matrix Or Zero Matrix :-** A Null matrix is a any matrix in which all the elements have zero value. It is usually denoted as zero.

eg:- $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & & \end{bmatrix}$

3) **Symmentric Matrix :-** A symmentric Matrix is a square matrix in which $a_{ij} = a_{ji}$

$$A_{ij} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} = A_{ji} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A_{ij} = \begin{bmatrix} 1 & 3 & 7 & 9 \\ 3 & 4 & 2 & 10 \\ 7 & 2 & 7 & 8 \\ 9 & 10 & 8 & 11 \end{bmatrix} \quad A_{ji} = \begin{bmatrix} 1 & 3 & 7 & 9 \\ 3 & 4 & 2 & 10 \\ 7 & 2 & 7 & 8 \\ 9 & 10 & 8 & 11 \end{bmatrix}$$

4) **Skew-symmentric Matrix :-**

A skew symmentric Matrix is a square matrix with all values in principle diagonal equal to zero & with all diagonal values equal to zero. With the diagonal value given such as that $a_{ij} = -a_{ji}$

$$\begin{bmatrix} 0 & 2 & 5 & -9 \\ -2 & 0 & -4 & -6 \\ 5 & 4 & 0 & 12 \\ 9 & 6 & -12 & 0 \end{bmatrix}$$

5) Transposed Matrix :-

Given Matrix A. The Transpose of A is given by A^T & Read A transpose is obtain by changing all the Rows by A into the columns A^T while preserving a orderer.

Hence the first Row of A becomes first Column of A^T; while second Row of A becomes second Column of A^T. And the last Row of A becomes the last Column of A^T. In terms of element.

$$a_{ij}^T = a_{ji}$$

If Matrix A has R row & C column then A^T will have C row & R column.

Note :- $(A^T)^T = A$

eg $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix} \quad AT = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

6) **Singular Matrix :-** A Square Matrix A is said to be a Singular Matrix if determi-nat A equal to 0.

$$|A| = 0$$

eg :- $\begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix} = 12 - 12 = 0$

7) **Non-Singular Matrix :-** A square Matrix A is said to be a Non-Singular Matrix if determinant A is not equal to zero

$$|A| \neq 0$$

$$\begin{bmatrix} 2 & 4 & -1 \\ 3 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix} = 2(0-2) - 4(9+4) - 1(3+0)$$

$= -4 - 52 + 3$

$= -~~56~~ - 1 - 52$

$= -53$

$|A| \neq 0$

8) **Square Matrix :-**

A Matrix having same no. of row & columns. The order of square matrix is of the form $n \times n$:. ~~And~~ A Matrix will be refer as Matrix of Matrix M.

eg:- $[15]_{1 \times 1}$

$$\begin{bmatrix} 2 & 3 \\ 415 & 1 \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ -7 & 8 & -9 \end{bmatrix}_{3 \times 3}$$

Remark :-

Let $A = [a_{ij}]_{n \times n}$ Here, square matrix of column N then element a_{11}, a_{22}, a_{33} a_{ii} ---- a_{nn} are called the diagonal element of Matrix N.

Note :- 1) The diagonal elements are define only for square Matrix.

2) Element a_{ij} ~~$i = j$~~ $i \neq j$ are called not diagonal element of Matrix 'a'.

3) The Elements a_{ij} where $i<j$ represent elements above the diagonal.

4) Elements a_{ij} where $i>j$ repront element below the diagonal.

9) Scalar Matrix :-

A diagonal Matrix in which all the diagonal elements are equal is called as a scalar Matrix.

eg :-

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} 57 & 0 & 0 \\ 0 & 57 & 0 \\ 0 & 0 & 57 \end{bmatrix}_{3\times 3}$$

10) **Column Matrix :—** A Matrix having only one column is called as a Column Matrix it is of order :-

$m \times 1$

$\begin{bmatrix} 3 \\ 2 \end{bmatrix}_{2\times 1}$ $\begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}_{3\times 1}$

Here m is greater than 1 $m \geq 1$

11) **Row Matrix :—**

$1 \times n$

$\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}_{1\times 3}$ $\begin{bmatrix} 3 & 2 \end{bmatrix}_{1\times 2}$

A Matrix having only one row is called as a row Matrix. It is of order $\underline{1 \times n}$

* Algebra Of Matrices :-

1) Equality Of Matrices :-

$$A = [a_{ij}]_{r \times c}$$

$$B = [b_{ij}]_{r \times c}$$

are equal if they have sent order & if $a_{ij} = a_{ij}$ for all i & j

eg :- $\begin{bmatrix} 5x & -3y \\ x & -2y \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

Implies that

$5x - 3y = 7$

$x - 2y = 1$

$a_{ij} + b_{ij} = c_{ij}$

2) Addition Of Matrix :-

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2+5 & 3+6 & 4+7 \\ 5+8 & 6+9 & 7+1 \\ 8+2 & 9+3 & 1+4 \end{bmatrix} = \begin{array}{c} c_{ij} \\ \begin{bmatrix} 7 & 9 & 11 \\ 13 & 15 & 8 \\ 10 & 12 & 5 \end{bmatrix} \end{array}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \\ 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 11 \\ 6 & 2 \\ 3 & 4 \\ 9 & 8 \end{bmatrix}$$

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$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 11 \\ 6 & 2 \\ 3 & 4 \\ 9 & 8 \end{bmatrix} = \begin{bmatrix} 2+5 & 3+11 \\ 4+6 & 5+2 \\ 6+3 & 7+4 \\ 8+9 & 9+8 \end{bmatrix} = \overset{c_{ij}}{\begin{bmatrix} 7 & 14 \\ 10 & 7 \\ 9 & 11 \\ 17 & 17 \end{bmatrix}}$$

*Substraction :-

$$c_{ij} = a_{ij} - b_{ij}$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-5 & 3-6 & 4-7 \\ 5-8 & 6-9 & 7-1 \\ 8-2 & 9-3 & 1-4 \end{bmatrix} = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & 6 \\ 6 & 6 & -3 \end{bmatrix}$$

* Multiplication Of Matrix :-

$$c_{ij} = a_{ij} \times b_{ji}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 9 & 2 \\ 3 & 4 & 7 \\ 4 & 9 & 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2\times5+3\times3+4\times4, & 2\times9+5\times4+4\times9 & 2\times2+2\times7+2\times3 \\ 5\times5+6\times3+7\times4 & 5\times9+6\times4+7\times9 & 5\times2+6\times7+7\times3 \\ 8\times5+9\times3+1\times4 & 8\times9+9\times4+1\times9 & 8\times2+9\times7+1\times3 \end{bmatrix}$$

$$\begin{bmatrix} 10+9+16 & 18+12+36 & 4+14+6 \\ 25+18+28 & 45+24+63 & 10+42+21 \\ 40+27+4 & 72+36+9 & 16+63+3 \end{bmatrix}$$

$$c_{ij} = \begin{bmatrix} 35 & 66 & 37 \\ 71 & 132 & 73 \\ 71 & 117 & 82 \end{bmatrix}$$

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* Multiplication Of Matrix by Scalar:-

The Multiplication of a Matrix 'A' by a Scalar 'S' has the effect of Multiplying each element by a_{ij} in the matrix by the scalar. The resulting element of Matrix 'B' can be express as b_{ij}

$$\boxed{b_{ij} = S a_{ij}}$$

i.e $\boxed{B = SA}$

eg $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 9 & 10 & 11 \end{bmatrix}$ if $S = 5$

$\rightarrow$ $5\begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 9 & 10 & 11 \end{bmatrix} = \begin{bmatrix} 5(2) & 5(4) & 5(6) \\ 5(1) & 5(3) & 5(5) \\ 5(9) & 5(10) & 5(11) \end{bmatrix} = \begin{bmatrix} 10 & 20 & 30 \\ 5 & 15 & 25 \\ 45 & 50 & 55 \end{bmatrix}$

eg 4 $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$ $S = \frac{2}{3}$ Find SA

$\rightarrow$ $\frac{2}{3}\begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 4/3 & 8/3 \\ 2 & 10/3 \\ -2/3 & -4/3 \end{bmatrix}$

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Note:— If A is any Matrix & $s = -1$ then $-A$ is a scalar multiple matrix of A & is called the Negative of Matrix A. Thus if $A = [a_{ij}]_{m \times n}$

$$-A = [-a_{ij}]_{m \times n}$$