MATHEMATICS-II

Time Allowed: 2.5 Hours   Full Marks : 60

Answer the following questions from Group A, B & C as directed

Group A

1. **Choose the correct alternatives(any ten) :** 1×10

i) The value of $\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix}$ is   (a) 2   (b) 0   (c) -1   (d) none of these.

ii) Determinant value of any skew symmetric matrix of order 2x2 is

   (a) strictly positive   (b) strictly negative   (c) 0   (d) none of these.

iii) The co-factor of the element 5 in $D = \begin{vmatrix} 1 & 3 & 4 \\ -1 & 0 & 2 \\ 3 & 5 & 8 \end{vmatrix}$ is   (a) 5   (b) 6   (c) 0   (d) -6.

iv) The value of k for which the points (1, 2), (k, 4) and (3, 6) are collinear is

   (a) 1   (b) 2   (c) 3   (d) none of these.

v) The value of k for which the line $2x + 3y + 4 + k(6x - y + 11) = 0$ is parallel to the y-axis is

   (a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 3   (d) none of these.

vi) Coordinates of centre of the circle $2x^2 + 2y^2 - 8x - 5 = 0$ are

   (a) $(0, 2)$ (b) $(2, 0)$ (c) $(4, 0)$ (d) $(-4, 0)$

vii) $\int e^{1-\log x} \, dx$ is equal to   (a) ex + c   (b) $e \log|x| + c$ (c) $x \log e + c$ (d) none of these.

viii) $\int \frac{\cos x}{\sqrt{1 + \cos 2x}} \, dx =$ (a) x + c   (b) $\frac{x}{2} + c$ (c) $\frac{x}{\sqrt{2}} + c$ (d) none of these.

ix) The order and degree of the differential equation $\left(\frac{d^2 y}{dx^2}\right)^5 - x^3 \left(\frac{dy}{dx}\right)^3 = 0$ are

   (a) 1 and 2   (b) 2 and 5   (c) 2 and 3   (d) none of these.

x) The Integrating Factor (IF) of the differential equation $\frac{dy}{dx} + \frac{1}{x} y = \sin x$ is

   (a) x   (b) $\frac{1}{x}$ (c) $\log_e x$ (d) none of these .

xi) General solution of the differential equation $\frac{dy}{dx} = 1$ is

   (a) y = c   (b) x = 0   (c) y = x   (d) none of these.

xii) The differential equation whose general solution is $y = A \cos x + B \sin x$ is

   (a) $\frac{d^2 y}{dx^2} = y$ (b) $\frac{d^2 y}{dx^2} = -y$ (c) $\frac{d^2 y}{dx^2} = 0$ (d) none of these.

xiii) If $u = 5x^2 + 7xy + 3y^2$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ equals to

   (a) u   (b) 2u   (c) 4u   (d) none of these.

xiv) The median of the data 6, 4, -1, 0, 4, 8, 9 is

    (a) 0   (b) 4   (c) 6   (d) none of these.

xv) Two coins are tossed. The probability of getting at least one head is

    (a) $\dfrac{3}{4}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) none of these.

2. **Fill in the blanks(any ten) :**　　　　　　　　　　　　　1×10

    i) For a matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 0 & 5 \end{pmatrix}$, $A^{T} = $ _____.

    ii) Matrix $(1 \quad 2)\begin{pmatrix} -2 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is equal to _____.

    iii) The area bounded by the curves $4x + 3y = 12$, $y = 0$ and $x = 0$ is _____.

    iv) If the length of the diameter of the circle $x^2 + y^2 + 4x - 6y - k = 0$ is 4 then value of k is

    _____.

    v) $x^2 - y^2 = 2022$ represents a _____.

    vi) If $\dfrac{d}{dx}[\varphi(x)] = f(x)$, then $\int f(x)\,dx$ is _____.

    vii) $\int_{0}^{1} |x|\,dx = $ _____.

    viii) $\int_{0}^{1} \dfrac{1-x}{1+x}\,dx = $ _____.

    ix) Particular Integral(PI) of the differential equation $\dfrac{d^2y}{dx^2} + 4y = 5e^{2x}$ is _____.

    x) If the differential equation $(2x^2 + 4y)dx + k(4x + y - 1)dy = 0$ is exact then value of k is

    _____.

    xi) The Complementary Function(CF) of $\dfrac{d^2y}{dx^2} - 16 = xe^{3x}$ is _____.

    xii) $y = c_1 + c_2 e^x$ is the solution of the differential equation _____.

    xiii) If $u = e^{xy}$, then the value of $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y}$ is _____.

    xiv) If the mean of $x - 2$, 10, $x + 3$, 7 is 9, then $x = $ _____.

    xv) The probability of getting an even number from the numbers up to 50(Including 50)

    is _____.

3. **Answer the following questions(any ten) :**　　　　　　　1×10

    I) Find x and y such that $\begin{pmatrix} 3 & x+1 \\ y+2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$.

    II) What is symmetric matrix?

    III) Find the value of x for which $A = \begin{pmatrix} 2 & x \\ 3 & 6 \end{pmatrix}$ is singular matrix.

    iv) Find slope of the line be $2x - 3y = 3$.

    v) Find length of major axis of the ellipse $9x^2 + 16y^2 = 144$.

    vi) Integrate : $\int \cot^2 x\,dx$.

vii) Evaluate : $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x \cos^5 x \, dx$.

viii) Evaluate: $\displaystyle\int_1^2 \log x \, dx$.

ix) If the roots of Auxiliary Equation(AE) of a linear differential equation of 2nd order with constant coefficients be $2 \pm 3i$, find the Complimentary Function(CF).

x) Find $\dfrac{1}{D^2 + 5}(\sin 2x)$.

xi) If $u = 7xy^2$, find $\dfrac{\partial^2 u}{\partial x \partial y}$.

xii) State Euler's theorem on homogeneous function.

xiii) If $y = 2x - 5$ and $\bar{x} = 7$, find the value of $\bar{y}$.

xiv) Find the mode of the data: 7, -1, 2, 3, 5, 0, 2, 3, 2, 3, 0, 4, 2, 5, 1

xv) Let A and B be two events and $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.4$, find the value of $P(A \cup B)$.

Group B

4. **Answer the following questions(any six) :**   2×6

i) If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, find A(adj A).

ii) Show that $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix} = - \begin{vmatrix} \alpha^2 & \beta^2 & \gamma^2 \\ \alpha & \beta & \gamma \\ 1 & 1 & 1 \end{vmatrix}$

iii) Transfer the Cartesian form of the equation $x^2 + y^2 = 2ax$ to its polar form.

iv) If the lines joining the points (a, -2) and (4, -a) be parallel to the line joining (2, 1) and (3, 4), find the value of a.

v) Integrate: $\displaystyle\int \dfrac{dx}{1 - \cos x}$.

vi) Evaluate : $\displaystyle\int_0^1 \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}} dx$.

vii) Solve : $xdy - ydx = (x^2 + y^2)dx$

viii) Verify Euler's Theorem for the function $f(x, y) = 3x^2 + 4y^2$.

ix) Find the variance of : 5, 6, 7, 8, 10.

x) An urn contains 12 white balls and 6 red balls. A ball is drawn at random. What is the probability of being it to be a white?

5. Answer the following question(any one) :

   i)

   a) If A be a skew symmetric matrix then prove that A^2 is symmetric.   2

   b) Evaluate the determinant $\begin{vmatrix} 0 & -1 & -1 & 1 \\ 4 & 5 & 1 & 1 \\ 3 & 9 & 4 & 1 \\ 4 & 4 & 4 & 1 \end{vmatrix}$ using Chio's method.   4

   ii)

   a) Without expanding prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$.   2

   b) If $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$, prove that $A^2 - 4A - I = 0$, where I is the unit matrix of order 2

   and hence find A^{-1}.   4

   iii) A straight line passes through the point (7, 3) and has intercepts on the axes in equal magnitude but opposite in sign. Find its equation. Also find the co-ordinates of the point on the line at which the abscissa is triple the ordinate.   6

6. Answer the following question(any one) :

   i) Evaluate : a) $\int \dfrac{e^{4\log x} - e^{3\log x}}{e^{\log x} - 1}\,dx$ b) $\int_0^{\frac{\pi}{2}} \dfrac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$.   2 + 4

   ii) Solve : $\dfrac{d^2y}{dx^2} - 4a\dfrac{dy}{dx} + 4a^2 y = x^2 e^{2ax}$.   6

   iii) Mark the area bounded by $y^2 = 2x$, x = 1 and x = 4 and find the enclosed area.   6

7. Answer the following question(any one) :

   i) A distribution consists of three components with frequencies 200, 250, 300 having means 25, 10 and 15; standard deviation 3, 4 and 5 respectively. Prove that the mean of the combined distribution is 16 and its standard deviation is 7.2 approximately.   6

   ii) A coin and a die are thrown simultaneously. Find the probabilities of occurrence of
   a) head and even face   b) tail and multiple of three.   6

   iii) If $u = \cos^{-1} \dfrac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{2}\cot u = 0$.   6

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