Indeterminate forms:-

L'Hospital Rule:-

If $\lim\limits_{x \to a} f(x) = f(a) = 0$

$\lim\limits_{x \to a} g(x) = g(a) = 0$

~~then~~ $\cancel{\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}}$

then $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$

Note: $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$

if $f'(a) = 0,\ g'(a) = 0$

then $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f''(x)}{g''(x)}$

we repeat this procedure until we get find value of limit.

(2)

(1) Evaluate $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$

$$L=\lim_{x\to 0} \frac{e^x-1-x}{x^2} \quad \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0} \frac{e^x-0-1}{2x} = \lim_{x\to 0} \frac{e^x-1}{2x} \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0} \frac{e^x-0}{2(1)} = \lim_{x\to 0} \frac{e^x}{2}$$

$$=\frac{e^0}{2}=\frac{1}{2}$$

(2) $\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x}$

$$L=\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x} \quad \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0} \frac{\tan x - x}{x^2 . x . \frac{\tan x}{x}}$$

$$=\lim_{x\to 0} \frac{\tan x - x}{x^3 (1)}$$

$$=\lim_{x\to 0} \frac{\tan x - x}{x^3}$$

$$\boxed{\lim_{x\to 0} \frac{\tan x}{x} = 1}$$

$$= \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} \quad \left(\frac{0}{0}\right)$$

$\sec^2 0 = 1$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$$= \lim_{x\to 0} \frac{\tan^2 x}{3x^2}$$

$$= \frac{1}{3}(1) = \frac{1}{3}$$

$\lim_{x\to 0} \frac{\tan x}{x} = 1$

$\therefore \lim_{x\to 0} \frac{\tan^2 x}{x^2} = 1^2 = 1$

8. Ex: $\lim_{x\to a} \frac{\log(x-a)}{\log(e^x - e^a)}$

$\log 0 = \infty$

$$l = \lim_{x\to a} \frac{\log(x-a)}{\log(e^x - e^a)} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to a} \frac{\frac{1}{x-a}(1-0)}{\frac{1}{e^x - e^a}(e^x - 0)}$$

$$= \lim_{x\to a} \frac{e^x - e^a}{e^x(x-a)} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to a} \frac{e^x - 0}{e^x(x-a) + e^x(1-0)}$$

$$= \lim_{x\to a} \frac{e^x}{e^x(x-a) + e^x} = \frac{e^a}{e^a(a-a) + e^a}$$

$$= \frac{e^a}{0 + e^a} = \frac{e^a}{e^a} = 1$$

Ex $\lim_{x\to 0} \log_{\tan x} \tan 2x$

$$\log_y^x = \frac{\log_e^x}{\log_e^y}$$

$$L = \lim_{x\to 0} \frac{\log_e \tan 2x}{\log_e \tan x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to 0} \frac{\frac{1}{\tan 2x} \cdot \sec^2 2x \cdot 2(1)}{\frac{1}{\tan x} \cdot \sec^2 x}$$

$$= \lim_{x\to 0} \frac{\tan x \cdot \sec^2 2x \cdot 2}{\tan 2x \cdot \sec^2 x}$$

$$= \lim_{x\to 0} \frac{x \times \left(\frac{\tan x}{x}\right) \cdot \sec^2 2x \cdot 2}{2x \times \left(\frac{\tan 2x}{2x}\right) \sec^2 x}$$

$$\lim_{x\to 0} \frac{\tan x}{x} \to 1$$

$$= \lim_{x\to 0} \frac{\cancel{x} \cdot \sec^2 2x \cdot \cancel{2}}{\cancel{2}\cancel{x} \cdot \sec^2 x}$$

$$= \frac{\sec^2 0}{\sec^2 0} = \frac{1}{1} = 1$$

Ex: $\lim_{x\to 1} (1-x) \tan \frac{\pi x}{2}$

$$L = \lim_{x\to 1} (1-x) \tan \frac{\pi x}{2} \quad (0 \times \infty)$$

$$= \lim_{x\to 1} \frac{(1-x)}{\cot \frac{\pi x}{2}}$$

$$\frac{1}{\cot x} = \tan x$$

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$$= \lim_{x\to 1} \frac{0-1}{-\operatorname{cosec}^2 \frac{\pi x}{2} \cdot \frac{\pi}{2}}$$

$$= \frac{1}{\operatorname{cosec}^2 \frac{\pi}{2}} \cdot \frac{2}{\pi}$$

$$= \frac{1}{1^2} \cdot \frac{2}{\pi} = \frac{2}{\pi}$$

Ex $\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

$$= \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right) \quad (\infty - \infty)$$

$$= \lim_{x\to 0} \left(\frac{x - \sin x}{x \cdot \sin x}\right) \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \left(\frac{x - \sin x}{x \cdot x \cdot \frac{\sin x}{x}}\right) \quad \Big| \; \lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x\to 0} \left(\frac{x - \sin x}{x^2}\right) \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{1-\cos x}{2x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{0-(-\sin x)}{2(1)}$$

$$= \lim_{x\to 0} \frac{\sin x}{2} = \frac{\sin 0}{2} = \frac{0}{2} = 0$$

Ex $\lim_{x\to 0} (\cot x)^x$

$L = \lim_{x\to 0} (\cot x)^x \quad (\infty^0)$

$\log_e x^n = n \log_e x$

$\log_e L = \lim_{x\to 0} \log_e (\cot x)^x$

$= \lim_{x\to 0} x \cdot \log_e \cot x$

$= \lim_{x\to 0} \dfrac{\log_e \cot x}{\frac{1}{x}} \quad \left(\dfrac{\infty}{\infty}\right)$

$= \lim_{x\to 0} \dfrac{\frac{1}{\cot x} \cdot (-\operatorname{cosec}^2 x)}{-1/x^2}$

$= \lim_{x\to 0} \dfrac{x^2 \operatorname{cosec}^2 x}{\cot x}$

$\dfrac{1}{\sin x} = \operatorname{cosec} x$

$= \lim_{x\to 0} \dfrac{1}{\cot x} \cdot x^2 \cdot \dfrac{1}{\sin^2 x}$

$= \lim_{x\to 0} \tan x \cdot \dfrac{x^2}{\sin^2 x}$

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

$\lim_{x\to 0} \dfrac{x}{\sin x} = 1$

$= \lim_{x\to 0} \tan x (1)$

$= \tan 0$

$= 0$

$\log_e L = 0 \qquad \therefore L = e^0 = 1$

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Ex evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$

$$L=\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}} \quad (1^\infty)$$

$$\log_e L=\lim_{x\to 0}\log_e\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\frac{1}{x}\cdot\log_e\left(\frac{a^x+b^x+c^x}{3}\right)$$

$$=\lim_{x\to 0}\frac{\log\left(\frac{a^x+b^x+c^x}{3}\right)}{x} \quad \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{\frac{1}{\left(\frac{a^x+b^x+c^x}{3}\right)}\left(\frac{a^x\log a+b^x\log b+c^x\log c}{3}\right)}{1}$$

$$=\frac{1}{\left(\frac{a^0+b^0+c^0}{3}\right)}\left(\frac{a^0\log_e a+b^0\log_e b+c^0\log_e c}{3}\right)$$

$$=\frac{1}{\left(\frac{1+1+1}{3}\right)}\left(\frac{\log_e a+\log_e b+\log_e c}{3}\right)$$

$$=\frac{1}{(3/3)}\frac{\log_e(abc)}{3}$$

$\log_e x+\log_e y$
$=\log_e(xy)$

$$=\frac{1}{3}\log_e(abc)$$

$\log_e x^n=n\log_e x$

$$\log_e L=\log_e(abc)^{1/3}$$

$$\therefore L=(abc)^{1/3}$$

i.e $L=\sqrt[3]{abc}$