Curve Tracing

* ~~Curves~~ Tracing of Cartesian Curve

(1) Symmetry
(2) Origin & tangent at origion (0,0)
(3) Intercept
(4) Asymptote
(5) ~~Region~~ Region
(6) Curve

* Procedure:

(1) Symmetry :

(i) Symmetry about X-axis :

Replace y by $-y$, if eq^n remains unchanged then there is symmetry about X-axis.

or if curve contains all even powers of y ;

(ii) Symmetry about Y-axis :

Replace x by $-x$, if eq^n remains unchanged then there is symmetry about Y-axis.

or

If curve contains all even powers of x.

(iii) Symmetry about origin :

Replace x by $-x$ & y by $-y$, if eq^n remains unchanged then there is symmetry about origin.

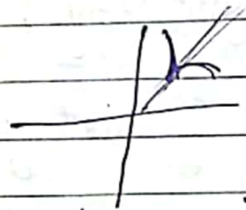
(2) Origin & Tangent at origin:

- The Curve passing through the origin if the equation of the Curve does not Contein the Constant term.

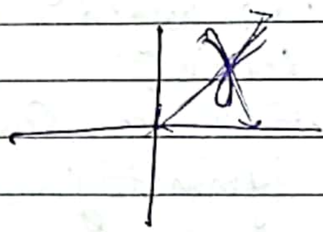
For the tangent, Take

lowest degree term = 0

if we have two tangent which are real & Coincident ~~distinct~~ (equal) then it's a cusp.



if we have two tangent which are real & distinct then it's a Node.



(3) Intercept:

X-intercept: Put $y=0$ & find x.

Y-intercept: put $x=0$ & find y.

(4) Asymptotes :

(i) Asymptotes $\parallel^{al}$ to X-axis :

Take coeff. of heighest degree term of x $= 0$

(ii) Asymptote $\parallel^{al}$ to y-axis :

coeff. of highest degree term of y $= 0$

(5) Region :

(6) Curve :-

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Ex:-1 Sketch the Curve $xy^2 = 4a^2(2a-x)$

Ex:-2 Sketch the Curve $y^2(a+x) = x^2(a-x)$

Sol^n. $ay^2 + xy^2 - ax^2 + x^3 = 0$

(I) Symmetry:

Given Curve is symmetric w.r.t. X-axis as it Contains all even power terms of y.

there is no symmetry about y-axis & origin.

(II) Origin & tangent at origin

Since, there is no Constant term in the given eq^n of curve.
So Curve passes through origin

Tangent at Origin:

lowest degree term = 0

$$ay^2 - ax^2 = 0$$
$$y^2 = x^2$$

$$\boxed{y = \pm x}$$ (real & distinct)

∴ Double point is a node.

(III) Intercept :

X - intercept : put $y=0$

$$x^3 - ax^2 = 0$$
$$x^2(x-a) = 0$$
$$\Rightarrow x = 0 \text{ or } x = a$$

$\therefore (0,0)$ & $(a,0)$ are intercept

Y - intercept : put $x=0$

$$ay^2 = 0 \Rightarrow y = 0$$

$\therefore (0,0)$ is intercept.

(IV) Asymptotes :

Asymptotes $\parallel^{al}$ to X-axis :

Coeff. of highest degree term of $x = 0$

$\Rightarrow 1 = 0$ ✗

$\therefore$ asymptote $\parallel^{al}$ to X-axis not possible.

Asymptote $\parallel^{al}$ to Y-axis

Coeff of highest deree term of $y = 0$

$$a + x = 0$$

$\boxed{x = -a}$ is asymptote $\parallel^{al}$ to Y-axis.

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(V) Region :

$$y^2 = \frac{x^2(a-x)}{a+x}$$

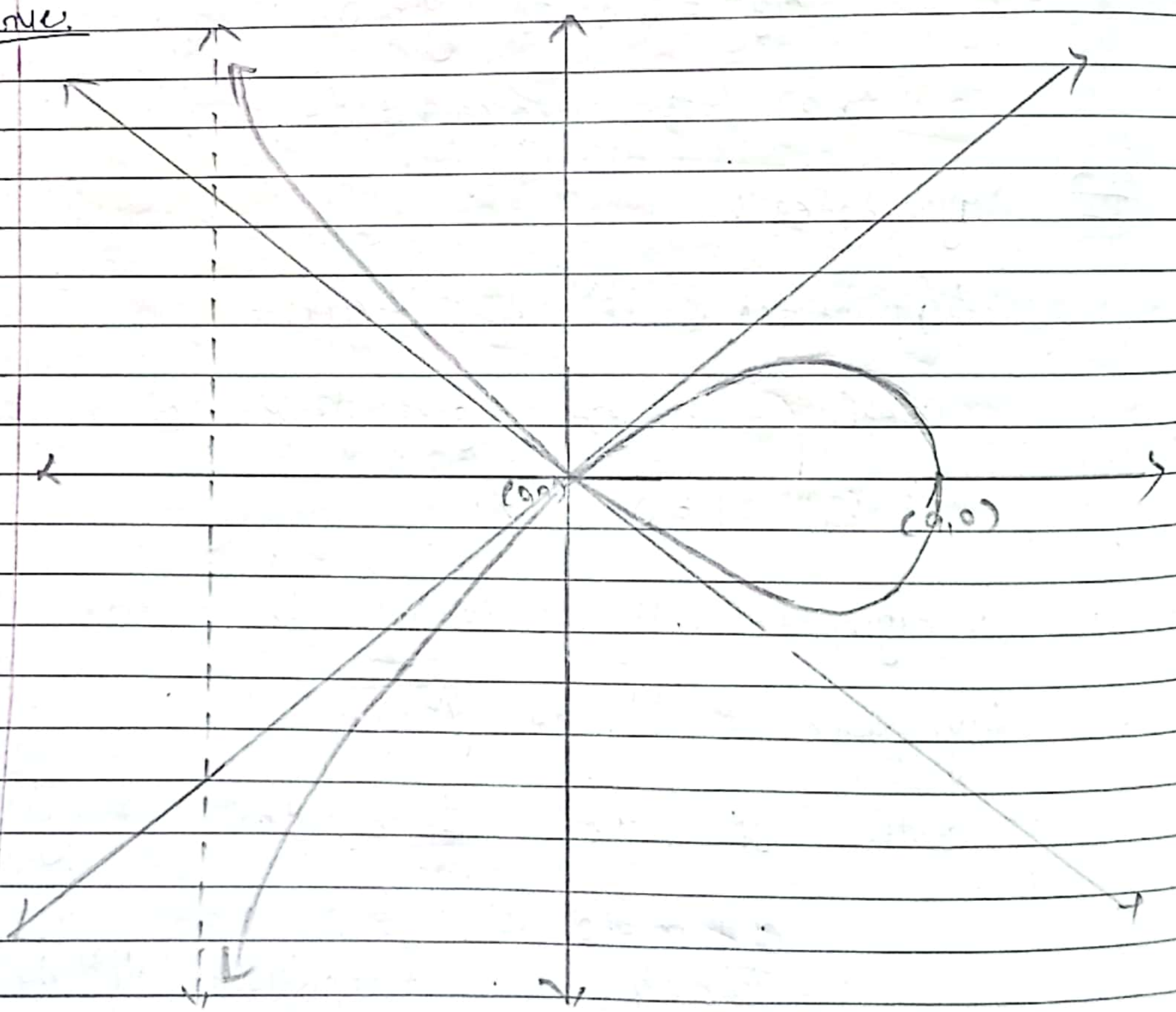
$$y^2 \geq 0 \Rightarrow \frac{x^2(a-x)}{(a+x)} \geq 0$$

if $a-x \geq 0$ & $a+x \geq 0$

i.e $x \leq a$ & $x \geq -a$

i.e $-a \leq x \leq a$

Curve.



Ex: 3 sketch the curve $y^2(2a-x) = x^3$ $(a>0)$

Sol: We have $y^2(2a-x) = x^3$

$$\Rightarrow 2ay^2 - xy^2 - x^3 = 0 \quad \text{--- (1)}$$

(I) Symmetry:

equation contains only even powers of y, so the curve has symmetry about X-axis.

There is no symmetry about y-axis & origin.

(II) Origin & tangent at origin.

Since,

There is no constant term in the eqn so the curve passes through origin

Tangent at origin:

lowest degree term $= 0$

$2ay^2 = 0$

$y^2 = 0$

$y = 0, 0$, real & coincident tangent

$\therefore$ Origin is a cusp.

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(III) Intercepts:

X-intercept : put $y=0 \Rightarrow x=0$
$\therefore (0,0)$ is intersecting point.

Y-intercept : put $x=0 \Rightarrow y=0$
$\therefore (0,0)$ is intersecting point.

(IV) Asymptotes:

Asymptote $\parallel$ to X-axis:

coeff of highest degree term of $x = 0$

$-1 = 0$ $\times$ $\therefore$ not possible

Asymptote $\parallel$ to Y-axis:

coeff. of highest degree term of $y = 0$

$\Rightarrow 2a - x = 0$

$\Rightarrow x = 2a$

$\therefore$ Curve has vertical asymptote $x = 2a$

(V) Region:

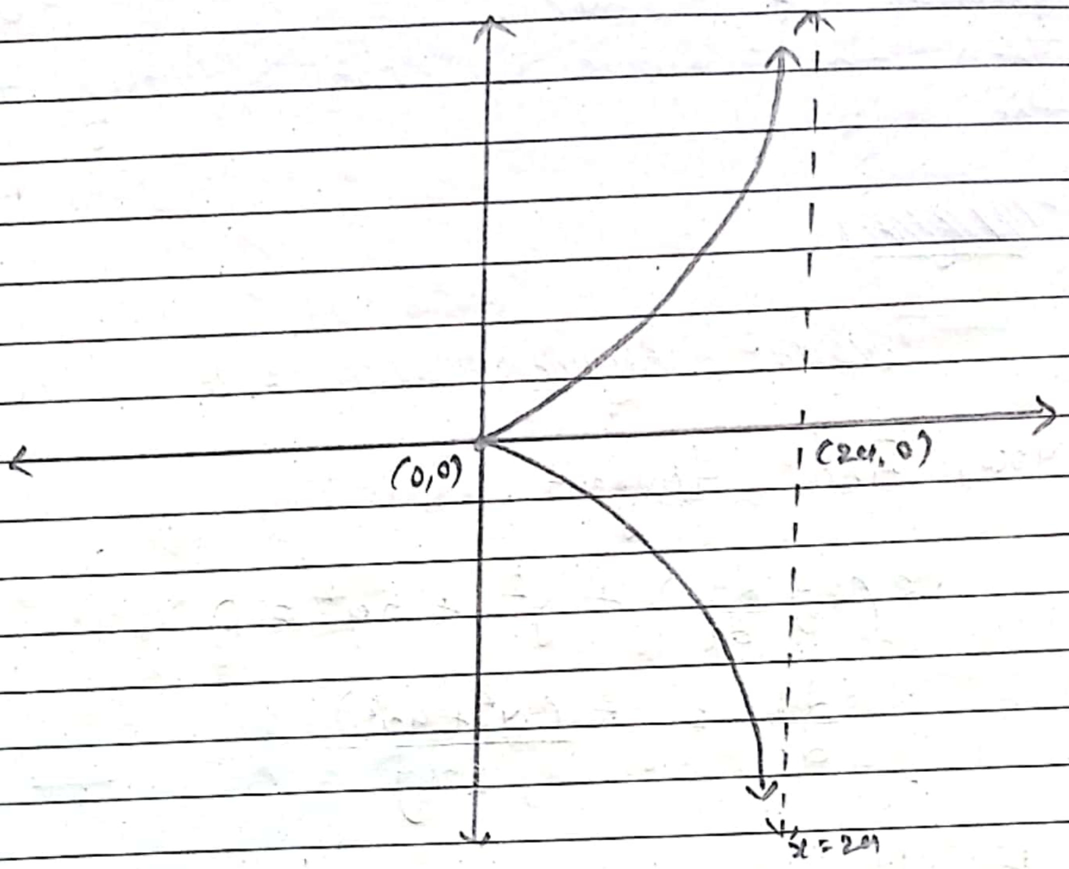
$$y^2 = \frac{x^3}{2a-x}$$

Since, $y^2 \geq 0$, $\dfrac{x^3}{2a-x} \geq 0$

$\Rightarrow x \geq 0$ & $2a - x > 0$

i.e. $x \geq 0$ & $x < 2a$

$\therefore$ Curve lies in $0 \leq x < 2a$



Ex:- Trace the curve $xy^2 = 4a^2(2a - x)$, $a > 0$.

Sol: Here, $xy^2 - 8a^3 + 4a^2x = 0$ —— (1)

(I) Symmetry:

About X-axis: eqn contains only even power of y. therefore curve has symmetry about X-axis.

There is no symmetry about Y-axis & origin.

(II) Origin & tangent :

There is a Constant term in given equation of Curve. Then the Curve is not passing through the origin.

~~[illegible]~~

~~[illegible]~~ ~~degree~~ ~~[illegible]~~ 4 ~~[illegible]~~

Now, Find tangent.

$$x\left(2y\frac{dy}{dx}\right) + y^2 + 4a^2 = 0$$

$$\therefore \frac{dy}{dx} = \frac{-(y^2+4a^2)}{2xy}$$

if $\frac{dy}{dx} \to 0$ it gives horizontal tangent.

if $\frac{dy}{dx} \to \infty$ it gives Vertical tangent.

If $\frac{dy}{dx} \to \infty$

i.e $x = 0$ or $y = 0$

If $x = 0$ form eqn (1)
we Can not find any value of y.

if $y = 0$ from eqn (1)

$$4a^2 x = 8a^3 \qquad \boxed{x = 2a}$$

$\therefore$ ~~$x = a$ is vertical tangent.~~

$\therefore$ at point $(2a, 0)$ there is a vertical tangent

Now, if $\frac{dy}{dx} \to 0$ i.e. $y^2 + 4a^2 = 0$

$y^2 = -4a^2$ ✗.

(III) Intercepts: X-intercept: put $y = 0$

we get $x = 2a$, point $(2a, 0)$

Y-intercept: put $x = 0$

$-8a^3 = 0$ ✗.

(IV) Asymptotes:

(i) $\parallel^{al}$ to X-axis:

coeff. of highest degree term of $x = 0$

$y^2 + 4a^2 = 0 \Rightarrow y^2 = -4a^2$ ✗.

(ii) $\parallel^{al}$ to Y-axis:

coeff. of highest degree term of $y = 0$

$\boxed{x = 0}$ is Asymptote. $\parallel^{al}$ to Y-axis.

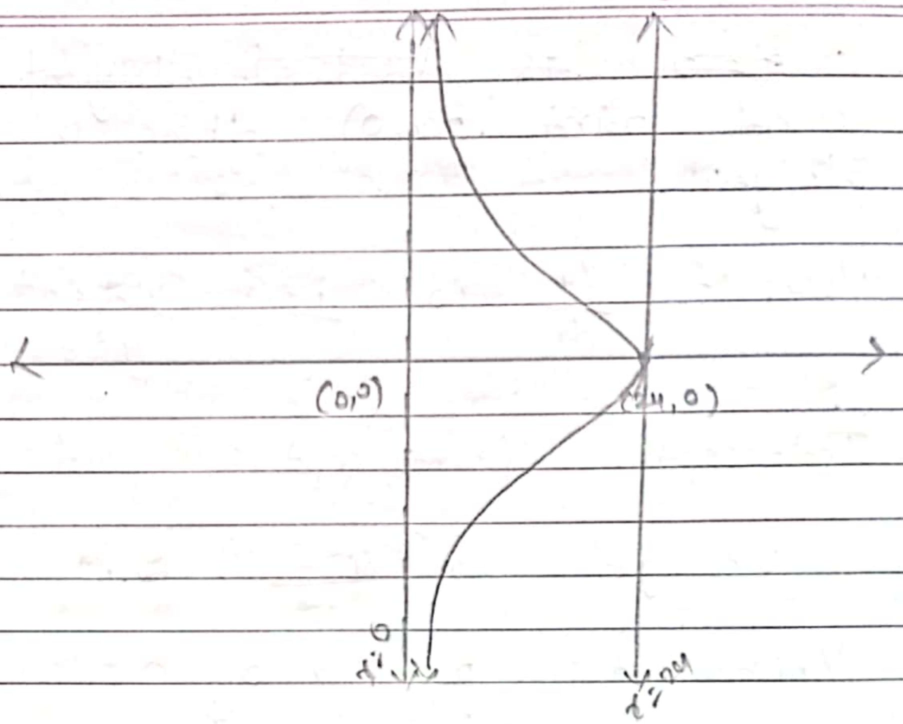
(V) Region: Here, $y^2 = \frac{4a^2(2a - x)}{x}$

$y^2 \geq 0$ ~~i.e~~ $\frac{4a^2(2a - x)}{x} \geq 0$

i.e $x \geq 0$ & $2a - x \geq 0$

$x \leq 2a$

$\therefore 0 \leq x \leq 2a$



Ex:- Sketch the Curve $a^2x^2 = y^3(2a-y)$ $(a>0)$

Sol^n:- Here given that,

$$a^2x^2 - 2ay^3 + y^4 = 0 \quad \text{--- (1)}$$

(I) Symmetry :

Since, equation Contains only even power of x then Curve has Symmetry about y-axis.

There is no symmetry about X-axis & origin

(II) Origin & tangent to the origin:

Since, there is no Constant term in the given eq^n of the Curve.

$\therefore$ The Curve passing through the origin

Tangent to the origin:

lowest degree term $=0$

$$a^2x^2=0$$

$$\Rightarrow x=0,0 \quad \text{(real \& coincident)}$$

$\therefore$ There is a cusp at origin.

(III) Intercepts:

X-intercept: Put $y=0$,

$$a^2x^2=0$$
$$\Rightarrow x=0 \qquad (0,0) \text{ is intercept.}$$

Y-intercept: put $x=0$

$$y^4-2ay^3=0$$
$$y^3(y-2a)=0$$

$$\Rightarrow y=0 \text{ or } y=2a$$

$\therefore (0,0)$ & $(0,2a)$ are intercept.

(IV) Asymptotes:

$\parallel^{al}$ to X-axis:

Coeff. of highest power term of $x=0$

$a^2=0$ $\times$

$||^{al}$ to y-axis :

Coeff. of highest power term of $y = 0$

$1 = 0$ ✗.

$\therefore$ Vertical asymptote are not possible

(V) **Region** :

$$x^2 = \frac{y^3(2a-y)}{a^2}$$

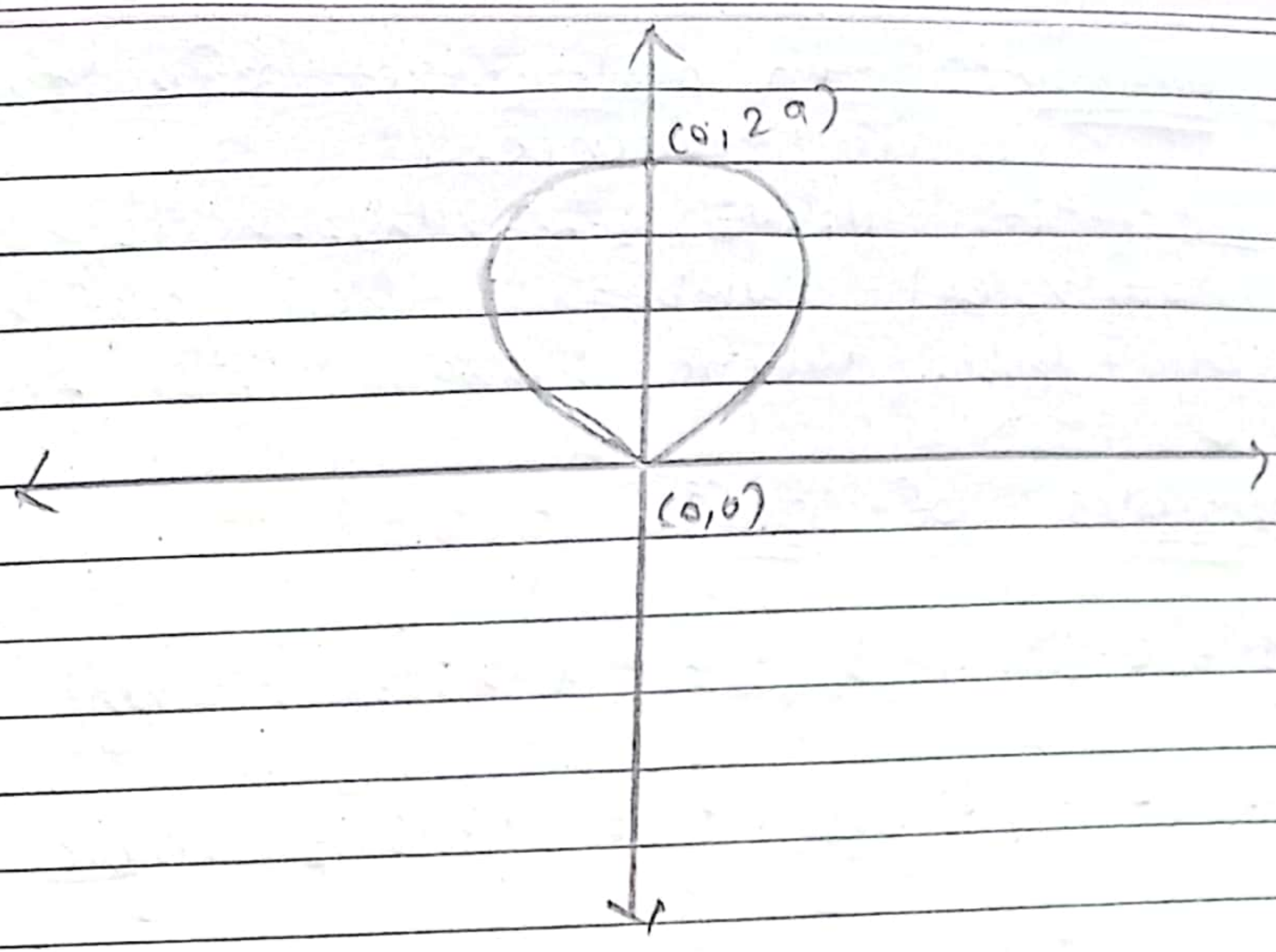
Since, $x^2 \geq 0$

$$\Rightarrow \frac{y^3(2a-y)}{a^2} \geq 0$$

$\Rightarrow y \geq 0$ or $2a - y \geq 0$

$y \leq 2a$

$\Rightarrow 0 \leq y \leq 2a$


(0, 2a)
(0,0)