Gradient: The gradient of a function $z = f(x,y)$ is written as grad f and defined as

$$\text{grad} f = \nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y}$$

similarly if $u = f(x,y,z)$ then

$$\text{grad} u = \nabla u = i\frac{\partial u}{\partial x} + j\frac{\partial u}{\partial y} + k\frac{\partial u}{\partial z}$$

Ex: $g(x,y) = \frac{x^2}{2} - \frac{y^2}{2}$ find gradient at $(\sqrt{2}, -1)$

$$\text{grad} g = \nabla g = i\frac{\partial g}{\partial x} + j\frac{\partial g}{\partial y}$$

$$= i\left(\frac{2x}{2} - 0\right) + j\left(0 - \frac{2y}{2}\right)$$

$$\nabla g = ix - jy$$

$$\therefore (\nabla g)(\sqrt{2}, -1) = i\sqrt{2} - j(-1)$$

$$= i\sqrt{2} + j$$

Ex: $\phi(x,y,z) = \ln(x^2+y^2+z^2)$ find grad ϕ

$$\text{grad}\phi = \nabla\phi = i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{\partial y} + k\frac{\partial \phi}{\partial z}$$

Here ϕ is function of x, y, z

Here $\phi(x,y,z) = \ln(x^2+y^2+z^2)$

i.e $\phi(x,y,z) = \log_e(x^2+y^2+z^2)$

$$\therefore \nabla\phi = i\frac{1}{x^2+y^2+z^2}(2x+0+0) + j\frac{1}{x^2+y^2+z^2}(0+2y+0) + k\frac{1}{x^2+y^2+z^2}(0+0+2z)$$