$$= \frac{p_1 r[\gamma r^{\gamma-1}(\rho-1)-(\rho^{\gamma-1})]}{(r-1)(\gamma-1)}$$

Example 5.1 *A heat engine receives heat at the rate of 1500 kJ/min and gives an output of 8 kW. Calculate (a) the thermal efficiency, and rate of heat rejection.*

Solution.

Given : $Q_1 = 1500 \text{ kJ/min} = \frac{1500}{60} = 25 \text{ kJ/s}, w = 8 \text{ kW or } 8 \text{ kJ/s}$

(*a*) $\eta_{th} = \frac{W}{Q_1} = \frac{8}{25} = 0.32 \text{ or } 32\%$

(*b*) Rate of heat rejection, $Q_2 = Q_1 - W = 25 - 8 = 17 \text{ kJ/s}$

Example 5.2 *Determine the coefficient of performance and heat transfer rate in the condenser of a refrigerator in kJ/h which has a refrigeration capacity of 15 MJ/h when power input is 1.0 kW.*

Solution.

Refrigeration capacity, $Q_2 = 15 \text{ kJ/h}$

Work input, $\omega = 1 \text{ kW} = 1 \times 3600 = 3600 \text{ kW/h or } 3.6 \text{ MJ/h}$

Coefficient of performance, $(COP)_{ref} = \frac{Q_2}{W}$

$$= \frac{15}{3.6} = 4.167$$

Heat transfer rate in condenser, $Q_1 = Q_2 + W$

$$= 15 + 3.6 = 18.6 \text{ MJ/h}$$

Example 5.3 *A domestic food refrigerator maintains a temperature of −10°C. The ambient air temperature is 30°C. If heat leaks into the freezer at the rate of 2 kJ/s, determine the least power necessary to pump this heat out continuously.*

Solution.

Freezer temperature, $T_2 = 273 - 10 = 263 \text{ K}$

Ambient air temperature, $T_1 = 273 + 30 = 303 \text{ K}$

Rate of heat leakage into the freezer, $Q_2 = 2 \text{ kJ/s}$

For minimum power requirement, $\oint \frac{\delta Q}{T} = 0$

or $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$$Q_1 = 2 \times \frac{303}{263} = 2.304 \text{ kJ/s}$$

$\therefore$ $W = Q_1 - Q_2 = 2.304 - 2 = 0.304 \text{ kJ/s}$

Least power required to pump the heat = 0.304 kW

Example 5.4 *A house requires 200 MJ/h for heating in winter. Heat pump is used to absorb heat from outside cold air in winter and send heat to the house. Work required to operate the heat pump is 30 MJ/h. Determine :*

(a) Heat abstracted from outside.

(b) Coefficient of performance.

Solution.

(*a*) Heat to be rejected, $Q_1 = 200$ MJ/h

Required heat pump work, $W_p = 30$ MJ/h

$$Q_1 = W + Q_2$$

where Q_2 = heat abstracted from outside air

$$200 = 30 + Q_2$$

$$Q_2 = 170 \text{ mJ/h}$$

(*b*)
$$(\text{COP})_{hp} = \frac{Q_1}{Q_1 - Q_2}$$

$$= \frac{200}{200-170} = \frac{200}{30} = 6.67$$

Example 5.5 *A Carnot cycle operates between source and sink temperatures of 300°C and −20°C. If the system receives 100 kJ from the source, find : (a) Efficiency of the system, (b) The net work transfer, and (c) Heat rejected to sink.*

Solution.

Heat source temperature, $T_1 = 273 + 300 = 573$ K

Heat sink temperature, $T_2 = 273 - 20 = 253$ K

Heat received, $Q_1 = 100$ kJ

(*a*)
$$\eta_{\text{carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{253}{573} = 0.5585 \text{ or } 55.85\%$$

(*b*) Net work transfer, $W = Q_1 \times \eta_{\text{Carnot}} = 100 \times 0.5585 = 55.85$ kJ

(*c*) Heat rejected to sink, $Q_2 = Q_1 - W = 100 - 55.85 = 44.15$ kJ

√**Example 5.6** *In an engine working on ideal Otto cycle, the temperature at the beginning and end of compression are 45° C and 370° C. Find the compression ratio air-standard efficiency of the engine. Assume $\gamma = 1.4$.*

Solution.

Compression ratio,
$$r = \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$T_1 = 45 + 273 = 318\ K$$
$$T_2 = 370 + 273 = 643\ K$$

$$r = \left(\frac{643}{318}\right)^{\frac{1}{0.4}} = 5.814$$

(*b*) Air standard efficiency,
$$\eta_a = 1 - \frac{1}{r^{\gamma-1}}$$

$$= 1 - \frac{1}{5.814^{0.4}} = 0.505 \text{ or } 50.5\%$$

Example 5.7 *In an Otto cycle, air at 15°C and 1 bar is compressed adiabatically until the pressure is 15 bar. Heat is added at constant volume until the pressure rises to 40 bar. Calculate (a) the air-standard efficiency, (b) the compression ratio, and (c) the mean effective pressure for the cycle. Assume c_v = 0.718 kJ/kg. K, γ = 1.4 and R = 8.314 kJ/kmol.K.*

Solution.

The Otto cycle is shown in Fig. 5.15.

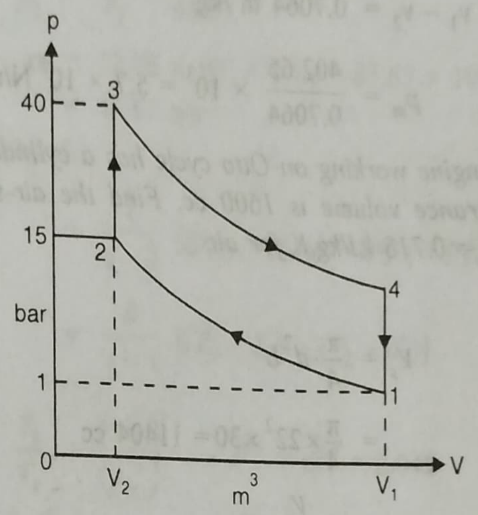

Fig. 5.15

For isentropic process 1-2,

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

Compression ratio, $$r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = 15^{\frac{1}{1.4}} = 6.92$$

Air standard efficiency, $$\eta_a = 1 - \frac{1}{r^{\gamma-1}}$$

$$= 1 - \frac{1}{(6.92)^{0.4}} = 0.539 \quad \text{or} \quad 53.9\%$$

$$T_1 = 15 + 273 = 288 \text{ K}$$

$$T_2 = \left(\frac{p_2 V_2}{p_1 V_1}\right) \cdot T_1 = \frac{15 \times 288}{6.92} = 624.3 \text{ K}$$

For constant volume process 2-3 :

$$T_3 = \frac{p_3}{p_2} T_2 = \frac{40}{6.92} \times 624.3 = 1664.7 \text{ K}$$

Heat supplied $$q_s = c_v (T_3 - T_2)$$

$$= 0.718 (1664.7 - 624.3) = 747.03 \text{ kJ/kg}$$

Work done , $$w = \eta_a q_s$$

$$= 0.539 \times 747.03 = 402.65 \text{ kJ/kg}$$

Mean effective pressure, $$p_m = \frac{\text{Work done}}{\text{Swept volume}} = \frac{w}{v_1 - v_2}$$

Specific Volume, $v_1 = \dfrac{m\,RT_1}{p_1} = \dfrac{8314 \times 288}{29 \times 1 \times 10^5} = 0.8257\ \text{m}^3/\text{kg}$

$$v_2 = \frac{v_1}{r} = \frac{0.8257}{6.92} = 0.1193\ \text{m}^3/\text{kg}$$

$$v_1 - v_2 = 0.7064\ \text{m}^3/\text{kg}$$

$$p_m = \frac{402.65}{0.7064} \times 10^3 = 5.7 \times 10^5\ \text{N/m}^2 \quad \text{or} \quad 5.7\ \text{bar}$$

Example 5.8 *A gas engine working on Otto cycle has a cylinder of diameter 220 mm and stroke 300 mm. The clearance volume is 1600 cc. Find the air-standard efficiency. Assume c_p = 1.004 kJ/kg.K and c_v= 0.718 kJ/kg.K for air.*

Solution.

Swept volume, $V_s = \dfrac{\pi}{4} \cdot d^2 L$

$$= \frac{\pi}{4} \times 22^2 \times 30 = 11404\ \text{cc}$$

Compression ratio, $r = 1 + \dfrac{V_s}{V_c}$

$$= 1 + \frac{11404}{1600} = 8.13$$

Adiabatic index, $\gamma = \dfrac{c_p}{c_v} = \dfrac{1.004}{0.718} = 1.4$

Air standard efficiency, $\eta_a = 1 - \dfrac{1}{r^{\gamma-1}}$

$$= 1 - \frac{1}{(8.13)^{0.4}} = 0.5675 \text{ or } 56.75\%$$

Example 5.9 *A gas engine working on ideal Otto cycle has a compression ratio of 6.1. The pressure and temperature at the commencement of compression are 1 bar and 27°C. The heat supplied during the constant volume combustion process is 1200 kJ/kg. Determine the peak pressure and temperature, work output per kg of air and air-standard efficiency. Assume c_v= 0.718 kJ/kg.K and γ = 1.4 for air.*

Solution.

Isentropic compression process 1-2 :

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma} = 6^{1.4} = 12.28$$

$$p_2 = 1 \times 12.28 = 12.28\ \text{bar} = 12.28 \times 10^5\ Pa$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r^{\gamma-1} = 6^{0.4} = 2.048$$

$$T_2 = 2.048 \times 300 = 614.3\ \text{K} = 341.3°\text{C}$$

Constant Volume heat addition process 2-3 :

$$q_s = q_{2\text{-}3} = c_v (T_3 - T_2)$$
$$1200 = 0.718 (T_3 - 614.3)$$
$$T_3 = 2285.6 \text{ K}$$

$$\frac{p_3}{p_2} = \frac{T_3}{T_2} = \frac{2285.6}{614.3} = 3.72$$

$$p_3 = 12.28 \times 10^5 \times 3.72 = 45.67 \times 10^5 \; Pa$$
$$= 45.67 \text{ bar}$$

Work output, w = area 1-2-3-4

$$= \frac{1}{\gamma - 1}[(p_3 v_3 - p_4 v_4) - (p_2 v_2 - p_1 v_1)]$$

$$= \frac{R}{\gamma - 1}[(T_3 - T_4) - (T_2 - T_1)]$$

$$\frac{T_3}{T_4} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1} = r^{\gamma-1} = 6^{0.4} = 2.048$$

$$R = c_p - c_v = 1.004 - 0.718 = 0.286 \text{ kJ/kg. K}$$

$$w = \frac{0.286}{0.4}[(2285.6 - 1116.0) - (614.3 - 300)]$$

$$w = 611.54 \text{ kJ/kg}$$

Air standard efficiency, $\eta_a = 1 - \frac{1}{r^{\gamma-1}}$

$$= 1 - \frac{1}{6^{0.4}} = 0.5116 \quad \text{or} \quad 51.16\%$$

Example 5.10 *The pressure limit in a Otto cycle are 1 bar and 20 bar. The compression ratio is 5. Calculate (a) thermal efficiency, and (b) mean effective pressure. Assume $\gamma = 1.4$ for air.*

Solution.

(*a*) $\eta_{th} = 1 - \frac{1}{r^{\gamma-1}}$

$$= 1 - \frac{1}{5^{0.4}} = 0.4747 \quad \text{or} \quad 47.47\%$$

(*b*) $\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma} = 5^{1.4} = 9.5182$

$$p_2 = 1 \times 9.5182 = 9.5182 \text{ bar}$$
$$p_3 = 20 \text{ bar}$$

Pressure Ratio, $\alpha_p = \frac{p_3}{p_2} = \frac{20}{9.5182} = 2.1$

$$p_m = p_1 r \left[\frac{(\alpha_p - 1)(r^{\gamma-1} - 1)}{(r-1)(\gamma - 1)} \right]$$

$$= 1 \times 5 \left[\frac{(2.1-1)(5^{0.4}-1)}{(5-1)(1.4-1)} \right]$$

$$= 3.1 \text{ bar}$$

Example 5.11 *An air-standard Otto cycle has a compression ratio of 6. The temperature at the start of compression process is 25°C and the pressure is 1 bar. If the maximum temperature of the cycle is 1150°C, calculate (a) the heat supplied per kg of air, (b) network done per kg of air, and (c) thermal efficiency of cycle. Assume $\gamma = 1.4$, $c_v = 0.778$ kJ/kg.K for air.*

Solution.

Process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = 6^{0.4} = 2.0476$$

$$T_1 = 25 + 273 = 298 \text{ K}$$

$$T_2 = 298 \times 2.0476 = 610.2 \text{ K}$$

Process 3-4 :

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \left(\frac{1}{6}\right)^{0.4} = 0.4884$$

$$T_3 = 1150 + 273 = 1423 \text{ K}$$

$$T_4 = 1423 \times 0.4884 = 694.9 \text{ K}$$

(*a*) Heat supplied, $q_s = c_v (T_3 - T_2) = 0.718\,(1423 - 610.2) = 583.59$ kJ/kg

(*b*) Heat Rejected, $q_r = c_v (T_4 - T_1) = 0.718\,(694.9 - 298) = 284.97$ kJ/kg

Work done per cycle, $w = q_s - q_r = 583.59 - 284.97 = 298.62$ kJ/kg.

Thermal Efficiency, $\eta_{th} = \frac{w}{q_s} = \frac{298.62}{583.59} = 0.5117$ or 51.17 %

Example 5.12 *A petrol engine with compression ratio of 5 develops 20 kW indicated power and consumes 8 litres of fuel per hour. The specific gravity of fuel is 0.78 and its calorific value is 44 MJ/kg. Calculate the indicated thermal efficiency and relative efficiency. Take $\gamma = 1.4$.*

Solution.

Air standard efficiency, $\eta_a = 1 - \frac{1}{r^{\gamma-1}}$

$$= 1 - \frac{1}{5^{0.4}} = 0.4747 \text{ or } 47.47\%$$

Fuel consumption $= 8 \times 0.78 \times 1 = 6.24$ kg/h

$$= \frac{6.24}{3600} = 1.733 \times 10^{-3} \text{ kg/s}$$

Indicated thermal efficiency,

$$(\eta_{th})_i = \frac{\text{Indicated Work}}{\text{Heat Supplied}}$$

$$= \frac{20}{44 \times 10^3 \times 1.733 \times 10^{-3}} = 0.2622$$

$$= 26.22\ \%$$

$$\text{Relative efficiency} = \frac{(\eta_{th})_i}{\eta_a} = \frac{0.2622}{0.4747} = 0.5524 \quad \text{or} \quad 55.24\%$$

Example 5.13 *A diesel engine has a compression ratio of 20 and cut-off takes place at 5% of the stroke. Find the air-standard efficiency. Assume $\gamma = 1.4$.*

Solution.

$$\frac{V_1}{V_2} = r = 20$$

$$V_1 = 20\ V_2$$

$$V_s = V_1 - V_2 = 20\ V_2 - V_2 = 19\ V_2$$

$$V_3 = 0.05\ V_s + V_2 = 0.05 \times 19\ V_2 + V_2$$

$$= 1.95\ V_2$$

Cut-off ratio,

$$\rho = \frac{V_3}{V_2} = 1.95$$

$$\eta_a = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\gamma\ (\rho - 1)}\right]$$

$$\eta_a = 1 - \frac{1}{20^{0.4}}\left[\frac{1.95^{1.4} - 1}{1.4\ (1.95 - 1)}\right]$$

$$= 0.649 \quad \text{or} \quad 64.9\ \%$$

Example 5.14 *In an engine working on diesel cycle, inlet pressure and temperature are 1 bar and 20°C. Pressure at the end of adiabatic compression is 40 bar. The ratio of expansion after constant pressure heat addition is 5. Calculate the heat supplied, heat rejected and the efficiency of the cycle. Assume c_p = 1.004 kJkg.k and c_v = 0.717 kJ/kg.K.*

Solution.

Adiabatic index,

$$\gamma = \frac{c_p}{c_v} = \frac{1.004}{0.717} = 1.4$$

Process 1-2 :

$$r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = (40)^{\frac{1}{1.4}} = 13.942$$

Cut-off ratio,

$$\rho = \frac{V_3}{V_2} = \frac{V_3}{V_1} \times \frac{V_1}{V_2} = \frac{13.942}{5} = 2.788$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (40)^{\frac{1.4-1}{1.4}} = 40^{0.286} = 2.872$$

$$T_2 = 2.872 \times 293 = 841.5\ \text{K}$$

Process 2-3 :

$$T_3 = \frac{T_2 V_3}{V_2} = 841.5 \times 2.788 = 2346.1\ \text{K}$$

Process 3-4 :

$$T_4 = T_3\left(\frac{V_3}{V_2}\right)^{\gamma-1} = T_3\left(\frac{V_3}{V_1}\right)^{\gamma-1}$$

$$= 2346.1\left(\frac{1}{5}\right)^{0.4} = 1232.4 \text{ K}$$

Heat supplied,

$$q_s = c_p(T_3 - T_2) = 1.004\,(2346.1 - 841.5)$$
$$= 1510.62 \text{ kJ/kg}$$

Heat rejected,

$$q_r = c_v(T_4 - T_1) = 0.717\,(1232.4 - 293)$$
$$= 673.55 \text{ kJ/kg}$$

$$\eta_a = \frac{q_s - q_r}{q_s} = 1 - \frac{673.55}{1510.62} = 0.5541 \quad \text{or} \quad 55.41\%$$