Unit 1

Section 1 Acoustics

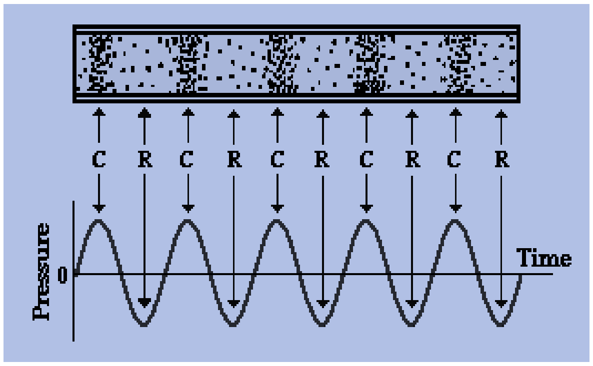


Figure 1

Sound: Sound is a pressure wave. Which propagates in a medium by compression (sanghanan) and rarefaction (vighanan). It is an adiabatic (samoshmi) process where heat transfer will be zero. It requires medium for propagation figure 1.

Period (τ) = the time between successive waveform peaks

Frequency (in cycles/sec or Hertz) = 1/period, f = 1/τ

Acoustics is the study of sound which deals with,

- Origin Properties
- Propagation (by contraction and rarefaction)
- Action on an obstacle [absorption , reflection ,transmission

Architectural Acoustics: It is a science of producing good quality sound by developing aspects of any architectural building

- **Classification of sound on the basis of frequency:**

1) Infrasonic (subsonic)	f<20 Hz
2) Audio	20Hz<f<20 kHz
3) Ultrasonic(supersonic)	f>20 kHz

Characteristics Of Sound:

1) Pitch (Related to frequency): It is a subjective sensation perceived when a tone of given frequency is sounded. It enables to classify a music note as high or low and to distinguish between shrill sound and flat sound of the same intensity sounded on the same musical instrument. Pitch is directly proportional to frequency. As frequency increases, pitch increases and vice-versa. Frequency is physical quantity so can be measured accurately while pitch is physiological quantity which is merely a sensation experienced by a listener so varies from person to person.

2) **Loudness (Related to intensity):** Loudness signifies how far and to what extent sound is audible. It is a subjective quantity so varies from person to person.

Weber-Fechner law: Perceived loudness/brightness is proportional to log of actual intensity measured with an accurate nonhuman instrument.

$L=k\log_{10}I$, where k is proportionality constant depends on sensitivity of ear and quality of sound. Let I_1 be the initial intensity which produces loudness L_1 then,

$$L_1 = k \log_{10} I_1 \qquad (1)$$

If the intensity is doubled, I_1 becomes $2I_1$. So the loudness L_2 can be given by,

$$L_2 = k\ \log_{10} 2I_1 = k \log_{10} 2 + k\ \log_{10} I_1 \tag{2}$$

$$L_2 = \ k \log_{10} 2 + L_1$$

$$\therefore L_2 - L_1 = k \log_{10} 2 \tag{3}$$

Equation (3) implies that loudness increases by the same amount whenever intensity is doubled, irrespective of the initial intensity.

3) **Timbre (Related to quality):** Timber is a subjective quantity which can be used to distinguish between the same note played on different instruments or sung by different singers. If two instruments have the same fundamental frequency but different overtones, they are said to have two different timbre.

Note: Higher frequencies are interpreted as a higher pitch. For example, when you sing in a high-pitched voice you are forcing your vocal chords to vibrate quickly.

Audible sound can be further classified according to their frequency spectrum as (1) Musical sound and (2) Noise.

Sr. no	**Musical sound (Voice)**	**Noise**
1.	They produce pleasing sensation	They produce irritating sensation
2.	They are periodic vibrations	They are non-periodic vibrations.
3.	They have line spectrum of frequency	They have complex spectrum of frequency
4.	Sudden changes in amplitude do not occur.	Sudden changes in amplitude do occur.
5.	Musical instruments like drum, harmonium, sitar etc.	Traffic, firing of crackers, flying of aeroplane etc.

Acoustical property comparisons

Property	Air	Water	Rock
Speed (m/s)	340	1500	2000-5000
Density (g/cm^3)	0.001	1	2-3

Intensity of sound in w/m^2	**Sound intensity level in** dB	**Sound Pressure in** Pa
1	120 (Threshold of feeling)	20
10^{-2}	100	2
10^{-4}	80	0.2
10^{-6}	60	0.02
10^{-8}	40	0.002
10^{-10}	20	0.0002

10^{-12} (Threshold of hearing /Standard Intensity)	0	0.00002

Standard Intensity: The minimum sound which can be heard by human ears.
Threshold of feeling: The sound which offers painful effect by human ears.
Physical significance of a decibel: The smallest change in intensity level that the human ear can detect is 1 dB.

$$1 \text{ dB} = 10 \log_{10} \frac{I}{I_0}$$

$$\therefore \log_{10} \frac{I}{I_0} = \frac{1}{10} = 0.1$$

$$\therefore \frac{I}{I_0} = 10^{0.1} = 1.26$$

$$\therefore \text{I} = 1.26 I_0$$

It means that change in intensity by 26% increases the intensity by 1 dB.
If $\text{I} = 100 I_0$, $\text{L} = 10 \log 10^2 = 20\ dB$
If $\text{I} = 1000 I_0$, $\text{L} = 10 \log 10^3 = 30\ dB$
If $\text{I} = 10000 I_0$, $\text{L} = 10 \log 10^4 = 40\ dB$
That means when two sounds differ by 20dB, the louder of them is 100 times more intense.
Reverberation: The persistence or prolongation of a sound in a hall even the source of sound is cut off is called reverberation.
Reverberation time: It is the time required for the sound intensity to fall below one millionth of its original intensity once the source of sound is cut off.
Sabine's formula for reverberation time: Wallance C. Sabine(1868-1919), professor in physics, Harward University U.S.A. was assigned to improve the acoustics of Art Museum hall of the university. For this work he studied different factors like loudness, echelon effect, reverberation time etc. He found that reverberation time is one of the most important factor for the architectural acoustics.

$$Reverberation\ time\ T \propto \frac{Volume\ of\ the\ hall\ V}{Absorption\ A}$$

$Reverberation\ time\ T = k \frac{V}{A}$ where k= proportionality constant and its value determined by experiments was 0.167

$T = k \frac{V}{\sum_1^n a_n s_n}$ Where $a_1, a_{2,,} \ldots are\ absorption\ coefficent\ of\ material\ and\ s_1, s_{2,} \ldots$ are their surface areas.
Absorption coefficient a: It is the ratio of sound energy absorbed by it to the total energy incident on it. An area of 1m^2 open window is standard unit of absorption as all the energy falling on it passes through it.

Or It is the reciprocal of the area of sound absorbing material which absorbs the same amount of sound energy as that of $1m^2$ of an open window. eg. if sound absorbing material of $5m^2$ absorbs the same amount of sound energy as absorbed by $1m^2$ of an open window. Absorption coefficient of the material is given by 1/5 =0.20sabine.

Measurement of Absorption Coefficient a: Let T_1 be the reverberation time of an empty hall of volume V, absorption coefficient a and area s.

$$T_1 = \frac{0.167\,V}{\Sigma as} \quad (1)$$

Let T_2 be the reverberation time of the hall after inserting the sound absorbing material of absorption coefficient a_1 and surface area s_1.

$$T_2 = \frac{0.167\,V}{\Sigma as + a_1\,s_1} \quad (2)$$

Taking reciprocal of equations (1) and (2),

$$\frac{1}{T_1} = \frac{\Sigma as}{0.167\,V} \quad (3)$$

$$\frac{1}{T_2} = \frac{\Sigma as}{0.167\,V} + \frac{a_1\,s_1}{0.167\,V} \quad (4)$$

Taking the difference of equation (3) and (4)

$$\frac{1}{T_2} - \frac{1}{T_1} = \frac{a_1\,s_1}{0.167\,V} \quad (5)$$

$$a_1 = \frac{0.167V}{S_1}\left[\frac{1}{T_1} - \frac{1}{T_2}\right] \quad (6)$$

The absorption coefficient of the hall can be calculated by knowing the values of reverberation times T_1, T_2 and surface area s_1 and volume of the given hall.

Factors affecting Acoustics of Building: Seven factors can affect the acoustics of a building. They are

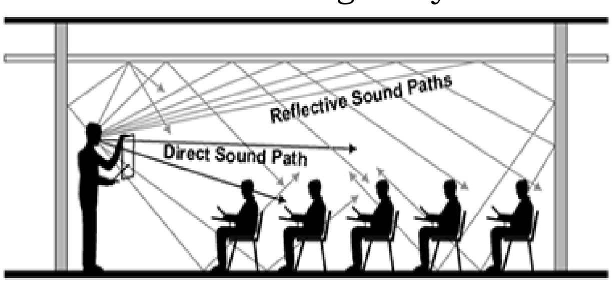


Figure 2

1) **Reverberation Time:** It is the time required for the sound intensity to fall below one millionth of its original intensity once the source of sound is cut off (figure 2). It has to be maintained at optimum value. For a hall of 10,000 m^3, reverberation period is between 1.0 to 1.5 seconds and 1.5 to 2.0 second for music.

It can be optimized by providing adequate windows and doors, Walls being coated by absorbent materials.

2) **Loudness:** Large grounding boards behind the speakers and facing the audience, by using public address system like loud speakers.

3) Focusing due to walls and ceilings and interference Effects: Curved surfaces should be avoided. If curved surfaces are present, they should be covered by suitable sound absorbing materials

4) Echo: Echo is a reflected sound wave coming from the same source with time interval of 1/T second as shown in figure 3. If the time interval between the direct and the reflected sound is less than 1/15th of a second, the reflected sound is helpful in increasing the loudness but those sounds arriving later than this cause confusion.

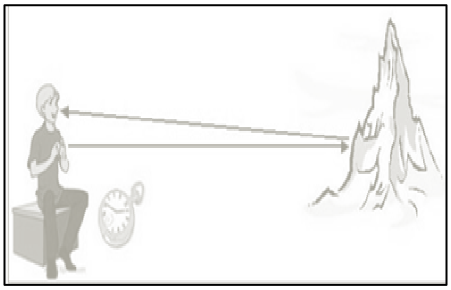


Figure 3

The wall should be covered with absorbent materials to avoid this.

5) Echelon Effect: It refers to the generation sound due to multiple echoes as shown in figure 4. A set of railings or railings or any regular reflecting surface surface produces the echelon effect. This This echelon effect affects the quality of the of the original sound.The remedy to avoid avoid echelon effect is to cover such surfaces surfaces with sound absorbing materials.

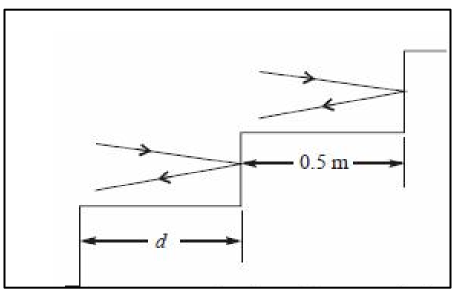


Figure 4

6) Resonance: Resonance occurs due to the matching of frequency. In case, if the window panels and sections of wooden portions have not been tightly fitted they may start vibrating creating sound in addition to the sound produced in the hall or the room.

Remedy: The resonance may be avoided by fixing the window properly. Any other vibrating object which may produce resonance can be placed over a suitable sound absorbing material.

7) Focusing and interference Effect: The presence of any curved surface (figure 5) in the hall makes the sound concentrated at the focus point and due to this the loudness decreases at other regions. This region is known as dead region.

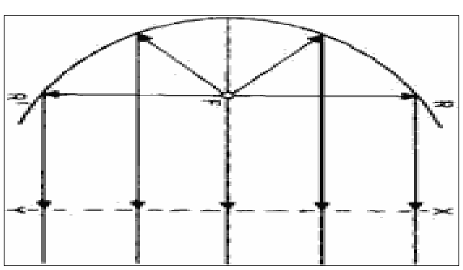


Figure 5

8) Noise: The hall or room should be properly insulated from external and internal noises. In general, there are three types of noises.

- **Air borne noise:** Extraneous noises which are coming from outside through open windows, doors and ventilators are known as air borne noise. The air borne noise can be avoided by following the below remedies.

Remedies: The hall or room can be made air conditioned. By using doors and windows with separate frames having proper sound insulating material between them.

- **Structure borne noise:** The noise which is conveyed through the structure of the building is called structure borne noise. The structural vibration may occur due to street traffic, operation of heavy machines etc.

Remedies: This noise can be eliminated by using double walls with air space between them. By using antivibration mounts this type of noise can be reduced. By covering the floor and walls with proper sound absorbing material this noise can be eliminated.

- **Inside noise:** The noises which are produced inside the hall or room is called inside noise. The inside noise may be produced due to machineries like air conditioners, refrigerators, generators, fans, typewriters etc.

Remedies: The sound producing machineries can be placed over sound absorbing materials like carpet, pads, wood, felt etc. By using curtains of sound absorbing materials. By covering the floor, wall and ceiling with sound absorbing materials. **Sound absorbing material helps in obtaining optimum reverberation time of a hall or a theatre.**

Properties of sound absorbing materials:

1) They should be highly porous.
2) They should be cheap and easily available.
3) They should be easy to fix and good looking.
4) They should be light in weight and durable.
5) They should be water proof and fire proof.
6) They should be efficient over a wide range of frequencies.

Classification of sound absorbing materials:

1) **Porous absorbent:** When sound waves strike a porous material, a small part of the wave gets reflected, while the major part of sound enters into the porous material and gets converted into heat energy hence it becomes inaudible and do not create interference.

e. g. fiber board, soft plastic, rock wood, wool wood, glass silk etc.

2) **Cavity resonator:** A chamber with a small opening is known as a cavity resonator. The sound waves which enter the cavity gets multiple reflections become inaudible. These materials are used in air condition plants.

3) **Resonant absorbent / Panel absorbent:** When sound energy is incident on these materials, it gets converted into heat due to flexural vibration of the panel.e. g. Gypsum board, hard board panels, wood board, suspended plasters, rigid plastic board panels. Composite type absorbent

4) **Composite type absorbent:** When the functions of all the three types described above are combined in a single unit, it is known as Composite type absorbent. When the sound energy strikes the panel, it passes through it and is damped by resonance of air in the cavity. e.g. Empty jars and bottles, perforated cardboard boards etc.

Noise Pollution: The unwanted sound waves which are dumped into the atmosphere are known as noise pollution. These sound waves produce unpleasant effect and create adverse effects to human health. Noise pollution is of mainly two types: industrial noise which is produced by movement of vehicles, explosions etc and non-industrial noise is produced by loud speakers, radio, construction work etc.

Noise control in machines:

1) Insulating the source with sound reducing houses
2) Providing a dynamic balance to vibrating machine
3) Using large work area consisting of good sound absorbing materials
4) Reducing structure born noise using double doors or walls with air spaces between them.
5) Providing sound reducing ear muffs and plugs to all the workers.
6) Isolating machine area from offices, show rooms etc.

Solved examples:

1. **If the intensity of the sound wave is increased to 40 times its value, by how many decibel the intensity level increase?**

Given: $I = 40I_{0,}\ I_l =?$

$\mathrm{I_l = k\ log_{10}\frac{I}{I_0}\ = 10\ log_{10}\frac{40I_0}{I_0}}$ = 10x1.6020=**16.20 dB**

1. Calculate the intensity level of plane just leaving the runway having sound intensity of about 10,000Wm^{-2}.

Given: I=10,000Wm^{-2}, I_l =?

We know that $I_0 = 10^{-12}Wm^{-2}$

$$\mathrm{I_l = k\ log_{10}\frac{I}{I_0}\ = 10\ log_{10}\frac{10{,}000}{10^{-12}} = 10\ log_{10}10^{16}\ = \mathbf{160dB}}$$

2. Calculate the intensity level in dB at distance of 10m from the source which radiates energy at the rate of 4W. The reference intensity is 100Wm^{-2}.

Given: I_0=100Wm^{-2}, I_l =?, r=10m,$\frac{Q}{t} = 4W$, I=?

$\mathrm{I = \frac{Q}{At} = \frac{4}{4\pi r^2} = \frac{4}{4x3.14x(10^2)} = \frac{4}{1256} = 0.0032Wm^{-2}}$

$\mathrm{I_l = k\ log_{10}\frac{I}{I_0}\ = 10\ log_{10}\frac{0.0032}{100} = \mathbf{-44.94dB}.}$

Note: Intensity level can have negative value.

3. A source of sound has frequency of 512 Hz and an amplitude of 0.2m. Calculate the flow of energy across 1m² per second if the velocity of sound in air is 340 ms^{-1} and density of air is 1.30kgm^{-3}.

Given: f=512 Hz,a=0.2m, A=1m²,v=340ms^{-1}, ρ =1.30kgm^{-3}.

$I = 2\Pi^2 f^2 a^2 \rho v = 2x(3.14)^2x(512)^2x1x1.30x340 = \mathbf{22.8x10^8 Wm^{-2}}$

4. A hall of volume 1,000m³ has sound absorbing surface of 400m². If the average absorption coefficient of the hall is 0.1 sabine, what is the reverberation time of the hall?

Given: V=1,000m³, S=400m²,a=0.1 sabine, T=?

$$T_1 = \frac{0.167\ V}{\Sigma a_1 s_1} = \frac{0.167x1000}{0.1x400} = \mathbf{4.175\ s}$$

5. The volume of hall is 1,000m³. The wall area of the room is 200 m². The floor area is 100 m² and the ceiling area is 100 m².The average sound absorbing coefficient of the wall is 0.2 sabine, for the ceiling is 0. 2 sabine and for the floor is 0.01 sabine. Calculate the average absorbent coefficient and reverberation time.

Given: V=1,000m³, S_1=200m²,S_2=100m² , S_3=100m²,a_1=0.2 sabine,a_2= 0.2 sabine a_3=0.01sabine, $\bar{a}$,T=?

$$\bar{a} = \frac{a_1\,S_1+a_2\,S_2+a_3\,S_3}{S_1+S_1+S_1} = \frac{0.2x200+0.2x100+0.01x100}{200+100+100} = \frac{70}{400} = \mathbf{0.175\ sabine}$$

$$T_1 = \frac{0.167\ V}{\Sigma a_1 s_1} = \frac{0.167x1000}{70} = \mathbf{2.39\ s}$$

6. A hall has a volume of 12,500 m³ and reverberation time of 1.5 sec. If 200 cushioned chairs are additionally placed in the hall, what will be the new reverberation time of the hall? The absorption of each chair is 1.0 O.W.U.

Given, V = 12,500 m³; T_1 = 1.5 s, a_2s_2 = 200 sabine-m²; $\sum a_1s_1$ = ?; T_2 = ?

Let $T_1 = \frac{0.167\ V}{\Sigma a_1 s_1}$ be the reverberation time before placing cushioned chairs.

So $\Sigma a_1 s_1 = \frac{0.167\ X\ 12.500}{1.5}$

$\sum a_1s_1$ = 1391.66 sabine-m².

Let the reverberation time after placing the cushioned chairs be

$$T_2 = \frac{0.167\ V}{\Sigma a_1 s_1 + \Sigma a_2 s_2} = \frac{0.167\ X\ 12,500}{1391.66 + 200} = \mathbf{1.3115\ s}$$

Therefore, the new reverberation time after placing the cushioned chairs will be reduced to 1.3115 sec.

7. What is the resultant sound level in bel, when a 9 bel sound is added to a 90 dB sound?

Given: I_{L1}= 9bel = 90dB; I_{L2}= 9bel= 90dB

$$I_{L1} = k \log \frac{I_1}{I_0}$$

$90 = 10 \log \frac{I_1}{I_0}$

$9 = \log_{10} (I_1/I_0)$

$\frac{I_1}{I_0} = 10^9$

$I_1 = 10^9 I_0$

$I_{L2= k \log \frac{I_2}{I_0}}$

$90 = 10 \log \frac{I_2}{I_0}$

$9 = \log_{10} \frac{I_2}{I_0}$

$\frac{I_2}{I_0} = 10^9$

$I_2 = 10^9 I_0$

$I = I_1 + I_2$

$= (10^9 + 10^9) I_0$

$I = 2\times10^9 I_0$

$I_{L= k \log \frac{I}{I_0}}$

$I_{L=} 10 \times \log_{10} (I / I_0) = 10 \times \log_{10}(2 \times 10^9)$

I_L=93.010dB

Therefore, the resultant intensity is 9.3010 bel

8. The reverberation time is found to be 1.5 sec for an empty hall and it is found to be 1.0 sec when a curtain cloth of 20 m² is suspended at the centre of the hall. If the dimension of the hall are 10*8*6 m³. Calculate the coefficient of absorption of curtain cloth.

Given T_1= 1.5 sec, T_2= 1.0 sec, S= 20 m^2,V=10*8*6m^3, a_1=?

$$a_1 = \frac{0.167V}{S_1}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

=0.167*480(1.5-1) / 20*1*1.5

=1.336 O.W.U.

9. An auditorium has a volume of 80,000m³.It has reverberation time of 1.5 second. What is the average absorbing power of the surface if the absorbing surface is 10000m²?

Given V=80,000m^3, S=10000m^2, T=1.5, ā =?

$$T_1 = \frac{0.167\, V}{\sum as}$$

$$\sum as = \frac{0.167x80{,}000}{1.5} = \frac{13360}{1.5} = 8906.66\, sabine.m^2$$

$$\bar{a} = \frac{\sum as}{\sum s} = \frac{\mathbf{8906.66}}{\mathbf{10000}} = 0.89 sabine$$

Assignment questions:

I. Multiple Choice Question

1) Audible sound has the frequency range of
a. 0 Hz to 100 kHz
b. 20 Hz to 20 kHz
c. 20 kHz to 200 kHz
d. 0 Hz to 20 Hz
2) The velocity of sound
a. does not depend on the temperature of the medium
b. does not depend on the nature of the medium
c. depends on nature and temperature of the medium
d. depends on hardness of the medium
3) Which one of the following sound waves have the frequencies less than those of audible waves?
a. supersonic
b. ultrasonic
c. infrasonic
d. infrared
4) Which of the following sound waves have the frequencies greater than that of audible waves?
a. ultrasonic
b. infrasonic
c. ultraviolet
d. infrared
5) The velocity of sound in air
a. decreases with increase in temperature
b. decreases with decrease in temperature
c. increases with decrease in temperature
d. does not depend on temperature
6) The intensity level of threshold hearing is
a. 0 dB
b. 1 dB
c. 60 dB
d. 0.1 dB
7) Reverberation of sound in a hall is
a. a desirable effect
b. not a desirable effect
c. not possible
d. always zero
8) Reverberation time is
a. inversely proportional to volume
b. not related to volume

c. directly proportional to volume
d. directly proportional to total area
9) Sabine's formula for reverberation gives T equal to
a. $\frac{0.061\,V}{\Sigma aS}$
b. $\frac{0.161\,V}{\Sigma aS}$
c. $\frac{0.116\,V}{\Sigma aS}$
d. $\frac{0.061}{V}\,\Sigma as$
10) Reverberation time of a room is decreased when
a. absorbing materials are added
b. the windows are all closed
c. absorbing materials are all removed
d. doors are closed

II. Answer the following questions.

1. Classify sound waves with respect to the basis of frequency.
2. What is the difference between musical sound and noise?
3. State the frequency range of infrasonic waves?
4. What are the characteristics of musical sounds?
5. Define noise.
6. Give the units of measurement of loudness and intensity.
7. What is meant by reverberation? Is it desirable to have it in a building?
8. What is the physical significance of acoustics?
9. List the factors affecting acoustics of building with their remedies.
10. Give the classifications of sound absorbing materials.
11. List the properties of good sound absorbing material.

III. PROBLEMS FOR PRACTICE

1. If the amplitude of the sound is tripled; by how many dB will the intensity level increase? [Ans. 9.54dB]
2. Calculate the relative intensity of a fighter plane just leaving the runway having sound intensity of about 2500Wm^{-2}. [Ans. 153.98dB]
3. The intensity of sound in a street during heavy traffic is 10 Wm^{-2}. Calculate the intensity in bell and dB. [Ans. 130dB]
4. Calculate the intensity level in dB at a distance of 10 m from the source of sound which emits energy at the rate of 4W. The reference intensity is 200Wm^{-2}. [Ans. -47.98 dB]
5. Obtain the resultant sound intensity when 8 bell sound is added to 80dB sound. [Ans. 83.01dB]
6. A source of sound has a frequency of 500Hz and amplitude of 0.05cm. Calculate the flow of energy across 1 square meter per second if the velocity of

the sound in air is 342m/s and the density of air is 1.29kg/m^3. [Ans. 543.73Wm^{-2}]

7. A hall has volume of 1000m^3 and sound absorbing surface area of 200m^2. If the average absorption coefficient of the hall is 0.4, what is the reverberation time of the hall? [Ans. 2.09 s]

8. The volume of a theater is 20000m^3. Its reverberation time is 1.5 second. If the average absorption coefficient of the interior surface is 0.8 sabine, find the area of the interior surface. [Ans. 2783.3 m^2]

9. An auditorium has a volume of 8000m^3. What should be the total absorption in the hall if the reverberation time of 1.5 is to be maintained? [Ans. 890.6 sabine m^{-2}]

10. A hall has a volume of 10,000 m^3 and reverberation time of 1.5 second. If 100 cushioned chairs are additionally placed in the hall what will be the new reverberation time of the hall? The absorption of each chair is 2 sabine. [Ans. 1.27 s]

11. A hall has volume of 50,000m^3. It has reverberation time of 1.5 second. What is the average absorbing power of the surface if the total absorbing surface is 10,000m^2? [Ans. 0.56 sabine]

12. A hall of volume 1000 m^3 has total absorption equivalent to 400 m^2 of open window. What will be the new reverberation time if the audience fill the hall and thereby increase the absorption by another 400 square meter of open window? [Ans. 0.2088 s]

13. A class room has dimension of 20 x 15 x 5 m^3. The reverberation time is 2 second. Calculate the total absorption of the surface of class room. [Ans. 125.25 sabine m^2]

Section 2 Ultrasonic

Properties of ultrasonic (Supersonic) sound waves:

1) They have frequency higher than 20 kHz.
2) They are highly energetic.
3) They propagate by compression and rarefaction.
4) They are highly directional so can travel longer distances.
5) They can travel with constant speed in homogeneous medium.
6) The speed of propagation increases with the density and temperature.
7) Due to very short wavelength they have high penetration power.
8) They set up standing wave pattern in liquids and produce an acoustical grating.
9) They are not easily absorbed by water so can be used for under water communication.

Generation of ultrasonic sound:

Ultrasonic sound waves can be generated by mainly two methods.

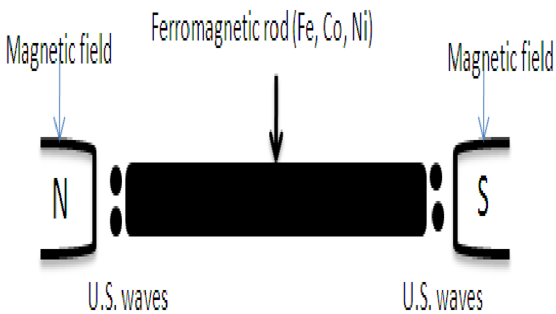


Figure 6

1) Magnetostriction effect and Magnetostriction oscillator for production of ultrasonic waves

Magnetostriction effect:

When a ferromagnetic material in the form of a rod is subjected to alternating magnetic field parallel to its length as shown in figure 6, the rod undergoes alternate contractions and expansions at a frequency of applied magnetic field it is called magnetostriction effect.

Magnetostriction oscillator:

Principle: Magnetostriction Effect

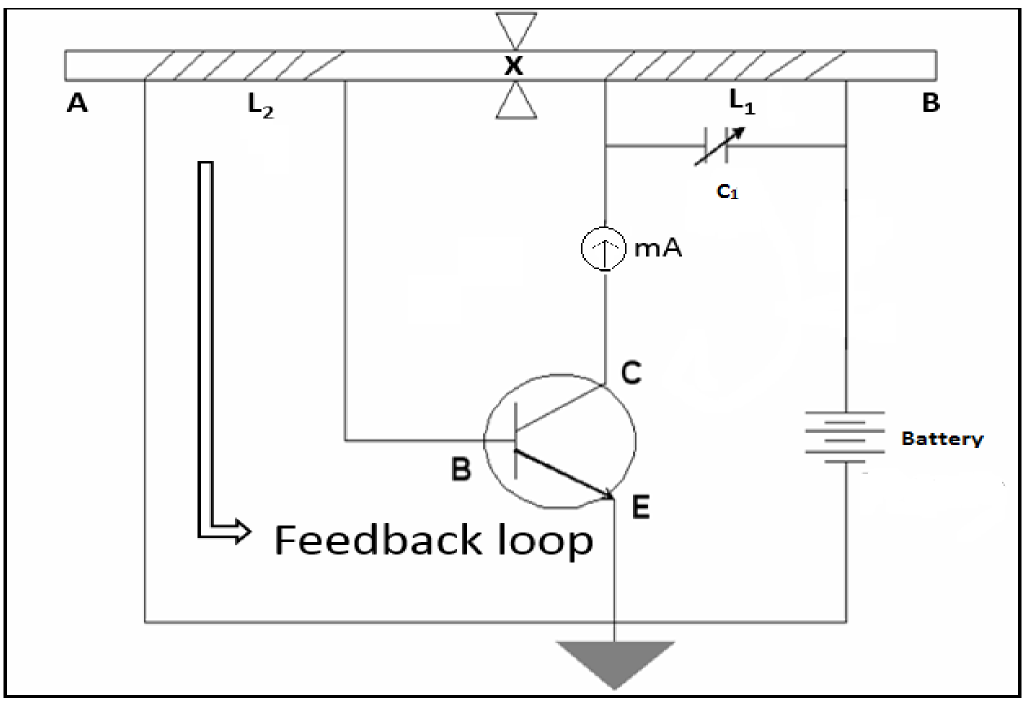


Figure 7 Magnetostriction Oscillator

Construction:

- A ferromagnetic rod is fixed by clamps from its center. Two coils L_1 and L_2 are wound at two ends of rod AB as shown in figure 7.
- A variable capacitor connected in parallel with coil L_1 forms a tank circuit or resonant circuit.
- One end of the tank circuit is connected with the emitter of the transistor through a battery).

- Second end of the tank circuit is connected with the collector of the transistor through milliameter.
- Coil L_3 is connected between base and emitter of the transistor T forms a feedback loop.

Working:

- When the battery is switched on, the resonant circuit L_1C_1 in the collector circuit of the transformer sets up on alternating current of frequency.

$$f= \frac{1}{2\pi\sqrt{L_{1}C_{1}}} \qquad (1)$$

- As a result, the rod gets magnetized by the collector current. Any change in the collector current brings about a change in the magnetization, and consequently a change in the length of the rod this gives rise to change in flux in coil L_2 in the base circuit , thereby inducing an emf in the thereby maintaining the oscillations.
- By varying the capacitor C_1, the frequency of the tank circuit of oscillator gets varied. If the frequency of tank matches with the natural frequency of the material, then due to resonance the rod vibrates vigorously producing ultrasonic waves the ends of the rod.
- The millimeter reading gives maximum value at the resonance condition.
- The frequency of ultrasonic waves produced by this method depends upon the length (l), density (ρ) and Modulus of elasticity (E) of the rod.

$$f = \frac{1}{2l}\sqrt{\frac{E}{\rho}} \qquad (2)$$

- Thus, by varying length and elastic constant of the rod, ultrasonic waves can be generated at any desired frequency. Hence the resonance condition.

$$\frac{1}{2\pi\sqrt{L_{1}C_{1}}} = \frac{1}{2l}\sqrt{\frac{E}{\rho}} \qquad (3)$$

Merits:

1) The design of the oscillator is very simple.
2) The production cost of the oscillator is very low.
3) Large output power is possible without damaging the oscillatory circuit.
4) It can generate frequency as high as 3MHz.

Demerits:

1) It is not possible to generate ultrasonic waves having frequency above 3MHz.
2) The frequency of the oscillator depends on greatly on temperature.
3) Eddy current losses and hysteresis loses makes it less efficient.

2) Piezoelectric Effect: If pressure or stress is applied to one pair of opposite faces of the crystal, equal and opposite charges are generated on the remaining opposite faces of the crystal. This phenomenon is known as piezoelectric effect.

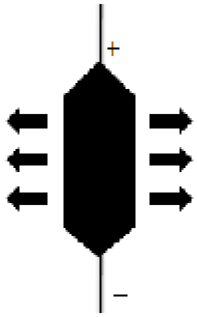


Figure 8

Piezoelectric oscillator /Electrostriction oscillator

- If an alternating voltage is applied to one pair of opposite faces of the crystal, alternatively mechanical contractions and expansions are produced in the crystal and the crystal starts vibrating. This phenomenon is known as inverse piezoelectric effect or electrostriction effect.
- If the frequency of the applied alternating voltage is equal to the vibrating frequency of the crystal, then the crystal will be thrown into resonant vibration producing ultrasonic waves as shown in figure 8

Piezoelectric Oscillator/ Electrostriction oscillator:

Principle: Inverse Piezoelectric Effect

Construction:

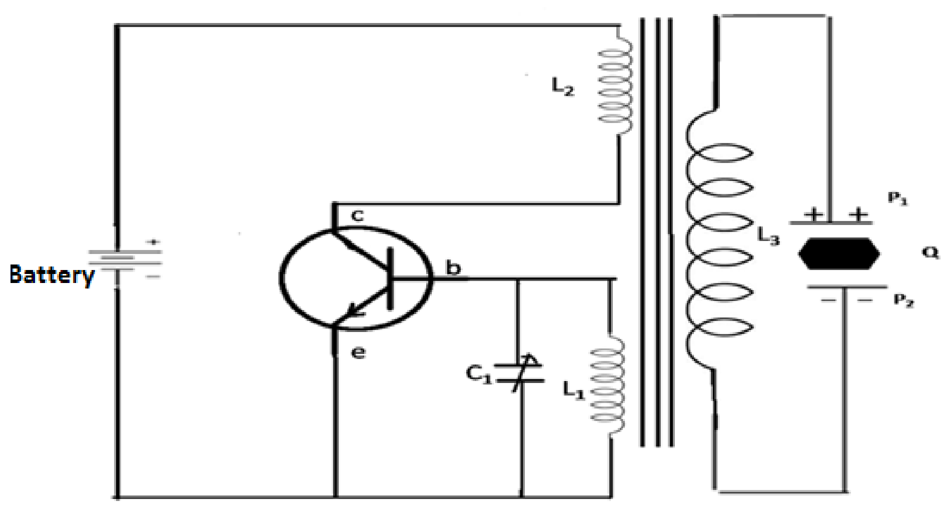


Figure 9 Piezoelectic Oscillator

- The circuit diagram is shown in figure .The quartz crystal Q is placed between two metal plates A and B. The plates A and B are connected to the coil L_3. The coil L_1,L_2 and L_3 are inductively coupled to the oscillatory circuit of a transistor.
- Coil L_1 with a variable capacitor C_1 forming the tank circuit is connected between the base and the emitter. While the coil L_2 is connected to the collector circuit, the battery is connected between free end of L_2 and the emitter of transistor.

Working:

- When the battery is switched on, the oscillator produces high frequency alternating voltage given by

$$f_1 = \frac{1}{2\pi\sqrt{L1C1}} \qquad (1)$$

- The frequency of oscillator can be controlled by the variable capacitor c_1.
- Due to the transformer action an emf is induced in the secondary coil L_3. This emf is impressed on plates A and B and thus excites the quartz crystal into vibrations.
- By adjusting the variable capacitor C_1, the crystal is set into one of the modes of resonant conditions. Thus; the vibrating crystal produces longitudinal ultrasonic waves in the surrounding air. The frequency of vibration of the crystal is

$$f_2 = \frac{P}{2l}\sqrt{\frac{E}{\rho}} \qquad (2)$$

Where, E is the Modulus of elasticity (Young's modulus), ρ is the density of the material and p=1,2,3,...for fundamental frequency first overtone ,second overtone.........respectively.

- At resonance condition,

Frequency of the oscillatory circuit = Frequency of the vibrating circuit

$$\frac{1}{2\pi\sqrt{L_1C_1}} = \frac{P}{2l}\sqrt{\frac{E}{\rho}} \qquad (3)$$

Merits:

1) It is more efficient than magnetostriction oscillator.
2) Ultrasonic frequency as high as $5X10^8$ Hz can be obtained with this arrangement.
3) The output of this oscillator is very high.
4) It is not affected by temperature and humidity.

Demerits:

1) The cost of piezoelectric quartz is very high.
2) Cutting and shaping of the quartz crystal are very complex.

Detection of Ultrasonic waves and velocity measurement:

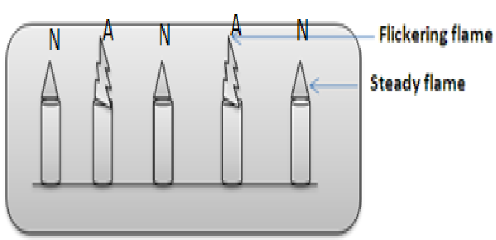


Figure 10

There are mainly four methods for detection of ultrasonic waves;

1) Sensitive flame method: Ultrasonic waves are detected by moving a sensitive flame in the medium. This is a qualitative method. According to figure 10, the periodic steady flame (at antinodes A) and flickering flame (at nodes N) will give idea about the exact location of nodes and antinodes.

2) Quartz crystal method: When the pair of faces of quartz crystal is exposed to ultrasonic waves, equal and opposite charges are generated on the other pair of opposite faces. These charges are amplified and detected using electronic circuits.

3) Thermal detection method: In this method a Callender Griffiths bridge is used for detection of ultrasonic waves as shown in figure 10. Out of its four arms three are connected with resistance and the fourth arm is connected with temperature sensitive Platinum-wire. If the ultrasonic waves are present in the chamber the resistance remains constant at the antinodes and varies periodically at nodes as a result there will be periodic fluctuates in the galvanometer

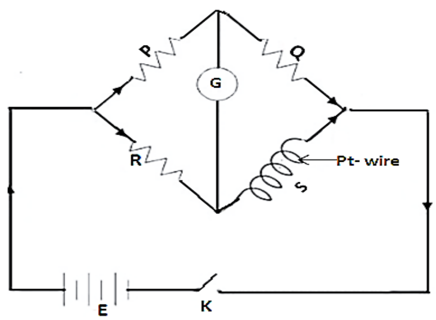


Figure 10

4) Kundt's tube method: Kundt's tube apparatus consists of a long glass tube of length nearly 1 m and diameter 5cm. One end of the tube is fixed with an adjustable piston rod which is connected with a cork and at the other end a quartz crystal Q is placed between two metallic plates as shown in figure 11. The glass tube is thoroughly dried and lycopodium powder is evenly spread along the entire length of the tube. If ultrasonic waves are presents in the tube, the lycopodium powder will be blown off and forms heaps at nodal points. The distance between two consecutive nodal points is equal to half of the wavelength of the ultrasonic waves λ_u.

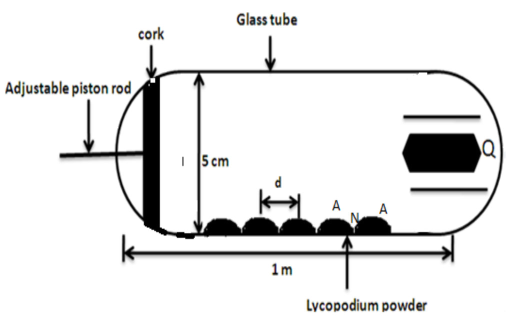
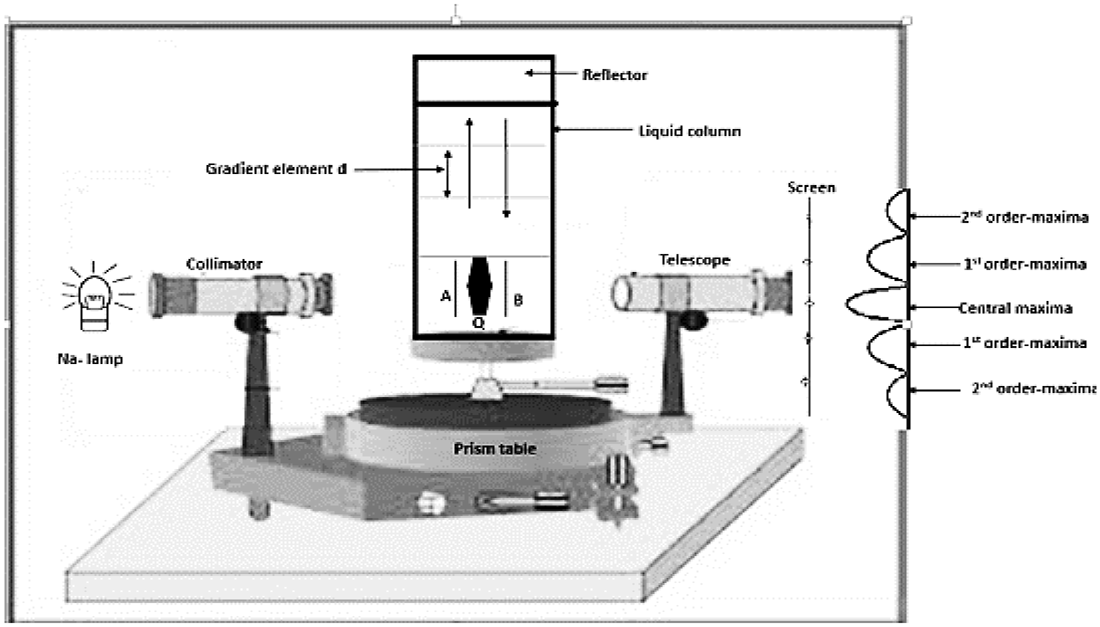


Figure 11

$$d = \frac{\lambda_u}{2} \quad (1)$$

$$v = f\lambda_u = 2fd \quad (2)$$

Acoustical grating method: Ultrasonic waves propagate through medium by compression and rarefaction so they produce different density and refractive index. According to figure 12, when a Na-lamp is switched on, parallel beam of monochromatic light passes through this medium, it behaves like a grating. This grating is known as acoustical grating. In liquid column quartz crystal Q placed between two metallic plates produces ultrasonic waves when connected



with an oscillatory circuit. These waves travel in the upward direction and get reflected back by the reflector. As a result superposition occurs and a stationary wave pattern can be observed through the telescope. The distance between two maxima or minima can be measured with the spectrometer. This distance d is known as gradient element. Using Bragg's law for diffraction pattern we have,

$$2d\sin\theta = n\lambda \tag{1}$$

Here n is the order of maxima or minima and θ is angle of diffraction.
For the first maxima, n= 1 and θ = 90°, therefore ultrasonic wavelength λ_u can be given by,

$$\lambda_u = 2d \tag{2}$$

From eq (1) and (2) we can write,

$$\lambda_u \sin\theta = n\lambda \tag{3}$$

$$\lambda_u = \frac{n\lambda}{\sin\theta} \tag{4}$$

But we know that,

$$v = f\lambda_u \tag{5}$$

We can calculate velocity of ultrasonic sound by equation (5).

Applications of ultrasonic sound:

Industrial applications:

Ultrasonic sound waves are used for Cutting, welding, drilling and cleaning

1) **Welding:** Both the work pieces are first cleaned and kept besides each

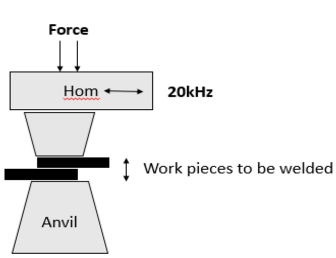


Figure 12

other. Ultrasonic beam is made to focus on the contact area of the two materials. Due to high temperature both the materials get melt and make a bond together and will get joined (figure 13)

Advantages: Less deformation takes place in materials. It is very accurate method so structure will not get disturbed.

Figure 13

2) **Drilling:** Ultrasonic waves produce vibratory mechanical energy due to compression and rarefaction. This energy is used for drilling the surface of metals, plastics, rubber sheets, stones etc.

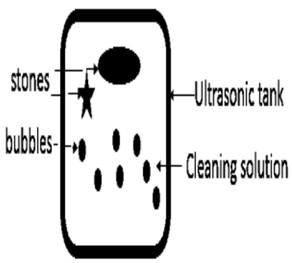


Figure 14

3) **Cleaning:** Cavitation effect: Formation and blasting of minute vacuum bubbles is known as cavitation which is shown in figure 14. When ultrasonic waves are passed through the ultrasonic tank, high pressure and thrust is generated on the substance to be cleaned which is dipped into the cleaning solution. Ultrasonic waves are used for cleaning of jewelry, watches, lenses, electronic components and surgical instruments etc.

4) **SONAR (Sound Navigation and Ranging):** Certain properties of ultrasonic sound enable its use for navigation in water and object identifier. They are highly directional so can cover longer distance. They are not easily absorbed by water. They get reflected whenever medium gets changed. Velocity can be measured by taking the ratio of distance traveled by time taken for the same as shown in figure 15.

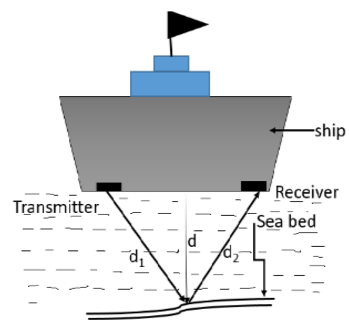


Figure 15

$$v = \frac{d_1+d_2}{t} \quad (1)$$

$$2d \approx d_1 + d_2 \quad (2)$$

$$\therefore v = \frac{2d}{t} \quad (3)$$

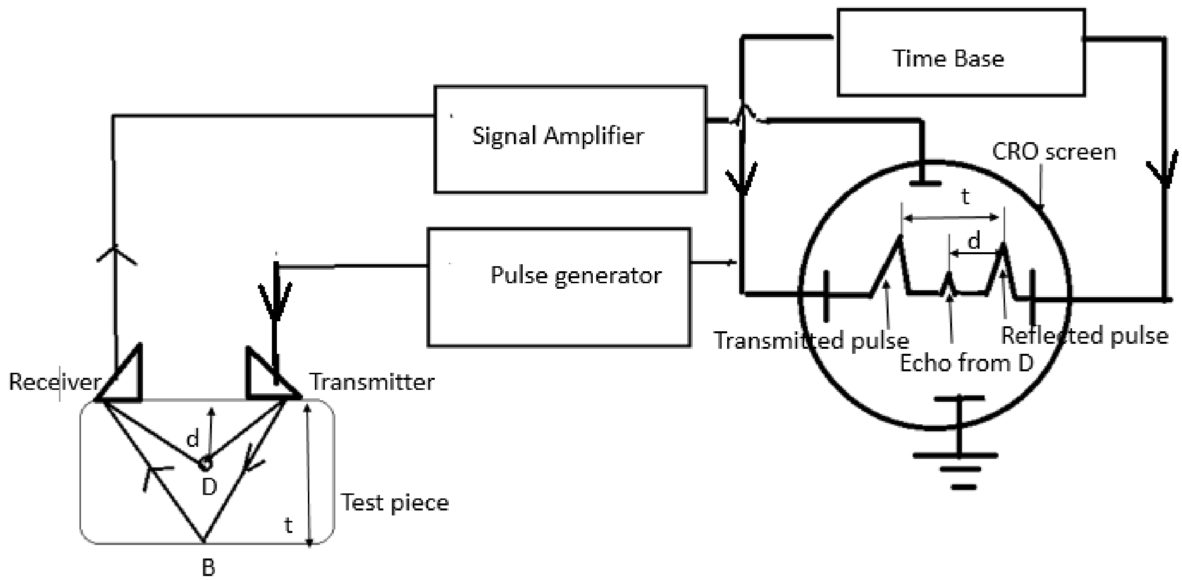
$$\therefore d = \frac{vt}{2} \quad (4)$$

Thus with the help of equation (4), we can find the depth of sea or distance of iceberg or rocks or shoal of fish.

Note: The distance of moving object can be obtained by an instrument echometer or fathometer which works on the principle of Doppler Effect.

Medical Applications:

1) In neurosurgery ultrasonic beams are used as they are highly focused.
2) Ultrasonic Doppler Testing: In the study of heart pulses and blood flow measurement ultrasonic waves are used.
3) For investigation of human organ this method is used.



4) Therapeutic applications: They are used to produce heat for muscle settlement after dislocation and are also used for healing of the damaged tissues.
5) They are used to detect tumors in human body.
6) Ultrasonography: They are used to study the fetus growth in womb.

Ultrasonic Non Destructive Testing (NDT): It is a method to inspect any object without damage.

Ultrasonic flaw detector (Through transmission System):

- The transmitted ultrasonic waves get reflected when they encounter a crack or cavity in the test pieceThe reflected pulse received by the transducer is converted into electrical signal and then amplified by the signal amplifier as shown in figure.
- The transmitted ultrasonic waves get reflected when they encounter a crack or cavity in the test piece.
- The reflected pulse received by the transducer is converted into electrical signal and then amplified by the signal amplifier.
- The amplified signal is sent to the Y-axis of the cathode ray oscilloscope (CRO).
- The time taken for transmission and reflection can be noted from the x-axis of the CRO.
- If the velocity v of the ultrasonic waves in the test piece is known then the position of the crack or cavity (can be calculated by the given equation.$d = \frac{vt}{2}$

Advantages:

1) It is more accurate than radiography.

2) It is cheap.
3) It has high speed of operation.
4) Large specimen can be detected in a very short time duration.
5) Deep seated defects can also be detected.
6) Location, nature and size of the defect can be accurately determined.
7) It can be used for metals, non-metals like rubber, plastic etc.

Disadvantages:

1) No permanent record of the flaw can be obtained.
2) Only skilled technicians can operate the instruments.
3) It is difficult to inspect rough and irregular shaped specimen.

Solved examples:

1. A piezoelectric X-cut crystal plate has a thickness of 2 mm. If the velocity of propagation of sound waves along the X-direction is 5600m/s, calculate the fundamental frequency of crystal.

Given: Thickness=2mm=2x 10^{-3} m, velocity=5600 m/s

In the lowest mode of vibration, the distance between the two faces of the crystal of thickness t will be, $t = \frac{\lambda}{2}$

$\therefore \lambda = 2t = 2x2x\,10^{-3} = \mathbf{4x10^{-3}\ m}$

$$v = f\lambda$$

$$\therefore f = \frac{v}{\lambda} = \frac{6400}{4x10^{-3}} = 1.6x10^{5}\ \text{Hz} = \mathbf{0,16\ MHz}$$

2. Calculate the capacitance to produce ultrasonic waves of $10^6/\pi$ Hz with an inductance of 1 henry.

Given: Frequency $=10^6/\pi$ Hz, L=1 henry

we know that,

$$f = \frac{1}{2\pi\sqrt{L_1C_1}}$$

$$C_1 = \frac{L_1}{4\pi^2 f^2} = \frac{1}{4\pi^2 f(\frac{10^6}{\pi})^2} = 0.25x10^{-12} = \mathbf{0.25\ pF}$$

3. Calculate the frequency to which piezoelectric oscillator circuit should be tuned so that a piezoelectric crystal of thickness 0.5 cm vibrates in its fundamental mode generating ultrasonic waves. Given that Y=800 GPa and density = 2000kg/m³.

Given: t=0.5 cm=0.5x10^{-2} m, Y=800GP=800x10^{10} Pa, ρ= 2000 kg/m³

Frequency of vibration $f = \frac{p}{2t}\sqrt{\frac{E}{\rho}} =$

$=1/2x0.5x10^{-2}\ \sqrt{80x10^{10}}\ /2000$

f $=2x10^6$ Hz

1. Find the frequency of the first and second mode of vibration for quartz crystal. The velocity of longitudinal waves in quartz crystal is 5 km/second. The thickness of quartz crystal is 50 cm.

Given: $v=5$ km/s $=5\text{x}10^3$ m, $t=50\text{cm}=50\text{x}10^{-2}$ m

We know that $\lambda=2t$

$$\lambda=2\text{x}50\text{x}10^{-2} = 1 \text{ m}$$

frequency of first mode => $f_1 = v/\lambda = 5\text{x}10^3 /1 = 5$ kHz

frequency of second mode => $f_2 = 2 f_1$

$$f_2 = 2\text{x}5$$

$$f_2 = 10 \text{ kHz}$$

2. The pulse arrival times from the steel bar of 30 cm thickness during the detection of possible defects using pulse echo method are 30 µs and 60 µs. Find out the distance of defect in a steel bar from the entrance end of ultrasonic wave.

Given $t_1=30\mu s=30\times10^{-6}s;t_2=60\times10^{-6},d_2=40cm=40\times10^{-2}m;d_1=?$

$T=2d/v$

$3\times10^{-6}=2\times d_1/v$ ...(1)

$6\times10^{-6}=2\times30\times10^{-2}/v$ ...(2)

Dividing (2)/(1),we get

$\frac{3\times10^{-6}}{6\times10^{-6}} = \frac{2\times30\times10^{-2}}{v} \times \frac{v}{2\times d1}$

$2=\frac{30\times10^{-2}}{d1}$

$d_1=15\times10^{-2}$ m

Assignment questions:

I. Multiple choice questions

1) Ultrasonic waves have frequencies

a. above 20 kHz

b. below 20 kHz

c. below 20 Hz

d. from 20 Hz to 20 kHz

2) Ultrasonic waves are produced by using

a. electromagnetic waves

b. tuning forks

c. piezoelectric effect

d. inverse piezoelectric effect

3) Ultrasonic waves are

a. electromagnetic waves

b. mechanical waves

c. matter waves

d. gravitational waves

4) Ultrasonic waves travel in air with a velocity
a. equal to the velocity of sound waves
b. equal to the velocity of electromagnetic waves
c. less than the velocity of sound waves
d. more than the velocity of sound waves
5) The velocity of ultrasonic waves is
a. greater in solids than in air
b. lower in solids than in air
c. equal in solids and air
d. greatest in vacuum.
6) The wavelength of ultrasonic waves in air is of the order
a. 1Å
b. 1μ
c. 1 nm
d. 1 cm
7) The frequency f of ultrasonic waves produced by an oscillator is given by
a. $\frac{m}{2L}\sqrt{\frac{\rho}{Y}}$
b. $\frac{2m}{L}\sqrt{\frac{Y}{\rho}}$
c. $\frac{p}{2l}\sqrt{\frac{Y}{\rho}}$
d. $\frac{2L}{m}\sqrt{\frac{Y}{\rho}}$
8) Velocity of ultrasonic waves in water is about __________ m/s at Temperature T= 25˚C
a. 340
b. 1500
c. 3400
d. 5400
9) Piezoelectric effect is observed in
a. Nickel
b. Copper
c. Iron
d. Quartz
10) Ultrasonic waves can travel long distances in sea water due to its
a. high intensity
b. larger wavelength
c. high frequency
d. low frequency

II. Answer the following questions

1. What are ultrasonic wave?
2. Why ultrasonic waves are used for detection of objects submerged in sea?
3. Which crystal is used more generally in the production of ultrasonic waves?
4. Define the natural frequency of a crystal.
5. What is meant by nondestructive testing?
6. What is sonar?
7. How ultrasonic cleaners are used to clean mechanical conductors and semiconductor chips, lens etc.?
8. Explain the direct and inverse piezoelectric effects?
9. Give the principle of piezoelectric Oscillator.
10. Describe different methods of detection of ultrasonic waves.
11. State the properties of ultrasonic waves?
12. What is an acoustic grating? How is it used in determining velocity of ultrasound?
13. Discuss at least three applications of ultrasonic in engineering.
14. Explain the principle of sonar.
15. What is Magnetostriction effect? Draw the diagram of Magnetostriction oscillator and explain its working.

III. Problems For Practice

1. Calculate the frequency to which piezoelectric oscillator circuit should be tuned so that a piezoelectric crystal of thickness 0.12 cm vibrates in its fundamental mode generating ultrasonic waves. Given that Y=80GPa and density= 2654 kg/m^3. [Ans. 2.29×10^6 Hz]

2. Find the frequency of the first and second mode of vibrations for quartz crystal. The velocity of longitudinal waves in quartz crystal is 5km/second. The thickness of quartz crystal is 55cm. [Ans. 4.54 kHz, 9.09 kHz]

3. An ultrasonic source of 90 kHz sends down a pulse towards the seabed which returns after 0.5 second. The velocity of sound in water is 1500m/s. Calculate the depth of the sea and wavelength of pulse. [Ans. 375 m, 0.016 m]

4. An ultrasonic source of 70 kHz sends down a pulse towards the seabed which returns after 0.65 second. The velocity of the ultrasonic sound in the sea is 1700m/s. Calculate the depth of the sea and the wavelength of the ultrasonic sound. [Ans. 552.5 m, 0.0243 m]

5. Calculate the natural frequency of the iron rod of 0.03m length. The density of iron is 23230 kg/m^3 and Young's modulus is $11.6\text{x}10^{10}$ N/m^2. [Ans. 37.24 kHz]

6. A quartz crystal of thickness 0.001 m is vibrating at resonance. Calculate its fundamental frequency if Young's modulus of quartz is $7.9\text{x}10^{10}$ N/m^{2}and the density is 2650 kg/m^3. [Ans. 2.73 MHz]

7. A Piezo- electric quartz plate (x-cut) has thickness of 1 mm. If the velocity of propagation of ultrasonic waves in the crystal along X-direction be 5760m/s, calculate the fundamental frequency of the crystal.[Ans. 2.88 MHz]

8. A wrought iron rod of length 6 cm is used in the oscillator. Obtain the frequency of the produced ultrasonic waves if modulus of elasticity = 20×10^{10} $N/m^2$and the density is 7850 kg/m^3. [Ans. 13.3 kHz]

9. Find the frequency of the first and second mode of vibration for a quartz crystal of Piezoelectric oscillator. The velocity of the longitudinal waves in quartz crystal is 5500 m/s. Thickness of quartz crystal is 0.05 m. [Ans. 55 kHz, 110 kHz]

Unit 3 Superconductivity

Metals are good conductors of electricity as they contain large number of free electrons. The low resistance offered by them to the flow of electric current is due to scattering of free electrons by vibrating ions of the lattice. When the temperature increases, the amplitude of vibrations also increases which will cause more scattering of electrons causing more resistance. On the other hand if the temperature is decreased, the resistance can be decreased up to a certain limit.

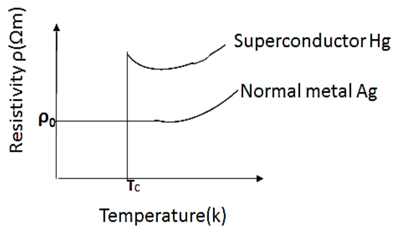


Figure 1

In 1911, H.K.Onnes verified the behavior of metals at low temperature and he discovered that the electrical resistivity of highly purified mercury dropped abruptly to zero at temperature of 4.2 K (figure 1). Upon heating mercury above its transition temperature, it regained its resistivity. This was a reversible transition. Onnes named this phenomena as superconductivity.

After 1980, ceramic materials were found to exhibit superconductivity at higher temperatures of about 120 K.

Superconductivity: It is a phenomenon in which metals, alloys and ceramics conduct electricity without resistance when it is cooled below a certain temperature. This temperature is known as critical temperature and these materials are known as superconductors.eg: Hg, Zn, Pb, Sn, CuS etc.

Properties of superconductors:

1) **Electrical resistivity ρ**: The electrical resistivity of a superconducting material is very low and is of the order of $10^{-7}\Omega$m

2) **Effect of impurities**: When impurities are added to superconducting elements, the superconducting property is not lost, but the $T_c$value is lowered.

3) **Effect of pressure and stress:** certain materials are found to exhibit the superconductivity of phenomena on increasing the pressure over them. For example cesium is found to exhibit the superconductivity phenomena at T_c= 1.5 K on applying a pressure of 110 kbar**.** **(**Increase in stress results in increase of the T_c value.)

4) **Isotope effect:** The critical temperature value of Superconductor is found to vary with its isotope. This variation in T_c with its isotope mass is called the isotope effect.$T_c \alpha \frac{1}{\sqrt{M}}$ where M is isotope mass.

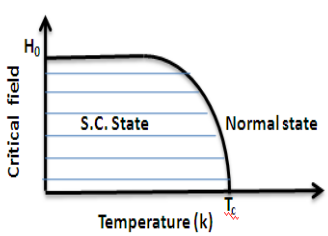


Figure 2

5) **Magnetic field effect:** If a sufficiently strong magnetic field is applied to a superconductor at any

temperature below its critical temperature T_c, the superconductor is found to undergo a transition from the superconducting state to the normal effect as shown in figure 2. As the temperature increases the critical field decreases and becomes zero at T=T_c.

$$H_C = H_0 \left[1 - \left(\frac{T}{T_C}\right)^2\right] \quad H_0 = \text{Critical magnetic field at } T = 0k$$

6) **Critical current I_c and Critical current density J_c:** For a thin long superconducting wire of radius r and cross-sectional area A, the relation between critical current Ic and critical magnetic field can be given by,

$$Ic = 2\pi r H_C \qquad \& \qquad J_c = \frac{Ic}{A}$$

Critical current density J_c is the maximum current density that can be passed through a superconductor without destroying its superconductivity.

7) **Persistent current:** The steady flow of current in superconductor without any significant loss is known as persistent current.

8) **Meissner effect (Diamagnetic property):** The complete expulsion or rejection of magnetic field lines is observed when T<T_C and H<H_C. This is known as Meissner effect which is shown in figure 3.

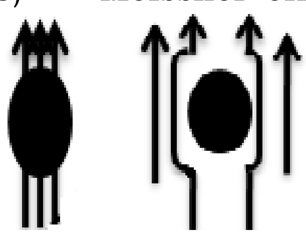


Figure 3

This process occurs due to surface current which results in development of magnetization M within the super conducting material which is equal and opposite to applied field. So the resultant field within the substance becomes zero. The magnetic induction inside the specimen is given by,

$B = \mu_0(M + H) = \mu_0(1 + \chi)H$ where H= Externally applied magnetic field

M=Magnetization produced within the specimen

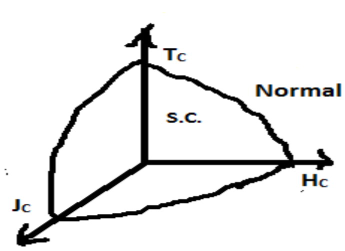
When T<T_C H<H_C, $B = 0$

As $\mu_0 \neq 0, M + H = 0$

$\therefore\ \chi = \frac{M}{H} = -1$

The specimen is therefore diamagnetic and the state in which the magnetization cancels the external magnetic field completely is termed as perfectly diamagnetism.

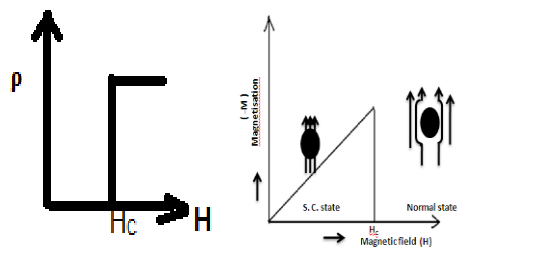
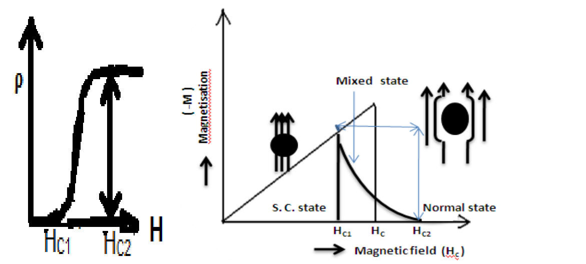
9) **Entropy:** Entropy is a measure of disorder of a system. Generaly entropy decreases with decreasing temperature. In superconductor, entropy decreases linearly upto critical temperature and more remarkably below the critical temperature. It indicates that superconducting state is more ordered than normal state.



10) **Relationship between T_C, J_c and H_C:** All these three parameters are dependent on each other. On and within this phase diagram (figure 4)the material behaves like superconductor but outside it, material becomes normal. Since the critical current falls with temperature, the critical magnetic field will also decrease as the transition temperature is approached. The variation of critical current density J_c **and critical magnetic field** H_C with temperature is shown in the figure.

Figure 4

Difference between Type-I and Type-II Superconductor:

Sr. no.	Type I Superconductors	Type II superconductors
1)		
2)	These superconductors are called soft superconductors.	These superconductors are called hard superconductors.
3)	They have only one critical field	They have two critical fields H_{C1} and H_{C2}.
4)	The critical field value is very low (0.1 T).	The critical field value is very high (30 T).
5)	They exhibit complete Meissner effect	They do not exhibit complete Meissner effect
6)	These materials have limited technical applications because of very low field strength value.	These materials have wider technological applications because very high field strength value.
7)	Ex:Pb,Zn,Hg	Ex:Nb_3Ge,NB_3Si, YBCO

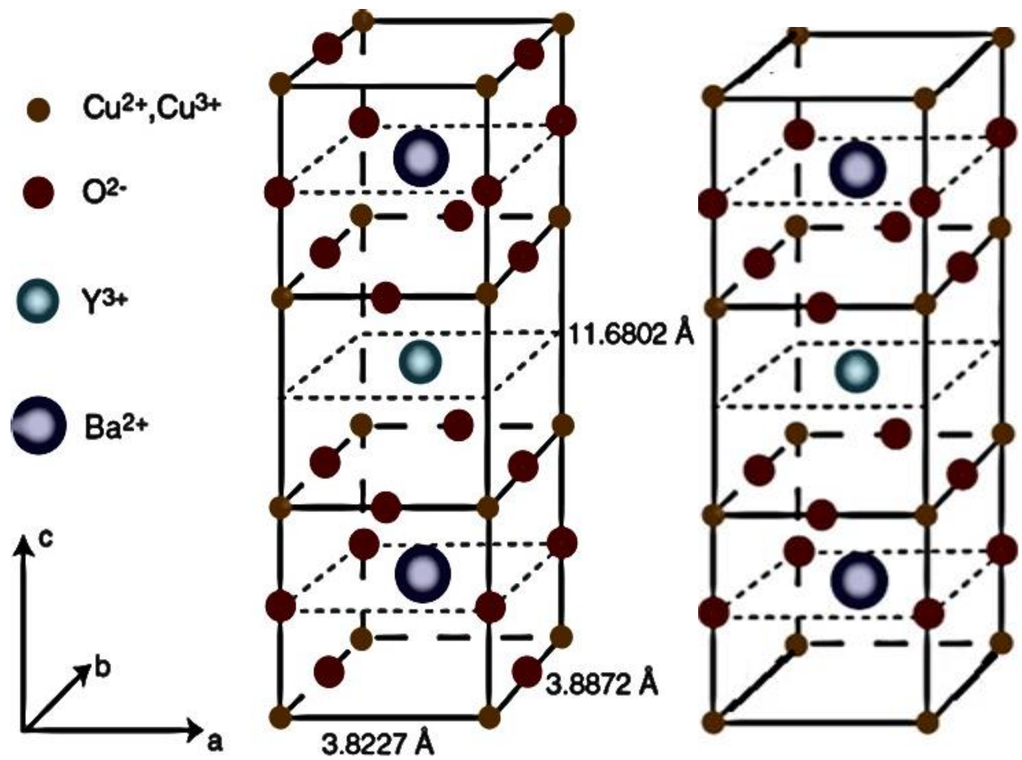
On the basis of their critical temperatures, superconductors are classified mainly into two types.

1) **Low Temperature superconductor (LTS):** In LTS, the superconductors are having T_c below 24 K and cooling is generally obtained with liquid helium. While in HTS, are having T_c above 24 K and cooling is generally obtained with liquid nitrogen.

2) **High Temperature superconductor (HTS):** In 1986, oxides of lanthanum barium copper ($La_1Ba_2Cu_3O_7$) was found which had T_c value of about 30 K. After one year, Chu and his co-workers replaced lanthanum by

yittrium and prepared $Y_1Ba_2Cu3O_7$ with T_c value about 95 K. These superconductors are also known as 123 compounds and belong to type II super conductors have very high critical magnetic field value (150 to 200T).

These oxides also contains deficiency of oxygen with the chemical formula $Y_1Ba_2Cu_3O_{7-\delta}$. The absence of Oxygen plays an important role in superconductors. $Y_1Ba_2Cu_3O_7$ and $Y_1Ba_2Cu_3O_6$ are shown in the figure 5. Continuous search is going on to discover materials that can exhibit superconducting state around room temperature.



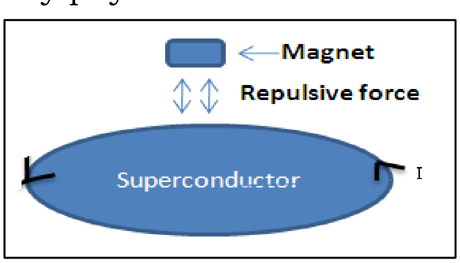
$Y_1Ba_2Cu_3O_7$ $Y_1Ba_2Cu_3O_6$

Figure 5

Applications of Superconductors:

Maglev:

➢ Maglev is a process in which object is suspended above another without any physical contact.

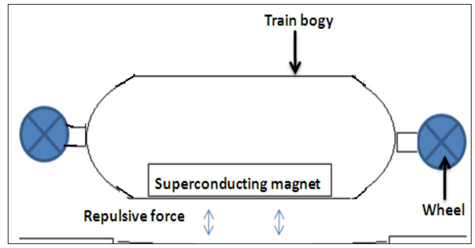


➢ Phenomenon of maglev is based on Meissner's effect.

➢ Maglev is brought by enormous amount of repulsion between two highly powerful magnetic fields.

➢ If a small magnet is brought near a superconductor, it repels (figure 6).

Figure 6

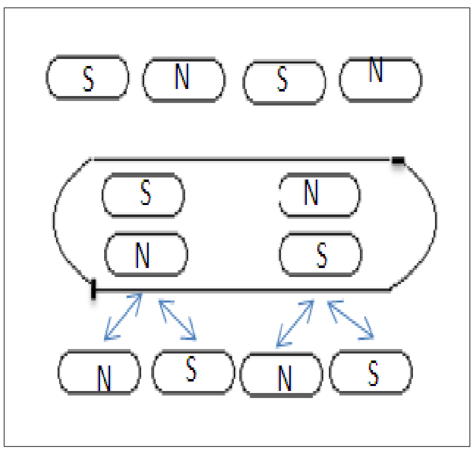


Repulsion takes place due to induce current in superconductor which is being generated by magnetic field of the magnet. Because of zero resistance property the current persists and thus the field induced due to this induced current repels the field due to magnet. As a result the magnet floats above the superconductor.

➢ Superconducting magnets are built into the base of its carriages as shown in figure 7. An aluminium guideway is laid on the ground and carries electric current. The repulsion between two powerful magnetic fields levitate the train about 10 to 15 cm above the guideways.

➢ We know that a diamagnetic substance repels magnetic field. Thus the perfect diamagnetic properties of superconductors make them suitable for achieving motion in motor and bearing. When the train is to be halted, the wheels are drawn out similar to the wheels of an airplane and the train descends slowly on to the guideways and runs forward till it stops (figure 8).

Figure 7

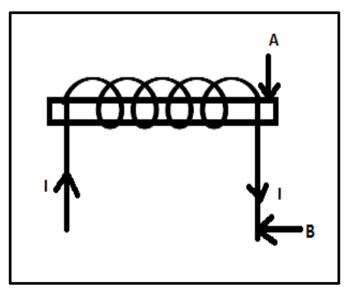


➢ There are mainly two types of suspensions in maglev train. Electromagnetic suspension (EMS) and Electro dynamic suspension (EDS).

➢ Maglev trains use magnets to levitate and propel the trains. The trains are virtually impossible to derail and collisions between trains are unlikely because computers are controlling the trains movements. Since there is no friction these trains can reach high speeds (500km/hr).

Figure 8

Cryotron: It is a relay or switch consists of two superconducting materials A of bar shape and B of coil shape. Superconductor B is wound over A as shown in figure 9



➢ The critical magnetic fields of superconductor A and B are H_{CA} and H_{CB} respectively. The selection of superconducting materials should be such that, $H_{CA} < H_{CB}$.

➢ If current I is passed through superconductor B, the

Figure 9

current will induce a magnetic field H. When this field H exceeds H_{CA} it destroys the superconductivity of A and converts it into normal material. As a result material A will start offering resistance to the flow of electric current flowing through it. When resistivity increases, the contact is broken and the switch behaves like open switch.

➢ Here current in A is controlled by the current in B and hence the system can be operated as a switch or relay. In addition, these switches consume very less current.

Josephson's Effect: There are two types of Josephson Effects:

1) **DC Josephson effect:** Two superconductors A and B are separated by a thin insulating layer of 10 nm thickness (figure 10). When the thickness of the insulating material is only 1 nm thick, they become a system of coupled conductors. The cooper pair of electrons will tunnel through the barrier and behave like a wave function. The effect of an insulating material is to introduce a phase difference between the wave function of cooper pairs on one side of insulating barrier and the other side. Because of this phase difference, supper current will appear across the junction even though the applied voltage is zero.

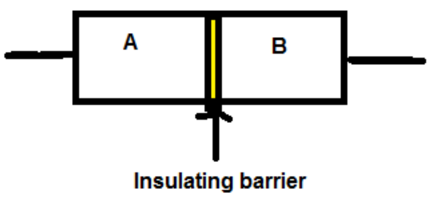


Figure 10

Here the supercurrent can be given by $\mathbf{I_S = I_C \sin \emptyset_0}$ where $\emptyset_0$ = Phase difference between the wave functions and I_C = Critical current at zero voltage condition.

2) AC Josephson Effect: If we apply a dc voltage across the Josephson junction, it introduces an additional phase on Cooper pairs during tunneling (figure 11). As a result new phenomenon will be observed. The dc voltage generates an ac current I given by,

$$I = I_C \sin(\emptyset_0 + \Delta\emptyset) \tag{1}$$

If voltage V is applied across the barrier, the energy of the Cooper pairs on both the sides of the barrier differ by 2eV.

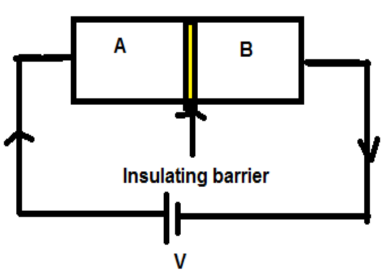


Figure 11

$$\Delta\emptyset = 2\pi t\left(\frac{2eV}{h}\right) \tag{2}$$

$$I = I_C \sin\left[\emptyset_0 + 2\pi t\left(\frac{2eV}{h}\right)\right] \tag{3}$$

From equation (3), the frequency of the current can be obtained,

$$f = \frac{2eV}{h} \tag{4}$$

At V=1μV ac current of frequency 483.6MHz is produced.

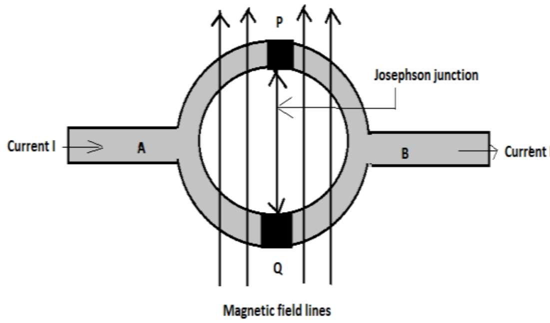


Figure 12

SQUID (SuperConducting Quantum Interference Device): It is a basically sensitive magnetometer which can measure very low magnetic fields and voltages with high accuracy. There are mainly two types of SQUID. (1) DC Squid and (2) AC Squid (RF Squid). **Construction:** It is fabricated from Lead or pure Niobium. The device is cooled using liquid helium. The schematic diagram of DC SQUID is shown in the figure 12. It consists of two Josephson Junctions arranged in parallel so that the electrons tunneling through the junction demonstrates quantum interference.

Working: A dc supercurrent is applied to SQUID. This current enters the device trough arm A and gets divided into two paths as I_1 and I_2 again merge and leaves through arm B.

When magnetic field passes through the loop, the supercurrent will interfere with each other. The phase difference between reunited current is directly proportional to the magnetic flux φ passing through the ring (loop). The flux φ is related to the magnetic field B and area A of the loop through the relation, $\Phi = BA$

Applications:

1) SQUID is used to measure very small magnetic fields 10^{-15}T.
2) Geologists use them to measure rock magnetism and continental drift.
3) In medicine it is used to study muscular or neural activity. It can measure the small current, voltages and magnetic fields developed in human brains and hearts.
4) It is used in NDT (Non Destructive Testing) for testing corrosion and defects developed in the wings of air craft.

Other applications of superconductors:

- Superconductors are used in power transmission.
- They are used in high efficient (99%) generators.
- In medical science for diagnostic areas and research.
- They can be used in electronic switches and can allow researchers to build 4-bit computers microchip to operate computer speed 500 times more than the current one.

Solved examples:

1. Calculate the critical current for a superconducting wire of lead having a diameter of 1 mm at 4.2 K. Critical temperature for lead is 7.18 K and H_c (0) = 6.5 X 10^4 A/m.

Given: H_0 = 6.5 X 10^4 A/m;Tc = 7.18 K; r = 0.5 X 10^{-3} m; T = 4.2 K; Tc = ?; Hc = ?

$$H_C = H_0 \left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$H_C = 6.5 \text{ X } 10^4 \left|1 - \frac{(4.2)^2}{(7.8)^2}\right| = 6.5 \text{ X } 10^4(1 - 0.34217) = 6.5 \text{ X } 10^4 \text{x} 0.65783 =$$

45.758 kA/m

$$\text{Ic} = 2\pi \text{r} H_C = 2 \text{ X} 3.14 \text{ X } 0.5 \text{ X } 10^{-3} \text{X } 42.758 \text{ X } 10^3 = \mathbf{134.26\ A}.$$

2. For mercury of mass number 202, the α value is 5 and T_c is 4.2 K . Find the transition temperature for the isotope of the mercury of mass number 200.

Given:Mass no. M_1= 202,α = 0.5,T_{c1} = 4.2 K, Mass no. M_2 = 200, T_{c2} = ?

We know that, $M^\alpha * T_{c1}$ = constant

$$M_1{}^\alpha x T_{c1} = M_2{}^\alpha x T_{c2}$$

$$T_{c2} = [\tfrac{M_1}{M_2}]^\alpha T_{c1} = [\tfrac{202}{200}]^{0.5} x\ 4.2 = 1.004987 \text{ x } 4.2 = 4.2209 \text{ K}$$

3. A critical temperature of Nb is 9 K . At 0 k the critical field is 0.2 T. Calculate the critical field at 6 K.

Given: T_c = 9 K, T = 6 K , H_0 = 0.2 T, H_c = ?

$$H_C = H_0 \left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$H_C = 0.2\left[1 - \left(\frac{6}{9}\right)^2\right] = 0.1111 \text{ T}$$

4. Calculate the critical current through a long thin superconducting wire of radius 0.5 mm. The critical magnetic field is 7 kA / m.

Given: H_c = 7 x10^3 A/m, r = 0.5 x 10^{-3} m

I_c = $2\pi r H_c$

= 2 x (22/7) x 0.5 x 10^{-3}x 7 x 10^3 = $22A$

5. Calculate the critical current for a superconducting wire of lead having a diameter of 1 mm at 4.2 K . Critical temperature for lead is 8.4 K. H_0 is 7 x 10^4 A/m.

Given: $H_0 = 7 * 10^4$ A/m, T_c = 8.4 K,r = 0.5 * 10^{-3} m,T = 4.2 K,I_c = ?, H_c = ?

$$H_C = H_0 \left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$= 7x10^4\left[1 - \left(\frac{4.2}{8.4}\right)^2\right] = 5.25x10^4 T$$

I_c = $2\pi r H_c$

= 2 x (22/7) x 0.5 x 10^{-3}x 5.25 x 10^4 = $164A$

6. The critical temperature T_C for Hg with isotopic mass 199.5 is 4.185 K. Calculate the critical temperature for its isotopic mass 204.5.

Given: Mass number M_1= 199.5, M_2 = 204.5, T_{c1} = 4.185K,T_{c2} = ?, α = 0.5

$$T_{c2} = [\frac{M_1}{M_2}]^{\alpha} T_{c1} \ = [\frac{199.5}{204.5}]^{0.5} x\, 4.185 = 0.98769 \times 4.185 = 4.13333\text{K}$$

7. A voltage of 6.62 μV is applied across Josephson junction. What is the frequency of radiation emitted by the junction.

Given: V = 6.62 * 10^{-6} V, f = ?

$$f = \frac{2eV}{h}$$

$$= \frac{2\, x 1.6\, x 10^{-19}\ x 6.62\, x 10^{-6}}{6.62\, x 10^{-34}} = 3.2 \text{ x } 10^9 \text{ Hz}$$

Assignment questions:

I. Multiple Choice Question

1) The electrical conductivity of a superconductor is

a. zero
b. finite
c. infinite
d. negative

2) Superconductors are

a. diamagnetic
b. paramagnetic
c. ferromagnetic
d. antiferromagnetic

3) The transition temperature of mercury to superconductor state is

a. 4.2°C
b. 4.2 K
c. 7.1 K
d. 4.2°F

4) The relation between critical current and critical magnetic field is

a. $H_C = \frac{I_C}{\pi r^2}$
b. $H_C = \frac{I_C}{2\pi r}$
c. $H_C = \frac{I}{\pi r^2}$
d. $H_C = \frac{I_C}{2r}$

5) The critical magnetic field H_C (T) of a superconductor is given by

a. $H_C (0) [1 - \frac{T}{T_C}]$
b. $H_C (0) [1 - \frac{T_C}{T}]$
c. $H_C (0) [1 - \frac{T_C^2}{T}]$
d. $H_C (0) [1 - \frac{T^2}{T_C^2}]$

6) In a superconducting state

a. entropy alone changes
b. electronic specific heat alone changes
c. both change
d. entropy remains constant

7) Type-1 superconductors are known as

a. semiconductors
b. soft conductors
c. hard conductors
d. insulators

8) Type-2 superconductors are known as

a. semiconductors
b. soft conductors
c. hard conductors
d. insulators

9) In type-2 superconductors the magnetic flux

a. passes through the entire material
b. does not pass through the entire material
c. flux decreases exponentially
d. is constant

10) The quantum of magnetic flux is given by

a. h/2e
b. h/2π e
c. 2h/e
d. 2πh/e

11) The magnetic susceptibility of a superconductor is

a. -1
b. +1
c. 0
d. infinite

12) In a particular superconductor, if transition from superconducting phase to normal phase takes place gradually, it belongs to

a. Type-I
b. Type-II
c. Type- I and Type –II both
d. insulator

13) Hard superconductors observe

a. break down of Silsbee's rule
b. complete Meissner effect
c. low critical field

d. low transition temperature

II. Answer the following questions:

1. The resistance of a superconductor is practically zero. True or False?
2. What is the importance of isotope effect in superconductivity?
3. What are cooper pairs?
4. What is the effect of impurities on critical temperature of super conductor?
5. What is the effect of stress on superconductors?
6. What is an isotopic effect?
7. Define critical current density.
8. Define critical magnetic field.
9. What is persistent current?
10. What is Meissner effect?
11. Give the applications of Meissner effect.
12. What are Josephson effects? Explain in detail.
13. Give the difference between Type -1 and Type -2 superconductors.
14. What is SQUID? Give its applications.
15. How cryotron works? Explain in detail.
16. How does the Maglev vehicle works? Explain in detail.
17. List the applications of superconducting materials.

III. Problems for Practice

1. A superconducting tin has a critical temperature T_c of 3.7K at zero magnetic field and critical field of 0.0306 T at 0K. Find out the critical field at 3K. [Ans. 0.01 T]
2. The critical temperature (T_c) for mercury with isotopic mass 199.5 is 4.185 K. Calculate the value of T_c when its mass changes to 203.4. [Ans. 4.144 K]
3. The superconducting material has T_c = 4.8K at zero magnetic field and critical magnetic field H_o = 0.0708 Tesla at 0K. Find out the critical field at 4K. [Ans. 0.0216 T]
4. Find out value of critical current for superconducting wire with diameter of 3 mm at 2.3K. Given that critical temperature = 3.7 K and H_0 =0.0305 T. [Ans. 0.176 mA]
5. Calculate critical current density for a superconducting wire of Lead having diameter of 1.5 mm at 5.3K. The value of critical temperature of Lead is 7.8 K and critical magnetic field at 0 K is 6.5 x 10^4 A/m. [Ans. 93.6 × 10^6 Am^{-2}]
6. The critical temperature of mercury is 4.152 K for its one isotope of mass 200.59 amu. Calculate the critical temperature of mercury for its one isotope of mass 204 amu.[Ans. 4.118 K]

Unit 4 Non Linear Optics:
Section 1 Laser

Laser technology is one of the most rapidly developing areas in modern technology. Laser was invented in 1960 and then used in different areas of medicine, communication, military and industries etc.

In Indian mythology light is considered as knowledge (gyan) and truly said that "Tamaso ma Jyotir gamay" means take us to light from the darkness. So, let us throw light on light and enjoy the powerful light of Laser in this chapter.

LASER: Light Amplification by Stimulated Emission of Radiation

Characteristics of LASER:

1) Highly monochromatic:

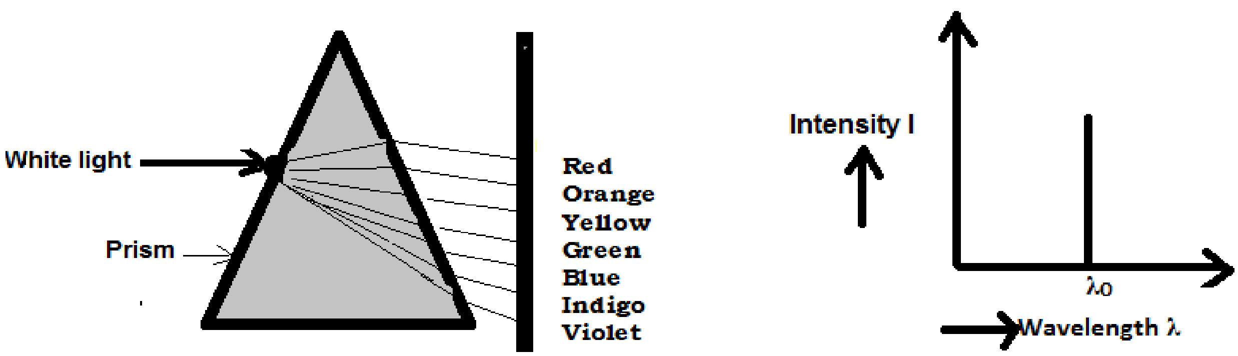


Figure 1

As shown in figure 1 an ordinary white light is polychromatic (Saptarangi Prakash) i.e. combination of many different wavelengths (vibgyor colours).In contrast, Laser light is a monochromatic light having only one colour that can be shown by a spectral line of wavelength λ_0.

2) Highly directional:

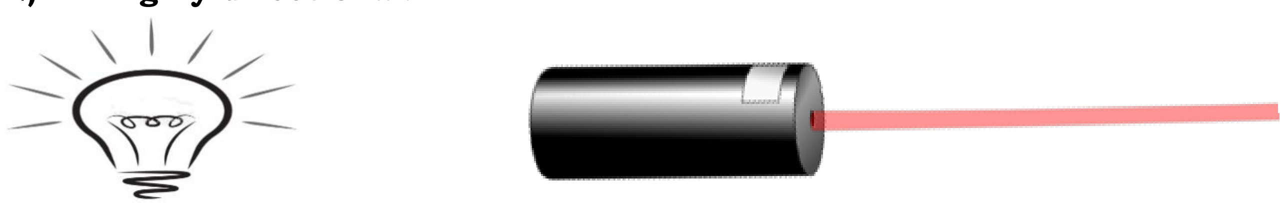


Figure 2

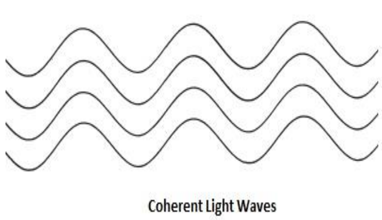


Figure 3

Ordinary light, such as coming from the sun, a light bulb, or a candle, is emitted in all directions away from the source while Laser light is emitted as a relatively narrow beam in a specific direction (figure 2). So it can travel longer distance without significant spreading (only 1mm/m).

3) **Highly coherence:** Light is an electromagnetic

wave so it can be described in wave theory as the superposition of sine waves as a function of time. Coherent waves are travelling with same phase or their phase difference is constant as shown in figure 3.

4) High Intensity:

Due to coherent nature of laser, it can focus over a very small area 10^{-6} cm^2 so extremely high concentration of its energy over a small area.

Types of LASER:

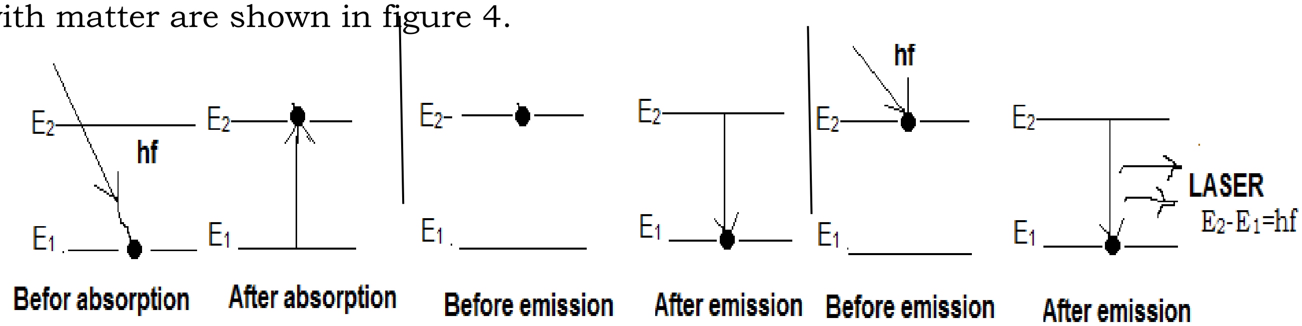
1) **Solid state lasers** have lasing material distributed in a solid matrix, e.g., the ruby and neodymium-yttrium aluminum garnet (Nd- YAG) lasers. The Nd-YAG laser emits infrared light at 1.064 micrometers.

2) **Gas lasers** (helium-neon, HeNe) have a primary output of a visible red light. CO_2 lasers emit energy in the far-infrared, 10.6 micrometers, and are used for cutting hard materials.

3) **Dye lasers** use complex organic dyes like rhodamine 6G in liquid solution or suspension as lasing media. They are tunable over a broad range of wavelengths.

4) **Semiconductor lasers** are not solid-state lasers. These electronic devices are generally very small and use low power. They may be built into larger arrays, e.g., the writing source in some laser printers or compact disc players.

Einstein's Theory: In 1916, Einstein showed the existence of equilibrium between matter and radiation required a new radiation process called the stimulated radiation. It needs the help of external radiation and generates very powerful stimulated radiation known as LASER. Different interactions of light with matter are shown in figure 4.



Absorption	Spontaneous Emission	Stimulated Emission
R_{12} α $N_1\rho$	$R_{21(sp)}$ α N_2	$R_{21(st)}$ α $N_2\rho$
$R_{12} = B_{12}N1\rho$	$R_{21(sp)} = A_{21}N_2$	$R_{21(st)} = B_{21}N_2\rho$

Figure 4

Lasing Action:

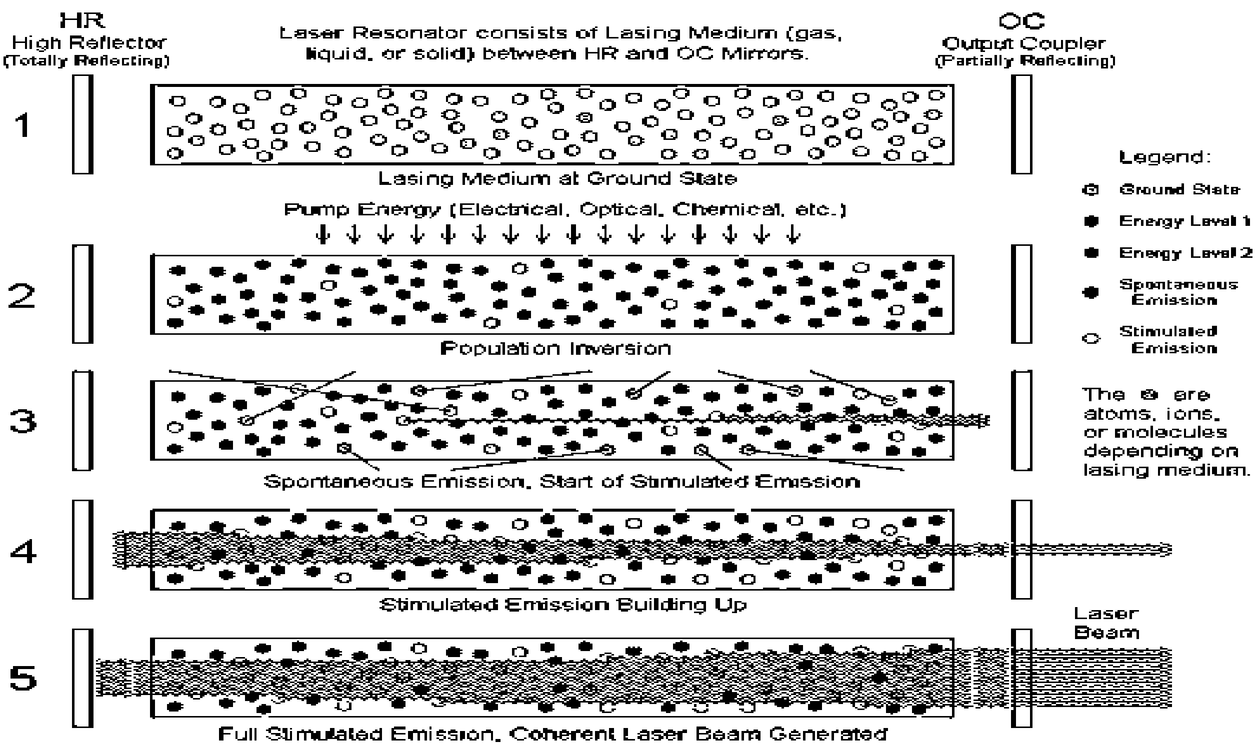
Figure shows the active medium which has been enclosed in optical resonator and being excited by pumping. The steps of the lesing action are shown in figure 5.

Step :1 Pumping

The atoms in the medium are in the ground state initially. By supplying energy from the external source they are excited from the ground state to the excited state.

Step :2 Population Inversion

The life time of the atoms in the exicited state is very small (10^{-9} S). Therefore the atoms drop spontaneously from the excited state to the metastable state. As the life time of the metastable state is relatively longer (10^{-3} S), the atoms go on accumulating in the metastable state. As soon as the number of atoms in



metastable state exceeds that of the ground state, the medium goes into the state of population inversion.

Step :3 Spontaneous emission

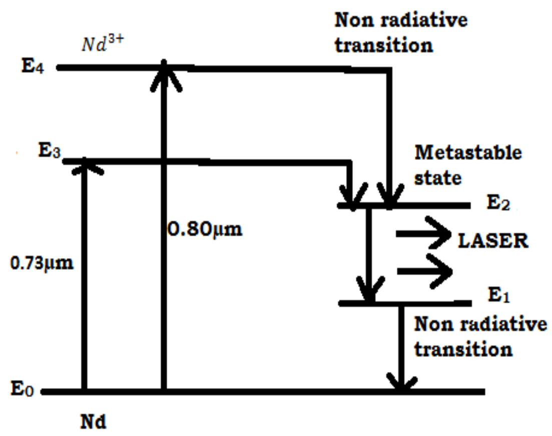
Some of the excited atoms at the metastable state may emit photons spontaneously in various directions and will be lost forever.

Step: 4 Amplification

The majority of photons travelling along the axis cause stimulated emission and are reflected back by both the mirrors and hence build up their strength and thus the amplification of light takes place. These mirrors operate as an oscillator and provide positive feedback of light into the medium.

Step: 5 Oscillations

Figure 5



At each reflection by front and back mirror, light is partially transmitted through it causing loss of energy from the resonator. When the losses at the mirrors and within the medium balance the gain, a steady and strong laser beam will emerge from the front end mirror.

Nd- YAG (Neodymium Yittrium Aluminium Garnet) laser:

➢ It is a four level device in which the active medium is YAG dopped with Nd^{+3} ions. And active center is Nd^{+3} ions.

➢ The optical pumping is used for population inversion by xenon lamp and two mirrors M1 and M2.

Construction:

➢ The schamtic diagram of ND-YAG laser is shown in figure. Nd YAG rod and a krypton flash lamp are enclosed inside an ellipsoidal reflector.

➢ In order to make the entire flash radiation to focus on the laser rod, the Nd-YAG rod is placed at one focal axis and the flash lamp at the other focal axis of the ellipsoidal reflector.

➢ The flash lamp is connected with a battery and a capacitor.

Working: As the flash lamp is switched on, the optical pumping excites Nd atoms from the ground energy state E_0 to the higher energy level E_3 and E_4 by absorbing radiations of wavelength 0.80μm and 0.73 μm, respectively.

➢ The excited Nd^{3+} ions then make a transition from these energy levels. The state E_2 is metastable state.

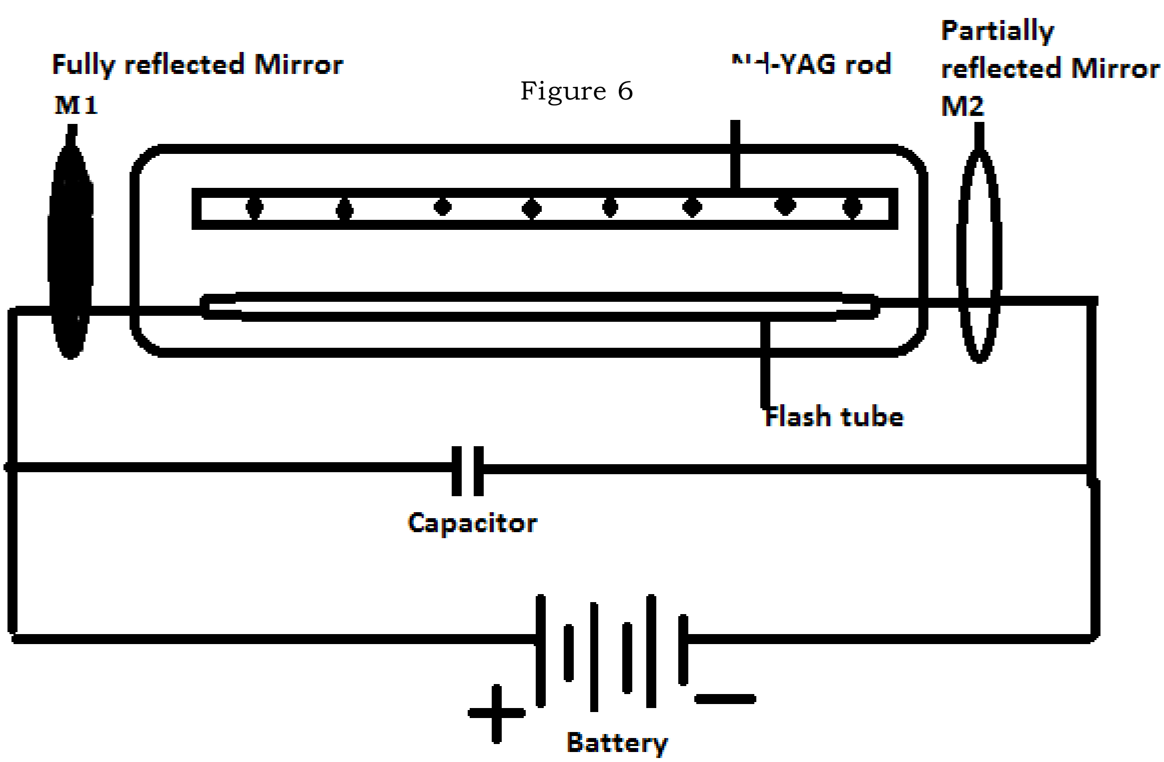


Figure 6

- Upon continuous excitation, population inversion of Nd^{3+} ions is achieved at the metastable state E_2. The energy level diagram is shown in figure 7.

Figure 7

Applications:

Some of the applications of laser are as follows:

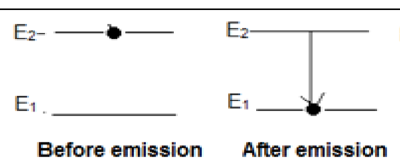
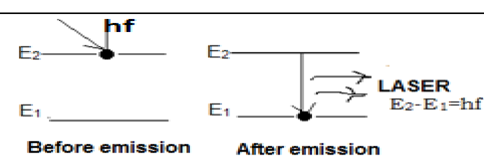
1) In Industry
- For welding and melting.
- For cutting and drilling holes.
- To test the quality of material.
- For heat treatment of metallic materials.

2) In Medical Science
- It is used for cataract surgery.
- It is used in performing micro surgery.
- It used to cure cancer and skin tumors in humans.

3) For Military Purposes
- The laser beam can be used in war as a weapon. It can destroy the big size objects like aero planes in few seconds. For this reason it can be even call as death ray.
- Laser beam can be used to determine the exact distance, velocity, direction as well as size of the object by means of reflected signal. This is known as LIDAR (Light Detection and Ranging.)

4) In Science and Engineering field
- It is used in fiber optic communication.
- It is used in holography.
- It is used in underwater communication between submarines, as they are not easily absorbed by water.
- It is used to create new chemical compounds by destroying atomic bonds between molecules.

Difference between Spontaneous and Stimulated Emission

Sr. no.	**Spontaneous Emission**	**Stimulated Emission**
1.		
2.	It does not take help of an external agency (photon)	It takes help of an external agency
3.	It is a random process	It is not a random process
4.	It is uncontrollable process	It is a controllable process

5.	Photons are emitted in all directions.	Photons emitted in the process travel only in one direction and hence are highly directional.
6.	Photons generated do not have same frequency and hence the light is not monochromatic	Photons generated have same frequency and hence the light is monochromatic
7.	The photons generated are not in same phase so they are incoherent.	The photons generated are in the same phase so they are coherent.
8.	In this process multiplication of photons does not take place so there is no amplification of light.	In this process multiplication of photons takes place so there is amplification of light.
9.	Light from the source is unpolarized.	Light from the source is polarized.

Solved Examples:

1. Find the ratio of population of the two states in a He-Ne laser that produces light of wavelength 698.3Å at 27˚C.

Given: T=27˚C= 300k, λ=698.3 Å, h=6.625x10^{-34}JS, k=1.38x10^{-23}J/k, C=3x10^{10}m/s

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

$$E_2 - E_1 = \frac{hc}{\boldsymbol{\lambda}}$$

$$\frac{N_2}{N_1} = e^{-\frac{\mathbf{6.625x10^{-34}x3x10^{10}}}{\mathbf{698.3x10^{-10}}}} = e^{-68.748} = \mathbf{1.3892x10^{-30}}$$

2. **A 10 mW He-Ne laser has efficiency of 1%. Assume that all input energy is utilized in pumping the atoms from the ground state to the excited state, which is 20 eV above the ground state. Find how many atoms are promoted to the excited state in 1 second.**

Given: Efficiency=1%=0.01

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\text{Power input} = \frac{\text{Power output}}{\eta} = \frac{10\text{mW}}{0.01} = \mathbf{1W}$$

Therefore energy input in one second= 1 J

No. of atoms excited in one second=$\frac{1J}{20eV} = \frac{1J}{20x\ 1.602x\ 10^{-19}} = \mathbf{3.12x10^{17}}$

3. A 10 mW laser has a beam diameter of 1.6mm. What is the intensity of the light. Assume that it is uniform across the beam.

Given: d=1.6mm, $r=0.8x10^{-3}$,$P=10mW=10x10^{-3}W$

$$I = \frac{Power\ of\ the\ laser}{Cross\ sectional\ Area\ of\ a\ beam} = \frac{P}{A} = \frac{10x10^{-3}w}{3.14x(0.8x10^{-3}m)^2} = \mathbf{4.97kW/m^2}$$

Assignment questions:

I. Multiple choice questions

1) If an atom jumps from a lower energy level to higher energy level, the process is known as
a. induced emission
b. induced absorption
c. spontaneous emission
d. spontaneous absorption

2) Emission of a photon by an excited atom due to interaction of external energy is called
a. stimulated emission
b. spontaneous emission
c. absorption
d. light amplification

3) The life time of an atom at an excited level is of the order of
a. few seconds
b. a millisecond
c. a nanosecond
d. unlimited

4) Light amplification is possible because of
a. stimulated emission
b. spontaneous emission
c. absorption
d. diffraction

5) The rate of absorption process is proportional to
a. photon density of incident radiation
b. population of the ground level
c. population of the excited level radiation and population of the ground level
d. photon density of incident sound

6) The rate of spontaneous emission is proportional to
a. population of the ground level
b. population of the excited level
c. photon density of incident radiation
d. electron density around the active center.

7) The rate of stimulated emission is proportional to
a. population of the ground level

b. population of the excited level

c. photon density of incident radiation

d. photon density of incident radiation and population of the excited level

8) Which of the following processes produce coherent light?

a. stimulated absorption

b. spontaneous emission

c. stimulated emission

d. both b and c

9) The population densities of lower and upper energy levels E_1 and E_2 are N_1 and N_2 respectively. Condition for population inversion between levels E_1 and E_2 is given by

a. $N_1 > N_2$

b. $N_1 = N_2$

c. $N_1 < N_2$

d. $N_1 \propto N_2$

10) The optical cavity consists of

a. cylindrical mirrors

b. three sets of plane mirrors

c. a pair of plane mirrors

d. a cylindrical vessel

II. **Answer the following questions**

1. How does a laser differ from a point source?
2. What are the processes through which interaction of radiation with matter can take pace?
3. Define spontaneous and stimulated emission of radiation
4. Explain the concept of population inversion in the context of laser.
5. What is metastable state? Why is it required in operation of laser?
6. What does pumping mean?
7. Explain the pumping methods used for lasers.
8. Distinguish between ground state, metastable state and excited state.
9. What is population inversion? How is it achieved by optical pumping?
10. Explain Nd-YAG LASER with level diagram in detail.

III. PROBLEMS FOR PRACTICE

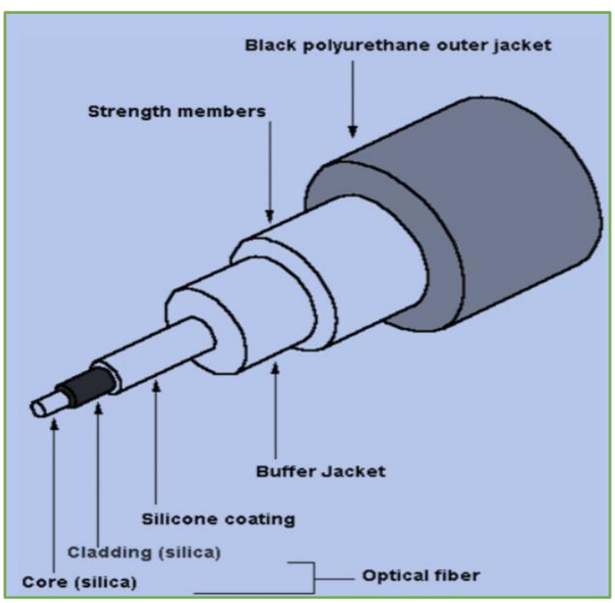
1. Find out the population inversion for two states in laser that produces light of wavelength 6328 Å at 27°C. [Ans. 1.1×10^{-33}]
2. A 10 mW laser has a beam diameter of 1.6 mm. What is the intensity of the light? [Ans. 4.97 kWm^{-2}]
3. Determine the wavelength of the radiation given out by a laser with energy of 3 eV. h= 6.63×10^{-34} Js and c= 3×10^{8} m/s.[Ans. 4143.8 Å]

Section 2 Fiber Optics

Optical Fiber System: It is the communication system in which information is sent in the form of light through a guided fiber cable (glass or plastic) from source to destination.

List of advantages of using optical fiber cable:

1) **Extremely large band width:** The information carrying capacity of a communication system is directly proportional to its bandwidth. The large bandwidth offers large number of signals (may be audio, video or both simultaneously) transmission. The light frequencies carried out by the fiber covers the range of 10^4 to 4×10^{14} Hz.
2) **Low losses:** Fibers have very low loss of information, only 0.2 dB/km compared to 20 dB/km in metallic cable.
3) **Immunity to electrostatic interference :** Not affected by natural noises like lightning or thundering i.e. noise free transmission
4) **Elimination of cross talk:** Absence of electromagnetic coupling between two adjacent wires eliminates the possibility of cross talking.
5) **Lighter in weight and smaller in size :** Easy transportation for ships, air crafts and high rise buildings
6) **Lower cost:** They are made up of SiO_2 or silica glass which is easily available in market and it is the second most available component in the earth. This cuts down its production cost.
7) **Security:** If anyone can try to trap your information beeping alarm will start at user's end.
8) **Greater safety:** Dielectric nature of optical fiber eliminates the possibility of spark hazards



9) **Corrosion:** Optical fiber made up of glass and plastic which are non-corrosive materials.
10) **Longer life span and less maintenance:** life span of optical fiber cable is longer than metallic cables also its maintenance is very less compared to metallic cable communication.

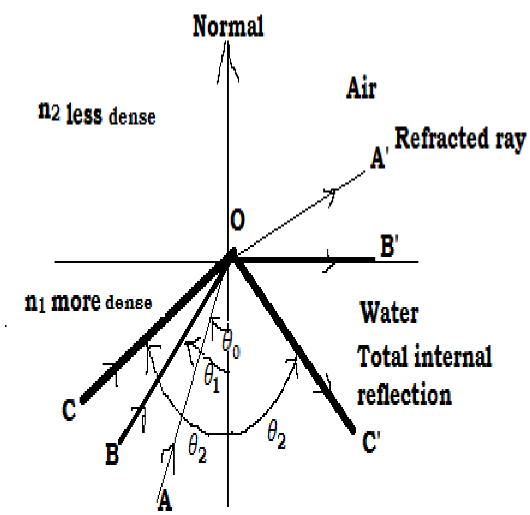
Optical fiber construction:

- The cable include core, cladding, buffer jacket, Kevlar, black poly urethane jacket as shown in figure 8.

Figure 8

- The core is the center of optical fiber made up having refractive index n_1
- Cladding is second sheath made up of glass with refractive index n_2
- Cladding in turn is covered by a buffer jacket (provide protection to fiber from external mechanical influences)
- Surrounding a buffer jacket there is layer known as Kevlar (a yarn type material) which increases tensile strength of the cable.

Working Principle of optical fiber:



When the ray of light is incident from optically denser medium to optically rarer medium three cases are observed as in figure 9.

Case :1 When the angle of incedence is less than critical angle ($\theta_0 < \emptyset_C$)

Ray AOA' gets refracted when it travels from the denser medium water to rarer medium air, it gets refracted and travels away from the normal with sbsequent loss of information.

Case :2 When the angle of incedence is equal to critical angle ($\theta_1 = \emptyset_C$)

Ray BOB' gets refracted and emerges parallel to the interface between core and cladding.

Case :3 When the angle of incedence is greater than critical angle ($\theta_2 > \emptyset_C$)

Figure 9

Ray COC' will have total internal reflection and information will remain in core, which is highly desirable.

Principle: When a light signal is directed at one end of the fiber at a suitable angle, it undergoes repeated total internal reflection along the length of the fiber and finally comes out at the other end.

Conditions for Total Internal Reflection:

1) Refractive index of the core is greater than that of the cladding then only total internal reflection of light will occur i.e.$n_1 > n_2$
2) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell's law)

$$\text{if } \theta_1 = \emptyset_c \text{ then } \theta_2 = \frac{\pi}{2} \quad \text{(by the definition of critical angle)}$$

Acceptance angle and Numerical aperture: Optical fiber will only propagate light that enters the fiber within a certain cone, known as acceptance cone of the fiber according to figure 10. The half-angle of this cone is called the acceptance angle $\sin \emptyset_{in(max)}$.

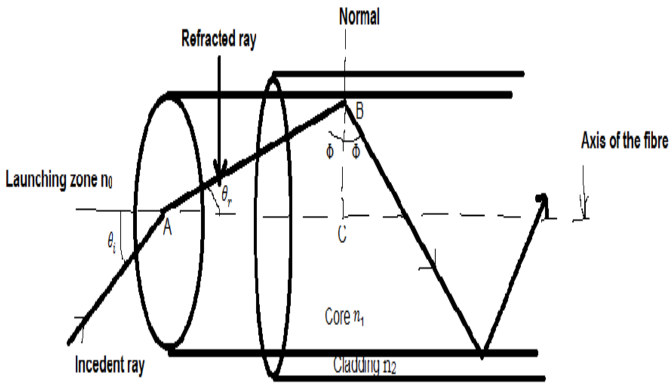


Figure 10

➢ In order to get total internal reflection of light, it must strike the internal core /cladding interface at an angle that is greater than the critical angle $\emptyset_c$.

➢ Let us consider a step index optical fiber into which light is launched at one end as shown in fig.

➢ Let n_1 and n_2 be the refractive indices of core and cladding respectively where $n_1 > n_2$.

➢ Let n_0 be the refractive index of the medium from where light is launched into the fiber.

➢ Assume that the light beam enters the fiber at an angle θ_i to the axis of the fiber. The ray refracts at an angle θ_r and strikes the core cladding interface at an angle Φ. If Φ is greater than Φ_C, the ray undergoes total internal reflection.

Applying Snell's law to the launching surface of the fiber,

$$n_0 \sin\theta_i = n_1 \sin\theta_r \tag{1}$$

If θ_i is increased beyond limit, Φ will drop below the critical value Φ_C and the ray escapes from the sidewalls of the fiber. The largest value of θ_i occurs when $\Phi = \Phi_C$.

From Δ ABC, it is seen that,

$$\sin\theta_r = \sin(90-\emptyset) = \cos\emptyset \tag{2}$$

Using equations (1) and (2), we get,

$$n_0 \sin\theta_i = n_1 \sin\theta_r = \cos\emptyset \tag{3}$$

$$\text{When } \Phi = \Phi_C,\ \sin\theta_{i(max)} = \frac{n_1}{n_0}\cos\emptyset_c \tag{4}$$

But, $\sin\emptyset_c = \frac{n_2}{n_1}$ and $\cos^2\emptyset_c = 1 - \sin^2\emptyset_c$

$$\therefore \cos\emptyset_c = \sqrt{1 - \frac{{n_2}^2}{{n_1}^2}} \tag{5}$$

$$\sin\theta_{i(max)} = \frac{\sqrt{1-\frac{{n_2}^2}{{n_1}^2}}}{n_0 x n_1} \tag{6}$$

$$\sin\theta_{i(max)} = \frac{\sqrt{(n_1^2-n_2^2)}}{n_0} \tag{7}$$

If the incedent ray is launched from air medium, $n_0 = 1$, Designeting $\theta_{i(max)} = \theta_0$, equation (7) becomes,

$$\sin\theta_0 = \sqrt{(n_1^2 - n_2^2)} \tag{8}$$

$$\therefore \theta_0 = \sin^{-1}\sqrt{(n_1^2 - n_2^2)} \tag{9}$$

Equation (9) gives the acceptance angle of the fiber. It is the maximum angle that a light ray can have relative to the axis of the fiber and propagate through it.

Numerical Aperture

It is the measure of the light gathering capacity of the fiber and is defined as the sine of the acceptance angle

Relative index Δ: It is the ratio of difference of refractive index of core or cladding to refractive index of core

$$\Delta = \frac{n_1 - n_2}{n_1} \tag{1}$$

➢ Prove that refractive index is independent from dimension of the optical fiber.

We know that,

$$\sin\theta_0 = \sqrt{(n_1^2 - n_2^2)}$$

Numerical aperture can also be defined as the sine of the acceptance angle.

$\therefore NA = \sin\theta_0$

$$NA = \sqrt{n_1^2 - n_2^2} \tag{2}$$

$n_1^2 - n_2^2 = (n_1 + n_2)(n_1 - n_2) = \frac{(n_1+n_2)}{2} \times \frac{(n_1-n_2)}{n_1} \times 2n_1$ (3) (From equation (1))

Taking approximation, $\frac{(n_1+n_2)}{2} \approx n_1$ then, $n_1^2 - n_2^2 = 2n_1^2\,\Delta$

Numerical aperture becomes,

$$NA = n_1\sqrt{2\Delta} \tag{4}$$

Equation (4) indicates that numerical aperture (NA) depends on the refractive indices of core and cladding and does not depend on dimensions of optical fiber. Large NA implies that the fiber will collect large amount of light from the source. That means large amount of information can be transmitted through it. The value of NA ranges from 0.13 to 0.50. Fibers with different NA are shown in figure 11.

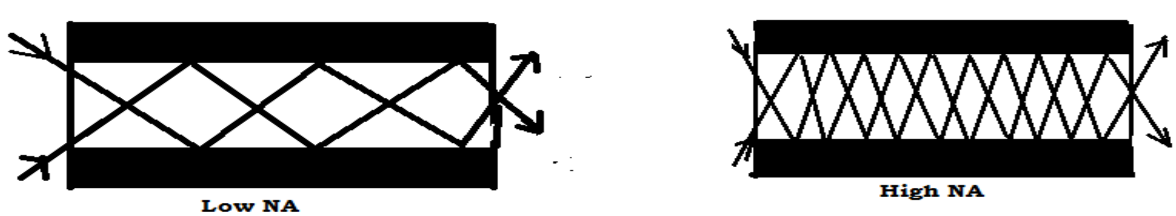


Figure 11

Classification of Optical fiber cable: optical fiber cables can be classified based upon several criteria as shown in the diagram in figure 12.

Types of material used for construction of optical fiber:

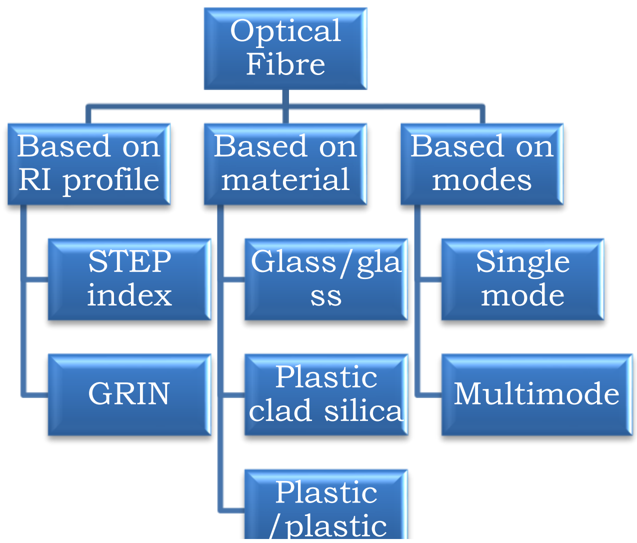


Figure 12

- High content silica glass

Advantage : useful in mass communication

- Multi components & silica glass

Disadvantages: attenuation or dispersion is very high

PCP (Plastic core & Plastic cladding)

Advantages:

- More flexible
- Easy to install
- It can withstand stress
- Less expensive
- Lighter in weight

Disadvantage: Very high attenuation

Application: Short distance computer (information capability 6 mbps)

PCS (Glass core & Plastic cladding)

Advantages:

- law attenuation
- Less affected by radiation

Disadvantage: Costly compared to pcp

Application: military

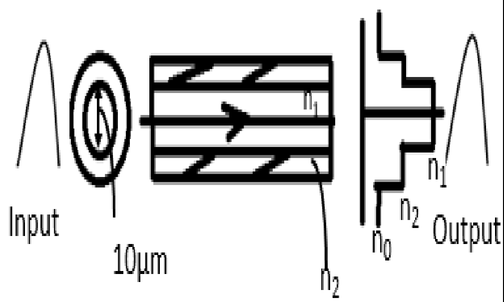
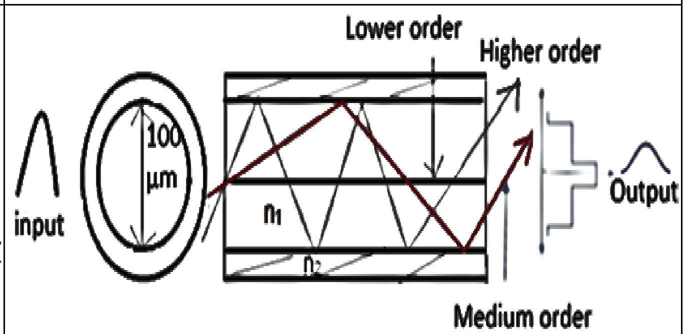
SCS (Glass core & Glass cladding)

Advantages:

- Attenuation is slightly better than pcs (Least attenuation)
- Best propagation
- e.g.- SiO_2 core ,P_2O_5- SiO_2cladding
- P_2O_5- SiO_2 core, SiO_2 cladding

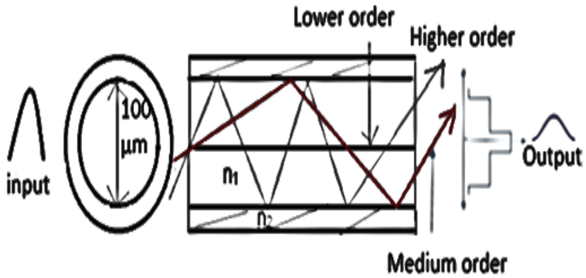
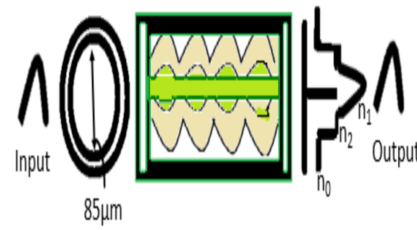
Disadvantage: Breakable, more weight

Difference between Single Mode Step Index (SMSI) Fiber amd Multi Mode Step Index (MMSI) Fiber:

Sr. No.	**Single Mode(SMSI)**	**Multi Mode (MMSI)**
1.		
2.	It offers only one path for propagation	It offers more than one path for propagation
3.	The core diameter is very small(10 μm)	The core diameter is very large(100 μm)
4.	There is no degradation or dispersion of the signal	There is degradation or dispersion of the signal.
5.	Launching information is very difficult	Launching information is very easy
6.	Fabrication is not easy .	Fabrication is easy.
7.	Numerical aperture is very law	Numerical aperture is very high
8.	Application: long haul communication	Application : short distance communication(LAN)

Difference between Multi Mode Step Index (MMSI) and Graded Index (GRIN) Fiber:

Sr. No.	**MMSI Cable**	**GRIN cable**

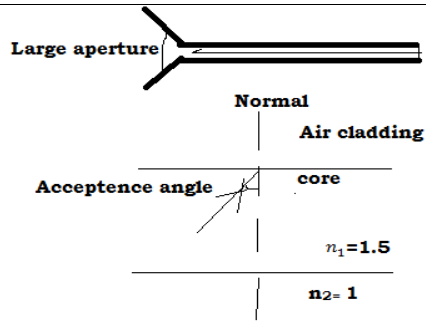
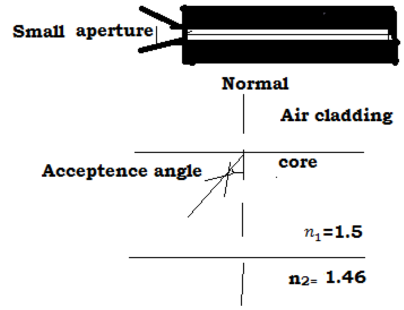
1.		
2.	Refractive index of core and cladding is uniform	Refractive index of core and cladding is not uniform
3.	The shape of the profile looks like step. So it is called step index	The refractive index is maximum at the center and decrease gradually with distance so it is called graded index and the shape of the profile is parabolic
4.	Light propagates by reflection	Light propagates by refraction
5.	Pulse dispersion is more	Pulse dispersion is less
6.	The light follow zigzag path and for every reflection will cross the fiber axis	The light follow helical path will not cross the fiber axis
7.	Refractive index of core and cladding is uniform	Refractive index of core and cladding is not uniform
8.	It offers low band width	It offers high band width
9.	Numerical aperture is low	Numerical aperture is high
10.	$N_{step} = 4.9x\left(\frac{dxNA}{\lambda}\right)^2$ Where d = core diameter λ = Optical wavelength NA = Numerical aperture	$N_{graded} = \frac{4.9}{2}x\left(\frac{dxNA}{\lambda}\right)^2 = \frac{N_{step}}{2}$ Where d = core diameter λ = Optical wavelength NA = Numerical aperture

Note:

- V number: the number of modes separated for propagation is defined by a parameter V number
- $V = \frac{2\pi a}{\lambda} NA$
- Maximum number of mode in step index fiber $Nm = \frac{v^2}{2}$
- Maximum number of mode in graded index fiber $Nm = \frac{v^2}{4}$
- For SMSI (V< 0.495) or For MMSI (V >0.495)

Difference between SMSI with air cladding and SMSI with glass cladding

Sr. no.	SMSI Cable	SMSI with glass cladding

1.		
2.	$\sin \emptyset_c = \frac{n_2}{n_1}$ $\emptyset_c = \sin^{-1}\frac{n_2}{n_1} =$ $\sin^{-1}\frac{1}{1.5} = 41°8'$ $\emptyset_{c(max)} = 90° - 41°8' = 48°2'$	$\sin \emptyset_c = \frac{n_2}{n_1}$ $\emptyset_c = \sin^{-1}\frac{n_2}{n_1}$ $= \sin^{-1}\frac{1.46}{1.5} = 76°7'$ $\emptyset_{c(max)} = 90° - 76°7' = 13°13'$
3.	To glass core & air cladding acceptance angle is high ~ 48˚2' so it is easy to launch the light & couple with output device	To glass core & glass cladding acceptance angle is less ~ 13˚3' so it is difficult to launch the light & couple with output device
4.	The fiber is very weak and hence has a limited application	The fiber is physically very strong and hence has a large number of applications

Applications of Optical fiber:

1) In communication: Because of large bandwidth it can transmit about 10^6 programs at a time. The communication system is classified into two groups.(1) LAN: Local Area Network which operates over a short distance of about 1 to 2 km and (2) Long Haul Communication. These systems are used for long distances (10km or more). e.g. telephone cables

2) Medical applications: Optic endoscope is used to inspect internal organs for diagnosis purpose.

3) Military applications: Ships and aircrafts need tons of copper wire for communication and control mechanism. In place of copper wires if optical fiber is used then it can reduce the weight remarkably.

4) Guided missiles are used in recent wars.

Solved Examples:

1. A silica optical fiber has a refractive index of 1.55 and a cladding of refractive index of 1.47. Determine (a) the critical angle at the core cladding interface (b) the numerical aperture for the fiber and (c) the acceptance angle in air for the fiber.

Given, $n_1 = 1.55$; $n_2 = 1.47$; $\emptyset_c = ?$; NA = ?; Øin(max) = ?

$$\emptyset_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.47}{1.55} = \mathbf{71°30'}$$

The numerical aperture for the fiber is

$$NA = \sqrt{(n_1^2 - n_2^2)} = \sqrt{1.552 \; - 1.472} \; = \mathbf{0.4915}$$

The acceptance angle in air for the fiber is

$$Øin(max) = \sin^{-1}(NA) = \sin^{-1}(0.4915) = \mathbf{29°26'}.$$

2. An optical fiber has numerical aperture of 0.2 and cladding refractive index of 1.59. Determine the acceptance angle for the fiber in water which has refractive index 1.33.

Given: n_2=1.55 , NA=0.20 , n_0=1.33, n_1=?; NA in water=?, Øin(max)=?

$NA = \sqrt{(n_1^2 - n_2^2)}$

$n_1 = \sqrt{n_2^2 + NA^2} \qquad = \sqrt{1.55^2 + 0.20^2} = \mathbf{1.5628}$

When fiber is in water n_0=1.33

Therefore, $NA = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0} = \frac{0.1996}{1.33} = \mathbf{0.15}$

Hence acceptance angle is

Øin(max) = $\sin^{-1}$ NA=**8˚37'**

3. A refractive index of core for step index fiber is 1.52, diameter is 2.9 µm and a fractional difference of refractive index is 0.0007. It is operated at a wavelength of 1.3 µm. Find the number of modes the fiber will support.

Given: d = 2.9 µm, n_1 = 1.52, Δ= 0.0007,λ = 1.3 µm

$$NA = n_1\sqrt{2\Delta} = 1.52\sqrt{2x0.0007} = 0.0086$$

$$N_{step} = 4.9x\left(\frac{dxNA}{\lambda}\right)^2 = \frac{4.9\,x(2.9x\,10^{-6})x0.0086}{1.3x10^{-6}} = 1.0766$$

4. A fiber cable has an acceptance angle of 30˚and the refractive index of core is 1.4. Calculate the refractive index of the cladding.

Given: n_1=1.4, θ_0 =30˚, n_2=?

$$\sin^2\theta_0 = \sqrt{(n_1^2 - n_2^2)}$$

$$\sin^2\theta_0 = n_1^2 - n_2^2$$

$$n_2^2 = n_1^2 - \sin^2\theta_0 = (1.4)^2 - \sin^2 30° = 1.96 - 0.25 = 1.71$$

$$\therefore \mathbf{n_2 = 1.308}$$

5. Calculate the fractional index change for a given fiber if the refractive indices of core and cladding are 1.502 and 1.498 respectively.

Given: n_1 =1.502 and n_2 = 1.498

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.502 - 1.498}{1.502} = \frac{0.065}{1.563} = \mathbf{0.0415}$$

6. Calculate the refractive indices of core and cladding if numerical aperture is 0.22 and relative refractive index is 0.012.

Given: NA=0.22 and Δ=0.012

$$\mathrm{NA} = n_1\sqrt{2\Delta}$$

$$0.22 = n_1\sqrt{2x0.012} = 0.155n_1$$

$$n_1 = \frac{0.22}{0.155} = \mathbf{1.42}$$

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.42 - n_2}{1.42} = 0.012$$

$$n_2 = 1.42 - 1.42x0.012 = \mathbf{1.403}$$

Assignment questions:

I. Multiple Choice Questions

1) Light propagates through an optical fiber due to

a. reflection

b. refraction

c. interference

d. total internal reflection

2) Total internal reflection occurs when a light ray travels from

a. rarer to denser medium

b. denser to rarer medium

c. denser to denser medium

d. rarer to rarer medium

3) A light ray travels from a medium of refractive index n_1 to another medium of refractive index n_2 ($n_1 > n_2$) and undergoes total internal reflection. Then Snell's law reduces to

a. $n_1 n_2 = \sin i_c \sin r$

b. $n_1 \sin i_c = n_1 \sin r$

c. $\sin i_c = n_2/n_1$

d. $\sin i_c = n_1/n_2$

4) An optical fiber is made up of

a. metal

b. semiconductor

c. dielectric

d. composite material

5) In an optical fiber light propagates in the

a. core

b. buffer

c. cladding

d. Air

6) The refractive index of the core is

a. higher than that of cladding

b. lower than that of cladding

c. same as that of cladding

d. same as that of buffer

7) The condition for propagation of light through the optical fiber is that

a. the angle of incidence of the light ray at the core-cladding interface must be smaller than the critical angle

b. the angle of incidence of the light ray at the core-cladding interface must be greater than the critical angle

c. the refractive index of the cladding should be smaller than that of the core

d. The refractive index of the cladding should be greater than that of the cladding and the angle of incidence of the light ray at the core-cladding interface must be greater than the critical angle.

8) If the angle of incidence of the light ray at the core-cladding interface is smaller than the critical angle then the ray travels

a. in the core

b. in the buffer

c. in the cladding

d. along the interface

9) The fractional refractive index change Δ is given by,

a. $\frac{n_2 - n_1}{n_1}$

b. $\frac{n_1 - n_2}{n_1}$

c. $\frac{n_1 - n_2}{n_2}$

d. $\frac{n_2 - n_1}{n_2}$

10) Numerical aperture depends on

a. diameter of the fiber

b. propagation angle

c. acceptance angle

d. critical angle

11) Choose the correct expression for numerical aperture NA

a. $\frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

b. $\sin \theta_C$

c. $\frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

d. $n_2 \sqrt{2\Delta}$

12) Optical fibers are classified on the basis of refractive index variation as

a. single mode fiber and GRIN fiber

b. step-index fiber and GRIN fiber

c. multimode fiber and GRIN fiber

d. glass-glass and glass-plastic fiber

13) Single mode step index fibers are superior to other fiber because of

a. low attenuation of the signals

b. lower numerical aperture

c. lower dispersion

d. lower data transfer rate

14) Distance bandwidth product is maximum in

a. GRIN fiber

b. single mode fiber

c. multimode fiber

d. step index fiber

15) The refractive indices of core and cladding are 1.50 and 1.44 respectively. Then the acceptance angle is

a. 22°

b. 23 °

c. 24 ° 50’

d. 25 ° 35’

16) The refractive index of cladding of a fiber with core refractive index of 1.5 and numerical aperture 0.244 is

a. 1.230

b. 1.320

c. 1.480

d. 1.650

II. Answer the following questions

1. What is an optical fiber?
2. Describe the structure of a typical optical fiber.
3. What is necessary for cladding?
4. Explain the basic principle used in optical fiber for transmission of light.
5. What is meant by refractive index profile?

6. What is a step index fiber? How is light propagated in a step index fiber?

7. What is a GRIN fiber? How is light propagated in a GRIN fiber?

8. Explain with diagram, the distribution of different modules in a step index fiber.

9. The number of modes propagating in an optical fiber increases as the acceptance angle increases. Why?

10. The information carrying capacity of electromagnetic radiation is very high in the optical region. Why?

11. What is V-number? Give its significance.

12. Define critical angle.

13. State Snell's Law.

III. Problems for Practice

1. An optical fiber core and its cladding have refractive indices of 1.54 and 1.40 respectively. Calculate the critical angle $\emptyset_c$,acceptance $\emptyset_{in\,(max)}$ and numerical aperture. [Ans. 65°22', 39° 54', 0.64]

2. The refractive index of the core and cladding materials of an optical fiber are 1.59 and 1.51 respectively. Calculate the numerical aperture of the optical fiber.[Ans. 0.497]

3. A silica optical fiber has a core of refrective index 1.55 and a cladding of refractive index of 1.45. Determine (a) the critical angle at the core cladding interface (b) the numerical aperture for the fiber and (c) the acceptance angle in air for the fiber.[Ans. 69°18', 0.55, 33°12']

4. Calculate the refractive indices of the core and cladding material of a fiber. Given, NA = 0.22 and refractive index difference$\Delta = 0.012$. [Ans.n_1= 1.42, n_2 = 1.40]

5. Calculate the NA and acceptance angle of the fiber having n_1=1.48 and n_2=1.43. [Ans. 0.38, 22°25']

6. An optical fiber core and its cladding have refractive indices of 1.545 and 1.495 respectively. Calculate the critical angle, acceptance angle and numerical aperture.[Ans.75°23', 22°56', 0.3896]