Lecture Notes
UNIT-1
CALCULUS

1 Indeterminate Forms:

★ There are total 7 types of indeterminate forms :

(I) $\dfrac{0}{0}$ (II) $\dfrac{\infty}{\infty}$ (III) $0 \times \infty$ (IV) $\infty - \infty$ (V) 1^{∞} (VI) ∞^{0} (VII) 0^{0}

To solve this type of indeterminate forms we can used the following rule:

★ **L'Hospital's Rule**:

If $f(x)$ & $g(x)$ are two function of x which can be expanded by taylor's series about $x = a$ and if $\lim\limits_{x\to a} f(x) = f(a) = 0$ & $\lim\limits_{x\to a} g(x) = g(a) = 0$ then,

$$\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$$

i.e. Simply derivative of numerator and derivative of denominator individually.

(1) $\mathbf{\dfrac{0}{0}}$ Form :

[**1**] $\lim\limits_{x\to 0} \dfrac{\mathbf{(1+x)^n - 1}}{\mathbf{x}}$

Solution:- $l = \lim\limits_{x\to 0} \dfrac{(1+x)^n - 1}{x}$ $\left[\dfrac{0}{0}\ form\right]$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{n(1+x)^{n-1}}{1} = n$$

[**2**] $\lim\limits_{x\to 0} \dfrac{\mathbf{e^x - 1 - x}}{\mathbf{x^2}}$

Solution:- $l = \lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$ $\left[\dfrac{0}{0}\ form\right]$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{e^x - 1}{2x} \quad \left[\frac{0}{0}\ form\right]$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$$

[**3**] $\lim\limits_{x\to\ 0} \dfrac{\log(\mathbf{1}+\mathbf{x^3})}{\sin^{\mathbf{3}}\mathbf{x}}$

Solution:- $l = \lim\limits_{x\to\ 0} \dfrac{\log(1+x^3)}{x^3\dfrac{\sin^3 x}{x^3}}$

$= \lim\limits_{x\to\ 0} \dfrac{\log(1+x^3)}{x^3} \qquad \left[\dfrac{0}{0}\ form\right]$

By L'Hospital's Rule

$= \lim\limits_{x\to\ 0} \dfrac{\dfrac{1}{1+x^3}\cdot 3x^2}{3x^2}$

$= \lim\limits_{x\to\ 0} \dfrac{1}{1+x^3} = 1$

[**4**] $\lim\limits_{x\to\ a} \dfrac{\mathbf{x^a}-\mathbf{a^x}}{\mathbf{x^x}-\mathbf{a^a}}$

Solution:- $l = \lim\limits_{x\to\ a} \dfrac{x^a-a^x}{x^x-a^a} \qquad \left[\dfrac{0}{0}\ form\right]$

By L'Hospital's Rule

$= \lim\limits_{x\to\ a} \dfrac{ax^{a-1}-a^x\log a}{x^x(1+\log x)-0}$

$= \dfrac{aa^{a-1}-a^a\log a}{a^a(1+\log a)}$

$= \dfrac{a^a-a^a\log a}{a^a(1+\log a)}$

$= \dfrac{1-\log a}{1+\log a}$

(2) $\dfrac{\infty}{\infty}$ **Form :** Evaluate this type of problems by same as above $\dfrac{0}{0}$ form .

[**1**] $\lim\limits_{x\to\ 0} \dfrac{\log(\sin\mathbf{x})}{\cot\mathbf{x}}$

Solution:- $l = \lim\limits_{x\to\ 0} \dfrac{\log(\sin x)}{\cot x} \qquad \left[\dfrac{\infty}{\infty}\ form\right]$

By L'Hospital's Rule

$= \lim\limits_{x\to\ 0} \dfrac{\dfrac{1}{\sin x}\ \cos x}{-\operatorname{cosec}^2 x}$

$= \lim\limits_{x\to\ 0} \dfrac{-\cos x}{\operatorname{cosec} x}$

$= \lim\limits_{x\to\ 0}\ -\cos x\cdot\sin x = 0$

[**2**] $\lim\limits_{x\to\ a} \dfrac{\log(\mathbf{x}-\mathbf{a})}{\log(\mathbf{a^x}-\mathbf{a^a})}$

Solution:- $l = \lim\limits_{x\to\ a} \dfrac{\log(x-a)}{\log(a^x-a^a)} \qquad \left[\dfrac{\infty}{\infty}\ form\right]$

By L'Hospital's Rule

$$= \lim_{x\to\ a} \frac{\frac{1}{x-a}}{\frac{1}{a^x-a^a}\ a^x\log a}$$
$$= \lim_{x\to\ a} \frac{a^x-a^a}{(x-a)a^x\log a}$$
$$= \lim_{x\to\ a} \frac{a^x-a^a}{x-a}\ \lim_{x\to\ a} \frac{1}{a^x\log a}$$
$$= \frac{1}{a^a\log a}\quad \lim_{x\to\ a} \frac{a^x-a^a}{x-a} \qquad \left[\frac{0}{0}\ form\right]$$

By L'Hospital's Rule

$$= \frac{1}{a^a\log a}\quad \lim_{x\to\ a} \frac{a^x\ \log a}{1}$$
$$= \frac{1}{a^a\log a}\ \ a^a\log a$$
$$= 1$$

[**3**] $\lim\limits_{x\to\ 0}\ \log_{\tan\mathbf{x}}(\tan\mathbf{2x})$ **(Ans. 1)**

Solution:- $l = \lim\limits_{x\to\ 0}\ \log_{\tan x}(\tan 2x)$

$$= \lim_{x\to\ 0} \frac{\log(\tan 2x)}{\log(\tan x)} \qquad \left[\frac{\infty}{\infty}\ form\right]$$

By L'Hospital's Rule

$$= \lim_{x\to\ 0} \frac{\dfrac{1}{\tan 2x}\ \sec^2 2x\ 2}{\dfrac{1}{\tan x}\ \sec^2 x}$$
$$= \lim_{x\to\ 0} \frac{\tan x\ \sec^2 2x\ 2}{\tan 2x\ \sec^2 x}$$
$$= \lim_{x\to\ 0} \frac{\dfrac{\tan x}{x}\ \sec^2 2x}{\dfrac{\tan 2x}{2x}\ \sec^2 x}$$
$$= \lim_{x\to\ 0} \frac{\sec^2 2x}{\sec^2 x}$$
$$= 1$$

(3) $0 \times \infty$ Form : If , $\lim\limits_{x\to a} f(x) = 0$ & $\lim\limits_{x\to a} g(x) = \infty$ then , $\lim\limits_{x\to a} f(x)\cdot g(x)$ $(0\times\infty\ form)$

is convert in the form of,

$$\boxed{\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} \frac{f(x)}{\frac{1}{g(x)}}} \qquad \left(\frac{0}{0}\ form\right)$$

<u>or</u>

$$\boxed{\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} \frac{g(x)}{\frac{1}{f(x)}}} \qquad \left(\frac{\infty}{\infty}\ form\right)$$

Evaluate this form same as above.

[**1**] $\lim\limits_{x\to 0} \mathbf{x}\cdot\log\mathbf{x}$

Solution:- $l = \lim\limits_{x\to 0} x\cdot\log x \quad [0\times\infty\ form]$

$$= \lim_{x\to 0} \frac{\log x}{\frac{1}{x}} \qquad \left[\frac{\infty}{\infty}\ form\right]$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0} (-x) = 0$$

[**2**] $\lim\limits_{x\to 0} \frac{\mathbf{1}}{\mathbf{x}}(\mathbf{1}-\mathbf{x}\cot\mathbf{x})$

Solution:- $l = \lim\limits_{x\to 0} \frac{1}{x}(1-x\cot x)$

$$= \lim_{x\to 0} \left(1-\frac{x}{\tan x}\right)\frac{1}{x} \qquad [0\cdot\infty\ form]$$

$$= \lim_{x\to 0} \frac{\left(1-\frac{x}{\tan x}\right)}{\frac{1}{x}}$$

$$= \lim_{x\to 0} \frac{\tan x - x}{x\tan x}$$

$$= \lim_{x\to 0} \frac{\tan x - x}{x^2\frac{\tan x}{x}}$$

$$= \lim_{x\to 0} \frac{\tan x - x}{x^2} \qquad \left[\frac{0}{0}\ form\right] \qquad \left(\because \lim_{x\to 0}\frac{\tan x}{x} = 1\right)$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{\sec^2 x - 1}{2x} \qquad \left[\frac{0}{0}\ form\right]$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{2\sec^2 x \tan x}{2}$$

$$= 0$$

[**3**] $\lim_{x\to 0} \sin \mathbf{x} \cdot \log \mathbf{x}$

Solution:- $l = \lim_{x\to 0} \sin x \cdot \log x \qquad [0 \times \infty\ form]$

$$= \lim_{x\to 0} \frac{\log x}{\frac{1}{\sin x}} \qquad \left[\frac{\infty}{\infty}\ form\right]$$

$$= \lim_{x\to 0} \frac{\log x}{\operatorname{cosec} x}$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{-\operatorname{cosec} x\ \cot x}$$

$$= \lim_{x\to 0} -\sin x \cdot \frac{\tan x}{x}$$

$$= \lim_{x\to 0} -\sin x \qquad \left[\because \lim_{x\to 0} \frac{\tan x}{x} = 1\right]$$

$$= 0$$

(4) $\infty - \infty$ **Form :**

In this form we are generally taking LCM and convert in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and evaluate this form by applying L'Hospital's rule.

[**1**] $\lim_{x\to 0} \left[\frac{\mathbf{1}}{\mathbf{x}} - \frac{\mathbf{1}}{\mathbf{x^2}} \log(\mathbf{1}+\mathbf{x})\right]$

Solution:- $l = \lim_{x\to 0} \left[\frac{1}{x} - \frac{1}{x^2}\log(1+x)\right] \qquad [\infty - \infty\ form]$

$$= \lim_{x\to 0} \left[\frac{x^2 - x\log(1+x)}{x^2}\right]$$

$$= \lim_{x\to 0} \left[\frac{x - \log(1+x)}{x}\right] \qquad \left(\frac{0}{0}\ form\right)$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \left[\frac{1-\dfrac{1}{1+x}}{2x}\right] \qquad \left(\frac{0}{0}\ form\right)$$

By L'Hospital's Rule

$$= \lim_{x\to 0} \frac{\left(\dfrac{1}{1+x}\right)^2}{2}$$

$$= \frac{1}{2}$$

[**2**] $\lim_{x\to \frac{\pi}{2}} \left(\tan \mathbf{x} - \dfrac{\mathbf{2x}\sec \mathbf{x}}{\pi}\right)$

Solution:- $l = \lim_{x\to \frac{\pi}{2}} \left(\tan x - \dfrac{2x\sec x}{\pi}\right) \quad [\infty - \infty\ form]$

$$= \lim_{x\to \frac{\pi}{2}} \left(\frac{\sin x}{\cos x} - \frac{2x}{\pi\ \cos x}\right)$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{\pi \sin x - 2x}{\pi \cos x} \qquad \left[\frac{0}{0}\ form\right]$$

By L'Hospital's Rule

$$= \lim_{x\to \frac{\pi}{2}} \frac{\pi \cos x - 2}{-\pi \sin x} = \frac{2}{\pi}$$

(5) $\mathbf{1^\infty}$, $\infty^{\mathbf{0}}$, $\mathbf{0^0}$ **Forms :** To solve this type of limit $\lim_{x\to a} [f(x)]^{g(x)}$

let l = $\lim_{x\to a} [f(x)]^{g(x)}$

Taking log on both the side,we get,

$\Rightarrow \log l = \lim_{x\to a} \log[f(x)]^{g(x)} \ \Rightarrow \log l = \lim_{x\to a} g(x)\cdot \log[f(x)]$

this is the $0\times\infty$ form and evaluate same as above and suppose it is a then ,

$\log l = a$

1taking exponential on both the side the we get,

$l = e^a$

[**1**] $\lim_{x\to 0} (\mathbf{e^{3x}} - \mathbf{5x})^{\dfrac{\mathbf{1}}{\mathbf{x}}}$

Solution:- $l = \lim_{x\to 0} (e^{3x} - 5x)^{\dfrac{1}{x}} \quad [1^\infty\ form]$

Taking log on othe the side, we get,

$$\log l = \lim_{x\to\ 0} \frac{1}{x} \log(e^{3x} - 5x) \qquad [0 \cdot \infty \ form]$$

$$\log l = \lim_{x\to\ 0} \frac{\log(e^{3x} - 5x)}{x} \qquad \left(\frac{0}{0}\ form\right)$$

By L'Hospital's Rule

$$\log l = \lim_{x\to\ 0} \frac{\frac{1}{(e^{3x} - 5x)} \cdot (3e^{3x} - 5)}{1}$$

$$\log l = \lim_{x\to\ 0} \frac{(3e^{3x} - 5)}{(e^{3x} - 5x)}$$

$$\log l = -2$$

$$l = e^{-2}$$

[**2**] $\lim_{x\to\ 0} (\cot \mathbf{x})^{\sin \mathbf{x}}$

Solution:- $l = \lim_{x\to\ 0} (\cot x)^{\sin x} \qquad [\infty^0\ form]$

Taking log on both the side,we get,

$$\log l = \lim_{x\to\ 0} \sin x\ \log(\cot x) \qquad [0 \cdot \infty \ form]$$

$$\log l = \lim_{x\to\ 0} \frac{\log(\cot x)}{\operatorname{cosec} x} \qquad \left[\frac{\infty}{\infty}\ form\right]$$

By L'Hospital's Rule

$$\log l = \lim_{x\to\ 0} \frac{\frac{1}{\cot x} \cdot (-\operatorname{cosec}^2 x)}{-\operatorname{cosec} x \cdot \cot x}$$

$$\log l = \lim_{x\to\ 0} \frac{\operatorname{cosec} x}{\cot^2 x}$$

$$\log l = \lim_{x\to\ 0} \frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x}$$

$$\log l = \lim_{x\to\ 0} \frac{\sin x}{\cos^2 x}$$

$$\log l = 0$$

Taking exponential on both the side, we get,

$$l = e^0$$

$$l = 1$$

[**3**] $\lim_{x\to\infty} \left(\frac{\mathbf{1}}{\mathbf{x}}\right)^{\frac{\mathbf{1}}{\mathbf{x}}}$

Solution:- $l = \lim_{x\to\infty} \left(\frac{1}{x}\right)^{\frac{1}{x}} \quad [0^0 \ form]$

Taking log on both the side,we get,

$$\log l = \lim_{x\to\infty} \log\left(\frac{1}{x}\right)^{\frac{1}{x}}$$

$$\log l = \lim_{x\to\infty} \frac{1}{x}\log\left(\frac{1}{x}\right)$$

$$\log l = \lim_{x\to\infty} \frac{\log\left(\frac{1}{x}\right)}{x} \quad \left[\frac{\infty}{\infty}\ form\right]$$

By L'Hospital's Rule

$$\log l = \lim_{x\to\infty} \frac{x\cdot -\frac{1}{x^2}}{1}$$

$$\log l = \lim_{x\to\infty} -\frac{1}{x}$$

$$\log l = 0$$

Taking exponential on both the side, we get,

$$l = e^0$$

$$l = 1$$

[**4**] $\lim_{x\to\frac{\pi}{2}} (\cos\mathbf{x})^{\frac{\pi}{2}-\mathbf{x}}$ (**Ans.1**)

Solution:- $l = \lim_{x\to\frac{\pi}{2}} (\cos x)^{\frac{\pi}{2}-x} \quad [0^0 \ form]$

Taking log on both the side,we get,

$$\log l = \lim_{x\to\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right)\log(\cos x) \quad [0\cdot\infty \ form]$$

$$\log l = \lim_{x\to\frac{\pi}{2}} \frac{\log(\cos x)}{\frac{1}{\left(\frac{\pi}{2}-x\right)}} \quad \left[\frac{\infty}{\infty}\ form\right]$$

By L'Hospital's Rule

$$\log l = \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\cos x}(-\sin x)}{-\frac{1}{\left(\frac{\pi}{2}-x\right)^2}(-1)}$$

$$\log l = \lim_{x \to \frac{\pi}{2}} \frac{-\tan x}{\frac{1}{\left(\frac{\pi}{2}-x\right)^2}}$$

$$\log l = \lim_{x \to \frac{\pi}{2}} -\tan x \left(\frac{\pi}{2}-x\right)^2$$

$$\log l = \lim_{x \to \frac{\pi}{2}} -\frac{\left(\frac{\pi}{2}-x\right)^2}{\cot x} \qquad \left[\frac{0}{0}\ form\right]$$

By L'Hospital's Rule

$$\log l = \lim_{x \to \frac{\pi}{2}} \frac{-2\left(\frac{\pi}{2}-x\right)(-1)}{-\operatorname{cosec}^2 x}$$

$$\log l = \lim_{x \to \frac{\pi}{2}} 2\left(x-\frac{\pi}{2}\right)\ \sin^2 x$$

$$\log l = 0$$

Taking exponential on both the side, we get,

$$l = e^0$$

$$l = 1$$

2 Successive Differentiation

Let $f(x)$ be a differentiable function then it's successive derivative is denoted by
$f'(x),\ f''(x),\ f'''(x),\ ...\ ...\ ,f^{(n)}(x)$.

Also if $y = f(x)$ be the differentiable function then

1^{st} derivative is denoted by - y' *or* y_1 *or* $\dfrac{dy}{dx}$ *or* Dy

2^{nd} derivative is denoted by - y'' *or* y_2 *or* $\dfrac{d^2y}{dx^2}$ *or* D^2y

3^{rd} derivative is denoted by - y''' *or* y_3 *or* $\dfrac{d^3y}{dx^3}$ *or* D^3y

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n^{th} derivative is denoted by - $y^{(n)}$ *or* y_n *or* $\dfrac{d^ny}{dx^n}$ *or* D^ny

⋆ n^{th} derivative of some standard function :

1. $y = e^{ax}$
   $y_1 = ae^{ax}$
   $y_2 = a^2e^{ax}$
   $y_3 = a^3e^{ax}$
   .
   .
   .
   $\boxed{y_n = a^ne^{ax}}$

2. $y = a^{bx}$
   $y_1 = ba^{bx}\log a$
   $y_2 = b^2a^{bx}(\log a)^2$
   $y_3 = b^3a^{bx}(\log a)^3$
   .
   .
   .
   $\boxed{y_n = b^na^{bx}(\log a)^n}$

3. $y = (ax+b)^m$
   $y_1 = ma(ax+b)^{m-1}$
   $y_2 = m(m-1)a^2(ax+b)^{m-2}$
   $y_3 = m(m-1)(m-2)a^3(ax+b)^{m-3}$
   .
   .
   .

$$\boxed{y_n = m(m-1)(m-2)\cdots(m-(n-1))a^n(ax+b)^{m-n}}$$

$\underline{case-1}$ $m > 0$ & $m > n$, then

$$\boxed{y_n = \frac{m!}{(m-n)!}a^n(ax+b)^{m-n}}$$

$\underline{case-2}$ $m = n$

$$\boxed{y_n = n!\ a^n}$$

$\underline{case-3}$ $n > m$

$$\boxed{y_n = 0}$$

$\underline{case-4}$ $m = -1$ i.e $y = \dfrac{1}{ax+b}$

$$\boxed{y_n = \frac{(-1)^n\ n!\ a^n}{(ax+b)^{n+1}}}$$

4. $y = \log(ax+b)$

$$y_1 = \frac{a}{ax+b}$$
$$y_2 = -\frac{a^2}{(ax+b)^2}$$
$$y_3 = \frac{2!\ a^3}{(ax+b)^3}$$

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$$\boxed{y_n = (-1)^{n-1}\frac{(n-1)!\ a^n}{(ax+b)^n}}$$

5. $y = \sin(ax+b)$

$$y_1 = a\cos(ax+b) = a\sin\left(ax+b+\frac{\pi}{2}\right)$$
$$y_2 = a^2\sin\left(ax+b+\frac{2\pi}{2}\right)$$
$$y_3 = a^3\sin\left(ax+b+\frac{3\pi}{2}\right)$$

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$$\boxed{y_n = a^n\sin\left(ax+b+\frac{n\pi}{2}\right)}$$

Similarly ,

6. $y = \cos(ax+b)$

$$\boxed{y_n = a^n \cos\left(ax+b+\frac{n\pi}{2}\right)}$$

7. $y = e^{ax} \cdot \sin(bx+c)$

$$\boxed{y_n = (a^2+b^2)^{\frac{n}{2}}\ e^{ax}\ \sin\left(bx+c+n\tan^{-1}\frac{b}{a}\right)}$$

If $r = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\dfrac{b}{a}$

$$\boxed{y_n = r^n\ e^{ax}\ \sin(bx+c+n\theta)}$$

8. $y = e^{ax} \cdot \cos(bx+c)$

$$\boxed{y_n = (a^2+b^2)^{\frac{n}{2}}\ e^{ax}\ \cos\left(bx+c+n\tan^{-1}\frac{b}{a}\right)}$$

If $r = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\dfrac{b}{a}$

$$\boxed{y_n = r^n\ e^{ax}\ \cos(bx+c+n\theta)}$$

Que. Find n^{th} derivative of the following :

1. $y = \log(2x+3)$
   solution:- W.K.T if $y = \log(ax+b)$ then $y_n = (-1)^{n-1}\dfrac{(n-1)!\ a^n}{(ax+b)^n}$
   From this we have,

$$y_n = (-1)^{n-1}\frac{(n-1)!\ 2^n}{(2x+3)^n}$$

2. $y = \dfrac{2x-1}{x^2-5x+6}$
   Solution:- $y = \dfrac{2x-1}{x^2-5x+6} = \dfrac{2x-1}{(x-3)(x-2)}$
   $y = \dfrac{2x-1}{(x-3)(x-2)} = \dfrac{A}{(x-3)} + \dfrac{B}{(x-2)}$

$\because A(x-2)+B(x-3)=2x-1$

put $x=2 \;\Rightarrow \boxed{\text{B = -3}}$

put $x=3 \;\Rightarrow \boxed{\text{A = 5}}$

Then,

$y=\dfrac{5}{(x-3)}-\dfrac{3}{(x-2)}$

w.k.t if $y=\dfrac{1}{ax+b}$ then $y_n=\dfrac{(-1)^n\ n!\ a^n}{(ax+b)^{n+1}}$

Then,

$$\boxed{y_n=5\frac{(-1)^n\ n!}{(x-3)^{n+1}}-3\frac{(-1)^n\ n!}{(x-2)^{n+1}}}$$

3. $y=\sin(2x+5)$

   Solution:- $y_n=2^n\sin\left(2x+5+\dfrac{n\pi}{2}\right)$

4. $y=\cos(5x-3)$

   Solution:- $y_n=5^n\cos\left(5x-3+\dfrac{n\pi}{2}\right)$

5. $y=\sin 6x\cdot\cos 4x$

   Sol:- $y=\dfrac{1}{2}[2\sin 6x\cdot\cos 4x]$

   $y=\dfrac{1}{2}[\sin 10x+\sin 2x]$ $\qquad(\because 2sc=s+s)$

   Then by standard formula,

   $$y_n=\frac{1}{2}\left[10^n\sin\left(10x+\frac{n\pi}{2}\right)+2^n\sin\left(2x+\frac{n\pi}{2}\right)\right]$$

   Note:- Some Standard formula

   $2\sin x\cos y=\sin(x+y)+\sin(x-y)$

   $2\cos x\sin y=\sin(x+y)-\sin(x-y)$

   $2\cos x\cos y=\cos(x+y)+\cos(x-y)$

   $-2\sin x\sin y=\cos(x+y)-\cos(x-y)$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

6. $y = e^{-x} \cdot \sin^2 x$

   Sol:- $y = e^{-x} \cdot \left(\frac{1-\cos 2x}{2}\right)$

   $y = \frac{1}{2}[e^{-x} - e^{-x}\cos 2x]$

   Then by standard formula,

   $$y_n = \frac{1}{2}(-1)^n e^{-x} - \ \frac{5^{\frac{n}{2}}}{2}\ e^{-x}\ \cos\left(2x + n\tan^{-1}(-2))\right)$$

7. $y = e^{2x} \cdot \cos 2x \cdot \cos x$

   Sol:- $y = e^{2x}\frac{1}{2}[2\cos 2x \cdot \cos x]$

   $y = \frac{1}{2}\ e^{2x}\ [\cos 3x + \cos x]$

   $y = \frac{1}{2}\ [e^{2x}\cos 3x + e^{2x}\cos x]$

   Then by standard formula,

   $$y_n = \frac{1}{2}\left[(2^2+3^2)^{\frac{n}{2}} e^{2x}\cos\left(3x + n\tan^{-1}\frac{3}{2})\right) + \ (2^2+1^2)^{\frac{n}{2}} e^{2x}\cos\left(x + n\tan^{-1}3)\right)\right]$$
   $$y_n = \frac{13^{\frac{n}{2}}}{2}e^{2x}\cos\left(3x + n\tan^{-1}\frac{3}{2}\right) + \ \frac{4^{\frac{n}{2}}}{2}e^{2x}\cos\left(x + n\tan^{-1}3\right)$$

3 LEIBNITZ'S RULE

If u & v are the function of x such that their n^{th} derivative are exist, then the n^{th} derivative of their product is given by

$$(u \cdot v)_n = u_n \cdot v + \binom{n}{1}u_{n-1} \cdot v_1 + \binom{n}{2}u_{n-2} \cdot v_2 + \cdots + \binom{n}{r}u_{n-r} \cdot v_r + \cdots + u \cdot v_n$$

Where, u_r & v_r represents r^{th} derivative of u & v.

Que.1 Find n^{th} derivative of $\mathbf{x}\log\mathbf{x}$.

Sol:- Let, $u = \log x$ and $v = x$

$$u_n = \frac{(-1)^{n-1}(n-1)!}{x^n} \qquad\qquad v_1 = 1$$

$$u_{n-1} = \frac{(-1)^{n-2}(n-2)!}{x^{n-1}} \qquad\qquad v_2 = 0$$

Using Leibnitz's Rule,we have,

$$(u\cdot v)_n = u_n\cdot v + \binom{n}{1}u_{n-1}\cdot v_1 + \binom{n}{2}u_{n-2}\cdot v_2 + \cdots + \binom{n}{r}u_{n-r}\cdot v_r + \cdots + u\cdot v_n$$

$$(x\log x)_n = \frac{(-1)^{n-1}(n-1)!}{x^n}\cdot x + \binom{n}{1}\frac{(-1)^{n-2}(n-2)!}{x^{n-1}}\cdot 1 + 0 + \cdots$$

$$= \frac{(-1)^{n-1}(n-1)(n-2)!}{x^{n-1}} + n\frac{(-1)^{n-2}(n-2)!}{x^{n-1}}$$

$$= \frac{(-1)^{n-2}(n-2)!}{x^{n-1}}[-(n-1)+n]$$

$$= \frac{(-1)^{n-2}(n-2)!}{x^{n-1}}$$

Que.2 Find n^{th} derivative of $\mathbf{x^2}\sin\mathbf{4x}$.

Sol:- Let, $u = \sin 4x$ and $v = x^2$

$$u_n = 4^n\ \sin\left(4x + \frac{n\pi}{2}\right) \qquad\qquad v_1 = 2x$$

$$u_{n-1} = 4^{n-1}\ \sin\left(4x + \frac{(n-1)\pi}{2}\right) \qquad\qquad v_2 = 2$$

$$u_{n-2} = 4^{n-2}\ \sin\left(4x + \frac{(n-2)\pi}{2}\right) \qquad\qquad v_3 = 0$$

Using Leibnitz's Rule,we have,

$$(u\cdot v)_n = u_n\cdot v + \binom{n}{1}u_{n-1}\cdot v_1 + \binom{n}{2}u_{n-2}\cdot v_2 + \cdots + \binom{n}{r}u_{n-r}\cdot v_r + \cdots + u\cdot v_n$$

$$(x^2\sin 4x)_n = 4^n\ \sin\left(4x + \frac{n\pi}{2}\right)\cdot x^2 + \binom{n}{1}4^{n-1}\ \sin\left(4x + \frac{(n-1)\pi}{2}\right)\cdot 2x + \binom{n}{2}4^{n-2}\ \sin\left(4x + \frac{(n-2)\pi}{2}\right)\cdot 2 + 0 + \cdots$$

$$= 4^{n-2}\left[16x^2\sin\left(4x+\frac{n\pi}{2}\right) + 8nx\ \sin\left(4x+\frac{(n-1)\pi}{2}\right) + n(n-1)\ \sin\left(4x+\frac{(n-2)\pi}{2}\right)\right]$$

Que.3 Find n^{th} derivative of $\mathbf{x^2e^{5x}}$.

Sol:- Let, $u = e^{5x}$ and $v = x^2$

$u_n = 5^n e^{5x}$ $\qquad v_1 = 2x$

$u_{n-1} = 5^{n-1}e^{5x}$ $\qquad v_2 = 2$

$u_{n-2} = 5^{n-2}e^{5x}$ $\qquad v_3 = 0$

Using Leibnitz's Rule,we have,

$$(u \cdot v)_n = u_n \cdot v + \binom{n}{1}u_{n-1} \cdot v_1 + \binom{n}{2}u_{n-2} \cdot v_2 + \cdots + \binom{n}{r}u_{n-r} \cdot v_r + \cdots + u \cdot v_n$$

$$(x^2e^{5x})_n = 5^n e^{5x} \cdot x^2 + \binom{n}{1}5^{n-1}e^{5x} \cdot 2x + \binom{n}{2}5^{n-2}e^{5x} \cdot 2 + 0 + \cdots$$

$$= 5^n\ e^{5x} \cdot x^2 + n\ 5^{n-1}\ e^{5x} \cdot 2x + n\ (n-1)\ 5^{n-2}\ e^{5x}$$

$$= 5^{n-2}\ e^{5x}[25\ x^2 + 10\ n\ x + n\ (n-1)]$$

Que.4 Find n^{th} derivative of $\mathbf{x^3e^{3x}}$.

Sol:- Let, $u = e^{3x}$ and $v = x^3$

$u_n = 3^n e^{3x}$ $\qquad v_1 = 3x^2$

$u_{n-1} = 3^{n-1}e^{3x}$ $\qquad v_2 = 6x$

$u_{n-2} = 3^{n-2}e^{3x}$ $\qquad v_3 = 6$

$u_{n-3} = 3^{n-3}e^{3x}$ $\qquad v_4 = 0$

Using Leibnitz's Rule,we have,

$$(u \cdot v)_n = u_n \cdot v + \binom{n}{1}u_{n-1} \cdot v_1 + \binom{n}{2}u_{n-2} \cdot v_2 + \cdots + \binom{n}{r}u_{n-r} \cdot v_r + \cdots + u \cdot v_n$$

$$(x^3e^{3x})_n = 3^n e^{3x} \cdot x^3 + \binom{n}{1}3^{n-1}e^{3x} \cdot 3x^2 + \binom{n}{2}3^{n-2}e^{3x} \cdot 6x + \binom{n}{3}3^{n-3}e^{3x} \cdot 6 + 0 + \cdots$$

$$= 3^n\ e^{3x} \cdot x^3 + n\ 3^n\ e^{3x} \cdot x^2 + n\ (n-1)\ 3^{n-1}\ e^{3x} \cdot x + n\ (n-1)\ (n-2)\ 3^{n-3}\ e^{3x}$$

$$= 3^{n-3}\ e^{3x}[27\ x^3 + 27n\ x^2 + 9n\ (n-1)\ x + n\ (n-1)\ (n-2)]$$

Ex.1 If $y = \sin^{-1} x$ then prove that

(i) $(1-x^2)y_2 - xy_1 = 0$

(ii) $(1-x^2)y_{n+2} - (2n+1)x\ y_{n+1} - n^2 y_n = 0$

sol:- Let $y = \sin^{-1} x$

Differentiating w.r.t x, we get,

$$y_1 = \frac{1}{\sqrt{1-x^2}} \qquad \left(\because \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}\right)$$

$\Rightarrow\ \sqrt{1-x^2}\ y_1 = 1$

$\Rightarrow\ (1-x^2)\ y_1^2 = 1$ $\qquad$ ($\because$ by taking square)

Again differentiating w.r.t x, we get,

$\Rightarrow\ (1-x^2)\ 2\ y_1\ y_2 + (-2x)\ y_1^2 = 0$

$\Rightarrow\ (1-x^2)\ y_2 - x\ y_1 = 0$ $\qquad$ ($\because$ divide by $2y_1$)

Now, by using Leibnitz's rule, we have,

$\Rightarrow\ (1-x^2)\ y_{n+2} + \binom{n}{1}\ y_{n+1}\ (-2x) + \binom{n}{2}\ y_n\ (-2) - [x\ y_{n+1} + \binom{n}{1}\ y_n\ (1)] = 0$

$\Rightarrow\ (1-x^2)\ y_{n+2} - 2nx\ y_{n+1} - n(n-1)\ y_n - x\ y_{n+1} - n\ y_n = 0$

$\Rightarrow\ (1-x^2)\ y_{n+2} - (2n+1)x\ y_{n+1} - n^2\ y_n = 0$

Hence,proved.

Ex.2 If $y = e^{m\cos^{-1} x}$ then prove that

(i) $(1-x^2)y_2 - xy_1 = m^2 y$

(ii) $(1-x^2)y_{n+2} - (2n+1)x\ y_{n+1} - (n^2+m^2)y_n = 0$

sol:- Let $y = e^{m\cos^{-1} x}$

Differentiating w.r.t x, we get,

$$y_1 = e^{m\cos^{-1}x}\ \frac{-m}{\sqrt{1-x^2}} \qquad \left(\because \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow\ \sqrt{1-x^2}\ y_1 = -m\ y$$

$$\Rightarrow\ (1-x^2)\ y_1^2 = m^2y^2 \qquad (\because \text{by taking square })$$

Again differentiating w.r.t x, we get,

$$\Rightarrow\ (1-x^2)\ 2\ y_1\ y_2 + (-2x)\ y_1^2 = m^2 2yy_1$$

$$\Rightarrow\ (1-x^2)\ y_2 - x\ y_1 = m^2y \qquad (\because \text{divide by } 2y_1\ )$$

Now, by using Leibnitz's rule, we have,

$$\Rightarrow\ (1-x^2)\ y_{n+2} + \binom{n}{1}\ y_{n+1}\ (-2x) + \binom{n}{2}\ y_n\ (-2) - [x\ y_{n+1} + \binom{n}{1}\ y_n\ (1)] = m^2y_n$$

$$\Rightarrow\ (1-x^2)\ y_{n+2} - 2nx\ y_{n+1} - n(n-1)\ y_n - x\ y_{n+1} - n\ y_n = m^2y$$

$$\Rightarrow\ (1-x^2)\ y_{n+2} - (2n+1)x\ y_{n+1} - n^2\ y_n - m^2\ y = 0$$

$$\Rightarrow\ (1-x^2)\ y_{n+2} - (2n+1)x\ y_{n+1} - (n^2+m^2)\ y = 0$$

Hence,proved.

4 TAYLOR'S AND MACLAURIN'S SERIES

Let $f(x)$ be a function with n^{th} order derivative are exist in some interval containing a point a, then the **Taylor's serise** of f at $x = a$ is given by,

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \cdots\ \ \cdots$$

Put $a = 0$ then it's gives **Maclaurin's series** of f and is given by,

$$f(x) = f(0) + xf'(0) + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \cdots\ \ \cdots$$

Remark

Alternative form of Taylor's series :

Let $f(x)$ be a function which is n^{th} times differentiable at a point $x = a$, then it's taylor's series expansion is given by,

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \cdots\ \ \cdots$$

Now, Taking $x - a = h$ we get,

$$f(a+h) = f(a) + hf'(a) + h^2\frac{f''(a)}{2!} + h^3\frac{f'''(a)}{3!} + \cdots \quad \cdots \tag{1}$$

If $h = x$ in above formula we get,

$$f(a+x) = f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + \frac{f'''(a)}{3!}x^3 + \cdots \quad \cdots$$

If $a = x$ in (1) ,we get,

$$f(x+h) = f(x) + hf'(x) + h^2\frac{f''(x)}{2!} + h^3\frac{f'''(x)}{3!} + \cdots \quad \cdots$$

Ex.1 Find the Taylor's series of $f(x) = 2x^3 + 3x^2 - 8x + 7$ in terms of $(x-2)$.

Sol:- Here, $f(x) = 2x^3 + 3x^2 - 8x + 7$ and $a = 2$

By Taylor's series,

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \cdots \quad \cdots$$

Put $a = 2$

$$f(x) = f(2) + (x-2)f'(2) + (x-2)^2\frac{f''(2)}{2!} + (x-2)^3\frac{f'''(2)}{3!} + \cdots \quad \cdots \tag{1}$$

$f(x) = 2x^3 + 3x^2 - 8x + 7$ $\qquad$ $f(2) = 19$

$f'(x) = 6x^2 + 6x - 8$ $\qquad$ $f'(2) = 28$

$f''(x) = 12x + 6$ $\qquad$ $f''(2) = 30$

$f'''(x) = 12$ $\qquad$ $f'''(2) = 12$

$f^{(IV)}(x) = 0$ $\qquad$ $f^{(IV)}(2) = 0$

Then from (1) we get,

$$f(x) = 19 + (x-2)\cdot 28 + (x-2)^2\cdot\frac{30}{2!} + (x-2)^3\cdot\frac{12}{3!} + 0 + \cdots \quad \cdots$$

$$f(x) = 19 + 28\cdot(x-2) + 15\cdot(x-2)^2 + 2\cdot(x-2)^3$$

Ex.2 Expand $\log x$ in powers of $(x-1)$ up to three powers and hence evaluate $\log 1.1$ correct up to three decimal places.

Sol:- Let, $f(x) = \log x$ and $a = 1$

By Taylor's series,

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \cdots \cdots$$

Put $a = 1$

$$f(x) = f(1) + (x-1)f'(1) + (x-1)^2\frac{f''(1)}{2!} + (x-1)^3\frac{f'''(1)}{3!} + \cdots \cdots \tag{1}$$

$f(x) = \log x$ $\qquad f(1) = 0$

$f'(x) = \dfrac{1}{x}$ $\qquad f'(1) = 1$

$f''(x) = -\dfrac{1}{x^2}$ $\qquad f''(1) = -1$

$f'''(x) = \dfrac{2}{x^3}$ $\qquad f'''(1) = 2$

and so on,

Then from (1) we get,

$$f(x) = 0 + (x-1)\cdot 1 + (x-1)^2 \cdot \frac{-1}{2!} + (x-1)^3 \cdot \frac{2}{3!} + \cdots \cdots$$

$$f(x) = (x-1) - \frac{1}{2}\cdot(x-1)^2 + \frac{1}{3}\cdot(x-1)^3 + \cdots \cdots$$

$$f(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \cdots \cdots$$

Now put $x = 1.1$ and taking only 1^{st} three terms we get,

$$f(1.1) = \log 1.1 = 0.1 - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3}$$

$$\log 1.1 = 0.1 - 0.005 + 0.0003$$

$$\boxed{\log 1.1 = 0.0953}$$

Ex.3 Find the Taylor's series of $\tan\left(\dfrac{\pi}{4}+x\right)$ in powers of x up to x^4 terms and find the value of $\tan(50°)$

Sol:- Let, $f\left(x+\dfrac{\pi}{4}\right) = \tan\left(\dfrac{\pi}{4}+x\right)$ and $a = \dfrac{\pi}{4}$

Then, $f(x) = \tan x$

By Taylor's series,

$$f(x+a) = f(a) + f'(a)\ x + \frac{f''(a)}{2!}\ x^2 + \frac{f'''(a)}{3!}\ x^3 + \cdots\ \cdots$$

Take $a = \dfrac{\pi}{4}$

$$f\left(x+\frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right)+f'\left(\frac{\pi}{4}\right)\ x+\frac{f''\left(\frac{\pi}{4}\right)}{2!}\ x^2+\frac{f'''\left(\frac{\pi}{4}\right)}{3!}\ x^3+\cdots\ \cdots \tag{1}$$

$f(x) = \tan x$ $\qquad f\left(\dfrac{\pi}{4}\right) = 1$

$f'(x) = \sec^2 x$ $\qquad f'\left(\dfrac{\pi}{4}\right) = 2$

$f''(x) = 2\sec^2 x \tan x$ $\qquad f''\left(\dfrac{\pi}{4}\right) = 4$

$= 2(1 + \tan^2 x)\tan x$

$= 2\tan x + 2\tan^3 x$

$f'''(x) = 2\sec^2 x + 6\tan^2 x \sec^2 x$ $\qquad f'''\left(\dfrac{\pi}{4}\right) = 16$

$= 2(1 + \tan^2 x) + 6\tan^2 x + 6\tan^4 x$

$= 2 + 8\tan^2 x + 6\tan^4 x$

$f^{(IV)}(x) = 16\tan x \sec^2 x + 24\tan^3 x \sec^2 x$ $\qquad f^{(IV)}\left(\dfrac{\pi}{4}\right) = 80$

and so on,

Then from (1) we get,

$$f\left(x + \frac{\pi}{4}\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \tfrac{10}{3}x^4 + \cdots\ \cdots$$

Now,

$$\tan 50^\circ = \tan(45^\circ + 5^\circ) = f\left(\frac{\pi}{4} + \frac{5\pi}{180}\right) = f\left(\frac{\pi}{4} + 0.112\right)$$

Take $x = 0.112$ in (2) and considering only first 4 terms we get ,

$$\tan 50^\circ = 1 + 0.224 + 2 \times 0.01254 + \frac{8}{3} \times 0.0014 + \frac{10}{3} \times 0.00016$$

$\boxed{\tan 50^\circ = 1.2533}$

Ex.4 Find the approximate value of $\sqrt{25.15}$ correct up to 4 decimal places by using taylor's series expansion.

Sol:- Let, $f(x) = \sqrt{x}$

$\therefore f(x+h) = \sqrt{x+h}$

By Taylor's series expansion,

$$f(x+h) = f(x) + hf'(x) + h^2\frac{f''(x)}{2!} + h^3\frac{f'''(x)}{3!} + \cdots \quad \cdots$$

Put $x = 25$ and $h = 0.15$,we get,

$$f(x+h) = \sqrt{x+h} = \sqrt{25+0.15} = \sqrt{25.15}$$

$$\sqrt{25.15} = f(25) + (0.15)f'(25) + (0.15)^2\frac{f''(25)}{2!} + (0.15)^3\frac{f'''(25)}{3!} + \cdots \quad \cdots$$

$f(x) = \sqrt{x}$ $\qquad f(25) = 5$

$f'(x) = \dfrac{1}{2\sqrt{x}}$ $\qquad f'(25) = 0.1$

$f''(x) = -\dfrac{1}{4x^{\frac{3}{2}}}$ $\qquad f''(25) = -0.002$

Substituting this values in (1) and considering only first three terms, we get,

$$\sqrt{25.15} = 5 + (0.15)(0.1) + (0.15)^2\frac{(-0.002)}{2!}$$

$$\boxed{\sqrt{25.15} = 5.01497}$$

[Ex:] Find the Maclaurin's series expansion of the following.

1. $f(x) = e^x$

Sol:- W.K.T Maclaurin's series is

$$f(x) = f(0) + xf'(0) + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \cdots \quad \cdots \qquad (1)$$

Here,

$f(x) = e^x$ $\qquad f(0) = e^0 = 1$

$f'(x) = e^x$ $\qquad f'(0) = 1$

$f''(x) = e^x$ $\qquad f''(0) = 1$

$f'''(x) = e^x$ $\qquad f'''(0) = 1$

And so on,

Substituting this value in (1) ,we have

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

If we replace $x\ by\ -x$ in above series expansion ,we get,

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + (-1)^n \frac{x^n}{n!} + \cdots\ \cdot\cdot$$

2. $f(x) = \sin x$

Sol:- W.K.T Maclaurin's series is

$$f(x) = f(0) + xf'(0) + x^2 \frac{f''(0)}{2!} + x^3 \frac{f'''(0)}{3!} + \cdots\ \cdot\cdot \tag{1}$$

Here,

$f(x) = \sin x$ $\qquad f(0) = 0$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$

And so on,

Substituting this value in (1) ,we have

$$\sin x = 0 + x + 0 + \frac{x^3}{3!}(-1) + 0 + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\ \cdot\cdot$$

Similarly we get the power series expansion of $\cos x$.

3. $f(x) = \log(1 + x)$

Sol:- Here,

$f(x) = \log(1 + x)$ $\qquad f(0) = 0$

$f'(x) = \dfrac{1}{1 + x}$ $\qquad f'(0) = 1$

$f''(x) = -\dfrac{1}{(1 + x)^2}$ $\qquad f''(0) = -1$

$f'''(x) = \dfrac{2}{(1+x)^3}$ $\qquad$ $f'''(0) = 2$

$f^{(IV)}(x) = -\dfrac{6}{(1+x)^4}$ $\qquad$ $f^{(IV)}(0) = -6$

And so on,

Then by using Maclaurin's series expansion

$$f(x) = f(0) + xf'(0) + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \cdots \;\; \cdot\cdot$$

$$\log(1+x) = 0 + x - \frac{x^2}{2!} + \frac{x^3}{3!}(2) - \frac{x^4}{4!}(6) + \cdots \;\; \cdot\cdot$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \;\; \cdot\cdot$$

4. Arrange the following polinomial in powers of x using Maclaurin's Series $f(x) = 5+(x+3)+7(x+3)^2$.

Sol:- Here,

$f(x) = 5 + (x+3) + 7(x+3)^2$ $\qquad$ $f(0) = 71$

$f'(x) = 1 + 14(x+3)$ $\qquad$ $f'(0) = 43$

$f''(x) = 14$ $\qquad$ $f''(0) = 14$

$f'''(x) = 0$ $\qquad$ $f'''(0) = 0$

And so on,

Then by using Maclaurin's series expansion

$$f(x) = f(0) + xf'(0) + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \cdots \;\; \cdot\cdot$$

$$5 + (x+3) + 7(x+3)^2 = 71 + 43x - 7x^2$$

Which is required power series in terms of x.

★★★