.11 **A capacitor and a resistor are connected in series with a d.c. source of V volts. Derive an expression for the voltage across the capacitor after 't' seconds during discharging.**

Ans. Consider a capacitor of C farads connected in series with a resistor of R ohms and a switch S. When the switch is in position 'a', the capacitor gets charged to V volts.

When the switch S is closed to position 'b' the charge on the capacitor starts decreasing and so does the voltage across it.

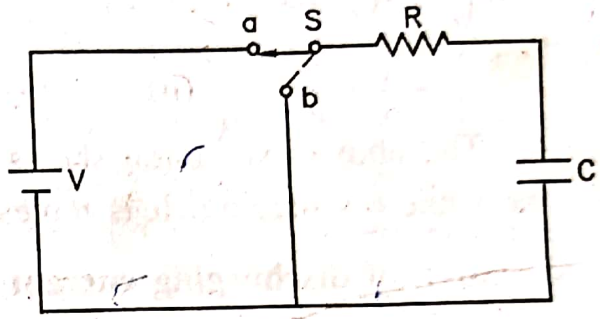

FIG. 2.18

Let at any time during discharging,

v_c = p.d. across the capacitor

i = discharging current

q = charge on the capacitor

$= C\, v_c$

According to kVL,

$$0 = v_c + i\,R$$

$$\therefore \quad 0 = v_c + RC\frac{dv_c}{dt} \qquad \left[i = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C\frac{dv_c}{dt}\right]$$

$$\therefore \quad \frac{dv_c}{v_c} = -\frac{dt}{RC}$$

Integrating both sides, we get,

$$\log_e v_c = -\frac{1}{RC}t + K_1 \qquad \text{... (i)}$$

Where K_1 is a constant of integration which can be determined from the initi conditions.

Initial conditions :

$t = 0, \quad v_c = V$

Substituting the initial conditions in equation (i)

$$\log_e V = 0 + K_1$$

$$\therefore \quad K_1 = \log_e V$$

Hence equation (i), becomes

$$\log_e v_c = \frac{-t}{RC} + \log_e V$$

or

$$\log_e \frac{v_c}{V} = \frac{-t}{RC}$$

$$\therefore \quad \frac{v_c}{V} = e^{-t/RC}$$

$$v_c = Ve^{-t/RC}$$

$$= V e^{-t/\lambda} \quad \text{... (ii)} \qquad [\because \lambda = RC = \text{time constant})$$

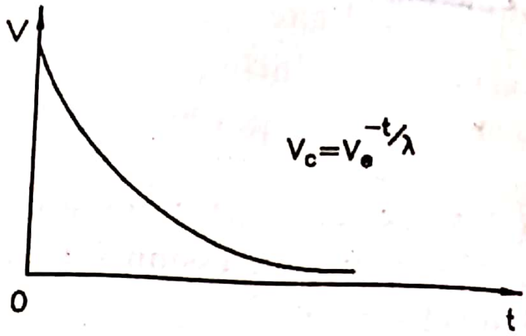

FIG. 2.19

The above expression shows the variation of voltage (v_c) across the capacitor with time while discharging. It is represented in fig. 2.19.

Variation of discharging current :

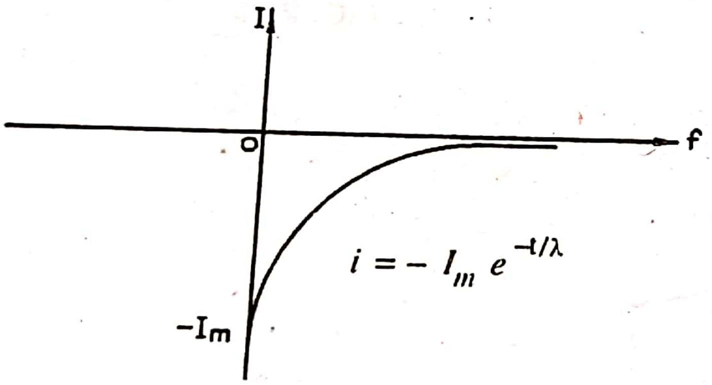

FIG. 2.20

We know that

$$O = v_c + iR$$

$$\therefore \quad iR = -v_c$$

$$= -V e^{-t/\lambda}$$

$$i = \frac{-V}{R} e^{-t/\lambda}$$

$$= -I_m e^{-t/\lambda} \quad \left[\because I_m = \frac{V}{R} = \text{initial current}\right]$$

The above expression shows the variation of discharging current with time. Negative sign icates that the current flows in the opposite direction with respect to the charging process. It hown graphically in the fig. 2.20.