5
Chapter

HEAT ENGINES

5.1 INTRODUCTION

An engine may be defined as a device which converts one form of energy into mechanical energy. Mechanical energy can be further easily converted into electrical energy which is the most required form of energy. Heat engine is a device which transforms heat energy into mechanical energy. In every heat engine, some form of fuel (solid, liquid, gas or nuclear) is used. The chemical energy or nuclear energy of fuel is converted into thermal or heat energy and that is further used to perform useful work.

The various types of fuels and prime movers have been discussed in chapter 1. The various prime movers that are used in practice are : Internal combustion engines, gas turbines, steam turbines, steam engines, etc.

5.2 HEAT ENGINES

A heat engine is a device which transforms the chemical energy of a fuel into thermal energy and uses this energy to produce mechanical work. Heat engines are classified into two broad categories :

1. External combustion engines, and
2. Internal combustion engines.

5.2.1 External Combustion Engines

In an external combustion engine, the products of combustion of air and fuel transfer heat to a second fluid which is the working fluid of the cycle, as in the case of a steam engine or a steam turbine plant. The heat of combustion is employed to generate steam which is used in a piston engine or a turbine. Another example of an external combustion engine is a closed cycle gas turbine plant in which heat of combustion in an external furnace is transferred to gas, usually air, which is used in a gas turbine.

5.2.2 Internal Combustion Engines

In an internal combustion engine, the products of combustion are directly the motive fluid. Petrol, gas, and diesel engines, open cycle gas turbines are examples of internal combustion engines. Jet engines and rockets are also internal combustion engines.

The classification of various types of heat engines along with their principal applications are given in Table 5.1.

next Chapter

Table 5.1 Classification of heat engines

Classification	*Name of Engines*	*Reciprocating or Rotary*	*Maximum size in kW*	*Principal Use*
(*a*) Internal combustion engines	1. Gasoline or petrol engine (SI)	Reciprocating	4000	Road vehicles, small, industrial, small marine (propulsion of ships), small aircrafts
	2. Gas engine (SI)	Reciprocating	4000	Industrial, electric power
	3. Diesel engine (CI)	Reciprocating	40,000	Road vehicles, industrial locomotives electric power, marine
	4. Wankel engine (SI, CI)	Rotary	400	Road vehicles, small aircrafts
	5. Open cycle gas turbine	Rotary	15,000	Electric power, aircraft
	6. Jet engine	Rotary	8000	Aircraft
	7. Rocket	No mechanism	Very big	Missiles, space travel
(*b*) External combustion engines	1. Steam engine	Reciprocating	4000	Locomotives, ships
	2. Steam turbine	Rotary	5,00,000	Electric power, large marine
	3. Stirling or hot air engine	Reciprocating	800	Experimental, power in space, vehicles
	4. Closed cycle gas turbine	Rotary	80,000	Electric power, marine

SI = spark ignition CI = compression ignition

5.3 ADVANTAGES OF HEAT ENGINES

The advantages of internal combustion engines are :

1. Greater mechanical simplicity.
2. Lower weight and bulk to output ratio.
3. Lower first cost.
4. Higher overall efficiency.
5. Lesser requirement of water for dissipation of energy through cooling system.

The advantages of external combustion engines are :

1. Use of cheaper fuels.
2. High starting torque.
3. Higher weight and bulk to output ratio.

5.4 COMPARISON BETWEEN INTERNAL AND EXTERNAL COMBUSTION ENGINES

The comparison between I.C. and E.C. engines is given in Table 5.2.

Table 5.2 Comparison between I.C. and E.C. engines

Parameter	*Internal combustion engine*	*External combustion engine (Steam turbine)*
1. Suitability	Small size upto 1 MW.	Large sizes
2. Size (weight and bulk)	Compact	Bulky
3. Fuel	Requires refined fuel	Cheaper fuels can be used.
4. Pollution level	Higher	Lower
5. Efficiency	Lower	Higher

6. Complexity of components	Less	
7. Expansion	Incomplete	More
8. Heat rejection	At higher temperature	Complete
		At atmospheric conditions

5.5 DEFINITIONS

Working substance. It is a substance used to operate a heat engine. It may be a gas, mixture of gases or a vapour. Burning or expansion of this substance generates huge quantity of heat to produce work. The properties of a working substance are :

1. A working substance produces mechanical work.
2. It is not destroyed through its state and only its heat content changes.
3. The thermal properties of a working substance gives its heat content.

Converting machine. It is a machine which converts heat energy of the working substance into mechanical work. For example : Petrol engine, diesel engine, steam engine, steam turbine, and gas turbine, etc.

Reciprocating machine. It is a machine in which the piston has to-and-fro motion in the cylinder by the action of working substance. For example : Petrol and diesel engines, steam engine.

Rotary machine. It is a machine in which the working substance produces circular motion of the machine due to its action on the blades or vanes fixed on the shaft. For example : Steam turbine, Gas turbine.

Jet machine. It is a machine in which the burning or expanding working substance is discharged through a nozzle at the rear in the form of a jet to produce the necessary reaction for its motion. For example : Jet engine, rockets.

Cycle. It may be defined as a series of processes performed in a definite sequence to convert heat energy into mechanical work. The working substance is either returned to its original state or exhausted to the atmosphere. For example : Otto cycle, diesel cycle, dual cycle, Brayton cycle, Rankine cycle, etc.

Direct cycle. A heat engine operating on a cycle to develop mechanical energy or work is called to be working on direct cycle. For example : Carnot cycle for heat engine.

Reversed cycle. If the sequence of processes in a direct cycle are reversed, it is said to be operating on reversed cycle. For example : Reversed carnot cycle for a heat pump and the refrigerator.

5.6 ESSENTIAL ELEMENTS OF A HEAT ENGINE

The essential elements of a heat engine are :

1. Working substance : It is a medium for receiving and rejecting heat.
2. Heat source : It is the reservoir from which the working substance receives heat.
3. Heat sink : It is the reservoir where the working substance rejects heat after developing work. It is at a lower temperature than the source temperature.
4. Expander : It is an enclosure where the working substance does work. It may be either a cylinder or casing.
5. Pump : It is a device used to raise the pressure of the working substance. It utilizes the work given out by the expander.

The essential elements of a direct cycle heat engine are shown in Fig. 5.1.

Q_s = heat supplied

Q_r = heat rejected

W = work output of cycle

W_p = pump work

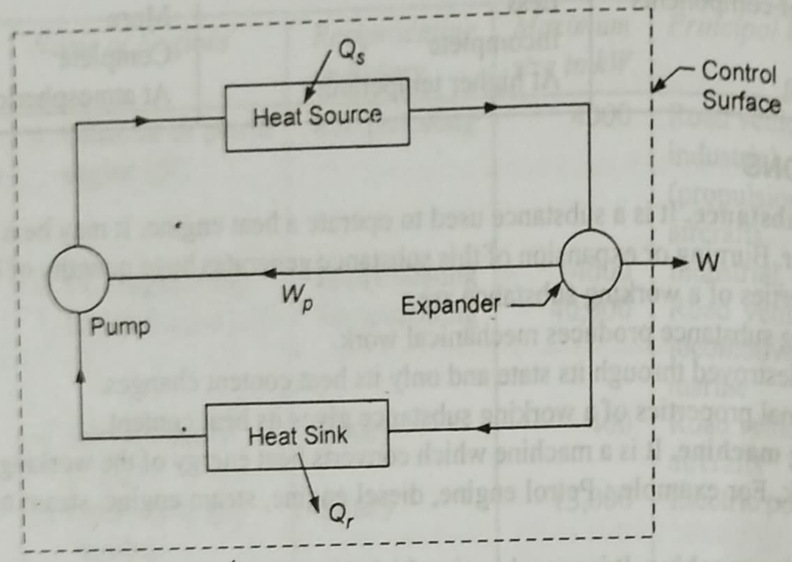


Fig. 5.1 Direct cycle heat engine.

5.7 THE CARNOT CYCLE

The working of the Carnot cycle is shown in Fig. 5.2. The working substance is enclosed in a cylinder having a frictionless piston. The walls of the cylinder and piston are taken as perfect insulators of heat. The cylinder head is assumed diathermic which permits the flow of heat. The heat source is at temperature T_1 and heat sink at temperature T_2. There is also a work reservoir.

5.7.1 Processes

The Carnot cycle consists of the following four reversible processes :

1. Isothermal expansion.
2. Adiabatic expansion.
3. Isothermal compression.
4. Adiabatic compression.

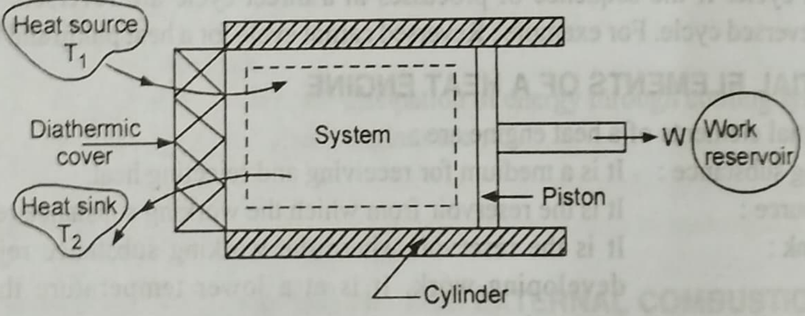


Fig. 5.2 Working of a Carnot cycle.

5.7.2 Assumptions

The following assumptions are made in the Carnot cycle :

1. The piston motion in the cylinder is frictionless.
2. The piston and cylinder walls are considered perfect insulators of heat.
3. The cylinder head is diathermic.
4. The transfer of heat does not change the temperature of source or sink.
5. The working substance is a perfect gas with constant specific heats.
6. The expansion and compression processes are reversible adiabatic, *i.e.*, isentropic.

The Carnot cycle is reversible, whereas the real heat engines are not due to friction, heat transfer to the insulating wall, etc.

In a Carnot cycle, since the processes are reversible, they are extremely slow, while in real life, the engines work faster.

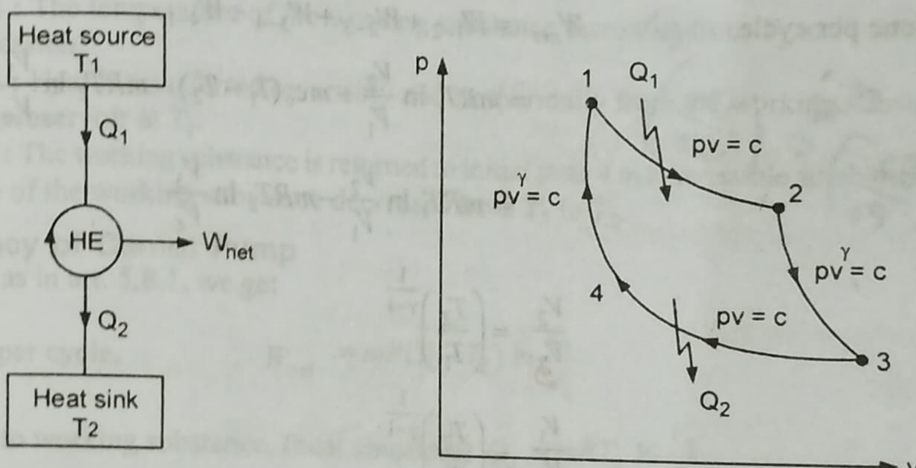
5.7.3 Limitations

The limitations of Carnot cycle are :

1. The isothermal expansion and compression processes should be carried out as slow as possible.
2. The adiabatic expansion and compression processes should be carried out as fast as possible.
3. It is not possible to move the piston slowly during isothermal processes and fast during adiabatic processes.
4. The piston rubs against the cylinder walls. Therefore the contact between piston and cylinder cannot be considered perfectly frictionless.
5. There has to be finite temperature difference for any heat transfer. Therefore the temperature of the working substance will always be either lower or higher than the temperature of the heat reservoirs.
6. It is impossible to construct cylinder walls which are totally insulating. ~~Hence five adiabatic processes are difficult to achieve.~~
7. The work output per cycle is very small, which may not be sufficient to overcome the friction of reciprocating parts.

5.8 CARNOT HEAT ENGINE

The principle of the Carnot heat engine is shown in Fig. 5.3(*a*) and the *p-V* diagram in Fig. 5.3(*b*). *m* kg of perfect gas is considered as the working substance.



(*a*) Principle of Carnot engine (*b*) p-V diagram

Fig. 5.3 The Carnot heat engine.

The Carnot heat engine consists of the following four processes :

Process 1-2 : Heat is transferred reversibly and isothermally to the working substance form a heat source at temperature T_1. The working substance is allowed to expand slowly and the quantity of heat Q_1 is withdrawn from the heat source. The work done is given by the area under path 1-2 of the p-V diagram shown in Fig. 5.3(*b*).

Process 2-3 : Reversible adiabatic expansion takes place. Since the cylinder-piston arrangement is thermally insulated, no heat transfer takes place. The temperature of working substances drops from T_1 to T_2. Work is delivered to the work reservoir and is represented by the area under path 2-3 of the *p-V* diagram.

Process 3-4 : Reversible isothermal heat rejection at constant temperature T_2 takes place by slowly compressing the working substance. Work is supplied by the work reservoir while Q_2 amount of heat flows into the sink. The work done in the process is represented by the area under path 3-4 of the *p-V* diagram.

Process 4-1: Reversible adiabatic compression takes places so that the working substance is returned to the original state 1. The temperature rises from T_2 to T_1.

5.8.1 Efficiency of Carnot Engine

The efficiency is defined as :

$$\eta_{engine} = \text{net work done per cycle/heat supplied per cycle} = \frac{W_{net}}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1}$$

Now $$W_{1-2} = p_1V_1 \ln\frac{V_2}{V_1} = mRT_1 \ln\frac{V_2}{V_1}$$

$$W_{2-3} = \frac{p_2V_2 - p_3V_3}{(\gamma-1)} = mc_v(T_1 - T_2)$$

$$W_{3-4} = -p_3V_3 \ln\frac{V_3}{V_4} = -mRT_2 \ln\frac{V_3}{V_4}$$

$$W_{4-1} = \frac{p_4V_4 - p_1V_1}{(\gamma-1)} = -mc_v(T_1 - T_2)$$

Work done per cycle, $$W_{net} = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

$$= mRT_1 \ln\frac{V_2}{V_1} + mc_v(T_1 - T_2) - mRT_2 \ln\frac{V_3}{V_4} - mc_v(T_1 - T_2)$$

$$= mRT_1 \ln\frac{V_2}{V_1} - mRT_2 \ln\frac{V_3}{V_4}$$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)$

Now $$\frac{V_2}{V_3} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{V_1}{V_4} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$\therefore$ $$\frac{V_2}{V_3} = \frac{V_1}{V_4}$$

or $$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$\therefore$ $$W_{net} = mR(T_1 - T_2) \ln\frac{V_2}{V_1}$$

Also $$Q_1 = mRT_1 \ln\frac{V_2}{V_1}$$

$\therefore$ $$\eta_{engine} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

Since $T_2 < T_1$, therefore, $\eta_{engine} < 100\%$.

5.11 RANKINE CYCLE

A simple steam plant is shown in Fig. 5.7. The various processes of Rankine cycle are shown in Fig. 5.8.

Process 3-4 : The water from the hot well at low pressure p_1 is pumped isentropically into the boiler at high pressure p_2.

Process 4-5 : This is the sensible heating of water along which the hot water is heated up to the saturation temperature T_1. The area *a*-3-4-5-*b*-*a* represents the heat supplied $(h_{f5} - h_{f4})$ during this process.

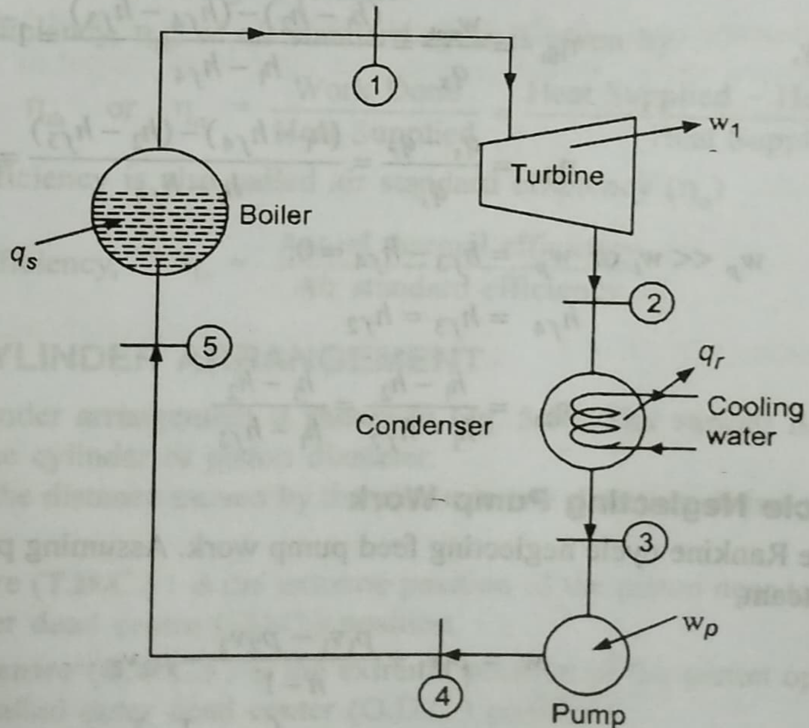


Fig. 5.7 A simple steam plant Rankine cycle.

Process 5-1 : The saturated water at pressure p_1 and temperature T_1 is completely vaporised into steam. The heat added $(h_1 - h_{f5})$ is represented by area b-5-1-c-b, *i.e.*, the latent heat of vaporization. The state point V shows wet steam, ĩ dry and saturated, and 1″ superheated condition of steam.

Process 1-2 : It is the isentropic expansion of steam in the turbine from p_1 to p_2.

Process 2-3 : The exhaust steam is condensed in the condenser at pressure p_2 and temperature T_2 giving latent heat to water.

Considering 1 kg of the working fluid, we have

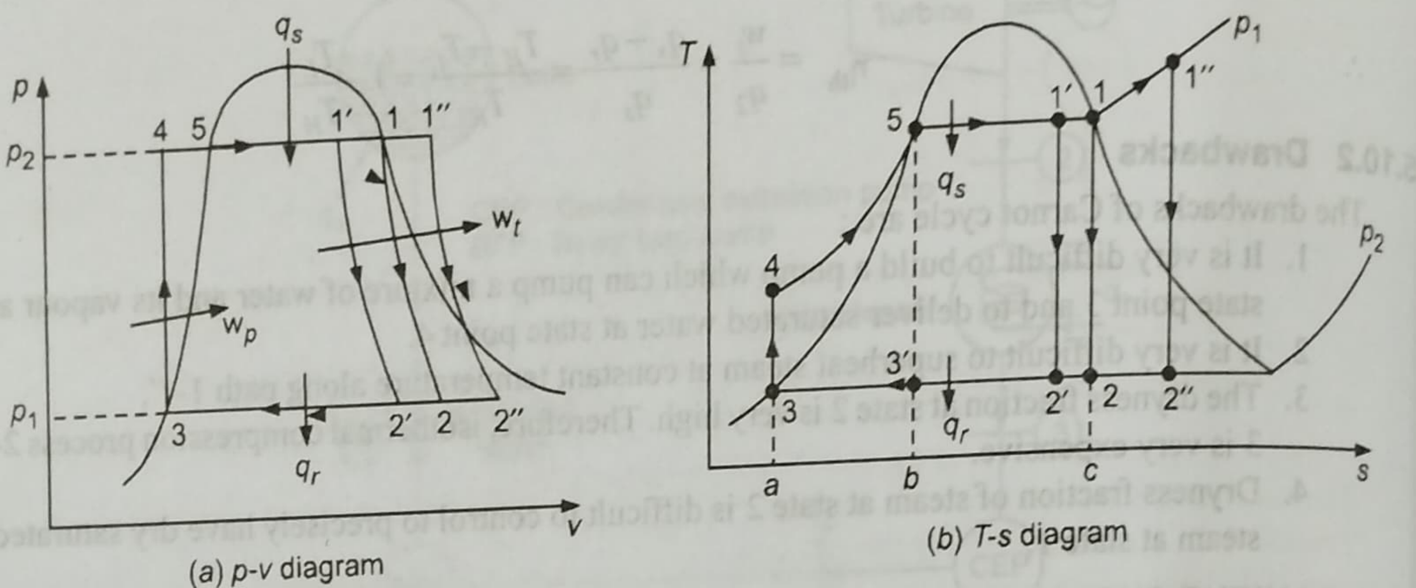


Fig. 5.8 Rankine cycle.

Heat supplied in the boiler, $\quad q_{4-1} = q_s = h_1 - h_{f4}$

Heat rejected in the condenser, $\quad q_{2-3} = q_r = h_2 - h_{f3}$

Work done by the turbine, $\quad w_{1-2} = w_t = h_1 - h_2$

Work done on the pump, $\quad w_{3-4} = w_p = h_{f4} - h_{f3} = v_{f2}(p_1 - p_2)$

Net work, $\quad w_{net} = w_t - w_p = (h_1 - h_2) - (h_{f4} - h_{f3})$

Thermal efficiency,
$$\eta_{th} = \frac{w_{net}}{q_s} = \frac{(h_1 - h_2) - (h_{f4} - h_{f3})}{h_1 - h_{f4}} = 1 - \frac{h_2 - h_{f3}}{h_1 - h_{f4}}$$

Also
$$\eta_{th} = \frac{q_s - q_r}{q_s} = \frac{(h_1 - h_{f4}) - (h_2 - h_{f3})}{h_1 - h_{f4}} = 1 - \frac{h_2 - h_{f3}}{h_1 - h_{f4}}$$

Now $\quad w_p << w_t$ or $w_p = h_{f3} - h_{f4} = 0.$

or $$h_{f4} = h_{f3} = h_{f2}$$

$\therefore$ $$\eta_{th} = \frac{h_1 - h_2}{h_1 - h_{f2}} = \frac{h_1 - h_2}{h_1 - h_{f3}}$$

Note that r_k is a **volume ratio** and should not be confused with the pressure ratio.

Another term frequently used in regard to reciprocating engines is the **mean effective pressure (mep)**, as mentioned earlier in Article 3.3 while studying indicator diagram. It is a fictitions pressure that, if it acted on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle.

$$W_{net} = \text{m.e.p.} \times \text{piston area} \times \text{stroke} = \text{m.e.p.} \times \text{displacement volume}$$

or, $$\text{m.e.p.},\ p_m = (W_{net})/[V_{max} - V_{min}]\ (\text{k Pa})$$

The mean effective pressure can be used as a parameter to compare the performance of reciprocating engines of equal size. The engine with a larger value of m.e.p. will deliver more net work per cycle and thus will perform better.

Reciprocating engines are classified as **spark-ignition (SI) engines** or **compression-ignition (CI) engines**, depending on how the combustion process in the cylinder is initiated. In SI engines, also called petrol or gasoline engines, the combustion of the air-fuel mixture is initiated by a spark plug. In CI engines, also called diesel engines, the air-fuel mixture is self-ignited as a result of compressing the mixture above its self-ignition temperature.

13.5 AIR STANDARD CYCLES

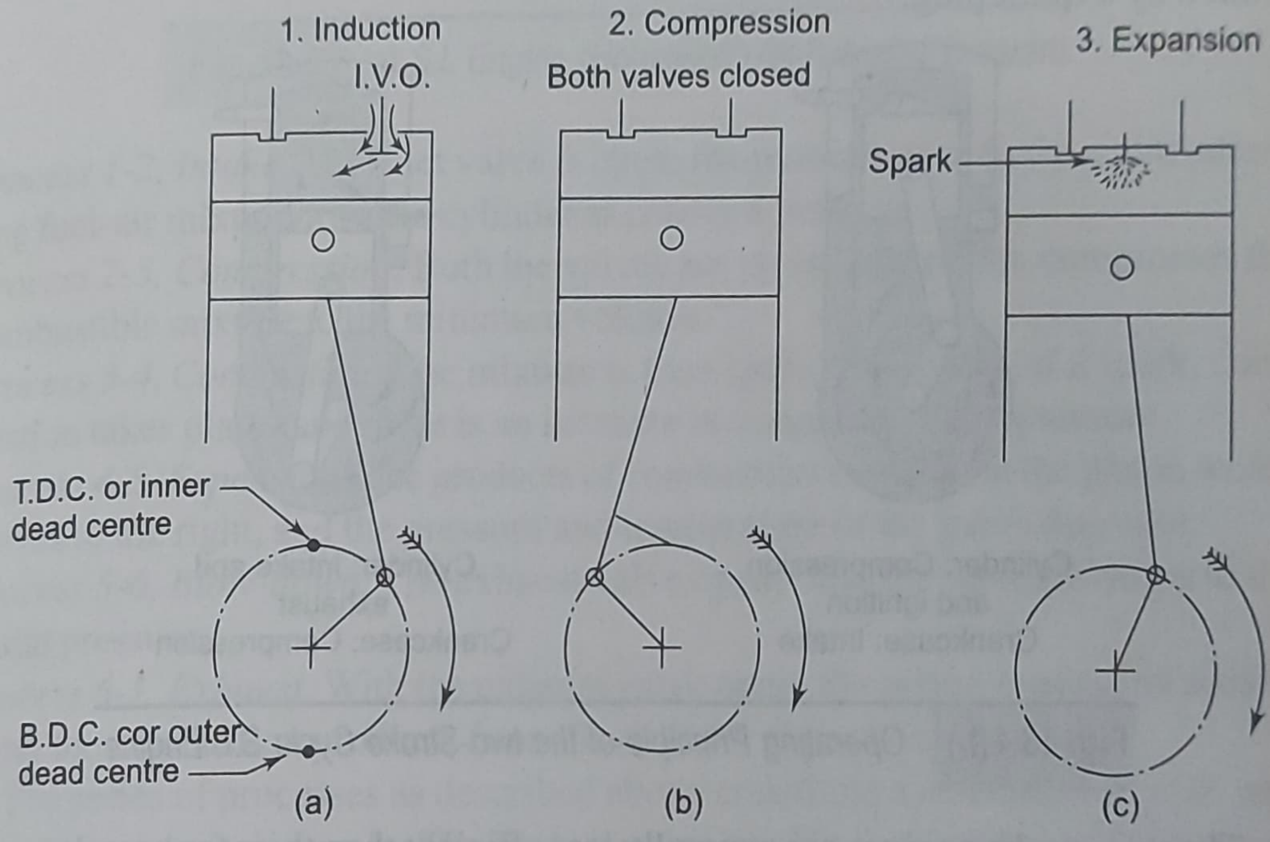
Internal combustion engines (Fig. 13.4) in which the combustion of fuel occurs in the engine cylinder itself are non-cyclic heat engines. The temperature due to the evolution of heat because of the combustion of fuel inside the cylinder is so high that the cylinder is cooled by water circulation around it to avoid rapid deterioration. The working fluid, the fuel-air mixture, undergoes permanent chemical change due to combustion, and the products of combustion after doing work are thrown out of the engine, and a fresh charge is taken. So the working fluid does not undergo a complete thermodynamic cycle.

To simplify the analysis of I.C. engines, *air standard cycles* are conceived. In an air standard cycle, a certain mass of air operates in a complete thermodynamic cycle, where heat is added and rejected with external heat reservoirs, and all the processes in the cycle are reversible. Air is assumed to behave as an ideal gas, and its specific heats are assumed to be constant. These air standard cycles are so conceived that they correspond to the operations of internal combustion engines.

13.6 OTTO CYCLE (1876)

The Otto cycle is the air standard cycle of the SI engine. It is named after Nikolaus A. Otto, a German engineer, who first built a successful four-stroke SI engine in 1876 using the cycle proposed by a Frenchman Alphouse Beau de Rochas in 1862. In most spark-ignition engines, the piston executes four complete strokes within the cylinder, and the crankshaft completes two revolutions for each thermodynamic cycle. These engines are called **four-stroke** internal combustion engines. A schematic of each stroke as well as *p-v* diagram for an actual four-stroke SI engine is given in Fig. 13.4 (a).

Initially, the inlet valve opens (I.V.O.) and fresh charge of fuel and air mixture is drawn into the cylinder. Then both the intake and exhaust valves are closed, and the piston is at its lowest position (BDC). During the compression stroke the piston moves upward, compressing the air-fuel mixture. Shortly before the piston reaches its highest position (TDC), the spark plug fires and the mixture ignites, increasing the pressure and temperature of the system. The high pressure gases force the piston down, which in turn forces the crankshaft to rotate, producing a useful work output during the **expansion or power stroke**. At the end of this stroke, the piston is at its lowest position, and the cylinder is filled with the combustion products. Next the piston moves upward again, purging the exhaust gases through the exhaust valve (the **exhaust stroke**), and down a second time, drawing in fresh air-fuel mixture through the intake valve (the **intake stroke**). It may be noted that the pressure in the cylinder is slightly above the atmospheric value during the exhaust stroke and slightly below it during the intake stroke.



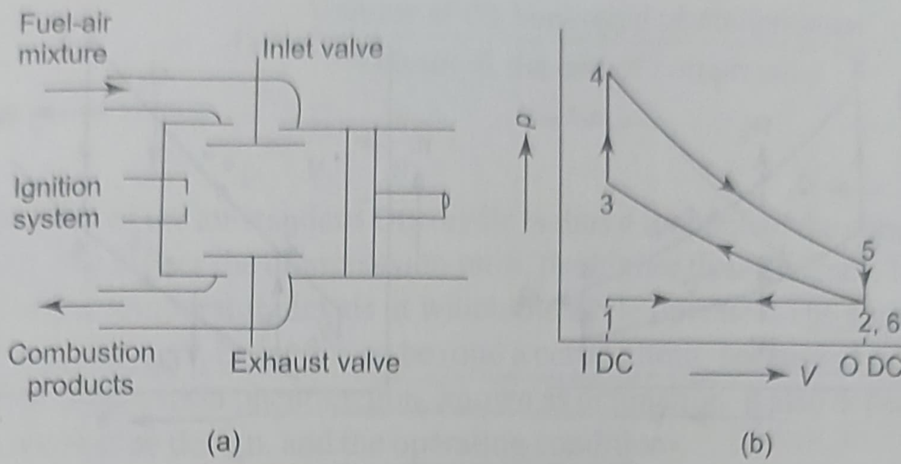


Fig. 13.5 *(a) S.I. Engine (Horizontal) (b) Indicator Diagram*

Process 1-2, Intake. The inlet valve is open, the piston moves to the right, admitting fuel-air mixture into the cylinder at constant pressure.
Process 2-3, Compression. Both the valves are closed, the piston compresses the combustible mixture to the minimum volume.
Process 3-4, Combustion. The mixture is then ignited by means of a spark, combustion takes place, and there is an increase in temperature and pressure.
Process 4-5, Expansion. The products of combustion do work on the piston which moves to the right, and the pressure and temperature of the gases decrease.
Process 5-6, Blow-down. The exhaust valve opens, and the pressure drops to the initial pressure.
Process 6-1, Exhaust. With the exhaust valve open, the piston moves inwards to expel the combustion products from the cylinder at constant pressure.

The series of processes as described above constitute a *mechanical cycle*, and not a thermodynamic cycle. The cycle is completed in four strokes of the piston.

Figure 13.5 (c) shows the air standard cycle (Otto cycle) corresponding to the above engine. It consists of: Two reversible adiabatics, one reversible isobar, and one reversible isochore.

Air is compressed in process 1-2 reversibly and adiabatically. Heat is then added to air reversibly at constant volume in process 2-3. Work is done by air in expanding reversibly and adiabatically in process 3-4. Heat is then rejected by air reversibly at constant volume in process 4-1, and the system (air) comes back to its initial state. Heat transfer processes have been substituted for the combustion and blow-down processes of the engine. The intake and exhaust processes of the engine cancel each other.

Let m be the fixed mass of air undergoing the cycle of operations as described above.

$$\text{Heat supplied } Q_1 = Q_{2-3} = mc_v\,(T_3 - T_2)$$

$$\text{Heat rejected } Q_2 = Q_{4-1} = mc_v\,(T_4 - T_1)$$

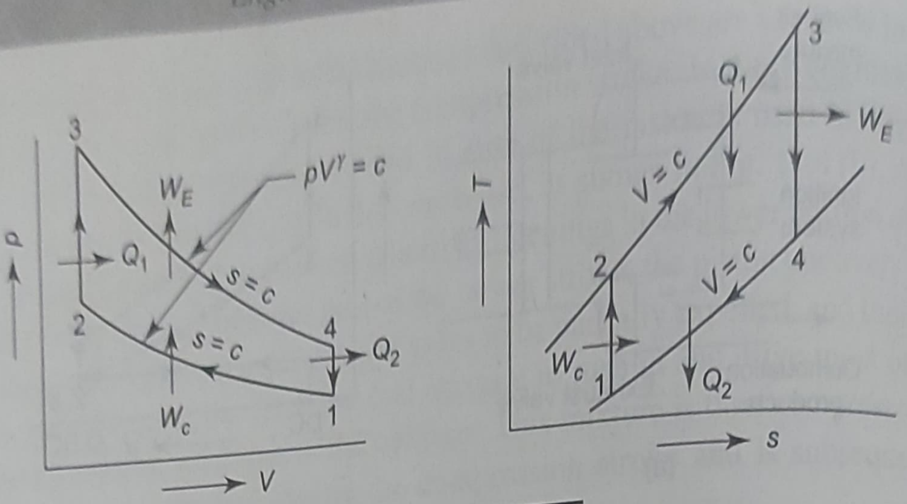


Fig. 13.5 (c) Otto Cycle

$$\text{Efficiency } \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad (13.4)$$

Process 1-2, $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2'}\right)^{\gamma-1}$$

Process 3-4, $$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$\therefore$ $$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

or $$\frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$\therefore$ $$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

$\therefore$ From Eq. (13.4), $\eta = 1 - \left(\dfrac{v_2}{v_1}\right)^{\gamma-1}$

or $$\eta_{\text{otto}} = 1 - \frac{1}{r_k^{\gamma-1}} \quad (13.5)$$

where r_k is called the compression ratio and given by

$$r_k = \frac{\text{Volume at the beginning of compression}}{\text{Volume at the end of compression}}$$

$$= \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

The efficiency of the air standard Otto cycle is thus a function of the compression ratio only. The higher the compression ratio, the higher the efficiency. It is independent of the temperature levels at which the cycle operates. The compression ratio cannot, however, be increased beyond a certain limit, because of a noisy and destructive combustion phenomenon, known as detonation. It also depends upon the fuel, the engine design, and the operating conditions.

13.7 DIESEL CYCLE (1892)

The limitation on compression ratio in the S.I. engine can be overcome by compressing air alone, instead of the fuel-air mixture, and then injecting the fuel into the cylinder in spray form when combustion is desired. The CI engine, first proposed by Rudolph Diesel in the 1890s, is very similar to the SI engine, differing mainly in the method of initiating combustion. In SI engines, a mixture of air and fuel is compressed during compression stroke, and the compression ratios are limited by the onset of autoignition or engine knock. In CI engines, only air is compressed during the compression stroke. Therefore, diesel engines can operate at much higher compression ratios, typically between 12 and 24. The spark plug and carburettor (for mixing fuel and air) are replaced by a fuel injector in diesel engines. The temperature of air after compression must be high enough so that the fuel sprayed into the hot air burns spontaneously. The rate of burning can, to some extent, be controlled by the rate of injection of fuel. An engine operating in this way is called a *compression ignition (C.I.) engine*. The sequence of processes in the elementary operation of a C.I. engine, shown in Fig. 13.6, is given below.

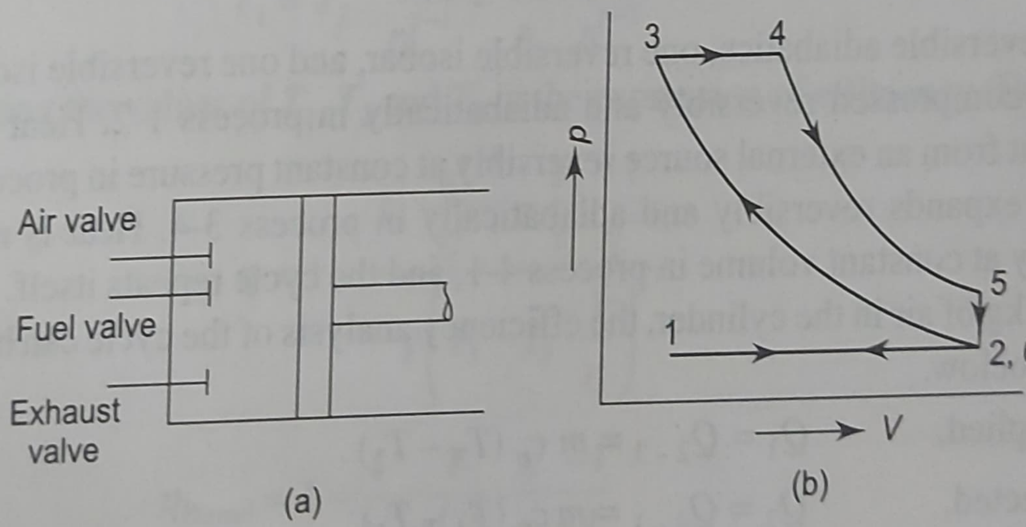


Fig. 13.6 *(a) C.I. Engine (b) Indicator Diagram*

Process 1-2, intake. The air valve is open. The piston moves out admitting air into the cylinder at constant pressure.
Process 2-3, Compression. The air is then compressed by the piston to the minimum volume with all the valves closed.
Process 3-4, Fuel injection and combustion. The fuel valve is open, fuel is sprayed into the hot air, and combustion takes place at constant pressure.
Process 4-5, Expansion. The combustion products expand, doing work on the piston which moves out to the maximum volume.

Process 5-6, Blow-down. The exhaust valve opens, and the pressure drops to the initial pressure.

Pressure 6-1, Exhaust. With the exhaust valve open, the piston moves towards the cylinder cover driving away the combustion products from the cylinder at constant pressure.

The above processes constitute an engine cycle, which is completed in four strokes of the piston or two revolution of the crank shaft.

Figure 13.7 shows the air standard cycle, called the *Diesel cycle*, corresponding to the C.I. engine, as described above. The cycle is composed of:

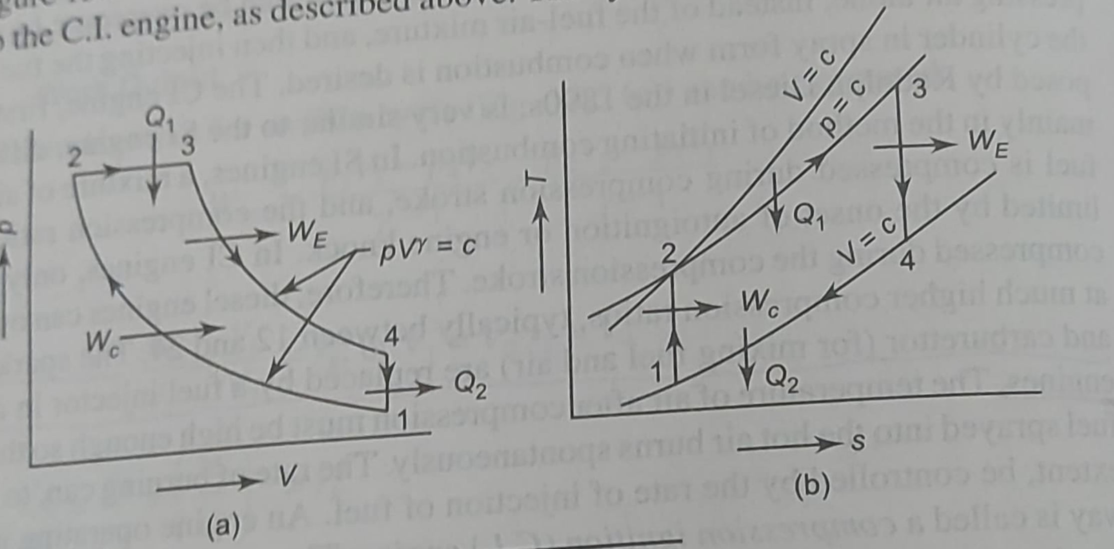


Fig. 13.7 *Diesel Cycle*

Two reversible adiabatics, one reversible isobar, and one reversible isochore. Air is compressed reversibly and adiabatically in process 1-2. Heat is then added to it from an external source reversibly at constant pressure in process 2-3. Air then expands reversibly and adiabatically in process 3-4. Heat is rejected reversibly at constant volume in process 4-1, and the cycle repeats itself.

For m kg of air in the cylinder, the efficiency analysis of the cycle can be made as given below.

Heat supplied, $\quad Q_1 = Q_{2-3} = m\, c_p\, (T_3 - T_2)$

Heat rejected, $\quad Q_2 = Q_{4-1} = m\, c_v\, (T_4 - T_1)$

Efficiency
$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)}$$

$$\therefore \qquad \eta = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)} \qquad (13.8)$$

The efficiency may be expressed in terms of any two of the following three ratios

Compression ratio, $\quad r_k = \frac{V_1}{V_2} = \frac{v_1}{v_2}$

Expansion ratio, $\quad r_e = \frac{V_4}{V_3} = \frac{v_4}{v_3}$

Cut-off ratio, $$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

It is seen that

$$r_k = r_e \cdot r_c$$

Process 3-4

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$

$$T_4 = T_3 \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}}$$

Process 2-3

$$\frac{T_2}{T_3} = \frac{p_2 v_2}{p_3 v_3} = \frac{v_2}{v_3} = \frac{1}{r_c}$$

$$\therefore \quad T_2 = T_3 \cdot \frac{1}{r_c}$$

Process 1-2

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \frac{1}{r_k^{\gamma-1}}$$

$$\therefore \quad T_1 = T_2 \cdot \frac{1}{r_k^{\gamma-1}} = \frac{T_3}{r_c} \cdot \frac{1}{r_k^{\gamma-1}}$$

Substituting the values of T_1, T_2 and T_4 in the expression of efficiency (Eq. 13.8)

$$\eta = 1 - \frac{T_3 \cdot \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}} - \frac{T_3}{r_c} \frac{1}{r_k^{\gamma-1}}}{\gamma\left(T_3 - T_3 \cdot \frac{1}{r_c}\right)}$$

$$\therefore \quad \eta_{\text{Diesel}} = 1 - \frac{1}{\gamma} \cdot \frac{1}{r_k^{\gamma-1}} \cdot \frac{r_c^{\gamma} - 1}{r_c - 1} \tag{13.9}$$

As $r_c > 1$, $\frac{1}{\gamma}\left(\frac{r_c^{\gamma} - 1}{r_c - 1}\right)$ is also greater than unity. Therefore, the efficiency of the Diesel cycle is less than that of the Otto cycle for the same compression ratio.