APPLICATION OF STOCHASTIC DIFFERENTIAL EQUATIONS IN R

Using the 2022 USD/KES Exchange Rates

Charity Wamwea

November 10, 2023

Contents

1 The Model

According to Christian and Christensen (2013), the foreign exchange rate dynamics, Q under the domestic measure $\mathbb{P}$ is given by:

$$\begin{cases} dQ_t &= \left(r_d - r_f\right) Q_t dt + \sigma Q_t dW_t \\ Q_0 &> 0 \end{cases} \tag{1.1}$$

where

r_d is the risk free rate of interest in the domestic market.

r_f is the risk free rate of interest in the foreign market.

σ is volatility.

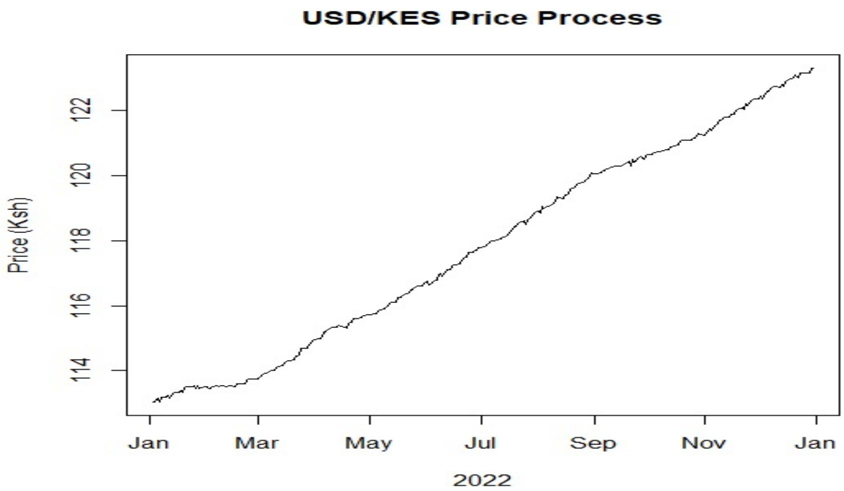
However, in our case, we would wish to estimate both the drift and diffusion terms and hence, we shall assume that

$$\begin{cases} dQ_t &= \mu Q_t dt + \sigma Q_t dW_t \\ Q_0 &= 113.05 \end{cases} \tag{1.2}$$

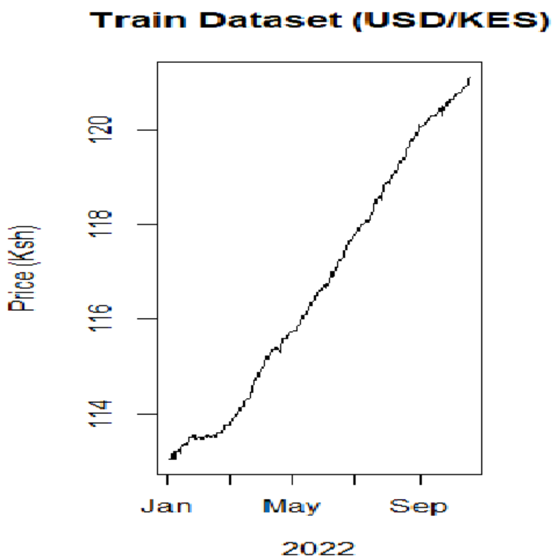
The model parameters (μ and σ) shall be estimated using the quasi maximum likelihood estimation (QMLE) method.

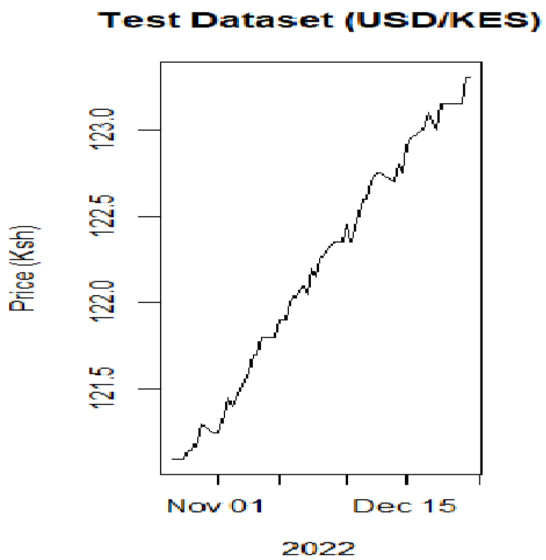
2 The Data

In this document, USD/KES foreign exchange rate spanning from 3^{rd} January 2022 to 30^{th} December 2022 shall be analysed. The USD/KES data stated above was collected from https://www.investing.com/currencies/usd-kes.



For analysis, the data shall be divided into two at a ratio of 4:1 to represent the train and test datasets respectively. The train data shall span from 3^{rd} **January 2022 to** 19^{th} **October 2022**. The test data shall span from 21^{st} **October 2022 to** 30^{th} **December 2022** as shown below:





Assumptions of the Study

- The study assumes that closing prices are sufficient in explaining the dynamics of the USD/KES foreign exchange rate.
- It is assumed that there are 252 trading days in a year.
- It is assumed that there is no bid-ask spread in the USD/KES prices.

3 Data Analysis in R

3.1 Loading the Required Packages into R

The following R packages are required for our analysis:

```
library(yuima)
library(quantmod)
library(xts)
```

In the event that the packages are not preinstalled into your R version, use the code:

```
install.packages(" ")
```

to install the required package into R.

3.2 Reading Data into R

```
#To call in your document directory, use:
file.choose()
USDKES=read.csv("FIle_Path",header=TRUE)
```

3.3 Extracting Important Information from USD/KES Rates

```
#Extracting the USD/KES Prices and Dates
price=USDKES$Price
Date=USDKES$Date
Date1<-as.Date(Date,format="%m/%d/%Y")
n=length(price)

#Rearranging data from 3/1/2022 to 30/12/2022
```

```
price=price[n:1]
date=Date1[n:1]
plot(price~date,type="l",main="USD/KES_Price_Process",xlab="2022",
ylab="Price(Ksh)")

#Describing the train data (80%) and test data (20%) data ratios
n1=round(0.8*n,0)
n2=n1+1
n3=n2:n
train=price[1:n1]
test=price[n2:n]
train.date=date[1:n1]
test.date=date[n2:n]

#Plotting the Test and Train Datasets
par(mfrow=c(1,2))
plot(train~train.date,type="l",main="Train_Dataset(USD/KES)",
xlab="2022",ylab="Price␣(Ksh)")
plot(test~test.date,type="l",main="Test_Dataset(USD/KES)",
xlab="2022",ylab="Price(Ksh)")
```

3.4 Fitting the USD/KES Train Data to the GBM Model

The USD/KES data over the train period was fitted in R to the GBM model described on equation (2.2) as follows:

```
data=setData(train,delta=1/252,t0=0)
model=setModel(drift="mu*x",diffusion="sigma*x",state.var="x",
time.var="t",solve.var="x",xinit=train[1])
D=setYuima(model=model,data=data)
```

```
initialise=list(mu=0.05,sigma=2)
lowbound=list(mu=-Inf,sigma=0)
upbound=list(mu=Inf,sigma=Inf)
mle=qmle(D,start=initialise ,lower=lowbound,upper=upbound)
summary(mle)

#Estimated Parameters
sigma=coef(mle)[1]
mu=coef(mle)[2]
```

The fitted Model parameters (for the train period) were estimated as follows:

Table 3.1: GBM Esimated Model Parameters

Parameter	Estimated Value	Standard Error
mu	0.0089	0.0002
sigma	0.0880	0.0100

3.5 Simulating in-sample Paths of the fitted model

Several sample paths of the fitted GBM model over the train period are simulated.These paths will enable us to adequately check whether our data has the same distribution as that of the simulated sample paths.

```
#Simulating 1000 sample paths from the fitted model
J=1000
V=matrix(0,nrow=n1,ncol=J)
for(j in 1:J)
{
grid=setSampling(delta=1/252,n=n1-1)
Y=simulate(model,true.param=list(mu=mu,sigma=sigma),sampling=grid)
sim.Data=Y@data@original.data
V[,j]=sim.Data[,]
}
```

3.6 Train Set Estimated Mean Equation and Confidence Intervals

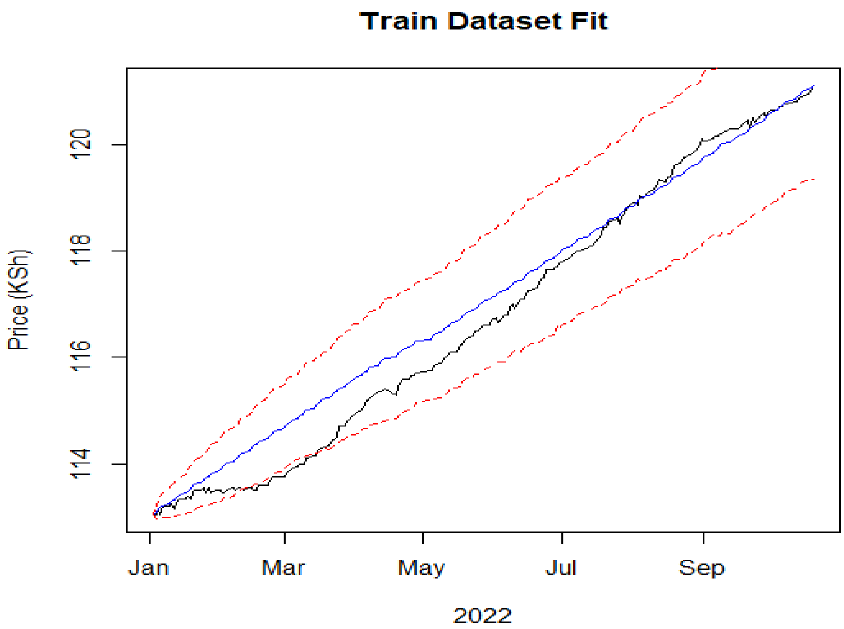
From the 1000 simulated paths, the mean equation and the 95% confidence interval were extracted as follows:

```
#Getting the mean equation from the 1000 fitted paths
train.est=0
train.uci=0
train.lci=0
for(i in 1:n1)
{
train.est[i]=mean(V[i,])
train.lci[i]=quantile(V[i,],0.025)
train.uci[i]=quantile(V[i,],0.975)
}

par(mfrow=c(1,1))
plot(train~train.date,type="l",main="Train Dataset Fit",xlab="2022",ylab="

lines(train.est~train.date,col="blue")
lines(train.uci~train.date,col="red",lty=2)
lines(train.lci~train.date,col="red",lty=2)
```

The actual data, mean sample path and the 95% confidence interval of the train dataset was plotted as shown below:



where:

Actual = Black Solid Line

Mean Path = Blue Solid Line

95% C.I = Red Dotted Line

3.7 Testing the Goodness of Fit of the Model

The mean sample path of the train dataset was be compared with the actual data to ascertain whether or not the dataset. To achieve this, the Kolmogorov-Smirnov Test was used. Where:
H_0 : Mean simulated sample path and the actual Train data come from same distribution
H_1 : Mean simulated sample path and the actual Train data come from different distributions

The Kolmogorov Smirnov test statistic is given by:

$$D = sup_x|F(x) - G(x)| \tag{3.1}$$

where:

F(x) is the empirical distribtuion of the simulated mean sample path.

G(x) is the empirical distribtuion of the actual train data.

The null hypothesis is rejected when the p-value<alpha. This was performed in R as follows:

```
ks.test(train.est, train)
```

The K-S test results were as summarised below:

Table 3.2: Train Dataset K-S Test Results

D-Statistic	P-Value
0.0817	0.9962

The KS-test results show that the fitted model is a good fit on the train dataset. Hence we can now use the fitted model parameters to forecast the future and compare our forecasted values with the test dataset.

3.8 Forecasting the Future USD/KES Prices

The fitted model was used to forecast the USD/KES prices from 21^{st} October 2022 to 30^{th} December 2022 as follows:

```
#Test Dataset
#The USD/KES price on 26/5/2023 was 138.25
#We need to forecast the values for the test period

model1=setModel(drift="mu*x",diffusion="sigma*x",state.var="x",
time.var="t",solve.var="x",xinit=price[n1])
```

3.9 Simulating out-of-sample Paths of the fitted model

Several sample paths of the fitted GBM model over the test period were simulated as follows.

```
#Simulating 1000 test data sample paths from the fitted model
J=1000
m=length(test)
```

```
M=matrix(0,nrow=m+1,ncol=J)
for(j in 1:J)
{
grid=setSampling(delta=1/252,n=m)
X=simulate(model1,true.param=list(mu=mu,sigma=sigma),sampling=grid)
test.Data=X@data@original.data
M[,j]=test.Data[,]
}

#Removing the initial value which is the last value of the train dataset
M=M[-1,]
```

3.10 Train Set Estimated Mean Equation and Confidence Intervals

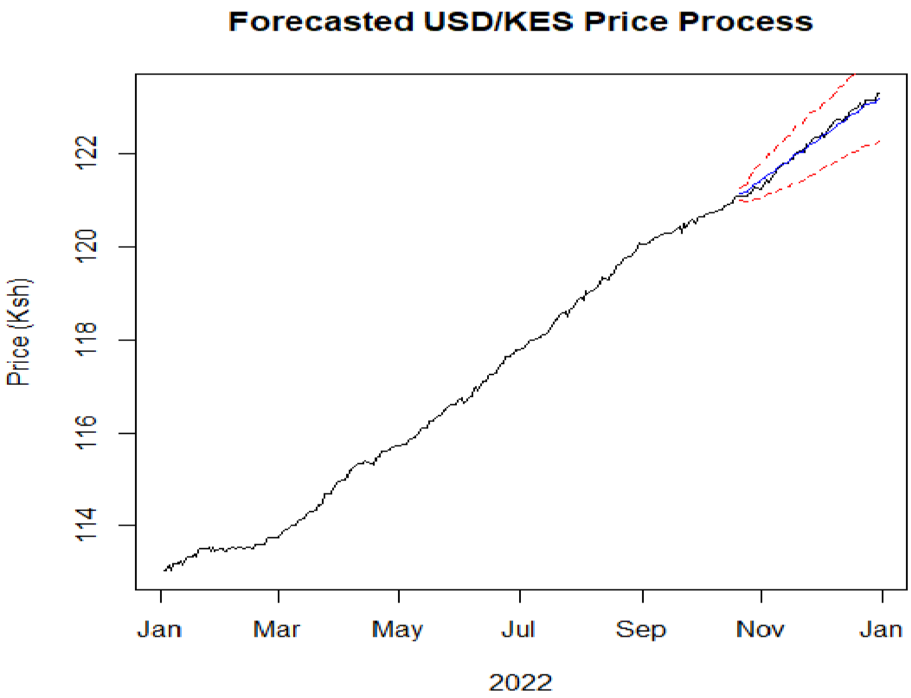
From the 1000 simulated paths of the test period, the mean equation and the 95% confidence interval were extracted as follows:

```
#Getting the mean equation and 95% C.I from the 1000 fitted paths
#Getting the mean equation from the 1000 fitted paths
test.est=0
test.uci=0
test.lci=0
for(i in 1:m)
{
test.est[i]=mean(M[i,])
test.lci[i]=quantile(M[i,],0.025)
test.uci[i]=quantile(M[i,],0.975)
}

plot(price~date,type="l",main="Forecasted USD/KES Price Process",
xlab="2022",ylab="Price (Ksh)")
```

```
lines(test.est~test.date, col="blue")
lines(test.uci~test.date, col="red", lty=2)
lines(test.lci~test.date, col="red", lty=2)
```

The plot of the USD/KES forecasted values were as shown below:



The summary statistics of the forecasted values were as shown below:

Table 3.3: Summary Statistics of Forecasted Values

			95% C.I	
Descriptive	Actual	Mean Sample Path	Lower	Upper
Min	121.1	121.1	121.0	121.3
Q_1	121.6	121.7	121.2	122.1
Median	122.2	122.2	121.5	122.8
Mean	122.2	122.2	121.6	122.8
Q_3	122.8	122.7	121.9	123.5
Max	123.3	123.2	122.3	124.1

3.11 Testing the Model Performance

The accuracy of the fitted model was tested using the mean absolute percentage errors (MAPE) calculated using the formula:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|A_t - F_t|}{|A_t|} \times 100 \tag{3.2}$$

The MAPE results were as summarized below:

Table 3.4: MAPE Results of Forecasted Values

MAPE (%)
0.0616

The KS-test was also performed to compare the forecasted values the actual values using

```
ks.test(test,test.est)
```

and the results were as summarised in the table below:

Table 3.5: Forecasted Values K-S Test Results

D-Statistic	P-Value
0.0816	0.9961

The KS-test results show that the forecasted model results at 5% level of significance are statistically similar to those of the actual values.

References

1. Jens Christian and Jore Christensen, 2013, *Dynamics of exchange rates and pricing of currency derivatives*, Thesis of degree of Master of Science, Faculty of Mathematics and Natural Sciences.