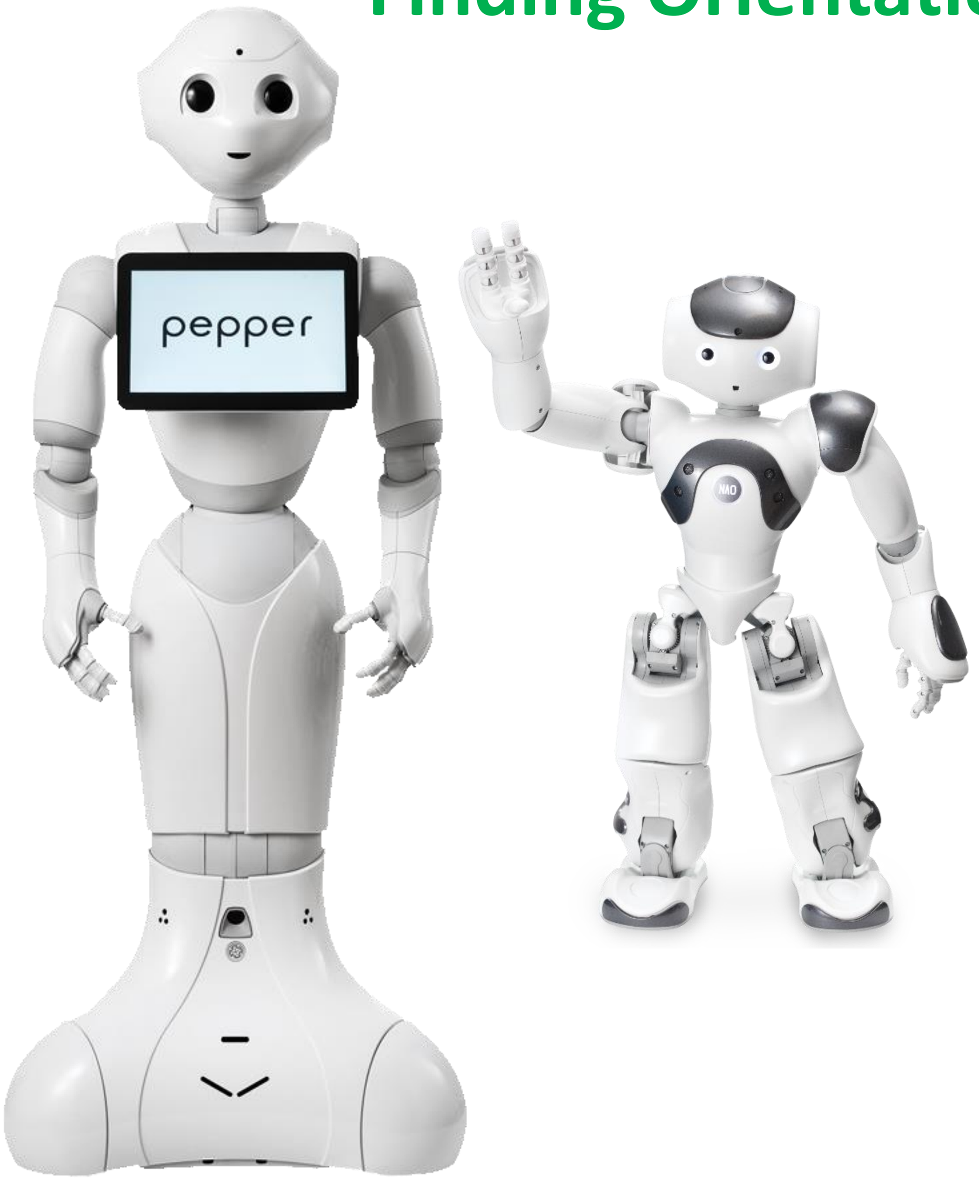
Fundamentals of Robotics

Robot Manipulators 07

Finding Orientation of a robot



Velocity of the end effector

Jacobian matrix

Table for filling Jacobian matrix

Jacobian matrix for Cartesian manipulator

Jacobian matrix for Articulated manipulator

Velocity of an end effector

- For a robot manipulator the position of an end effector is not only important. However, controlling the velocity of an end effect is vital.
- There are application where controlling velocity is very important e.g., welding, cutting, assembling, screwing etc.
- There is a difference between the velocities of joint and the end effector.
- To control the velocity of an end effector, we not only consider the joint velocities, besides we also consider joint variable values.

Jacobian matrix

- We figure out the velocities of the joints that will get the end effector to move with right velocity, taking into account the position of the manipulator at each point in time. This calculation is called Jacobian matrix.
- The Jacobian matrix helps you convert angular velocities of the joints (i.e., joint velocities) into the velocity of the end effector of a robotic arm.
- The Jacobian matrix relates the end effector velocities to joint velocities.
- For example, if the servo motors of a robotic arm are rotating at some velocity (e.g., in radians per second), we can use the Jacobian matrix to calculate how fast the end effector of a robotic arm is moving (both linear velocity x, y, z and angular velocity roll ω_x, pitch ω_y, and yaw ω_z).

Jacobian matrix

- For a robot that operates in three dimensions, the Jacobian matrix transforms joint velocities into end effector velocities using the following equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

V

- x, y, and z with the dots on top represent linear velocities (i.e., how fast the end effector is moving in the x, y, and z directions relative to the base frame of a robotic arm)

- ω_x, ω_y, and ω_z represent the angular velocities (i.e., how fast the end effector is rotating around the x, y, and z axis of the base frame of a robotic arm).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

Jacobian matrix

- q with the dot on top represents the joint velocities (i.e., how fast the joint is rotating for a revolute joint and how fast the joint is extending or contracting for a prismatic joint) .
- The size of this vector is n x 1, with n being equal to the number of joints (i.e., servo motors or linear actuators) in your robotic arm.
- Note that q can represent either revolute joints (typically represented as θ) or prismatic joints (which we usually represent as d, which means displacement)
- J is the Jacobian matrix. It is an m rows x n column matrix (m=3 for two dimensions, and m=6 for a robot that operates in three dimensions). n represents the number of joints.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

Example

- E.g., if you have a manipulator with first two joints revolute and the last one prismatic then the equation looks like:

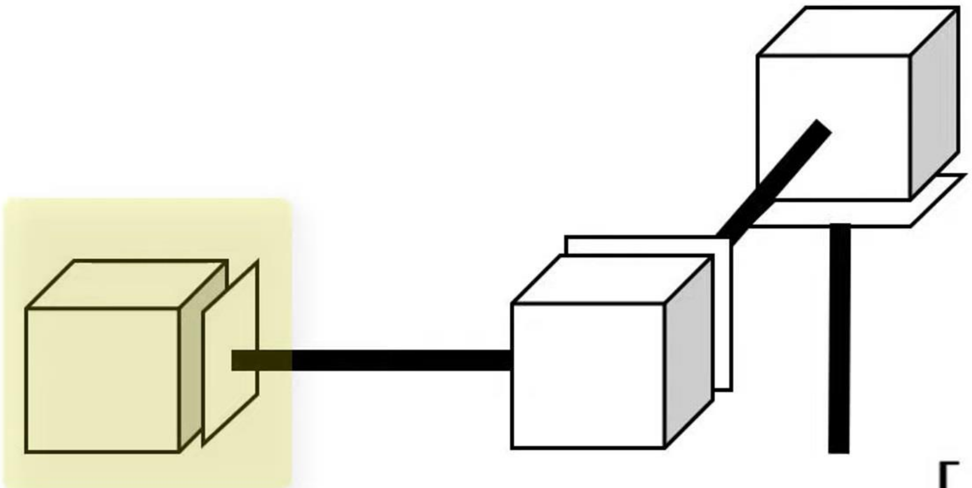
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}$$

Table for filling Jacobian matrix

- This table is used to fill the Jacobian matrix.

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times\left(d_n^0-d_{i-1}^0\right)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

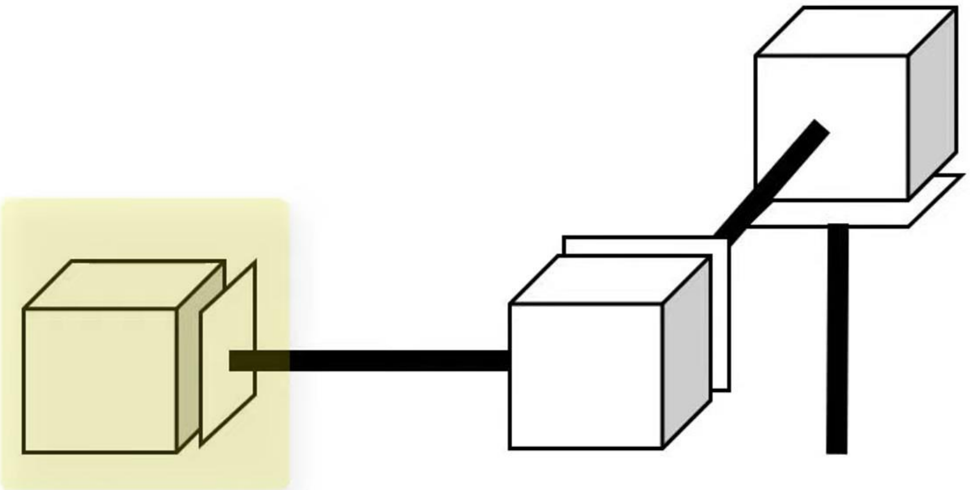
Cartesian manipulator example



	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times\left(d_n^0 - d_{i-1}^0\right)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}\square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\end{bmatrix}\begin{bmatrix}\dot{d}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix}$$

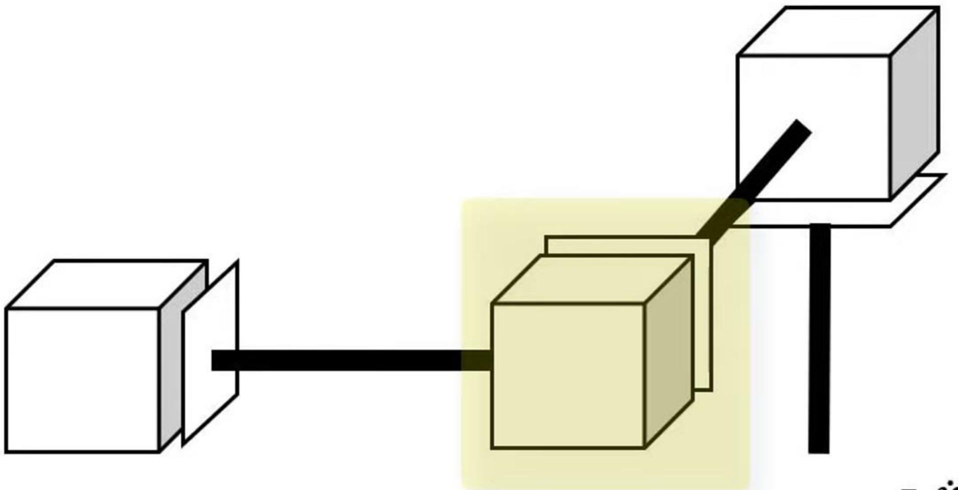
Cartesian manipulator example



	Prismatic	Revolute
Linear	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \square & \square \\ & \square & \square \\ & \square & \square \\ 0 & \square & \square \\ 0 & \square & \square \\ 0 & \square & \square \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

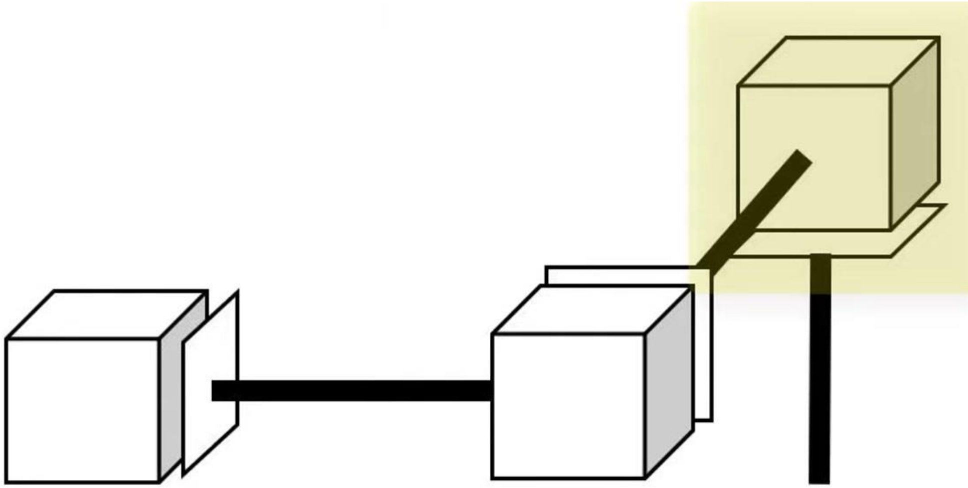
Cartesian manipulator example



	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times\left(d_n^0-d_{i-1}^0\right)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix}=\begin{bmatrix} R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{matrix}\square\\\square\\\square\end{matrix}\\ 0 & 0 & \square\\ 0 & 0 & \square\\ 0 & 0 & \square\end{bmatrix}\begin{bmatrix}\dot{d}_1\\\dot{d}_2\\\dot{d}_3\end{bmatrix}$$

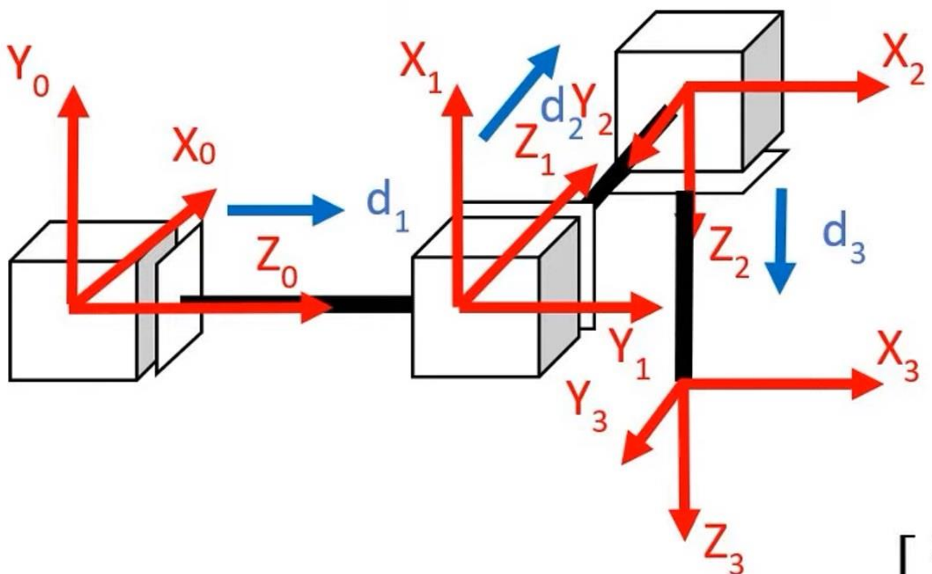
Cartesian manipulator example



	Prismatic	Revolute
Linear	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

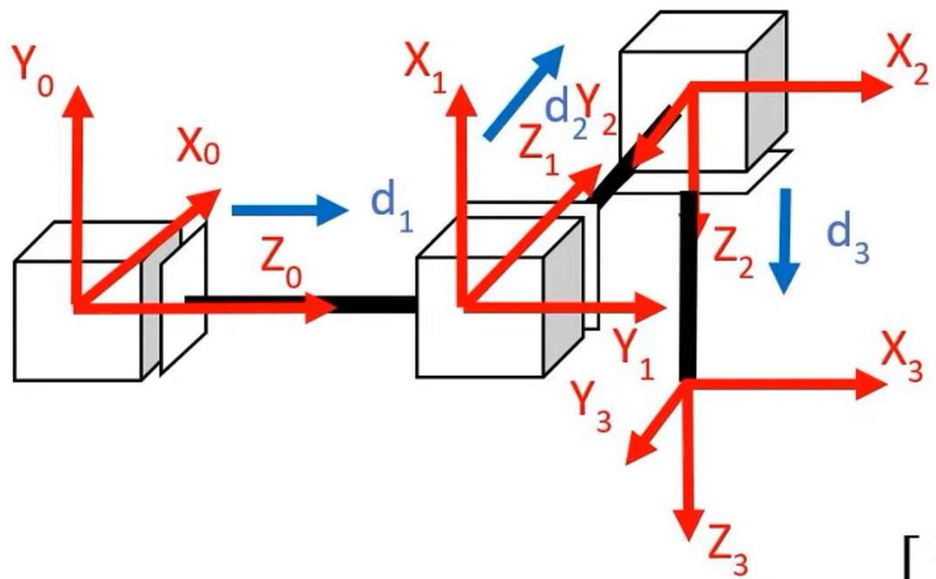
Cartesian manipulator example



$$R_0^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

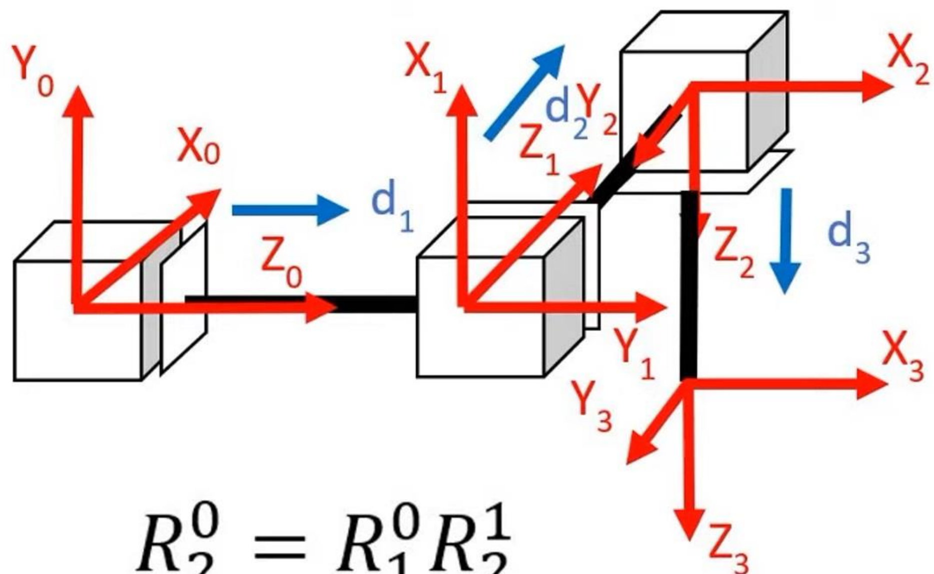
Cartesian manipulator example



$$R_1^0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Cartesian manipulator example



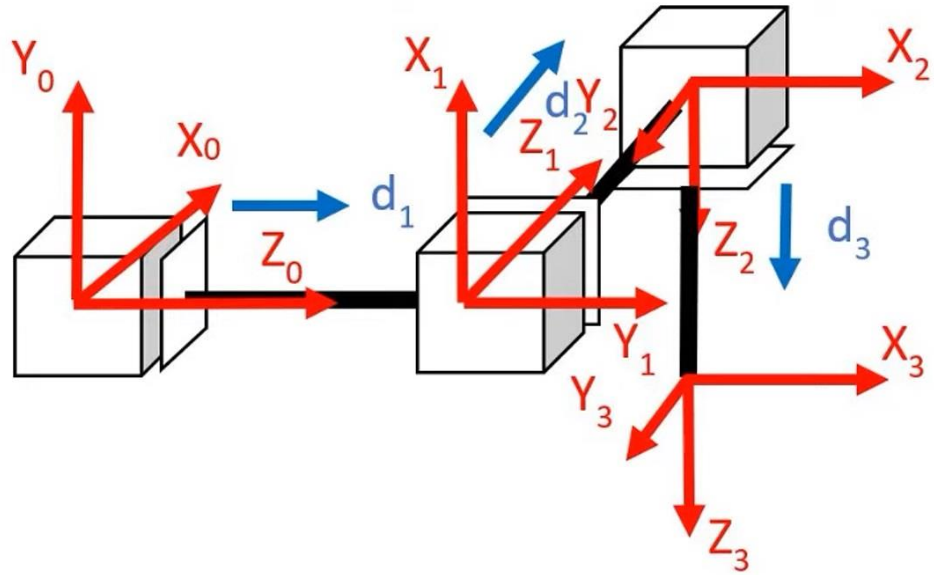
$$R_2^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R_2^0 = R_1^0 R_2^1$$

$$R_1^0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_2^1 = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

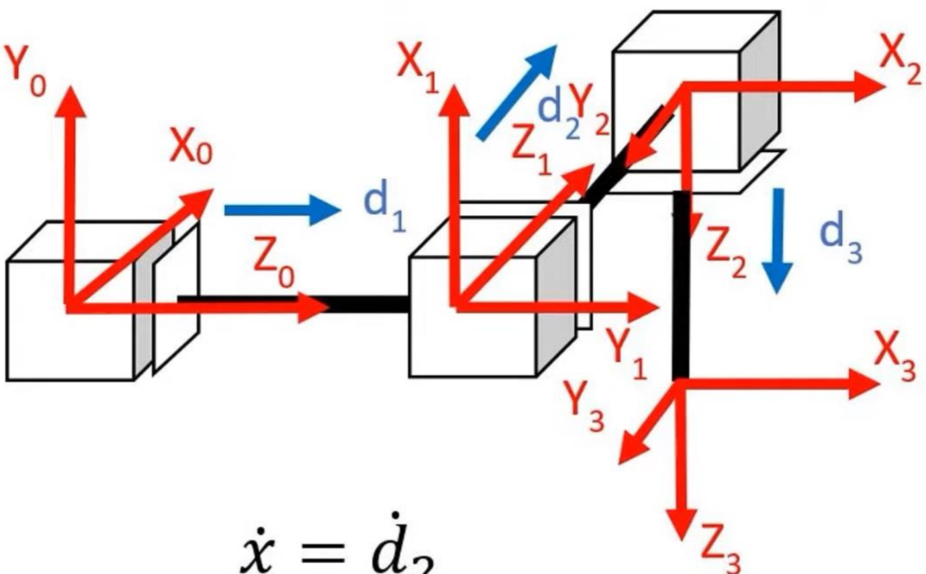
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & \\ 0 & 0 & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 1 & 0 & \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Cartesian manipulator example



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Cartesian manipulator example



$$\omega_x = 0$$

$$\omega_y = 0$$

$$\omega_z = 0$$

$$\dot{x} = \dot{d}_2$$

$$\dot{y} = -\dot{d}_3$$

$$\dot{z} = \dot{d}_1$$

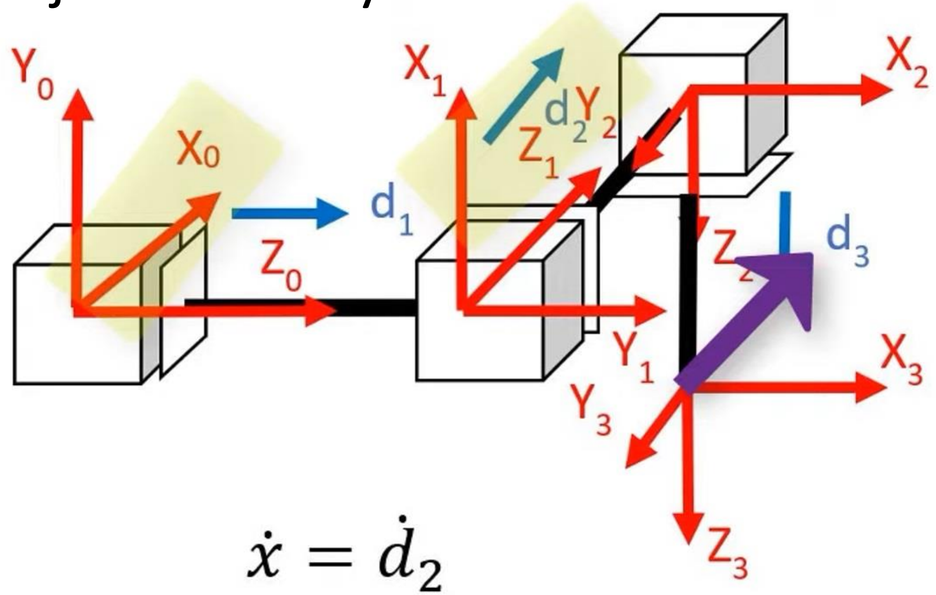
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Cartesian manipulator example

- If we look at the kinematic diagram, we can see if the equations (from Jacobian matrix) make sense. You can see that the speed of the end effector in the x0 direction is determined by the displacement of joint 2 (i.e., d2).
- You can see that the speed of the end effector in the y0 direction is determined by the negative displacement of joint 3 (i.e., d3).
- The speed of the end effector in the z0 direction is determined by the positive displacement of joint 1 (i.e., d1).
- The last three equations tell us that the end effector will never be able to rotate around the x, y, and z axes of frame 0, the base frame of the robot. This makes sense because all of the joints are prismatic joints. We would need to have revolute joints in order to get rotation.

Cartesian manipulator example

- The velocity of the end effector in x direction come from d2 joint velocity



$$\omega_x = 0$$

$$\omega_y = 0$$

$$\omega_z = 0$$

$$\dot{x} = \dot{d}_2$$

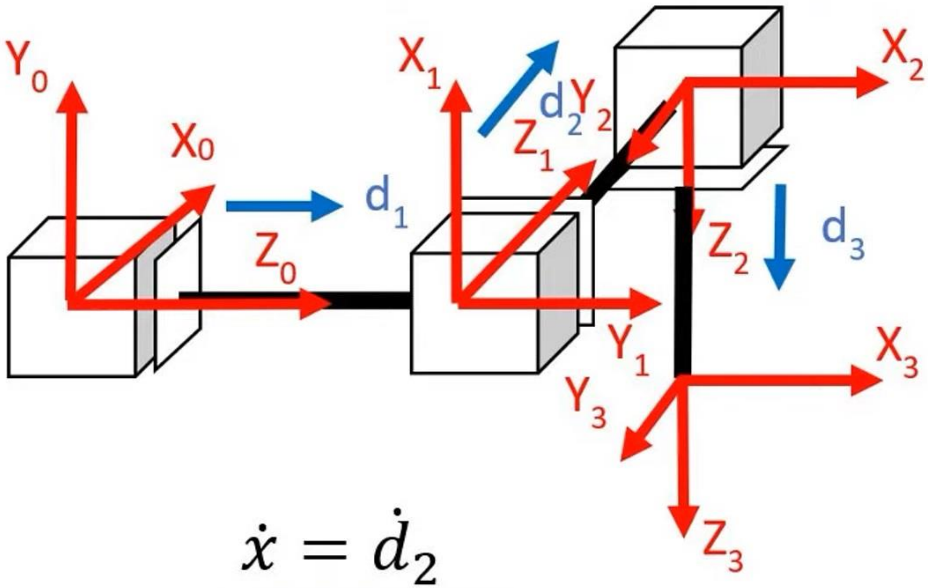
$$\dot{y} = -\dot{d}_3$$

$$\dot{z} = \dot{d}_1$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Cartesian manipulator example

- The velocity of the end effector in y direction opposite to the speed of third joint.



$$\omega_x = 0$$

$$\omega_y = 0$$

$$\omega_z = 0$$

$$\dot{x} = \dot{d}_2$$

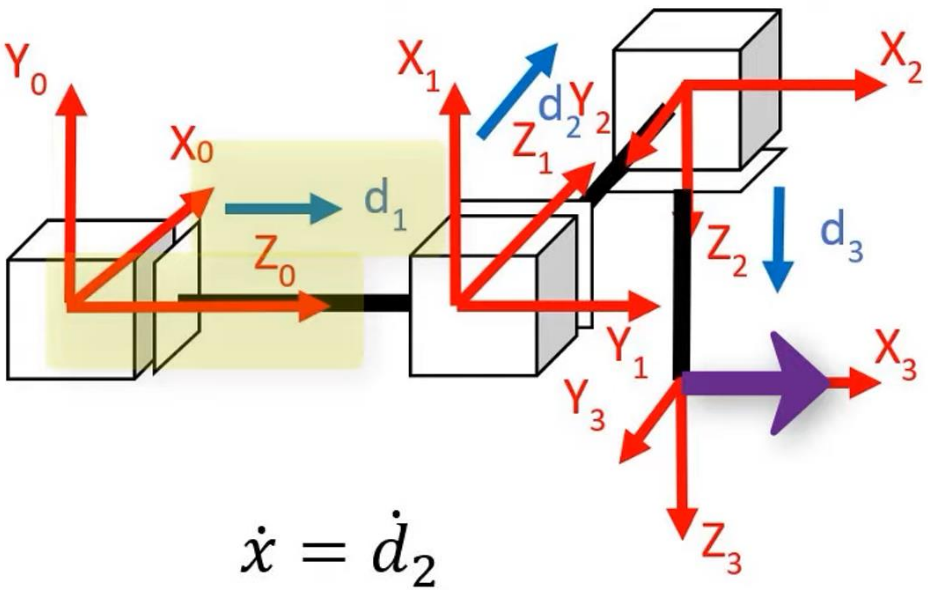
$$\dot{y} = -\dot{d}_3$$

$$\dot{z} = \dot{d}_1$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Cartesian manipulator example

- The speed of the end effector in z direction is only controlled by first joint.



$$\omega_x = 0$$

$$\omega_y = 0$$

$$\omega_z = 0$$

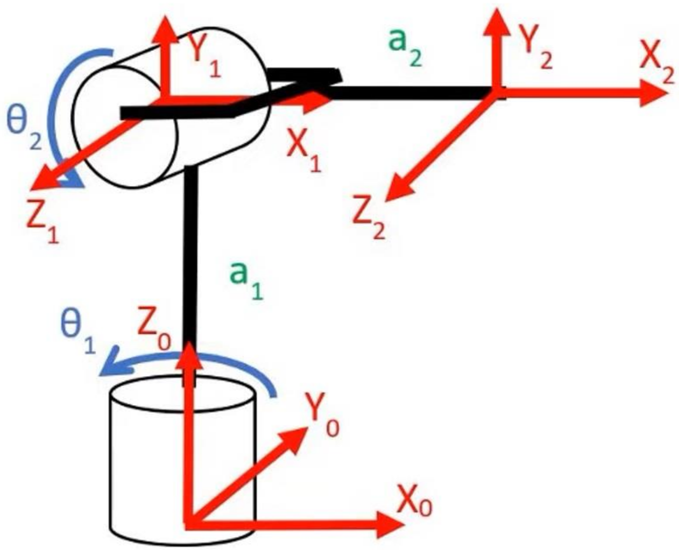
$$\dot{x} = \dot{d}_2$$

$$\dot{y} = -\dot{d}_3$$

$$\boxed{\dot{z} = \dot{d}_1}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

Articulated manipulator example

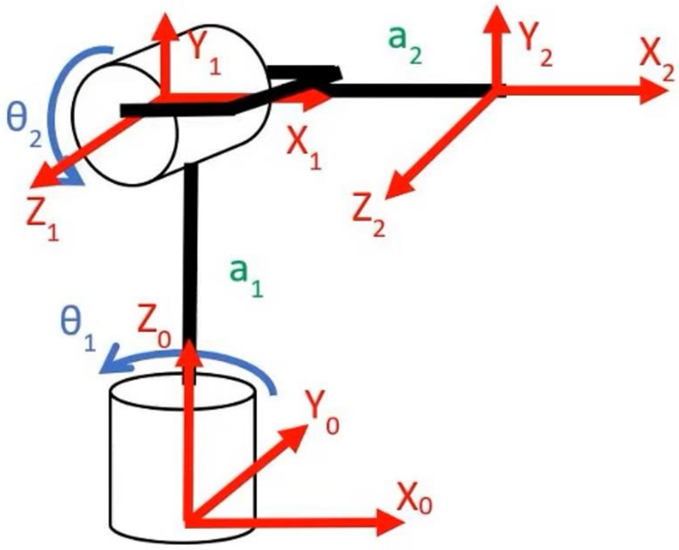


i=1
n=2

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix} = \begin{bmatrix}\square & \square\\\square & \square\\\square & \square\\\square & \square\\\square & \square\\\square & \square\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\end{bmatrix}$$

Articulated manipulator example

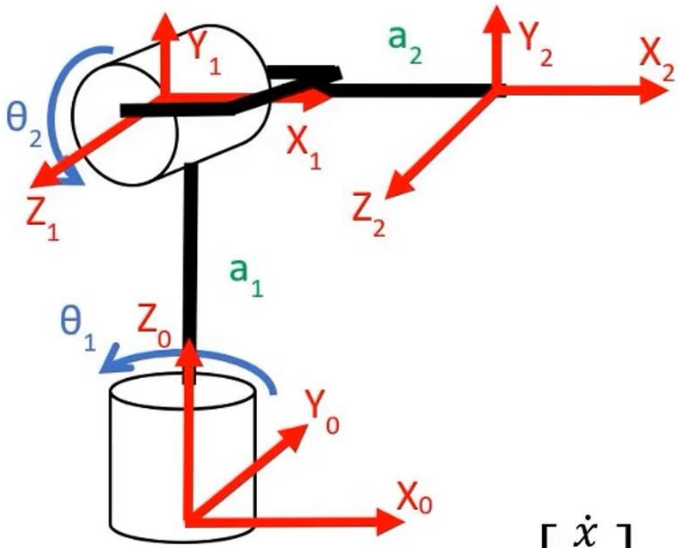


	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0-d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

i=1
n=2

$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix}=\begin{bmatrix}R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_2^0-d_0^0) & \square\\ & \square\\ & \square\\ R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix} & \square\\ & \square\\ & \square\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\end{bmatrix}$$

Articulated manipulator example



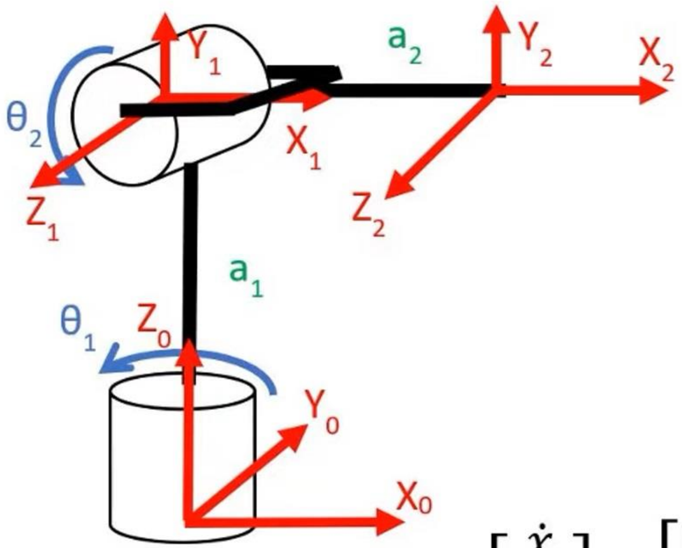
	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0-d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

i=2

n=2

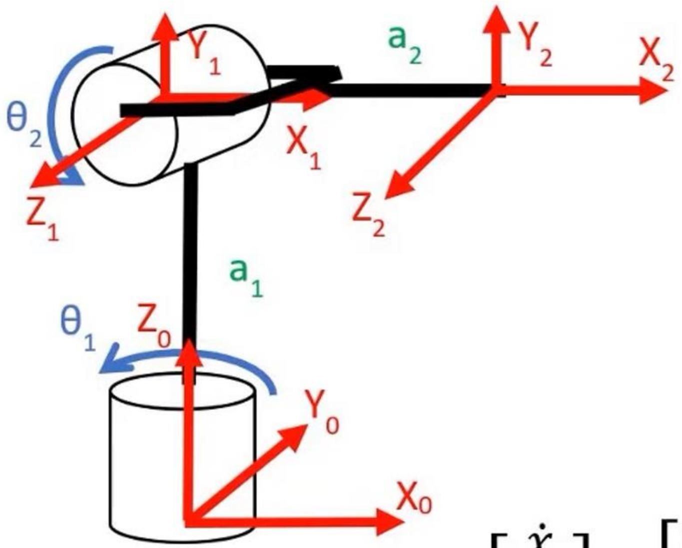
$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix}=\begin{bmatrix}R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_2^0-d_0^0) & R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_2^0-d_1^0)\\ R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\end{bmatrix}$$

Articulated manipulator example



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_2^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_2^0 - d_1^0) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

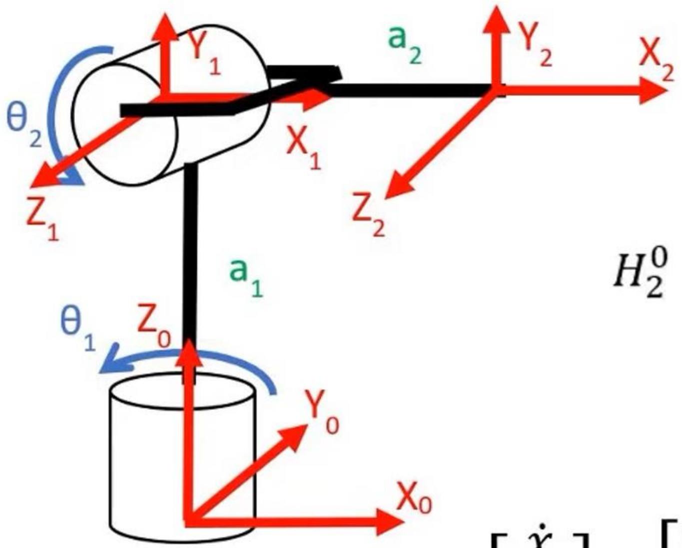
Articulated manipulator example



$$H_2^0 = \begin{bmatrix} R_2^0 & & d_2^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_2^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_2^0 - d_1^0) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

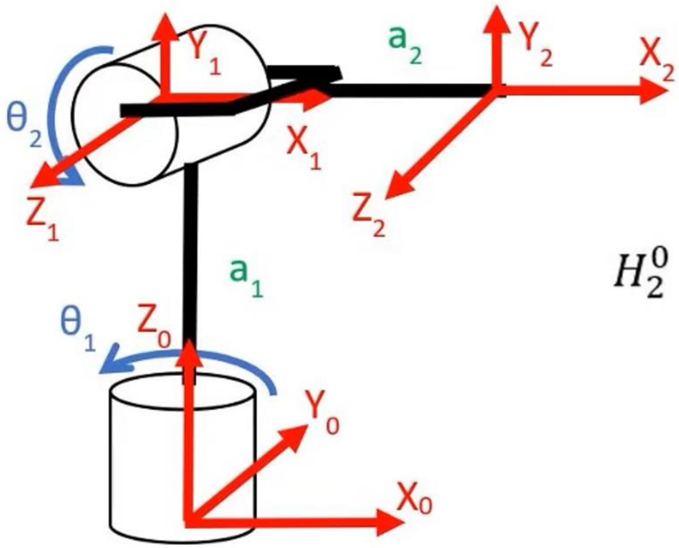
Articulated manipulator example



$$H_2^0 = \begin{bmatrix} C\theta_1 C\theta_2 & -C\theta_1 S\theta_2 & S\theta_1 & a_2 C\theta_1 C\theta_2 \\ S\theta_1 C\theta_2 & -S\theta_1 S\theta_2 & -C\theta_1 & a_2 S\theta_1 C\theta_2 \\ 0 & C\theta_2 & 0 & a_2 S\theta_2 + a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_2^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_2^0 - d_1^0) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

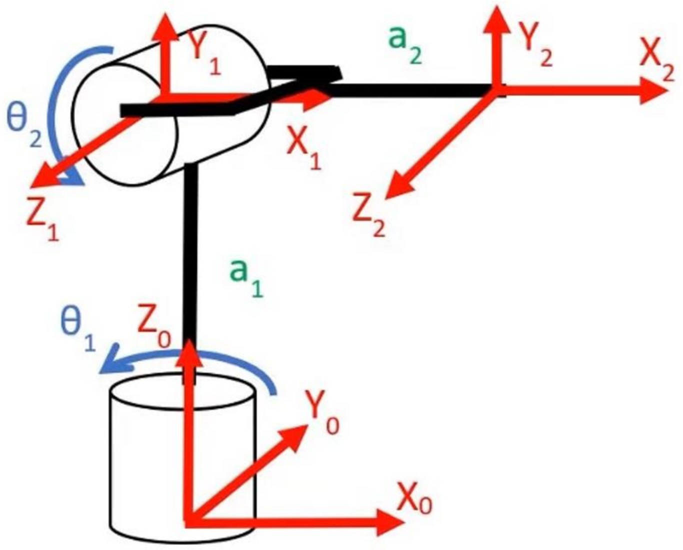
Articulated manipulator example



$$H_2^0 = \begin{bmatrix} C\theta_1 C\theta_2 & -C\theta_1 S\theta_2 & S\theta_1 & a_2 C\theta_1 C\theta_2 \\ S\theta_1 C\theta_2 & -S\theta_1 S\theta_2 & -C\theta_1 & a_2 S\theta_1 C\theta_2 \\ 0 & C\theta_2 & 0 & a_2 S\theta_2 + a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

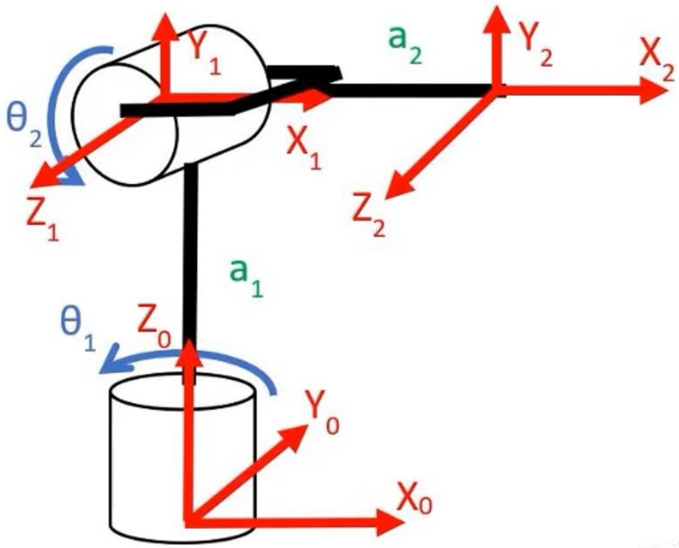
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} a_2 C\theta_1 C\theta_2 \\ a_2 S\theta_1 C\theta_2 \\ a_2 S\theta_2 + a_1 \end{bmatrix} - d_0^0 \right) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} a_2 C\theta_1 C\theta_2 \\ a_2 S\theta_1 C\theta_2 \\ a_2 S\theta_2 + a_1 \end{bmatrix} - d_1^0 \right) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Articulated manipulator example



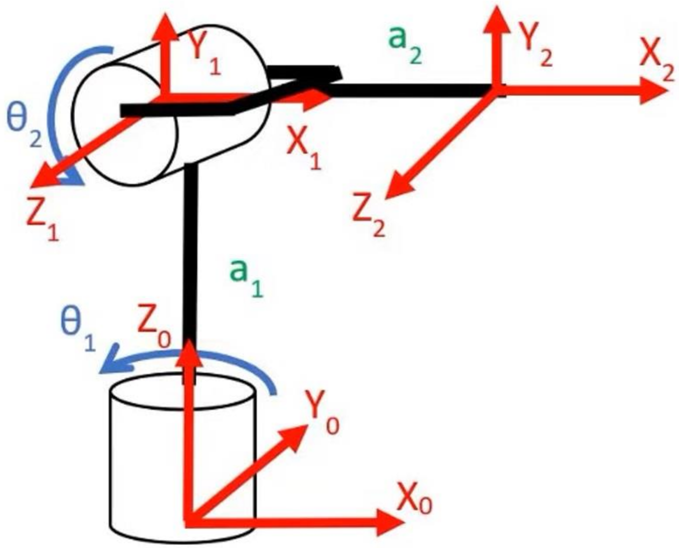
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} a_2 C\theta_1 C\theta_2 \\ a_2 S\theta_1 C\theta_2 \\ a_2 S\theta_2 + a_1 \end{bmatrix} \right) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} a_2 C\theta_1 C\theta_2 \\ a_2 S\theta_1 C\theta_2 \\ a_2 S\theta_2 + a_1 \end{bmatrix} - d_1^0 \right) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Articulated manipulator example



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{matrix} -a_2 S\theta_1 C\theta_2 \\ a_2 C\theta_1 C\theta_2 \\ 0 \end{matrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} a_2 C\theta_1 C\theta_2 \\ a_2 S\theta_1 C\theta_2 \\ a_2 S\theta_2 + a_1 \end{bmatrix} - d_1^0 \right) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

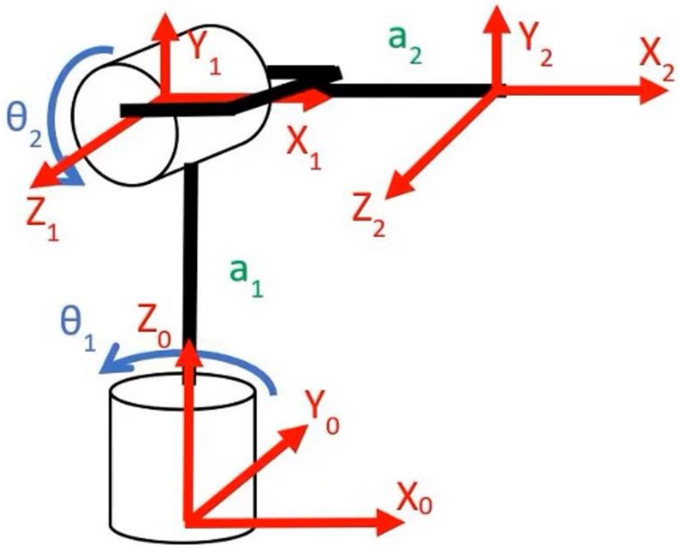
Articulated manipulator example



$$d_1^0 = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix}$$

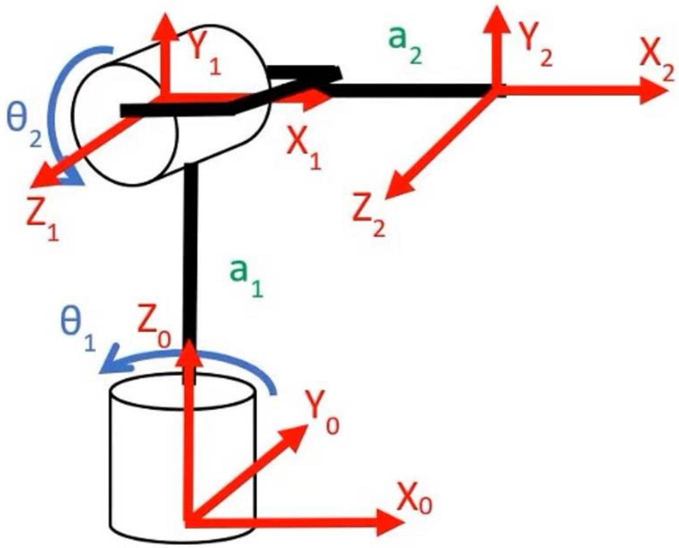
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -a_2 S\theta_1 C\theta_2 & -a_2 C\theta_1 S\theta_2 \\ a_2 C\theta_1 C\theta_2 & -a_2 S\theta_1 S\theta_2 \\ 0 & 2a_2 C\theta_2 \\ 0 & \\ 0 & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ 1 & \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Articulated manipulator example



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -a_2 S\theta_1 C\theta_2 & -a_2 C\theta_1 S\theta_2 \\ a_2 C\theta_1 C\theta_2 & -a_2 S\theta_1 S\theta_2 \\ 0 & 2a_2 C\theta_2 \\ 0 & S\theta_1 \\ 0 & -C\theta_1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Articulated manipulator example



$$\dot{x} = -a_2 S\theta_1 C\theta_2 \dot{\theta}_1 - a_2 C\theta_1 S\theta_2 \dot{\theta}_2$$

$$\dot{y} = a_2 C\theta_1 C\theta_2 \dot{\theta}_1 - a_2 S\theta_1 S\theta_2 \dot{\theta}_2$$

$$\dot{z} = a_2 C\theta_2 \dot{\theta}_2$$

$$\omega_x = S\theta_1 \dot{\theta}_2$$

$$\omega_y = -C\theta_1 \dot{\theta}_2$$

$$\omega_z = \dot{\theta}_1$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -a_2 S\theta_1 C\theta_2 & -a_2 C\theta_1 S\theta_2 \\ a_2 C\theta_1 C\theta_2 & -a_2 S\theta_1 S\theta_2 \\ 0 & 2a_2 C\theta_2 \\ 0 & S\theta_1 \\ 0 & -C\theta_1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

End