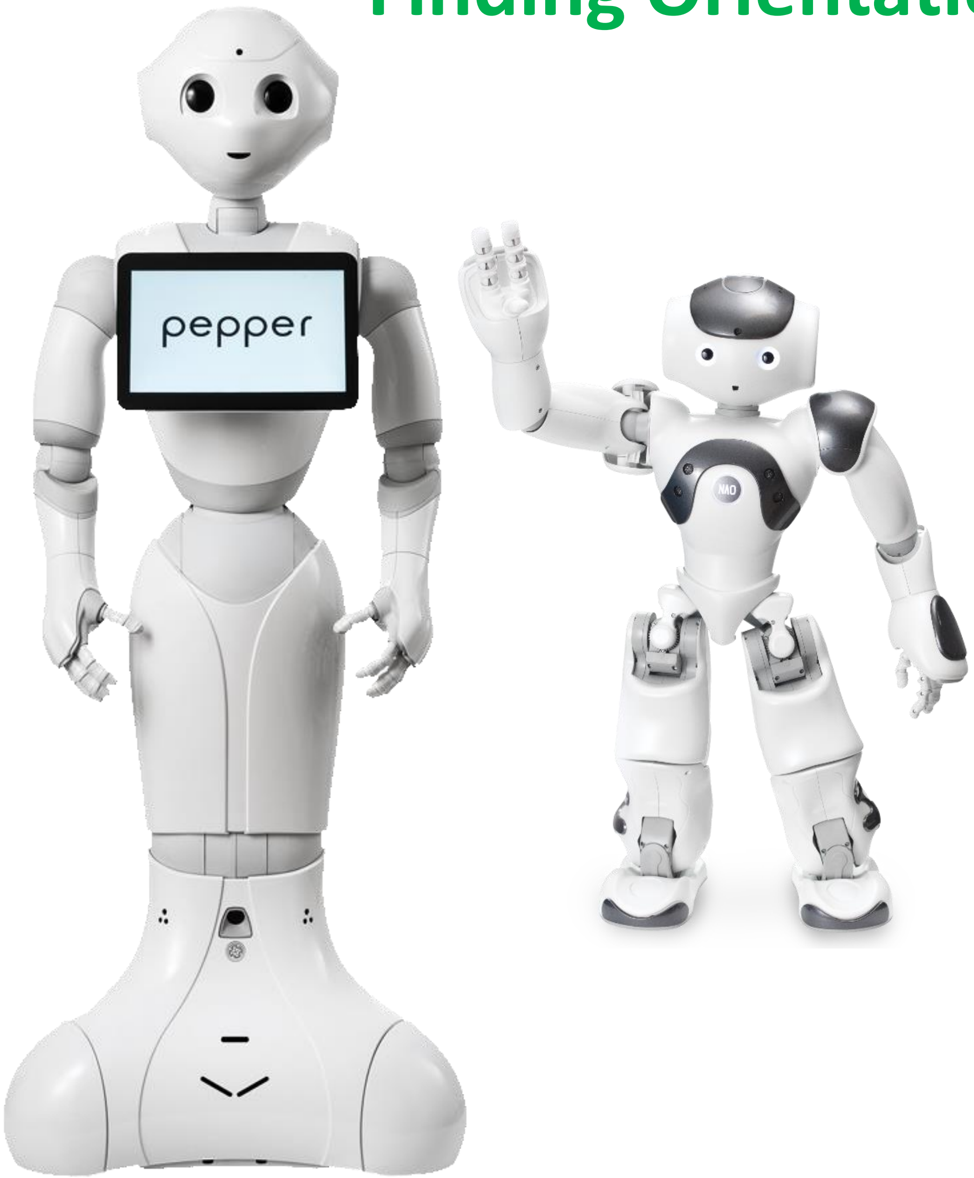
Fundamentals of Robotics

Robot Manipulators 06

Finding Orientation of a robot



Forward kinematics

Inverse Kinematics

Inverse kinematics for 2 DoF (R+P) manipulator

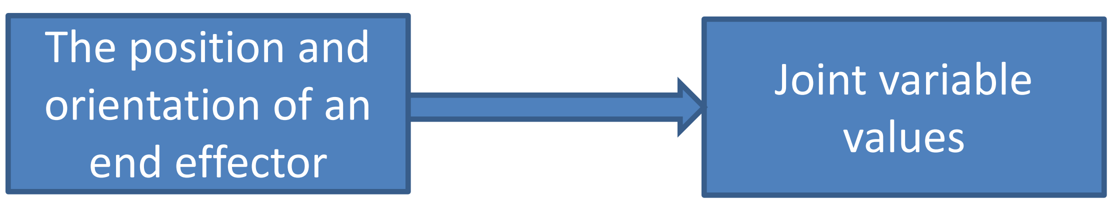
Inverse kinematics for 2R manipulator

Forward Kinematics

- When you know the joint variables of a joint and want to know the position and orientation of the end effector is called forward kinematics.
- Finding position of end-effector for given joint parameters & link dimensions
- e.g,. z = 0 is the board (when manipulator is mounted on a cardboard) in case of grid board experiment. Z = 0 means the end effector would collide with the board surface. Therefore, we need to avoid such joint variable values.

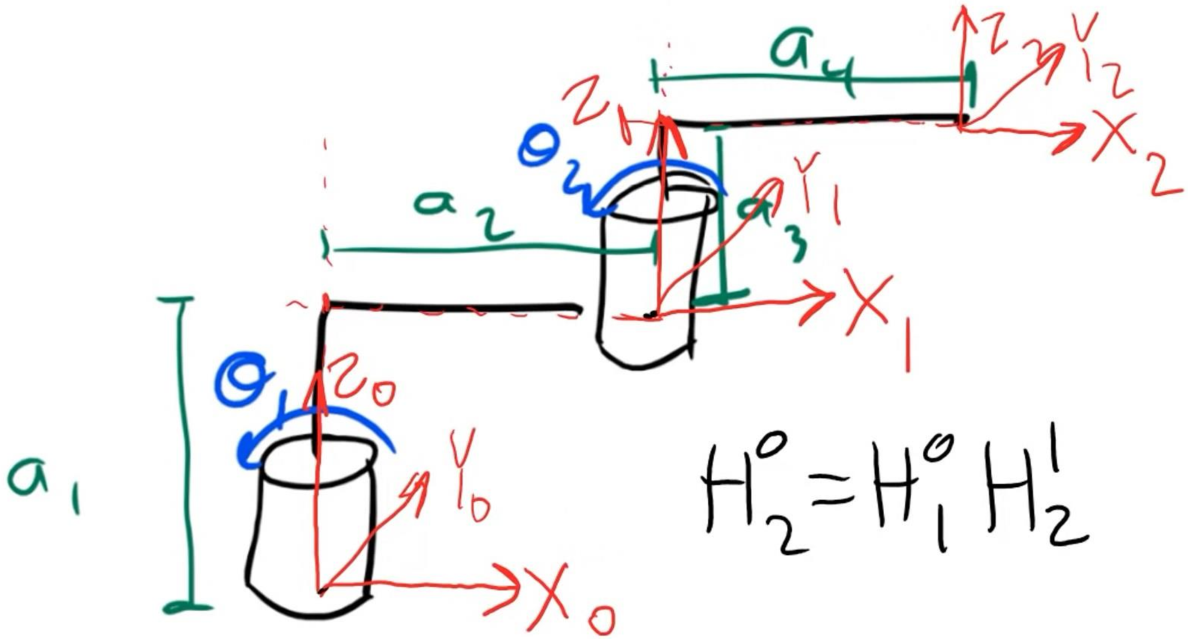
Inverse Kinematics

- We know the position and orientation of an end effector. However, we need to find the joint values that are required to reach the given position and orientation.
- Finding joint parameters, while end-effector position & link dimensions are provided
- Inverse kinematics is relatively difficult problem compare to forward.



Inverse Kinematics 2R manipulator

- We know 2R manipulator homogeneous matrix works by multiplying individual homogenous transformation matrices in sequence.



Inverse Kinematics 2R manipulator

$$H^0_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_2 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_2 S\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H^1_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_4 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_4 S\theta_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H^0_2 = \begin{bmatrix} C\theta_1 C\theta_2 - S\theta_1 S\theta_2 & -C\theta_1 S\theta_2 - S\theta_1 C\theta_2 & 0 & a_4 C\theta_1 C\theta_2 - a_4 S\theta_1 S\theta_2 + a_2 C\theta_1 \\ S\theta_1 C\theta_2 + C\theta_1 S\theta_2 & -S\theta_1 S\theta_2 + C\theta_1 C\theta_2 & 0 & a_4 S\theta_1 C\theta_2 + a_4 C\theta_1 S\theta_2 + a_2 S\theta_1 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Kinematics 2R manipulator

- In H_2^0, the upper left part of matrix is rotation matric.
- The upper right part represents displacement vector.

$$H_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_2 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_2 s\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2^1 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_4 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_4 s\theta_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} c\theta_1 c\theta_2 - s\theta_1 s\theta_2 & -c\theta_1 s\theta_2 - s\theta_1 c\theta_2 & 0 & a_4 c\theta_1 c\theta_2 - a_4 s\theta_1 s\theta_2 + a_2 c\theta_1 \\ s\theta_1 c\theta_2 + c\theta_1 s\theta_2 & -s\theta_1 s\theta_2 + c\theta_1 c\theta_2 & 0 & a_4 s\theta_1 c\theta_2 + a_4 c\theta_1 s\theta_2 + a_2 s\theta_1 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Kinematics 2R manipulator

- We create non-linear system of equations.
- To solve this system, we need numerical method.
- There is another easy method to solve these called graphical method

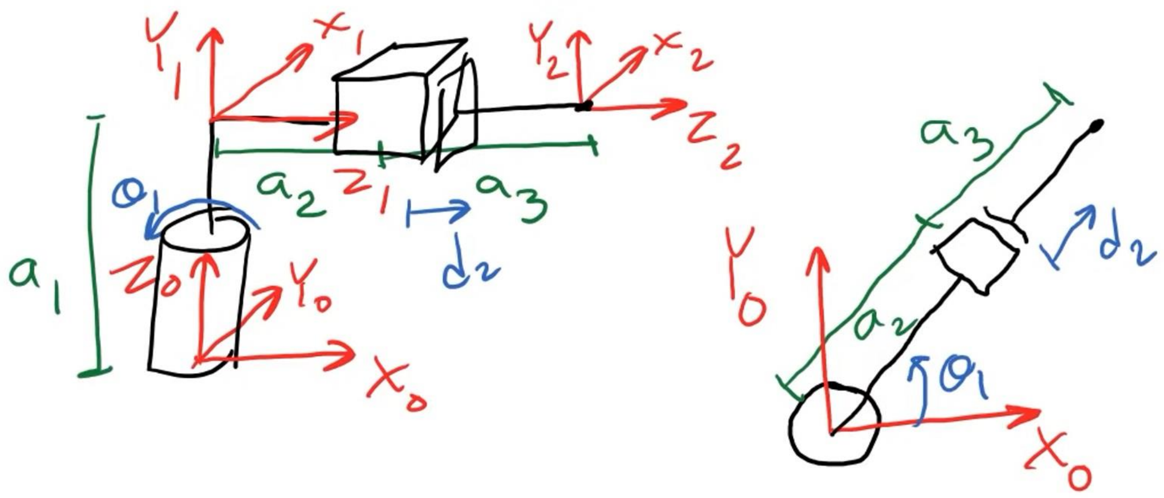
$$\begin{bmatrix} a_4 c\theta_1 c\theta_2 - a_4 s\theta_1 s\theta_2 + a_2 c\theta_1 \\ a_4 s\theta_1 c\theta_2 + a_4 c\theta_1 s\theta_2 + a_2 s\theta_1 \\ a_3 \\ 1 \end{bmatrix}$$

$$X_2^0 = a_4 c\theta_1 c\theta_2 - a_4 s\theta_1 s\theta_2 + a_2 c\theta_1$$

$$Y_2^0 = a_4 s\theta_1 c\theta_2 + a_4 c\theta_1 s\theta_2 + a_2 s\theta_2$$

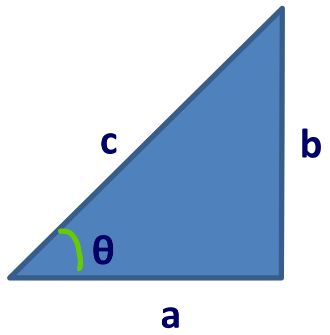
Inverse Kinematics 2DoF manipulator

- In graphical method, we draw top or side view of a kinematic diagram.
- In this case, we take top view.
- We also create angle of 45 degree to make method feasible for various equations.



Inverse Kinematics 2DoF manipulator

- Before we dugout more into this problem, we refresh few concepts.
- Pythagorean theorem is used with right angled triangle:
  - $a^2 + b^2 = c^2$
- SOHCAHTOA (Also need right angled triangle)
  - $\sin\theta = \frac{b}{c}$
  - $cos\,\theta = \frac{a}{c}$
  - $tan\,\theta = \frac{b}{a}$



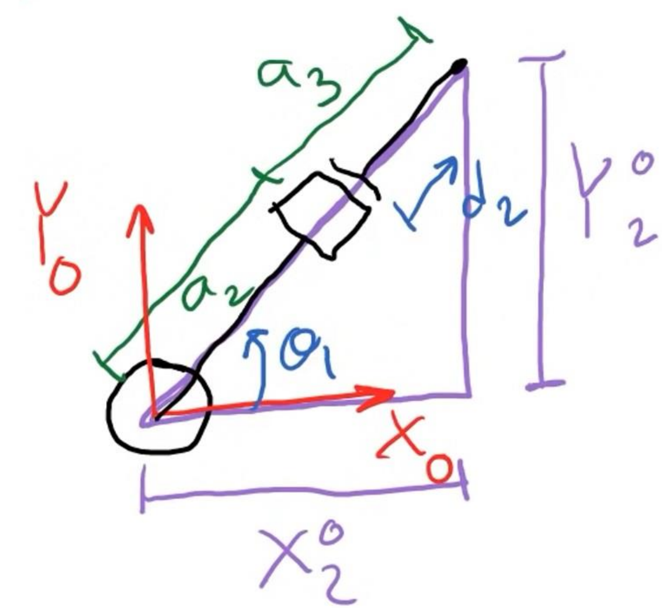
- Law of cosines:
  - This law doesn't need right angled triangle.
  - $a^2 = b^2 + c^2 - 2bc\,cos\,\theta$

Inverse Kinematics 2DoF manipulator

$$\begin{bmatrix} \boxed{\begin{matrix} a_4 C\theta_1 C\theta_2 - a_4 S\theta_1 S\theta_2 + a_2 C\theta_1 \\ a_4 S\theta_1 C\theta_2 + a_4 C\theta_1 S\theta_2 + a_2 S\theta_1 \\ a_3 \end{matrix}} \\ 1 \end{bmatrix}$$

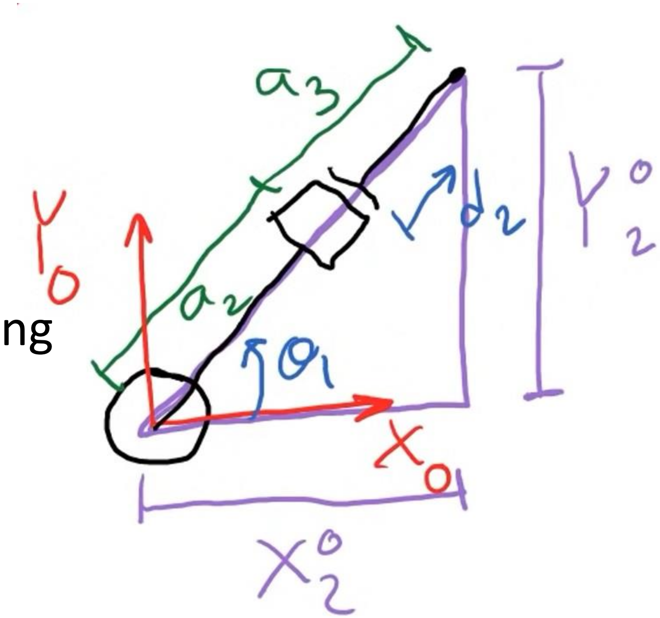
$$X_2^0 = a_4 C\theta_1 C\theta_2 - a_4 S\theta_1 S\theta_2 + a_2 C\theta_1$$

$$Y_2^0 = a_4 S\theta_1 C\theta_2 + a_4 C\theta_1 S\theta_2 + a_2 S\theta_2$$



Inverse Kinematics 2DoF manipulator

- The triangle is right angled
- We can used Pythagorean theorem
- In this way we have found an equation for finding



$$\begin{bmatrix} \boxed{\begin{matrix} a_4 C\theta_1 C\theta_2 - a_4 S\theta_1 S\theta_2 + a_2 C\theta_1 \\ a_4 S\theta_1 C\theta_2 + a_4 C\theta_1 S\theta_2 + a_2 S\theta_1 \\ a_3 \end{matrix}} \\ 1 \end{bmatrix}$$

$$X_2^0 = a_4 C\theta_1 C\theta_2 - a_4 S\theta_1 S\theta_2 + a_2 C\theta_1$$

$$Y_2^0 = a_4 S\theta_1 C\theta_2 + a_4 C\theta_1 S\theta_2 + a_2 S\theta_2$$

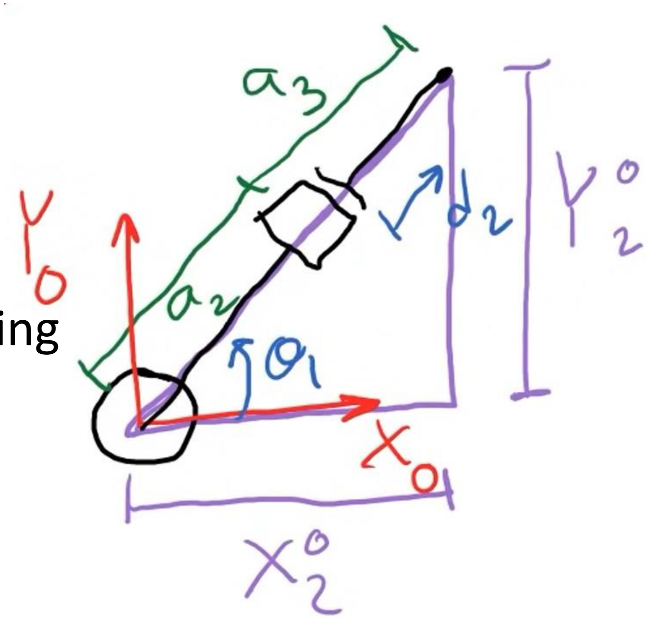
$$(X_2^0)^2 + (Y_2^0)^2 = (a_2 + a_3 + d_2)$$

$$a_2 + a_3 + d_2 = \sqrt{(X_2^0)^2 + (Y_2^0)^2}$$

$$d_2 = \sqrt{(X_2^0)^2 + (Y_2^0)^2} - a_2 - a_3$$

Inverse Kinematics 2DoF manipulator

- The triangle is right angled
- We can used Pythagorean theorem
- In this way we have found an equation for finding
  - d_2
  - θ_1



$$\tan\theta_1 = \frac{Y_2^0}{X_2^0}$$

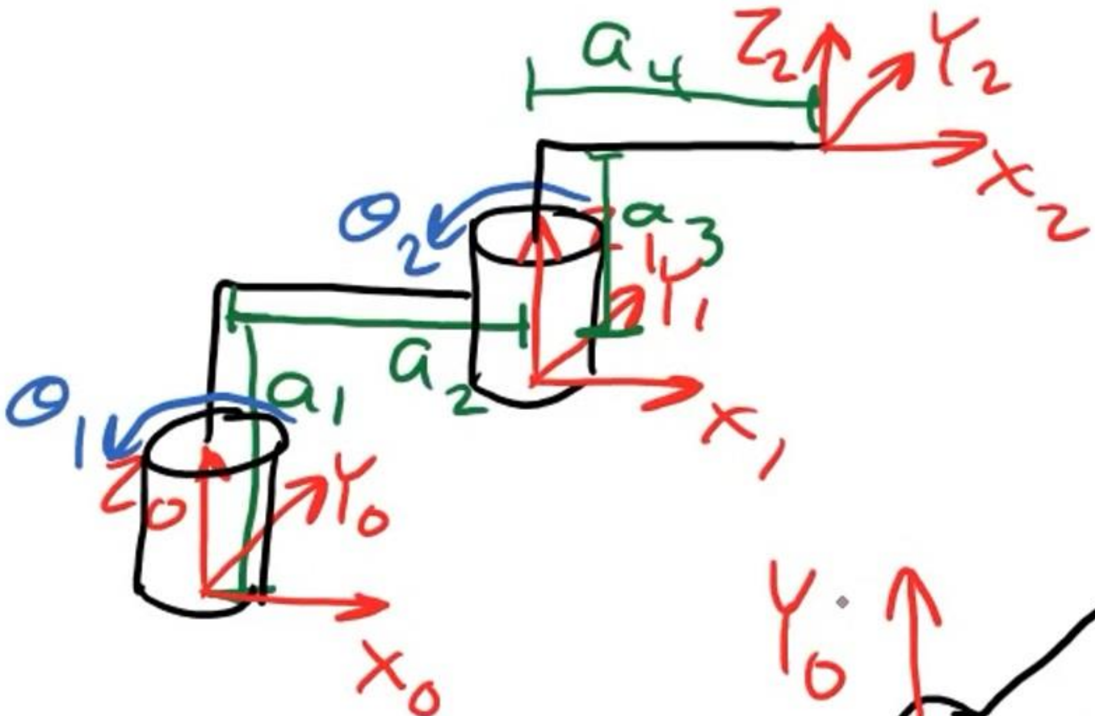
$$\boxed{\theta_1 = \tan^{-1}\left(\frac{Y_2^0}{X_2^0}\right)}$$

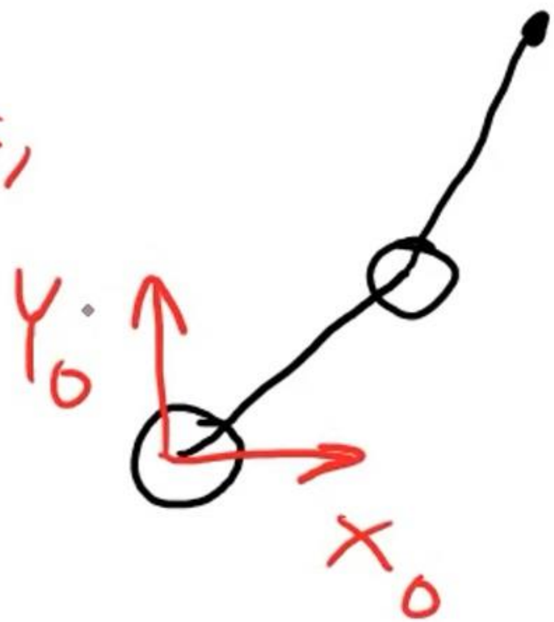
$$(X_2^0)^2 + (Y_2^0)^2 = (a_2 + a_3 + d_2)$$

$$a_2 + a_3 + d_2 = \sqrt{(X_2^0)^2 + (Y_2^0)^2}$$

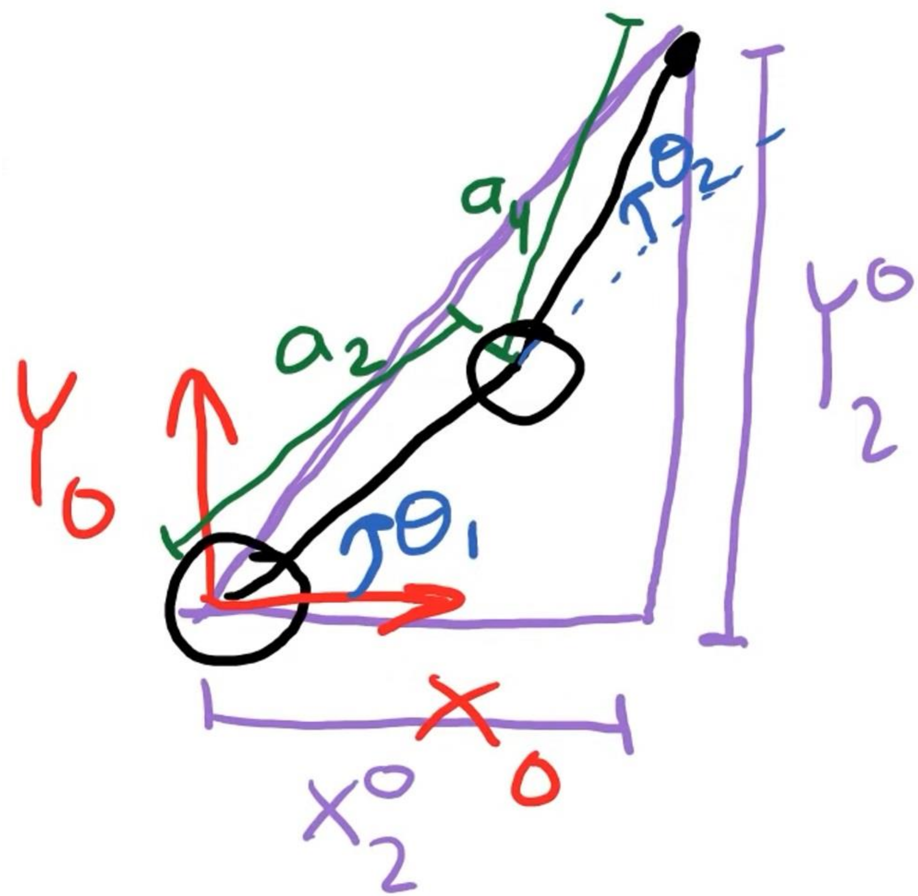
$$d_2 = \sqrt{(X_2^0)^2 + (Y_2^0)^2} - a_2 - a_3$$

Inverse Kinematics 2R manipulator

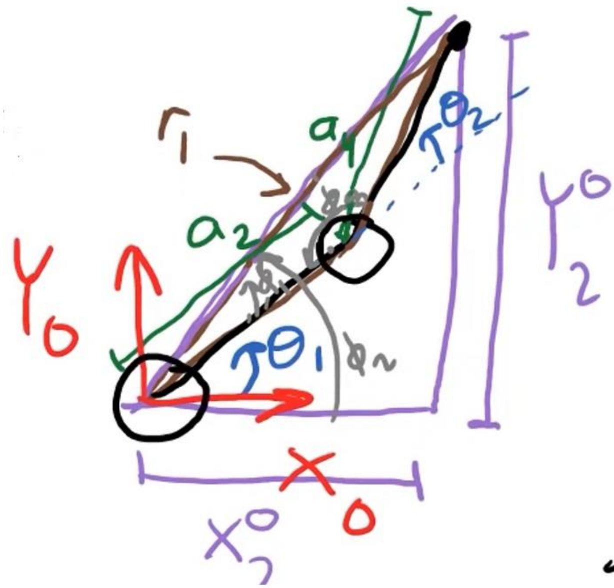




Inverse Kinematics 2R manipulator



Inverse Kinematics 2R manipulator



$$(X_2^0)^2 + (Y_2^0)^2 = r_1^2$$

$$\boxed{r_1 = \sqrt{(X_2^0)^2 + (Y_2^0)^2}} \quad (1)$$

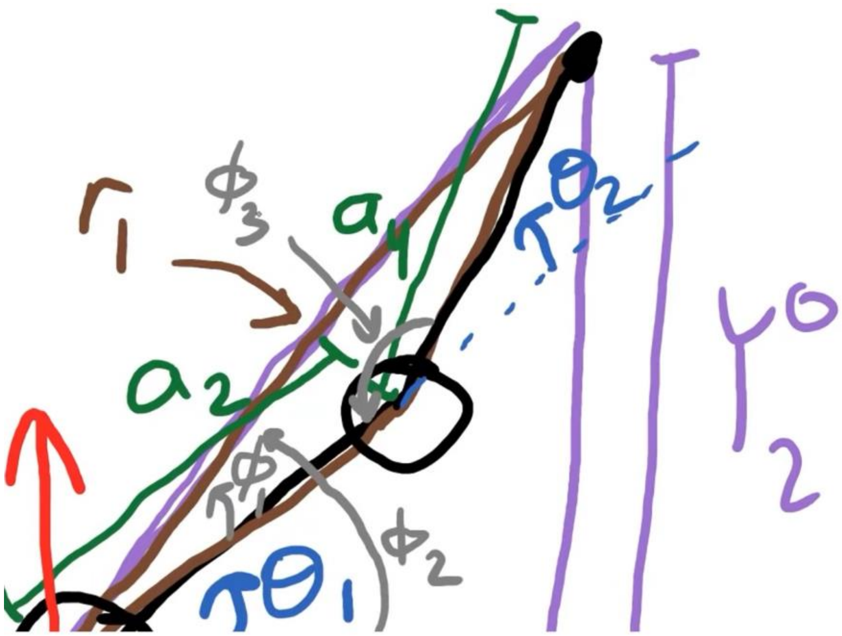
$$\theta_1 = \phi_2 - \phi_1$$

$$\tan\phi_2 = Y_2^0 / X_2^0$$

$$a_4^2 = a_2^2 + r_1^2 - 2a_2 r_1 \cos\phi_1$$

$$\phi_1 = \cos^{-1}\left(\frac{a_4^2 - a_2^2 - r_1^2}{-2a_2 r_1}\right) \qquad \phi_2 = \tan^{-1}\left(Y_2^0 / X_2^0\right)$$

Inverse Kinematics 2R manipulator



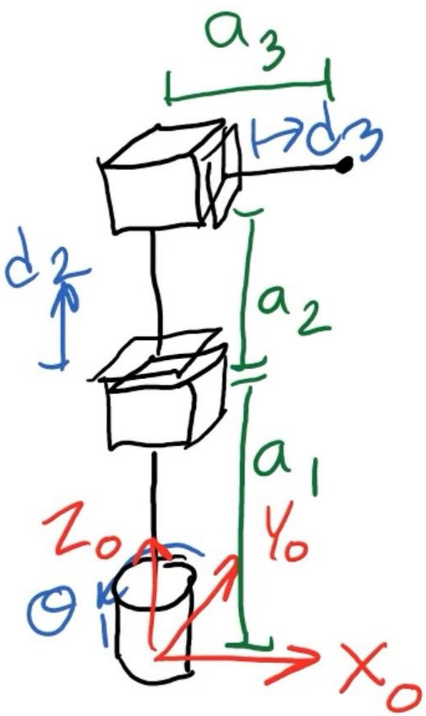
$$\phi_3 + \theta_2 = 180$$

$$\theta_2 = 180 - \phi_3$$

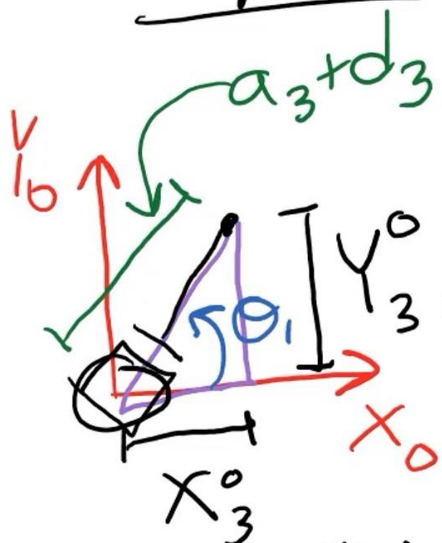
$$r_1^2 = a_2^2 + a_4^2 - 2a_2a_4\cos\phi_3$$

$$\boxed{\phi_3 = \cos^{-1}\left(\frac{r_1^2 - a_2^2 - a_4^2}{-2a_2a_4}\right)}$$

Inverse Kinematics 3 DoF manipulator



top view



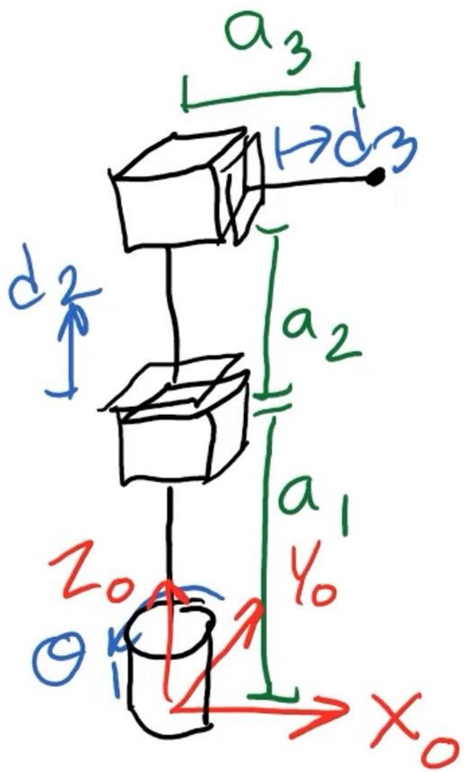
$$\boxed{\theta_1 = \tan^{-1}\left(\frac{Y_3^0}{X_3^0}\right)} \quad (1)$$

$$(X_3^0)^2 + (Y_3^0)^2 = (a_3 + d_3)^2$$

$$a_3 + d_3 = \sqrt{(X_3^0)^2 + (Y_3^0)^2}$$

$$\boxed{d_3 = \sqrt{(X_3^0)^2 + (Y_3^0)^2} - a_3} \quad (2)$$

Inverse Kinematics 3 DoF manipulator



side view

$$Z^0_3 = a_1 + d_2 + a_2$$

$$\boxed{d_2 = Z^0_3 - a_1 - a_2} \quad (3)$$

End