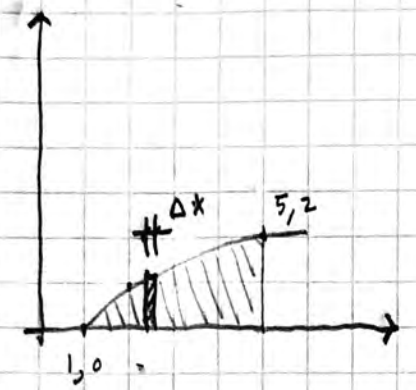
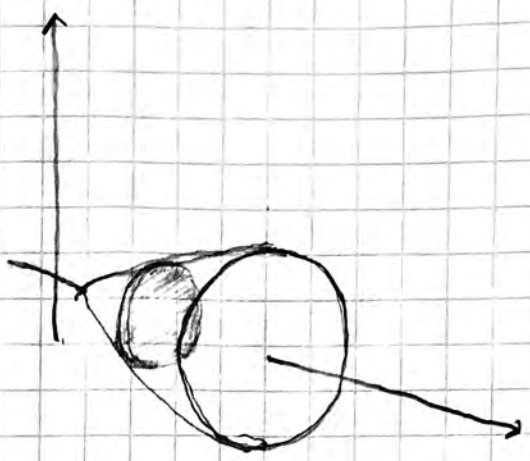
6.2.3 $y=\sqrt{x-1}$, $y=0$, $x=5$, about x axis





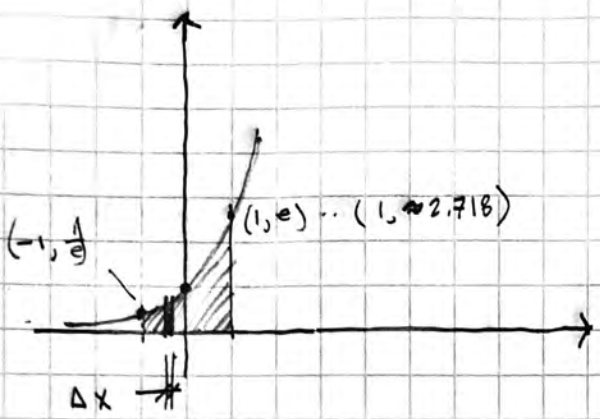
$$A(x) = \pi y^2 = \pi\left(\sqrt{x-1}\right)^2 = \pi(x-1) = \pi x - \pi$$

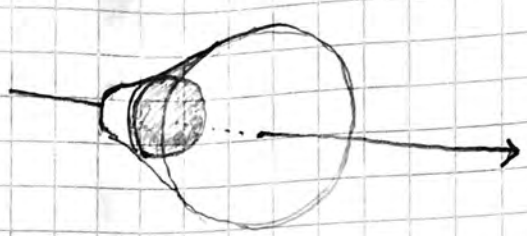
$$V = \int_1^5 A(x)\,dx = \int_1^5 \pi(x-1)\,dx = \pi\int_1^5 (x-1)\,dx$$

$$= \pi\left[\frac{x^2}{2} - x\right]_1^5 = \pi\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right] = \pi\left(\frac{25}{2} - \frac{10}{2} - \frac{1}{2} + \frac{2}{2}\right)$$

$$= \pi\left(\frac{16}{2}\right) = 8\pi$$

6.2.4 $y = e^x$, $y = 0$, $x = -1$, $x = 1$, about the x axis





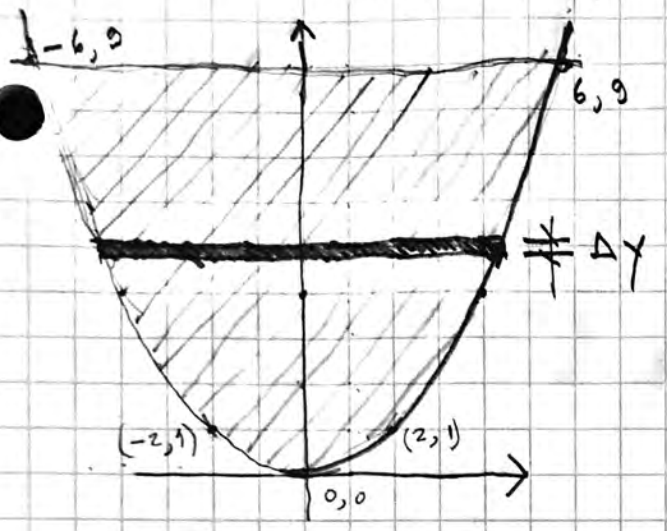
$$A(x) = \pi y^2 = \pi e^{2x}$$

$$V = \int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} \pi e^{2x} = \pi \int (e^{2x})\,dx$$

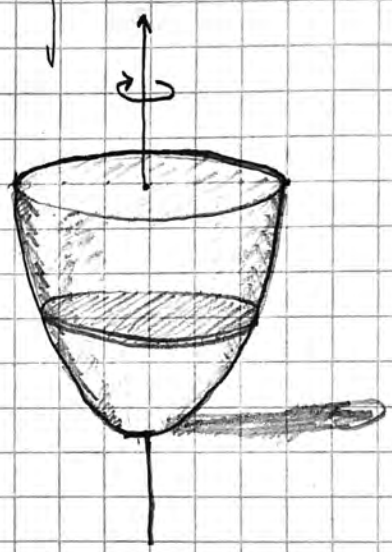
$$= \pi \left[\frac{e^{2x}}{2} \right]_{-1}^{1}$$

$$= \pi (e^2 - e^{-2})$$

6.2.5 $x = 2\sqrt{y}$, $x = 0$, $y = 9$, about y axis



x	y
0	0
± 2	1
± 4	4
± 6	9



$$A = \pi r^2 = \pi (2\sqrt{y})^2 = 4y$$

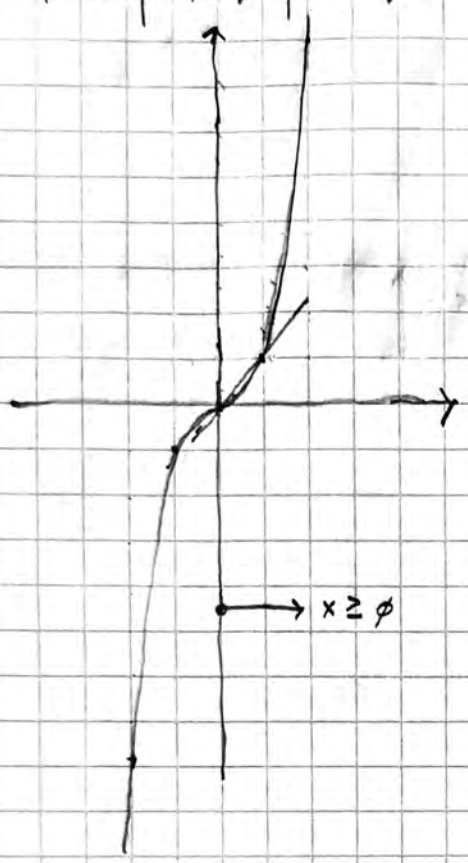
$$V = \int_0^9 (\pi\, 4y)\, dy$$

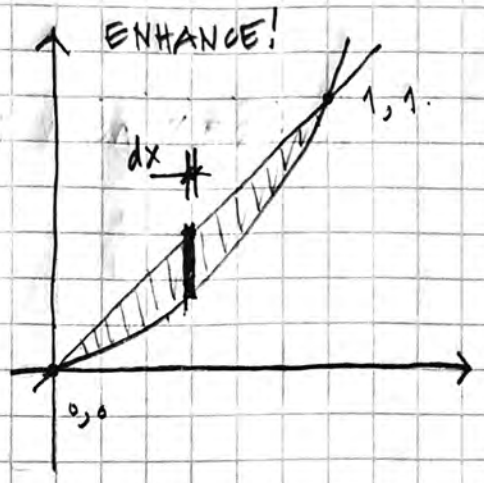
$$= \pi \int_0^9 (4y)\, dy$$

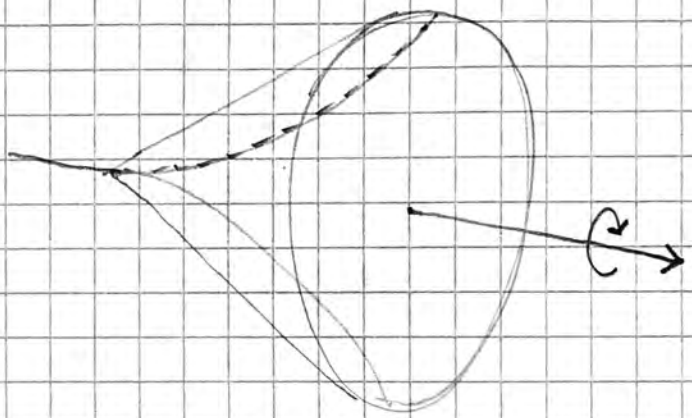
$$= 4\pi \int_0^9 y\, dy$$

$$= 4\pi \left[\frac{y^2}{2}\right]_0^9 = 4\pi \frac{81}{2} = 162\pi$$

6.2.7 $y = x^3$, $y = x$, $x \geq 0$, about x axis.





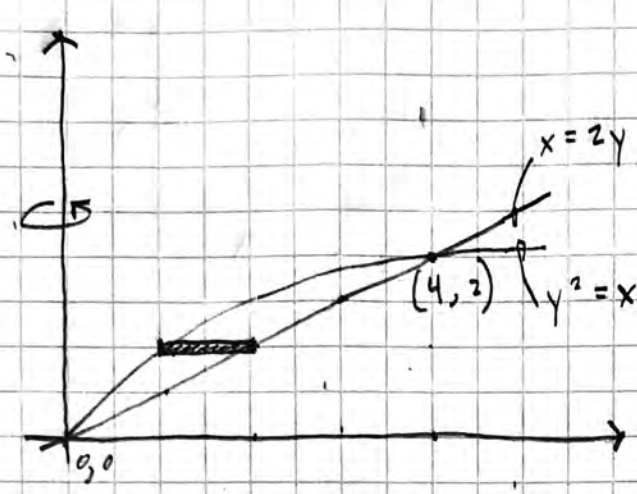


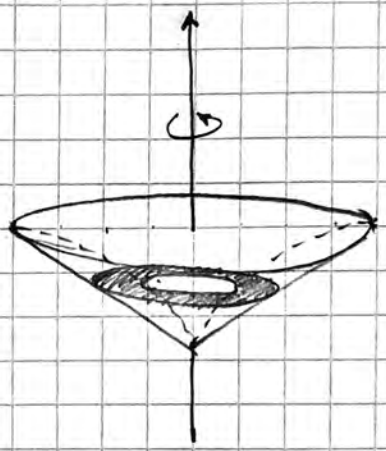
$$A = \pi x^2 - \pi x^6$$

$$V = \int_0^1 \pi x^2 - \pi x^6$$

$$= \left[\frac{\pi x^3}{3} - \frac{\pi x^7}{7}\right]_0^1 = \frac{7\pi(1^3)}{21} - \frac{3\pi(1^7)}{21} = \frac{4\pi}{21}$$

6.2.9 $y^2 = x$, $x = 2y$, about y





$$A = \pi(2y)^2 - \pi(y^2)^2$$

$$= \pi 4y^2 - \pi y^4$$

$$= \pi(4y^2 - y^4)$$

$$V = \int_0^2 \pi(4y^2 - y^4)\,dy = \pi \int_0^2 (4y^2 - y^4)\,dy$$

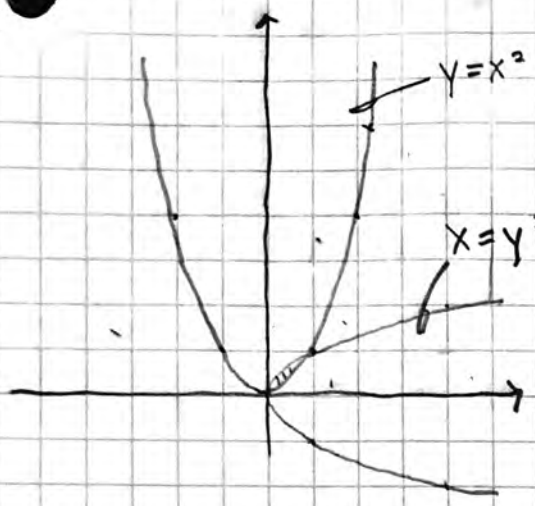
$$= \pi\left[\frac{4y^3}{3} - \frac{y^5}{5}\right]_0^2$$

$$= \pi\left(\frac{4(8)}{3} - \frac{32}{5}\right) = \pi\left(\frac{32}{3} - \frac{32}{5}\right)$$

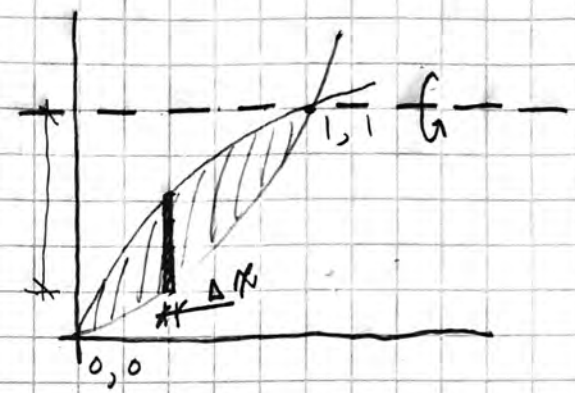
$$= \pi\left(\frac{160}{15} - \frac{96}{15}\right) = \frac{64\pi}{15}$$

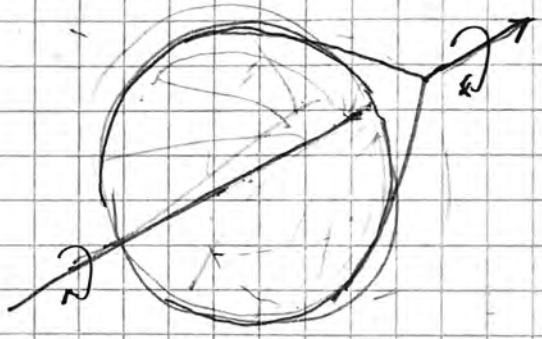
6.2.11 $y = x^2$, $x = y^2$, about $y = 1$

$x = \sqrt{y}$ $y = \sqrt{x}$



ENHANCE!





$A = \pi r_{out}^2 - \pi r_{in}^2$

$= \pi (1 - x^2)^2 - \pi (1 - \sqrt{x})^2$

$= \pi (1 + x^4 - 2x^2 - 1 - x + 2\sqrt{x})$

$= \pi (x^4 - 2x^2 - x + 2\sqrt{x})$

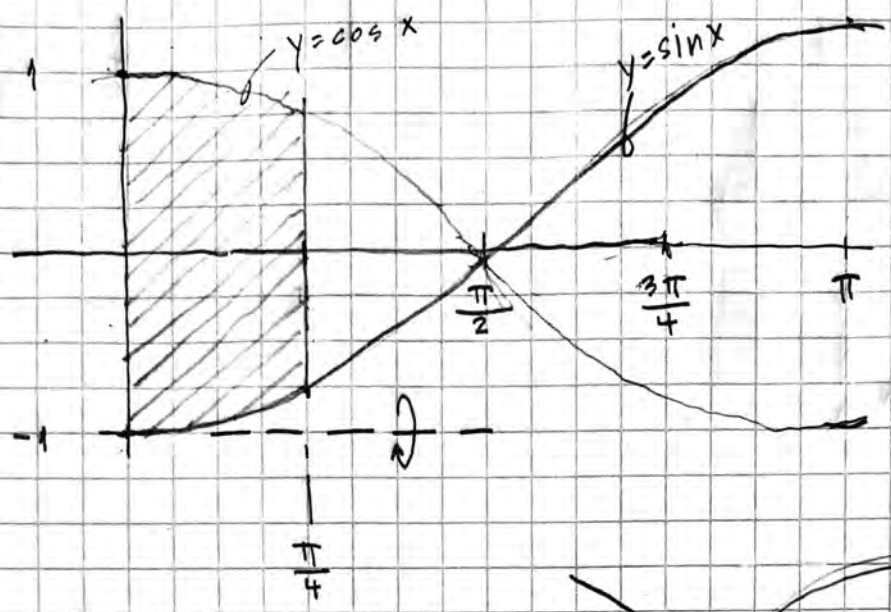
$V = \pi \int_0^1 (x^4 - 2x^2 - x + 2\sqrt{x})\, dx$

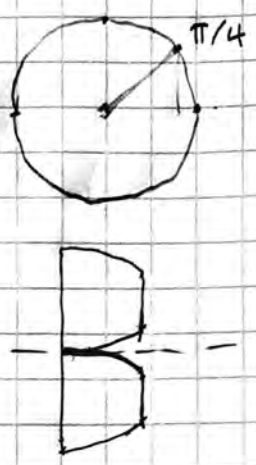
$= \pi \left[\frac{x^5}{5} - \frac{2x^3}{3} - \frac{x^2}{2} + \frac{4x^{3/2}}{3} \right]_0^1$

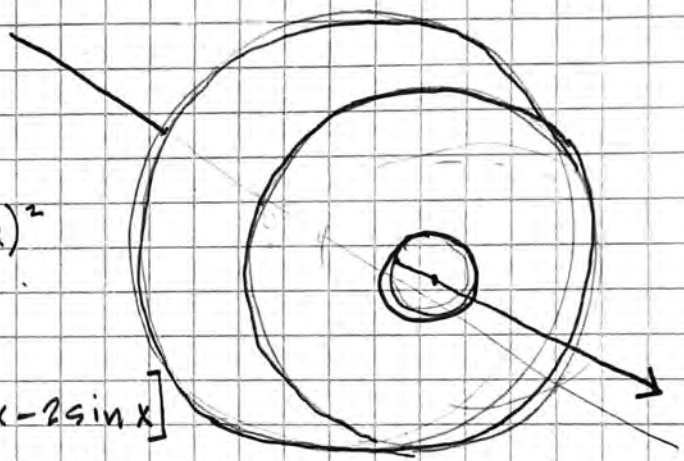
$= \pi \left[\left(\frac{1^5}{5} - \frac{2(1)^3}{3} - \frac{1^2}{2} + \frac{4(1)^{3/2}}{3} \right) - \emptyset \right]$

$= \pi \left[\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3} \right] = \pi \left[\frac{6}{30} - \frac{20}{30} - \frac{15}{30} + \frac{40}{30} \right] = \frac{11\pi}{30}$

6.2.14 $y = \sin x$, $y = \cos x$, $0 \le x \le \frac{\pi}{4}$, about $y = -1$







$$A = \pi r_{out}^2 - \pi r_{in}^2$$

$$= \pi(-1-\cos x)^2 - \pi(-1-\sin x)^2$$

$$= \pi\left[(1+\cos x)^2 - (1+\sin x)^2\right.$$

$$= \pi\left[1+\cos^2 x + 2\cos x - 1 - \sin^2 x - 2\sin x\right]$$

$$= \pi\left[\cos^2 x - \sin^2 x + 2\cos x - 2\sin x\right]$$

$$V = \pi \int_0^{\pi/4} \left[\cos^2 x - \sin^2 x + 2\cos x - 2\sin x\right] dx$$

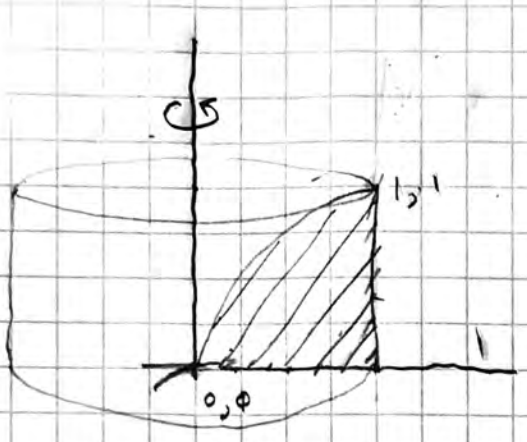
$$= \pi\left[\frac{\sin 2x}{2} + 2\sin x + 2\cos x\right]_0^{\pi/4}$$

$$= \pi\left[\left(\frac{\sin \pi/2}{2} + 2\sin \pi/4 + 2\cos \pi/4\right) - \left(\frac{\sin 0}{2} + 2\sin 0 + 2\cos 0\right)\right]$$

$$= \pi\left[\left(\frac{1}{2} + \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right) - (0+0+2)\right] = \pi\left[\frac{1}{2} + 2\sqrt{2} - 2\right] = \frac{(4\sqrt{2}-3)\pi}{2}$$

6.3.3 $y = \sqrt[3]{x}$, $y = 0$, $x = 1$ $\quad x = y^3$

x	y
0	0
1	1
2	8



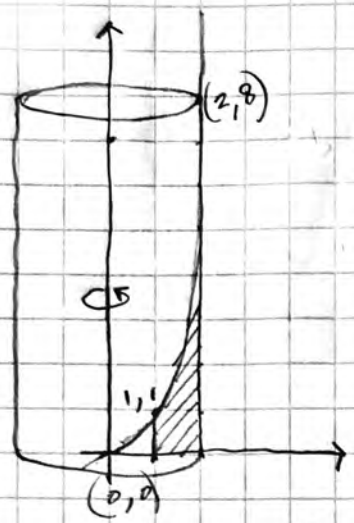
$$V = \int_0^1 2\pi x(\sqrt[3]{x})\,dx$$

$$= 2\pi \int_0^1 \left(x^{4/3}\right) dx$$

$$= \left[2\pi\left(\frac{x^{7/3}}{7/3}\right)\right]_0^1 = \left[2\pi\left(\frac{3x^{7/3}}{7}\right)\right]_0^1 = \frac{6\pi}{7}(1-0)$$

$$= \frac{6\pi}{7}$$

6.3.4 $y = x^3$, $y = 0$, $x = 1$, $x = 2$



$$V = \int_1^2 2\pi x(x^3)\,dx$$

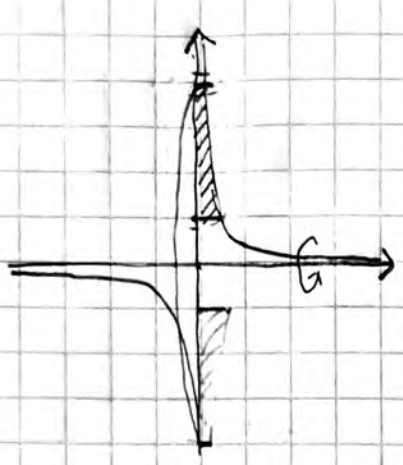
$$= 2\pi \int_1^2 x^4\,dx$$

$$= 2\pi \left[\frac{x^5}{5}\right]_1^2 = 2\pi \left[\frac{2^5}{5} - \frac{1^5}{5}\right]$$

$$= 2\pi \left[\frac{32}{5} - \frac{1}{5}\right]$$

$$= 2\pi \left(\frac{31}{5}\right) = \frac{62\pi}{5}$$

6.3.9. $xy = 1$, $x = 0$, $y = 1$, $y = 3$



$$V = \int_1^3 2\pi y \left(\frac{1}{y}\right) dy$$

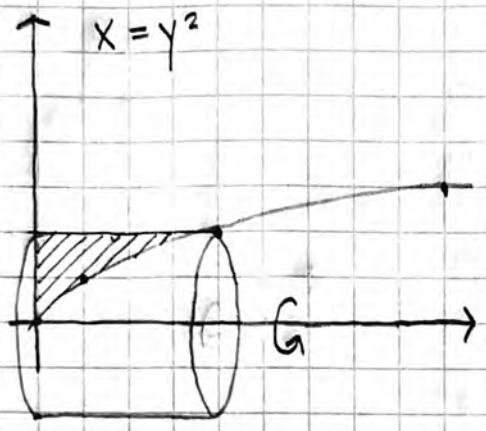
$$= 2\pi \int_1^3 dy$$

$$= 2\pi [y]_1^3$$

$$= 2\pi [3 - 1]$$

$$= 2\pi (2) = 4\pi$$

6.3.10 $y = \sqrt{x}$, $x = 0$, $y = 2$



$$V = \int_0^2 2\pi y (y^2)\, dy$$

$$= 2\pi \int_0^2 (y^3)\, dy$$

$$= 2\pi \left[\frac{y^4}{4} \right]_0^2$$

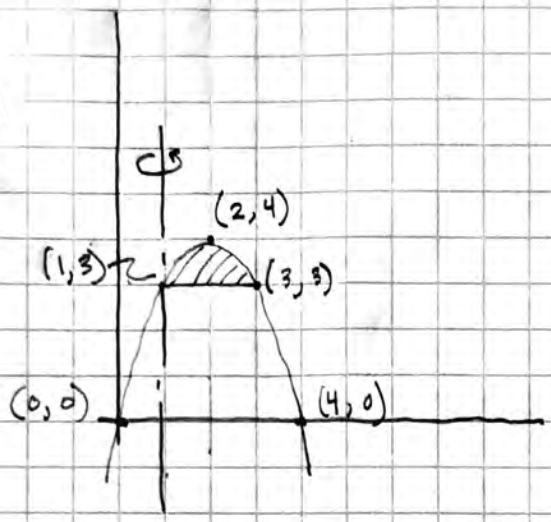
$$= 2\pi \left(\frac{2^4}{4} - 0 \right)$$

$$= 2\pi \left(\frac{16}{4} \right)$$

$$= 2\pi (4)$$

$$= 8\pi$$

6.3.17 $y = 4x - x^2$, $y = 3$, about $x = 1$



$$V = \int_1^3 2\pi (x-1)(4x - x^2 - 3)\,dx$$

$$= 2\pi \int_1^3 (4x^2 - x^3 - 3x - 4x + x^2 + 3)\,dx$$

$$= 2\pi \int_1^3 (-x^3 + 5x^2 - 7x + 3x)\,dx$$

$$= 2\pi \left[-\frac{x^4}{4} + \frac{5x^3}{3} - \frac{7x^2}{2} + 3x \right]_1^3$$

$$= 2\pi \left[\left(-\frac{3^4}{4} + \frac{5(3)^3}{3} - \frac{7(3)^2}{2} + 3(3) \right) - \left(-\frac{1^4}{4} + \frac{5(1)^3}{3} - \frac{7(1)^2}{2} + 3(1) \right) \right]$$

$$= 2\pi \left[\left(-\frac{81}{4} + \frac{135}{3} - \frac{63}{2} + 9 \right) - \left(-\frac{1}{4} + \frac{5}{3} - \frac{7}{2} + 3 \right) \right]$$

$$= 2\pi \left(\frac{9}{4} - \frac{11}{12} \right) = 2\pi \left(\frac{4}{3} \right) = \frac{8\pi}{3}$$