SCIENTIFIC SECTION

Mathematics

Applications

By a group of supervisors

Interactive exams using QR CODE

EXERCISES

FIRST TERM

2 SEC.

2024

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Mathematics

Applications

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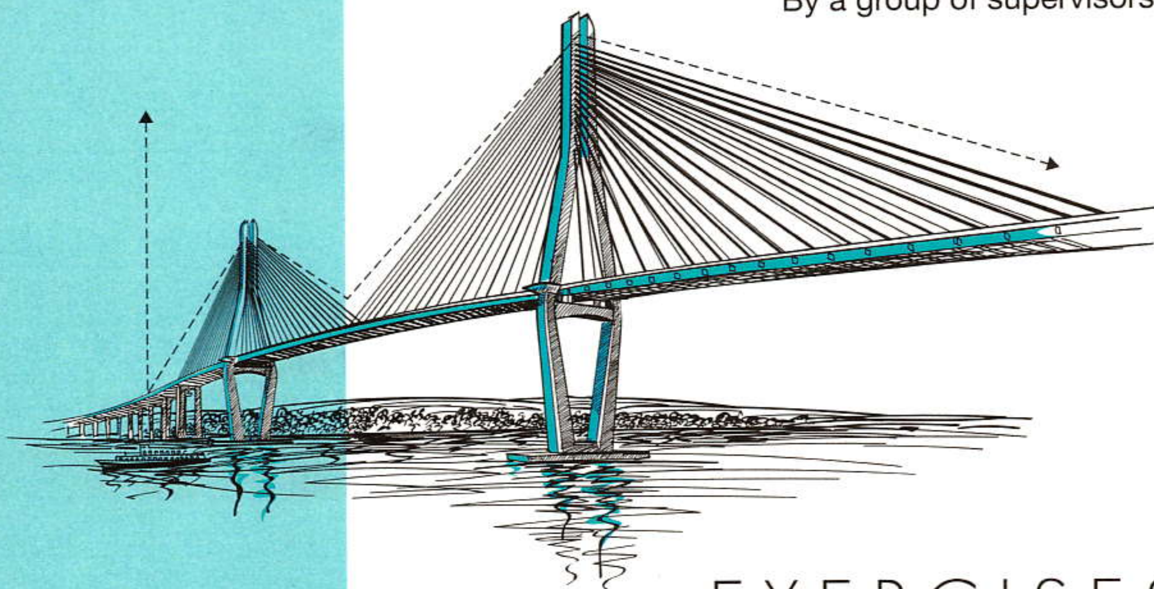

EXERCISES

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Preface

Thanks to God who helped us to introduce one of our famous series **"El Moasser"** in mathematics.

We introduce this book to our colleagues.

We also introduce it to our students to help them study mathematics.

In fact, this book is the outcome of more than thirty years experience in the field of teaching mathematics.

This book will make students aware of all types of questions.

We would like to know your opinions about the book hoping that it will win your admiration.

We will be grateful if you send us your recommendations and your comments.

The Authors

Solve

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QR Code

1

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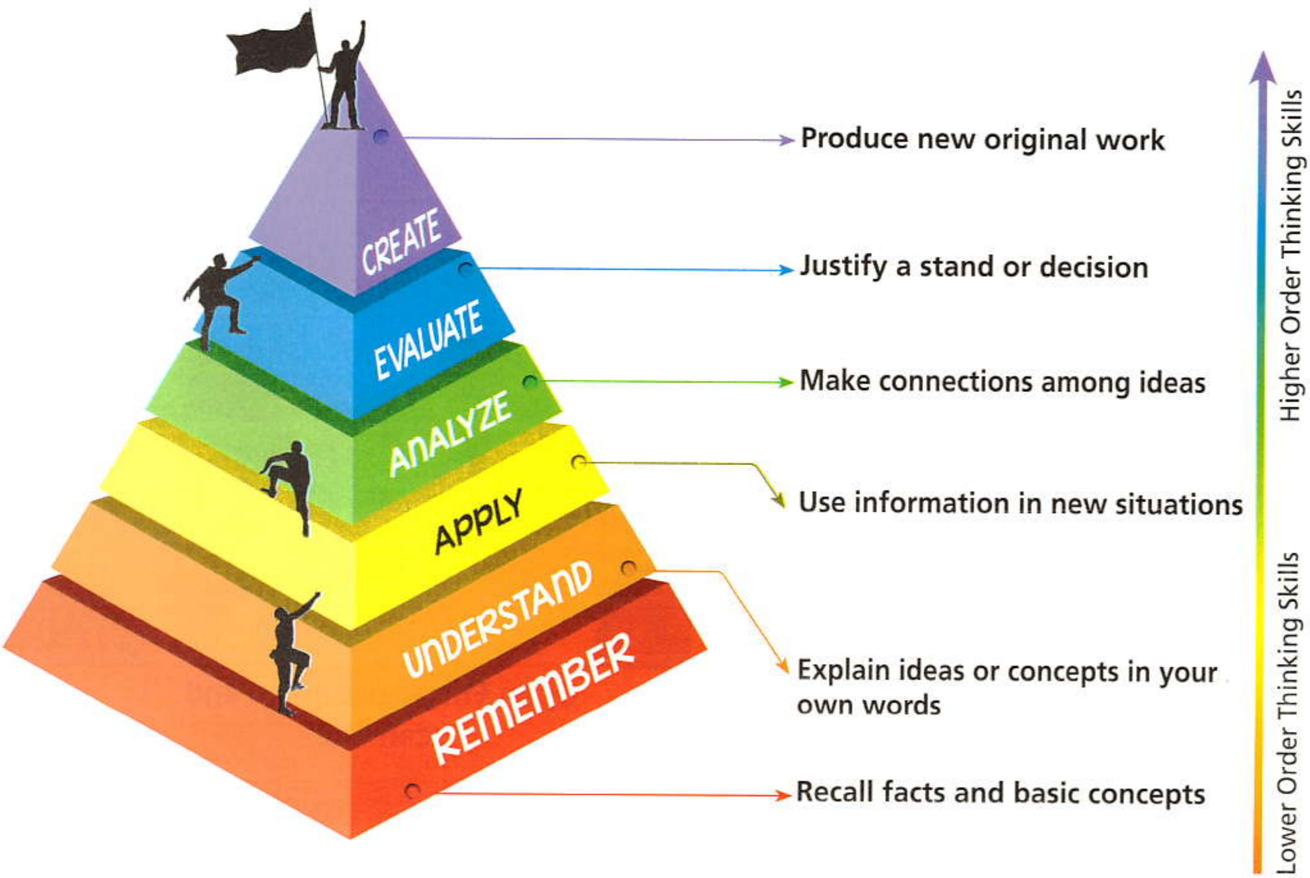
Bloom's Taxonomy Of Cognitive Levels

Bloom's Taxonomy is an educational classification created by Benjamin Bloom, it is often represented as a pyramid. This taxonomy was revised to include six cognitive levels graded form the lower level to the higher level as follows:

REMEMBER → UNDERSTAND → APPLY → ANALYZE → EVALUATE → CREATE



Bloom's Revised Pyramid

Note :

The questions within each exercise are classified according to the levels of Bloom's pyramid and are referred to as follows:

● REMEMBER ● UNDERSTAND ● APPLY ● PROBLEM SOLVING (ANALYZE - EVALUATE - CREATE)

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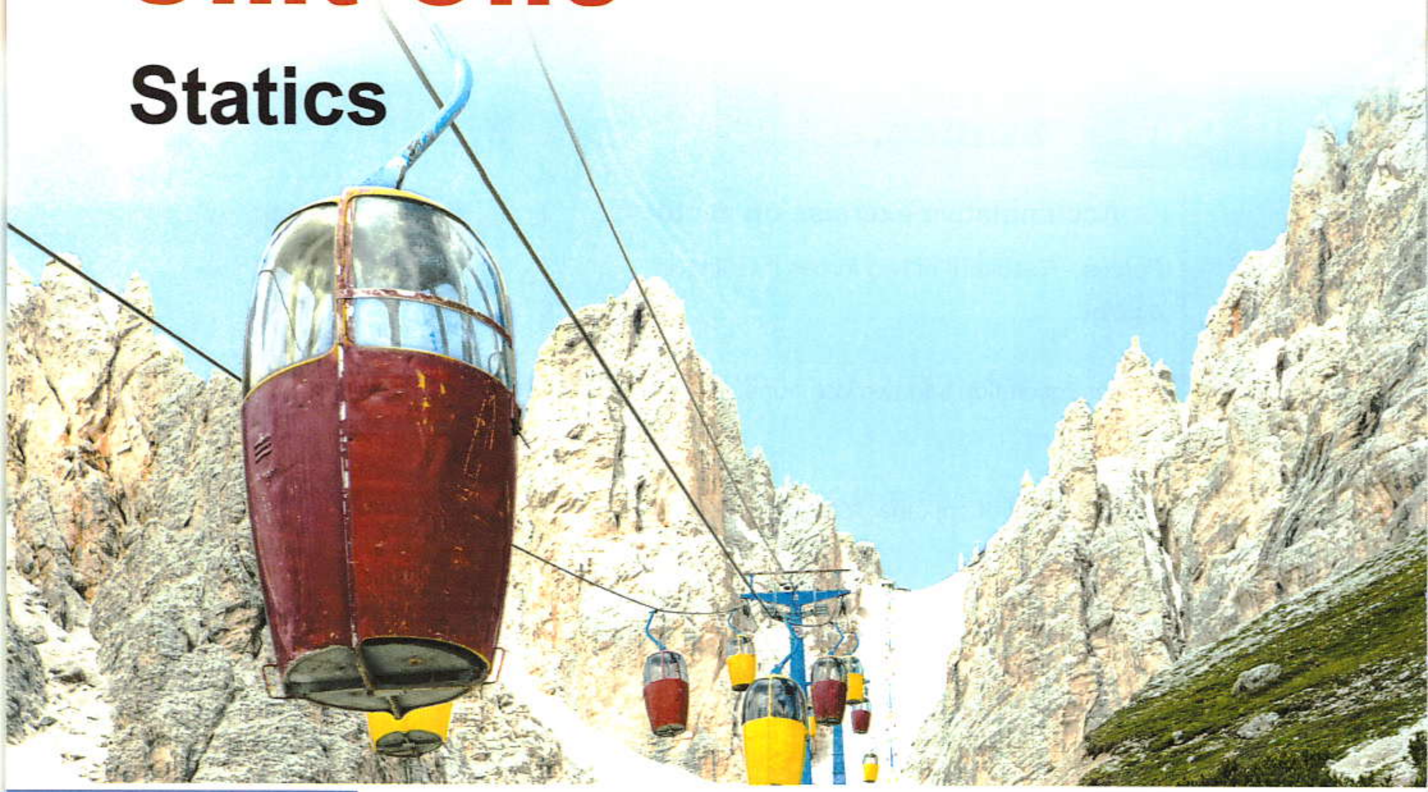
Unit One

Statics



• Accumulative exercise on vectors.

Exercise 1 Forces - Resultant of two forces meeting at a point.

Exercise 2 Forces resolution into two components.

Exercise 3 The resultant of coplanar forces meeting at a point.

Exercise 4 Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point.
(The triangle of forces rule - Lami's rule).

Exercise 5 Follow : The equilibrium
(Meeting lines of action of three equilibrium forces).

Accumulative exercise on vectors

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

1 Choose the correct answer from the given ones :

(1) Norm of the vector $\overrightarrow{A} = -3\,\overrightarrow{i} + 4\,\overrightarrow{j}$ equals .... length unit.

(a) 3 (b) 4 (c) 5 (d) 1

(2) The cartesian form of the vector $\overrightarrow{B} = \left(5\sqrt{2}\,,225^\circ\right)$ is ....

(a) $(5\,,5)$ (b) $(-5\,,-5)$ (c) $(5\,,-5)$ (d) $(-5\,,5)$

(3) Measure of the polar angle of the vector $\overrightarrow{B} = -\overrightarrow{i} + \sqrt{3}\,\overrightarrow{j}$ equals ....

(a) 60° (b) 90° (c) 120° (d) 150°

(4) The polar form of the vector $\overrightarrow{A} = \sqrt{2}\,\overrightarrow{i} + \sqrt{2}\,\overrightarrow{j}$ is ....

(a) $(2\,,135^\circ)$ (b) $(4\,,45^\circ)$ (c) $(2\,,45^\circ)$ (d) $(4\,,135^\circ)$

(5) The polar form of the vector $\overrightarrow{m} = 5\,\overrightarrow{i} + 12\,\overrightarrow{j}$ is ....

(a) $(17\,,67^\circ\,22'\,48'')$ (b) $(17\,,22^\circ\,37'\,12'')$

(c) $(13\,,67^\circ\,22'\,48'')$ (d) $(13\,,22^\circ\,37'\,12'')$

(6) The vector that represents a force of magnitude 20 kg.wt. in the direction 30° South of East is written as ....

(a) $\left(10\,,-10\sqrt{3}\right)$ (b) $\left(10\sqrt{3}\,,-10\right)$ (c) $\left(-10\,,10\sqrt{3}\right)$ (d) $\left(10\sqrt{3}\,,10\right)$

(7) If $\overrightarrow{F} = k\,\overrightarrow{i} + 2\sqrt{2}\,\overrightarrow{j}$ and $\|\overrightarrow{F}\| = 2\sqrt{3}$ newton , then $|k| =$ ....

(a) $6\sqrt{2}$ (b) $2\sqrt{6}$ (c) -2 (d) 2

(8) If $\overrightarrow{F_1} = (5 , -3)$, $\overrightarrow{F_2} = (7 , 4)$, then the resultant of the two forces $\overrightarrow{R} =$ ....

(a) $\vec{i} + 12\vec{j}$ (b) $9\vec{i} + 4\vec{j}$ (c) $35\vec{i} - 12\vec{j}$ (d) $12\vec{i} + \vec{j}$

(9) If $\overrightarrow{F_1} = 5\vec{i}$, $\overrightarrow{F_2} = 7\vec{i} - 5\vec{j}$, then $\|\overrightarrow{R}\| =$ .... force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$

(10) If $\overrightarrow{F_1} = 2\vec{i} + 3\vec{j}$, $\overrightarrow{F_2} = \vec{i} + \vec{j}$, then the magnitude of their resultant equals .... force unit.

(a) 3 (b) 4 (c) 5 (d) 7

(11) Two forces of magnitudes 5 newtons and 7 newtons acting in the direction of East , then thier resultant equals ....

(a) 12 newton due East. (b) 2 newton due East.

(c) 12 newton due West. (d) 2 newton due West.

(12) If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three forces in equilibrium and meeting at one point where : $\overrightarrow{F_1} = (2 , -5)$, $\overrightarrow{F_2} = (-3 , 2)$, then $\overrightarrow{F_3} =$ ....

(a) $(2 , 1)$ (b) $(-1 , -3)$ (c) $(1 , 3)$ (d) $(3 , 1)$

(13) If the set of forces $\overrightarrow{F_1} = a\vec{i} + 7\vec{j}$, $\overrightarrow{F_2} = -5\vec{i} - b\vec{j}$, $\overrightarrow{F_3} = \vec{i} + \vec{j}$ are in equilibrium , then $(a , b) =$ ....

(a) $(2 , 4)$ (b) $(1 , 2)$ (c) $(-4 , -8)$ (d) $(4 , 8)$

(14) If the set of forces $\overrightarrow{F_1} = 4\vec{i} - 5\vec{j}$, $\overrightarrow{F_2} = a\vec{i} + 3\vec{j}$, $\overrightarrow{F_3} = 7\vec{i} - b\vec{j}$ are in equilibrium , then $a + b =$ ....

(a) 13 (b) -13 (c) -11 (d) -2

(15) If the forces $\overrightarrow{F_1} = 4\vec{i} + 5\vec{j}$, $\overrightarrow{F_2} = a\vec{i} - 7\vec{j}$ and $\overrightarrow{F_3} = 3\vec{i} + b\vec{j}$ act at one point and the forces are in equilibrium , then $a + 2b =$ ....

(a) -5 (b) 5 (c) 7 (d) -3

(16) If $\overrightarrow{F_1} = 2\vec{i} - 2\vec{j}$, $\overrightarrow{F_2} = 4\vec{i} - 8\vec{j}$, their resultant $\overrightarrow{R} = 2a\vec{i} - 3b\vec{j}$, then $a + b =$ ....

(a) 3 (b) $3\frac{1}{3}$ (c) $6\frac{1}{3}$ (d) 12

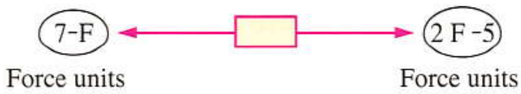
(17) If $\overrightarrow{F_1} = 5\vec{i} + 3\vec{j}$, $\overrightarrow{F_2} = a\vec{i} + 6\vec{j}$ and $\overrightarrow{F_3} = -14\vec{i} + b\vec{j}$ are three forces meeting at one point , $\overrightarrow{R} = \left(10\sqrt{2} , \frac{3}{4}\pi\right)$ then : $(a , b) =$ ....

(a) $(-1 , 1)$ (b) $(2 , 1)$ (c) $(-1 , 2)$ (d) $(1 , -1)$

(18) In the opposite figure :

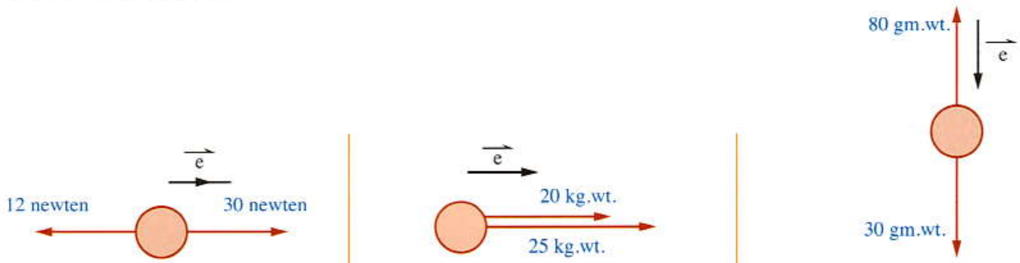
If the system is in equilibrium

, then F = ... force units.



(a) 4 (b) 7 (c) 2.5 (d) 3.5

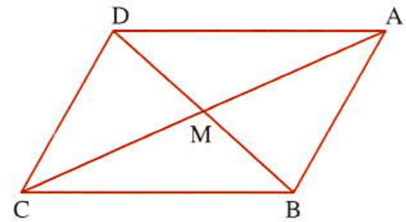
2 Write in terms of the unit vector $\overrightarrow{e}$ the resultant of the forces shown in each figure of the following figures :



The resultant is .... The resultant is .... The resultant is ....

3 **In the opposite figure :**

ABCD is a parallelogram , M is the point of intersection of its diagonals , then :



- $\overrightarrow{AB} + \overrightarrow{BC} =$ ....
- $\overrightarrow{DA} + \overrightarrow{DC} =$ ....
- $\overrightarrow{AM} + \overrightarrow{CM} =$ ....
- $\overrightarrow{AB} + 2\,\overrightarrow{BM} =$ ....
- $\overrightarrow{AB} - \overrightarrow{AM} =$ ....

Exercise 1

Forces - Resultant of two forces meeting at a point

From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The force is defined by ....

(a) its magnitude. (b) its direction.

(c) the point of action. (d) all the previous.

(2) Two forces act at a point. The magnitude of the two forces are 5 , 3 newton and the angle between them 60° , then the magnitude of their resultant = ... newton.

(a) 2 (b) 5 (c) 7 (d) 8

(3) Two forces act at a point the magnitude of the two forces $8\sqrt{3}$, 8 newton and the measure of the included angle between them 150° , then the magnitude of their resultant = ... newton.

(a) 64 (b) 32 (c) 16 (d) 8

(4) Two perpendicular forces act at a point. The magnitude of the two forces 12 , 5 newton , then the magnitude of their resultant = ... newton.

(a) 17 (b) 7 (c) 13 (d) 14

(5) Resultant of two forces 6 newton and 8 newton could be .... newton.

(a) 20 (b) 15 (c) 12 (d) 1

(6) The magnitude of two forces are 4 , 5 N. They act at a point and cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant R = ... newtons.

(a) 15 (b) 5 (c) 20 (d) 25

(7) Two forces act at a point. The magnitude of the two forces are 6 , 3 newton and their resultant is perpendicular to one of them , then the magnitude of their resultant = ... newton.

(a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$

(8) Two forces enclosing between them an angle of measure θ , then the magnitude of their resultant ....

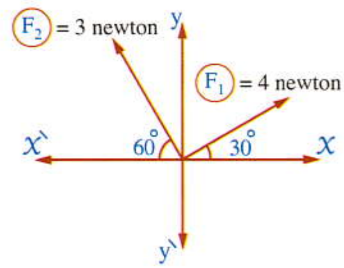
(a) increase as the value of θ increase.

(b) doubled as the value of θ doubled.

(c) increase as the value of θ decrease.

(d) don't change as change of the value of θ

(9) In the opposite figure :

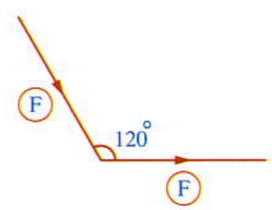


The magnitude of the resultant of the two forces in the figure = ... newton.

(a) 7 (b) 5

(c) 1 (d) $\sqrt{7}$

(10) In the opposite figure :

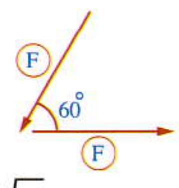


Magnitude of the resultant of the two forces = ... newton.

(a) 2 F (b) F

(c) $\sqrt{3}$ F (d) zero

(11) The magnitude of the resultant of the two forces shown in the opposite figure is ....



(a) $\frac{1}{2}$ F (b) F (c) $\sqrt{3}$ F (d) $\sqrt{5}$ F

(12) If the resultant of the two forces F_1 , F_2 bisects the angle between them. Which of the following statements is true ?

① $F_1 = F_2$ ② $\overrightarrow{F_1} = \overrightarrow{F_2}$ ③ $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

(a) only ① (b) only ① , ③

(c) only ② , ③ (d) All the previous.

(13) Two forces act at a point. The magnitude of the two forces are F , 2 newton and the measure of the angle between them is 60° , if their resultant equal $2\sqrt{3}$ newton , then F = ... newton.

(a) 2 (b) 4 (c) 8 (d) 12

(14) The magnitude of two forces F , 2 newton and the measure of their included angle = $\frac{2\pi}{3}$ and the magnitude of their resultant is F newton , then F = ... newton.

(a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$

(15) Two forces of equal magnitudes , enclosing between them an angle of measure $\frac{\pi}{2}$ If the magnitude of their resultant is 8 N. , then the value of each force measured in newton is ....

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

(16) Two equal forces in magnitude , the magnitude of their resultant = $7\sqrt{3}$ newton and the measure of the included angle is $\frac{\pi}{3}$, then the magnitude of each of them = ... newton.

(a) 3 (b) $5\sqrt{3}$ (c) 5 (d) 7

(17) The magnitude of two forces F , F kg.wt. , the magnitude of their resultant 24 kg.wt. and inclined to the first force by an angle of measure 30° , then F = ... kg.wt.

(a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12

(18) Two forces of magnitudes 8 and F gm.wt. The measure of the angle between them is $\alpha \in]0 , \pi[$, their resultant bisects the included angle between them , then F = ... gm.wt.

(a) 4 (b) 16 (c) $2\sqrt{2}$ (d) 8

(19) Two forces of magnitudes 3 , F newton and the measure of the angle between them is 120°. If their resultant is perpendicular to the first force , so the value of F in newton is ....

(a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6

(20) The magnitude of two perpendicular forces are (2 F – 5) and (F + 2) newton and the magnitude of their resultant if $3\sqrt{5}$ newton , then F = ... newton.

(a) 7 (b) 4 (c) 6 (d) 3

(21) Two forces of magnitudes 6 N. and 10 N. , if the magnitude of their resultant is 14 N. , then the measure of the angle between the forces is ....

(a) 15° (b) 30° (c) 60° (d) 45°

(22) Two equal forces , the magnitude of each of them is 6 N. , the magnitude of their resultant is 6 N. , then the angle between them equals ....

(a) 30° (b) 60° (c) 120° (d) 150°

(23) Two forces of magnitudes 6 N. and 8 N. , if the magnitude of their resultant is 2 N. , then the measure of the angle between the two forces is ....

(a) 30° (b) 90° (c) 180° (d) 270°

(24) Magnitude of resultant of two forces of magnitudes 6 , 2.5 newton is equal to 6.5 newton , then the angle between the two forces is ....

(a) an acute angle. (b) an obtuse angle.

(c) a right angle. (d) a straight angle.

(25) The magnitude of two forces are 2 F , 5 F newton and the measure of their included angle is θ and their resultant is 3 F , then θ = ...

(a) zero (b) 60° (c) 90° (d) 180°

(26) Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton , then the measure of the angle between them = ...

(a) 60° (b) 0° (c) 180° (d) 90°

(27) Two forces of magnitudes F and F act at a particle and their resultant is F , then the measure of the angle between the two forces = ...

(a) 120° (b) 60° (c) 45° (d) 90°

(28) The magnitude of two forces acting at a point F , $\sqrt{3}$ F newton. If the magnitude of their resultant is 2 F newton , then the measure of their included angle equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

(29) If $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$ and $\| \overrightarrow{R} \| = \| \overrightarrow{F_1} \| - \| \overrightarrow{F_2} \|$, then the measure of the angle between $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ equals ....

(a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(30) If the magnitude of the resultant of two forces act at a point is maximum value , then the measure of the angle between the two forces equal ....

(a) 180° (b) 120° (c) zero (d) 60°

(31) The measure of the angle between $\overrightarrow{F_1}$ and the resultant of the two forces $(\overrightarrow{F_1} + \overrightarrow{F_2})$ and $(\overrightarrow{F_1} - \overrightarrow{F_2})$ is ....

(a) zero (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(32) If $\overrightarrow{R_1}$ is the resultant of the two forces $(\overrightarrow{F_1}, \overrightarrow{F_2})$ and $\overrightarrow{R_2}$ is the resultant of the two forces $(\overrightarrow{F_1}, -\overrightarrow{F_2})$, $\|\overrightarrow{F_1}\| = \|\overrightarrow{F_2}\|$, then ....

(a) $\overrightarrow{R_1} \perp \overrightarrow{R_2}$ (b) $\overrightarrow{R_1} = \overrightarrow{R_2}$

(c) $\|\overrightarrow{R_1}\| = \|\overrightarrow{R_2}\|$ (d) $\overrightarrow{R_1} \,/\!/\, \overrightarrow{R_2}$

(33) Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90° , then the tangent of the angle between the resultant and the first force equal ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{6}}{2}$

(34) The magnitudes of two perpendicular forces are 6 , 8 newton then the measure of the angle between the resultant and the first force is ....

(a) $\sin^{-1} \frac{4}{3}$ (b) $\cos^{-1} \frac{4}{3}$ (c) $\tan^{-1} \frac{4}{3}$ (d) $\tan^{-1} \frac{3}{4}$

(35) Two forces of magnitudes F , 2 F newton act at a point , if the resultant of them is perpendicular to one of them , then R = ...

(a) $\sqrt{5}$ F (b) $\sqrt{3}$ F (c) 3 F (d) F

(36) Two forces of magnitudes $3\sqrt{2}$ and 6 newton and the measure of the angle between them is 135° , then the measure of the angle between their resultant and the second force is ....

(a) 30° (b) 45° (c) 60° (d) 90°

(37) Two forces of magnitudes 12 , 15 newton act at a particle and the measure of the enclosing angle between them is θ° , where $\cos \theta = \frac{-4}{5}$, then the measure of the included angle between the resultant and the first force = ...°

(a) zero (b) 30 (c) 90 (d) 36° 52`

(38) The magnitude of two forces acting on a particle are 5 , 8 newton , then the smallest value of their resultant = ... newton.

(a) 2 (b) 3 (c) 7 (d) 13

(39) Two forces of magnitudes 9 newton , 1000 dyne , the maximum value of their resultant ....

(a) 1009 dyne. (b) 1009 newton. (c) 9.01 dyne. (d) 9.01 newton.

(40) Two forces of magnitudes 5 , F newton , if the smallest resultant of them is 10 newton , $F > 5$, then $F =$ ·········· newton.

(a) 6 (b) 10 (c) 15 (d) 20

(41) Two forces act at a point. The magnitude of the two forces are 5 F , 3 F. If the maximum value of their resultant is 40 newton , then the minimum value of their resultant ·········· newton.

(a) 10 (b) 20 (c) 5 (d) zero

(42) Two forces act at a point. The magnitudes of the two forces are 5 , 3 newton , then the magnitude of their resultant measure by newton $\in$ ··········

(a) $[2 , 8]$ (b) $]2 , 8[$ (c) $[3 , 5]$ (d) $]3 , 5[$

(43) If θ is the angle between two forces of magnitudes 2 newton , 6 newton , $\theta \in]0 , \pi]$, then the magnitude of their resultant measured by newton $\in$ ··········

(a) $]4 , 8[$ (b) $[4 , 8[$ (c) $]4 , 8]$ (d) $[4 , 8]$

(44) Two forces of equal magnitude and the magnitude of their resultant equal 16 newton when the measure of the angle between the two forces is $\frac{\pi}{2}$, then the maximum value of their resultant equal ·········· newton.

(a) 32 (b) $8\sqrt{2}$ (c) $16\sqrt{2}$ (d) zero

(45) Two forces of magnitude F_1 , F_2 kg.wt. , where $F_1 > F_2$ and the magnitude of smallest and greatest resultant of them are 3 and 12 gm.wt. respectively , then $F_1^2 - F_2^2 =$ ··········

(a) 12 (b) 3 (c) 9 (d) 36

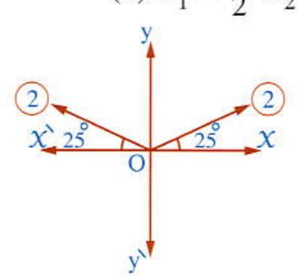
(46) The magnitude of two forces are 12 , 17 newton then the difference between the greatest and the smallest value of their resultant = ·········· newton.

(a) 29 (b) 5 (c) 14 (d) 24

(47) Two forces of magnitude F , $\sqrt{3}$ F newton meeting at a point and the magnitude of their resultant is R_1 when the measure of the angle between the two forces is 90° , and their resultant becomes R_2 when the measure of the angle between the two forces is 150° , then ··········

(a) $R_1 = R_2$ (b) $R_1 = 2 R_2$ (c) $R_1 = \frac{3}{5} R_2$ (d) $R_1 = \frac{1}{2} R_2$

(48) The direction of the resultant of the forces which represented in the opposite figure is ··········



(a) $\overrightarrow{Ox}$ (b) $\overrightarrow{Ox'}$

(c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$

(49) Two forces act at a point and the magnitude of smallest and greatest resultant of them are 0 and 12 newton respectively , then ....

(a) magnitude of one force is three times magnitude of the other.

(b) magnitude of one force is twice magnitude of the other.

(c) the two forces are equal in magnitude.

(d) the two forces are perpendicular.

Second Essay questions

1 Find the magnitude and the direction of the resultant of two perpendicular forces of magnitudes 8 and 15 kg.wt. acting at a particle. « 17 kg.wt. , $\theta = 61° \; 55' \; 39''$ »

2 The magnitude of the resultant of two perpendicular forces is 50 newton. If the resultant makes with the first force an angle of measure 30° , find the magnitude of each of these two forces. « $25\sqrt{3}$, 25 newton »

3 Two forces of magnitudes 30 and 16 newton act at a particle , if the magnitude of their resultant is 26 newton. Find the measure of the angle between these two forces. « 120° »

4 Two forces are of magnitudes 9 and 6 kg.wt. act at a particle. The measure of the included angle is α , find α if the magnitude of the resultant is $3\sqrt{7}$ kg.wt. , find the measure of the angle between the resultant and the great force. « $\alpha = 120°$, $\theta \simeq 40° \; 53' \; 36''$ »

5 Two forces acted at a point. If the magnitude of the first is 15 kg.wt. towards East and the second is of magnitude 18 kg.wt. in the direction 30° West of the North. Calculate the magnitude and the direction of the resultant. « $3\sqrt{31}$ kg.wt. , $\theta \simeq 68° \; 56' \; 54''$ »

6 Two forces of magnitudes 12 , F kg.wt. act on a point. The first force acts in direction of East and the second force acts in direction 60° South of the West. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° South of the East. « 6 kg.wt. , $6\sqrt{3}$ kg.wt. »

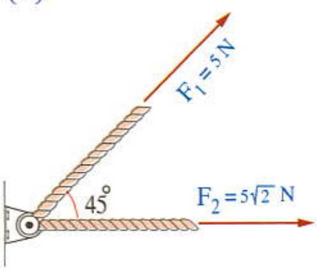
7 Two forces act at a particle and they include an angle of measure α where $\tan \alpha = \frac{-1}{\sqrt{3}}$

If the resultant is perpendicular to the small force and the magnitude of the great force equals 30 kg.wt. What is the magnitude of each of the small force and the resultant ?

« $15\sqrt{3}$ kg.wt. , 15 kg.wt. »

8 **Find the magnitude and the direction of the resultant in each of the following figures :**

(1)



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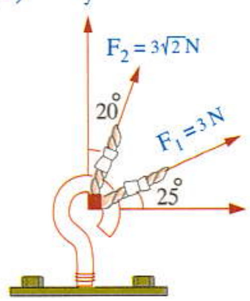
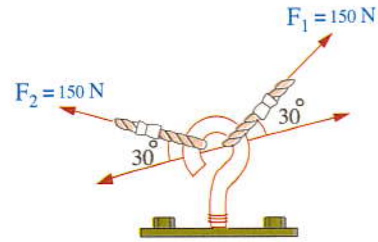


(3)



---

9 Two forces of magnitudes F , 4 newton act on a particle and the measure of the angle between their directions is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton. Find the magnitude of $\overrightarrow{F}$ and the measure of the angle that $\overrightarrow{R}$ from with $\overrightarrow{F}$ « 8 newton , 30° »

---

10 Two forces of magnitudes $\sqrt{3}$ F and 2 F act at a point. Find the measure of the angle included between them if their resultant is perpendicular to the small force and if F = 15 Find the magnitude of the resultant. « 150° , 15 newton »

---

11 Two forces of magnitudes $2\sqrt{2}$ and F newton act at a particle and the magnitude of their resultant is $\sqrt{2}$ newton. If the resultant is perpendicular to the second force , find F and the measure of the angle between the two forces. « $\sqrt{6}$ newton , 150° »

---

12 Two forces of magnitudes 16 and F kg.wt. act on a particle and the measure of the angle between them is 120°. If their resultant is inclined to the force 16 kg.wt. by an angle whose measure is 30° , find the magnitude of F and the resultant.

« 8 kg.wt. , $8\sqrt{3}$ kg.wt. »

---

13 Three forces of magnitudes 5 , 10 , $4\sqrt{7}$ N. act on a particle , if the measure of the angle between the first and the second forces equals 60° , find the magnitude of the maximum and the minimum resultant for the three forces. « $9\sqrt{7}$ newton , $\sqrt{7}$ newton »

---

14 **Two forces of magnitudes 2 F and 3 F newton. The angle between them is of measure θ , find the value of θ if the magnitude of their resultant is :**

(1) 3 F

(2) F

(3) 5 F

(4) $\sqrt{13}$ F

« 109° 28′ 16″ , 180° , zero , 90° »

15 **Two forces of magnitudes 2 , F newton , the angle between them is of measure 120° Find F in each of the two cases :**

(1) The direction of the resultant is perpendicular to the second force.

(2) The resultant inclines by 45° to the 2^{nd} force. « 1 , $\sqrt{3}+1$ newton »

---

16 F_1 and F_2 newton are magnitudes of two forces intersect at a point and their resultant equals R newton where $R \in [2 , 10]$, $F_1 > F_2$, find each of F_1 and F_2 , then find R when the measure of the angle between them is 120° « 6 , 4 , $2\sqrt{7}$ newton »

---

17 Two forces act at a point , the value of one is 3 N. more than the other. If the magnitude of their resultant is $3\sqrt{3}$ newton and is perpendicular to the smaller force. Find the magnitude of each force and the measure of the angle between them.

« 3 , 6 newton , $\alpha = 120°$ »

---

18 The resultant of two forces F_1 and F_2 is $\sqrt{10}$ newton when $F_1 \perp F_2$ and their resultant becomes $\sqrt{13}$ newton when the angle between F_1 and F_2 becomes 60° , find F_1 and F_2

« 1 , 3 newton »

---

19 Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force. « $3\sqrt{5}$, $3\sqrt{5}$ kg.wt. »

---

20 Two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ meet at a point. Their resultant is R gm.wt. The angle between them is of measure 120°. If the direction of $\overrightarrow{F_2}$ is reversed , the resultant will be $R\sqrt{3}$ gm.wt., prove that $F_1 = F_2$ and the resultant in the first case is perpendicular to the second case.

---

21 4 , F are two forces acting at a point and their resultant is 10 newton and makes an angle of measure 60° with the force 4 newton. Find the value of F. « $2\sqrt{19}$ newton »

---

22 The difference between the magnitudes of two forces acting at a point is 15 newton. and their resultant = 35 newton in magnitude when the measure of the angle between the two forces = 120° , find the magnitude of each of the two forces. « 40 , 25 newton »

---

23 The sum of magnitudes of two forces is 4 newton when the measure of the angle between them is 60° , then the resultant becomes $\sqrt{13}$ newton. Find the magnitude of each of the two forces. « 1 , 3 newton »

---

24 The sum of magnitudes of two forces acting at a point is 40 kg.wt. the magnitude of their resultant is 20 kg.wt. and it is perpendicular to the smaller force. Find the magnitude of each of the two forces and the cosine of the angle between them. « 15 , 25 kg.wt. , $-\frac{3}{5}$ »

25 Two forces of same magnitude F kg.wt. enclose between them an angle of measure $120°$. If the two forces are doubled and the measure of the angle between them became $60°$, then the magnitude of their resultant increases by 11 kg.wt., than the first case. Find the magnitude of F «$1 + 2\sqrt{3}$»

---

26 F, 2 F are two forces act on a particle and enclose between them an angle of measure α The magnitude of their resultant equals $\sqrt{5}\, F (m + 1)$ and if the measure of the angle between them becomes $(90° - \alpha)$, then the magnitude of the resultant will be $\sqrt{5}\, F (m - 1)$

Prove that : $\tan \alpha = \frac{m - 2}{m + 2}$

Third Higher skills

1 **Choose the correct answer from those given :**

(1) If the ratio between the maximum and the minimum values of the resultant of two forces is 7 : 3, then the ratio between the two forces = ...

(a) 7 : 4 (b) 7 : 3 (c) 5 : 3 (d) 5 : 2

(2) If the ratio among magnitudes of two forces and their resultant is $4 : 3 : \sqrt{13}$ respectively, then the measure of the angle between the two forces = ...

(a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

(3) If the resultant of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ is perpendicular on $\overrightarrow{F_1}$, then the measure of the angle between the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ equals ....

(a) $\cos^{-1}\left(\frac{F_1}{F_2}\right)$ (b) $\cos^{-1}\left(\frac{-F_1}{F_2}\right)$ (c) $\sin^{-1}\left(\frac{F_1}{F_2}\right)$ (d) $\sin^{-1}\left(\frac{-F_1}{F_2}\right)$

(4) If the resultant of two perpendicular forces makes an angle of measure θ to the greater force which of the following values could be a value of θ ?

(a) $90°$ (b) $70°$ (c) $45°$ (d) $10°$

(5) $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces acting at a point and their resultant is R. If $\overrightarrow{F_2}$ reversed then their resultant rotates with angle of measure $90°$, then ....

(a) $F_1 = F_2$ (b) $F_1 = 2 F_2$

(c) $F_1 = \frac{1}{2} F_2$ (d) nothing of the previous.

(6) The magnitudes of two forces acting at a point are 4, F newton and the measure of their included angle is $120°$, then F which makes the resultant minimum equals .... newton.

(a) 1 (b) 2 (c) 3 (d) 4

(7) If θ_1 is the measure of the angle between the resultant of two forces $\left(\overrightarrow{F_1}, \overrightarrow{F_2}\right)$ and the force $\overrightarrow{F_1}$ and θ_2 is the measure of the angle between the resultant of the two forces $\left(\overrightarrow{F_1}, 2\overrightarrow{F_2}\right)$ and the force $\overrightarrow{F_1}$, then ....

(a) $\theta_1 = \theta_2$ (b) $\theta_1 > \theta_2$ (c) $\theta_1 < \theta_2$ (d) $\theta_1 + \theta_2 = \frac{\pi}{2}$

(8) The magnitudes of two forces acting at a point are F , $\sqrt{3}$ F newton and the magnitude of their resultant is F newton and θ_1 is the measure of the angle between F , R and θ_2 is the measure between $\sqrt{3}$ F and R , then ....

(a) $\theta_1 = \theta_2$ (b) $\theta_1 = \frac{1}{2}\theta_2$ (c) $\theta_1 = 3\theta$ (d) $\theta_1 = 4\theta_2$

(9) The magnitudes of two forces acting at a point are F_1 , F_2 where : $3 \le F_1 \le 12$, $4 \le F_2 \le 16$ and the magnitude of their resultant is R and the measure of their included angle is 90° , then ....

(a) $5 \le R \le 20$ (b) $7 \le R \le 28$ (c) $0 \le R \le 18$ (d) $1 \le R \le 4$

(10) Two forces meet at a point , their magnitudes are F_1 , F_2 where $1 \le F_1 \le 9$, $3 \le F_2 \le 7$ and the magnitude of their resultant R , then ....

(a) $2 \le R \le 16$ (b) $4 \le R \le 16$ (c) $6 \le R \le 16$ (d) $0 \le R \le 16$

(11) The magnitudes of two forces acting at a point are F_1 , F_2 where $5 \le F_1 \le 20$, $12 \le F_2 \le 21$ and the magnitude of their resultant is R , the measure of the angle between them is θ where $0 \le \theta \le \frac{\pi}{2}$ then ....

(a) $13 \le R \le 29$ (b) $0 \le R \le 41$ (c) $13 \le R \le 41$ (d) $17 \le R \le 29$

---

2 One of two forces is half the other in magnitude , they have a certain resultant. If the small force increased by 4 kg.wt. and the great force becomes double, then their resultant stays in the same direction of the first case , find the magnitudes of the two forces and the ratio between the magnitudes of the two resultants in the two cases. « 4 , 8 kg.wt. , 1 : 2 »

---

3 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two forces meeting at a point and their resultant is R newton. If the direction of $\overrightarrow{F_2}$ becomes in the opposite direction , then the magnitude of the resultant becomes R$\sqrt{3}$ newton and the resultant becomes perpendicular to the first resultant. Find the measure of the angle between the two forces. « $\alpha = 120°$ »

Exercise

2

Forces resolution into two components

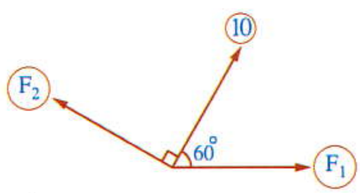
From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) In the opposite figure :

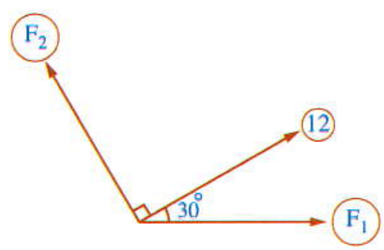


If the force of magnitude 10 N. is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ inclined to the force by two angles of measures 60° and 90° respectively , then F_2 = ... N.

(a) $5\sqrt{3}$ (b) 10

(c) $10\sqrt{3}$ (d) 20

(2) In the opposite figure :



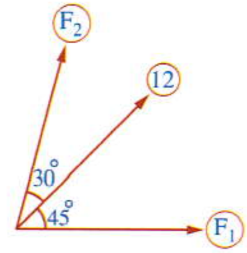
If the force of magnitude 12 N. is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ inclined to the force by two angles of measures 30° and 90° respectively, then F_2 = ... N.

(a) 10 (b) $10\sqrt{3}$

(c) $6\sqrt{3}$ (d) $4\sqrt{3}$

(3) In the opposite figure :

If the force of magnitude 12 N. is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, then $F_1 = \cdots\cdots$ newton.

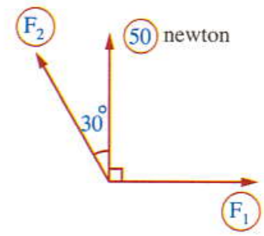


(a) 12 cos 75° (b) 12 cos 45°

(c) 6 csc 45° (d) 6 csc 75°

(4) In the opposite figure :

If the force of magnitude 50 newton is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, then $F_1 + F_2 = \cdots\cdots$ newton.

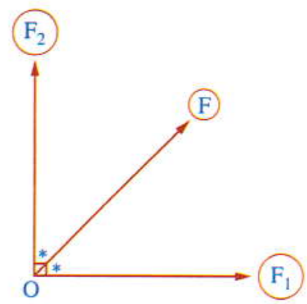


(a) 50 (b) 25

(c) $50\sqrt{2}$ (d) $50\sqrt{3}$

(5) In the opposite figure :

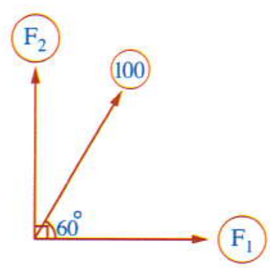
If the force $\vec{F}$ is resolved into the two perpendicular components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, the vector of the force $\vec{F}$ bisects the angle between the directions of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ and $\|\overrightarrow{F_1}\| = 6\sqrt{2}$ newton , then $\|\vec{F}\| = \cdots\cdots$ newton.



(a) 6 (b) $6\sqrt{2}$ (c) 12 (d) $12\sqrt{2}$

(6) In the opposite figure :

If the force of magnitude 100 newton is resolved into two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ and the force is measured by newton , then $(F_1 , F_2) = \cdots\cdots$

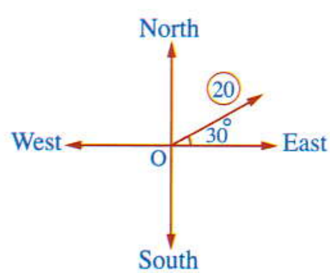


(a) $\left(50 , 50\sqrt{3}\right)$ (b) $\left(50\sqrt{3} , 10\right)$

(c) (50 , 50) (d) (10 , 10)

(7) In the opposite figure :

A force of magnitude 20 newton. acts in the direction 30° North of the East is resolved into two perpendicular components , then the magnitude of the component in North direction = ········ newton.

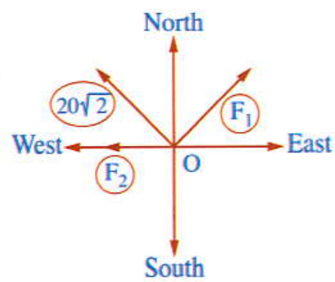


(a) $10\sqrt{3}$ (b) 20

(c) 10 (d) 5

(8) In the opposite figure :

A force of magnitude $20\sqrt{2}$ kg.wt. acts in the Western North direction , is resolved into two component. One of them of magnitude F_1 in the Eastern North direction and the other of magnitude F_2 in the direction of West , then $F_2 =$ .... kg.wt.

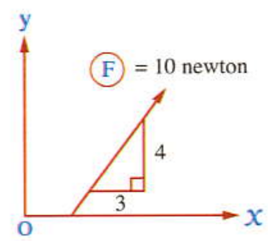


(a) 30 (b) 40

(c) 50 (d) $40\sqrt{2}$

(9) In the opposite figure :

If a force $\vec{F}$ is resolved into two components in the directions of the coordinate axes , then the magnitude of the component of this force in the direction of $\overrightarrow{Ox}$ equals .... newton.



(a) 10 (b) 6

(c) 8 (d) $\frac{40}{3}$

(10) A force of magnitude $10\sqrt{2}$ gm.wt. acts in the Eastern South direction, is resolved into two perpendicular components , then the magnitude of the component in the South direction = ... gm.wt.

(a) 5 (b) 10 (c) $10\sqrt{2}$ (d) $5\sqrt{2}$

(11) A force of magnitude 6 newton acts in direction of North. It is resolved into two perpendicular components , so its component in direction of the East of magnitude .... newton.

(a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6

(12) A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components , so its component in the direction of Northern East of magnitude .... newton.

(a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6

(13) The magnitude of a force is 6 newton and acts towards the North. It is resolved into two perpendicular components then its component in direction of Eastern North of magnitude .... newton.

(a) 6 (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) zero

(14) A force of magnitude $5\sqrt{3}$ newton acts in the direction 30° East of the North , is resolved into two perpendicular components , then the magnitude of its component in the East direction = ... newton.

(a) $\frac{5\sqrt{3}}{2}$ (b) $\frac{15}{2}$ (c) $\frac{15\sqrt{3}}{2}$ (d) $15\sqrt{3}$

(15) The magnitude of a force is 8 newton and acts in East direction. It is resolved into two components , the angle between the two components is 120° , then its component in South direction = ... newton.

(a) 16 (b) 8 (c) $8\sqrt{3}$ (d) $\frac{8\sqrt{3}}{3}$

(16) A force of magnitude 40 newton acts vertically upwards is resolved into two components one of them is horizontal of magnitude 20 newton , then the magnitude of the other = ... newton.

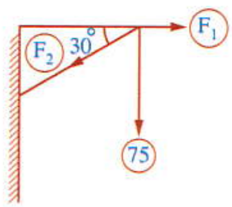
(a) 20 (b) $20\sqrt{3}$ (c) $20\sqrt{5}$ (d) $10\sqrt{3}$

(17) Force of magnitude F newton is resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ and they make angles of measure 60° , 90° respectively but on different sides from the line of action of $\overrightarrow{F}$, then F_1 = ...

(a) $2 F_2$ (b) $\frac{\sqrt{3}}{2} F_2$ (c) $\frac{2}{\sqrt{3}} F_2$ (d) $\frac{1}{2} F_2$

(18) In the opposite figure :

A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal of magnitude F_1 and the other is of magnitude F_2 , then F_2 = ... newton.

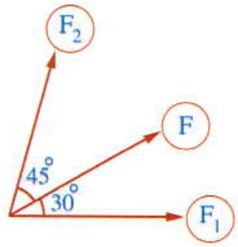


(a) 75 (b) $75\sqrt{3}$

(c) 150 (d) $150\sqrt{3}$

(19) In the opposite figure :

The force $\overrightarrow{F}$ is the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then $\frac{F_1 + F_2}{F}$ = ...



(a) sin 30° + sin 45° (b) $\frac{\sin 75° + \sin 30°}{\sin 75°}$

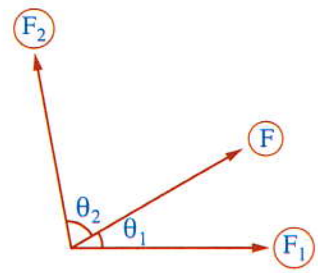
(c) $\frac{\sin 45° + \sin 30°}{\sin 75°}$ (d) $\frac{\sin 75°}{\sin 30°} + \frac{\sin 75°}{\sin 45°}$

(20) ABCDEF is a regular hexagon. A force of magnitude 20 newton acts in direction of $\overrightarrow{AD}$, then the magnitudes of the components of the force in direction of $\overrightarrow{AC}$, $\overrightarrow{AF}$ respectively are ....

(a) $10\sqrt{3}$, 10 (b) $5\sqrt{3}$, 10 (c) 10 , $10\sqrt{3}$ (d) $20\sqrt{3}$, 20

(21) In the opposite figure :

The force $\vec{F}$ has been resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then $\frac{F_1}{F_2}$ = ...



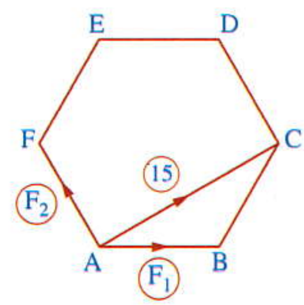
(a) $\frac{\sin\theta_2}{\sin\theta_1}$ (b) $\sin\left(\frac{\theta_2}{\theta_1}\right)$

(c) $\sin(\theta_1 + \theta_2)$ (d) $\frac{\sin\theta_1}{\sin\theta_2}$

(22) In the opposite figure :

ABCDEF is a regular hexagon. Force of magnitude 15 N. acts along $\overrightarrow{AC}$ and it has been resolved into two components $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ as shown in the figure

$F_1 : F_2$ = ...

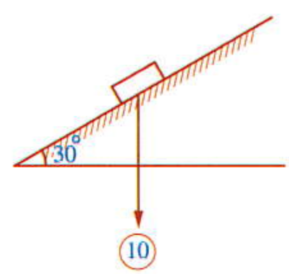


(a) $\sqrt{3} : 2$ (b) 2 : 1

(c) 1 : 2 (d) $1 : \sqrt{3}$

(23) In the opposite figure :

If a body of weight 10 newtons is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , then the component of the weight in direction of line of the greatest slope downward = ... N.



(a) $5\sqrt{2}$ (b) $5\sqrt{3}$

(c) 5 (d) $10\sqrt{3}$

(24) If a body of weight (W) is placed on a smooth plane inclined to horizontal by angle (θ) , so the component of its weight in direction of the plane equals ....

(a) W (b) W sin θ (c) W cos θ (d) W tan θ

(25) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the horizontal , then its weight component in the perpendicular direction of the plane is ....

(a) W sin θ (b) W cos θ (c) W tan θ (d) W csc θ

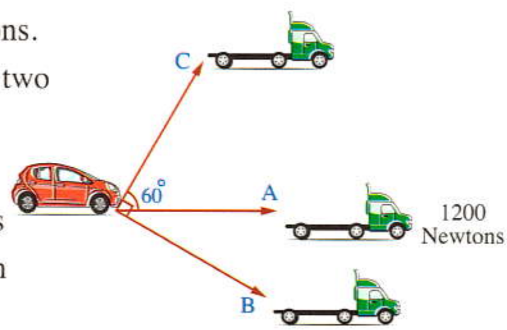
(26) If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the vertical , then its weight component in direction of the plane is ....

(a) W sin θ (b) W cos θ (c) W (d) W tan θ

(27) A body of weight (W) newton is placed on an inclined plane makes an angle of measure (θ) with the horizontal , then the components of its weight in direction line of greatest slope and its perpendicular are 7 , 24 newton respectively , then the magnitude of the weight (W) = ... newton.

(a) 7 (b) 24 (c) 25 (d) 31

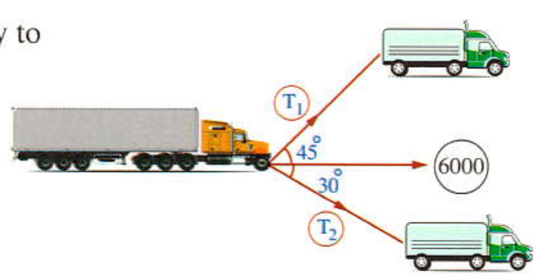
(28) A tractor drags a car with a force 1200 newtons. It's required to replace the tractor by another two tractors at B and C attached with two cables to the car and the angle between the two cables is 90°. If one of the two cables inclined to the tractor A at an angle 60° , then the tensions in the two cables B and C are .... newtons.



(a) 600 , 600 (b) 800 , 400

(c) $600\sqrt{3}$, 600 (d) 700 , 500

(29) A truck has broken down traffic officers try to pull the truck by using two draging cars. The resultant of their tensions is a horizontal tension of magnitude 6000 newtons as shown in the figure then T_2 = ... to the nearest newton.

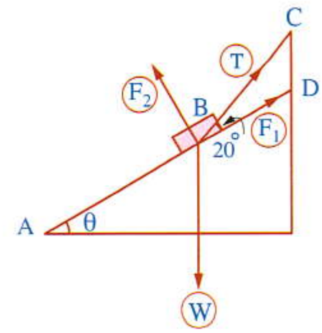


(a) 3105 (b) 3606

(c) 4392 (d) 4293

(30) In the opposite figure :

A body of weight (W) newtons is placed on a plane inclined to the horizontal at an angle of measure (θ). It is tied by a light string $\overline{BC}$ inclined to the plane at an angle of measure 20° above the plane. F_1 and F_2 are the components of the tension in direction of the plane and perpendicular to the plane then..........



(a) $F_2 = T\cos\theta$ (b) $F_1 = T\sin(20° + \theta)$

(c) $F_1 = T\cos(20° + \theta)$ (d) $T = F_1 \sec 20°$

Second Essay questions

1 A force of magnitude 600 kg. wt. acts on a particle. Find its two components in two directions making with the force two angles of measures 30° and 45° « 439.23 , 310.68 gm.wt. »

2 A force of magnitude 100 gm.wt. acts in the direction of Western North. Find its components in the North direction and in West direction. « $50\sqrt{2}$, $50\sqrt{2}$ gm. wt. »

3 A force of magnitude 12 kg. wt. acting in the direction of Eastern North was resolved into two components. One in the direction of East and the other in the direction of Western North. Find these two components. « $12\sqrt{2}$, 12 kg.wt. »

4 Resolve a horizontal force of magnitude 160 gm.wt. in two perpendicular directions. One of them inclined to the horizontal with an angle of measure 30° upwards.

« $80\sqrt{3}$, 80 gm.wt. »

5 A force of magnitude 300 dyne. acts in the North direction. Find the magnitudes of the two perpendicular components if one of them acts in the direction 30° North of East.

« 150 , $150\sqrt{3}$ dyne »

6 A force of magnitude 18 newton acts in the direction of South. Find its two components in the two directions 60° East of the South and the other direction towards 30° West of the South. « 9 , $9\sqrt{3}$ newton »

7 Resolve a force of magnitude 90 newton into two equal forces in magnitude and the measure of the angle between their lines of action is 60° « $30\sqrt{3}$ newton »

8 A body of weight 80 newton is placed on a horizontal plane. Find the two perpendicular components of the weight if one of them inclines to the horizontal with 30° downwards.

« 40 , $40\sqrt{3}$ newton »

---

9 Two forces act at a point. α is the angle between them and $\tan \alpha = -\frac{1}{\sqrt{3}}$,

If their resultant is perpendicular to the smaller force and the greater force 30 newton. Find the magnitude of the other force and the resultant.

« $15\sqrt{3}$, 15 newton »

---

10 Resolve a force of magnitude F newton in the North direction into two components , the first in the direction 30° North of East with magnitude 40 newton and the other is in the West direction. Find each of the magnitude of the force F and the magnitude of the other component. « 20 , $20\sqrt{3}$ newton »

---

11 A rigid body of weight 42 netwon is placed on a plane inclined to the horizontal with an angle of measure 60°. Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it. « $21\sqrt{3}$, 21 newton »

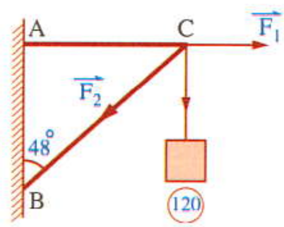
---

12 A body of weight 60 newton is placed on an inclined plane , at an angle of measure θ where $\tan \theta = \frac{3}{4}$, find the magnitudes of the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it. « 36 , 48 newton »

---

13 **In the opposite figure :**

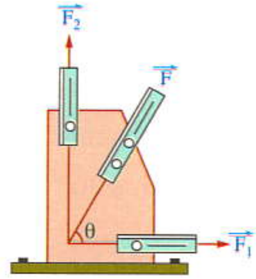
Resolve the vertical force of magnitude 120 gm.wt. into two components, one of them in the horizontal direction and the other inclined by an angle of measure 48° with the line of action of the force.



« 133.27 , 179.34 gm.wt. »

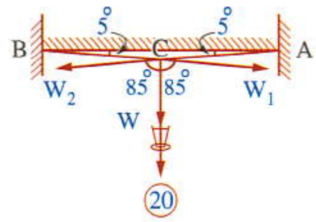
---

14 The opposite figure represents an angle of a bridge , the force $\overrightarrow{F}$ of magnitude 30 newton is resolved into two perpendicular components , the magnitude of one of them is $15\sqrt{3}$ newton Find the magnitude of the other component.



« 15 newton »

15 **In the opposite figure :**



A lamp of weight 20 newton suspended by two metal rods $\overline{AC}$, $\overline{BC}$ inclined to the horizontal by two equal angles , the measure of each is 5° :

(1) Resolve the weight of the lamp into two components in the directions $\overrightarrow{AC}$, $\overrightarrow{BC}$ approximating the result to the nearest netwon.

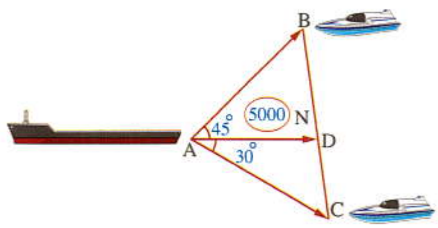
(2) What happens to the magnitude of the components of the weight in the directions of the two metal rods if the measure of the inclination angle to the horizontal decreased to be smaller than 5° ? And what do you expect to the components when the rods become horizontal ? Justify your answer. « 114.74 , 114.74 newton »

---

16 An inclined plane of length 130 cm. and height 50 cm. a rigid body of weight 390 gm.wt. is placed on it. Find the two components of the weight in the direction of the line of greatest slope of the plane and the perpendicular to it. « 150 , 360 gm.wt. »

---

17 **In the opposite figure :**



A cruiser is pulled by two ships B and C using two strands hanged to a point A on the cruiser , the measure of the angle between the two strands equals 75° , if the measure of the angle between one of the strands and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 newton and acts on $\overrightarrow{AD}$

Find the tension in the two strands. « 2588.2 , 3660.3 newton »

Exercise 3

The resultant of coplanar forces meeting at a point

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(where $\vec{i}$ and $\vec{j}$ are the two fundamental unit vectors in two perpendicular directions)

(1) If $\overrightarrow{F_1} = \vec{i} - \vec{j}$, $\overrightarrow{F_2} = 2\,\vec{i} - 4\,\vec{j}$, $\vec{R} = 2\,a\,\vec{i} - 3\,b\,\vec{j}$, then a + b = ...

(a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12

(2) If $\overrightarrow{F_1} = 3\,\vec{i} - 2\,\vec{j}$, $\overrightarrow{F_2} = a\,\vec{i} - \vec{j}$, $\overrightarrow{F_3} = 4\,\vec{i} - b\,\vec{j}$, $\vec{R} = 6\,\vec{i} - 4\,\vec{j}$, then (a , b) = ...

(a) (1 , – 1) (b) (– 1 , 1) (c) (– 1 , – 1) (d) (1 , 1)

(3) If $\overrightarrow{F_1} = 4\,\vec{i}$, $\overrightarrow{F_2} = 8\,\vec{i} - 5\,\vec{j}$, then $\|\vec{R}\|$ = ... force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$

(4) If $\overrightarrow{F_1} = 3\,\vec{i} + 2\,\vec{j}$, $\overrightarrow{F_2} = a\,\vec{i} + 7\,\vec{j}$, $\overrightarrow{F_3} = -12\,\vec{i} + b\,\vec{j}$ are three coplanar forces meeting at a point and the resultant $\vec{R} = \left(6\sqrt{2}\ ,\ \frac{3}{4}\pi\right)$, then a – b = ...

(a) – 3 (b) 3 (c) zero (d) 6

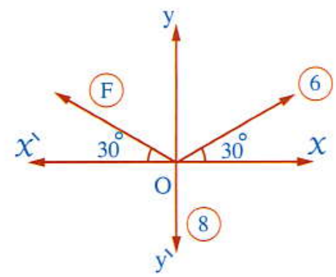
(5) Three coplanar forces $\overrightarrow{F_1} = 6\,\vec{i} + 7\,\vec{j}$, $\overrightarrow{F_2} = a\,\vec{i} - 9\,\vec{j}$, $\overrightarrow{F_3} = 5\,\vec{i} + b\,\vec{j}$ act at a particle and they are in equilibrium , then a + 2 b = ...

(a) – 9 (b) 5 (c) 7 (d) – 7

(6) If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three coplanar equilibrium forces meeting at a point , and $\overrightarrow{F_1} = 2\,\overrightarrow{i} - 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = 3\,\overrightarrow{i} + 5\,\overrightarrow{j}$, then $\overrightarrow{F_3} =$ ....

(a) $-5\,\overrightarrow{i} - 2\,\overrightarrow{j}$ (b) $-5\,\overrightarrow{i} + 2\,\overrightarrow{j}$ (c) $5\,\overrightarrow{i} + 2\,\overrightarrow{j}$ (d) $5\,\overrightarrow{i} - 2\,\overrightarrow{j}$

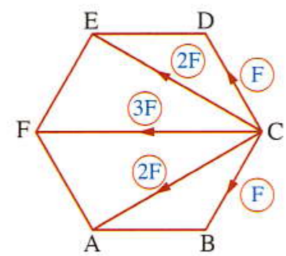
(7) If the resultant of the forces in the given figure acts in direction of y-axis , then F = ... force unit.



(a) 2 (b) 6

(c) 8 (d) 14

(8) The resultant of the forces in the opposite figure acts in direction of ....

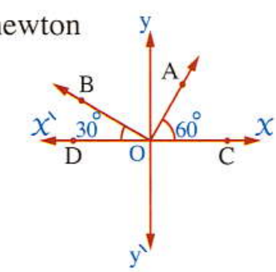


(a) $\overrightarrow{CD}$ (b) $\overrightarrow{CE}$

(c) $\overrightarrow{CF}$ (d) $\overrightarrow{CA}$

(9) In the opposite figure :

The magnitude of four coplanar forces are 1 , 2 , $4\sqrt{3}$, $3\sqrt{3}$ newton act at point O in the direction of $\overrightarrow{OX}$, $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OY}$, m (∠ AOC) = 60° , m (∠ BOD) = 30° , then the magnitude and the direction of the resultant of the forces is ....

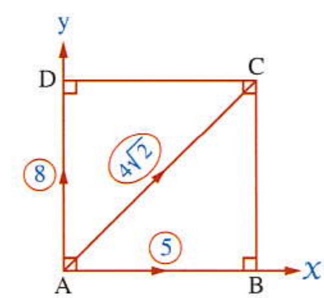


(a) (4 , 180°) (b) (4 , 0°)

(c) (3 , 0°) (d) (5 , 90°)

(10) In the opposite figure :

ABCD is a square , the forces of magnitudes 5 , 8 , $4\sqrt{2}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AC}$ respectively , then the polar form of the resultant is ....

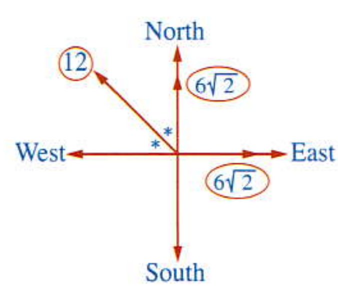


(a) (5 , 54°) (b) (15 , 60°)

(c) (15 , 53° 8`) (d) (13 , 90°)

(11) In the opposite figure :

The direction of the resultant of the forces is ....

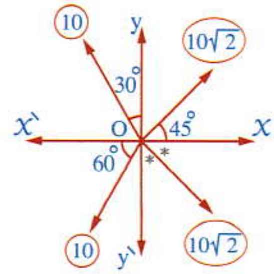


(a) South. (b) East.

(c) West. (d) North.

(12) In the opposite figure :

The magnitude of the resultant of the forces (R) = ... newton.

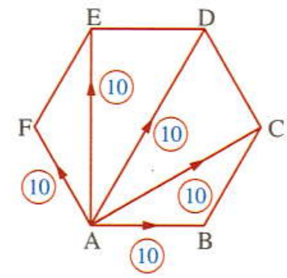


(a) 20 (b) $10\sqrt{2}$

(c) 10 (d) zero

(13) In the opposite figure :

Five equal forces each of magnitude 10 newton act at one vertex of a regular hexagon and in direction of the other vertices of the hexagon , then the magnitude of the resultant of these forces = ... newton.

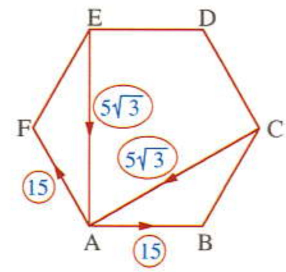


(a) 50 (b) 20

(c) $30\sqrt{3}$ (d) $20 + 10\sqrt{3}$

(14) In the opposite figure :

ABCDEF is a regular hexagon , the forces of magnitudes 15 , $5\sqrt{3}$, $5\sqrt{3}$, 15 newton act on $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$, $\overrightarrow{AF}$ respectively , then the magnitude of their resultant = ... newton.

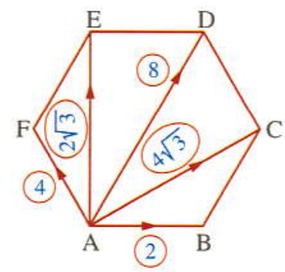


(a) 5 (b) 10

(c) 25 (d) zero

(15) In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ and $\overrightarrow{AF}$ respectively.



First : The magnitude of their resultant = ... kg.wt.

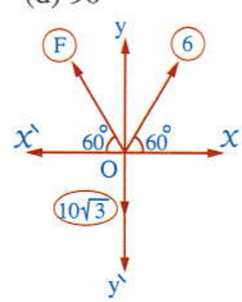
(a) $14 + 6\sqrt{3}$ (b) 20

(c) $20\sqrt{3}$ (d) $20 + \sqrt{3}$

Second : The direction of the resultant inclined by an angle of measure .... with $\overrightarrow{AB}$

(a) 30° (b) 45° (c) 60° (d) 90°

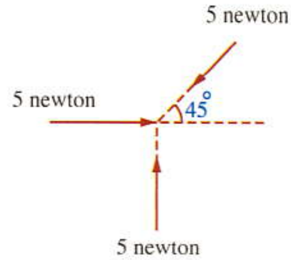
(16) If the resultant of the forces represented in the opposite figure acts in x-axis , then F = ... newton.



(a) 10 (b) 14

(c) 18 (d) 6

(17) The opposite figure represents some of forces meeting at a point , then the magnitude of the resultant of these forces = ... newton.



(a) $15\sqrt{2}$ (b) 5

(c) $5\sqrt{2}-5$ (d) zero

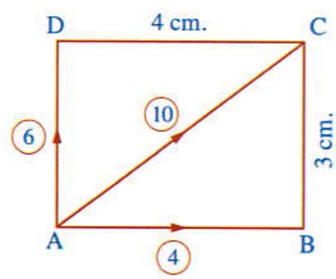
(18) Three coplanar forces meeting at a point , their magnitudes are 40 , 30 , 40 newton , the first is in direction 60° West of North , the second is towards West and the third in the direction 30° North of East , then the magnitude of their resultant equal .... newton.

(a) 30 (b) 110

(c) 60 (d) 50

(19) In the opposite figure :

ABCD is a rectangle AB = 4 cm. , BC = 3 cm. forces 4 N , 10 , 6 N acts along $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ respectively. The resultant of these forces makes with $\overrightarrow{AB}$ an angle of measure ....



(a) 45° (b) 60°

(c) 30° (d) $\sin^{-1}\left(\frac{3}{5}\right)$

(20) ABCD is a right trapezium at A and D , in which AD = CD = 4 cm. , AB = 7 cm. , M $\in \overline{AB}$ where AM = 4 cm. , a set of forces their magnitudes 25 , F and $15\sqrt{2}$ gm.wt. act at $\overrightarrow{CB}$, $\overrightarrow{CM}$ and $\overrightarrow{CA}$ respectively and the norm of the resultant of these forces equals 45 gm.wt. , then the value of F = ... gm.wt.

(a) 10 (b) 50

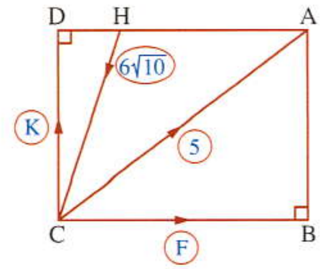
(c) 20 (d) 30

(21) The forces of magnitudes F , 12 , $8\sqrt{2}$, $10\sqrt{2}$, k newton act on a particle in the directions of East , North , Western North , Western South and South respectively. If the magnitude of the resultant = 4 newton due to North , then F – K = ... newton

(a) 24 (b) 27

(c) 12 (d) 6

(22) In the opposite figure :



The forces of magnitude F , 5 , K and $6\sqrt{10}$ N act in the rectangle ABCD in the directions $\overrightarrow{CB}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{HC}$

Such that : AB = 6 cm. , BC = 8 cm. , AH = 6 cm.

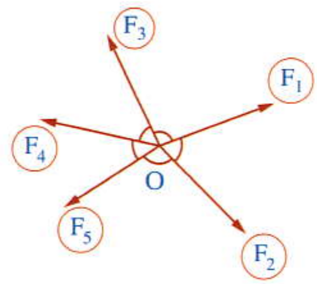
If these forces are in equilibrium , then K = ········· newton.

(a) 12 (b) 15 (c) 18 (d) 20

(23) The coplanar forces of magnitudes 5 , 4 , F , 3 , k , 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60° , if the system is in equilibrium , then F + 2 K = ··············· kg.wt.

(a) 21 (b) 6 (c) 9 (d) 15

(24) The opposite figure represents a set of forces meeting at a point (O)



Mohamed took (O) as an origin of coordinate system and the positive direction of X-axis in direction of $\overrightarrow{F_1}$

The magnitude of the resultant was R_1 and made angle of measure (θ_1) with the positive direction of X-axis and Ebrahim took (O) as an origin of coordinate system and the positive direction of X-axis in direction of $\overrightarrow{F_2}$, the magnitude of the resultant was R_2 and made an angle of measure (θ_2) with the positive direction of X-axis , then ··········

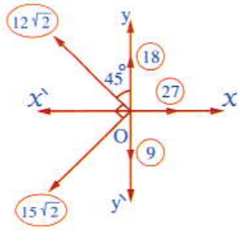
(a) $R_1 = R_2$, $\theta_1 = \theta_2$ (b) $R_1 = R_2$, $\theta_1 \neq \theta_2$

(c) $R_1 \neq R_2$, $\theta_1 = \theta_2$ (d) $R_1 \neq R_2$, $\theta_1 \neq \theta_2$

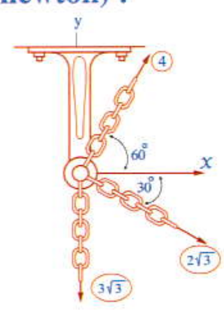
Second Essay questions

1 **Find the resultant (magnitude and direction) of the set of forces in each of the following figures (where each force magnitude is in newton) :**

(1)



(2)



2 Three coplanar forces of magnitudes 1 , 2 , $\sqrt{3}$ newton act at M , their directions are $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ respectively where m ($\angle$ AMB) = 60° , m ($\angle$ BMC) = 30° , m ($\angle$ AMC) = 90° , find the resultant. « 4 newton , in direction of $\overrightarrow{MB}$ »

3 The forces 8 , $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point , the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order.
Find the magnitude and direction of the resultant of these forces.

« 4 newton , in direction of 4th force »

4 The coplanar forces of magnitudes 2 , $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act at a point.
If the measures between the first force and the second force is 45° , the measure between the second and the third is 105° and the measure between the third and the fourth is 120° taken in the same cyclic order , find the resultant of these forces.

« $\sqrt{13}$ newton , 11° 19̀ with 2nd force»

5 Five coplanar forces meeting at a point , their magnitudes are 9 , 6 , $4\sqrt{2}$, $5\sqrt{2}$ and 5 newton act due to East , North , Western North , Western South and in the direction of South respectively. Prove that the set of forces are in equilibrium.

6 Three coplanar forces of magnitudes 60 , 88 and 60 gm.wt. act at a point , the 1st is towards North , the second is in the direction 30° South of West and the 3rd in the direction 30° South of East.
Find the magnitude of the resultant of these forces and its direction.

« 28 gm.wt. , 30° South of West »

7 Four coplanar forces act on a particle the first of magnitude 4 newton acts in the Eastern direction , the second of magnitude 2 newton , acts in direction 60° North of the East , the third of magnitude 5 newton , acts in direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° West of the South.
Find the magnitude and direction of their resultant. « 4 N. , 120° »

8 The forces of magnitudes 2 F , 3 F and 4 F newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order.
Find the magnitude and the direction of the resultant of these forces.

« $\sqrt{3}$ F newton , perpendicular to the force 3 F »

9 ABC is an equilateral triangle. M is the point of intersection of its medians.
the forces of magnitude 15 , 20 and 25 newton act on a particle at the point M in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$
Find the magnitude and the direction of the resultant of these forces.

« $5\sqrt{3}$ newton , 30° with $\overrightarrow{MA}$ »

10 ΔABC is an isosceles triangle where $m(\angle BAC) = 120°$, the forces of magnitudes 4 , $6\sqrt{3}$, 4 newton act at A in the directions $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CA}$ respectively.
Find the magnitude and the direction of the resultant of these forces.

« $10\sqrt{3}$ newton in the direction of $\overrightarrow{CB}$ »

---

11 Four coplanar forces of magnitude 2 , 1 , 4 and $3\sqrt{3}$ N. act at a point A in directions of $\overrightarrow{BC}$, $\overrightarrow{BA}$, $\overrightarrow{CA}$ and $\overrightarrow{AD}$ where ABC is an equilateral triangle and D is the midpoint of $\overline{BC}$
Find the magnitude and direction of their resultant. « 1 newton in the direction of $\overrightarrow{AC}$ »

---

12 ABCD is a rectangle where $AB = 4$ cm. , $BC = 3$ cm. the forces of magnitudes 2 , 5 and 3 kg.wt. act at the point A in the directions $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.
Find the resultant of these forces and the measure of its angle of inclination on $\overrightarrow{AB}$

« $6\sqrt{2}$ kg.wt. , $45°$ »

---

13 ABCD is a rectangle in which $AB = 8$ cm. , $BC = 6$ cm. , $E \in \overline{CD}$ where $ED = 6$ cm. , a set of forces their magnitudes 12 , 40 , $26\sqrt{2}$ and 4 newton act at $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{AE}$ and $\overrightarrow{AD}$ respectively.
Find the magnitude and the direction of the resultant of these forces.

« $6\sqrt{2}$ newton , $45°$ with $\overrightarrow{AB}$ »

---

14 ABCD is a rectangle in which : $AB = 21$ cm. , $BC = 9$ cm. The point $O \in \overline{AB}$ where $AO = 9$ cm. four forces of magnitudes 4 , 10 , 6 and $12\sqrt{2}$ kg.wt. act at the point O in the directions $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{BC}$ and $\overrightarrow{OD}$ respectively.
Find the magnitude of the resultant of these forces and prove that it is parallel to $\overrightarrow{BC}$

« 24 kg.wt. »

---

15 ABCDEF is a regular hexagon , the forces of magnitudes 8 , $6\sqrt{3}$, 5 , $4\sqrt{3}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant. « $\sqrt{651}$ newton , $40° \, 9'$ with $\overrightarrow{AB}$ »

---

16 ABCDHE is a regular hexagon. Forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AH}$, $\overrightarrow{AE}$ respectively.
Find the magnitude and the direction of their resultant. « 20 kg.wt. , $60°$ with $\overrightarrow{AB}$»

---

17 ABCDEF is a regular hexagon. M is the point of intersection of its diagonals.
the forces of magnitudes 4 , 1 , 4 , 5 , 2 and 3 gm.wt. act at M in the directions of $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$, $\overrightarrow{MD}$, $\overrightarrow{ME}$ and $\overrightarrow{MF}$
Find the resultant of these forces and prove that it is in the direction of $\overrightarrow{MD}$ « 2 gm.wt. »

18 ABC is a right-angled triangle at B where AB = 80 cm. , BC = 60 cm. , $D \in \overline{AC}$ where BD = DC

The four forces of magnitudes 8 , 12 , 15 and 10 newton act at the point B in the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ and $\overrightarrow{BD}$ respectively.

Find the resultant of these forces and prove that it acts in $\overrightarrow{BD}$ « 15 newton »

19 ABCD is a square of side length is 12 cm. $H \in \overline{BC}$ where BH = 5 cm.

forces of magnitudes 2 , 13 , $4\sqrt{2}$, 9 gm.wt. act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$ and $\overrightarrow{AD}$ respectively.

Find the magnitude of the resultant of these forces. « $10\sqrt{2}$ gm.wt. in direction of $\overrightarrow{AC}$ »

20 ABCD is a square of side length 6 cm. The point E is the midpoint of $\overline{BC}$ and F is the midpoint of $\overline{DC}$, the five forces of magnitudes 2 , $12\sqrt{5}$, $6\sqrt{2}$, $4\sqrt{5}$ and 4 kg.wt. act at the point A in the directions of $\overrightarrow{AB}$, $\overrightarrow{AE}$, $\overrightarrow{CA}$, $\overrightarrow{AF}$ and $\overrightarrow{AD}$ respectively.

Find the magnitude and the direction of the resultant of these forces. « 30 kg.wt. , 36° 52' 12" »

21 ABCD is a square , $E \in \overline{AD}$, four forces of magnitudes 4 , $4\sqrt{3}$, $10\sqrt{2}$, F kg.wt. act at point B in the directions $\overrightarrow{BA}$, $\overrightarrow{BE}$, $\overrightarrow{DB}$, $\overrightarrow{BC}$, if these forces are in equilibrium , find m (∠ ABE) and the value of F « 30° , $2\left(5-\sqrt{3}\right)$ kg.wt. »

22 The coplanar forces of magnitudes 5 , 4 , F , 3 , K and 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60°

Find the magnitude of F and K that makes the system in equilibrium. « 9 , 6 kg.wt. »

23 The forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$, K newton act on a particle in the directions of East , North , Western North , Western South and South respectively.

Find the values of F and K if the magnitude of the resultant = 2 newton due to North.

« 9 , 3 newton »

24 Forces of magnitudes F , $4\sqrt{3}$, $12\sqrt{3}$, 36 gm.wt. act at a particle. The last three forces are in the directions of North , 60° West of North , 60° South of East respectively. If the resultant of these four forces = 8 gm.wt. in magnitude in the direction of East.

Determine the value of F and its direction. « 16 gm.wt. , 60° North of East »

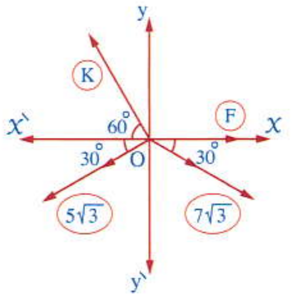
25 The forces of magnitudes F , 8 , K , 5 , $8\sqrt{3}$ newton act at a point in the directions of : East , 30° East of North , North , West and South respectively.

Find the values of F and K if the resultant is 4 newton in magnitude in the direction of 60° North of East. « 3 , $6\sqrt{3}$ newton »

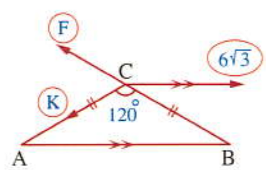
26 ABCD is a right trapezium at A and D , in which AD = CD = 40 cm. , AB = 70 cm. , $M \in \overline{AB}$ where AM = 40 cm. , a set of forces their magnitudes 25 , F , $10\sqrt{2}$ and 35 gm.wt. act at $\overrightarrow{CB}$, $\overrightarrow{CM}$, $\overrightarrow{CA}$ and $\overrightarrow{CD}$ respectively and the norm of the resultant of these forces equals 50 gm.wt. Find F « F = 10 gm.wt. »

27 **In each of the following figures find the magnitudes of F and K in newton that makes the system in equilibrium :**

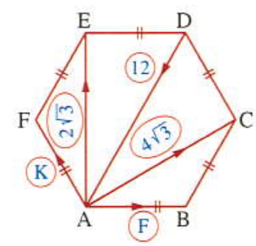
(1)



(2)



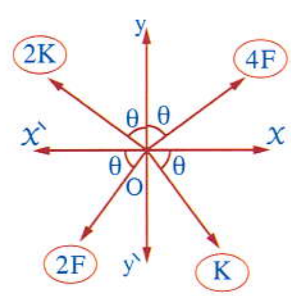
(3)



28 Coplanar forces of magnitudes F , $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act on a particle. The first force acts in the east direction. The angle between the first and the second force is of measure 45° , the angle between the second and the third force is of measure 105° , the angle between the third and the fourth force is of measure 120°. If the magnitude of their resultant is $3\sqrt{2}$ newton , then find the value of F and measure of the angle between the resultant and the first force. « 3 newton , 45° »

29 ABCDEF is a regular hexagon.
Forces of magnitudes 4 , $2\sqrt{3}$, F , $2\sqrt{3}$ and K kg.wt. act in the directions of $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ and $\overrightarrow{AF}$ respectively.
If the resultant of these forces is of magnitude 20 kg.wt. in the direction of $\overrightarrow{AD}$
Find the values of F , K « 10 , 4 kg.wt. »

30 **In the opposite figure :**
Four coplanar forces act at the point (O) in the directions shown in the figure where $\sin\theta = \frac{4}{5}$ and the resultant of these forces is $8\sqrt{2}$ N. and makes an angle of measure 135° with $\overrightarrow{OX}$, then find the values of F , K « 3 , 14 newton »



31 If $\overrightarrow{F_1} = 5\,\vec{i} + 3\,\vec{j}$, $\overrightarrow{F_2} = a\,\vec{i} + 6\,\vec{j}$, $\overrightarrow{F_3} = -14\,\vec{i} + b\,\vec{j}$ are three coplanar forces meeting at a point and their resultant is $\vec{R} = \left(10\sqrt{2} , 135°\right)$, then find the values of a and b
« a = −1 , b = 1 »

Exercise

4

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point
(The triangle of forces rule - Lami's rule)

From the school book

Remember Understand Apply Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the .... of the included angle between the other two forces.

(a) cosine (b) sine (c) tangent (d) cotangent

(2) If a body is in equilibrium under action of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, then ....

(a) $\overrightarrow{F_1} = \overrightarrow{F_2}$

(b) $F_1 = F_2$

(c) $\overrightarrow{F_1} + \overrightarrow{F_2} \neq 0$

(d) $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are not on the same line.

(3) If a body is kept in equilibrium under action of several forces , then the least number of forces could cause equilibrium equals ....

(a) 1 (b) 2 (c) 3 (d) 4

(4) The least number of coplanar unequal in magnitude forces could be in equilibrium is ....

(a) 1 (b) 2 (c) 3 (d) 14

(5) If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ = ...

(a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero

(6) Three equal forces in magnitude meeting at a point and they are in equilibrium , then the measure of the angle between each two forces = ...

(a) 60° (b) 90° (c) 120° (d) 150°

(7) If $\vec{F}$ is in equilibrium with two perpendicular forces of magnitudes 8 newton and 15 newton , then F = ... newton.

(a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$

(8) If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then F = ... newton.

(a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 6

(9) Which of the following sets of forces could be in equilibrium ?

① 8 newton , 8 newton , 8 newton.

② 8 newton , 8 newton , 16 newton.

③ 8 newton , 8 newton , 20 newton.

(a) ① only. (b) ② only. (c) ① , ② (d) ② , ③

(10) Which of the following systems of forces could not be in equilibrium ?

(a) 10 newton , 10 newton , 5 newton (b) 4 newton , 6 newton , 10 newton

(c) 11 newton , 7 newton , 8 newton (d) 8 newton , 4 newton , 14 newton

(11) Three coplanar forces not on the same straight line meeting at a point are in equilibrium , the magnitude of two forces of them are 7 and 3 newton , then the magnitude of the third could be .... newton.

(a) 10 (b) 4 (c) 5 (d) 3

(12) Three coplanar forces are in equilibrium act at a particle , the measure of the angle between the first two forces is 60° , and between the second and third forces is 150° then the ratio between forces is ....

(a) $1 : 1 : \sqrt{3}$ (b) $1 : 2 : \sqrt{3}$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $\sqrt{3} : \sqrt{3} : 1$

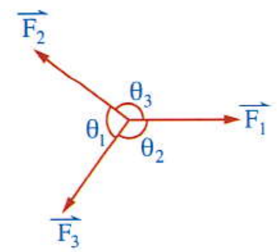
(13) The force which is in equilibrium with two perpendicular forces F , F newton makes with one of the two forces an angle of measure ....°

(a) 90 (b) 120 (c) 135 (d) 150

(14) Three coplanar forces of magnitudes 5 , 6 , 7 newton act at a particle. If the forces are in equilibrium , then the cosine of the angle between the second and the third force = ...

(a) $\frac{7}{5}$ (b) $\frac{-5}{7}$ (c) $\frac{15}{17}$ (d) $\frac{1}{2}$

(15) If a body is in equilibrium under action of three forces as shown in the figure.



Which of the following statements is true ?

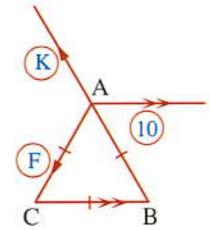
① $F_1 + F_2 + F_3 = \text{zero}$ ② $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \overrightarrow{\text{zero}}$

③ $\dfrac{F_1}{\sin\theta_1} = \dfrac{F_2}{\sin\theta_2} = \dfrac{F_3}{\sin\theta_3}$

(a) ① , ② only. (b) ② , ③ only.

(c) ③ only. (d) ① , ② , ③

(16) In the opposite figure :

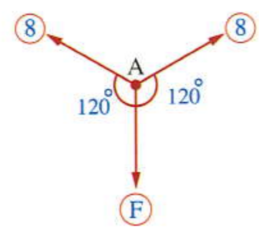


If the three coplanar forces meeting at a point and in equilibrium

, then F = ... N.

(a) 10 (b) 12 (c) 20 (d) 13

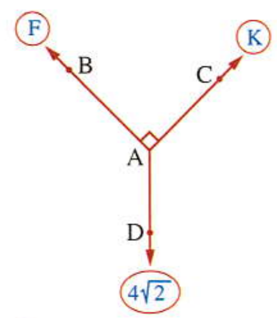
(17) Particle A is balanced under action of three forces as shown in the opposite figure where $\overrightarrow{F}$ is balanced with two forces the magnitude of each is 8 newton and makes an angle of measure 120° with each of them, then F = ... newton.



(a) zero (b) 8

(c) 16 (d) 8 sin 120°

(18) In the opposite figure :

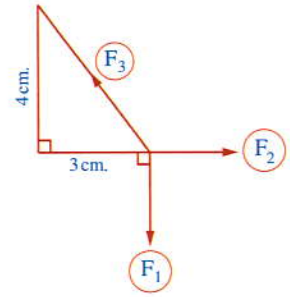


Three equilibrium forces of magnitudes F , k and $4\sqrt{2}$ newton , m (∠ BAC) = 90° , m (∠ BAD) = 135° , then (F , K) = ...

(a) (4 , 4) (b) $\left(4 , \sqrt{2}\right)$

(c) $\left(\sqrt{2} , 4\right)$ (d) (2 , 2)

(19) In the opposite figure :

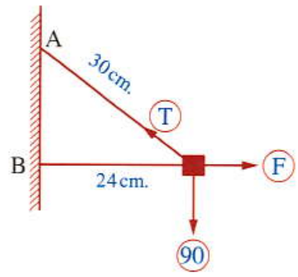


A body is in equilibrium under action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order , then $F_1 : F_2 : F_3 = $ ....

(a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

(20) In the opposite figure :

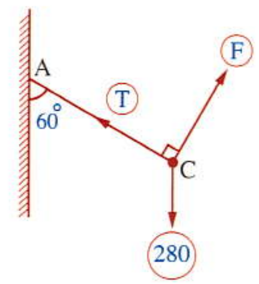
A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long. The body is pulled by a horizontal force. It comes to equilibrium when it is 24 cm. apart from the wall $\overline{AB}$ then T – F = ... gm.wt.



(a) 150
(b) 120
(c) 50
(d) 30

(21) In the opposite figure :

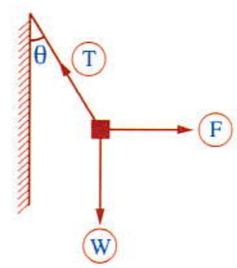
A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T}$ = ...



(a) 2
(b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{3}}$
(d) $\sqrt{3}$

(22) In the opposite figure :

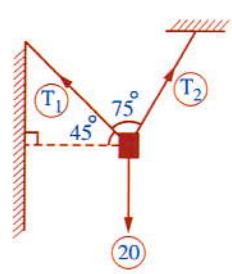
A body of weight (W) newton is suspended from the end of a string. The other end of the string is fixed to a vertical wall. The body is pulled by a horizontal force of magnitude (F) newton. The body is in equilibrium when the string makes an angle θ to the wall which of the following statements is false in case of equilibrium ?



(a) F = W tan θ
(b) $\overrightarrow{W} + \overrightarrow{F} + \overrightarrow{T} = \overrightarrow{Zero}$
(c) $T^2 = F^2 + W^2$
(d) T = F + W

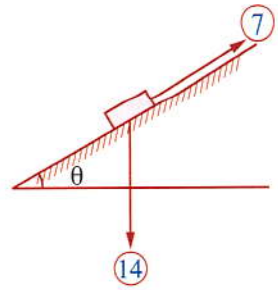
(23) In the opposite figure :

The weight of a body is 20 kg.wt , the body is in equilibrium then $\frac{T_1}{T_2}$ = ...



(a) $\frac{1}{2}$
(b) $\frac{1}{\sqrt{2}}$
(c) $\frac{2}{3}$
(d) $\frac{\sqrt{3}}{2}$

(24) In the opposite figure :

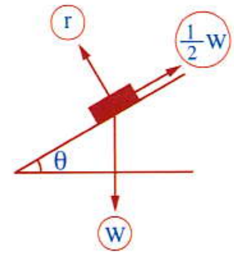


If the body is in equilibrium when it is placed on an inclined smooth plane , then m (∠ θ) = ...

(a) 60° (b) 45°

(c) 30° (d) 75°

(25) In the opposite figure :

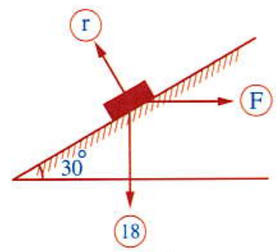


If the body is in equilibrium under action of forces shown , then m (∠ θ) = ...

(a) 30° (b) 60°

(c) 45° (d) 15°

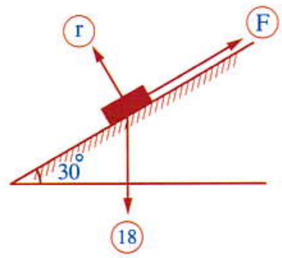
(26) In the opposite figure :



A body of weight 18 newton is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force of magnitude F newton , then F + r = ... newton.

(a) $6\sqrt{3}$ (b) $12\sqrt{3}$ (c) $18\sqrt{3}$ (d) $24\sqrt{3}$

(27) In the opposite figure :



A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° , it is kept in equilibrium by a force of magnitude F newton in the direction of the plane upward , then F + R = ... newton.

(a) $6\sqrt{3}$ (b) $9\sqrt{3}$ (c) $18\sqrt{3}$ (d) $9 + 9\sqrt{3}$

(28) The weight of a body is 6 kg.wt. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium by a horizontal force , then the magnitude of this horizontal force = ... kg.wt.

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) 6

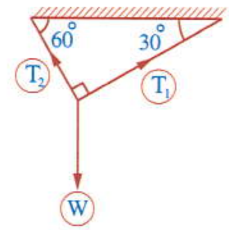
(29) The weight of a body is 6 newton. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium with a force of magnitude 49 newton which makes an angle of measure θ upwards the line of greatest slope of the plane , then cos θ = ...

(a) $\frac{3}{49}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

(30) The weight of a body is 20 kg.wt. It is placed on a smooth inclined plane makes an angle θ to the horizontal , where $\sin \theta = \frac{3}{5}$ and it prevent from sliding by a horizontal force F , then F = ... kg.wt.

(a) 30 (b) 15 (c) 10 (d) $5\sqrt{3}$

(31) In the opposite figure :



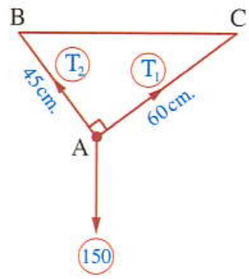
A body of weight (W) is hanged by two strings.

The two strings inclined to the horizontal as shown in the figure , then T_1 = ...

(a) $\frac{1}{3}$ W (b) $\frac{1}{2}$ W

(c) $\frac{\sqrt{3}}{3}$ W (d) $\frac{\sqrt{3}}{2}$ W

(32) In the opposite figure :



A body of weight 150 gm.wt. is in equilibrium by suspending it by two perpendicular strings of lengths 60 cm. and 45 cm. , and the other two ends C and B are on a horizontal line , then : $T_2 - T_1$ = ... gm.wt.

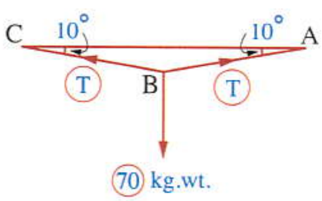
(a) 120 (b) 90

(c) 60 (d) 30

(33) A body of weight 28 kg.wt. is suspended by two perpendicular strings , if the measure of the angle between one strings and the line of the weight is 120° , then the magnitude of the tension of this strings equals .... kg.wt.

(a) 14 (b) 28 (c) $14\sqrt{3}$ (d) $28\sqrt{3}$

(34) In the opposite figure :



A man of weight 70 kg.wt. is walking on a rope.

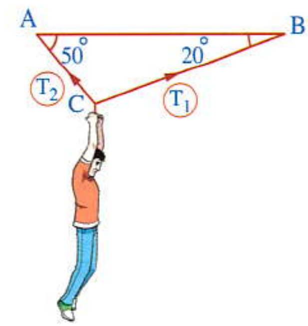
If the rope lowered 10° from the horizontal when the man becomes at the middle of the rope , then the tension in the rope (T) = ... kg.wt.

(a) $\frac{70 \sin 20°}{\sin 100°}$ (b) $\frac{70 \sin 100°}{\sin 160°}$

(c) $\frac{70 \cos 100°}{\sin 160°}$ (d) $\frac{\sin 100°}{70 \sin 160°}$

(35) In the opposite figure :

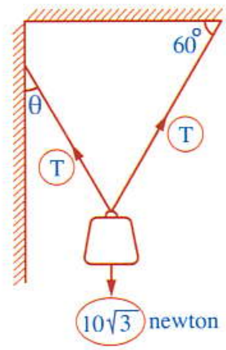
A man of weight (W) suspended vertically at C by two ropes $\overrightarrow{CB}$, $\overrightarrow{CA}$ as shown in the figure and $T_2 = 60$ kg.wt. then (W) = ... kg.wt.



(a) 87.7 (b) 70.6

(c) 60 (d) 49.8

(36) A body of weight $10\sqrt{3}$ newton is suspended by two strings as shown in the figure , then the value of θ which makes both tensions are equal is ....



(a) 15° (b) 30°

(c) 45° (d) 60°

Second Essay questions

1 Three forces of magnitudes F_1 , F_2 and 75 newton intersecting at one point they are represented by the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ of Δ ABC respectively where :
AB = 3 cm. , BC = 4 cm. and CA = 5 cm.
Find the value of each of F_1 and F_2 « 45 , 60 newton »

2 Three coplanar forces of magnitudes 60 , F and K newton meeting at a point and in equilibrium. If the angle between the 1st and the 2nd force measures 120° and between the 2nd and the 3rd measures 90°
Find the value of each of F and K « 30 , $30\sqrt{3}$ newton »

3 A body of weight 12 kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force.
Find the magnitude of each of the force and the reaction of the plane. « $4\sqrt{3}$, $8\sqrt{3}$ kg.wt. »

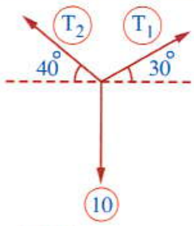
4 A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards. Find the magnitude of the weight W and the magnitude of the reaction of the plane. « 72 , $36\sqrt{3}$ newton »

5 The magnitudes of three coplanar concurrent forces are $F_1 = 8$ gm.wt. , $F_2 = 4\sqrt{3}$ gm.wt. and $F_3 = 4$ gm.wt. If these forces are in equilibrium , then find the measures of the angles between these forces. « 90° , 120° , 150° »

6 If M is the point of intersection of the two diagonals of a square ABCD , E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$ and F_1 , F_2 and 42 are the magnitudes of three forces in equilibrium act on $\overrightarrow{ME}$, $\overrightarrow{MF}$, $\overrightarrow{MD}$

Calculate the value of F_1 and F_2 « $21\sqrt{2}$, $21\sqrt{2}$ gm.wt. »

7 **In the opposite figure :**



A weight of magnitude 10 newton is suspended by two strings , the first is inclined by an angle of measure 30° to the horizontal and the second is inclined by an angle of measure 40° to the horizontal.

Find T_1 , T_2 in case of equilibrium. « 8.15 , 9.216 newton »

8 A string of length 40 cm. is fixed from its two ends at two points on a horizontal line where the distance between them is 32 cm. A body of weight 180 kg.wt. is suspended at the midpoint of the string. Find the values of the tensions in the two branches of the string. « 150 , 150 kg.wt. »

9 A body of weight 15 kg.wt. is placed on a smooth plane inclines to the horizontal at an angle of measure $\sin^{-1}\frac{1}{2}$, a force inclined to the horizontal at an angle of measure 60° acted on the body to keep it in equilibrium. Find the magnitude of each of the force and the reaction of the plane. « $5\sqrt{3}$, $5\sqrt{3}$ kg.wt. »

10 A body of weight (W) kg.wt. is placed on a smooth plane inclines to the horizontal at an angle of measure $\cos^{-1}\frac{1}{2}$, it is kept in equilibrium by means of a force inclined to the horizontal at an angle of measure 30° upwards. Find the magnitude of each of the force and the reaction of the plane in terms of (W) « F = r = W »

11 A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string in case of equilibrium. « 160 , 120 gm.wt. »

12 A body of weight 6.5 newton is suspended by two strings of lengths 0.5 and 1.2 m. the two other ends are fixed at two points on a horizontal line such that the strings are perpendicular to each other. Find the tension in each of the two strings in case of equilibrium. « 6 , 2.5 newton »

13 A weight of 50 gm.wt. is suspended by means of two perpendicular strings. If the tensions in the two strings are of magnitudes $25\sqrt{3}$, 25 gm.wt. Find the measures of the angles which the two strings are inclined to the vertical in case of equilibrium. « 30° , 60° »

14 A weight of 200 gm.wt. is suspended at the end of a light string , the other end of which is attached to the ceiling of a room. The weight is pulled by a horizontal force until the string is inclined to the vertical by an angle of measure 30°

Find the magnitude of each of the horizontal force and the tension in the string.

« $\frac{200\sqrt{3}}{3}$, $\frac{400\sqrt{3}}{3}$ gm.wt. »

---

15 A weight of 60 gm.wt. is suspended at the end of a string and the other end is fixed at a point of a vertical wall. A horizontal force of magnitude F acts on the weight in a perpendicular direction to the wall , the weight becomes in equilibrium when the string is inclined to the wall with an angle of measure θ where $\tan\theta = \frac{3}{4}$

Find the magnitude of each of F and the tension in the string. « 45 , 75 gm.wt. »

---

16 A weight of 16 newton is suspended at the end of a light string and the other end is fixed at a point of a vertical wall. A force of magnitude F acts on the weight in a perpendicular direction of the string till it become in equilibrium when the string is inclined to the wall with an angle of measure 30°

Find the magnitude of the force F and the tension of the string. « 8 , $8\sqrt{3}$ newton »

---

17 The ball of a pendulum of weight 600 gm.wt. is displaced until the string makes an angle of measure 30° with the vertical under the action of a force perpendicular to the string.

Find the magnitude of each of the force and the tension in the string. « 300 , $300\sqrt{3}$ gm.wt. »

---

18 A light string of length 170 cm. , its end A is fixed at a point of a ceiling of a room. From the other end B there is a lamp of weight 34 gm.wt. Find the magnitude of each of the tension and the required force to make the lamp in equilibrium at a distance 80 cm. down the ceiling in each of the following cases :

(1) If the force is horizontal. « 72.25 , 63.75 gm.wt. »

(2) If the force is perpendicular to $\overline{AB}$ « 16 , 30 gm.wt. »

---

19 A body of weight 6 N. is placed on a smooth plane inclines to the horizontal by an angle θ. The body is kept in equilibrium by means of a force of magnitude $2\sqrt{3}$ N. inclines to the line of greatest slope of the plane by an angle of measure θ up.

Find the value of θ and the magnitude of the normal reaction of the plane.

« $\theta = 30°$, $r = 2\sqrt{3}$ N. »

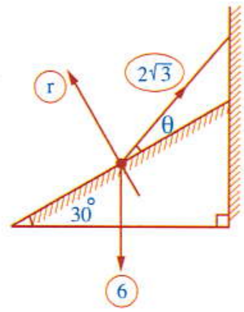
20 A body is in equilibrium on a smooth plane inclines to the horizontal at an angle under the action of a force acting in the direction of the plane upwards. Its magnitude equals half the magnitude of weight of the body. Find the measure of the angle of inclination of the plane and the magnitude of the reaction of the plane. « $30°$, $\frac{\sqrt{3}}{2}$ W »

---

21 **In the opposite figure :**

A body of weight 6 kg.wt. is placed on a smooth plane inclines to the horizontal by an angle of measure 30° The body is kept in equilibrium by a tension force (T) of magnitude $2\sqrt{3}$ kg.wt. The tension force acts along one end of the string of which is fixed by the body and the other end at a point on a vertical wall.

Find the measure of the angle of inclination of the string to the plane and the magnitude of the reaction of the plane on the body.



« $30°$, $2\sqrt{3}$ kg.wt. »

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22 A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ The body is prevented from sliding by a force form with the line of the greatest slope an angle of measure 30° upwards.

Find the magnitude of the force and the reaction of the plane. « $100\sqrt{3}$, $100\sqrt{3}$ gm.wt.»

---

23 A body of weight 800 gm.wt. is placed on a smooth plane inclines to the horizontal by an angle θ , where $\sin \theta = 0.6$ the body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

« 600 , 1000 gm.wt. »

---

24 A smooth string of length 30 cm. is attached by its end in the two points A , B such that $\overline{AB}$ is horizontally , AB = 18 cm. if a smooth ring of weight 150 gm.wt. slides on the string. Prove that in the case of equilibrium the lengths of the two parts of the strings are equal , then find the tension in each part. « 93.75 gm.wt. »

---

25 A body of weight 24 newton is suspended at one end of a string of length 130 cm. , the other end is fixed at a point of a vertical wall. A horizontal force acts on the body to become in equilibrium. Find the magnitudes of the force and the tension in the string.

(1) When the body is at a distance = 50 cm. from the wall. « 10 , 26 newton »

(2) When the string is inclined to the wall with an angle of measure 30° « $8\sqrt{3}$, $16\sqrt{3}$ newton »

26 A body of weight 72 gm.wt. is suspended at one end of a string. The other end of the string is fixed at a point A on a vertical wall. Another string is attached to the first one at a point B 25 cm. a part from A and pulled horizontally until the point B becomes 7 cm. apart the wall.

Find the tension in the horizontal string and in each part of the first string.

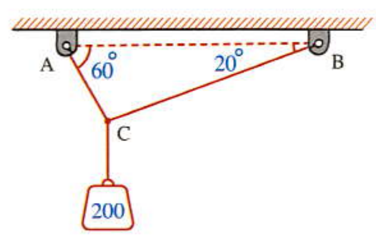
« 21 , 75 , 72 gm.wt. »

---

27 A particle of weight 200 gm.wt. is suspended by two light strings. One of them is inclined to the vertical by an angle of measure θ and the other inclined to the vertical by an angle of measure 30°. If the magnitude of the tension in the first string is 100 gm.wt. , then find θ and the magnitude of the tension in the second string. « 60° , $100\sqrt{3}$ gm.wt. »

---

28 The opposite figure represents a weight of magnitude 200 newton hanged vertically at a point C by two strings $\overline{BC}$ and $\overline{AC}$ which make with the horizontal the angles of measures 20° , 60° respectively find in the state of equilibrium the tension in the two strings to the nearest newton.



« 102 , 191 newton »

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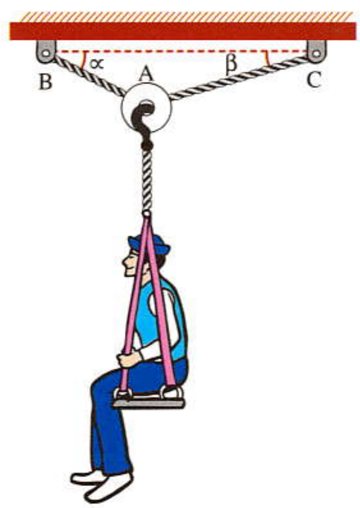
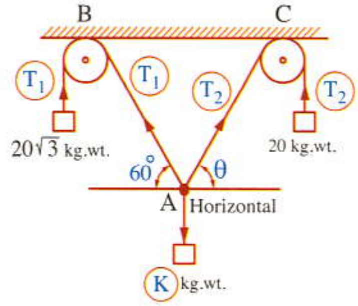
29 **Join with navigation :** The operation of saving a nautilus is done by using the captain chair which is hanged in a bully. Two ropes $\overline{AB}$ and $\overline{AC}$ are passing over the bully making two angles α , β with the horizontal whose measures are 25° , 15° respectively. If the tension in the rope $\overline{AB}$ equals 80 newton , find the weight of the nautilus and the chair together and the tension in the rope $\overline{AC}$ in the state of equilibrium.



« 53 , 75 newton »

30 **In the opposite figure :**

A weight of magnitude K is suspended by an end of a string , the other end is suspended by two strings passing over two smooth pulleys at B , C and carries two weights of magnitudes $20\sqrt{3}$ kg.wt. and 20 kg.wt.
Find the value of the weight K and the measure of angle θ in state of equilibrium.



« 40 kg.wt. , 30° »

Third Higher skills

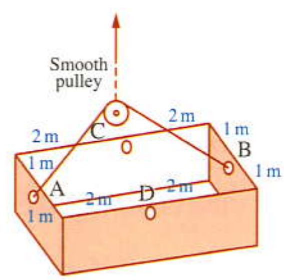
1 A body of weight 400 gm.wt. is suspended at point A by a string. From a point B on the string another string is attached and pulled horizontally by a second string $\overline{BC}$ passing over a smooth fixed pulley and carries at its other end a body of weight 300 gm.wt. Find the inclination of $\overline{AB}$ with the vertical and the tension in each of the two strings $\overline{AB}$, $\overline{BC}$

« 36° 52̀ , 500 , 300 gm.wt. »

2 $\overline{AB}$ is a light string , its two ends are fixed at two points on a horizontal line. C and D are two points of the string. Two weights K and 20 gm.wt. are suspended from C and D respectively. If the set of forces are in equilibrium when $\overline{CD}$ is horizontal and the two parts $\overline{AC}$ and $\overline{BD}$ of the string incline to the vertical by angles of measures 30° and 60° respectively. Find the magnitudes of tensions in the three parts of the string and the value of K

« $40\sqrt{3}$, $20\sqrt{3}$, 40 kg.wt. , K = 60 gm.wt. »

3 A box of weight 20 newton is suspended by a string as in the opposite figure. If the box can be fixed with the string through two methods one of them is A , B and the other is C , D which of these two methods can produce the less tension in the string to become the set in equilibrium ?



Exercise

5

Follow : The equilibrium

(Meeting lines of action of three equilibrium forces)

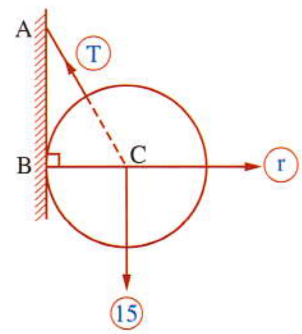
From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from the given ones :

(1) In the opposite figure :

A solid uniform sphere of weight 15 gm.wt. and radius length 10 cm. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then (r , T) = ...



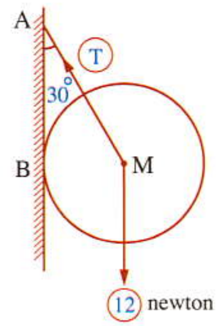
(a) $\left(4\sqrt{3} , 8\sqrt{3}\right)$ (b) $\left(5\sqrt{3} , 10\sqrt{3}\right)$

(c) (5 , 10) (d) $\left(5\sqrt{3} , 8\sqrt{3}\right)$

(2) In the opposite figure :

If the sphere is in equilibrium , then T – r = ... newton

(Where r is the magnitude of the wall reaction on the sphere)



(a) $8\sqrt{3}$ (b) $4\sqrt{3}$

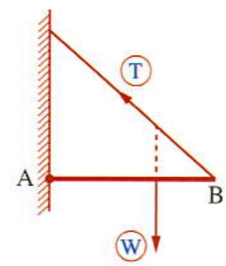
(c) 4 (d) 8

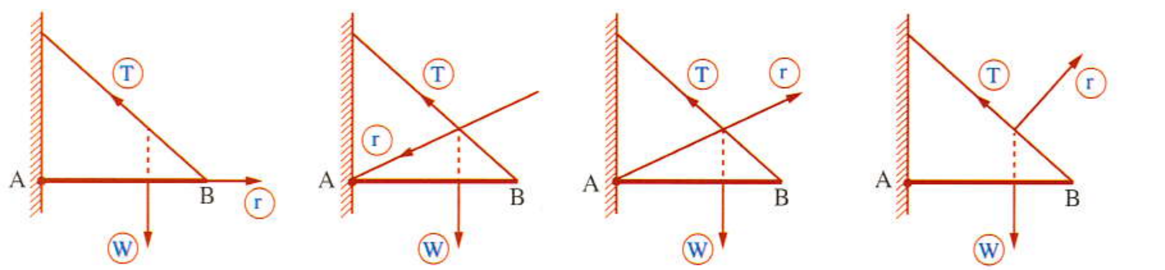
(3) A solid uniform sphere of weight 20 kg.wt. and its radius length 5 cm. If it is in equilibrium by a string of length 5 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then the reaction of the vertical plane r = ·········· kg.wt.

(a) $\frac{20}{\sqrt{3}}$ (b) 20 (c) $\frac{20}{\sqrt{5}}$ (d) zero

(4) In the opposite figure :

$\overline{AB}$ is a rod. The end A is attached to a hinge fixed on a vertical smooth wall , if the rod is in equilibrium , then which of the following figures represent the correct direction of the reaction of the hinge ?

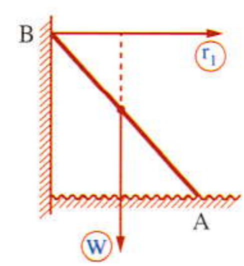


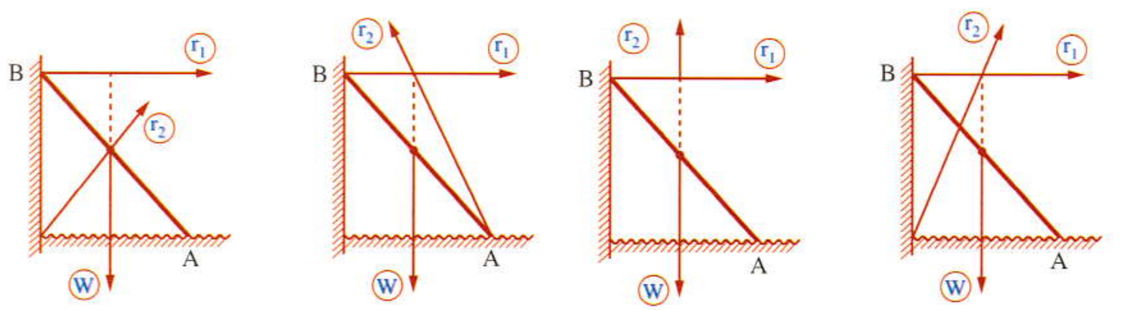


(a) (b) (c) (d)

(5) In the opposite figure :

$\overline{AB}$ is a uniform rod and its weight is wrests at the end A against a horizontal rough ground , and the end B on a vertical smooth wall. Then which of the following figures represent the correct direction of the reaction of the ground ?





(a) (b) (c) (d)

(6) In the opposite figure :

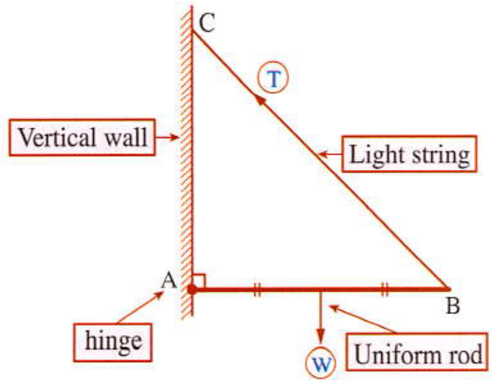
The direction of the reaction of the hinge on the rod at A ....

(a) in the direction of $\overrightarrow{AB}$

(b) in the direction of $\overrightarrow{AC}$

(c) bisects $\overline{BC}$

(d) perpendicular to $\overline{BC}$



(7) In the opposite figure :

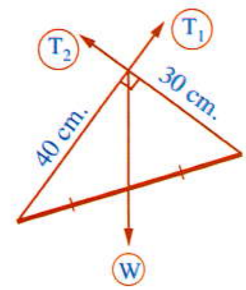
$T_1 : T_2 : W =$ ....

(a) 5 : 3 : 4

(b) 8 : 5 : 4

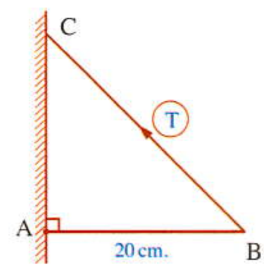
(c) 4 : 3 : 5

(d) 5 : 4 : 3



(8) In the opposite figure :

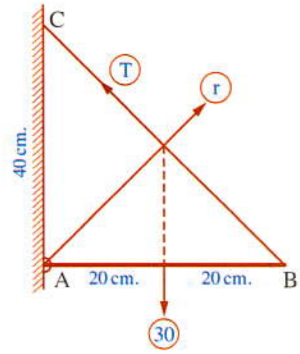
$\overline{AB}$ is uniform rod with length 20 cm. and weight 30 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B of length $20\sqrt{2}$ cm. fixed at a point C on the wall just above A , then magnitude of the reaction of the hinge = ... newton.



(a) $10\sqrt{2}$ (b) 10 (c) 15 (d) $15\sqrt{2}$

(9) In the opposite figure :

$\overline{AB}$ is a uniform rod with length 40 cm. and weight 30 newton is connected to a hinge at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and C where C is located on the wall just above A , AC = 40 cm.



First : The reaction of the hinge r = ... newton.

(a) 30 (b) 20

(c) $40\sqrt{2}$ (d) $15\sqrt{2}$

Second : The tension of the string T = ... newton.

(a) $15\sqrt{2}$ (b) 30 (c) 20 (d) $40\sqrt{2}$

(10) A uniform rod of weight 20 newton which is movable around a hinge at on its ends is pulled a side by a horizontal force of magnitude 10 newton acting on the other end , then the measure of the angle of inclination of the rod to the vertical when it is in equilibrium = ...

(a) 60° (b) 45° (c) 30° (d) 90°

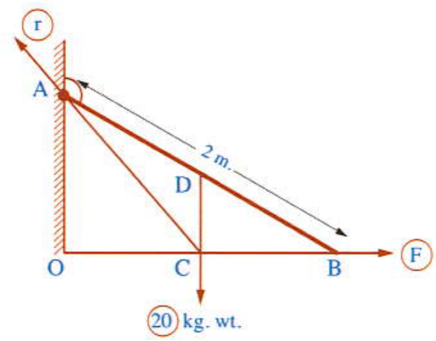
(11) A uniform rod of weight 24 newton is placed on two smooth planes inclined at two angles of measures 60° and 30° to the horizontal , then the magnitude of the pressure on each plane .... newton.

(a) 12 , 15 (b) 12 , $12\sqrt{3}$

(c) $12\sqrt{3}$, 10 (d) 15 , 13

(12) In the opposite figure :

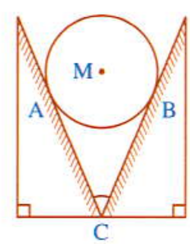
$\overline{AB}$ is a uniform rod of length 2 m. and weight 20 kg.wt. It is connected to a hinge fixed to a vertical wall at A. A horizontal force acts at B. If the rod is kept in equilibrium when it is inclined to the vertical at an angle of measure 60° , then the reaction of the hinge on the rod = ... kg.wt.



(a) $10\sqrt{3}$ (b) $10\sqrt{5}$ (c) $10\sqrt{7}$ (d) $20\sqrt{2}$

(13) The opposite figure represents a metal sphere , its weight (W) kg.wt. The sphere is placed such that it thouches two smooth planes each makes an angle of measure θ with the vertical.

If the sphere touches the planes at A and B , then the reaction of the plane at A equals .... kg.wt.



(a) $\frac{1}{2}$ W (b) W cos θ csc θ (c) W sin θ csc 2 θ (d) W cos θ csc 2 θ

Second Essay questions

1 A smooth sphere of radius length 30 cm. and of weight 200 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 20 cm. , one of its ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the touch point of the sphere and the wall.

Find the magnitudes of the tension in the string and the reaction of the wall in case of equilibrium. « 250 , 150 gm.wt. »

2 A smooth sphere of weight $10\sqrt{3}$ gm.wt. rests against a smooth vertical wall. It is suspended at a point of its surface by means of a string and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. If the string makes with the vertical an angle of measure 30°

Find the tension in the string and the reaction of the wall in case of equilibrium.

« 20 , 10 gm.wt. »

---

3 A smooth sphere of weight 15 newton is on a smooth vertical wall and suspended by a light string from a point on its surface. The other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere. If the length of the string equals the radius length of the sphere. Find the pressure on the wall and the tension in the string in case of equilibrium. « $5\sqrt{3}$, $10\sqrt{3}$ newton »

---

4 A metallic sphere of weight 15 kg.wt. is put such that it touches two smooth planes , one of them is vertical and the other inclines to the vertical by an angle of measure 30°

Find the reaction of each of the two planes. « $15\sqrt{3}$, 30 kg. wt. »

---

5 $\overline{AB}$ is a unifrom rod of length 100 cm. and weight 30 kg.wt. is suspended from its two ends A and B by means of two strings , their other ends are fixed at a pin in the ceiling at the point C , if the two strings are perpendicular and AC = 50 cm.

Find the tension in each of the two strings. « 15 , $15\sqrt{3}$ kg.wt. »

---

6 A uniform rod of length 130 cm. and weight 26 newton is suspended at its ends by two strings tied at one point. If the length of one of them is 50 cm. and the length of the other one is 120 cm. What is the position in which the rod is in equilibrium and what is the tension in each of the two strings ? « 24 , 10 newton »

---

7 $\overline{AB}$ is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod keept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall just above A and at a distance 60 cm. from A. Find the tension on the string and the reaction on the hinge at A.

« $20\sqrt{2}$, $20\sqrt{2}$ newton »

8 $\overline{AB}$ is a uniform rod of length 80 cm. and weight 24 kg.wt. The end A is attached to a hinge fixed on a vertical wall , and the end B is tied by a light string of length $80\sqrt{3}$ cm. fixed at a point C on the wall which lies directly above A and at a distance 80 cm. If the rod is in equilibrium, find the magnitude of the tension and the reaction of the hinge.

« $12\sqrt{3}$, 12 kg.wt. »

---

9 A homogeneous sphere rests on two parallel rods lie on the same horizontal plane. The distance between them equals the radius length of the sphere. Find the pressure on each rod if the weight of the sphere is 60 newton in case of equilibrium. « $20\sqrt{3}$, $20\sqrt{3}$ newton »

---

10 A sphere in which M is its centre and its radius length is 12 cm. and its weight is (W) newton rests at B against a smooth vertical wall , from a point C on its surface , it is tied by a string , its other end is fixed at A of the wall lies directly above B
If the tension in the string is 50 newton. Find the length of the string and the weight of the sphere when the reaction of the wall to the sphere equals 25 newton.

« 12 cm. , $25\sqrt{3}$ newton »

---

11 A uniform rod whose length is 80 cm. and its weight is 12 newton , the rod is freely suspended from its ends by means of two strings, and the other ends are attached to a fixed nail in the ceiling. If the two strings are perpendicular and one of them is of length 48 cm. Find in equilibrium the magnitude of the tension in each of the two strings.

« 7.2 , 9.6 newton »

---

12 $\overline{AB}$ is a uniform ladder of weight 36 kg.wt. rests at the end A against a vertical smooth wall , and the other end B on a horizontal rough ground. If the ladder is in equilibrium when its end A is at a distance 3 metres from the ground and the end B is at a distance 2.5 metres from the wall. Find the reaction of each of the ground and the wall on the ladder. «15 , 39 kg.wt. »

---

13 $\overline{AB}$ is not a uniform rod of length 60 cm. and weight 16 kg.wt. acts at a point D on the rod where AD = 20 cm. The rod is attached to a hinge at A and the hinge is fixed on a vertical wall. The end B of the rod is tied by a light string its other end is fixed at a point C on the wall lying directly above A and at a distance 80 cm. from it , then the rod becomes in equilibrium such that it is perpendicular to the wall.
Find the tension in the string and the reaction of the hinge. « $6\frac{2}{3}$, $\frac{4\sqrt{73}}{3}$ kg.wt. »

14 $\overline{AB}$ is a uniform rod of length 2 L cm. and weight 8 kg.wt. acting at its midpoint. its end A is hinged at a point in a vertical wall where its end B is attached to a light string and the other end of the string is fixed to a point C on the wall situated vertically above A If AB = AC = BC and the rod is in equilibrium. Find the tension in the string and the reaction of the hinge at A « 4 kg.wt. , $4\sqrt{3}$ kg.wt. »

15 $\overline{AB}$ is a uniform rod of length 60 cm. and weight (W) kg.wt. The end A is attached to a hinge fixed on a vertical wall and the end B is tied by a string of length 80 cm. , its other end is fixed to a point on the wall vertically above A directly and at a distance 100 cm. of it , then the rod became in equilibrium. Find the tension in the string and the reaction of the hinge , also find the measure of the angle of inclination of the reaction of the hinge to the rod. « $\frac{2}{5}$ W , $\frac{\sqrt{13}}{5}$ W kg.wt. , 33° 41̀ »

16 $\overline{AB}$ is a uniform rod of length 90 cm. , and weight (W) kg.wt. Its end A is fixed to a vertical wall by a hinge and the rod is kept in equilibrium horizontally by means of a string of length 50 cm. , one of its ends is tied at the point C on the rod at a distance 30 cm. from A , the other end of the string is fixed at a point D on the vertical wall above A directly, calculate the tension in the string and the reaction of the hinge on the rod.

« $\frac{15}{8}$ W , $\frac{\sqrt{97}}{8}$ W kg.wt. »

17 $\overline{AB}$ is a uniform rod , its end A is attached by a hinge fixed in a vertical wall. A horizontal force acts at the end B to keep the rod in equilibrium while it is inclined to the wall by an angle of measure 45° , if the weight of the rod is 4 kg.wt. acts at its midpoint , then find the magnitude of the force and the reaction of the hinge.

« 2 , $2\sqrt{5}$ kg.wt. »

18 A uniform rod which is movable around one of its ends is pulled a side by a horizontal force acting on the other end and equals half the weight of the rod. Find the measure of the angle of inclination of the rod to the vertical when it is in equilibrium and also the reaction at the first end. « 45° , $\frac{\sqrt{5}}{2}$ of the weight of the rod »

19 A uniform rod of weight 4 newton is placed on two smooth planes inclined at 30° and 60° to the horizontal. Find the magnitude of the pressure on each plane and the measure of the angle of inclination of the rod to the horizontal in state of equilibrium.

« $2\sqrt{3}$, 2 newton , 30° »

20 A smooth iron sphere of weight (W) kg.wt. rests against a vertical smooth wall and a smooth plane inclines to the horizontal at an angle θ where $\cos\theta = \frac{3}{5}$, if the sphere is in equilibrium. Find the pressure on each of the wall and the inclined plane.

« $\frac{4}{3}$ W , $\frac{5}{3}$ W kg.wt. »

---

21 A uniform rod of weight 20 kg.wt. rests at one of its ends against a smooth vertical plane and at the other end on a smooth plane inclined to the vertical at an angle of measure 60° , in the state of equilibrium. Find the magnitude of each of the two reactions of the two planes , also find the measure of the angle at which the rod inclines to the vertical.

« $\frac{20\sqrt{3}}{3}$, $\frac{40\sqrt{3}}{3}$ kg.wt. , 49° 6̀ »

---

22 A uniform rod $\overline{AB}$ of weight 8 newton acting at its midpoint is placed on two smooth perpendicular planes that are inclined to the horizontal. Such that the vertical plane of the rod and the two lines of greatest slope of the two inclined planes is perpendicular to the intersection line of the two planes. If the magnitude of the pressure on the plane at the end B is 4 newton.

Find the magnitude of the pressure on the other plane and measures of the two inclination angles of the planes to the horizontal , in the state of equilibrium.

« $4\sqrt{3}$ newton , 30° , 60° »

---

23 A uniform hollow sphere of radius length (r) and weight $12\sqrt{3}$ kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is prevented from motion on the plane by means of a string fixed at a point on its surface , the length of the string equals the radius length of the sphere. The other end of the string is fixed at a point on the inclined plane. In the state of equilibrium , prove that the string is horizontal , then find the tension in the string and the reaction of the inclined plane upon the sphere.

« 12 , 24 kg.wt. »

---

24 $\overline{AB}$ is a uniform rod can move in a vertical plane freely around a hinge at A , the other end B is tied to a string passes over a smooth pulley C exactly above A and attached to a weight equals half the weight of the rod. Find the measure of the angle of inclination of the rod to the horizontal in state of equilibrium given that AC = AB « 30° »

25 $\overline{AB}$ is a uniform rod which is 40 cm. long and weight 12 N. The rod rests with its end A on a vertical smooth wall. It is kept in equilibrium by means of a light inextensible string , one of its ends is attached to point C where $C \in \overline{AB}$ and BC = 10 cm.
, and the other end is fixed to a point D on the wall directly vertical above A.
If the rod is inclined by an angle whose measure is 60° to the vertical , then find the magnitudes of the tension in the string and the reaction of the wall. « $8\sqrt{3}$, $4\sqrt{3}$ N. »

---

26 A uniform rod $\overline{AB}$ of length 6 metres and weight 8 kg.wt. is attached to a hinge fixed in a vertical wall at its end A. The rod is kept horizontally by attaching it at a point C on the rod (where AC = 4 metres) by a string which its other end is fixed at the point D on the wall above A exactly and at a distance 4 metres from it. Calculate the magnitude of the tension in the string and the reaction of the hinge in case of equilibrium.

« $6\sqrt{2}$, $2\sqrt{10}$ kg.wt. »

Use the resolution to solve

27 A body of weight 100 newton is placed on a smooth plane inclined to the horizontal at an angle of measure θ where $\sin\theta = \frac{3}{5}$, the body is kept in equilibrium by means of a force inclines to the line of the greatest slope at an angle of measure α where $\cos\alpha = \frac{12}{13}$
Find F and the reaction of the plane. « 65 , 55 newton »

---

28 A body is placed on a smooth plane inclined to the horizontal at an angle of measure 30°, it is kept in equilibrium by means of two forces , one of them in the direction of the line of the greatest slope upwards , its magnitude = 50 newton and the second inclines to the line of the greatest slope upwards with an angle of measure 30° and its magnitude is $20\sqrt{3}$ newton. Find each of the weight of the body and the reaction of the plane.

« 160 , $70\sqrt{3}$ newton »

---

29 A smooth ring and a string passes through it. The length of the string is 40 cm. its two ends are fixed at the two points A and B on the same horizontal line , the distance between them is 20 cm. A horizontal force $\vec{F}$ acts on the ring to be in equilibrium vertically down B and the string is in tension. Find the value of F and the magnitude of tension in the string given that the weight of the ring = 400 gm.wt. « 200 , 250 gm.wt »

Unit Two

Geometry and Measurement

Exercise 6 The straight lines and the planes in the space.

Exercise 7 The pyramid.

Exercise 8 The cone.

Exercise 9 The circle.

Exercise 6

The straight lines and the planes in the space

From the school book   ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(1) Number of straight lines that are passing through a given point is ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(2) Number of straight lines that are passing through two given points is ....

(a) 1 (b) 2

(c) 3 (d) an infinite number.

(3) Number of planes that are passing through two given points is ....

(a) 1 (b) 2

(c) 3 (d) an infinite number.

(4) Number of planes that are passing through three non-collinear points is ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(5) Number of planes that are passing through three collinear points is ....

(a) Zero (b) 1 (c) 3 (d) an infinite number.

(6) All of the following cases determine a plane except ....

(a) a straight line and a point doesn't belong to it.

(b) two parallel and not coincident straight lines.

(c) two intersecting straight lines.

(d) two skew straight lines.

(7) All of the following cases determine a plane except ....

(a) two intersecting straight lines.

(b) two different parallel straight lines.

(c) a straight line and a point belong to it.

(d) three non-collinear points.

(8) Number of planes that are passing through two different parallel straight lines = ...

(a) 1 (b) 2

(c) 3 (d) an infinite number.

(9) Two skew straight lines are two straight lines which are ....

(a) not intersect. (b) not perpendicular.

(c) not parallel. (d) not intersect and not parallel.

(10) The two straight lines are skew if they are ....

(a) not parallel. (b) not intersecting.

(c) not coincident. (d) not contained in the same plane.

(11) If the straight line L // the plane X and $A \in X$, then $L \cap X =$ ....

(a) $\varnothing$ (b) L (c) X (d) $\{A\}$

(12) If the straight line $L \subset$ the plane X and $A \in X$, then $L \cap X =$ ....

(a) $\varnothing$ (b) L (c) X (d) $\{A\}$

(13) If the two straight lines L_1 and L_2 are skew , then $L_1 \cap L_2 =$ ....

(a) $\varnothing$ (b) L_1 (c) L_2 (d) the plane contains L_1 and L_2

(14) The two not parallel planes are intersecting in ....

(a) a point. (b) a straight line.

(c) a plane. (d) ray.

(15) If X , Y are two planes where $X \cap Y = \varnothing$, then X .... Y

(a) $\perp$ (b) // (c) = (d) $\subset$

(16) The two planes are coincident if they have .... in common.

(a) only one point (b) two points

(c) three collinear points (d) three non-collinear points

(17) If a straight line and a plane have two point in common , then the straight line ....

(a) is parallel to the plane.

(b) intersects the plane in only one point.

(c) lies completely inside the plane.

(d) intersects the plane in only two points.

(18) If A , B and C are three points determine a plane , then ....

(a) $AB = BC = AC$ (b) $AB + BC = AC$

(c) $AB + BC > AC$ (d) $AB + BC < AC$

(19) All different vertical straight lines in the space are ....

(a) parallel. (b) skew.

(c) contained in the same plane. (d) intersecting.

(20) Relative position of two straight lines in one plane could be each of the following except ....

(a) parallel. (b) intersecting. (c) skew. (d) coincident.

(21) If X , Y and Z are planes in the space where $X \cap Y \cap Z = \{A\}$ and $X \cap Y =$ the straight line L , then which of the following is not true ?

(a) $A \in L$ (b) $L \cap Z = \{A\}$

(c) $L \,//\, Z$ (d) $A \in Z$

(22) If M is a point outside the plane that contains the three points A , B and C , then $\overleftrightarrow{MA}$ ....

(a) lies completely inside the plane. (b) intersects the plane at a point.

(c) intersects the plane at two points. (d) is parallel to the plane.

(23) If $\overleftrightarrow{AB} \subset$ plane X , $\overleftrightarrow{CD} \,//\,$ plane X , then $\overleftrightarrow{CD}$, $\overleftrightarrow{AB}$ are ....

(a) parallel only. (b) skew only.

(c) parallel or skew. (d) intersecting.

(24) X and Y are two parallel planes and straight line $L_1 \subset X$ and straight line $L_2 \subset Y$, then which of the following can not be happen ?

(a) $L_1 \,//\, L_2$ (b) L_1 and L_2 are skew.

(c) $L_1 \,//\, Y$ and $L_2 \,//\, X$ (d) L_1 and L_2 are intersecting.

(25) The least number of planes that determine a solid is ....

(a) 1 (b) 2 (c) 3 (d) 4

(26) ABCD $\grave{A}\grave{B}\grave{C}\grave{D}$ is a cuboid , how many straight lines carry edge from the edges of the cuboid and skew to $\overleftrightarrow{AB}$?

(a) not exist (b) one (c) two (d) four

(27) Which of the following statements is not true ?

(a) For any two points in the space , there is only one plane is passing through them.

(b) Any three non-collinear points in the space determine a plane.

(c) Vertices of the triangle determine a plane.

(d) For each two intersecting straight lines there is only one plane contains them.

(28) Which of the following statements is not true ?

(a) Any two different parallel straight lines determine a plane.

(b) For any two different intersecting straight lines there is only one point in common.

(c) The two skew straight lines can't be contain in the same plane.

(d) For any three non-collinear points there is one plane passing through them at least.

(29) Which of the following statements is not true ? (where L_1 and L_2 are two straight lines , X and Y are two planes) ?

(a) If $L_1 \cap L_2 = \varnothing$, then $L_1 // L_2$ or L_1 and L_2 are skew.

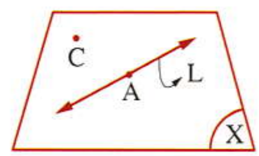
(b) If $L_1 \cap X = \varnothing$, then $L_1 // X$

(c) If $L_2 \cap X = L_2$, then $L_2 \subset X$

(d) If $L_2 \subset Y$, then $L_2 \cap Y = \varnothing$

(30) Using the opposite figure ,

which of the following statements is not true ?



(a) $L \subset X$

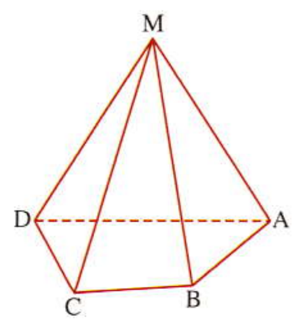
(b) $A \in L$, $A \notin X$

(c) $C \in X$, $C \notin L$

(d) $\overline{AC} \cap L = \{A\}$

(31) In the opposite figure :

The plane ABD $\cap$ The plane MCD = ...



(a) $\overleftrightarrow{AM}$

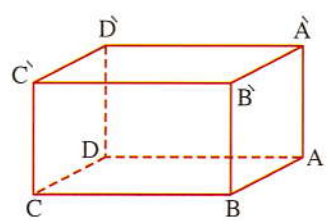
(b) $\overleftrightarrow{CD}$

(c) $\{D\}$

(d) $\overleftrightarrow{MC}$

(32) In the opposite figure :

The plane $AA'B'$ $\cap$ the plane $A'CC'$ = ...



(a) $\overleftrightarrow{AA'}$

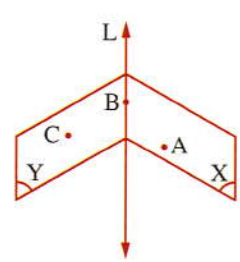
(b) $\overleftrightarrow{BB'}$

(c) $\overleftrightarrow{CC'}$

(d) $\overleftrightarrow{A'C}$

(33) In the opposite figure :

The plane X $\cap$ the plane Y = ...



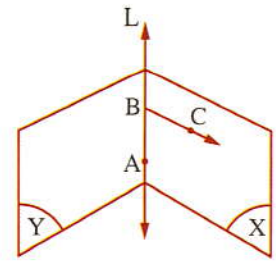
(a) $\{B\}$

(b) $\{A , B , C\}$

(c) the straight line L

(d) $\varnothing$

(34) In the opposite figure :



First : L X

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

Second : A X

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

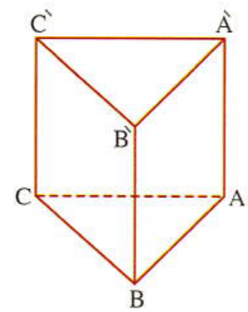
Third : C .... Y

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

Fourth : $\overleftrightarrow{BC}$ .... Y

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

(35) In the opposite figure :



First : The plane $ABB'A'$ $\cap$ the plane $BCC'B'$ = ...

(a) $\overleftrightarrow{BB'}$ (b) $\varnothing$ (c) $\{B'\}$ (d) $\overleftrightarrow{A'C'}$

Second : The plane ABC $\cap$ the plane $A'B'C'$ = ...

(a) $\overleftrightarrow{BB'}$ (b) $\varnothing$ (c) $\overleftrightarrow{AA'}$ (d) $\overleftrightarrow{A'B'}$

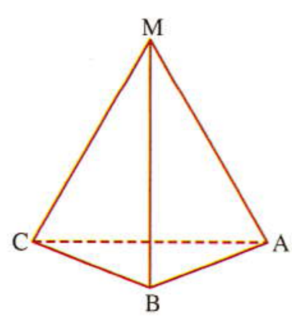
Third : $\overleftrightarrow{AC} \cap \overleftrightarrow{A'C'}$ = ...

(a) $\{A\}$ (b) $\{C'\}$ (c) $\overleftrightarrow{AA'} \cap \overleftrightarrow{BB'}$ (d) $\overleftrightarrow{AC}$

Fourth : $\overleftrightarrow{BB'}$ $\cap$ the plane ABC = ...

(a) $\overleftrightarrow{BB'}$ (b) $\{B'\}$ (c) $\{B\}$ (d) $\varnothing$

(36) In the opposite figure :



First : The plane MAB $\cap$ the plane MBC = ...

(a) $\overleftrightarrow{AB}$ (b) $\overleftrightarrow{MB}$ (c) $\varnothing$ (d) $\{M\}$

Second : The plane MBC $\cap$ the plane ABC = ...

(a) $\{B\}$ (b) $\varnothing$ (c) $\overleftrightarrow{AB}$ (d) $\overleftrightarrow{BC}$

Third : $\overleftrightarrow{MB}$ $\cap$ the plane ABC = ...

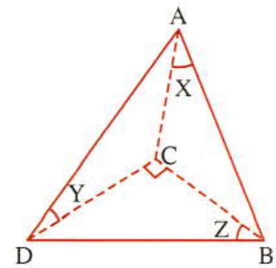
(a) $\overleftrightarrow{MB}$ (b) $\varnothing$ (c) $\{B\}$ (d) $\{M\}$

Fourth : The plane MAB $\cap$ the plane MBC $\cap$ the plane MAC = ...

(a) $\overleftrightarrow{MB}$ (b) $\overleftrightarrow{MC}$

(c) the solid MABC (d) $\{M\}$

(37) In the opposite figure :



If A $\notin$ the plane BCD , then :

First : $X \cap Y =$ ....

(a) $\overleftrightarrow{AC}$ (b) $\varnothing$ (c) $\{A\}$ (d) $\{C\}$

Second : $X \cap Z =$ ....

(a) $\varnothing$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{AC}$ (d) $\{C\}$

Third : $Y \cap Z =$ ....

(a) $\{C\}$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{CD}$ (d) $\varnothing$

Fourth : $\overleftrightarrow{AB} \cap X =$ ....

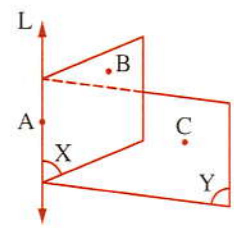
(a) $\overleftrightarrow{AB}$ (b) $\varnothing$ (c) $\overleftrightarrow{AC}$ (d) $\{B\}$

Fifth : Let m ($\angle$ BCD) = 90° , BC = 3 cm. , CD = 4 cm.

, then BD = ... cm.

(a) 6 (b) 5 (c) 4 (d) 7

(38) In the opposite figure :



X and Y are two intersecting planes at the straight line L

, $A \in L$, $B \in X$, $B \notin Y$, $C \in Y$, $C \notin X$:

First : The plane $X \cap$ the plane ABC = ...

(a) $\overleftrightarrow{AB}$ (b) $\overleftrightarrow{AC}$ (c) $\overleftrightarrow{BC}$ (d) L

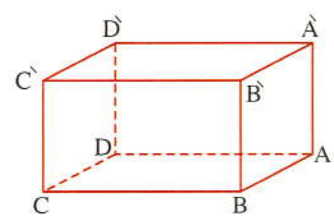
Second : The plane $Y \cap$ the plane ABC = ...

(a) $\overleftrightarrow{AB}$ (b) $\{A\}$ (c) $\overleftrightarrow{CB}$ (d) $\overleftrightarrow{AC}$

Third : The plane $X \cap$ the plane $Y \cap$ the plane ABC = ...

(a) $\varnothing$ (b) L (c) $\{A\}$ (d) $\{B\}$

(39) In the opposite figure :



First : The plane ABCD // the plane ....

(a) ABC` (b) A`B`D`

(c) ABB` (d) A`B`C

Second : The plane BCC`B` // the plane ....

(a) ABC (b) A`B`C` (c) ABD (d) AA`D

Third : The plane ABB`A` $\cap$ the plane ABCD = ...

(a) $\{B\}$ (b) $\{A , B\}$ (c) $\overline{AB}$ (d) $\overleftrightarrow{AB}$

Fourth : The plane $ABB'A'$ $\cap$ the plane $DCC'D'$ = ...

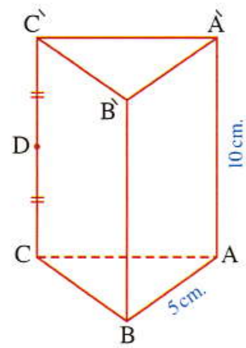
(a) $\overleftrightarrow{BC}$ (b) $\overleftrightarrow{AD}$ (c) $\varnothing$ (d) $\{C\}$

Fifth : The plane $DCC'D'$ $\cap$ the plane $ABCD$ $\cap$ the plane $A'ADD'$ = ...

(a) $\varnothing$ (b) $\overleftrightarrow{AB}$ (c) $\{C\}$ (d) $\{D\}$

(40) In the opposite figure :

$ABB'A'$, $BB'C'C$, $ACC'A'$ are three congruent rectangles , each pairs are intersecting , D is the midpoint of $\overline{CC'}$, if AB = 5 cm. , AA' = 10 cm.



First : The plane ADA' $\cap$ the plane BDB' = ...

(a) $\overleftrightarrow{CC'}$ (b) $\overleftrightarrow{BB'}$ (c) $\{D\}$ (d) $\overleftrightarrow{A'C'}$

Second : The plane ADB $\cap$ the plane ABC = ...

(a) $\varnothing$ (b) $\overleftrightarrow{AB}$ (c) $\{B\}$ (d) $\overleftrightarrow{AC}$

Third : The plane ADB $\cap$ the plane $BCC'B'$ = ...

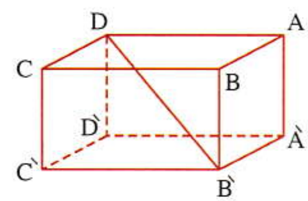
(a) $\{B\}$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{BD}$ (d) $\varnothing$

Fourth : m ($\angle BDB'$) = ...°

(a) 60 (b) 120 (c) 90 (d) 100

(41) In the opposite figure :

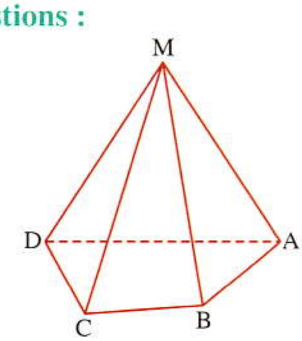
$ABCD\,A'B'C'D'$ is a cuboid. How many straight lines carrying edges of the cuboid and skew to the straight line $\overleftrightarrow{DB'}$?



(a) 2 (b) 4 (c) 6 (d) 8

Second Essay questions

1 Meditate the opposite figure and answer the following questions :



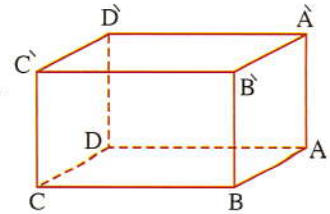
(1) How many lines which carry edges in the figure ?

(2) State the names of the straight lines which carry edges and passing through point A

(3) How many planes which carry faces in the figure ?

(4) State the names of three planes passing through point A

2 **Meditate the opposite figure and answer the following questions :**



(1) Write three straight lines passing through point A

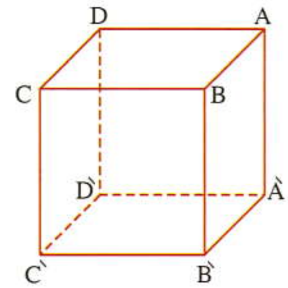
(2) Write the straight lines passing through points A and B together.

(3) Write three planes passing through point A

(4) Write three planes passing through points A and B together.

---

3 **The opposite figure represents a classroom , find :**



(1) The lines which carry edges and intersect with $\overleftrightarrow{AB}$

(2) The lines which carry edges and parallel to $\overleftrightarrow{AB}$

(3) The lines which carry edges and skew to $\overleftrightarrow{AB}$

---

4 **Write the number of planes which passing through :**

(1) One given point.

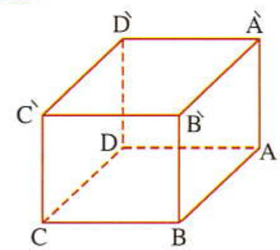
(2) Two different points.

(3) Three collinear points.

(4) Three non-collinear points.

---

5 **In the opposite figure , $ABCDA'B'C'D'$ is a cube of edge length 6 cm.**



(1) Identify the relative positions for each pair of the following straight lines :

① $\overleftrightarrow{AB}$, $\overleftrightarrow{DD'}$

② $\overleftrightarrow{AB}$, $\overleftrightarrow{D'C'}$

③ $\overleftrightarrow{AB}$, $\overleftrightarrow{A'D'}$

④ $\overleftrightarrow{AC}$, $\overleftrightarrow{A'C'}$

⑤ $\overleftrightarrow{AB}$, $\overleftrightarrow{DC'}$

⑥ $\overleftrightarrow{AC}$, $\overleftrightarrow{AA'}$

(2) Identify the relative positions for each pair of the following planes :

① $ABB'A'$, $DCC'D'$

② $A'B'BA$, $A'B'CD$

③ ABC , $D'B'D$

(3) If $\overleftrightarrow{AB} \perp \overleftrightarrow{AD'}$, Find the length of $\overline{BD'}$

---

6 **Draw the figures which represent the plane (X) , the straight line (L) and the point (A) in the following cases :**

(1) $A \in L$

(2) $L \subset X$

(3) $L \cap X = \{A\}$

(4) $L \,//\, X$

(5) $A \in X$, $A \notin L$, $L \subset X$

Exercise

7

The pyramid

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

● (1) The line segment joining the vertex of the pyramid and any vertex of its base vertices is called ....

(a) the height of the pyramid. (b) the slant height of the pyramid.

(c) the lateral edge of the pyramid. (d) the side of its base.

● (2) If MABCD is a regular quadrilateral pyramid , then this pyramid must be ....

(1) regular faces (2) its base is a square (3) right

(a) (1) , (2) (b) (2) , (3) (c) (1) only (d) (1) , (2) , (3)

● (3) Which of the following statements is true ?

(a) The lateral faces of the right pyramid are congruent.

(b) The regular pyramid is a right pyramid.

(c) The heights of the lateral faces of the right pyramid are equal.

(d) The least number of planes that can determine a solid = 3 planes.

● (4) Which of the following statements is not true ?

(a) The base of the right pyramid can be a surface of a rhombus.

(b) The triangular pyramid has three faces.

(c) The pentagonal pyramid has six faces.

(d) All lateral faces of the quadrilateral pyramid are surfaces of triangles.

(5) In the regular pyramid , which of the following is the right ascendingly order of the lengths ?

(a) The length of the lateral edge , the height , the slant height.

(b) The height , the slant height , the length of the lateral edge.

(c) The slant height , the height , the length of the lateral edge.

(d) The length of the lateral edge , the slant height , the height.

(6) The shape of the base of a regular pyramid must be ....

(a) parallelogram. (b) rhombus. (c) rectangle. (d) square.

(7) If MABCD is a regular quadrilateral pyramid , then all lateral edges are ....

(a) parallel. (b) congruent.

(c) perpendicular to the base. (d) mutually perpendicular.

(8) If MABC is a right triangular pyramid , N is the projection of the point M on the plane ABC , E is midpoint of $\overline{BC}$, then all the following triangles are right except ....

(a) Δ MNC (b) Δ MNE (c) Δ MBC (d) Δ MNA

(9) If MABC is a regular faces pyramid , N is the projection of the point M on the plane ABC , E is the midpoint of $\overline{AB}$, which of the following is an equilateral triangle ?

(a) Δ MNE (b) Δ MBE (c) Δ ACE (d) Δ MBC

(10) The number of all faces of a regular pentagonal pyramid is ....

(a) 5 (b) 6 (c) 7 (d) 10

(11) If the number of the faces of a pyramid = m and the number of its vertices = n , then the number of its edges = ...

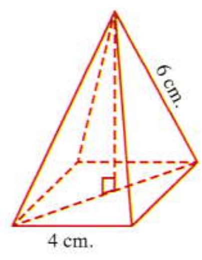
(a) m + n (b) m + n – 1 (c) m + n – 2 (d) m + n + 2

(12) In the hexagonal pyramid :

number of faces + number of vertices – number of edges = ...

(a) 1 (b) 2 (c) 3 (d) 4

(13) The opposite figure represents a regular quadrilateral pyramid of height = ... cm.



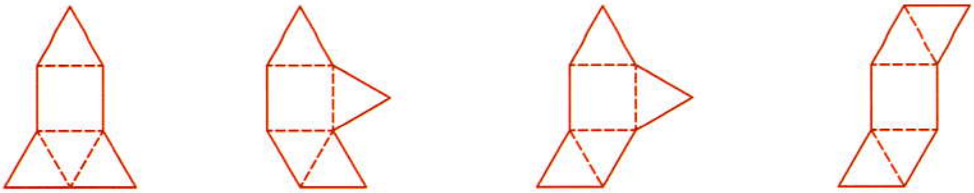
(a) $7\sqrt{2}$ (b) $2\sqrt{7}$

(c) $4\sqrt{2}$ (d) $2\sqrt{5}$

(14) A regular quadrilateral pyramid , if the length of its base side is 6 cm. , the length of its lateral edge is 8 cm. , then the length of its height = ... cm.

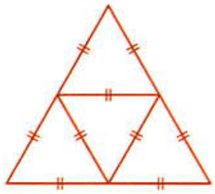
(a) $5\sqrt{2}$ (b) $\sqrt{46}$ (c) $\sqrt{85}$ (d) $\sqrt{48}$

(15) Which of the following nets does not make a regular quadrilateral pyramid when it folded ?



(a) (b) (c) (d)

(16) Which solid represents the opposite net ?



(a) Quadrilateral pyramid.

(b) Regular quadrilateral pyramid.

(c) Triangular regular faces pyramid.

(d) Otherwise.

(17) The ratio between the edge length of the triangular regular faces pyramid : its height = ...

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$

(18) The ratio between the length of the edge of the regular faces pyramid : its slant height = ...

(a) $2\sqrt{2} : \sqrt{3}$ (b) $2\sqrt{3} : 3$ (c) $\sqrt{6} : 2$ (d) $\sqrt{6} : 3$

(19) If we cut a regular quadrilateral pyramid by a plane parallel to its base , then the resulted section is ....

(a) triangle. (b) square. (c) rectangle. (d) circle.

(20) A right quadrilateral pyramid of height 10 cm. , its base is a rhombus whose diagonal lengths are 12 cm. and 8 cm. , then its volume = ... cm^3.

(a) 40 (b) 80 (c) 160 (d) 200

(21) A regular quadrilateral pyramid whose base perimeter is 36 cm. , and its height is 10 cm. , then its volume = ... cm^3.

(a) 810 (b) 180 (c) 360 (d) 270

(22) A regular hexagonal pyramid , the side length of its base = 8 cm. and its height = 10 cm. , then its volume equal .... cm^3.

(a) $320\sqrt{3}$ (b) $960\sqrt{3}$ (c) $\frac{320\sqrt{3}}{3}$ (d) 160

(23) A regular pyramid whose volume is 12 cm^3. , and its base area is 4 cm^2. , then its height = ... cm.

(a) 3 (b) 6 (c) 9 (d) 2

(24) A regular quadrilateral pyramid whose volume 64 cm^3. , and its height is 6 cm. , then its base perimeter = ... cm.

(a) 8 (b) $8\sqrt{2}$ (c) 16 (d) $16\sqrt{2}$

(25) A regular quadrilateral pyramid whose volume is 480 cm^3. , and its base length is 12 cm. , then the length of its height = ... cm.

(a) 10 (b) 20 (c) 30 (d) 15

(26) If the volume of a regular hexagonal pyramid equal $8\sqrt{3}$ cm^3. , and its height length equal 4 cm. , then the perimeter of its base = ... cm.

(a) 2 (b) 12 (c) 6 (d) $6\sqrt{3}$

(27) In a regular quadrilateral pyramid , the length of its base is 10 cm. , the length of its slant height is 13 cm. , then its lateral area equal .... cm^2.

(a) 260 (b) 360 (c) 130 (d) 520

(28) A regular quadrilateral pyramid , the area of its base = 100 cm^2. and its height is 12 cm. , then its lateral area equal .... cm^2.

(a) 260 (b) 520 (c) 130 (d) 360

(29) The total area of a right quadrilateral pyramid , its base is a regular polygon and its diagonal length = $10\sqrt{2}$ cm. and its height = $5\sqrt{3}$ cm. equals .... cm^2.

(a) 40 (b) 100 (c) 200 (d) 300

(30) A regular quadrilateral pyramid whose lateral area = 30 cm^2. , and its slant height = 5 cm. , then its base perimeter = ... cm.

(a) 6 (b) 12 (c) 24 (d) 36

(31) A triangular regular faces pyramid , its edge length 10 cm. , then its total area equal .... cm^2.

(a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$

(32) If the sum of edge lengths of a triangular regular faces pyramid equals 18 cm. , then its total area = ... cm^2.

(a) $\frac{27\sqrt{2}}{4}$ (b) $\frac{27\sqrt{3}}{4}$ (c) $9\sqrt{3}$ (d) $\frac{27\sqrt{3}}{2}$

(33) If the total area of a regular faces pyramid = $36\sqrt{3}$ cm^2, then the sum of its edges lengths = ... cm.

(a) 6 (b) 12 (c) 18 (d) 36

(34) If the total area of a triangular regular faces is $9\sqrt{3}$ cm^2, then the length of its edge .... cm.

(a) 3 (b) 9 (c) 27 (d) $\sqrt{3}$

(35) A regular triangular pyramid, its base length is $6\,\ell$ cm. and its height ℓ cm., then its lateral area = ... cm^2

(a) $27\sqrt{3}\,\ell^2$ (b) $18\,\ell^2$ (c) $9\sqrt{3}\,\ell^2$ (d) $36\,\ell^2$

(36) A triangular regular faces pyramid, its edge length 6 cm., then its volume = ... cm^3

(a) $27\sqrt{3}$ (b) $36\sqrt{3}$ (c) $54\sqrt{2}$ (d) $18\sqrt{2}$

(37) A triangular regular faces pyramid, if the sum of the lengths of its edges equal 18 cm., then its volume = ... cm^3

(a) $9\sqrt{2}$ (b) $\frac{9\sqrt{2}}{4}$ (c) $\frac{27\sqrt{2}}{5}$ (d) $9\sqrt{3}$

(38) If the slant height of a triangular regular faces pyramid equals $5\sqrt{3}$ cm., then the sum of areas of its faces = ... cm^2

(a) $\frac{50\sqrt{3}}{3}$ (b) $25\sqrt{3}$ (c) $100\sqrt{3}$ (d) $50\sqrt{3}$

(39) A right quadrilateral pyramid whose base is a rhombus of side length equals to one of the diagonals of the rhombus equals 6 cm., if the height of the pyramid = 12 cm., then its volume = ... cm^3

(a) $72\sqrt{3}$ (b) $216\sqrt{3}$ (c) 144 (d) 72

(40) $ABCD\grave{A}\grave{B}\grave{C}\grave{D}$ is a cube of edge length = 6 cm., then volume of the pyramid $\grave{B}ABC$ = ... cm^3

(a) 36 (b) 72 (c) $36\sqrt{3}$ (d) $18\sqrt{3}$

(41) A regular quadrilateral pyramid whose total area = 70 cm^2 and its lateral area = 45 cm^2, then its height = ... cm.

(a) 2.5 (b) 5 (c) $\sqrt{14}$ (d) 4.5

(42) In a regular quadrilateral pyramid the length of its base side 10 cm. and the area of one of its lateral faces 60 cm^2, then its total area equal .... cm^2

(a) 600 (b) 340 (c) 160 (d) 240

(43) The ratio between the lateral surface area of a triangular pyramid of regular faces to the area of its total surface area = ...

(a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2

(44) A quadrilateral regular pyramid , the length of its base side = its slant height , then the ratio between its lateral surface area to its total surface area = ...

(a) 2 : 3 (b) 3 : 4 (c) 1 : 2 (d) 3 : 5

(45) A quadrilateral regular pyramid , the area of any face from its lateral faces equals the area of its base , if the side length of the base of the pyramid is 6 cm. , then volume of the pyramid = ... cm^3.

(a) 36 (b) $6\sqrt{3}$ (c) $36\sqrt{15}$ (d) $216\sqrt{15}$

(46) If the side length of the base of quadrilateral regular pyramid is doubled but its height remains constant , then its volume ....

(a) is doubled. (b) will not change.

(c) become four times its first volume.

(d) become six times its first volume.

(47) In a regular quadrilateral pyramid , the side length of its base = 18 cm. , if its volume = 1296 cm^3. , then its lateral area = ... cm^2.

(a) 270 (b) 360 (c) 450 (d) 540

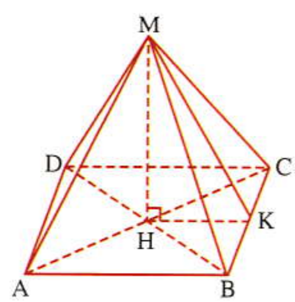
(48) A right pyramid whose base is a square , and all its eight edges are equal in length and each one = a cm. , then its lateral area = ...

(a) $3\ a^2$ (b) $4\ a^2$ (c) $\sqrt{3}\ a^2$ (d) $4\sqrt{3}\ a^2$

(49) In the triangular pyramid MABC , the vertex (M) is at a distance 15 cm. from its base ABC and the sides lengths of its base 5 , 6 , 7 cm.
, then its volume = ... cm^3.

(a) $15\sqrt{3}$ (b) $10\sqrt{6}$ (c) $30\sqrt{6}$ (d) 90

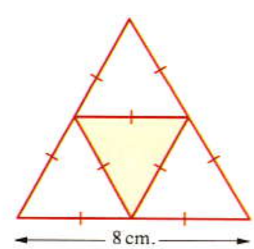
(50) In the opposite figure :



MABCD is a regular quadrilateral pyramid
, its volume 48 cm^3. , its height 4 cm.
, KC = KB , $\overline{AC} \cap \overline{BD} = \{H\}$,
m (∠ MHK) = m (∠ HKB) = m (∠ MKB) = 90°
, then its lateral area = ... cm^2.

(a) 18 (b) 24 (c) 36 (d) 60

(51) By using the opposite solid net :



The lateral area of the resulted solid = ... cm^2.

(a) $8\sqrt{3}$ (b) $12\sqrt{3}$

(c) $16\sqrt{3}$ (d) 24

Second Essay questions

1 **In the regular pentagonal pyramid :**

(1) What the number of its lateral faces ?

(2) What the number of its faces ?

(3) What the number of its lateral edges ?

(4) What the number of its edges ?

(5) The pyramid has one vertix regardless of the vertices of the base. What is the number of all vertices of pentagonal pyramid ? Is your answer prove Euler's rule for any solid , its base is a polygon.

---

2 MABCD is a regular quadrilateral pyramid , the length of its base side is 10 cm. , and its height is 12 cm. , find its slant height. « 13 cm. »

---

3 MABCD is a regular quadrilateral pyramid of height 20 cm. and slant height 25 cm. Find the length of its base side. « 30 cm. »

---

4 MABCD is a regular quadrilateral pyramid , its base as a square ABCD , if its height equals $4\sqrt{3}$ cm. , and its lateral edge length $MA = 4\sqrt{5}$ cm. , find the length of its base side. « 8 cm. »

---

5 MABC is a regular triangular pyramid whose base is the equilateral triangle ABC whose side length 12 cm. , if the height of the pyramid is 6 cm.
Find the length of its lateral edge. « $2\sqrt{21}$ cm. »

---

6 MABC is a regular triangular pyramid whose base ABC as an equilateral triangle of side length 3 cm. , if the length of its lateral edge is $\sqrt{7}$ cm. Find the height of the pyramid. « 2 cm. »

---

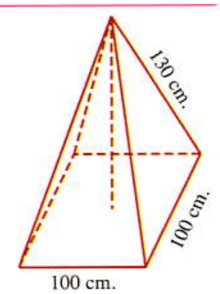
7 MABC is a triangular pyramid with regular faces , the length of its edge is 12 cm.
Find its height and its slant height. « $4\sqrt{6}$ cm. , $6\sqrt{3}$ cm. »

---

8 A regular hexagonal pyramid whose height 8 cm. , its base as a regular hexagon of perimeter $24\sqrt{3}$ cm. Find the length of its lateral edge and its slant height. « $4\sqrt{7}$ cm. , 10 cm. »

---

9 The opposite figure represents a water tank as a regular quadrilateral pyramid , use the given data to find the height of the lateral face and the height of the tank.



« 120 cm. , $10\sqrt{119}$ cm. »

10 **Each of the following figures represents a solid net. Describe the solid and find its height :**

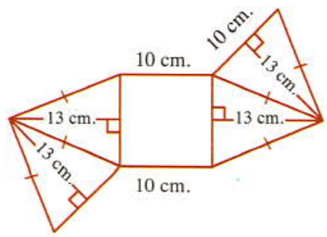


Fig. (1)

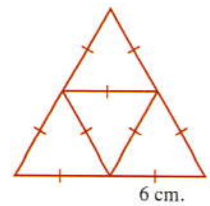


Fig. (2)

« 12 cm. , $2\sqrt{6}$ cm. »

11 The great pyramid of Giza (Khopho pyramid) is a regular quadrilateral pyramid the side length of its base is 232 metres and its slant height is 186 metres. Find height of the pyramid. « 145.4 m. »

12 MABC is a right triangular pyramid , the length of its edge MA = 25 cm. , and its base ABC as a right-angled triangle at A. If BA = 16.8 cm. , CA = 12.6 cm.
Find the height of the pyramid. « 24 cm. »

13 MABCD is a right quadrilateral pyramid , whose base is the rhombus ABCD where AC = 16 cm. , BD = 12 cm. , N is the point of intersection of its diagonals.
If the height of the pyramid MN = 10 cm. Find the lengths of its lateral edges.

« $2\sqrt{34}$ cm. , $2\sqrt{41}$ cm. »

14 A regular triangular pyramid whose height 12 cm. , and the side length of its base is 18 cm.
Find its volume. « $324\sqrt{3}$ cm^3. »

15 MABCD is a regular quadrilateral pyramid , its base ABCD where AB = 10 cm. , and the height of the pyramid = 12 cm.

Find : (1) The length of any slant height.
(2) The volume of the pyramid.
(3) The total area of the pyramid.

« 13 cm. , 400 cm^3. , 360 cm^2. »

16 A regular quadrilateral pyramid the length of its base is 20 cm. , and its height is $10\sqrt{3}$ cm.

Find : (1) Its lateral surface area. **(2)** Its volume. « 800 cm^2. , $\frac{4000}{3}\sqrt{3}$ cm^3. »

17 A regular quadrilateral pyramid , the length of its base diagonal is $24\sqrt{2}$ cm. , and its slant height = 20 cm. , find its total area and its volume. « 1536 cm^2. , 3072 cm^3. »

18 MABCD is right quadrilateral pyramid , its base is the square ABCD whose side length $8\sqrt{2}$ cm. , and the length of its lateral edge is $4\sqrt{6}$ cm.

Find : (1) The lateral surface area of the pyramid.
(2) The volume of the pyramid. « $128\sqrt{2}$ cm^2. , $\frac{512}{3}\sqrt{2}$ cm^3. »

19 MABCD is a regular quadrilateral pyramid , the side length of its base = 20 cm. and the length of its lateral edge is 26 cm.

Find : (1) The slant height of the pyramid. **(2)** The height of the pyramid.

(3) The lateral area of the pyramid. **(4)** The volume of the pyramid.

« 24 cm. , $2\sqrt{119}$ cm. , 960 cm^2. , $\frac{800}{3}\sqrt{119}$ cm^3. »

20 A triangular regular faces pyramid , its edge length = 12 cm. , find its height , volume and total area. « $4\sqrt{6}$ cm. , $144\sqrt{2}$ cm^3. , $144\sqrt{3}$ cm^2. »

21 MABCD is a right pyramid whose base ABCD as a square of side length = 18 cm. , MA = MB = MC = MD = 15 cm.

Find : (1) The total area. **(2)** The volume. « 756 cm^2. , $324\sqrt{7}$ cm^3. »

22 Calculate to the nearest tenth the volume of a regular pentagonal pyramid whose side length of its base = 16 cm. and its height = 12 cm. « 1761.8 cm^3. »

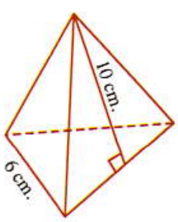
23 A regular hexagonal pyramid , the side length of its base = 12 cm. and its slant height = $10\sqrt{3}$ cm. ,

Find : (1) Its lateral area.

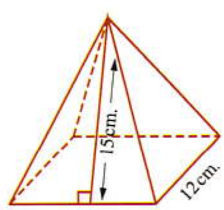
(2) Its total area. « $360\sqrt{3}$ cm^2. , $576\sqrt{3}$ cm^2. »

24 **Find the lateral area and the total area of each regular pyramid of the following :**

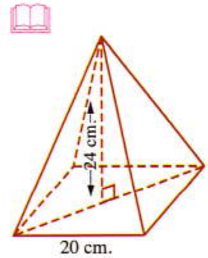
(1)



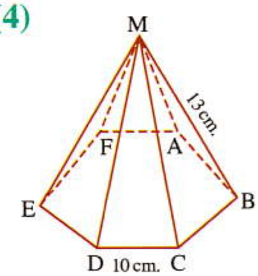
(2)



(3)

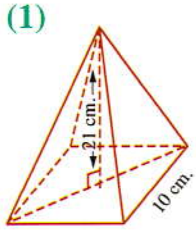


(4)

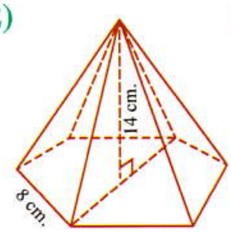


25 **Find the volume of each of the following regular pyramids :**

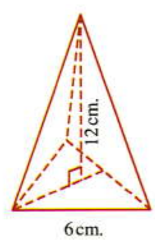
(1)



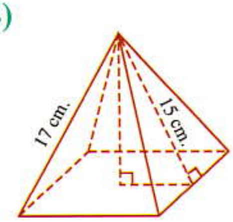
(2)



(3)



(4)



26 A quadrilateral pyramid whose height is 12 cm. , and its base as a rhombus of diagonals length 4 cm. and 8 cm. , prove that its volume equals the volume of a cube of edge length 4 cm.

27 MABC is a triangular pyramid whose vertex M at distance 15 cm. form its base ABC , and the side lengths of its triangular base are 5 , 6 and 7 cm. Find its volume « $30\sqrt{6}$ cm^3. »

28 A regular quadrilateral pyramid whose base area is 700 cm^2. and its slant height is 20 cm. Find its volume. « 3500 cm^3. »

29 A regular quadrilateral pyramid whose base area is 9 cm^2. and the length of its lateral edge is 5 cm. Find its volume. « 13.6 cm^3. »

30 A regular quadrilateral pyramid whose volume is 400 cm^3. and its height is 12 cm. Find its lateral area. « 260 cm^2. »

31 A regular quadrilateral pyramid , the side length of its base is 18 cm. , and its volume is 1296 cm^3.
Find its slant height and its lateral surface area. « 15 cm. , 540 cm^2. »

32 A regular quadrilateral pyramid , the side length of its base = 12 cm. , and its total area = 384 cm^2. Find its volume. « 384 cm^3. »

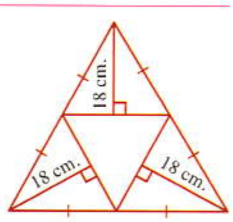
33 A right pyramid whose base is as a square of diagonal length = $10\sqrt{2}$ cm.
If its lateral area = 260 cm^2. Find the volume of the pyramid. « 400 cm^3. »

34 MABC is a regular triangular pyramid , the side length of its base is 3 cm. , and the length of its lateral edge = $\sqrt{7}$ cm. Find its volume and lateral area. « $\frac{3\sqrt{3}}{2}$ cm^3. , $\frac{9}{4}\sqrt{19}$ cm^2. »

35 A regular hexagonal pyramid whose height is 8 cm. and its base perimeter is $24\sqrt{3}$ cm. , Find its lateral and total area. « $120\sqrt{3}$ cm^2. , $192\sqrt{3}$ cm^2. »

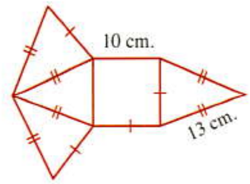
36 Find the volume of the right pyramid whose slant height is 10 cm. , and its base as an equilateral triangle drawn inside a circle of radius length 12 cm. « $288\sqrt{3}$ cm^3. »

37 Use the opposite net to discribe the solid , then find its total area.



« $432\sqrt{3}$ cm^2. »

38 **Connecting to industry :**



Products containers of a factory manufactured from cardboard by folding the net of the opposite figure.

(1) Find the area of the used cardboard to produce 1000 containers.

(2) Calculate the costs of the used cardboard if each square metre costs 15 pounds.

« 34 m^2 , 510 pounds »

---

39 MABCD is a right quadrilateral pyramid whose base is the square ABCD , if the length of each lateral edge equals $6\sqrt{5}$ cm. and height of the pyramid = $6\sqrt{3}$ cm.

Find : (1) The total area of the pyramid.

(2) The volume of the pyramid. « 432 cm^2 , $288\sqrt{3} \text{ cm}^3$ »

---

40 **Connecting to tourism :**

A model of the great pyramid (regular quadrilateral pyramid) is made of metallic alloy its density is 3.2 gm./cm^3 If the length of the model base side 11.5 cm. and its height 7 cm. , then calculate its mass to the nearest one decimal place. « 987. 5 gm. »

---

41 France cared about the ancient Egyptian monuments. So it transported some of them to Paris to be shown in their museums. It also set up a pyramid its side faces of transparent glass similar to the great pyramid (regular quadrilateral pyramid) to be a main entrance to the Louvre in Paris. If you know its height 21.6 metres , and the length of its base side 35 metre , then find the area of the glass used in its building to the nearest square cm. « 1946 m^2 »

Third Higher skills

1 A regular hexagonal pyramid , the side length of its base = 2ℓ , and its height = 3ℓ ,

Prove that : The lateral area of the pyramid equals twice of its base area.

---

2 MABCD is a regular quadrilateral pyramid , if the length of its lateral edge = length of the diagonal of its base = ℓ , prove that : the total area of the pyramid = $\frac{\ell^2}{2}\left(1+\sqrt{7}\right)$

---

3 A hollow circular cylinder , put inside it a triangular pyramid MABC whose base ABC is an equilateral triangle whose vertices lies on perimeter of the lower base of the cylinder , M (vertex of the pyramid) is the centre of the upper base of the cylinder.

Find the ratio between volume of the pyramid and volume of the cylinder. « $\frac{\sqrt{3}}{4\pi}$ »

---

4 A right pyramid whose base as a square , and all its eight edges are equal in length , if its total area = $\left(\sqrt{3}+1\right)$ A , find the length of its edge in terms of (A) « $\sqrt{A}$ »

Exercise 8

The cone

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The right circular cone is generated by folding a paper in the shape of ....

(a) an equilateral triangle. (b) a right-angled triangle.

(c) a circular segment. (d) a circular sector.

(2) The measure of the smallest rotation angle of an isosceles triangle around its axis of symmetry to form a right circular cone is ....

(a) 90° (b) 180° (c) 270° (d) 60°

(3) The right circular cone is formed from rotation of a right-angled triangle a complete rotation about ....

(a) its hypotenuse.

(b) one of its right sides.

(c) any straight line in the plane of the triangle.

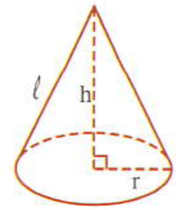
(d) any straight line passes through one of its vertices and parallel to the opposite side of this vertex.

(4) If a right circular cone intersected by a plane parallel to its base ,
then the resulted sector is ....

(a) an isosceles triangle. (b) an equilateral triangle.

(c) a circle. (d) a trapezoid.

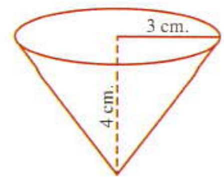
(5) The total area for the opposite right cone equals ....



(a) $\pi r \ell$ (b) $\frac{\pi}{3} \pi^2 h$

(c) $\pi r (r + \ell)$ (d) $\frac{\pi}{3} r (r h + 3 \ell)$

(6) In the opposite figure :

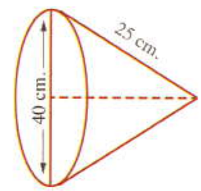


The length of the drawer = ... cm.

(a) 2 (b) 3

(c) 4 (d) 5

(7) In the opposite figure :



The height of the cone = ... cm.

(a) 15 (b) 20

(c) 25 (d) 40

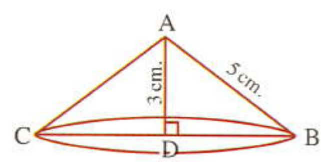
(8) In a right circular cone , if the length of its height 15 cm. , and the length of its drawer 17 cm. , then its radius length equal .... cm.

(a) 10 (b) 8 (c) 7 (d) 9

(9) In a right circular cone , the radius length of its base = 15 cm. and its height = 20 cm. , then its lateral area = ... cm^2.

(a) 375π (b) 600π (c) 1500π (d) 1875π

(10) In the opposite figure :



If AD = 3 cm. , AB = 5 cm.

, then the total area of the cone = ... cm^2.

(a) 8π (b) 24π (c) 48π (d) 36π

(11) If the length of the diameter of the base of a right circular cone is 12 cm. and its height 8 cm. , then its lateral area equal .... cm^2.

(a) 60π (b) 28π (c) 10π (d) 48π

(12) The height of a right circular cone is 6 cm. and the circumference of its base is 16π cm. , then its lateral area = ... cm.^{2}

(a) 144π (b) 64π (c) 60π (d) 80π

(13) A right circular cone , the length of its drawer equal the length of the diameter of its base , then its total area = ... cm^2.

(a) $3\pi r^2$ (b) $3\pi r^3$ (c) $4\pi r^2$ (d) $4\pi r^3$

(14) A right circular cone , its height 24 cm. , and the length of its drawer 26 cm. , then the area of its base .... cm^2.

(a) 25π (b) 100π (c) 20π (d) 50π

(15) The radius length of the base of a right circular cone where its total area = 616π cm^2. , and the length of its drawer is 30 cm. is .... cm.

(a) 44 (b) 14 (c) 30 (d) 34

(16) Lamp cover is in the form of a right circular cone , the circumference of its base circle = 88 cm. , its height = 20 cm. , then its lateral area ≃ .... cm^2. $\left(\pi = \frac{22}{7}\right)$

(a) 88 (b) 596 (c) 1074 (d) 1047

(17) A right circular cone , the radius length of its base = 6 cm. and the length of its drawer = 10 cm. , then its volume = ... cm^3.

(a) 32π (b) 64π (c) 96π (d) 228π

(18) A right circular cone where its height 4 cm. , the length of its drawer 5 cm. , then its volume .... cm^3.

(a) 36π (b) 15π (c) 24π (d) 12π

(19) ABC is an equilateral triangle. Its side length (ℓ) it turned around $\overline{BC}$ as a rotation axis a complete turn , then the volume of the generated solid in terms of π and ℓ is ....

(a) $\frac{\pi \ell}{4}$ (b) $\frac{\pi}{4} \ell^2$ (c) $\frac{\pi \ell^3}{4}$ (d) $\sqrt{3}\,\pi \ell^2$

(20) A right circular cone where its volume 27π cm^3. , and the circumference of its base 6π cm. , then its height equal .... cm.

(a) 27 (b) 18 (c) 9 (d) 6

(21) A right circular cone , the radius length of its base 5 cm. and its total area = 90π cm^2. , then its volume = ... cm^3.

(a) 105π (b) 95π (c) 100π (d) 120π

(22) The volume of a right circular cone , if the length of its drawer = 15 cm. and its total area = 216π cm^2. equal .... cm^3.

(a) 205π (b) 220π (c) 280π (d) 324π

(23) A right circular cone where the length of its drawer 25 cm. and its lateral area 550 cm^2. , then its volume = ... cm^3. where $\left(\pi = \frac{22}{7}\right)$

(a) 1223 (b) 1232 (c) 1322 (d) 3122

(24) If the volume of a right circular cone is 9π cm^3. and the length of its base radius equal the length of its height , then its base area = ... cm^2.

(a) 9π (b) 3π (c) 27π (d) 12π

(25) The volume of a right circular cone 100 cm^3, then its volume when the radius length of its base is doubled = ... cm^3

(a) 100 (b) 200 (c) 300 (d) 400

(26) In a right circular cone , if the length of its radius base increased to its double , and the length of the height is decreased to its half , then its volume ....

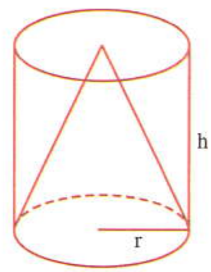
(a) don't change. (b) increased to its double.

(c) decreased to its half. (d) increased to its four times.

(27) The radius of the base of a right circular cone = twice its height = 6 cm. and uniform quadrilateral pyramid its base length = its height = 6 cm. , then the ratio between the volume of the cone : the volume of the pyramid = ... : ....

(a) $3 : \pi$ (b) $\pi : 3$ (c) $\pi : 2$ (d) $2 : \pi$

(28) In the opposite figure :

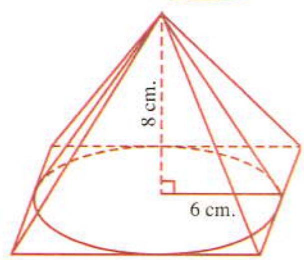


$\frac{\text{the volume of the cone}}{\text{the volume of the cylinder}} =$ ....

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{1}$

(29) In the opposite figure :



A right regular pyramid and right circular cone have common vertex and the circle of base of the cone touches sides of the base of the pyramid internally :

First : The lateral area of the right circular cone = ... cm^2

(a) 60 (b) 60π (c) 48 (d) 48π

Second : The total area of the regular pyramid equals .... cm^2

(a) 360 (b) 240 (c) 384 (d) 432

Third : The volume of the pyramid equals .... cm^3

(a) 64 (b) 96 (c) 480 (d) 384

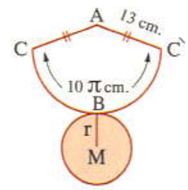
Fourth : The ratio between the volume of the pyramid and the volume of the cone equals ....

(a) $\pi : 3$ (b) $4 : \pi$ (c) $\pi : 4$ (d) $3 : \pi$

Fifth : The ratio between the lateral area of the pyramid and the lateral area of the cone equals ....

(a) $\pi : 3$ (b) $4 : \pi$ (c) $\pi : 4$ (d) $3 : \pi$

(30) The opposite net describes a solid its volume = ... cm^3.

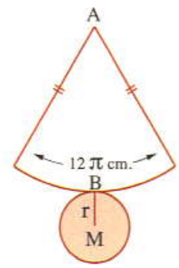


(a) 25π (b) 50π

(c) 75π (d) 100π

(31) The opposite net describes a solid its volume = 96π cm^3.

, then its total area = ... cm^2.



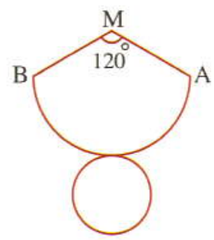
(a) 16π (b) 32π

(c) 48π (d) 96π

(32) The opposite figure represents the net of a solid

, MB = 3π cm. , m ($\angle$ AMB) = 120°

, then the volume of the solid = ... cm^3.

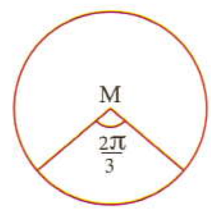


(a) $2\sqrt{2}\,\pi^2$ (b) $\frac{2\sqrt{2}}{3}\pi^4$

(c) $2\sqrt{2}\,\pi$ (d) $\frac{2\sqrt{2}}{3}\pi^3$

(33) In the opposite figure a circle is divided into two circular sectors such that they form two right cone nets

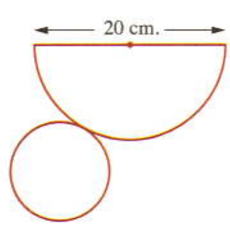
, then $\frac{\text{the lateral area of the smallest cone}}{\text{the lateral area of the greatest cone}}$ = ...



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(34) In the opposite figure :

If we fold the shown net it becomes a cone its base radius length is .... cm.

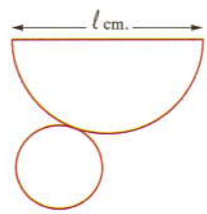


(a) 10 (b) 8

(c) 5 (d) 2.5

(35) In the opposite figure :

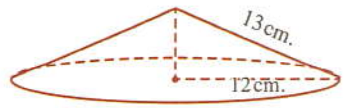
If we folded this net it becomes a cone its base radius is .... cm.



(a) $\frac{\ell}{2}$ (b) $\frac{\ell}{3}$

(c) $\frac{\ell}{4}$ (d) $\frac{\ell}{5}$

(36) The central angle of the sector if be folded it becomes the opposite cone is ....



(a) acute. (b) obtuse. (c) straight. (d) reflex.

(37) If we folded the circular sector it becomes a right circular cone , its drawer length 10 cm. and the radius length of its base 5 cm. , then the central angle of this sector is ....

(a) acute. (b) obtuse. (c) straight. (d) reflex.

(38) If we have a quarter circle , its radius length 16 cm. , then the radius length of the base of the cone which can be formed from the arc of the quarter circle = ... cm.

(a) 16 (b) 8 (c) 4 (d) 2

(39) The area of a circular sector : the total area of the circular solid cone which can be formed from folding this sector ....

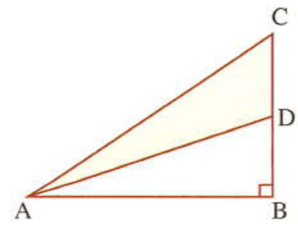
(a) > 1 (b) < 1 (c) $= 1$ (d) ≥ 1

(40) The ratio between the volume of a regular quadrilateral pyramid and the volume of the smallest circular cone contains the pyramid equals ....

(a) $2 : \pi$ (b) $4 : \pi$ (c) $6 : \pi$ (d) $8 : \pi$

(41) In the opposite figure :

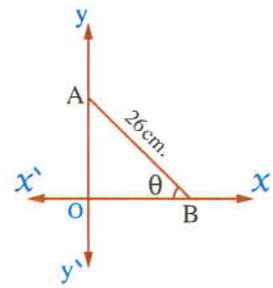
If AB = 3 cm. , BD = CD = 1 cm. , m (∠ ABC) = 90° , then the volume of the solid generated by turning the shaded part around $\overleftrightarrow{AB}$ as a rotation axis a complete turn = ...



(a) π (b) 2π (c) 3π (d) 4π

(42) In the opposite figure :

If $\tan \theta = \frac{5}{12}$, AB = 26 cm. , then the lateral area of the solid generated by rotating the triangle ABO a complete revolution about the X-axis = ... π cm.^{2}

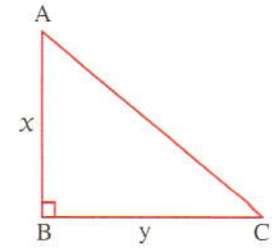


(a) 360 (b) 260π

(c) 260 (d) 360π

(43) In the opposite figure :

If v_1 is the volume of the cone produced by rotating the triangle ABC about $\overleftrightarrow{AB}$ a complete revolution , v_2 is the volume of the cone produced by rotating the triangle ABC about $\overleftrightarrow{BC}$ a complete revolution , then $\frac{V_1}{V_2} = \cdots\cdots\cdots$

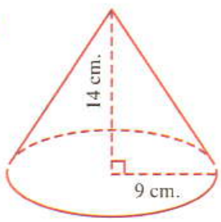


(a) $\frac{1}{2}$ (b) $\frac{\pi}{2}$ (c) $\frac{x}{4}$ (d) $\frac{y}{x}$

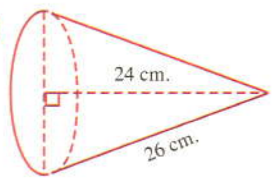
Second Essay questions

1 **Find the volume of the right circular cone shown in each figure using the given data :**

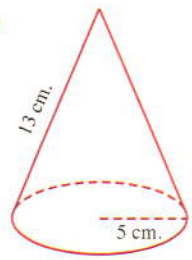
(1)



(2)

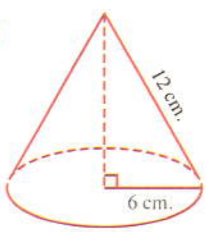


(3)

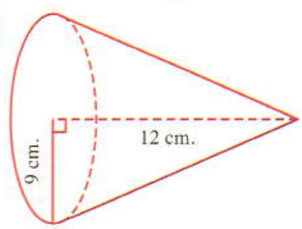


2 **Find the lateral and the total areas of each right circular cone due to the given data :**

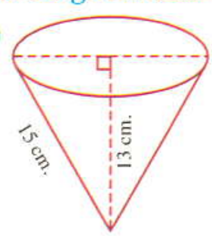
(1)



(2)



(3)



3 A right circular cone , its drawer length = 17 cm. its height = 15 cm. **Find :**

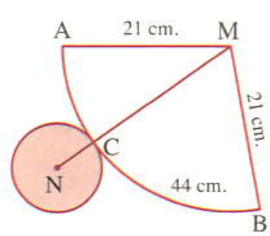
(1) Its lateral area. **(2)** Its total area. **(3)** Its volume.

« 136 π cm^2. , 200 π cm^2. , 320 π cm^3. »

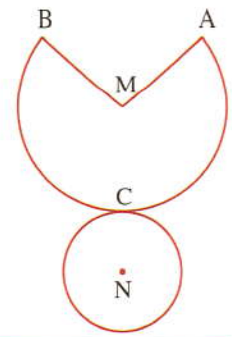
4 Find in terms of π the circumference and the area of the base of a right circular cone whose height is 24 cm. , and the length of its drawer is 26 cm. « 20 π cm. , 100 π cm^2. »

5 The opposite figure shows a net of a right cone , use the given data to find its height $\left(\pi = \frac{22}{7}\right)$

« $14\sqrt{2}$ cm. »

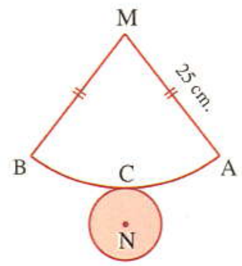


6 The opposite figure represents a right cone net form from a circular sector whose area is 20π cm^2, the length of its arc $\overset{\frown}{ACB} = 8\pi$ cm. Find the height of the solid.



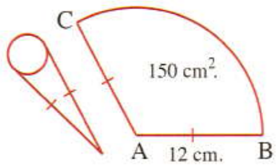
« 3 cm. »

7 The opposite figure represents a solid net. Describe the resulting solid of the folding process and find its height if MA = MB = 25 cm., area of the circle N = 49π cm^2.



« 24 cm. »

8 The frozen milk is encapsulated (kept) on a right circular cone by folding a piece of healthy - insulated paper in the form of circular sector the length of its radius is 12 cm. and its area is 150 cm^2, where the two radii of the circle $\overline{AB}$, $\overline{AC}$ become in contact. Find the height of the cone to the nearest one decimal.



« 11.3 cm. »

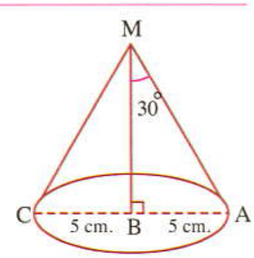
9 Find to the nearest tenth, the total area of the right circular cone in which the diameter length of its base is 10 cm. and its height is 12 cm. « 282.7 cm^2. »

10 Find the volume of the right circular cone where the circumference of its base is 44 cm. and its height is 25 cm. « 1283.8 cm^3. »

11 **In the opposite figure :**

A right circular cone in which m (∠ AMB) = 30°, the radius length of the base = 5 cm.

Calculate its lateral area and also the total area.



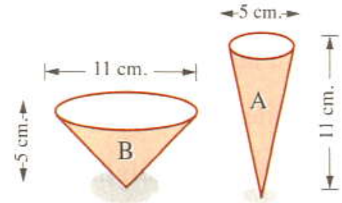
« 50π cm^2, 75π cm^2. »

12 A right circular cone, the radius length of its base is 8 cm. and its lateral area = 96π cm^2. Find to the nearest one decimal the volume of this cone. « 599.5 cm^3. »

13 **In the opposite figure :**

A , B are two cups for drinking which of them has greater capacity ?

Find the difference between their capacities.



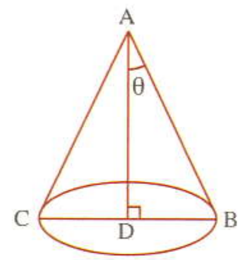
« The capacity of B is the greater. , $\frac{55}{2}\pi$ cm^3. »

---

14 **In the opposite figure :**

If $\sin\theta = \frac{3}{5}$, and the height of the cone = 12 cm. ,

Find the total area of the cone.



« 216π cm^2. »

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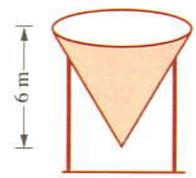
15 **Civil engineering :**

The opposite figure shows a water tank in the shape of a right circular cone , its volume = 32π m^3. and its height = 6 m.

Find the radius length of its base and its total area.



« 4 m. , $\left(16 + 8\sqrt{13}\right)\pi$ m^2. »

---

16 Which is greater in volume ?

A right circular cone in which the radius length of its base is 15 cm. and its height is 20 cm. or a regular quadrilateral pyramid whose height is 40 cm. and its base perimeter = 48 cm.

---

17 A right circular cone , its height = h and its volume = πh^3. Prove that its lateral area equals the lateral area of a right circular cylinder which is common with the cone in the base and the height.

---

18 **Connecting to physics :**

A cylindrical shaped vessel contain water , a metal body in the form of a right cone , its height is 12 cm. and the length of its base radius is 2 cm. and is completely immersed in it raising the surface of the water in the vessel with the value 1 cm.

Find the length of base diameter of the vessel. « 8 cm. »

---

19 A cube made of wax , its edge length = 20 cm. it is melted and converted to a right circular cone of height 21 cm. Find the radius length of the base of the cone given that 12% from wax had been lost during melting and reforming. $\left(\pi = \frac{22}{7}\right)$ « $8\sqrt{5}$ cm. »

---

20 A container in the shape of a right cone of capacity 2.2 litre and its height = 21 cm.

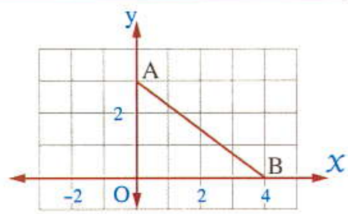
Find the radius length of its base. $\left(\pi = \frac{22}{7}\right)$ « 10 cm. »

21 A circular sector MAB , the radius length of its circle is 18 cm. and the measure of its central angle = 60° , it is folded and their radii are connected to form greatest lateral area of a right circular cone. Find the volume of this cone. « 167.3 cm³. »

22 AMB is a quadrant of a circle of centre M and its radius length = 20 cm. It is converted to the surface of a right circular cone where M is its vertex such that $\overline{MA}$ coincide $\overline{MB}$
Find the radius length of the base of the cone also find its volume in π
« 5 cm. , $\frac{125\sqrt{15}}{3}\pi$ cm³. »

23 ABC is a right-angled triangle at B in which AB = 6 cm. , BC = 8 cm.
Find the volume of the solid generated by turning Δ ABC a complete turn around :
(1) $\overleftrightarrow{BC}$ **(2)** $\overleftrightarrow{AC}$ « 96 π cm³. , 76.8 π cm³. »

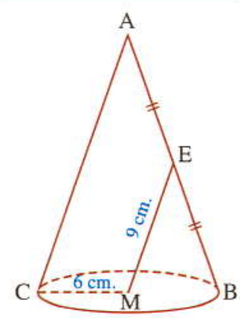
24 The opposite figure shows a coordinate perpendicular plane. Calculate in terms of π the volume of solid generated when revolving triangle ABO one complete revolution around :



(1) The X-axis **(2)** The y-axis « 12 π cubic units , 16 π cubic units »

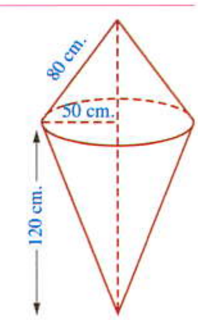
25 ABC is an isosceles triangle in which AB = AC = 10 cm. and BC = 12 cm. It turned around the base $\overline{BC}$ a complete turn. Calculate the volume of the generated solid. « 256 π cm³. »

26 **In the opposite figure :**
Find the lateral area and the total area and the volume of the right circular cone.



« 108 π cm². , 144 π cm². , 144$\sqrt{2}$ π cm³. »

27 **Marine navigation :**
The opposite figure shows a guide sign (Shamandora) (Buoy) to determine the waterway , and it is in the form of two right cones have a common base.
Find the costs of its painting with a material which resists erosion factor , note that each square metre of its costs 300 pound.



« 990 pounds »

28 **Connecting with industry :**

A regular pentagon pyramid made of copper , the side length of its base = 10 cm. and its height = 42 cm. it is melted and converted to a right circular cone the radius length of its base = 15 cm. given that 10% of copper has been lost during melting and converting it. Find the height of the cone to the nearest one decimal. « 9.2 cm. »

29 **Critical thinking :**

A right circular cone of volume 100 cm^3. Find its volume when :

(1) Its height is doubled. | **(2)** The length of its radius is doubled.

(3) Its height is doubled and the length of its radius is doubled.

What you conclude ? Explain your answer. « 200 cm^3. , 400 cm^3. , 800 cm^3. »

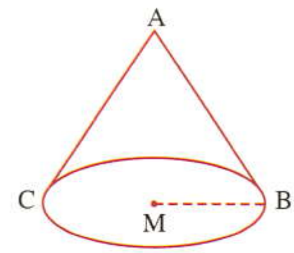
Third Higher skills

1 **Choose the correct answer from those given :**

(1) If the volume of hemisphere with raduis (r) equals the volume of cone with base raduis length (r) and height (h) , then ....

(a) $h = \frac{2}{3} r$ (b) $h = 2 r$ (c) $h = 2 r^2$ (d) $h = 4 r$

(2) **In the opposite figure :**



The volume of a right circular cone is 96π cm^3.

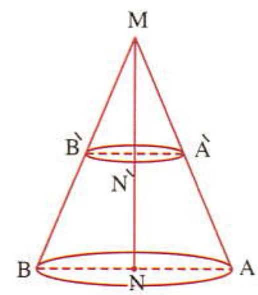
and $\frac{MB}{AB} = \frac{3}{5}$, then its total surface area = ... cm^2.

(a) 24π (b) 48π (c) 96π (d) 192π

(3) The arc length of a circular sector that if it is folded it becomes a right circular cone whose volume is 49π cm^3. and height 3 cm. equals .... cm.

(a) 2π (b) 4π (c) 8π (d) 14π

(4) **In the opposite figure :**



If a plane is drawn perpendicular to the cone axis and intersects it at midpoint of $\overline{MN}$, then

First : $\frac{\text{The volume of the smaller cone}}{\text{The volume of the greater cone}}$ = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

Second : $\dfrac{\text{The lateral area of the smaller cone}}{\text{The lateral area of the greater cone}} = \cdots\cdots\cdots$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(5) The ratio between the volume of a regular triangular pyramid and the volume of the greatest right circular cone can fit inside of the pyramid equals ....

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

(6) The ratio between the volume of a regular triangular pyramid and the volume of the smallest right circular cone can contain it equals ....

(a) $\frac{3\sqrt{3}}{\pi}$ (b) $\frac{3\sqrt{3}}{2\pi}$ (c) $\frac{\sqrt{3}}{\pi}$ (d) $\frac{3\sqrt{3}}{4\pi}$

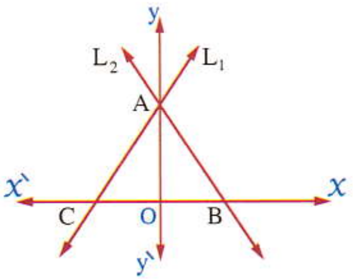
(7) The volume of a right circular cone is (v). If its base radius length is increased 50 % and its height is increased 50 % and its volume after increase is ($\grave{v}$) , then ....

(a) $\grave{v} = 150\ \%\ v$ (b) $\grave{v} = 225\ \%\ v$ (c) $\grave{v} = 337.5\ \%\ v$ (d) $\grave{v} = 450\ \%\ v$

2 **In the opposite figure :**

The equation of the straight line L_1 is $3x - \sqrt{3}\,y + 6 = 0$ and the equation of the straight line L_2 is $\sqrt{3}\,x + y - 2\sqrt{3} = 0$

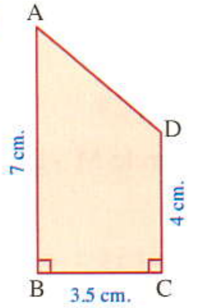
Find the volume of the body generated from turning ΔABC a complete turn around x-axis.



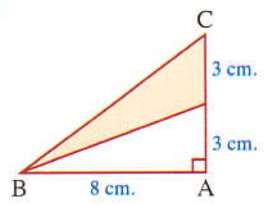
« 16 π cube units »

3 **Find the volume of the solid generated by turning the shaded part a complete turn around $\overleftrightarrow{AB}$ as an axis of rotation in each of the following figures :**

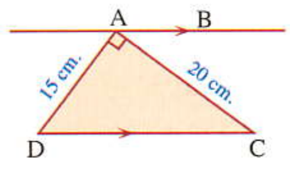
(1)



(2)



(3)



« 192.4 , 226.2 , 7539.8 cm^3 »

Exercise

9

The circle

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The centre of the circle in which its diameter is $\overline{AB}$ where $A = (-1 , 3)$, $B = (5 , -3)$ is ....

(a) $(4 , 0)$ (b) $(2 , 0)$ (c) $(-6 , -6)$ (d) $(0 , 4)$

(2) The radius length of the circle whose equation $x^2 + y^2 - 4x + 2y - 4 = 0$ is .... length unit.

(a) 2 (b) 4 (c) 3 (d) 9

(3) The radius length of the circle whose equation $(x + 2)^2 + y^2 + 2y = 0$ is .... length unit.

(a) zero (b) 1 (c) 2 (d) 4

(4) The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals .... length unit.

(a) 3 (b) 6 (c) 12 (d) 24

(5) If the two straight lines $y = -6$, $y = 8$ are two tangents to the circle M , then its radius length = ... length unit.

(a) 1 (b) 2 (c) 7 (d) 14

(6) If the straight line $y = 2$ touches the circle M whose center is $(6 , 9)$, then its diameter length = ... length unit.

(a) 6 (b) 7 (c) 14 (d) 15

(7) The radius length of the circle $(n+3)\,x^2 + y^2 - 4y + (m-2)\,xy + (m-n)\,x - 8 = 0$ is .... length unit.

(a) 2 (b) 4 (c) 6 (d) $2\sqrt{2}$

(8) The area of the circle whose equation is $(x-5)^2 + (y+4)^2 = 7$ equals .... square unit.

(a) $3.5\,\pi$ (b) $7\,\pi$ (c) $12.25\,\pi$ (d) $49\,\pi$

(9) If the equation $2x^2 + ay^2 + bxy - 5 = 0$ represents a circle ,

then its area = ... square unit.

(a) $5\,\pi$ (b) $\sqrt{5}\,\pi$ (c) $\frac{5}{2}\,\pi$ (d) $5\sqrt{2}\,\pi$

(10) The circumference of the circle whose equation $x^2 + y^2 + 2x - 2y - 2 = 0$ is .... length unit.

(a) π (b) $2\,\pi$ (c) $4\,\pi$ (d) $8\,\pi$

(11) The circumference of the circle whose equation is $x^2 + y^2 = 8$ is .... length units.

(a) $8\,\pi$ (b) $64\,\pi$ (c) $2\sqrt{2}\,\pi$ (d) $4\sqrt{2}\,\pi$

(12) If the two straight lines : $x = -3$, $x = 4$ touch the circle M ,

then its circumference = ... length units. where $\left(\pi = \frac{22}{7}\right)$

(a) 22 (b) 44 (c) 12 (d) 14

(13) If $(x \quad y \quad 8)\begin{pmatrix} x \\ y \\ -2 \end{pmatrix} = \boxed{}$, then the obtained equation represents a circle with diameter length = ... length unit. (Where $\boxed{}$ is the zero matrix)

(a) 2 (b) 4 (c) 6 (d) 8

(14) The equation : $\begin{vmatrix} x & yi \\ yi & x \end{vmatrix} - 49 = 0$ represents the equation of a circle with radius length .... length unit.

(a) 49 (b) 14 (c) 9 (d) 7

(15) Which of the following equations represent a circle ?

(a) $x^2 - y^2 + x - y = 6$ (b) $2x^2 + y^2 - x + y = 5$

(c) $x^2 + y^2 - x = 6$ (d) $x^2 + y^2 - xy = 6$

(16) If the equation : $2x^2 + (a-1)\,y^2 + 5x - 3y = 7$ represent a circle , then a = ...

(a) 1 (b) 2 (c) 3 (d) 4

(17) If $x^2 + y^2 + 2(\cos\theta)x - 2(\sin\theta)y - 8 = 0$ represent an equation of a circle , then r = ... length unit.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) 8

(18) The center of the circle whose equation $(x-2)^2 + (y+3)^2 = 16$ is ....

(a) (2 , 3) (b) (2 , – 3) (c) (13 , 16) (d) (4 , 9)

(19) The centre of the circle whose equation $x^2 + y^2 - 6x + 8y = 0$ is the point ....

(a) (3 , – 4) (b) (4 , – 3) (c) (– 3 , 4) (d) (– 4 , 3)

(20) The centre of the circle whose equation $2x^2 + 2y^2 + 12x - 16y = 0$ is ....

(a) (3 , – 4) (b) (– 6 , 8) (c) (– 3 , 4) (d) (6 , – 8)

(21) The circle $(x+2)^2 + y^2 + 2y = 0$ its centre is the point ....

(a) (2 , 2) (b) (– 2 , – 1) (c) (2 , – 1) (d) (– 2 , 0)

(22) Centre of the circle passes through the origin and the two points A (– 6 , 0) , B (0 , 8) is ....

(a) (4 , – 3) (b) (– 5 , 5) (c) (5 , 5) (d) (– 3 , 4)

(23) If any circle touches the two coordinate axes and it is drawn in the first quadrant , then its centre lies on the straight line ....

(a) $y = x$ (b) $y = -x$ (c) $y = x + 1$ (d) $y = x - 1$

(24) How many circles whose centre (3 , – 5) and touches one of the two axes ?

(a) 1 (b) 2 (c) 3 (d) 4

(25) The point (2 , 2) lies .... the circle whose equation $x^2 + y^2 = 9$

(a) on (b) inside (c) outside (d) in the centre of

(26) The point (2 , 0) lies on ....

(a) x-axis. (b) y-axis.

(c) the straight line $y = 2x$ (d) the circle $x^2 + y^2 = 9$

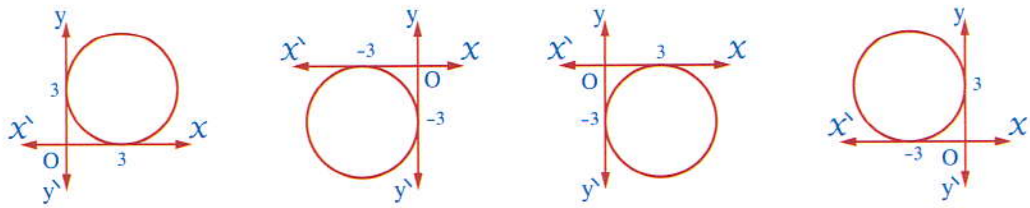
(27) The point which lies on the circle : $(x-2)^2 + y^2 = 13$ is ....

(a) (2 , 3) (b) (3 , – 2) (c) (2 , 5) (d) (4 , 3)

(28) The circle whose equation : $(x-1)^2 + (y+2)^2 = 5$ passes through the point ....

(a) (0 , 0) (b) (3 , – 1) (c) (2 , – 4) (d) All the previous.

(29) The circle C : $(x + 3)^2 + (y - 3)^2 = 9$ is represented by the figure ....



(a) (b) (c) (d)

(30) The general form of the equation of a circle if its centre is (2 , – 1) and its radius length = 3 cm. is ....

(a) $x^2 + y^2 - 4x + 2y - 9 = 0$ (b) $x^2 + y^2 - 4x + 2y - 4 = 0$

(c) $x^2 + y^2 + 2x - y + 3 = 0$ (d) $x^2 + y^2 + 2x - y + 9 = 0$

(31) The equation of the circle whose centre (4 , 3) and touches x-axis is ....

(a) $(x - 3)^2 + (y - 4)^2 = 16$ (b) $(x - 4)^2 + (y - 3)^2 = 9$

(c) $(x + 3)^2 + (y + 4)^2 = 9$ (d) $(x + 3)^2 + (y - 4)^2 = 16$

(32) The equation of the circle whose centre (– 4 , 4) and touches the two coordinate axes is ....

(a) $x^2 + y^2 + 8x - 8y + 16 = 0$ (b) $x^2 + y^2 = 16$

(c) $x^2 + y^2 - 8x + 8y + 16 = 0$ (d) $x^2 + y^2 = 8$

(33) The equation of the circle which is the image of the circle $x^2 + y^2 - 12x + 6y + 20 = 0$ by translation $(x + 2 , y - 2)$ is ....

(a) $(x + 8)^2 + (y + 5)^2 = 25$ (b) $(x - 8)^2 + (y + 5)^2 = 25$

(c) $(x - 8)^2 + (y - 5)^2 = 25$ (d) $(x + 5)^2 + (y - 8)^2 = 25$

(34) The equation of the circle whose centre (– 4 , 3) and passes through the origin point is ....

(a) $(x + 4)^2 + (y - 3)^2 = 5$ (b) $(x - 4)^2 + (y + 3)^2 = 25$

(c) $(x + 4)^2 + (y - 3)^2 = 625$ (d) $(x + 4)^2 + (y - 3)^2 = 25$

(35) The equation of the circle whose centre (1 , 2) and touches the line : $3x + 4y + 9 = 0$ is ....

(a) $x^2 + y^2 - 2x - 4y = 16$ (b) $x^2 + y^2 + 2x + 4y - 11 = 0$

(c) $x^2 + y^2 + 2x + 4y - 16 = 0$ (d) $x^2 + y^2 - 2x - 4y = 11$

(36) The circle equation whose centre lies on the straight line $y = \frac{1}{2}x$ and touches x-axis could be ....

(a) $(x-2)^2 + (y-1)^2 = 4$ (b) $(x-4)^2 + (y-2)^2 = 16$

(c) $(x-2)^2 + (y-4)^2 = 16$ (d) $(x-4)^2 + (y-2)^2 = 4$

(37) The equation of the circle which concentric with the circle whose equation $x^2 + y^2 - 6x + 2y - 6 = 0$ and passes through the point $(-3, 4)$ is ....

(a) $(x+3)^2 + y^2 = 16$ (b) $(x-3)^2 + (y+1)^2 = 25$

(c) $(x-3)^2 + (y+1)^2 = 16$ (d) $(x-3)^2 + (y+1)^2 = 61$

(38) In the following equations : The circle whose centre lies on the y-axis and does not intersect the x-axis is ....

(a) $x^2 + (y-1)^2 = 4$ (b) $x^2 + (y-5)^2 = 25$

(c) $x^2 + (y+5)^2 = 9$ (d) $(x+5)^2 + y^2 = 16$

(39) The equation of circle whose centre $(-4, -3)$ and its surface area is 25π cm^2 is ....

(a) $x^2 + y^2 - 8x + 6y - 25 = 0$ (b) $x^2 + y^2 + 8x + 6y = 0$

(c) $x^2 + y^2 + 4x + 3y + 25 = 0$ (d) $x^2 + y^2 + 8x - 6y = 0$

(40) ABCD is a rectangle in which $A = (-1, 4)$, $B = (7, 8)$, $C = (9, 4)$, $D = (1, 0)$, then the equation of the circumcircle of the rectangle is ....

(a) $(x-4)^2 + (y-4)^2 = 25$ (b) $(x-4)^2 + (y-4)^2 = 16$

(c) $(x+4)^2 + (y+4)^2 = 25$ (d) $(x-4)^2 + (y+4)^2 = 16$

(41) The geometrical centre of square ABCD is the origin and its side length is $2\sqrt{3}$, then the equation of the circle that touches its sides is ....

(a) $x^2 + y^2 = 3$ (b) $x^2 + y^2 = 12$

(c) $x^2 + y^2 = 6$ (d) $\left(x - \sqrt{3}\right)^2 + \left(y - \sqrt{3}\right)^2 = 3$

(42) The equation of the circle passes through the vertices of a regular hexagon that has area $6\sqrt{3}$ cm^2 and the centre of the circle is the origin is ....

(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 9$ (d) $x^2 + y^2 = 16$

(43) The circle whose equation is $(x-a)^2 + (y-b)^2 = a^2$ where $(a \neq b)$ ....

(a) touches x-axis. (b) touches y-axis.

(c) touches the two coordinates axes. (d) does not touch any of the two axes.

(44) If y-axis is a tangent to the circle $x^2 + y^2 + 4x + my + 4 = 0$, then m = ...

(a) 4 (b) – 4 (c) 0 (d) ± 4

(45) If the circle whose equation $x^2 + y^2 - 6x + 8y + c = 0$ touches x-axis ,
then c = ...

(a) – 9 (b) 9 (c) 6 (d) – 6

(46) If x-axis touches the circle $x^2 + y^2 + mx + 4y + 7 - 3m = 0$, then m = ...

(a) 2 or 14 (b) – 2 or – 14 (c) 2 or – 14 (d) – 2 or 14

(47) If the straight line $3x - 4y - 12 = 0$ touches the circle $(x + 3)^2 + (y - 1)^2 = r^2$,
then the circumference of the circle = ... length unit (in terms of π)

(a) 5π (b) 10π (c) 15π (d) 20π

(48) If the straight line $y = mx$ touches the circle $(x - 2)^2 + (y - 6)^2 = 4$, then m = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$

(49) The straight line $y = 5 - 2x$ .... the circle whose equation :
$x^2 + y^2 - 8x - 4y + 15 = 0$

(a) touch (b) intersect (c) outside (d) passes the centre

(50) The two circles $c_1 : (x + 2)^2 + (y - 1)^2 = 4$, $C_2 : (x - 5)^2 + (y - 3)^2 = 9$ ....

(a) distant. (b) touching externally.

(c) touching internally. (d) intersecting.

(51) The two circles $C_1 : (x + 2)^2 = 1 - y^2$, $C_2 : x^2 + y^2 - 2x - 8y - 19 = 0$ are ....

(a) intersecting. (b) touching internally.

(c) distant. (d) touching externally.

(52) If the straight line L : $3x + 4y + 9 = 0$ touches the circle M :
$x^2 + y^2 - 22x - 4y - c = 0$, then c = ...

(a) 15 (b) – 20 (c) 25 (d) – 25

(53) The length of the tangent segment to the circle : $x^2 + y^2 = 9$ from the point (5 , 0)
equals .... length unit.

(a) 14 (b) 3 (c) 5 (d) 4

(54) If $\overleftrightarrow{AB}$ is a tangent to the circle $x^2 + y^2 + 6x - 8y + 15 = 0$ at the point A (– 2 , 1) ,
then the equation of $\overleftrightarrow{AB}$ is ....

(a) $x - 3y + 5 = 0$ (b) $x - 3y = 5$ (c) $3x - y - 5 = 0$ (d) $3y - x + 5 = 0$

(55) If x-axis intersects the circle whose equation $x^2 + y^2 = 49$ at the two points A and B , then AB = ... length unit.

(a) 49 (b) 7 (c) 2 (d) 14

(56) The intersection point of the circle $(x-2)^2 + y^2 = 16$ with the x-axis is ....

(a) (6 , 0) , (– 2 , 0) (b) (– 6 , 0) , (2 , 0)

(c) (4 , 0) , (– 4 , 0) (d) (2 , 0) , (– 2 , 0)

(57) If the line y = 2 intersects the circle whose equation $(x-3)^2 + (y-2)^2 = 25$ at the two points A and B , then AB = ... length unit.

(a) $\sqrt{13}$ (b) 7 (c) 8 (d) 10

(58) If the straight line : $y - 2x + 5 = 0$ cuts the circle $x^2 + y^2 - 4x - 8y = 0$ at the two points A and B , then the distance between the centre and the chord $\overline{AB}$ = ...

(a) 3 (b) 4 (c) 5 (d) $\sqrt{5}$

(59) A circle , its centre M = (5 , 4) and its radius length = 5 length units and it intersects x-axis at the two points A and B , then the area of Δ MAB = ... square units.

(a) 6 (b) 9 (c) 12 (d) 18

(60) If the straight line $\overleftrightarrow{AB}$ is the axis of symmetry of the circle whose equation : $x^2 + y^2 = k^2$, and A , B $\in$ the circle where A = (– 2 , 5) , then B = ...

(a) (2 , – 5) (b) (2 , 5) (c) (0 , 0) (d) (5 , – 2)

(61) Area of the square whose vertices lie on the circle : $x^2 + y^2 - 4x + 6y + 4 = 0$ is .... square units.

(a) 6 (b) 9 (c) 12 (d) 18

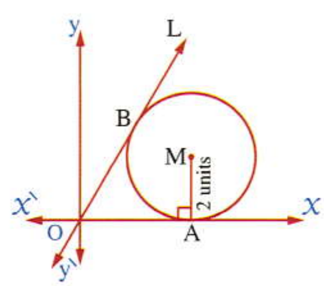
(62) If a circle with radius length 4 cm. and it passes through the vertices of a regular hexagon , then the area of the hexagon = ... cm.^{2}

(a) $8\sqrt{3}$ (b) $16\sqrt{3}$ (c) 16 (d) $24\sqrt{3}$

(63) In the opposite figure :

If OB = 5 length unit

, then the equation of the circle M is ....



(a) $(x-2)^2 + (y-5)^2 = 25$

(b) $(x-2)^2 + (y-5)^2 = 4$

(c) $(x-5)^2 + (y-2)^2 = 25$

(d) $(x-5)^2 + (y-2)^2 = 4$

(64) The equation of the circle that touches the straight lines $X = 4$, $X = -2$, $y = 0$ could be ....

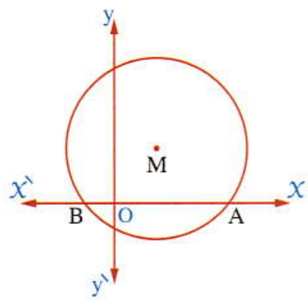
(a) $(X + 2)^2 + (y - 4)^2 = 36$

(b) $(X - 1)^2 + (y - 3)^2 = 36$

(c) $(X - 1)^2 + (y + 3)^2 = 9$

(d) $(X + 1)^2 + (y + 3)^2 = 9$

(65) In the opposite figure :



If the equation of the circle is

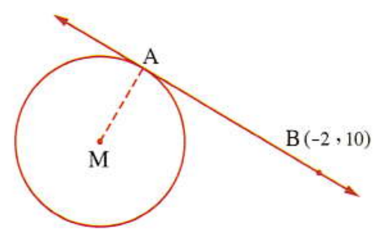
$(X - 2)^2 + (y - 3)^2 = 25$

, then AB = ... length unit.

(a) 8 (b) 4

(c) 6 (d) 5

(66) In the opposite figure :



If the equation of the circle M is

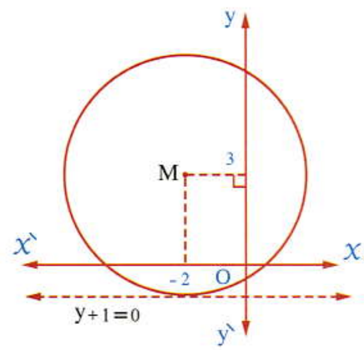
$(X - 3)^2 + (y + 2)^2 = 25$,

$\overleftrightarrow{AB}$ is a tangent to the circle M at A where

B = (– 2 , 10) , then AB = ... length unit.

(a) 13 (b) $\sqrt{194}$ (c) 12 (d) 5

(67) Which of the following circle equations does represent the circle in the opposite figure ?



(a) $(X - 3)^2 + (y + 2)^2 = 16$

(b) $(X + 2)^2 + (y - 3)^2 = 16$

(c) $(X + 2)^2 + (y - 3)^2 = 4$

(d) $(X + 2)^2 + (y - 3)^2 = 9$

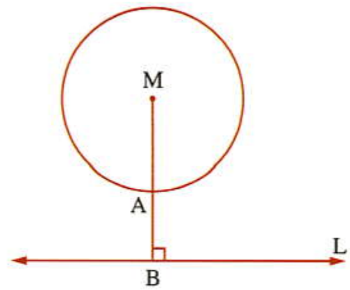
(68) If M is a circle where its circumference = 10π length unit , intersects X-axis at the two points A (2 , 0) , B (8 , 0) , then the equation of the circle M could be ....

(a) $(X + 5)^2 + (y + 4)^2 = 25$ (b) $(X - 5)^2 + (y - 4)^2 = 25$

(c) $(X - 5)^2 + (y - 4)^2 = 9$ (d) $(X - 5)^2 + (y - 4)^2 = 36$

(69) In the opposite figure :

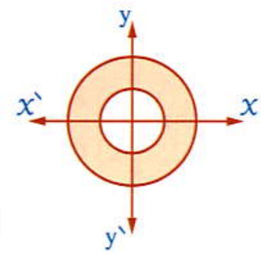
If the equation of the circle M is $x^2 + y^2 - 6x + 4y - 12 = 0$, $\overline{MB} \perp$ the straight line L where the equation of L is $3x - 4y + 23 = 0$, $A \in \overline{MB}$, then the length of $\overline{AB}$ = ... length unit.



(a) 3 (b) 4 (c) 5 (d) 2.5

(70) The opposite figure represents a disc of a machine.

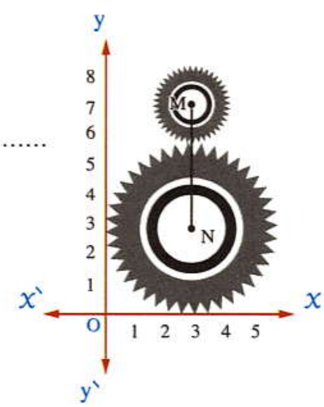
It is required to make another one like it, if the cost of one square unit from the surface of the disc is 5 pounds and the equation of the smaller disc is $x^2 + y^2 = 4$ the length of the diameter of the greater circle is 10 length units, then the cost of the disc ≃ .... pounds.



(a) 440 (b) 660 (c) 220 (d) 330

(71) The opposite figure represents two gears in a machine, their centres are M and N, $\overline{MN}$ // the y-axis.

If the radius of the smaller gear = $\frac{1}{3}$ the radius of the greater gear, then the equation of the smaller gear is ....



(a) $(x - 3)^2 + (y - 1)^2 = 9$

(b) $(x - 3)^2 + (y - 7)^2 = 1$

(c) $x^2 + y^2 - 6x - 14y + 58 = 0$

(d) $(x - 1)^2 + (y - 1)^2 = 1$

Second Essay questions

1 **Find the equation of the circle whose centre is (M) and its radius length (r) unit in each of the following cases :**

(1) M = (2, 3), r = 5

(2) M = (0, 0), r = 3

(3) M = (0, – 1), $r = 2\sqrt{3}$

(4) M = (– 4, – 3), $r = \frac{3}{2}$

2 **Write the general form of the equation of the circle if :**

(1) Its centre M $(-2 , 3)$ and its diameter length equals 8 length unit.

(2) Its centre M $(5 , -12)$ and it passes through the origin point.

(3) Its centre M $(7 , -5)$ and passes through the point $A = (3 , 2)$

(4) $\overline{AB}$ is a diameter in the circle where $A = (6 , -4)$ and $B = (0 , 2)$

(5) Its centre is the point $(-3 , -2)$ and touches X-axis.

(6) Its centre is the point $(3 , 0)$ and touches y-axis.

(7) Its centre is the point $(5 , -5)$ and touches the two coordinate axes.

(8) It passes through the two points $A = (6 , 2)$, $B = (0 , -1)$ and the two tangents to the circle at A and B are parallel.

(9) Its centre lies on X-axis and it passes through the two points $(2 , 0)$, $(8 , 0)$

(10) Its radius length = 6 length units and it touches the two axes given that the circle lies in the fourth quadrant.

3 **Find the coordinates of the centre , also find the radius length for each of the following circles :**

(1) $X^2 + y^2 - 8 = 0$

(2) $(X + 3)^2 + (y - 5)^2 = 49$

(3) $(X + 4)^2 + y^2 = 9$

(4) $X^2 + (y + 7)^2 = 24$

(5) $X^2 + y^2 - 4X + 6y - 12 = 0$

(6) $X^2 + y^2 - 8X = 12$

4 **Show which two circles in the following are congruent and why ?**

(1) $X^2 + y^2 - 4X + 8y = 0$, $X^2 + y^2 + 12y + 16 = 0$

(2) $X^2 + y^2 + 14y = 1$, $X^2 + y^2 + 10X - 25 = 0$

(3) $X^2 + y^2 - 2X + 4y - 3 = 0$, $X^2 + y^2 + 6X - 11 = 0$

5 The opposite figure shows the two circles C_1 and C_2

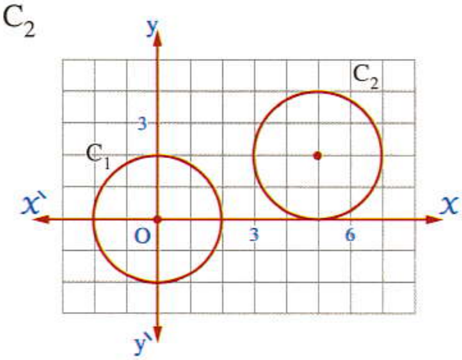
Prove that the two circles are congruent

, then find the equation of each of them.

If the circle C_3 is the image of the circle C_1

by translation $(-4 , 3)$

Find the equation of the circle C_3



6 **Show with reasons which of the following equations represents a circle and which of them does not represent a circle :**

(1) $x^2 + x y + y^2 = 25$

(2) $x^2 + y^2 + 8x - 16y - 1 = 0$

(3) $x^2 + 2y^2 + 6x - 5y = 0$

(4) $2x^2 + 2y^2 + 3y - 8 = 0$

(5) $(x + y)^2 - 3x + 6y - 4 = 0$

(6) $x^2 + y^2 + x + 2y + 7 = 0$

(7) $x^2 + y^2 + 2x - 4y + 5 = 0$

(8) $\frac{1}{4}x^2 + \frac{1}{4}y^2 + x - 8 = 0$

(9) $x^2 + x = y^2 + y + 7$

7 M_1 and M_2 are the two centers of two circles where $M_1 = (2, -1)$, $M_2 = (-1, 3)$

Find the equation of each circle given that each of them passes through the centre of the other.

8 Prove that the two circles : $x^2 + y^2 - 2x + 6y + 1 = 0$, $4x^2 + 4y^2 - 8x + 24y + 15 = 0$ are concentric and find the radius length of each of them. « 3 , 2.5 length unit »

9 Show which of the following points belongs to the circle C whose equation : $(x - 6)^2 + (y + 1)^2 = 25$, then determine the position of each of the other points with respect to the circle C where : A (9 , 3) , B (7 , 5) , C (3 , 3) , D (2 , – 3)

10 A circle of centre (2 , – 1) passes through the point A = (– 1 , 3). Show the positions of the following points with respect to the circle M : B = (2 , 4) , C = (– 3 , 1) , D = (1 , 2)

11 Determine the position of the straight line with respect to the circle $(x + 3)^2 + (y - 4)^2 = 9$

If the equation of the straight line is :

(1) $L_1 : 3x - 4y + 5 = 0$ (2) $L_2 : 6x - 8y + 23 = 0$ (3) $L_3 : 3x - 4y + 10 = 0$

12 Determine the position of the circle $C_1 : (x - 5)^2 + (y + 2)^2 = 4$ with respect to the circle , $C_2 : (x + 7)^2 + (y - 3)^2 = 1$

13 Are the two circles $C_1 : x^2 + y^2 - 10x - 8y + 16 = 0$ and $C_2 : x^2 + y^2 + 14x + 10y - 26 = 0$ touching externally ? Explain your answer.

14 If the two circles $C_1 : (x + 2)^2 + (y + 11)^2 = k$, $C_2 : (x - 3)^2 + (y - 1)^2 = 16$ are touching. Find the value of k « 81 or 289 »

15 Prove that the two circles : $x^2 + y^2 - 6x - 4y + 12 = 0$, $x^2 + y^2 + 2x - 4y - 4 = 0$ touch each other and find the coordinates of the point of tangency, then find the circle equation whose centre is the point of tangency and passes through the center of the second circle.

16 Write the equation of the unit circle and if the point $(2a\cos\theta , 2a\sin\theta)$ belongs to this circle. Find the real values of a (*i.e.* $a \in \mathbb{R}$)

17 **Find the value of $h \in \mathbb{R}$ which makes each of the following represents an equation of circle :**

(1) $x^2 + y^2 - 2x - 4y - h + 2 = 0$

(2) $x^2 + y^2 + 4x - 6y - h^2 + 4 = 0$

(3) $x^2 + y^2 - 4hx - 2hy + 10(h-1) = 0$

(4) $x^2 + y^2 + 6x + 8y + h^2 - 3h + 15 = 0$

(5) $x^2 + y^2 + 2hx - 6hy - 2h^2 + 12h - 3 = 0$

18 **Find the value of a in the equation :**

$x^2 + y^2 - 2x + 4y + 2a - 3 = 0$ in each of the following cases :

(1) The equation represents a circle.

(2) The equation represents a circle passing through the origin point.

(3) The equation represents a circle touching x-axis.

(4) The equation represents a circle touching y-axis.

(5) The equation represents a circle touching the straight line : $3x + 4y + 15 = 0$

(6) The equation represents a circle of diameter length 14 length unit.

19 **Write the general form of the equation of a circle if :**

(1) Its centre M $(5 , 4)$ and touches the straight line $x = 2$

(2) Its centre M $(5 , 3)$ and touches the straight line passing through the two points $(3 , 7) , (-1 , 3)$

(3) Its centre M lies in the first quadrant and its radius length = 3 length unit and the two straight lines $x = 1 , y = 2$ are tangents to it.

(4) Its radius length = 5 length unit and touches x-axis at the point $(4 , 0)$

(5) Its radius length = $3\frac{1}{2}$ unit and touches y-axis at the point $(0 , -4)$

(6) Touches the two coordinate axes and passes through the point $(-2 , -4)$

(7) Touches x-axis at the point $(-3 , 0)$ and touches also y-axis

(8) Touches x-axis at the point $(-2 , 0)$ and intercepts from the positive part of y-axis a chord of length $4\sqrt{3}$ length unit.

(9) Touches y-axis at the point $(0 , -1)$ and intercepts from the negative part of x-axis a chord of length $4\sqrt{6}$ length unit.

(10) Touches the x-axis and passes through the two points $(2 , 1)$, $(-5 , 2)$

(11) Touches y-axis and passes through the two points $(-4 , 2)$, $(-1 , 2)$

(12) Its centre lies on x-axis and passes through the two points $A (1 , 3)$, $B (2 , -4)$

(13) Passes through the origin point and intercepts from the two positive parts of the x-axis and y-axis two parts of lengths 12 , 16 length units respectively.

(14) Its centre lies on the straight line : $y - x = 1$ and passes through the two points $A = (-2 , 4)$, $B = (6 , 8)$

(15) Its radius length $= \sqrt{85}$ length unit and passes through the two points $A = (-1 , 2)$, $B = (3 , 4)$

(16) Its diameter $\overline{AB}$ where A and B are the points of intersection between the circle $x^2 + y^2 + 2x + 4y = 0$ and x-axis.

---

20 Find the area of the equilateral triangle which its circumcircle is :

$x^2 + y^2 + x - 4y - 2 = 0$ « $\frac{75\sqrt{3}}{16}$ square units »

---

21 Find to the nearest cm^2 the surface area of a regular pentagon. If the circle :

$x^2 + y^2 + 6x - 12y + 5 = 0$ passes through its vertices knowing that each unit in the coordinate plane represents 5 cm. « $2378\ cm^2$ »

---

22 Find the surface area of the regular hexagon which its circumcircle is :

$x^2 + y^2 - 10x + 6y + 25 = 0$ « $\frac{27\sqrt{3}}{2}$ square units »

---

23 Find the surface area of a regular polygon of 12 sides and the circle :

$x^2 + y^2 - 16 = 0$ passes through its vertices. « 48 square units »

---

24 Find the equation of the circle whose radius length = 5 length unit and the equations of two straight lines carrying two diameters in it are : $3x + y + 2 = 0$, $4x - y - 16 = 0$, then prove that the point $(5 , -4)$ belongs to the circle.

---

25 Find the equation of the circle whose radius length equals the radius length of the circle :

$x^2 + y^2 - 2x \cos\theta - 2y \sin\theta - 8 = 0$

and the equations of two straight lines carrying two diameters in it are

$x + y = 0$, $\vec{r} = (1 , 5) + k (1 , 2)$

26 Find the equation of the circle which passes through the two points of intersection of the two circles $x^2 + y^2 - 10x = 0$ and $x^2 + y^2 + 2x - 12 = 0$ and whose center

(1) The origin (2) The point $(2, 0)$

---

27 Prove that the points : $A = (0, -1)$, $B = (-1, 0)$, $C = (-9, 0)$ lie on circle whose centre is $M = (-5, -5)$, then find the equation of this circle.

---

28 If the points : $A = (3, -2)$, $B = (3, 8)$, $C = (-1, 0)$ belong to one circle, prove that $\overline{AB}$ is a diameter in it , then write the general form of its equation.

---

29 Prove that the triangle whose vertices are $A (8, 0)$, $B (0, 6)$, $C (0, 0)$ is right-angled , then find the equation of the circle which passes through its vertices.

---

30 Prove that the points : $A = (-2, 0)$, $B = (4, 0)$, $C = \left(1, 3\sqrt{3}\right)$ are the vertices of the equilateral triangle ABC , then find the equation of the circumcircle of ΔABC

---

31 Find the equation of the circle which passes through the points : $A = (2, -1)$, $B = (-2, 0)$, $C = (0, -9)$ and determine its centre and its radius length.

---

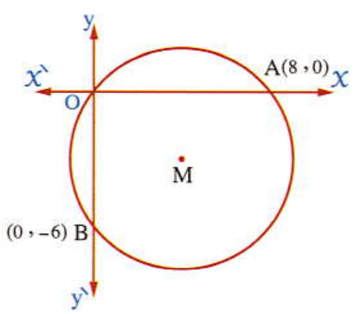
32 If $A = (3, 0)$, $B = (0, 9)$, $C = (0, 1)$, $D = (-1, 2)$

Prove that the quadrilateral ABCD is cyclic.

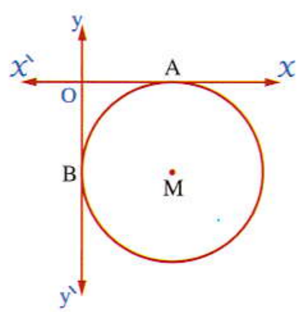
---

33 **Find the general form of the equation of the circle M in each of the following figures :**

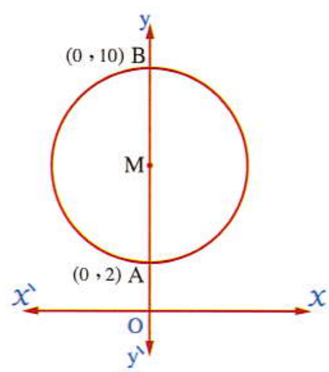
(1) The circle passes through the origin point and passes through the two points A and B



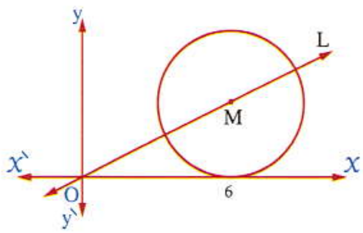
(2) The circle touches the two coordinate axes at A and B and the length of $\overline{MO} = 2\sqrt{2}$



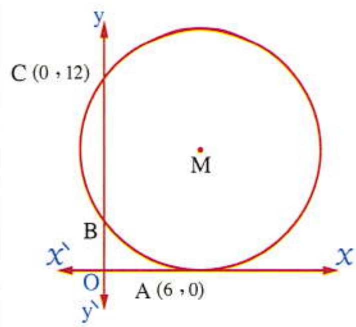
(3) The centre of the circle lies on y-axis and the circle intersects y-axis at A and B



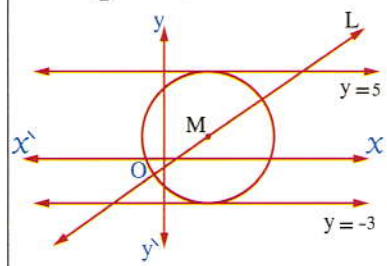
(4) The straight line whose equation is $x - 3y = 0$ passes through the centre of the circle and the origin point



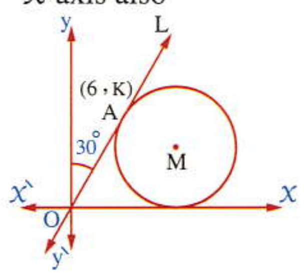
(5) The circle touches x-axis at A and intersects y-axis at B and C



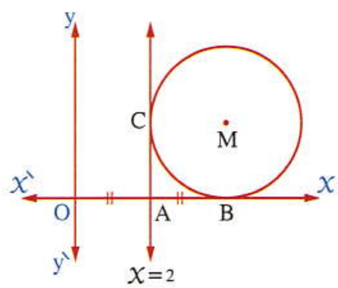
(6) The straight line L : $2x - 3y = 1$ passes through the centre of the circle and the two straight lines $y = 5$, $y = -3$ are tangents to the circle.



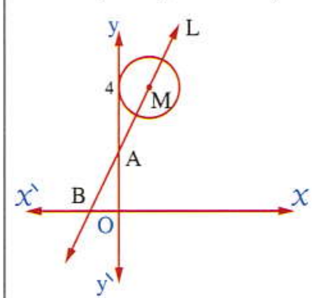
(7) The straight line L touches the circle at A (6 , k) and makes an angle of measure 30° with the positive direction of y-axis and the circle touches x-axis also



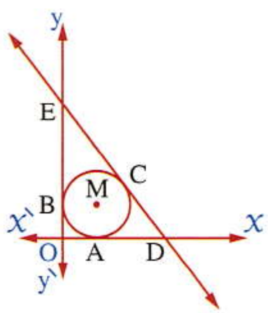
(8) The circle touches x-axis at B and touches the straight line $x = 2$ at C



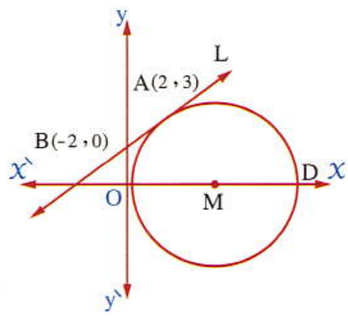
(9) The circle touches y-axis at the point (0 , 4) and the straight line L passes through the centre of the circle and the two points A (0 , 2) and B (– 1 , 0)



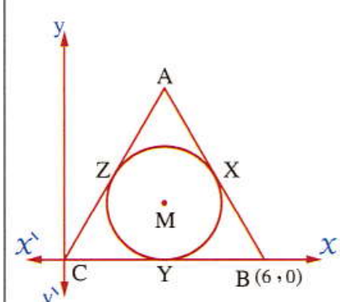
(10) The two coordinate axes touch the circle M at A and B. If the straight line $4x + 3y - 12 = 0$ is a tangent to the circle M at C



(11) The straight line L touches the circle at A (2 , 3) and intersects x-axis at B (– 2 , 0)



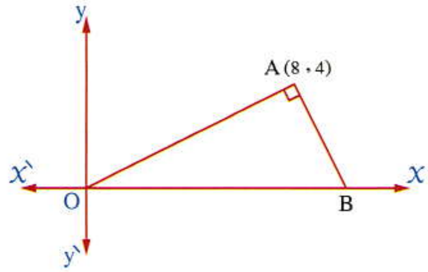
(12) ABC is an equilateral triangle its sides touch the circle M , B = (6 , 0)



34 **In the opposite figure :**

If $\overline{OA} \perp \overline{AB}$, A (8 , 4)

Find the equation of the circle which passes through the points A , B and O



Third Higher skills

1 **Choose the correct answer from those givens :**

(1) The equation : $(k-2)\,x^2+(2-k)\,y^2-k\,x+3\,k\,y-25=0$ ....

(a) represents a circle when $k=2$

(b) represents a circle when $k \neq 2$

(c) represents a circle when $k \in \mathbb{R}$

(d) does not represent a circle whatever the value of k.

(2) The height of a right circular cone is 6 length units and the equation of its circular base is $x^2+y^2=64$ in the $x\,y$ – plane , then the volume of the cone = ... cubic units.

(a) 96π (b) $\frac{640}{3}\pi$ (c) 128π (d) $\frac{128}{3}\pi$

(3) The least distance between the y-axis and a point on the circle whose equation : $(x-7)^2+(y-5)^2=16$ is .... length units.

(a) 11 (b) 3 (c) 5 (d) 7

(4) Number of circles touch the coordinate axes and their centres lie on the circle : $x^2+y^2=25$ equals ....

(a) zero (b) 1 (c) 2 (d) 4

(5) **In the opposite figure :**

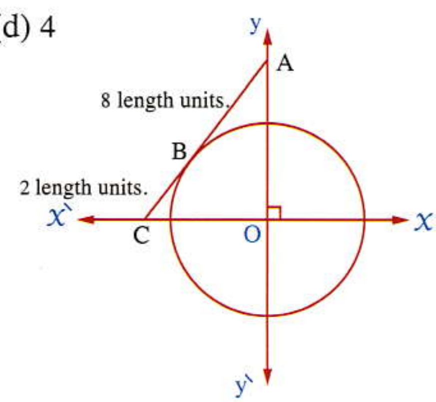
The equation of the circle is ....

(a) $x^2+y^2=4$

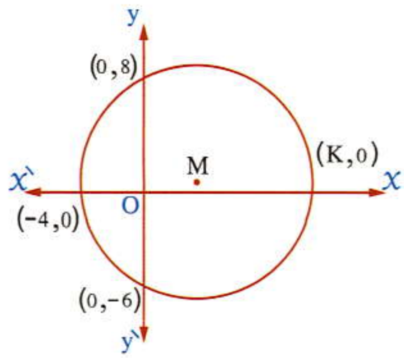
(b) $x^2+y^2=16$

(c) $x^2+y^2=64$

(d) $x^2+y^2=100$



(6) In the opposite figure :



The equation of the circle is ....

(a) $(X+4)^2+(y+1)^2=65$

(b) $(X-6)^2+(y-2)^2=64$

(c) $(X-4)^2+(y-1)^2=65$

(d) $(X-4)^2+(y-2)^2=64$

(7) If O is the origin , $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two tangents to the circle $X^2+y^2-10X+4y+6=0$, then the centre of the circumcircle of Δ AOB is ....

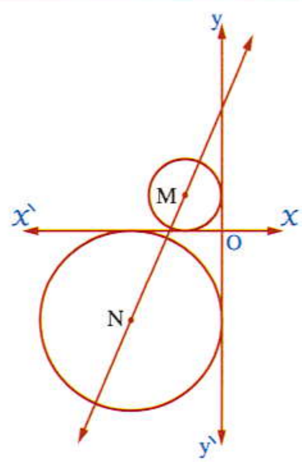
(a) $\left(\frac{7}{4}, 2\right)$ (b) $\left(\frac{5}{2}, -1\right)$ (c) $\left(\frac{7}{4}, -1\right)$ (d) $\left(\frac{5}{2}, 2\right)$

(8) The length of the common chord of the two circles $X^2+y^2-10X-10y=0$ and $X^2+y^2+6X+2y-40=0$ equals .... length unit.

(a) $5\sqrt{2}$ (b) 10 (c) 12 (d) $10\sqrt{2}$

---

2 In the opposite figure :



If each of the two circles M and N touches the two coordinate axes and the equation of the line of centres $\overleftrightarrow{MN}$ is : $y=2X+1$

Find the equation of each of the two circles M and N

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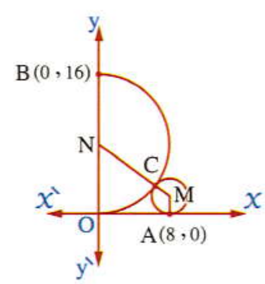
3 In the opposite figure :



A semicircle , its centre N lies on y-axis and touches a circle M at C and the circle M touches X-axis at A where A = (8 , 0) If B = (0 , 16)

Find the general form of the equation of the circle M

Life applications

1 **Town designing :**

In the drawing for one of the cities in a perpendicular coordinate axes plane , where each unit in it represents 5 metres. It is found that the circle : $x^2 + y^2 - 6x + 8y + 11 = 0$ represents one of its squares. Find to the nearest squared metre the area of the square $\left(\pi = \frac{22}{7}\right)$ « 1100 m^2 »

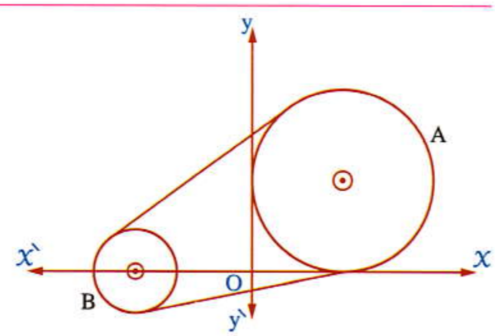
2 **Marine navigation :** A radar is located in the position A (7 , – 9) and cover a circular region. The length of its radius equals 30 length unit. Write the equation of the circle that determine the radar range in the coordinates plane. Can the radar observe a ship in the position B (25 , – 30) ? Explain your answer.

3 **Architectural design :**

An architect designs a building in the form of a regular octagon. Its vertices passes by a circle $x^2 + y^2 - 4x + 12y - 60 = 0$ Calculate the area of the building to the nearest squared unit. « $200\sqrt{2}$ square units »

4 **Industry :**

The opposite figure shows a pulley A in a machine touching the two coordinate axes , it rotates by a wire passing on a small pulley B which the equation of its circle is : $x^2 + y^2 + 14x + 45 = 0$



Find :

(1) The equation of the circle of the pulley A given that its radius length = 5 units.

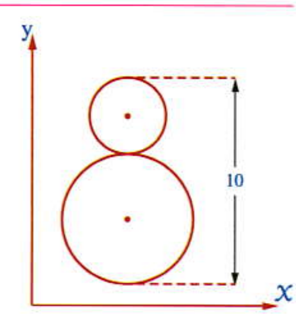
(2) The distance between the two centres of the two pullies if the plane unit represents 6 cm. « 78 cm. »

5 **Industry :**

The opposite figure shows two gears in a machine such that their centres lie on a straight line parallel to y-axis and the maximum distance between their edges is 10 units.

Find the equation of the circle of the small gear given that the equation of the great gear is :

$x^2 + y^2 - 10x - 8y + 32 = 0$



Notes

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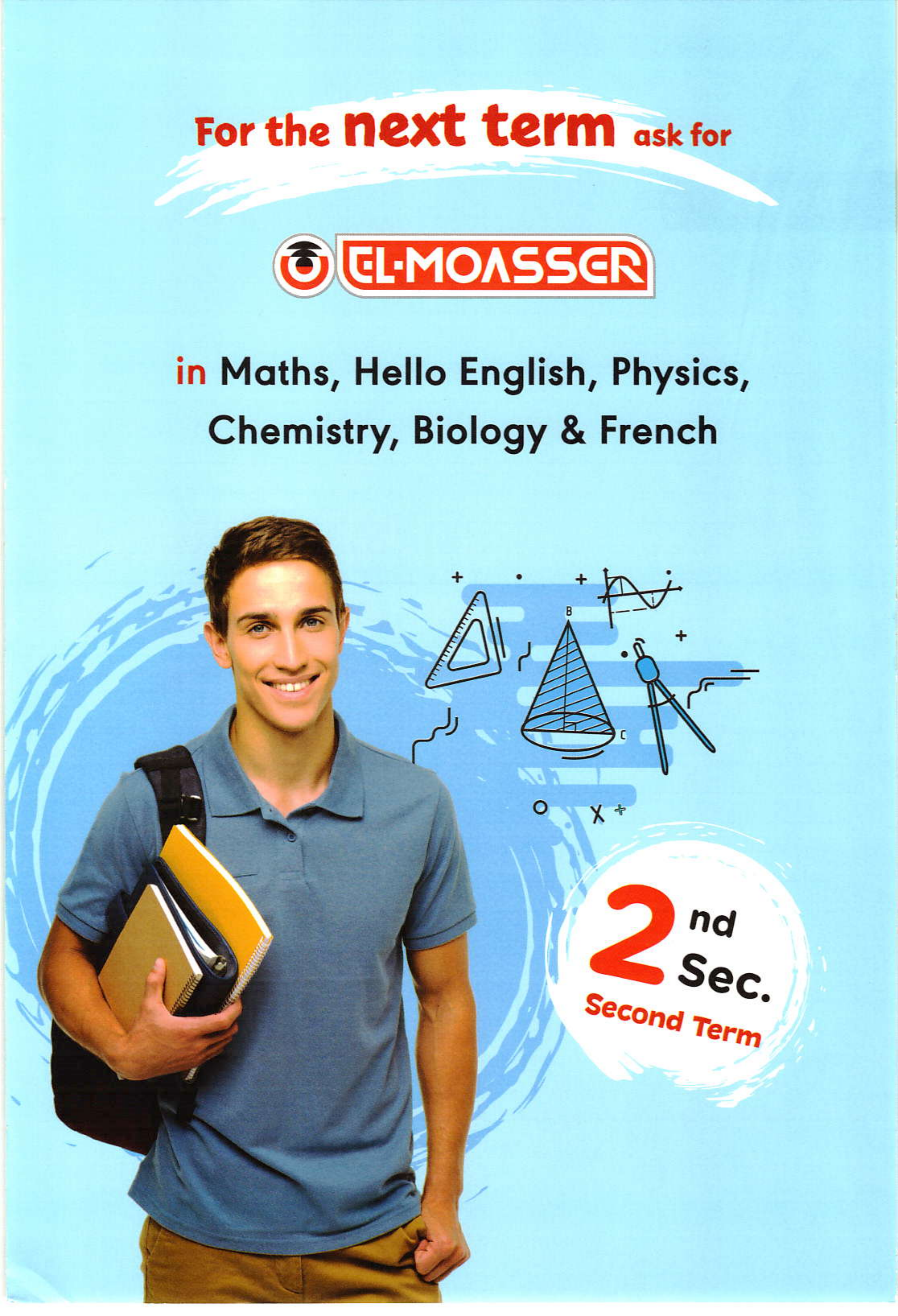
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