Name of the School	School of Basic and Applied Sciences	Name of the Department	Department of Mathematics
Name of the Program	B Tech	Course Code- Course Name	UBS 1003M Maths I / UBS 1006M Engg. Math.1
Session	2024-25	Branch, Year & Semester	CSE/ECE/R&AI , 1st, 1st
Time/Maximum Marks	90 Minutes/50	Set	A

Note: Attempt all questions.

Q No.	QUESTIONS	MARKS	CO
	SECTION-A		
1	Verify Rolle's theorem for $f(x) = \sin x + 2$ for $0 \leq x \leq 2\pi$.	2	1
2	If $y = (2x + 3)^{-1}$, find y_{30}.	2	1
3	If $z = \dfrac{x^{-\frac{9}{2}} + y^{-\frac{9}{2}}}{x+y}$, find $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$.	2	2
4	If $u = e^{-r\sin\theta}$, find $\dfrac{\partial u}{\partial r}$ and $\dfrac{\partial u}{\partial \theta}$.	2	2
5	Evaluate $\int_0^1 \int_0^1 x^2 e^y \, dx \, dy$.	2	3
6	Find $\int_0^5 \int_{2-x}^{2+x} dy \, dx$.	2	3
	SECTION-B		
7	Find the nth derivative of $y = e^{3x}\sin^2 x$.	5	1
8	If $\log y = \tan^{-1} x$, show that $(1 + x^2)y_{n+2} - \{2(n+1)x - 1\}y_{n+1} + n(n+1)y_n = 0$.	7	1
9	If $x^x y^y z^z = c$, show that at $x = y = z$ $\dfrac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$.	7	2
10	If $u = \log\left(\dfrac{x^2+y^2}{\sqrt{x}+\sqrt{y}}\right)$, find the value of (i) $\quad x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ (ii) $\quad x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$	7	2
11	Evaluate $\int_0^a \int_0^{a-z} \int_0^{a-y-z}(x^2 + y^2 + z^2)\, dx \, dy \, dz$	5	3
12	By changing of order of integration, evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \dfrac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\, dy \, dx$.	7	3

Name of the School	School of Basic & Applied Sciences	Name of the Department	Mathematics
Name of the Program	B.Tech.	Course Code/ Course Name	UBS1003M [Maths I] /UBS1006M [Engg. Mathematics I]
Session	2024 – 25	Branch, Year & Semester	CSE, R&AI, EEE
Time/Max Marks	3 Hours/100	Set	A

Note:
1) Attempt all sections (A, B & C).
2) Attempt all sections and assume any missing data.
3) Appropriate marks are allotted to each question, answer accordingly.

Q No.	QUESTIONS	MARKS	CO
	SECTION-A: Attempt All of the following Questions in brief.	(10x2=20)	
Q1(a)	Find the nth derivative of $y = \sin 2x \cos 3x$	2	1
Q1(b)	Test the applicability of Rolle's theorem for $f(x) = x^2 - 3x + 4$ on $[0, 2]$	2	1
Q1(c)	Find $\frac{dy}{dx}$ if $y^2 + xy - 3x - 3 = 0$ at $(-1, 1)$	2	2
Q1(d)	Find Jacobian if $x = r\cos\theta$ & $y = r\sin\theta$	2	2
Q1(e)	Evaluate $$\int_0^3 \int_0^x y\, dy dx$$	2	3
Q1(f)	Evaluate $$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx$$	2	3
Q1(f)	Describe in brief the Cauchy – Riemann equations for the analyticity of a complex function.	2	4
Q1(h)	Define the continuity of a complex function.	2	4
Q1(i)	Discuss the nature of the series $$2 - 2 + 2 - 2 + 2 - \ldots\ldots\ldots\ldots\infty$$	2	5
Q1(j)	What do you mean by infinite series? Write an example of a positive infinite series.	2	5
	SECTION-B: Attempt all Questions.	(5x6=30)	
Q2(a)	Expand $e^x \sin x$ in powers of $(x - 1)$ up-to third degree term.	6	1
Q2(b)	Find the extreme values of the function $x^3 + y^3 + 3axy$	6	2
Q2(c)	Change the order of integration in $$\int_0^4 \int_y^4 \frac{x\, dx\, dy}{x^2 + y^2}$$ & evaluate the same.	6	3
Q2(d)	Show that $u = e^x \cos y$ is harmonic function. Find its harmonic conjugate.	6 → 3	4
Q2(e)	Examine the convergence of the series $$\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \ldots\ldots\ldots\ldots + \infty$$	6	5
	SECTION-C: Attempt ANY ONE of the following Questions.	(1x10=10)	

Q3(a)	If $y = x \log\left(\frac{x-1}{x+1}\right)$ then show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$	10 ✓	1
Q3(b)	Verify Lagrange's mean value theorem $f(x) = x^3 - 6x^2 + 9x + 1$ in $[1,4]$	10	1

SECTION-C: Attempt ANY ONE of the following Questions. (1x10=10)

Q4(a)	Verify Euler's theorem on homogenous function for the function $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$	10 - 6	2
Q4(b)	If $u = xyz$, $v = xy + yz + zx$ & $w = x + y + z$ then compute the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	10	2

SECTION-C: Attempt ANY ONE of the following Questions. (1x10=10)

Q5(a)	Evaluate $$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\, dz\, dy\, dx$$	10 ✓	3
Q5(b)	Change the order of integration and hence evaluate $$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2 x^2}}\, dy\, dx$$	10 ✓	3

SECTION-C: Attempt ANY ONE of the following Questions. (1x10=10)

Q6(a)	Determine analytic function using Milne Thomson's method whose real part is $$e^x(x \cos y - y \sin y)$$	10	4
Q6(b)	Prove that $u = x^4 - 6x^2 y^2 + y^4$ is harmonic. Also find the analytic function $f(z) = u(x,y) + iv(x,y)$	10	4

SECTION-C: Attempt ANY ONE of the following Questions. (1x10=10)

Q7(a)	Discuss the convergence of infinite series $$\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n \; ; x > 0$$	10 ✓	5
Q7(b)	Discuss the convergence of infinite series $$\frac{x}{1} + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot3}{2\cdot4}\cdot\frac{x^5}{5} + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{x^7}{7} + \dots\dots\dots\dots + \infty$$	10	5