QUESTION BANK

SECTION A

SHORT ANSWER TYPE QUESTIONS

1. Solve

i. $(9D^3 + 3D^2 - 5D + 1)y = 0$

ii. $\frac{d^4y}{dx^4} + 4x = 0$

iii. $\frac{d^2y}{dx^2} + (a+b)\frac{dy}{dx} + aby = 0$

iv. $D^2y - 5Dy + 6y = 0$

v. $D^2y - 6Dy + 9y = 0$

vi. $D^2y + y = 0$

vii. $D^2y - 5Dy + 6y = e^x + 2$

viii. $D^2y + y = \sin 2x$

Solution

i. $\boldsymbol{c_1e^{-x} + c_2e^{\frac{x}{3}} + c_3xe^{\frac{x}{3}}}$

ii. $\boldsymbol{c_1e^{-\sqrt{2}x} + c_2e^{\sqrt{2}x} + c_3\cos\sqrt{2}x + c_4\sin\sqrt{2}x}$

iii. $\boldsymbol{c_1e^{-ax} + c_2e^{-bx}}$

iv. $\boldsymbol{c_1e^{2x} + c_2e^{3x}}$

v. $\boldsymbol{c_1e^{3x} + c_2xe^{3x}}$

vi. $\boldsymbol{c_1\cos x + c_2\sin x}$

vii. $\boldsymbol{c_1e^{3x} + c_2xe^{3x} + \frac{e^x}{2} + \frac{1}{3}}$

viii. $\boldsymbol{c_1\cos x + c_2\sin x - \frac{1}{3}\sin 2x}$

2.

Solve the simultaneous differential equations:

a) $\frac{dx}{dt} = 3x + 2y, \frac{dy}{dt} = 5x + 3y$

b) $Dx = -\omega y, Dy = \omega x.$ *Also show that the point* (x, y) *lies on a circle.*

c) $Dx + 5x - 2y = t, Dy + 2x + y = 0;$ *where* $D = \frac{d}{dt}$

Solution

a) $\boldsymbol{x = e^{3t}(c_1 \cosh\sqrt{10}t + c_2 \sinh\sqrt{10}t); y = \frac{\sqrt{10}}{2}e^{3t}(c_2 \cosh\sqrt{10}t + c_1 \sinh\sqrt{10}t)}$

b) $\boldsymbol{x = c_1 \cos\omega t + c_2 \sin\omega t\,;\ y = c_1 \sin\omega t - c_2 \cos\omega t\ \&\ x^2 + y^2 = c_1^2 + c_2^2}$

c) $\boldsymbol{x = -\frac{1}{27}(1+6t)e^{-3t} + \frac{1}{9}\left(t + \frac{1}{3}\right); y = -\frac{2}{27}(2+3t)e^{-3t} - \frac{2}{9}\left(t - \frac{2}{3}\right)}$

3. Solve second order linear differential equation with variable coefficients:

i. $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

ii. $x^6\frac{d^2y}{dx^2} + 3x^5\frac{dy}{dx} + 4y = 0$

iii. $y'' + y'\tan x + y\cos^2 x = 0$

Solution

i. $\boldsymbol{c_1 \cos(2\tan^{-1}x) + c_2 \sin(2\tan^{-1}x)}$

ii. $\boldsymbol{c_1 \cos\frac{a}{2x^2} + c_2 \sin\frac{a}{2x^2} + \frac{1}{a^2x^2}}$

iii. $\boldsymbol{c_1 \sin\sin x + c_2 \cos\sin x}$

4. Find Wronskian of following:

i. e^{2x} & e^{5x}

ii. e^{3x} & xe^{3x}

iii. $\sin 7x$ & $\cos 7x$

Solution

i. $\boldsymbol{-3e^{7x}\ or\ 3e^{7x}}$

ii. $\boldsymbol{e^{6x}\ or - e^{6x}}$

iii. $\boldsymbol{7\ or - 7}$

SECTION B

LONG ANSWER TYPE QUESTIONS

5. Solve

i. $(D^3 - 3D^2 + 4)y = e^{2x}$

ii. $y''' - y'' + 4y' - 4y = \sin 3x$

iii. $(D^2 + D + 1)y = (1 + sin\, x)^2$

iv. $(D^2 + 5D + 4)y = x^2 + 7x + 9$

v. $(D^2 - 4D + 3)y = e^x \cos 2x$

vi. $(D - 2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

vii. $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

Solution

i. $\boldsymbol{c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x} + \frac{x^2 e^{2x}}{6}}$

ii. $\boldsymbol{c_1 e^x + c_2 \cos 2x + c_3 \sin 2x + \frac{1}{50}(3\cos 3x + \sin 3x)}$

iii. $\boldsymbol{y''' - y'' + 4y' - 4y = sin\, 3x}$

iv. $\boldsymbol{(D^2 + D + 1)y = (1 + sin\, x)^2}$

v. $\boldsymbol{(D^2 + 5D + 4)y = x^2 + 7x + 9}$

vi. $\boldsymbol{(D^2 - 4D + 3)y = e^x cos\, 2x}$

vii. $\boldsymbol{(D - 2)^2 y = 8(e^{2x} + sin\, 2x + x^2)}$

viii. $\boldsymbol{(D^2 - 4D + 4)y = 8x^2 e^{2x} sin\, 2x}$

6. Solve the simultaneous differential equations:

i. $\boldsymbol{D^2 x + y = \sin t, D^2 y + x = \cos t; where\ D = \frac{d}{dt}}$

$$\boldsymbol{Sol}: x = c_1 e^t + \frac{c_2}{e^t} + c_3 \cos t + c_4 \sin t + \frac{t}{4}(\sin t - \cos t)$$

$$y = -c_1 e^t - \frac{c_2}{e^t} + c_3 \cos t + c_4 \sin t + \frac{t}{4}(\sin t - \cos t) + \frac{1}{2}(\sin t - \cos t)$$

ii. $\boldsymbol{Dx + 4x + 3y = t, Dy + 2x + 5y = e^t; where\ D = \frac{d}{dt}}$

$$\boldsymbol{Sol}: x = c_1 e^{-2t} + \frac{c_2}{e^{7t}} + \frac{5}{14}t - \frac{31}{196} - \frac{1}{8}e^t$$

$$y = -\frac{2}{3}c_1 e^{-2t} + \frac{c_2}{e^{7t}} - \frac{1}{7}t - \frac{9}{98} - \frac{5}{24}e^t$$

iii. $\boldsymbol{Dx + Dy + 3x = \sin t, Dx + y - x = \cos t; where\ D = \frac{d}{dt}}$

$$\boldsymbol{Sol}: x = c_1 e^{-t} + c_2 e^{3t} - \frac{1}{5}(\cos t - 2\sin t)$$

$$y = 2c_1 e^{-t} - 2c_2 e^{3t} + \frac{1}{5}(2\cos t + \sin t)$$

iv. $\boldsymbol{D^2 x - 4Dx + 4x = y\ and\ D^2 y + 4Dy + 4y = 25x + 16e^t; where\ D = \frac{d}{dt}}$

$$\textbf{Sol}: x = c_1 e^{3t} + \frac{c_2}{e^{3t}} + c_3 \cos t + c_4 \sin t + 8e^t$$

$$y = c_1 e^{3t} + 25\frac{c_2}{e^{3t}} + (3c_3 - 4c_4)\cos t + (4c_3 + 3c_4)\sin t + 8e^t$$

7. By changing the independent variable, solve second order linear differential equation with variable coefficients:

i. $\left[\boldsymbol{D^2 - \frac{1}{x}D + 4x^2}\right]\boldsymbol{y = x^4}$

$$\textbf{Sol}: y = c_1 \cos x^2 + c_2 \sin x^2 + \frac{x^2}{4}$$

ii. $\boldsymbol{y'' + (3\sin x - \cot x)y' + 2y\sin^2 x = e^{-\cos x}\sin^2 x}$

$$\textbf{Sol}: c_1 e^{\cos x} + c_2 e^{-\cos x} + \frac{e^{-\cos x}}{6}$$

iii. $\boldsymbol{(1+x)^2 y'' + (x+1)y' + y = 4\cos\log(1+x)}$

$$\textbf{Sol}: c_1 \cos\log(1+x) + c_2 \sin\log(1+x) + 2\log(1+x)\sin\log(1+x)$$

iv. $\left[\boldsymbol{(1+x^2)^2 D^2 + 2x(x^2+1)D + 4}\right]\boldsymbol{y = 0}$

$$\textbf{Sol}: c_1 \cos(2\tan^{-1}x) + c_2 \sin(2\tan^{-1}x)$$

v. $\boldsymbol{\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x}$

$$\textbf{Sol}: c_1 e^{\sqrt{2}\sin x} + c_2 e^{-\sqrt{2}\sin x} + \sin^2 x$$

vi. $\boldsymbol{x\frac{d^2y}{dx^2} + (4x^2 - 1)\frac{dy}{dx} + 4x^3 y = 2x^3}$

$$\textbf{Sol}: \frac{c_1 + c_2 x^2}{e^{x^2}} + \frac{1}{2}$$

8. Solve by the method of variation of parameters:

i. $\boldsymbol{\frac{d^2y}{dx^2} + y = \tan x}$

$$\textbf{Sol}: c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)$$

ii. $\boldsymbol{(D^2 - 1)y = 2(1 - e^{-2x})^{-1/2}}$

$$\textbf{Sol}: c_1 e^x + c_2 e^{-x} - \frac{\sqrt{(e^{2x} - 1)}}{e^x} - e^x \sin^{-1}\frac{1}{e^x}$$

iii. $\boldsymbol{\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}}$

$$\textbf{Sol}: \left[\log\left(\frac{1+e^x}{e^x}\right) - e^x + c_1\right]e^x + [c_2 - \log(1+e^x)]e^{-x}$$

iv. $\boldsymbol{\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}}$

$$\textbf{Sol}: e^x[\log(e^{-x}+1)+c_1]+[\log(1+e^{-x}) \ -(1+e^{-x})+c_2]e^{2x}$$

v. $\boldsymbol{x^2y''+xy'-y=x^2e^x}$

$$\textbf{Sol}: c_1x+\frac{c_2}{x}+\left(1-\frac{1}{x}\right)e^x$$

vi. $\boldsymbol{y''+(1-\cot x)y'-y\cot x=\sin^2 x}$

$$\textbf{Sol}: e^{-x}\left[\frac{e^x}{20}(3\sin 2x-\cos 2x-5)+c_1\right]+\left(c_2-\frac{\cos x}{2}\right)(\sin x-\cos x)$$

9. Solve

i. $\boldsymbol{x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x}$

Sol: $\frac{c_1}{x}+\frac{c_2}{x^2}+\frac{e^x}{x^2}$

ii. $\boldsymbol{x^2y''+xy'-y=x^3e^x}$

Sol: $c_1x+\frac{c_2}{x}+\left(x-3+\frac{3}{x}\right)e^x$

iii. $\boldsymbol{x^2D^2y+xDy+y=(\log x)\sin(\log x)}$

Sol: $c_1\cos\log x+c_2\sin\log x+\frac{\log x}{4}[\sin\log x-\log x\cos\log x]$

iv. $\boldsymbol{x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)}$

Sol: $\frac{c_1}{x}+x\,(\,c_2\cos\log x+c_3\sin\log x\,)+5x+\frac{2}{x}\log x$

v. $\boldsymbol{x^3y'''+3x^2y''+xy'+y=x+\log x}$

Sol: $\frac{c_1}{x}+\sqrt{x}\left[c_2\cos\frac{\sqrt{3}}{2}(\log x)+c_3\sin\frac{\sqrt{3}}{2}(\log x)\right]+\frac{x}{2}+\log x$