Question Bank (Unit: 1)

2 Marks Questions

1. Find the n^{th} derivative of $y = \sin 2x \cos 3x$.

   $y_n = \frac{1}{2} \cdot 5^n \sin\left(5x + n\frac{\pi}{2}\right) - \frac{1}{2} \sin\left(x + n\frac{\pi}{2}\right)$

2. Find the n^{th} derivative of $y = x^2 e^{3x}$.

   $y_n = e^{3x}\left(3^n x^2 + n \times 3^{n-1} \times 2x + n(n-1)\, 3^{n-2}\right)$

3. Test the applicability of Rolle's theorem for $f(x) = x^2 - 3x + 4$ on $[0, 2]$.

   Not applicable as $f(0) \neq f(2)$

4. Test the applicability of Lagranges's mean value theorem for $f(x) = x^2 - 3x + 4$ on $[0, 2]$.

   Applicable

5. Define Leibnitz's theorem for successive differentiation.

   $D^n\ (uv) = \sum_{k=0}^{n} {}^nC_k\ u_{n-k}\ v_k$

6 Marks Questions

1. If $y = a\cos(\ln x) + b\sin(\ln x)$, show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)\, xy_{n+1} + (n^2+1)\, y_n = 0$.

2. If $\ln y = m\cos^{-1} x$, show that $(1-x^2)\, y_2 - 2xy_1 = m^2 y$ and $(1-x^2)\, y_{n+2} - (2n+1)\, xy_{n+1} - (n^2+m^2)\, y_n = 0$.

3. Verify Rolle's theorem for $f(x) = (x-1)(x+3)\, e^x$ in $[-3, 1]$.

   $c = -2 + \sqrt{5}$

4. Expand $e^x \sin x$ in powers of $(x-1)$ upto third degree term.

   $e^x \sin x = e\sin 1 + e(\sin 1 + \cos 1)(x-1) + e\cos 1(x-1)^2 + \frac{e(\cos 1 - \sin 1)}{3}(x-1)^3 + ...$

5. Expand $4x^3 - x^2 + 3x - 1$ in powers of $(x+1)$.

   $4x^3 - x^2 + 3x - 1 = 4(x+1)^3 - 13(x+1)^2 + 17(x+1) - 9.$

10 Marks Questions

1. If $y = x\ln\left(\frac{x-1}{x+1}\right)$, show that $y_n = (-1)^n (n-2)! \left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$.

2. If $y = x^n \ln x$, show that $xy_{n+1} = n!$.

3. Verify Lagrange's mean value theorem $f(x) = x^3 - 6x^2 + 9x + 1$ in $[1, 4]$.

   $c = 3$

4. Expand $\log(1+x)$ in powers of x. Hence find the series for $\log\left(\frac{1-x}{1+x}\right)$.

   $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 + ...$

   $\log\left(\frac{1-x}{1+x}\right) = -2x - \frac{2}{3}x^3 - \frac{2}{5}x^5 + ...$

5. Find Maclaurian series for $e^{\sin x}$ upto powers of x^4.

   $e^{\sin x} = 1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + ...$