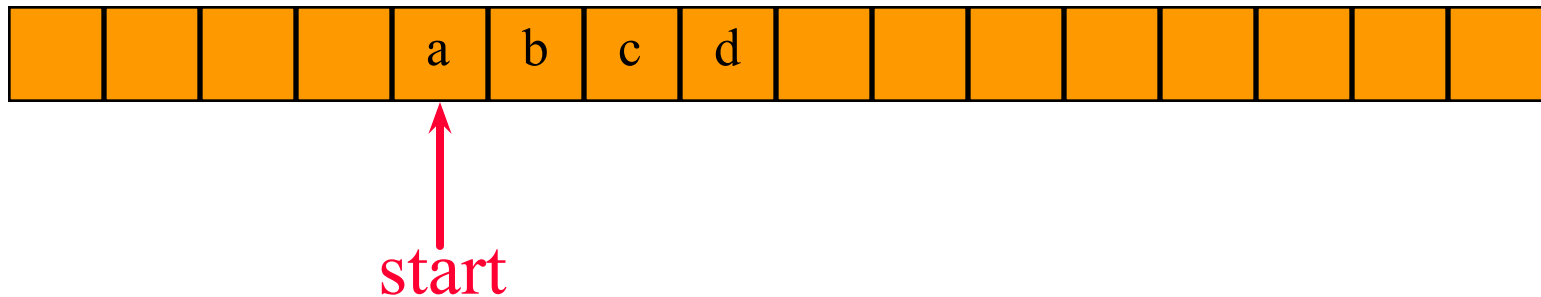


# Arrays



# 1D Array Representation In C++

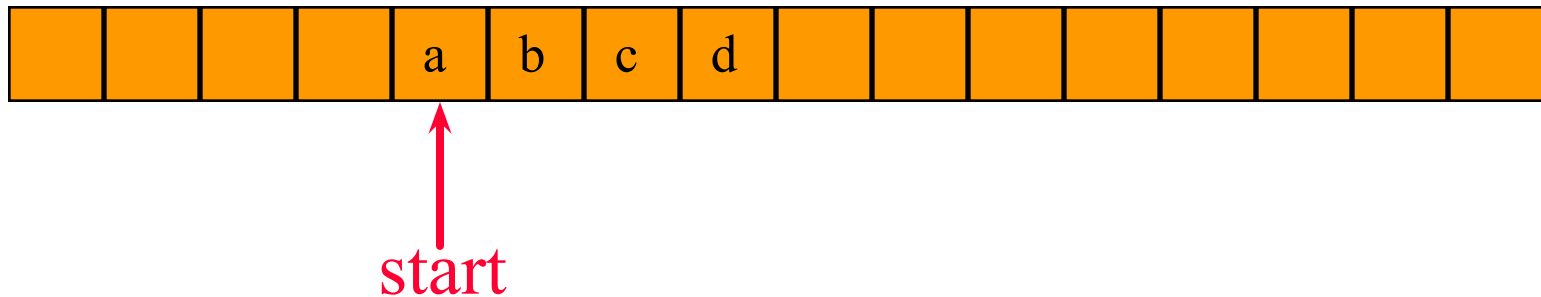
Memory



- 1-dimensional array  $x = [a, b, c, d]$
- map into contiguous memory locations
- $\text{location}(x[i]) = \text{start} + i$

# Space Overhead

Memory



space overhead = 4 bytes for **start**

(excludes space needed for the elements of **x**)

# 2D Arrays

The elements of a 2-dimensional array **a**  
declared as:

```
int [][]a = new int[3][4];
```

may be shown as a table

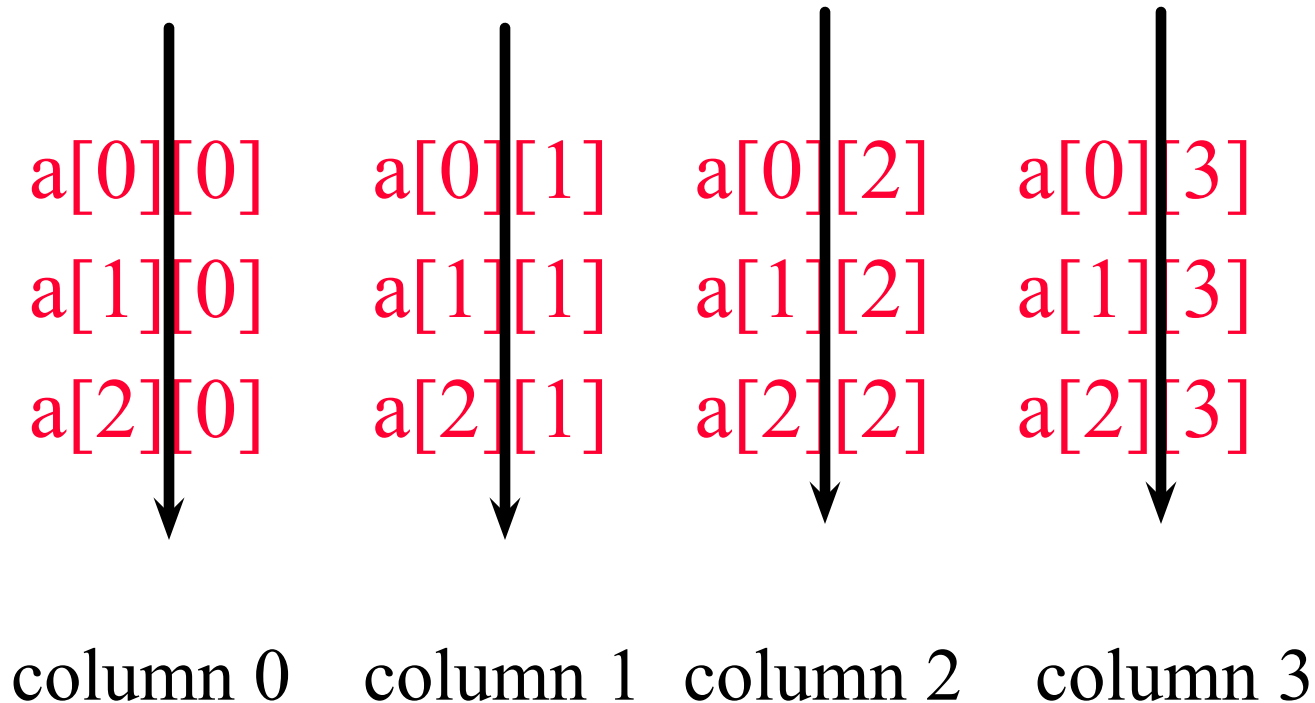
|         |         |         |         |
|---------|---------|---------|---------|
| a[0][0] | a[0][1] | a[0][2] | a[0][3] |
| a[1][0] | a[1][1] | a[1][2] | a[1][3] |
| a[2][0] | a[2][1] | a[2][2] | a[2][3] |

# Rows Of A 2D Array

The diagram illustrates the rows of a 2D array. It consists of three horizontal lines, each representing a row. Each line has four red text labels representing array indices: `a[0][0]`, `a[0][1]`, `a[0][2]`, and `a[0][3]` for the first row; `a[1][0]`, `a[1][1]`, `a[1][2]`, and `a[1][3]` for the second row; and `a[2][0]`, `a[2][1]`, `a[2][2]`, and `a[2][3]` for the third row. To the right of each line is a black arrow pointing to the right, followed by the text "row 0", "row 1", and "row 2" respectively.

|                      |                      |                      |                      |   |       |
|----------------------|----------------------|----------------------|----------------------|---|-------|
| <code>a[0][0]</code> | <code>a[0][1]</code> | <code>a[0][2]</code> | <code>a[0][3]</code> | → | row 0 |
| <code>a[1][0]</code> | <code>a[1][1]</code> | <code>a[1][2]</code> | <code>a[1][3]</code> | → | row 1 |
| <code>a[2][0]</code> | <code>a[2][1]</code> | <code>a[2][2]</code> | <code>a[2][3]</code> | → | row 2 |

# Columns Of A 2D Array



# 2D Array Representation In C++

2-dimensional array **x**

a, b, c, d

e, f, g, h

i, j, k, l

view 2D array as a 1D array of rows

**x** = [row0, row1, row 2]

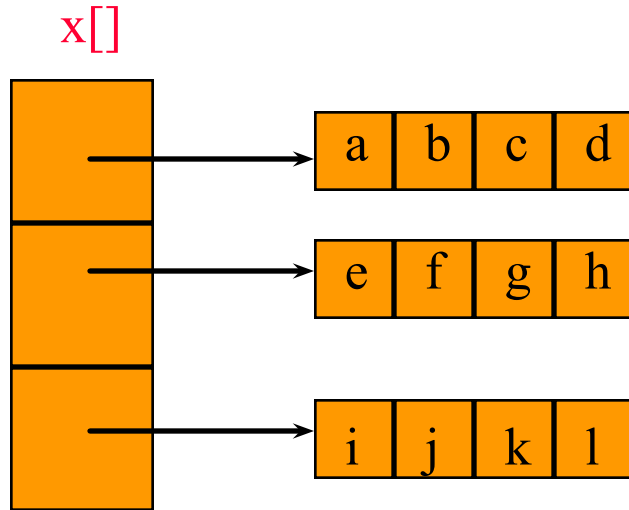
row 0 = [a,b, c, d]

row 1 = [e, f, g, h]

row 2 = [i, j, k, l]

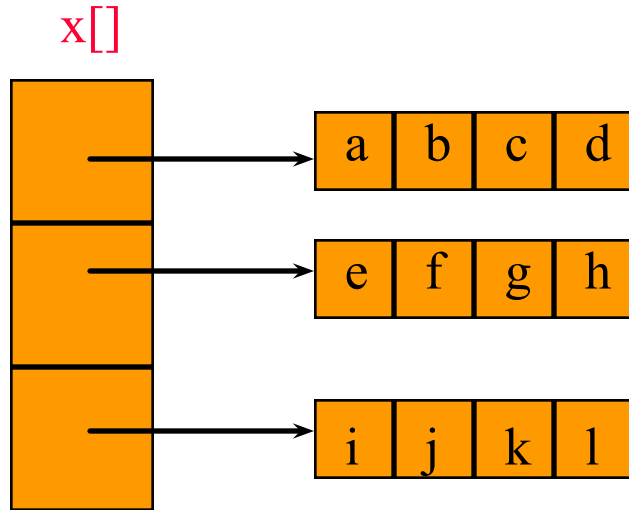
and store as **4** 1D arrays

# 2D Array Representation In C++



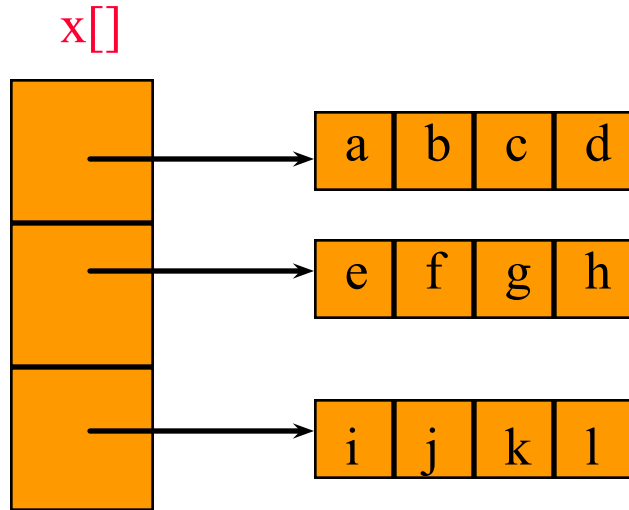


# Space Overhead



space overhead = overhead for 4 1D arrays  
=  $4 * 4$  bytes  
= 16 bytes  
= (number of rows + 1) x 4 bytes

# Array Representation In C++



- This representation is called the **array-of-arrays** representation.
- Requires contiguous memory of size 3, 4, 4, and 4 for the 4 1D arrays.
- 1 memory block of size **number of rows** and **number of rows** blocks of size **number of columns**

# Row-Major Mapping

- Example 3 x 4 array:

a b c d

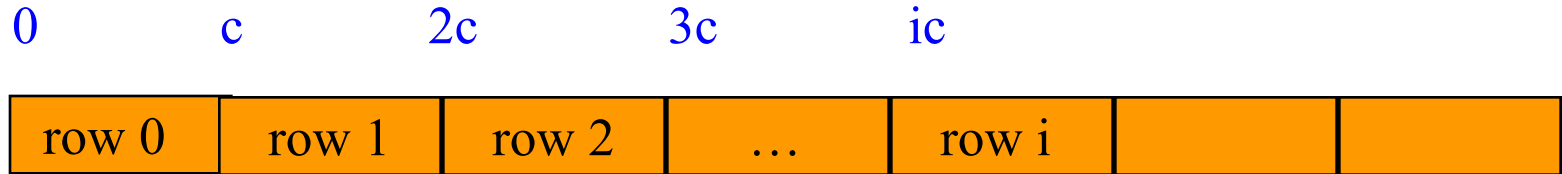
e f g h

i j k l

- Convert into 1D array  $y$  by collecting elements by rows.
- Within a row elements are collected from left to right.
- Rows are collected from top to bottom.
- We get  $y[] = \{a, b, c, d, e, f, g, h, i, j, k, l\}$

|       |       |       |     |       |  |  |
|-------|-------|-------|-----|-------|--|--|
| row 0 | row 1 | row 2 | ... | row i |  |  |
|-------|-------|-------|-----|-------|--|--|

# Locating Element $x[i][j]$



- assume  $x$  has  $r$  rows and  $c$  columns
- each row has  $c$  elements
- $i$  rows to the left of row  $i$
- so  $ic$  elements to the left of  $x[i][0]$
- so  $x[i][j]$  is mapped to position  
 $ic + j$  of the 1D array

# Space Overhead

|       |       |       |     |       |  |  |
|-------|-------|-------|-----|-------|--|--|
| row 0 | row 1 | row 2 | ... | row i |  |  |
|-------|-------|-------|-----|-------|--|--|

4 bytes for **start** of 1D array +  
4 bytes for **c** (number of columns)  
= 8 bytes

# Disadvantage

Need contiguous memory of size **rc**.

# Column-Major Mapping

a b c d

e f g h

i j k l

- Convert into 1D array  $y$  by collecting elements by columns.
- Within a column elements are collected from top to bottom.
- Columns are collected from left to right.
- We get  $y = \{a, e, i, b, f, j, c, g, k, d, h, l\}$

# Matrix

Table of values. Has rows and columns, but numbering begins at 1 rather than 0.

a b c d      row 1

e f g h      row 2

i j k l      row 3

- Use notation  $x(i,j)$  rather than  $x[i][j]$ .
- May use a 2D array to represent a matrix.



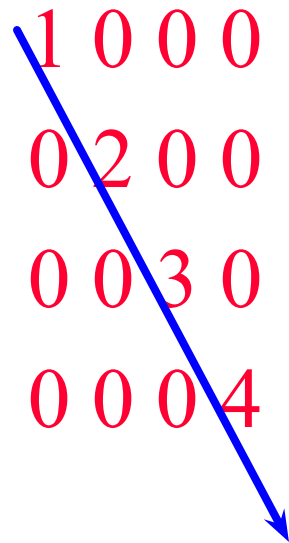
# Shortcomings Of Using A 2D Array For A Matrix

- Indexes are off by 1.
- C++ arrays do not support matrix operations such as **add**, **transpose**, **multiply**, and so on.
  - Suppose that **x** and **y** are 2D arrays. Can't do **x + y**, **x - y**, **x \* y**, etc. in Java.
- Develop a class **Matrix** for object-oriented support of all matrix operations.

# Diagonal Matrix

An  $n \times n$  matrix in which all nonzero terms are on the diagonal.

# Diagonal Matrix



A 4x4 diagonal matrix is shown with red numbers. The diagonal elements are 1, 2, 3, and 4. All other elements are 0. A blue arrow points from the top-left element (1) to the bottom-right element (4), indicating the diagonal.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

- $x(i,j)$  is on diagonal iff  $i = j$
- number of diagonal elements in an  $n \times n$  matrix is  $n$
- non diagonal elements are zero
- store diagonal only vs  $n^2$  whole

# Lower Triangular Matrix

An  $n \times n$  matrix in which all nonzero terms are either on or below the diagonal.

1 0 0 0

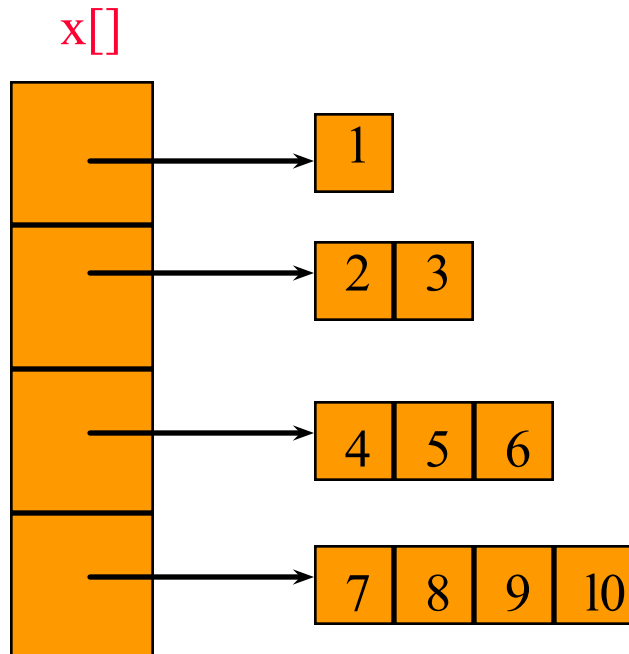
2 3 0 0

4 5 6 0

7 8 9 10

- $x(i,j)$  is part of lower triangle iff  $i \geq j$ .
- number of elements in lower triangle is  $1 + 2 + \dots + n = n(n+1)/2$ .
- store only the lower triangle

# Array Of Arrays Representation



Use an irregular 2-D array ... length of rows is not required to be the same.

# Creating And Using An Irregular Array

// declare a two-dimensional array variable

// and allocate the desired number of rows

```
int ** irregularArray = new int* [numberOfRows];
```

// now allocate space for the elements in each row

```
for (int i = 0; i < numberOfRows; i++)
```

```
    irregularArray[i] = new int [length[i]];
```

// use the array like any regular array

```
irregularArray[2][3] = 5;
```

```
irregularArray[4][6] = irregularArray[2][3] + 2;
```

```
irregularArray[1][1] += 3;
```

# Map Lower Triangular Array Into A 1D Array

Use row-major order, but omit terms that are not part of the lower triangle.

For the matrix

1 0 0 0

2 3 0 0

4 5 6 0

7 8 9 10

we get

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

# Index Of Element [i][j]

|     |    |    |     |       |  |  |
|-----|----|----|-----|-------|--|--|
| 0   | 1  | 3  | 6   |       |  |  |
| r 1 | r2 | r3 | ... | row i |  |  |

- Order is: row 1, row 2, row 3, ...
- Row i is preceded by rows 1, 2, ..., i-1
- Size of row i is i.
- Number of elements that precede row i is  
 $1 + 2 + 3 + \dots + i-1 = i(i-1)/2$
- So element (i,j) is at position  $i(i-1)/2 + j - 1$  of the 1D array.