Double Integral Intro

(2)

Q1 $\int_0^3 \int_0^2 (4-y^2)\,dy\,dx = \int_0^3 4y - \frac{y^3}{3}\Big|_0^2 dx = \int_0^3 4(2) - \frac{8}{3}\,dx$

$= \cancel{8x-8}\ \frac{16x}{3}\Big|_0^3 = 16$

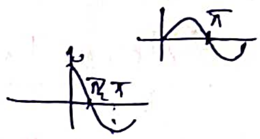
Q2 $\int_0^3 \int_{-2}^0 (x^2y - 2xy)\,dy\,dx = \int_0^3 \frac{x^2y^2}{2} - \frac{2xy^2}{2}\Big|_{-2}^0 dx = \int_0^3 -2x^2+4x\,dx$

$= \frac{2x^3}{3} - 2x^2\Big|_0^3 = 0$

Q3 $\int_\pi^{2\pi} \int_0^\pi (\sin x + \cos y)\,dx\,dy = \int_\pi^{2\pi} -\cos x + (\cos y)\cdot x\Big|_0^\pi = \int_\pi^{2\pi} -\cos\pi + \cos y\cdot\pi - (-\cos 0 + 0)\,dy$

$= \int_\pi^{2\pi} (-(-1) + \pi\cos y + 1)\,dy = 2y + \pi\sin y\Big]_\pi^{2\pi} = 2(2\pi) + 0 - 2\pi$

$= 2\pi$



Q4 $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\,dx\,dy = \int_0^1 3y^3\frac{e^{xy}}{y}\Big|_0^{y^2} dy = \int_0^1 (3y^2 e^{y^3} - 3y^2)\,dy$

$= e^{y^3} - y^3\Big|_0^1 = \cancel{e}\ e-1-1 = e-2$

$y^3 = t$

$3y^2\,dy = dt$

①

Q1 $\int_0^1 \underbrace{\int_0^y xy\, e^{-x^2} dx\, dy}$ $= \int_0^1 \cancel{-y\frac{x^2}{2}\,e^{-x^2}}$ ILATE (I, E) $x^2 = t$, $2x\,dx = dt$, $x\,dx = \frac{1}{2}dt$

$\int x e^{-x^2} dx$, $\int \frac{1}{2} e^{-t} dt = \frac{1}{2}\frac{e^{-t}}{-1}$

$\int_0^1 y \left[\frac{e^{-x^2}}{-2}\right]_0^y dy = \int_0^1 y \left[\frac{e^{-y^2}}{-2} - \frac{1}{-2}\right] dy.$

$= \frac{-1}{2}\int (y e^{-y^2} - y)\, dy = -\frac{1}{2}\left[\frac{e^{-y^2}}{-2} - \frac{y^2}{2}\right]_0^1 = \frac{1}{4}[e^{-1} - 1 + 1]$

$= \frac{1}{4e}$ //

Q2 $\int_0^1 \underbrace{\int_0^{x^2} e^{y/x}\, dy}\, dx = \int_0^1 \left.\frac{e^{y/x}}{1/x}\right]_0^{x^2} dx = \int_0^1 \underset{\mathrm{I}\ \mathrm{II}}{x e^x}\, dx - \int_0^1 x\, dx$

$= x e^x - e^x \Big|_0^1 - \frac{x^2}{2}\Big|_0^1 = e - e + 1 - \frac{1}{2} = \frac{1}{2}$ //

Q3 $\int_1^{\log 8} \underbrace{\int_0^{\log y} e^{x+y}\, dx}\, dy = \int_1^{\log 8} e^y \left(\int_0^{\log y} e^x\, dx\right) dy.$

$= \int_1^{\log 8} e^y [y - 1]\, dy = y e^y - e^y - e^y \Big|_1^{\log 8}$

$= (\log 8)\, 8 - 8 - 8 - \cancel{e} + \cancel{e} + e$

$= 8 \log 8 - 16 + e$

Q4 $\int_0^1 \int_0^1 \frac{dx\, dy}{\sqrt{(1-x^2)(1-y^2)}} = \int_0^1 \frac{1}{\sqrt{1-y^2}}\left(\int_0^1 \frac{1}{\sqrt{1-x^2}}\right) dx\, dy$

$= \int_0^1 \frac{1}{\sqrt{1-y^2}} [\sin^{-1} x]_0^1\, dy = \int_0^1 \frac{1}{\sqrt{1-y^2}} \left[\frac{\pi}{2} - 0\right] dy$

$= \frac{\pi}{2} [\sin^{-1} y]_0^1 = \frac{\pi}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{4}$

Q5 $\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2 y^2\, dx\, dy$ Ans $\frac{856}{945}$