	Name of the School	School of Basic & Applied Sciences	**Name of the Department**	Mathematics
	Name of the Program	B.Tech, CSE	**Course Code- Course**	UBS 1003M
	Session	2024-25	**Branch, Year & Semester**	CSE , 1^{st}, 1^{st}

UNIT 4: Complex Analysis

Questions bank

2 marks questions

1. Write Cauchy-Riemann equations in the polar coordinates.
2. Find the limit of $\lim_{z\to\infty} \frac{z}{2-iz}$
3. Check the continuity of $f(z) = \bar{z}$ about origin.
4. Using Cauchy-Riemann equations show that $f(z) = sin\, z$ is analytic.
5. Check whether the function $u(r,\theta) = r^2 * \cos 2\theta$ is harmonic or not.

6 marks questions

1. Show that the function $f(z) = |z|^2$ is differentiable at the origin and nowhere else.
2. Find the constants a, b, c such that the function $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$ is analytic.
3. Prove that the $u = e^x cosy$ is a harmonic function. Find its complex conjugate.
4. Using the Cauchy-Riemann equations , show that
   (i) $f(z) = |z|^2$ is not analytic at any point.
   (ii) $f(z) = 1/z, z \neq 0$ is analytic at all points except at the origin.

10 marks questions

1. Check the continuity of the function $f(z)$ about the origin, where

   (i) $$f(z) = \begin{cases} \frac{Im(z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

   (ii) $$f(z) = \begin{cases} \frac{Re(z^2)}{|z|^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

2. Prove that the function $f(z) = \left\{\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}\right\}, z \neq 0, f(0) = 0$

   satisfies Cauchy-Riemann equations, yet it is not analytic about the origin.

3. Prove that the function $u = x^4 - 6x^2y^2 + y^4$ is harmonic. Find the analytic function $f(z) = u(x, iy) + iv(x, y)$.

4. Determine the analytic function by Milne Thomson's method

   (i) whose real part is $e^{2x}(x * cos2y - y * sin2y)$

   (ii) whose imaginary part is $e^{-x}(x * cosy + y * siny)$

5. Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function.

   (i) If $u(x, y) = e^{-2xy} * \sin(x^2 - y^2)$

   (ii) If $\mathrm{v}(x, y) = log(x^2 + y^2) + x - 2y$

   Construct the corresponding analytic function in terms of z for both.

Solutions

2 marks questions

1. $u_r = \frac{1}{r} v_\theta$ & $u_\theta = -r v_r$
2. i
3. continuous
5. harmonic

6 marks questions

2. $a = -\frac{1}{2}, b = -2, c = \frac{1}{2}$
3. $v = e^x siny + c$

10 marks questions

1. (i) not continuous
   (ii) not continuous

3. $v = 4x^3y - 4xy^3 + c, f(z) = z^4 + ic$
4. (i) $f(z) = z^4 + C$
   (ii) $f(z) = iz * e^{-z} + C$
5. (i) $f(z) = -ie^{-iz^2} + C$
   (ii) $f(z) = (i - 2z) + 2i * \log z + C$