QUESTION BANK

SECTION A

SHORT ANSWER TYPE QUESTIONS

Ques 1. Form the PDE by eliminating the arbitrary constants:

(i) $z = ax + by + ab$ (Ans. $z = px + qy + pq$)

(ii) $az + b = a^2x + y$ (Ans. $pq = 1$)

Ques 2. Form the PDE by eliminating the arbitrary function:

(i) $z = f(x^2 - y^2)$ (Ans. $py + qx = 0$)

(ii) $z = x + y + f(xy)$ (Ans. $px - qy = x - y$)

Ques 3. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

(Ans. $z = e^y \cos x + \sin x$)

Ques 4. Solve the following PDEs using Lagrange subsidiary equations

(i) $yzp - xzq = xy$ (Ans. $\phi(x^2 + y^2, x^2 - z^2) = 0$)

(ii) $x^2p + y^2q = (x + y)z$ (Ans. $\phi\left(\frac{1}{x} - \frac{1}{y}, \frac{x-y}{z}\right) = 0$)

(iii) $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ (Ans. $\sqrt{x} - \sqrt{y} = f(\sqrt{x} - \sqrt{z})$)

Ques 5. Solve the homogeneous PDEs:

(i) $(D^4 + D'^4)z = 0$.

(Answer: $\phi_1(y + \alpha x) + \phi_2(y + \bar{\alpha}x) + \phi_3(y + \beta x) + \phi_4(y + \bar{\beta}x)$; where $\alpha = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, \beta = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$)

(ii) $DD'(D + D')z = 0$

(Answer: $\phi_1(y) + \phi_2(y - x) + \phi_3(x)$)

(iii) $2r + 5s + 2t = 0$.

(Answer: $z = \phi(2y - x) + \psi(y - 2x)$)

Ques 6. Solve the Non-homogeneous linear PDEs:

(i) $(D - D')(D + D' - 3)z = 0$ (Ans. $z = \phi_1(y + x) + e^{3x}\phi_2(y - x)$)

(ii) $(D^2 - a^2D'^2 + 2abD + 2a^2bD')z = 0$ (Ans. $z = \phi_1(y - ax) + e^{-2abx}\phi_2(y + ax)$)

SECTION B

LONG ANSWER TYPE QUESTIONS

Ques 7. Solve using Lagrange subsidiary equations

(i) $(mz - ny)p + (nx - lz)q = ly - mx$ (Ans. $x^2 + y^2 + z^2 = f(lx + my + nz)$)

(ii) $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$ (Ans. $\phi(x^2 + y^2 - 2z, xyz)$)

(iii) $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$ (Ans. $\phi(x - y - z, \frac{x^2 - y^2}{z^2})$)

(iv) $yp + xq = xyz^2(x^2 - y^2)$ (Ans. $\phi(x^2 - y^2, \frac{x^4}{4} - \frac{y^4}{4} + \frac{1}{z})$)

Ques 8. Use Cauchy Method of characteristic to find the solution of following PDEs:

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x + y$; $u(x, 0) = 0$ (Answer: $u(x, y) = xy$)

(ii) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \sinh x + \sinh x$; $u(0, y) = 1 + \cosh y$ (Answer: $u(x, y) = \cosh x + \cosh y$)

(iii) $u_x + yu = 0$; $u(0, y) = 1$ (Answer: $u(x, y) = e^{-xy}$)

(iv) $u_x + u_y = 1 + \cos y$; $u(0, y) = \sin y$ (Answer: $u(x, y) = x + \sin y$)

Ques 9. Solve the homogeneous linear PDEs:

(i) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$

(Answer: $z = \phi_1(y - x) + \phi_2(y + 2x) + x\phi_3(y + 2x) + \frac{1}{27}e^{x+2y}$)

(ii) $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$

(Answer: $z = \phi_1(2y + x) + x\phi_2(2y + x) + \frac{x^2}{8}e^{x+2y}$)

(iii) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$

(Answer: $z = \phi_1(y) + \phi_2(y + x) + \frac{1}{2}\sin(x + 2y) - \frac{1}{6}\sin(x - 2y)$)

(iv) $\left(D^2 + DD' - 6D'^2\right)z = \cos(2x + y)$

(Ans. $z = \phi_1(y + 2x) + \phi_2(y - 3x) + \frac{x}{5}\sin(2x + y)$)

(v) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$

(Ans. $z = \phi_1(y) + x\phi_2(y) + \phi_3(y + 2x) + \frac{1}{60}(15e^{2x} + 3x^5 y + x^6)$)

(vi) $\left(D^2 + DD' - 6D'^2\right)z = y \cos x$

(Ans. $z = \phi_1(y + 2x) + \phi_2(y - 3x) + \sin x - y \cos x$)

(vii) $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$

(Ans. $z = \phi_1(y - x) + \phi_2(y + 2x) + \sin xy$)

Ques 10. Solve the Non-homogeneous linear PDEs:

(i) $(D + 1)(D + D' - 1)z = \sin(x + 2y)$

(Ans. $z = e^{-x}\phi_1(y) + e^{x}\phi_2(y - x) - \frac{1}{10}[\cos(x + 2y) + 2\sin(x + 2y)]$)

(ii) $[D^2 - D'^2 + D + 3D' - 2]z = x^2y$

(Ans. $z = e^{-2x}\phi_1(y + x) + e^{x}\phi_2(y - x) - \frac{1}{2}\left(x^2y + \frac{3x^2}{2} + xy + \frac{3y}{2}\right)$)

(iii) $r + 2s + t = 2(y - x) + \sin(x - y)$

(Ans. $z = \phi_1(y - x) + x\phi_2(y - x) + x^2y - x^3 + \frac{x^2}{2}\sin(x - y)$)

(iv) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = xy + e^{x+2y}$

(Ans. $z = \phi_1(y + x) + e^{3x}\phi_2(y - x) - \frac{1}{3}\left(\frac{x^2y}{2} + \frac{x^3}{6} + \frac{x^2}{3} + \frac{xy}{3} + \frac{2x}{9}\right) - xe^{x+2y}$)