Unit 2 Imp points

① $f(x, y)$. $f_x = \frac{\partial f}{\partial x}$ Partial derivative of f^n f w.r.t x i.e keep y as a constant

$f_y = \frac{\partial f}{\partial y}$ Partial derivative of f^n f w.r.t y i.e keep x as a constant

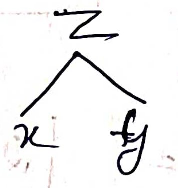
$$f_x = \lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h} \; ; \; f_y = \lim_{h \to 0} \frac{f(x_0, y_0 + h) - f(x_0, y_0)}{h}$$

eg ① $x^2 e^{y/x}$ at $(4,2)$ $\quad f_x\big|_{(4,2)} = 6\sqrt{e}$ $\quad f_y\big|_{(4,2)} = 4\sqrt{e}$

② $\dfrac{x}{\sqrt{x^2+y^2}}$ at $(6,7)$ $\quad f_x\big|_{(6,7)} = \dfrac{49}{(85)^{3/2}}$ $\quad f_y\big|_{(6,7)} = \dfrac{-42}{(85)^{3/2}}$

Total differential

$z = f(x,y)$



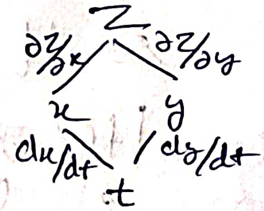
$$dz = f_x\,dx + f_y\,dy \quad \text{③}$$

just calculate f_x & f_y and put the value's in the formula above ③. eg ① $z = \log(x^2+y^2)$

$$dz = \frac{2}{x^2+y^2}(x\,dx + y\,dy)$$

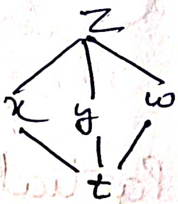
Chain Rule

① $z = f(x,y)$, $x = h(t)$, $y = g(t)$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

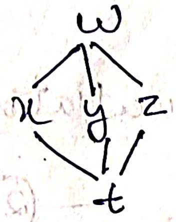
② $z = f(x,y,w)$ $\quad x = h(t)$ $\quad y = g(t)$ $\quad w = l(t)$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$$

eg ① $w = 2ye^x - \ln z$ where $x = \ln(t^2+1)$, $y = \tan^{-1}(t)$, $z = e^t$

find $\dfrac{dw}{dt}\Big|_{t=1}$



$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$$

$$\frac{dw}{dt}\Big|_{t=1} = \pi + 1$$

Don't get confused by variable names you have to built formula according to tree chart

Implicit f^n

① $f(x,y) = c$ (constant) $\quad \dfrac{dy}{dx} = -f_x/f_y \quad f_y \neq 0$

make note here you are asked to calculate $\dfrac{dy}{dx}$

i.e here y is a differentiable f^n of x.

$\|^{ly}$ for $\dfrac{dx}{dy} = -f_y/f_x$.

② $f(x,y,z) = c$ then $\dfrac{\partial y}{\partial x} = -\dfrac{f_x}{f_y}$; $\dfrac{\partial z}{\partial y} = \dfrac{-f_y}{f_z}$; $\dfrac{\partial z}{\partial x} = \dfrac{-f_x}{f_z}$

eg $x^y + y^x = \alpha$

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{(yx^{y-1} + y^x \log y)}{(x^y \log x + xy^{x-1})}$$

Homogeneous function $f(x,y)$, replace $x \to \lambda x$ $y \to \lambda y$.
$f(\lambda x, \lambda y) = \lambda^n f(x,y)$ n – degree.

eg $\frac{1}{x^2+y^2}$ $f(\lambda x, \lambda y) = \frac{1}{\lambda^2}(f(x,y))$ $n = -2$.

Euler's Theorem $f(x,y)$ – hom. f^n of deg. n
cont. first & second order partial derivatives

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf \qquad x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$$

* If f_x, f_y, f_{xy}, f_{yx} are defined at (a,b) and are all continuous then $f_{xy}\big|_{(a,b)} = f_{yx}\big|_{(a,b)}$.

Euler's Thm when you are taking something to LHS

① $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{F(u)}{F'(u)}$

$Z = F(u) = h(x,y)$ where $h(x,y)$ is a hom. f^n of x & y.

② $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)\left[g'(u)-1\right]$ where

$g(u) = n\frac{F(u)}{F'(u)}$

eg $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ not hom. f^n. But if we take $\tan^{-1}$ to Left hand side (LHS)

$$\underbrace{\tan u}_{F(u)} = \underbrace{\frac{x^2+y^2}{x+y}}_{h(x,y)}$$

here $h(x,y)$ is a hom f^n of x & y so now use the above formula.

*Remark Always First check the f^n as it is if it is not hom. f^n then only see if you can take something to LHS to get a hom f^n of x & y