IILM University, Greater Noida

Unit-III

Short Answer Questions — 2 Marks

1. Evaluate $\int_0^3 \int_x^{4x-x^2} y dy dx$ Ans: $\frac{54}{5}$
2. Evaluate $\int_0^a \int_0^x \frac{x dy dx}{x^2+y^2}$ Ans: $\frac{\pi a}{2}$
3. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx$ Ans: $\frac{1}{6}$
4. Prove that $\int_0^1 \int_{2y}^2 dx dy = \int_0^2 \int_0^{x/2} dy dx$, by changing the order of integration.
5. If $\int_0^2 \int_0^{\sqrt{4-y^2}} \sqrt{x^2+y^2} dx dy = \int_0^{\pi/2} \int_0^2 r^2 dr d\theta$, by changing into polar co-ordinates. Then, evaluate $\int_0^{\pi/2} \int_0^2 r^2 dr d\theta$. Ans: $\frac{8\pi}{6}$

Long Answer Questions — 6 Marks

1. Change the order of integration in $\int_0^4 \int_y^4 \frac{x dx dy}{x^2+y^2}$ and evaluate the same. Ans: π
2. Evaluate $\iint (x^2+y^2) dx dy$ over the region in the positive quadrant for which $x+y \le 1$. Ans: $\frac{1}{6}$
3. Find the area lying between the parabola $y = 4x - x^2$ and above the line $y = x$. Ans: $\frac{9}{2}$
4. Find the volume of the region bounded by the surface $y = x^2, x = y^2$ and the planes $z = 0, z = 3$. Ans: 1

E-Long Answer Questions — 10 Marks

1. Evaluate the following integration
   (a) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dz dy dx$ Ans: $\frac{\pi^2}{8}$
   (b) $\iiint_R (x+y+z) dx dy dz$ where $R: 0 \le x \le 1, 1 \le y \le 2, 2 \le z \le 3$. Ans: $\frac{9}{2}$
2. Change the order of integration and hence evaluate

$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2x^2}} dy dx$$

   Ans: $\frac{\pi a^2}{6}$
3. Evaluate $\int\int xy(x+y) dx dy$ over the area between $y = x^2$ and $y = x$. Ans: $\frac{3}{56}$

4. Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dydx$. Hence show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. Ans: $\frac{\pi}{4}$

5. A triangular thin plate with vertices $(0,0)$, $(2,0)$, and $(2,4)$ has density $\rho = 1 + x + y$. Then find:

   (a) the mass of the plate Ans: $\frac{44}{3}$

   (b) the position of its center of gravity G. Ans: $\frac{18}{11}$