Unit-I

Short Answer Questions

1. Find n^{th} differentiation of following functions:

   (a) $\dfrac{x^2}{(x-1)^3(x-2)} = \dfrac{4}{x-2} - \dfrac{4}{x-1} - \dfrac{3}{(x-1)^2} - \dfrac{1}{(x-1)^3}$

   (b) $\dfrac{1}{1+x+x^2+x^3} = \dfrac{1}{2\,(x+1)} + \dfrac{1}{2}\dfrac{1-x}{x^2+1}$

2. Find n^{th} differentiation of following functions:

   (a) $\dfrac{ax+b}{cx+d}$

   (b) $\ln\left(ax+x^2\right)$

   (c) $\ln\left(x^2-a^2\right)$

   (d) $\log\sqrt{\dfrac{2x+1}{x-2}}$

   (e) $\dfrac{1}{x^2+a^2}$

   (f) $\ln\left(x^2+a^2\right)$

   (g) $\tan^{-1}\left(\dfrac{x}{c}\right)$

   (h) $x\tan^{-1}x$

   (i) $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

   (j) $e^{2x}+e^{-2x}$

   (k) $\cos^4 x$

   (l) $\sin 2x\cos 3x$

   (m) $\exp\left(ax\right)\sin^2 x\sin 2x$

   (n) $e^x\sin 4x\cos 6x$

   (o) $a^x\cos x$

3. Find n^{th} derivative of

   (a) x^3e^x

   (b) $e^x\log x$

   (c) $x^2\sin x$ at $x=0$. .

4. Expand a^x and e^x in powers of x.

5. Expand $\ln x$ in powers of $(x-1)$ upto third degree term.

6. Examine if Rolle's theorem is applicable for $f\left(x\right)=\sec x$ in $[0,2\pi]$.

7. Examine if mean value theorem (Lagrange's mean value theorem) is applicable for $f\left(x\right)=\frac{2x-1}{3x-4}$ in $[1,2]$.

Long Answer Questions

1. Expand e^x in powers of $x+2$.

2. Expand $4x^2+7x+5$ in powers of $(x-3)$

3. Verify Rolle's theorem for following:

   (a) x^2-6x+8 in $[2,4]$

   (b) $e^x\sin x$ in $[0,\pi]$

4. Verify Lagrange's mean value theorem for following

(a) $\sqrt{x^2-4}$ in $[2,4]$ (b) $\ln x$ in $[1,e]$

5. If $y = x\ln\left(\dfrac{x-1}{x+1}\right)$, then show that $y_n = (-1)^{n-2}(n-2)!\left[\dfrac{x-n}{(x-1)^n} - \dfrac{x+n}{(x+1)^n}\right]$.

6. If $y = (x^2-1)^n$ show that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

7. If $y = x^{n-1}\log x$, show that $y_n = \dfrac{(n-1)!}{x}$.

8. If $\cos^{-1}\dfrac{y}{b} = \log\left(\dfrac{x}{n}\right)^n$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$.

9. If $y = a\cos\ln x + b\sin\ln x$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

10. If $y = \sin\left(m\sin^{-1}x\right)$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$.

11. If $y = \ln\left[x+\sqrt{1+x^2}\right]$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$.

12. If $y = \ln\left[x+\sqrt{1+x^2}\right]^2$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$.