# Question Bank Unit 2

1. Find $\frac{dy}{dx}$ if $y^2 + xy - 3x - 3 = 0$ at $(-1, 1)$. 2M
2. Find $\frac{dy}{dx}$ if $x^2 + xy + y^2 - 7 = 0$ at $(1, 2)$. 2M
3. Find jacobian if $x = r\cos\theta,\ y = r\sin\theta$. 2M
4. If $\tan u = \frac{x^3 + y^3}{x - y}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. 2M
5. Develop the chain rule for $w = f(x, y, z),\ x = g(r, s),\ y = h(r, s),\ z = k(r, s)$. 2M
6. Find local extreme of the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$. 6M
7. Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ upto second degree using Taylor's Theorem. 6M
8. Find the maximum and minimum value of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ by using Lagrange method of multipliers. 6M
9. Use the Jacobian to prove that the functions $u = \frac{x - y}{x + y},\ v = \frac{xy}{(x + y)^2}$ are functionally dependent. Find the relation between them. 6M
10. Write the Maclaurin series of function $f(x, y) = e^x ln(1 + y)$ upto second degree. 10M
11. Find absolute maxima and minima of $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0,\ y = 2,\ y = 2x$ in the first quadrant. 10M
12. Find the relative maximum and minimum points of the function $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$. 10M
13. If $u = \sin^{-1}(\frac{x + y}{\sqrt{x} + \sqrt{y}})$, prove that
    1. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$
    2. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$. 10M
14. If $u = x(1 - r^2)^{-1/2},\ v = y(1 - r^2)^{-1/2},\ w = z(1 - r^2)^{-1/2}$, where $r^2 = x^2 + y^2 + z^2$, then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = (1 - r^2)^{-5/2}$ 10M

**Answers:**

1. 2
2. $-\frac{4}{5}$
3. r
4. $\sin 2u$
5. $\dfrac{\partial w}{\partial r} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial r} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial r}; \quad \dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial s} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial s}$
6. $(0,0)$ local minimum and $(1,-1)$ saddle point
7. $f(x,y) = -10 - 4(x-1) + 4(y+2) - 2(x-1)2 + 2(x-1)(y+2)$
8. $f(\frac{3}{5}, \frac{4}{5}) = 5$ maximum; $f(-\frac{3}{5}, -\frac{4}{5}) = -5$ minimum
9. $4v = 1 - u^2$
10. $f(x,y) = y + \frac{1}{2}(2xy - y^2)$
11. $f(1,2) = -5$ absolute minima, $f(0,0) = 1$ absolute maxima
12. Saddle points $(0,0),\ (1,1)\ (1,-1)\ (-1,1), (-1,-1)$; local minima $(0,1),\ (0,-1)$; local maxima $(1,0),\ (-1,0)$