Tutorial Sheet 1

1. Solve

i. $\frac{d^2y}{dx^2} + y = 0$

ii. $\frac{d^2y}{dx^2} + (a+b)\frac{dy}{dx} + aby = 0$

iii. $D^2y + 4Dy + 4y = e^x$

iv. $D^2y + y = \sin x$

v. $(D^2 + 2D + 1)y = x\ cosx$

vi. $x^2D^2y + 4xDy + 2y = e^x$

vii. $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

viii. $y'' + y'\tan x + y\cos^2 x = 0$

Answers:

i. $y = c_1\cos(\mathrm{x}) + c_2\sin(\mathrm{x})$.

ii. $y = c_1e^{ax} + c_2e^{bx}, a \neq b, a, b \in R.$

iii. $y = c_1e^{-2x} + c_2xe^{-2x} + \frac{1}{9}e^x.$

iv. $y = c_1\cos(\mathrm{x}) + c_2\sin(\mathrm{x}) - \frac{x}{2}\cos(\mathrm{x})$.

v. $y = (c_1 + c_2\mathrm{x})e^{-x} + \frac{1}{2}\cos x + \frac{1}{2}(\mathrm{x}-1)\sin x$

vi. $y = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{1}{x^2}e^x$

vii. $y = c_1\cos(2\ tan^{-1}\mathrm{x}) + c_2\sin(2\ tan^{-1}\mathrm{x})$

viii. $y = c_1\cos(\sin x) + c_2\sin(\sin x)$

Tutorial Sheet 2

2. Solve

i. $\frac{dx}{dt} = 3x + 2y, \frac{dy}{dt} = 5x + 3y$

ii. $Dx = -2y, Dy = 2x. Also\ show\ that\ the\ point\ (x, y)\ lies\ on\ a\ circle.$

iii. $x^3D^3y + 3x^2D^2y + xDy + y = x + \log x$

iv. $x^2\frac{d^2y}{d^2x} - x\frac{dy}{dx} + y = 2\ logx$

v. $x^2y'' + xy' - y = x^2e^x$

vi. $(D^2+1)y = \tan x$

vii. $y'' - 3y' + 2y = \frac{e^x}{1+e^x}$

Answers:

i. $x = e^{3t}\left(c_1 \sinh\sqrt{10}\,t + c_2 \cosh\sqrt{10}\,t\right),\ y = \frac{\sqrt{10}}{2} e^{3t}\left(c_1 \sinh\sqrt{10}\,t + c_2 \cosh\sqrt{10}\,t\right)$

ii. $x^2 + y^2 = a^2 + b^2$

iii. $y = \frac{c_1}{x} + \sqrt{x}\left[c_2 \cos\frac{\sqrt{3}}{2}(\log x) + c_3 \sin\frac{\sqrt{3}}{2}(\log x)\right] + \frac{x}{2} + \log x$

iv. $y = x(c_1 + c_2 \log x) + 2\log x + 4$

v. $y = c_1 x + \frac{c_2}{x} + \left(1 - \frac{1}{x}\right) e^x$

vi. $y = c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)$

vii. $y = [\log(e^{-x} + 1) + c_2]e^x + [\log(1 + e^{-x}) - (1 + e^{-x}) + c_2]e^{2x}$