IILM University, Greater Noida

Worksheet-5; Mathematics for Computing (UCS 2005)

UNIT: 5

SECTION A

1. Obtain the rank of the matrix A= $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

2. Obtain the value of P for which the matrix $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is of rank 1.

3. Obtain the eigen values of $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

4. if λ_1, λ_2 and λ_3 are eigen values of a non-singular matrix A, then compute the latent roots of $7A^3 - 5A^2 + \frac{1}{3}A' - 4A^{-1} + 3I$.

5. If the eigenvalues of a Matrix $A = \begin{bmatrix} a & b \\ 2 & 4 \end{bmatrix}$ are 5 and 2, then determine the values of a and b.

6. If 2 is a latent root of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ then compute the other two.

7. Evaluate the determinant of the matrix: $X = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$.

8. State the Rank-Nullity theorem.

SECTION B

1. Find the rank of the matrix by reducing it into echelon form: $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 5 \\ 6 & 9 & 8 \end{bmatrix}$.

2. Find the rank of the following matrices by reducing them into Echelon form.

(i) $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$

(iii) $\begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$

3. Find the rank of the following matrix by reducing it into upper triangular matrix: $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

4. Test the consistency of the following system of linear equations:

   $5x + 3y + 7z = 4,\ 3x + 26y + 2z = 9,\ 7x + 2y + 11z = 5.$

5. Test the consistency of the following system of linear equations and hence obtain the solution, if exists : $4x - y = 12,\ -x + 5y - 2z = 0,\ -2y + 4z = -8.$

6. Investigate, for what values of m and n do the system of equations :

   $x + y + z = 6, x + 2y + 3z = 10, x + 2y + mz = n,$

   have **(i)** no solution, **(ii)** unique solution **(iii)** more than one solutions ?

7. Solve the system of linear equations: $x + 3y + 2z = 0, 2x - y + 3z = 0, 3x - 5y + 4z = 0,$

   $x + 17y + 4z = 0.$

8. Obtain the real value of w for which the system of equations:

   $x + 2y + 3z = wx, 3x + y + 2z = wy, 2x + 3y + z = wz$ have non-trivial solution.

9. Discuss the consistency of the following system of equations and hence obtain the solution if consistent :

   (i) $x + y + z = 4,\ 2x + 5y - 2z = 3$

   (ii) $5x + 3y + 7z = 4,\ 3x + 26y + 2z = 9,\ 7x + 2y + 10z = 5.$

10. Examine, whether or not the following set of vectors are linearly dependent or independent

    (i) $(3, 2, 4), (1, 0, 2), (1, -1, -1)$ **(ii)** (1, 2, 3), (2, -2, 0)

    (iii) (3, 1, -4), (2, 2, -3), (0, -4, 1) **(iv)** (1, 1, 1, 1), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1)

11. Show that the vectors: X_1 = (1, 2, 4), X_2 = (2, – 1, 3), X_3 = (0, 1, 2) and X_4 = (– 3, 7, 2) are linearly dependent and hence obtain the relation between them.

12. Compute the eigen values and eigen vectors of $A = \begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$.

13. Compute the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

14. Obtain the latent roots and latent vectors of $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

Answers

Section A

1. 2
2. $P = 3$
3. $\lambda = 1, 1, 3$
4. $7{\lambda_1}^3 - 5{\lambda_1}^2 + \frac{1}{3}\lambda_1 - \frac{4}{\lambda_1} + 3, 7{\lambda_2}^3 - 5{\lambda_2}^2 + \frac{1}{3}\lambda_2 - \frac{4}{\lambda_2} + 3, 7{\lambda_3}^3 - 5{\lambda_3}^2 + \frac{1}{3}\lambda_3 - \frac{4}{\lambda_3} + 3$
5. $a = 3,\ b = 1$
6. 2, -2
7. -2
8. If A is an $m \times n$ order matrix over some field then: Rank (A) +nullity $(A) = n$

Section B

1. 3

2. **(i)** 2 **(ii)** 2 **(iii)** 3 **(iv)** 3

3. 3

4. Consistent

5. Consistent; $x = -\frac{32}{15},\ y = -\frac{4}{15},\ z = -\frac{44}{15}$

6. **(i)** $m = 3, n \neq 10$ **(ii)** $m \neq 3, n$ may have any value **(iii)** $m = 3, n = 10$

7. Infinite number of non-trivial solutions given by: $x = 11k,\ y = k,\ z = -7k,$ where k is arbitrary constant.

8. $w = 6$

9. (i) Given system of equations are consistent and have infinite number of solutions given by

$x = \frac{17-7k}{3}, y = \frac{4k-5}{3}k,\ z = k,$ where k is arbitrary constant.

(ii) Given system of equations are consistent and have infinite number of solutions given by

$x = \frac{7-16k}{11}, y = \frac{k+3}{11}k,\ z = k,$ where k is arbitrary constant.

10. (i) Linearly independent (ii) Linearly independent

(iii) Linearly dependent (iv) Linearly independent

11. The given vectors are linearly dependent and the relation between them is given as:

$9X_1 - 12X_2 + 5X_3 - 5X_4 = 0$

12. $\lambda_1 = 6; v_1\ (eigen\ vector) = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ and $\lambda_2 = -1; v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

13. $\lambda_1 = -2; v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \lambda_2 = 6; v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\lambda_3 = 3; v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

14. $\lambda_1 = 1;\ v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix},\ \lambda_2 = 2; v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\lambda_3 = 3; v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$