IILM University, Greater Noida

Unit-II

Short Answer Questions

1. If $u = x^3 + e^{xy} + y^{-3}$, find $\partial u/\partial x$ and $\partial u/\partial y$.
2. Find 1^{st} order partial derivatives of function $u = \cos^{-1}\left(\frac{x}{y}\right)$.
3. If $f = x^3y - xy^3$, find $[\partial f/\partial x + \partial f/\partial y]_{x=1,y=2}$.
4. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then find the value of $xu_x + yu_y$.
5. If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, show that $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$.
6. If $u = x^3 + y^3$, where $x = 2\cos t,\ y = 3\sin t$, find $\frac{\partial u}{\partial t}$.
7. If $z = f(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

Long Answer Questions

1. If $v = (x^2 + y^2 + z^2)^{m/2}$, then find the value of m $(m \neq 0)$ which will satisfy

$$v_{xx} + v_{yy} + v_{zz} = 0.$$

2. If $x^x y^y z^z = c$, show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$

3. Verify Euler's theorem for $z = \frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}$.
4. If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, prove that

   (a) $xu_x + yu_y = 2u$ (b) $u_{xy} = \frac{x^2-y^2}{x^2+y^2}$

5. State Euler's theorem on homogeneous function. Using it show that

   (a) $xu_x + yu_y = \sin 2u$
   (b) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 2\sin u \cos 3u$

   if $u = \tan^{-1}(x^2 + 2y^2)$.

6. If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

7. If $u = f(x^2 + 2yz, y^2 + 2xz)$, then find the value of

$$(y^2 - xz)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z}.$$