Date: / /20

Q n^{th} derivative of $\frac{1}{a^2-x^2}$

$$y = \frac{1}{a^2-x^2} = \frac{1}{(a-x)(a+x)}$$ $a^2-x^2 = (a-x)(a+x)$

$$\frac{1}{(a-x)(a+x)} = \frac{A}{(a-x)} + \frac{B}{(a+x)}$$ [By partial fraction]

$$\frac{1}{\cancel{(a-x)(a+x)}} = \frac{A(a+x)+B(a-x)}{\cancel{(a-x)(a+x)}}$$

compare R.H.S & L.H.S coefficient wise

$aA + Ba = 1 \Rightarrow A+B = 1/a$

$(A-B)x = 0.x \Rightarrow A-B=0 \Rightarrow \boxed{A=B}$

$2A = 1/a$

$\Rightarrow \boxed{A = \frac{1}{2a}}$ $\boxed{B = \frac{1}{2a}}$

$$y = \frac{1}{(a-x)(a+x)} = \frac{1}{2a(a-x)} + \frac{1}{2a(a+x)}$$

$$y_1 = \frac{1}{2a}\left[\frac{-(-1)}{(a-x)^2} + \frac{-1}{(a+x)^2}\right]$$

$$= \frac{1}{2a}\left[\frac{1}{(a-x)^2} - \frac{1}{(a+x)^2}\right]$$

$$y_2 = \frac{1}{2a}\left[\frac{-(-2)}{(a-x)^3} + \frac{2}{(a+x)^3}\right]$$

$$y_3 = \frac{1}{2a}\left[\frac{(-2)(-3)}{(a-x)^4} - \frac{(2)(3)}{(a+x)^4}\right]$$

$\vdots$

$$y_n = \frac{1}{2a}\left[\frac{n!}{(a-x)^{n+1}} + \frac{(-1)^n \, n!}{(a+x)^{n+1}}\right]$$

$$= \frac{n!}{2a}\left[\frac{1}{(a-x)^{n+1}} + \frac{(-1)^n}{(a+x)^{n+1}}\right]$$

Q If $y = \log(x + \sqrt{1+x^2})$ P.T. $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$

Sol^n $y = \log(x + \sqrt{1+x^2})$

$$y_1 = \frac{1}{x+\sqrt{1+x^2}}\left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}\right) = \frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}\,\cancel{(x+\sqrt{1+x^2})}}$$

$$= \frac{1}{\sqrt{1+x^2}}$$

$\sqrt{1+x^2}\cdot y_1 = 1$

Squaring $(1+x^2)(y_1)^2 = 1$

differentiating

$$(1+x^2)(2y_1 y_2) + 2x y_1^2 = 0.$$

divide with $2y_1$

$$(1+x^2)y_2 + xy_1 = 0.$$

$${}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots + {}^nC_n u v_n$$

By Leibnitz theorem

n^{th} derivative of $\underset{v}{(1+x^2)}\underset{u}{y_2}$

$u = y_2$ $\quad$ $v = 1+x^2$

$u_1 = y_3 = y_{2+1}$ $\quad$ $v_1 = 2x$

$u_2 = y_4 = y_{2+2}$ $\quad$ $v_2 = 2$

$\vdots$

$u_n = y_{n+2}$

$$y_{n+2} \cdot (1+x^2) + n\, y_{n+1} (2x) + \frac{n(n-1)}{2} \cdot (2) y_n \quad \text{—— (1)}$$

n^{th} derivative of $\underset{v}{x}\underset{u}{y_1}$

$u = y_1$, $u_1 = y_2$, $u_2 = y_3$ $\dots$ $u_n = y_{n+1}$

$v = x$ $\quad v_1 = 1$ $\quad v_2 = 0$.

$$y_{n+1} \cdot x + n\, y_n \cdot 1 \quad \text{—— (2)}$$

adding (1) & (2)

$$(1+x^2) y_{n+2} + (2n+1) y_{n+1} + n^2 y_n = 0.$$

Q If $u(x,y,z) = \log(\tan x + \tan y + \tan z)$

S.T. $\sin 2x\, u_x + \sin 2y\, u_y + \sin 2z\, u_z = 2$.

$$u_x = \frac{\sec^2 x}{\tan x + \tan y + \tan z}$$

$$u_y = \frac{\sec^2 y}{\tan x + \tan y + \tan z}$$

$$U_z = \frac{\sec^2 z}{\tan x + \tan y + \tan z}$$

L.H.S. $\sin 2x\, u_x + \sin 2y\, u_y + \sin 2z\, U_z$

$$= \frac{\sin 2x \sec^2 x}{\tan x + \tan y + \tan z} + \frac{\sin 2y \sec^2 y}{\tan x + \tan y + \tan z} + \frac{\sin 2z \sec^2 z}{\tan x + \tan y + \tan z}$$

$$= \frac{2\sin x \cancel{\cos x} \cdot \frac{1}{\cos^{\cancel{2}} x} + 2\sin y \cdot \cancel{\cos y} \cdot \frac{1}{\cos^{\cancel{2}} y} + 2\sin z \cancel{\cos z} \cdot \frac{1}{\cos^{\cancel{2}} z}}{\tan x + \tan y + \tan z}$$

$$= \frac{2(\cancel{\tan x + \tan y + \tan z})}{\cancel{\tan x + \tan y + \tan z}} = 2$$

Q ~~If $u = xy$ S.T. $\frac{\partial^3 u}{\partial x^2}$~~

Q If $z(x+y) = x^2+y^2$ S.T. $(z_x - z_y)^2 = 4(1 - z_x - z_y)$

$z = (x^2+y^2)/(x+y)$

$$z_x = \frac{(x+y)(2x) - (x^2+y^2)}{(x+y)^2} = \frac{2x^2+2xy-x^2-y^2}{(x+y)^2}$$

$$= (x^2+2xy-y^2)/(x+y)^2$$

$$z_y = \frac{(x+y)(2y) - (x^2+y^2)}{(x+y)^2} = \frac{2xy+2y^2-x^2-y^2}{(x+y)^2}$$

$$= (y^2+2xy-x^2)/(x+y)^2$$

L.H.L $(z_x - z_y)^2$

$$z_x - z_y = \frac{x^2+2xy-y^2}{(x+y)^2} - \frac{(y^2+2xy-x^2)}{(x+y)^2}$$

$$= \frac{x^2+2xy-y^2-(-x^2+2xy+y^2)}{(x+y)^2}$$

$$= \frac{2x^2-2y^2}{(x+y)^2} = \frac{2(x^2-y^2)}{(x+y)^2}$$

$$= \frac{2(x-y)}{(x+y)}$$

$$(z_x - z_y)^2 = \frac{4(x-y)^2}{(x+y)^2}$$

R.H.S. $4(1-z_x-z_y) = 4\left(1-\frac{(x^2+2xy-y^2)}{(x+y)^2} - \frac{(-x^2+2xy+y^2)}{(x+y)^2}\right)$

$$= 4\left(\frac{(x+y)^2 - \cancel{x^2} - 2xy + \cancel{y^2} + \cancel{x^2} - 2xy - \cancel{y^2}}{(x+y)^2}\right)$$

$$= 4\left(\frac{x^2+2xy+y^2-4xy}{(x+y)^2}\right)$$

$$= \frac{4(x-y)^2}{(x+y)^2}$$

L.H.S = R.H.S