## Unit – II (LAPLACE TRANSFORM)

**1.** Find the Laplace transform of the following functions

**(i)** $\sin(a+bt)$ **(iii)** $sin^3 2t$

**(ii)** $\sin 3t \cos 2t$ **(iv)** $(1+\text{te}^{-t})^3$

**Ans: (i)**$\sin a \frac{s}{s^2+b^2} + \cos a \frac{b}{s^2+b^2}$**(ii)**$\frac{1}{2}\left(\frac{5}{s^2+25}+\frac{1}{s^2+1}\right)$ **(iii)**$\frac{48}{(s^2+4)(s^2+36)}$ **(iv)**$\frac{1}{s}+\frac{3}{(s+1)^2}+\frac{6}{(s+2)^3}+\frac{6}{(s+3)^4}$

**2.** Find the Laplace transform

**(i)** $t^2 e^t \sin 4t$ **(iii)** $\frac{sin^2 t}{t}$ **(v)** $\frac{e^{-t}\sin t}{t}$

**(ii)** $\frac{e^{-at}-e^{-bt}}{t}$ **(iv)** $\frac{e^{at}-\cos bt}{t}$

**Ans:** **(i)**$\frac{8(3s^2-6s-13)}{(s^2-2s+17)^3}$**(ii)**$\log\frac{s+b}{s+a}$**(iii)**$\frac{1}{4}\, log\, \frac{s^2+4}{s^2}$**(iv)**$\frac{1}{2}\, log\, \frac{s^2+b^2}{(s-a)^2}$**(v)**$\cot^{-1}(s+1)$

**3.** Evaluate the following integrals.

**(i)** $\int_0^\infty \frac{\sin t}{t}\, dt$ **(iii)** $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\, dt$ **(v)** $\int_0^\infty \frac{e^{-t} sin^2 t}{t}\, dt$

**(ii)** $\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}\, dt$ **(iv)** $\int_0^\infty e^{-2t} sin^3 t\, dt$

**Ans:(i)**$\frac{\pi}{2}$ **(ii)**$log\,\frac{b}{a}$ **(iii)**$log\,\frac{2}{3}$**(iv)**$\frac{6}{65}$ **(v)**$\frac{1}{4}\log 5$

**4.** Find its Laplace transform

**(i)** $f(\text{t}) = \begin{cases} t & , \quad 0<t<\pi \\ \pi - t & , \quad \pi < t < 2\pi \end{cases}$

**(ii)** $f(\text{t}) = \begin{cases} 1 & , \quad 0 \le t \le a/2 \\ -1 & , \quad a/2 < t < a \end{cases}$

**(iii)** $f(\text{t}) = \begin{cases} t & for \quad 0<t\le a \\ 2a-t & for \quad a<t<2a \end{cases}$

**(iv)** $f(\text{t}) = \begin{cases} t & for \quad 0<t\le 1 \\ 1 & for \quad 1<t<2 \end{cases}$

**(v)** $f(\text{t}) = \begin{cases} \frac{h}{a}t & for \quad 0<t\le a \\ \frac{h}{a}(2a-t) & for \quad a<t<2a \end{cases}$

**Ans:(i)** $\boldsymbol{\frac{-\pi s e^{-\pi s}+1-e^{-\pi s}}{s^2(1+e^{-\pi s})}}$ **(ii)** $\frac{1}{s}\tanh\frac{as}{4}$**(iii)**$\frac{1}{s^2}\tanh\frac{as}{2}$**(iv)**$\frac{1}{s^2(1-e^{-2s})}[1-e^{-s}-se^{-2s}]$**(v)**$\frac{h}{as^2}\tan h\,\frac{as}{2}$

**5.** Find the Laplace transform of **(i)** $(t-1)^2 U(t-1)$ **(ii)** $\sin t\; U(t-\pi)$ **(iii)** $e^{-3t}U(t-2)$

**Ans:(i)**$\frac{2e^{-s}}{s^3}$ **(ii)**$-\frac{e^{-\pi s}}{s^2+1}$ **(iii)**$\frac{e^{-2(s+3)}}{s+3}$

**6.** Find the Laplace transform of following functions:

**(i)** $f(\text{t}) = \begin{cases} t^2 & 0<t<2 \\ t-1 & 2<t<3 \\ 7 & t>3 \end{cases}$

**(ii)** $f(\text{t}) = \begin{cases} 1 & 0<t<1 \\ t & 1<t<2 \\ t^2 & t>2 \end{cases}$

**(iii)** $f(\text{t}) = \begin{cases} t/\omega, & 0<t<\omega \\ 1-(t/\omega) & \omega<t<2\omega \\ 1 & t>2\omega \end{cases}$

**(iv)** $f(\text{t}) = \begin{cases} e^{t-a} & t>a \\ 0 & t<a \end{cases}$

**Ans: (i)**$\frac{2}{s^3} - \frac{e^{-2s}}{s^3}(2+3s+3s^2) + \frac{e^{-3s}}{s^2}(5s-1)$ **(ii)**$\frac{1}{s}+\frac{2e^{-2s}}{s}+\frac{e^{-s}}{s^2}+\frac{3e^{-2s}}{s^2}+\frac{2e^{-2s}}{s^3}$

**(iii)** $\boldsymbol{\frac{1}{s}}\left[\frac{1}{s\omega} - e^{-s\omega}\left(1+\frac{2}{s\omega}\right) + e^{-2s\omega}\left(2+\frac{1}{s\omega}\right)\right]$**(iv)** $\boldsymbol{\frac{e^{-as}}{s-1}}$

**7.** Express the following functions in terms of unit step function & hence find its Laplace transform

**(i)** $f(t) = \begin{cases} t-1 \text{ when } 1<t<2 \\ 3-t\,,\ when\ 2<t<3 \end{cases}$

**(ii)** $f(t) = \begin{cases} 4 & \text{when } 0<t<1 \\ -2 & when\ 1<t<3 \\ 5 & when\quad t>3 \end{cases}$

**Ans:**(i) $\frac{e^{-s}}{s^2}(1-e^{-s})^2$**(ii)**$\frac{4-6e^{-s}+7e^{-3s}}{s}$

**8.** Find the inverse Laplace transform of the following

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**(i)** $\frac{2s+1}{s^2-4}$ **(ii)** $\frac{s^3}{s^4-a^4}$ **(iii)** $\frac{s}{s^4+4a^4}$

**(iv)** $\frac{1}{\sqrt{2s+3}}$ **(v)** $\frac{s}{s^2+6s+25}$ **(vi)** $\frac{s+8}{s^2+4s+5}$

**(vii)** $\frac{5s+3}{(s-1)(s^2+2s+5)}$ **(viii)** $\frac{s}{s^4+s^2+1}$ **(ix)** $\frac{1}{(s^2+1)^3}$

**(x)** $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$

**Ans:(i)**$2\cos h2t+\frac{1}{2}\sin h2t$**(ii)**$\frac{1}{2}(\cos hat+\cos at)$**(iii)**$\frac{1}{2a^2}(\sin at\sin hat)$**(iv)**$\frac{1}{\sqrt{2\pi t}}e^{-\frac{3t}{2}}$**(v)**$e^{-3t}\left(\cos 4t-\frac{3}{4}\sin 4t\right)$

**(vi)**$e^{-2t}(\cos t+6\sin t)$ **(vii)**$e^t-e^{-t}\left(\cos 2t-\frac{3}{2}\sin 2t\right)$**(viii)**$\frac{2}{\sqrt{3}}\sinh\frac{t}{2}\sin\frac{\sqrt{3}}{2}t$

**(ix)**$\frac{1}{8}[(3-t^2)\sin t-3t\cos t]$**(x)**$\frac{e^t}{2}-e^{2t}+\frac{5e^{3t}}{2}$

**9.** Find the inverse Laplace Transform of the following functions.

**(i)** $\frac{e^{-2\pi s}}{s(s^2+1)}$ **(ii)** $\frac{e^{-s}-3e^{-3s}}{s^2}$

**Ans:(i)**$(1-cos\ t)\ U(t-2\pi)$**(ii)**$(t-1)u(t-1)-3(t-3)u(t-3)$

**10.** Find inverse Laplace for the following: **(i)** $\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}$ **(ii)** $\frac{s\,e^{-\frac{s}{2}}+\pi e^{-s}}{s^2+\pi^2}$

Ans. **(i)**$\frac{4\,(t-3)^{\frac{3}{2}}e^{-4(t-4)}}{3\sqrt{\pi}}\ u(t-3)$**(ii)** $\sin\pi t\left[u\left(t-\frac{1}{2}\right)-u(t-1)\right]$

**11.** Use convolution theorem to find the inverse Laplace transform of following functions

**(i)** $\frac{s}{(s^2+4)^2}$ **(ii)** $\frac{s}{(s^2+1)(s^2+4)}$ **(iii)** $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ **(iv)** $\frac{1}{s^3(s^2+1)}$ **(v)** $\frac{1}{s^2(s+1)^2}$

**Ans:(i)**$\frac{t}{4}\sin 2t$ **(ii)**$\frac{\cos t-\cos 2t}{3}$ **(iii)**$\frac{asinat-bsinbt}{a^2-b^2}$ **(iv)**$\frac{t^2}{2}+\cos t-1$**(v)**$(t+2)e^{-t}+t-2$

**<u>Questions on Application of Laplace Transform</u>**

**12.** Solve the following initial value problems using Laplace transform

**(i)** $y''(t) + 4y'(t) + 4y(t) = 6e^{-t}$ y(0) = -2, y'(0) = 8.

**(ii)** $\frac{d^2y}{dx^2}+9y=6\cos 3x$, y(0) = 2, y'(0) = 0.

**(iii)** $\frac{d^2y}{dt^2}+y=t\cos 2t,\ t>0$ given that y = $\frac{dy}{dt}=0$ for t = 0.

Ans. **(i)** $y=6e^{-t}-(8+2t)e^{-2t}$ **(ii)**$y=xsin\,3x+2\cos 3x$ **(iii)**$y=-\frac{5}{9}\sin t+\frac{4}{9}\sin 2t-\frac{t}{3}\cos 2t$

**13.** Solve the following differential equations:

**(i)** $\frac{d^3y}{dt^3}-3\frac{d^2y}{dt^2}+3\frac{dy}{dt}-y=t^2e^t$ where $y(0)=1, y'(0)=0, y''(0)=-2.$

**(ii)** $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+8y=\sin x$ where y(0) = 0, y'(0) = 1.

**(iii)** $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$ where y(0) = 1, y'(0) = 0.

**Ans: (i)**$y=\left(1-t-\frac{t^2}{2}+\frac{t^5}{60}\right)e^t$**(ii)**$y=\frac{e^{-2x}}{65}(4\cos 2x+33\sin 2x)-\frac{1}{65}(4\cos x-7\sin x)$

**(iii)** $\boldsymbol{y} = \left(1 - x + \frac{x^2}{2}\right) e^x$

**14.** Solve the following simultaneous differential equations by Laplace transform

$Dx - y = e^t$, $Dy + x = \sin t$, given that $x(0) = 1$, $y(0) = 0$.

**15.** $\frac{dx}{dt} + y = \sin t;\ \ \frac{dy}{dt} + x = \cos t$ given that x = 2, y = 0 at t = 0.

**Ans:(14)** $x = \frac{1}{2}\{e^t + \cos t + 2\sin t - t\cos t\}, y = \frac{1}{2}\{t\sin t - e^t + \cos t - \sin t\}$ **(15)** $x = 2\ \cosh t\,, y = \sin t + 2\sinh t$