IILM University, Greater Noida

School of Basic & Applied Sciences	Department of Mathematics	B.Tech, CSE	Mathematics for Computing (UCS2005)

Question Bank- Unit 4 (Fourier Series)

Section A

1. Write the formula for the Fourier coefficients a_n, b_n for $f(x)$ in $(-\pi, \pi)$.

2. If $f(x)$ is an even function in $(-\pi, \pi)$, then write the Fourier coefficients.

3. If $f(x) = x^2 + x$ is expressed as a Fourier series in $(-2, 2)$, then find $f(2)$.

4. Write the half range sine series for $f(x) = x$ in $(0, \pi)$.

5. Write the Dirchlet's conditions.

6. Write the period of $\cos 3x$.

7. If $x = c$ is a point of discontinuity then the Fourier series of $f(x)$ at $x = c$ is gives $f(x)$ as.

8. Find the value of $f(0)$ if $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$.

9. The function $f(x) = \begin{cases} 1 - x & -\pi < x < 0 \\ 1 + x & 0 < x < \pi \end{cases}$ is an odd or even function?

10. If $f(x) = x \sin x$ in $(-\pi, \pi)$ then find the value of b_n.

11. Consider the function f defined for x on the interval $-\pi \leq x < \pi$ as

$$f(x) = \begin{cases} 1 & -\pi \leq x < 0 \\ 1 + x & 0 \leq x < \pi/2 \\ 2x & \pi/2 \leq x < \pi \end{cases}$$

and for all other x function is defined by the periodicity condition $f(x + 2\pi) = f(x)$. without finding its Fourier series, determine the value where Fourier series of this function converges to at points $x = 0,\ x = \pi/2$ and $x = \pi$.

12. $xcosx$ is even or odd function?

Section B

1. Find the Fourier series expansion of the periodic function with period 2π

$$f(x) = \begin{cases} \pi + x & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases}$$

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2. Find the Fourier series expansion of

   (a)
   $$f(x) = \begin{cases} k & -\pi < x < 0 \\ 0 & 0 \leq x < \pi. \end{cases}$$

   (b)
   $$f(x) = \begin{cases} \pi & -\pi < x < 0 \\ \pi - x & 0 \leq x < \pi. \end{cases}$$

   (c)
   $$f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 \leq x < \pi. \end{cases}$$

   (d)
   $$f(x) = \begin{cases} 0 & -\pi < x < -\pi/2 \\ k & -\pi/2 \leq x \leq \pi/2 \\ 0 & \pi/2 < x < \pi. \end{cases}$$

   (e)
   $$f(x) = \begin{cases} 2 & -\pi < x < 0 \\ 4 & 0 \leq x < \pi. \end{cases}$$

   (f)
   $$f(x) = \begin{cases} -(\pi + x) & -\pi < x < 0 \\ -(\pi - x) & 0 \leq x < \pi. \end{cases}$$

3. Find the Fourier series expansion of the periodic function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 \leq x < \pi \end{cases}$$

   with period 2π. Hence show that $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots$ and $\dfrac{\pi^2}{12} = 1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} \ldots$

4. A periodic function of period 2 is defined as $f(x) = 1 + x, \quad -1 < x < 1$. Obtain the Fourier series expansion of $f(x)$ and hence show that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7}$

5. Find the half range Fourier sine and cosine series of $f(x) = x(\pi - x);\ 0 \leq x \leq \pi$.

6. Show that constant c can be expanded in the form of an half range sine series $\frac{4c}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right]$ in the range $0 < x < \pi$.

7. If

$$f(x) = \begin{cases} \sin x & -\pi \le x \le \pi/4 \\ \cos x & \pi/4 \le x \le \pi/2 \end{cases}$$

then expand $f(x)$ in a half range cosine series.

8. Find the Fourier cosine series expansion of the periodic function

$$f(x) = \begin{cases} x^2 & 0 \le x \le 2 \\ 4 & 2 \le x \le 4. \end{cases}$$

Answers:

Section A

1. $a_n = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x)\cos nx \; dx \qquad b_n = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x)\sin nx \; dx$

2. $a_n = \frac{2}{\pi}\int\limits_{0}^{\pi} f(x)\cos nx \; dx$ and $b_n = 0$

3. 4

4. $2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \dots\right)$

5. Dirchlet's conditions

6. $\frac{2\pi}{3}$

7. $\frac{1}{2}[f(c-0) + f(c+0)]$

8. $\frac{-\pi}{2}$

9. even function

10. 0

11. $1,\ \dfrac{3\pi}{2}+\dfrac{1}{2},\ \pi+\dfrac{1}{2}$

12. odd

Section B

1. $f(x)=\dfrac{\pi}{4}+\dfrac{2}{\pi}\left[\dfrac{\cos x}{1^2}+\dfrac{\cos 3x}{3^2}+\dots\right]-\left[\dfrac{\sin x}{1}+\dfrac{\sin 2x}{2}+\dots\right]$

2. (a) $\dfrac{k}{2}-\dfrac{k}{\pi}\sum\left[\dfrac{1}{n}(1-(-1)^n)\sin nx\right]$.

   (b) $\dfrac{3\pi}{4}+\sum\left[\dfrac{1}{\pi n^2}(1-(-1)^n)\cos nx+\dfrac{1}{n}\cos nx\ \ \sin nx\right]$.

   (c) $\dfrac{2k}{\pi}\sum\left[\dfrac{1}{n}(1-(-1)^n)\sin nx\right]$.

   (d) $\dfrac{k}{2}+\dfrac{2k}{\pi}\sum\left[\dfrac{1}{n}\sin(\dfrac{n\pi}{2})\cos nx\right]$.

   (e) $3+\dfrac{2}{\pi}\sum\left[\dfrac{1}{n}(1-(-1)^n)\sin nx\right]$.

   (f) $-\dfrac{\pi}{2}-\dfrac{2}{\pi}\sum\left[\dfrac{1}{n^2}(1-(-1)^n)\cos nx\right]$.

3. $f(x)=\dfrac{\pi^2}{6}+\sum\left[\dfrac{2}{n^2}(-1)^n\cos nx+\left\{\dfrac{\pi}{n}(-1)^{n+1}-\dfrac{2}{\pi n^3}[1-(-1)^n]\right\}\sin nx\right]$
   Set $x=0$(point of continuity), Set $x=\pi$ (point of discontinuity).

4. $1-\dfrac{2}{\pi}\sum\left[\dfrac{1}{n}\cos n\pi\sin n\pi x\right]$. set $x=\frac{1}{2}$.

5. $\displaystyle\sum_{n=1}^{\infty}\frac{4}{\pi n^3}[1-(-1)^n]\sin nx$ and $\displaystyle\frac{\pi^2}{6}-\sum_{n=1}^{\infty}\frac{21+(-1)^n\cos nx}{n^2}$

6. $\dfrac{4c}{\pi}\left[\sin x+\dfrac{\sin 3x}{3}+\dfrac{\sin 5x}{5}+\dots\right]$

7. $\displaystyle\frac{4-2\sqrt{2}}{\pi}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1+(-1)^n-\sqrt{2}\cos(\frac{n\pi}{2})\cos 2nx}{1-4n^2}.$

8. $\displaystyle f(x)=\frac{8}{3}+\frac{32}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\left(\frac{n\pi}{2}\right)-\left(\frac{2}{n\pi}\right)\sin\left(\frac{n\pi}{2}\right)\right]\cos\left(\frac{n\pi x}{4}\right).$