Assignment-3-QMM

Dev

2023-09-20

QUESTION:

The Weigelt Corporation has three branch plants with excess production capacity.Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large,medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

```
df1=data.frame(Profits=c("$420","$360","300"),
               space_required=c(20,15,12),
               Atmost_sales=c(900,1200,750),
               row.names=c("Large","Medium","Small"))
df1
```

```
       Profits space_required Atmost_sales
Large     $420             20          900
Medium    $360             15         1200
Small      300             12          750
```

```
df2=data.frame(Capacity=c(750,900,450),
               Space_available=c(13000,12000,5000),
               row.names=c("Plant-1","Plant-2","Plant-3"))
df2
```

```
        Capacity Space_available
Plant-1      750           13000
Plant-2      900           12000
Plant-3      450            5000
```

Defining decision variables

Let,

$$Number\ \ of\ \ different\ \ sized\ \ products\ \ produced\ \ by\ \ plant-1: L_1, M_1, S_1$$

$$Number\ \ of\ \ different\ \ sized\ \ products\ \ produced\ \ by\ \ plant-2: L_2, M_2, S_2$$

$$Number\ \ of\ \ different\ \ sized\ \ products\ \ produced\ \ by\ \ plant-3: L_3, M_3, S_3$$

$$Hence,\ \ decision\ \ variables\ \ are: L_1, M_1, S_1, L_2, M_2, S_2, L_3, M_3, S_3$$

Formulating a linear programming model for this problem

-Weigelt corporation wants to maximize profit.So, the objective function can be mathematically represented as:

$$Z = 420L_1 + 360M_1 + 300S_1 + 420L_2 + 360M_2 + 300S_2 + 420L_3 + 360M_3 + 300S_3$$

-Following constraints can be deduced out of the given problem:

Sales Constraints:

$$L_1 + L_2 + L_3 \leq 900$$

$$M_1 + M_2 + M_3 \leq 1200$$

$$S_1 + S_2 + S_3 \leq 750$$

Capacity constraint:

$$L_1 + M_1 + S_1 \leq 750$$

$$L_2 + M_2 + S_2 \leq 900$$

$$L_3 + M_3 + S_3 \leq 450$$

Storage constraint:

$$20L_1 + 15M_1 + 12S_1 \leq 13000$$

$$20L_2 + 15M_2 + 12S_2 \leq 12000$$

$$20L_3 + 15M_3 + 12S_3 \leq 5000$$

To avoid layoffs:

$$For\ \ plant-1\ \ and\ \ plant-2: (L_1 + M_1 + S_1)/750 = (L_2 + M_2 + S_2)/900$$

$$i.e.,\ \ 900L_1 + 900M_1 + 900S_1 - 750L_2 - 750M_2 - 750S_2 = 0$$

$$For\ \ plant-2\ \ and\ \ plant-3: (L_2 + M_2 + S_2)/900 = (L_3 + M_3 + S_3)/450$$

$$i.e,\ \ 450L_2 + 450M_2 + 450S_2 - 900L_3 - 900M_3 - 900S_3 = 0$$

$$For\ \ plant-1\ \ and\ \ plant-3: (L_1 + M_1 + S_1)/750 = (L_3 + M_3 + S_3)/450$$

$$i.e,\ \ 450L_1 + 450M_1 + 450S_1 - 750L_3 - 750M_3 - 750S_3 = 0$$

Non-negativity of variables:

$$L_1, M_1, S_1, L_2, M_2, S_2, L_3, M_3, S_3 \geq 0$$

Solving the problem

Applying lpsolve library:

```r
library(lpSolve)
```

Making objective function:

```
f.obj=c(420,360,300,420,360,300,420,360,300)
```

Making constraint matrix:

```
f.const=matrix(c(1,0,0,1,0,0,1,0,0,
                 0,1,0,0,1,0,0,1,0,
                 0,0,1,0,0,1,0,0,1,
                 1,1,1,0,0,0,0,0,0,
                 0,0,0,1,1,1,0,0,0,
                 0,0,0,0,0,0,1,1,1,
                 20,15,12,0,0,0,0,0,0,
                 0,0,0,20,15,12,0,0,0,
                 0,0,0,0,0,0,20,15,12,
                 900,900,900,-750,-750,-750,0,0,0,
                 0,0,0,450,450,450,-900,-900,-900,
                 450,450,450,0,0,0,-750,-750,-750),
               nrow=12,
               byrow=TRUE)
```

Specifying direction of constraint equations:

```
f.dir=c("<=","<=","<=","<=","<=","<=","<=","<=","<=","=","=","=")
```

Specifying rhs values of equations:

```
f.rhs=c(900,1200,750,750,900,450,13000,12000,5000,0,0,0)
```

Applying lpsolve to get solutions:

```
optimum=lp(direction="max", f.obj, f.const, f.dir, f.rhs)
```

Maximum value of profit is :

```
optimum$objval
```

```
[1] 696000
```

Number of large, medium , small sized products to be produced by plant-1 are:

```
optimum$solution[1]
```

```
[1] 516.6667
```

```
optimum$solution[2]
```

```
[1] 177.7778
```

```r
optimum$solution[3]
```

```
[1] 0
```

Number of large, medium , small sized products to be produced by plant-2 are:

```r
optimum$solution[4]
```

```
[1] 0
```

```r
optimum$solution[5]
```

```
[1] 666.6667
```

```r
optimum$solution[6]
```

```
[1] 166.6667
```

Number of large, medium , small sized products to be produced by plant-1 are:

```r
optimum$solution[7]
```

```
[1] 0
```

```r
optimum$solution[8]
```

```
[1] 0
```

```r
optimum$solution[9]
```

```
[1] 416.6667
```