Assignment -2 QMM

Dev

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Linear Programming Problems

Question-1:

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

```
df=data.frame(Material_requirement=c(3,2),
              Labor_requirement=c(45,40),
              Profits=c("$32","$24"),
              row.names=c("Collegiate","Mini"))
df
```

```
           Material_requirement Labor_requirement Profits
Collegiate                    3                45     $32
Mini                          2                40     $24
```

a) Clearly define decision variable

Let,

$$Number\ \ of\ \ collegiate\ \ bags\ \ produced\ \ in\ \ one\ \ week = C_b$$
$$Number\ \ of\ \ mini\ \ bags\ \ produced\ \ in\ \ one\ \ week = M_b$$

$$Hence,\ \ decision\ \ variables\ \ are : C_b, M_b$$

b) What's the objective function?

The objective function is to maximize the profits of Back Savers company.This objective function can be mathematically represented as the following equation:

$$Z = 32C_b + 24M_b$$

c) What are the constraints?

Following constraints can be deduced out of the given question:

$$Material\ \ requirement\ \ constraint : 3C_b + 2M_b \leq 5000\ \ sq.ft$$
$$Labor\ \ requirement\ \ constraint\ \ \ : 45C_b + 40M_b \leq 60 * 35 * 40 = 84000$$
$$Atmost\ \ sales\ \ constraint : C_b \leq 1000,\ \ M_b \leq 1200$$
$$Non - negativity\ \ of\ \ variables : C_b \geq 0,\ \ M_b \geq 0$$

d) Write down full mathematical formulation of this LP problem.

-Decision variables are:

$$C_b, M_b$$

-Objective function can be mathematically represented as the following equation:

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-Following constraints can be deduced out of the given question:

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$$Non - negativity\ \ of\ \ variables : C_b \geq 0,\ \ M_b \geq 0$$

QUESTION-2

The Weigelt Corporation has three branch plants with excess production capacity.Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large,medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

```
df1=data.frame(Profits=c("$420","$360","300"),
               space_required=c(20,15,12),
               Atmost_sales=c(900,1200,750),
               row.names=c("Large","Medium","Small"))
df1
```

```
       Profits space_required Atmost_sales
Large     $420             20          900
Medium    $360             15         1200
Small      300             12          750
```

```r
df2=data.frame(Capacity=c(750,900,450),
               Space_available=c(13000,12000,5000),
               row.names=c("Plant-1","Plant-2","Plant-3"))
df2
```

```
        Capacity Space_available
Plant-1      750           13000
Plant-2      900           12000
Plant-3      450            5000
```

a)Define the decision variables:

Let,

$$Number\ \ of\ \ different\ \ sized\ \ products\ \ produced\ \ by\ \ plant-1: L_1, M_1, S_1$$
$$Number\ \ of\ \ different\ \ sized\ \ products\ \ produced\ \ by\ \ plant-2: L_2, M_2, S_2$$
$$Number\ \ of\ \ different\ \ sized\ \ products\ \ produced\ \ by\ \ plant-3: L_3, M_3, S_3$$
$$Hence,\ \ decision\ \ variables\ \ are: L_1, M_1, S_1, L_2, M_2, S_2, L_3, M_3, S_3$$

b)Formulate a linear programming model for this problem

-Decision variables are:

$$L_1, M_1, S_1, L_2, M_2, S_2, L_3, M_3, S_3$$

-Weigelt corporation wants to maximize profit.So, the objective function can be mathematically represented as:

$$Z = 420L_1 + 360M_1 + 300S_1 + 420L_2 + 360M_2 + 300S_2 + 420L_3 + 360M_3 + 300S_3$$

-Following constraints can be deduced out of the given problem:

Sales Constraints:

$$L_1 + L_2 + L_3 \leq 900$$
$$M_1 + M_2 + M_3 \leq 1200$$
$$S_1 + S_2 + S_3 \leq 750$$

Capacity constraint:

$$L_1 + M_1 + S_1 \leq 750$$
$$L_2 + M_2 + S_2 \leq 900$$
$$L_3 + M_3 + S_3 \leq 450$$

Storage constraint:

$$20L_1 + 15M_1 + 12S_1 \leq 13000$$
$$20L_2 + 15M_2 + 12S_2 \leq 12000$$
$$20L_3 + 15M_3 + 12S_3 \leq 5000$$

To avoid layoffs:

$$For\ \ plant-1\ \ and\ \ plant-2: (L_1 + M_1 + S_1)/750 = (L_2 + M_2 + S_2)/900$$
$$i.e.,\ \ 900L_1 + 900M_1 + 900S_1 - 750L_2 - 750M_2 - 750S_2 = 0$$
$$For\ \ plant-2\ \ and\ \ plant-3: (L_2 + M_2 + S_2)/900 = (L_3 + M_3 + S_3)/450$$
$$i.e,\ \ 450L_2 + 450M_2 + 450S_2 - 900L_3 - 900M_3 - 900S_3 = 0$$
$$For\ \ plant-1\ \ and\ \ plant-3: (L_1 + M_1 + S_1)/750 = (L_3 + M_3 + S_3)/450$$
$$i.e,\ \ 450L_1 + 450M_1 + 450S_1 - 750L_3 - 750M_3 - 750S_3 = 0$$

Non-negativity of variables:

$$L_1, M_1, S_1, L_2, M_2, S_2, L_3, M_3, S_3 \geq 0$$